5

Demand factors and the production function

5.1. Production function as a source of demand

Demand for production factors is exerted by the organizers of production. These organizers of production may either be private entrepreneurs or managers of publicly owned plants. They combine quantities of the various production factors in order to obtain certain quantities of product. Whatever the criterion guiding the organizers – maximum profit or the satisfaction of a given demand at given prices – they have to know what product they can obtain with the aid of given quantities of production factors. Depending on the prices at which they can attract the production factors needed they will combine the latter in different proportions.

For quite some time the production factors considered in macro-economic production functions were *land*, *labour* and *capital* only; and, to begin with, only their quantities were considered. Since for the economy as a whole the quantity of land can be considered constant, land has often been left out altogether. A first improvement made consisted of the introduction of technical development as an additional factor which increases exponentially over time. As a further improvement *technological development* was expressed as quality changes in capital or in labour or in both. In order to study income distribution, especially between various types of labour, we have to introduce these *types of labour* separately into a macro-economic production function. Recently this has been undertaken by a number of authors, in particular by Bowles [5], Dougherty [22], Kuipers [37] and Ullman [74]. In this study a slightly different approach will be followed, but we shall on occasion compare it with the approaches of the authors mentioned. Since for the time being the best data available and relevant to our problem are those about schooling, we will restrict ourselves to this characteristic. But even if we stick to the use of only one aspect of the quality of labour, namely the quantity of schooling, we must make a difference between the schooling normally *required* for the execution of a given productive task and the *actual* schooling of the person engaged for that task. In this simple first approach only three levels of education will be considered, namely, the first, the second and the third level, as usually distinguished. The quantities of persons engaged will be expressed by a symbol ϕ_{sv}, where ϕ is the portion of the total active population carrying out tasks for which preferably the level s (1, 2 or 3) is required but for which people with level v are being used. Knowing that they will not always succeed in attracting people with $v = s$, the organizers of production will also try to obtain people with $v \neq s$ and they are supposed to know what the contribution to production of such people will be; this is expressed in the production function. In a situation where fewer people of highest schooling are available than could be used in the production process a rational behaviour of both the demand and the supply side of the market will imply that $v \leq s$. The total active population can be represented by the matrix of Table 5.I.

Table 5.I
Matrix of active population, assuming (a) full employment, (b) scarcity of educated manpower and (c) three levels of education.

Schooling required	Actual schooling v			Total
s	1	2	3	
1	ϕ_{11}	.	.	$\phi_{1.}$
2	ϕ_{21}	ϕ_{22}	.	$\phi_{2.}$
3	.	ϕ_{32}	ϕ_{33}	$\phi_{3.}$
Total	$\phi_{.1}$	$\phi_{.2}$	$\phi_{.3}$	1

Although it is conceivable that $\phi_{31} \neq 0$, it is not rational and the actual figure is small enough to be neglected. This means that, apart from capital and land, we have five production factors in the realm of labour.

A more general approach is conceivable if we assume that both s and v can assume any values between a minimum of zero and some maximum S or V, meaning that s and v are considered as continuous variables. Instead of the matrix of Table 5.I we would then use a two-dimensional frequency distribution with densities $\phi(s, v)$, and total manpower could then be written as an integral,

$$1 = \int_0^S \mathrm{d}s \int_0^V \mathrm{d}v\, \phi(s, v). \tag{5.1}$$

Such a procedure might be fruitful if ϕ can be written as a not too complicated function of s and v, for instance, as the normal or the log-normal distribution. Moreover, the productive contribution of the element $\phi(s, v)\,\mathrm{d}s\,\mathrm{d}v$ should be known explicitly and be close to a not too complicated function of s and v. Exercises with figures for a number of production sectors did not disclose a simple shape for either of these functions and discouraged B. Herman and myself from following this procedure. These exercises actually referred to the two production factors of capital (including human capital) and ('pure', that is, unskilled) labour. In the present essay I therefore stuck to the much simpler discrete-value system expressed in Table 5.I.

5.2. Two alternative Cobb–Douglas production functions

In a first attempt [63] to use a Cobb–Douglas-like production function in ϕ_{sv} I introduced (for two values of s and v each) the function

$$y = C\phi_{11}^{\varrho_{11}}\phi_{12}^{\varrho_{12}}\phi_{21}^{\varrho_{21}}\phi_{22}^{\varrho_{22}}. \tag{5.2}$$

This function has the inconvenience that $y = 0$ for ϕ_{12} or $\phi_{21} = 0$, which is completely unrealistic and the function (5.2) has therefore to be rejected. As factors (in the mathematical meaning of that term) we have to introduce sums of ϕ's of either the same s or the same v. Accordingly, in the case described by our Table 5.I, two alternatives can be formulated. If we combine ϕ's with the same s we consider as one production factor the group of people having the same job; if we combine ϕ's with the same v we consider as one production factor the group of people with the same education. In both cases we cannot simply take the unweighted sums, that is, either $\phi_{21} + \phi_{22}$ or $\phi_{11} + \phi_{21}$, since the productivity of an individual with an education not intended for the job will differ from the productivity of an individual with the appropriate level of education for the job considered. Of the two alternatives the former seems better if data are available about incomes by jobs (or job groups), whereas the latter is to be preferred if data are available on incomes by education received. An additional condition for the application of the former alternative is that jobs are classified according to education required. The absence of this sort of data for most countries made me choose in favour of the second alternative, implying the production function as specified in equation (5.3),

$$y = C(\phi_{11} + \pi_{21}\phi_{21})^{\rho_1}(\phi_{22} + \pi_{32}\phi_{32})^{\rho_2}\phi_{33}^{\rho_3}. \quad (5.3)$$

Here π_{21} constitutes the productivity ratio between individuals with education 1 on jobs 2 and 1; and π_{32}, that ratio between persons with education 2 on jobs 3 and 2. Both π will be > 1, with an upper limit such that the (marginal) productivity on job 2 of an individual with education 1 will be lower than, or at most equal to the (marginal) productivity of an individual with education 2. These assumptions were also made in a previous article on the same subject [63].

In addition I assume that, in a configuration with a very small ϕ_{21}, π_{21} will be very little above 1 and similarly for ϕ_{32} and π_{32}. This assumption is based on the underlying assumption that in reality job 1 stands for a group of jobs, of which the most productive one is very close to the least productive job belonging to group 2. It has to be admitted that a more precise elaboration of this point of view would be useful.

In (5.3) the value of C depends on the units chosen for y and for labour as well as on the contribution to production by capital. Also the sum total of the ρ's equals the portion of national product to be attributed to labour. For the Netherlands the contribution of capital to national product has been taken as equal to 0.2, implying that $\rho_1 + \rho_2 + \rho_3 = 0.8$. For 1962 I estimated $\rho_1 = 0.648$, $\rho_2 = 0.088$ and $\rho_3 = 0.064$. Expressing y in thousands of guilders per employed person I found $C = 15$ and y close to 9.1.

The use of a (generalized) Cobb–Douglas production function implies a choice among many different forms available, differing mainly in the degree of substitutability of the factors of production considered. In most of the more sophisticated studies of production functions now available, using only the quantity and the quality of labour and capital, more complicated forms such as the CES function or even a variable elasticity of substitution have been preferred. In quite a few cases, however, elasticities close to one have been found. Since we want to concentrate on the implications of our distinction between schooling available and schooling required we stick to a Cobb–Douglas function of the form shown, which implies some characteristics of the degree of substitutability between different types of labour. Education planning, during its short period of life, has mostly assumed a low degree of substitution possibility; as an approximation, a fixed mix of the types of labour as characterized by its education, was often assumed, similar to the assumption that for many industries there is a fixed ratio between the quantities of capital and labour. For many industries in fact the substitution of capital for labour or vice versa was shown to be difficult (cf. Boon [4]). Studies on income distribution were mostly based on the assumption of more flexibility in the ratios between the quantities of different types of labour used; partly, however, because of *indirect substitution*, that is, because of the possibilities of changing the industry mix of a country over time or of different mixes prevailing in countries with different endowments of capital per head.

The need for better knowledge of substitution is now being felt by both groups of economists mentioned; a usual measure for substitution possibilities is the substitution elasticity. In this section the demand substitution elasticity is the one that will be discussed. The substitution elasticity on the side of the supply of different types of labour has been discussed elsewhere, for instance by Freeman [24], by the human capital school [44] and by this author [67].

5.3. Two types of substitution elasticity of demand

An important preliminary question to be answered first is in what way various types of labour have to be defined. In order to estimate the demand elasticity of types of labour the economist will tend to follow the categorization of labour used by statisticians. This can be found, first of all, in census figures, and for some countries also in income distribution figures, whether taken from tax statistics or from special sample surveys.

Census figures provide, first of all, a broad, one-digit classification, and in addition, finer subdivisions. The trouble with this classification is, however, that it is based on two principles, and as yet not shown in a two-entry table or matrix. One principle is a subdivision of people according to *education received* (for instance in the categories professionals and technicians); the other principle is the *type of work* done (as in the categories administrators and managers). For the description of the population according to education received extensive statistics are available, but these do not mention the type of work done (cf. also [49]). In tax statistics we now find, for quite a few countries, incomes according to job categories, but no data on education received.

As a consequence of this state of affairs, the classifications used in some recent empirical research suffer from this duality in census surveys.

In the preceding section I tried to use an, admittedly very crude, two-dimensional classification of income earners, making an explicit distinction between the *job* performed and the *education* received. To be sure, the jobs were defined by the education required, but it was assumed, I think realistically, that education required and education received, are not necessarily identical.

Once this two-dimensionality is assumed, we can distinguish *two different types of substitution.* The first and most usual one is the substitution, for a given job, of a person with one type of education for a person with another type of education. Here we expect the elasticity σ of substitution to be negative; if a category of given education becomes 'cheaper', the organizer of production will exert a larger demand for it, with the limiting case of no change in demand for production with a rigid mix of persons with different levels of education. The '*education substitution elasticity*' is expected to be negative (cf. [69]).

In the second type of substitution, where a person of given education is considered for a job requiring higher education than the job he had so far, he will become more productive (although not as productive as those educated for that higher job). The change in quantity of labour in the new job will then be positive (one man more), and also the change in wage, giving a positive '*job substitution elasticity*'.

The reader may wonder whether the above argument is not too simple, since only one quantity of labour and one price are supposed to change, whereas substitution elasticities refer to the relative changes of two quantities and prices. For simplicity's sake we have kept constant, however, the quantities and prices of the original positions; and so the relative changes have the same sign as the absolute changes in the new positions. In the subsequent section the precise calculations will be shown.

What I want to emphasize here is the essential difference between the two types of substitution and the difference to be expected in the *algebraic signs* of these two types.

A simple example will now be given where the two elasticities of substitution just discussed are computed. The production function used is that of (5.3),

$$y = 15(\phi_{11} + \pi_{21}\phi_{21})^{0.648}(\phi_{22} + \pi_{32}\phi_{32})^{0.088}\phi_{33}^{0.064}, \tag{5.4}$$

where the $\phi_{hh'}$ are the frequencies already defined, y is total national product (including the share of capital) per member of the labour force, and the figures are constants found for the Netherlands in 1962.

The production function could be called semi-empirical since its mathematical form was assumed *a priori*, but the figures were then derived from statistical evidence.

The coefficients π_{21} and π_{32} (assumed to be $1 + 2.3\phi_{21}$ and $1 + 5\phi_{32}$ respectively, if the equilibrium values of ϕ_{21} and ϕ_{32} change) need somewhat further explanation. They represent productivity ratios between the groups 21 and 11, and 32 and 22, respectively. For the individual production organizer they are considered constants, just as in the theory of a competitive market prices are considered given by the individual buyer or seller. As a consequence of the joint action of all individuals ϕ's may change in the longer run; and then π_{21} and π_{32} also change, in the way indicated. The formulae for this change have been chosen in such a way that π_{21} becomes 1 for $\phi_{21} = 0$ and π_{32} becomes 1 for $\phi_{32} = 0$, as already observed in the preceding section.

5.4. Short-term elasticities of substitution

Corresponding to this difference between short-term or micro-economic constancy of the π's and long-term or macro-economic variation of them, a set of two values can be calculated for each of the elasticities. The first is valid for individual decisions of competing production managers, and the second for central planning of government measures.

Let us now calculate the two elasticities of substitution, and begin with the *short-run* elasticities. As an illustration of the substitution in a given job 2 of a person with education 1 for a person with education 2, we suppose a change in ϕ_{21} and constancy of ϕ_{22}. The relevant elasticity σ_e^s now equals the expression

$$\sigma_e^s = \mathrm{d}\frac{\phi_{21}}{\phi_{22}} \Big/ \mathrm{d}\frac{l_{21}}{l_{22}} \cdot \frac{l_{21}}{l_{22}} \Big/ \frac{\phi_{21}}{\phi_{22}}. \tag{5.5}$$

Since the l's are equal to the corresponding marginal productivities, we have

$$l_{21} = \frac{0.648\pi_{21}y}{\phi_{11} + \pi_{21}\phi_{21}}, \tag{5.6}$$

$$l_{22} = \frac{0.088y}{\phi_{22} + \pi_{32}\phi_{32}}, \tag{5.7}$$

and consequently,

$$\frac{l_{21}}{l_{22}} = \frac{7.4\pi_{21}(\phi_{22} + \pi_{32}\phi_{32})}{\phi_{11} + \pi_{21}\phi_{21}}. \tag{5.8}$$

Since the only change is an increase $\mathrm{d}\phi_{21}$ in ϕ_{21}, we have

$$\mathrm{d}\frac{l_{21}}{l_{22}} = -\frac{7.4\pi_{21}^2(\phi_{22} + \pi_{32}\phi_{32})\,\mathrm{d}\phi_{21}}{(\phi_{11} + \pi_{21}\phi_{21})^2}, \tag{5.9}$$

and

$$\mathrm{d}\frac{\phi_{21}}{\phi_{22}} = \frac{\mathrm{d}\phi_{21}}{\phi_{22}}, \tag{5.10}$$

hence

$$\sigma_e^s = -\frac{\phi_{11} + \pi_{21}\phi_{21}}{\pi_{21}\phi_{21}}. \tag{5.11}$$

This education substitution elasticity turns out to be negative, as expected; and for the numerical values for the Netherlands around 1962, it is equal to

$$-\frac{0.79 + 0.15}{0.15} = -6.3. \tag{5.12}$$

For the computation of the short-run *job* elasticity we again assume ϕ_{21} to change, with ϕ_{11} constant. The job elasticity σ_j^s will be

$$\sigma_j^s = \mathrm{d}\frac{\phi_{21}}{\phi_{11}}\bigg/\mathrm{d}\frac{l_{21}}{l_{11}} \cdot \frac{l_{21}}{l_{11}}\bigg/\frac{\phi_{21}}{\phi_{11}}. \tag{5.13}$$

Since $l_{21}/l_{11} = \pi_{21}$, we have

$$\sigma_j^s = \frac{\mathrm{d}\phi_{21}}{\mathrm{d}\pi_{21}}\,\frac{\pi_{21}}{\phi_{21}} = \infty. \tag{5.14}$$

5.5. Long-term elasticities of substitution

For the calculation of the *long-run* elasticities we can use formulae (5.8), (5.10) and (5.13), taking into account now that also π varies. *The education elasticity* now has to be obtained from

$$\mathrm{d}\frac{l_{21}}{l_{22}} = \frac{7.4(\phi_{22} + \pi_{32})\,\mathrm{d}\pi_{21}}{\phi_{11} + \pi_{21}\phi_{21}} - \frac{7.4\pi_{21}(\phi_{22} + \pi_{32}\phi_{32})(\pi_{21}\mathrm{d}\phi_{21} + \phi_{21}\mathrm{d}\pi_{21})}{(\phi_{11} + \pi_{21}\phi_{21})^2} \tag{5.15}$$

from which we obtain

$$\sigma_e^l = \frac{(\phi_{11} + \pi_{21}\phi_{21})\pi_{21}}{\phi_{21}\{(\phi_{11} + \pi_{21}\phi_{21})\alpha_1 - \pi_{21}^2 - \alpha_1\pi_{21}\phi_{21}\}}. \tag{5.16}$$

This expression *will not always be negative;* in fact for the Netherlands we found it to be, for substitution between education levels 1 and 2, ca 50.

This finding is somewhat paradoxical: why would the planner of production be willing to pay a higher wage when he puts more people with education 1 on jobs requiring education 2? In as far as our model (that is, production function) is correct, he does so because the additional (21) people are taken for lack of (22) people (whose education is geared to the jobs); and he engages the additional (21) people for more difficult jobs, having a higher productivity: π_{21} rises with ϕ_{21}. Since the expression in the denominator of dl_{21} also contains negative terms, which reflect the forces economists ordinarily emphasize, these latter have not been neglected and are even preponderant in our case of substituting education 3 for education 2 or vice versa.

For substitution between levels 2 and 3, we found, indeed, $\sigma_e^l = -2.1$.

The long-term *job elasticity* can be found as follows:

$$\sigma_j^l = \mathrm{d}\frac{\phi_{21}}{\phi_{11}} \bigg/ \mathrm{d}\frac{l_{21}}{l_{11}} \cdot \frac{l_{21}}{l_{11}} \bigg/ \frac{\phi_{21}}{\phi_{11}} = \frac{1 + \alpha_1 \phi_{21}}{\alpha_1 \phi_{21}}. \tag{5.17}$$

For the Dutch figures for the substitution of job 2 for job 1, we found this elasticity to be 15; for the substitution of job 2 for job 3 it became 7.7.

Summarizing our numerical results for the semi-empirical case considered we give, in Table 5.II, a survey.

Table 5.II
Short-term and long-term education and job demand elasticities of substitution between labour of different kinds.

	Levels of substitution			
	Short-term		Long-term	
	1 vs 2	2 vs 3	1 vs 2	2 vs 3
Education elasticities	−6.3	−1.9	50	−2.1
Job elasticities	∞	∞	15	7.7

Comparing these results with some empirical figures mentioned by Psacharopoulos and Hinchliffe [52], our short-term education substitution elasticities are very close to theirs; they find −4.8 and −2.2 (minus sign added by me because of the difference in algebraic sign between their and my definition). They add an elasticity of substitution between persons with primary and without any education; but this figure of -50 is highly uncertain: if we add or subtract one standard deviation from their regression coefficient we obtain elasticity values of -13 and $+29$, respectively.

5.6. Statistical estimation of the elasticity of substitution between graduate and other labour: introductory remarks

As a test of the generalized Cobb–Douglas production function in Sections 5.2–5.5 statistical estimates will be discussed of the demand elasticity of substitution, in national production, between third-level educated or graduate labour and other labour. This elasticity has been chosen because it refers to the only type of labour which has been introduced as a separate category.

For the interstate American material only the highest educational level has been considered third-level (more than 3 years college).

For a clear analysis of the problems of substitution between various labour types, substitution on the demand side – exerted by the organizers of production in the widest sense – should be distinguished, as we already said in Chapter 3, from substitution on the supply side, where many individuals have a choice within a range of occupations. Their willingness to change their job will be determined by their preference functions in which not only the income attached to each of the possible jobs enters, but also the satisfaction or dissatisfaction going with each job. In part, this satisfaction (positive or negative) will depend on each persons' level of education. This, by the way, implies the desirability of describing an individual's position vis-à-vis his or her job with at least two indicators, one for the education required to do the job adequately and one for the education actually received, as set out in the preceding sections.

Some of the authors quoted give more explicit attention to this difference between demand side and supply side than others and some remarks on their methods will be made later (cf. Section 5.7).

This author was struck by the high elasticity figures obtained by several others and wondered how to interpret them. Since the question is of particular relevance to the credibility of some calculations, to be presented in Chapters 6 and 7, on the possibilities of reducing income differences, a few alternative attempts were undertaken, using, among other material, quite a few figures collected by Messrs. Bowles and Dougherty. I am particularly grateful to Dr. Dougherty who most generously provided me with a vast amount of material collected by him [22].

The question whether one can determine, with the aid of figures on *prices* and *quantities* exchanged, the demand or the supply function is an old one; various aspects of it were discussed by Frisch in 1933 [25]. It is irrelevant whether prices and quantities refer to one commodity or to the ratios between two commodities; in fact, the price of one commodity is a price ratio for the price of that good relative to the price of money. The simplest illustration of the dilemma is the situation in which both the supply and the demand curve (or line) in the price–quantity diagram have shifted in one direction. The observed points are then not lying on the demand or on the supply curve but on the '*historical path*', the slope of which can be anything. Another possible situation for which the same conclusion applies is the one of random shifts dealt with by Frisch.

There are various ways out of the dilemma. One is that only one of the curves has shifted; then the observed points are all situated on the other curve. This is what Bowles, Dougherty, Psacharopoulos and Ullman have assumed, in order to estimate the short-term demand curve. The elasticities found in this way are reliable only, however, if the correlation coefficient between the price variable and the quantity variable is high; otherwise the regression coefficient found highly depends on whether the first or the second regression has been determined. Our authors take the regression where the quantities are assumed to be given and the price ratios dependent. For a short-term demand curve this can be accepted, but for a long-term demand function this choice is debatable.

Typically *short-term* reactions are reactions where not only the numbers of people employed, but also and especially, the durable production equipment cannot be changed. As a complement, long-term reactions will contain *changes in industries* and in *technology.* It seems natural to me that in the latter type of decisions the organizers of production will start from their information on prices of products as well as production factors, including those of the various types of labour. For this reason I submit that, for long-run studies, relative quantities also on the demand side should be considered to be the dependent and not the independent variables; of course there will not be a large difference between the alternative results whenever a correlation coefficient close to 1 is obtained. This appears not to be so in the cases of simple correlation presented by two of our authors. Since Bowles' simple correlation coefficient is -0.55 and Dougherty's -0.42, their elasticities would have to be multiplied by $0.55^2 = 0.30$ and $0.42^2 = 0.176$, respectively, if the other simple regression had been taken. This drastically reduces the elasticities.

Another way of solving the dilemma is to add at least one more independent variable to each of the equations linking price and quantity variables. As already observed in Chapter 3, these additional variables have often been called *demand* or *supply 'factors'*; they are supposed to co-determine the quantities actually exchanged, looked at from the demand or the supply side, respectively. Both Bowles and Ullman apply this method; Bowles adds, on the demand side, the percentage of active population in agriculture. Ullman, on the demand side, adds the qualities of the types of labour, represented by dummies for the human capital invested in each individual of the two categories – certainly a highly interesting enrichment –; and on the supply side she adds income and cost of education. Bowles' elasticity of demand reduces from 8 to 6 for the substitution between second- and third-level on the one hand

and lower-level manpower on the other hand; the correlation coefficient (for this substitution) improves from 0.85 to 0.90. For the substitution between third-level educated labour only and all other labour, I calculated, from his figures, but using another additional independent variable (cf. below), an improvement from 0.55 to 0.9.

A third way of separating the demand and supply equation consists of introducing a *time lag* for one of the relations (or a different time lag for both). Clearly this only makes sense if such a lag actually exists and is of sufficient length. For the supply of university graduates this is not an unrealistic assumption and was successfully applied by Freeman for college-trained technicians. Such a lag implies the development over time as shown in the *cobweb theorem.* Fluctuations of this kind are common to coffee, pigs and graduates, probably *bien étonnés de se trouver ensemble*!

5.7. Alternative results from Bowles' (cross-nation) and Dougherty's (cross-state) material

Since for income inequality reduction the substitution of third-level educated manpower (or womanpower, for that matter) by all others is more relevant than any other substitution (as far as I experienced in my attempts in [67]), I tried to derive the relevant long-term demand and supply elasticities from the two relations:*

Demand:

$$\frac{L_1 + L_2}{L_3} = b_1 \frac{w_1 + w_2}{w_3} + b_2 u + b_3, \tag{5.18}$$

Supply:

$$\frac{L_1 + L_2}{L_3} = a_1 \frac{w_1 + w_2}{w_3} + a_2 L_2 + a_3. \tag{5.19}$$

* For the cross-nation material the shape chosen here gives a much better fit than a linear relation between the inverse quantity and price ratios. This does not apply to Dougherty's material.

Here L_i (in Bowles' notation) stands for the labour force with education i, w_i for earnings of category i; $i = 3$ stands for more than 11 years of schooling, which is an overestimation of L_3 and an underestimation of w_3, in comparison to my own approaches. Further, u stands for the per mille of the active population in utilities, health services, transportation and communication (ISIC 5 and 7); this is admittedly an incomplete measure of the services sector, since education and government are not included. The a and b are regression coefficients and their values, together with the corresponding standard deviations (*), elasticities (ε) and corrected multiple correlation coefficients ($\bar{R}$) are given in Table 5.III. The upper half of the table gives coefficients estimated with the aid of the least-squares method for (5.18) and (5.19) in succession, the lower half gives coefficients estimated with the aid of reduced-form equations.

Table 5.III
Values found for coefficients in (5.18) and (5.19), c, standard deviations * and elasticities ε.

	a_1	a_2	a_3	$\bar{R}_{sup}$	b_1	b_2	b_3	$\bar{R}_{dem}$
c	−12	−0.236	33.2	0.70	−14.3	−0.375	48.7	0.89
*	(9)	(0.095)	(8.7)		(5.6)	(0.070)	(6.2)	
ε	−1	−1			−1.2			
c	109	−0.68		$R_p = 0.47$	−7.1	−0.35		$R_q = 0.85$
ε	8.5				−0.6			

Note: R_p and R_q are multiple correlation coefficients for price and quantity equations.

According to the least-squares estimates the supply elasticity is not significantly different from 0; since its algebraic sign is negative, it may be interpreted as a small consumptive aspect of supply but does not leave much room for the investment in human capital aspect. The theory behind the L_2 term is that in countries with a large number of people with secondary education one has a stronger tendency to be induced to continue one's education. The algebraic sign of a_2 implies no rejection of this theory. The demand equation, in which we are mainly interested, behaves according to expectations: both signs are correct and the values of the coefficients are significant at the 1 per cent level. The corrected multiple correlation coefficient is satisfactory. The demand elasticity is not significantly different from unity, implying support for the generalized Cobb–Douglas production function used in my earlier models.

The reduced-form estimates yield a strongly positive supply elasticity and a demand elasticity half as low even as the least-squares estimate. The multiple correlation coefficients of the price equation (R_p) and the quantity equation (R_q) – both using the demand factor and the supply factor as explanatory variables – are 0.47 and 0.85, respectively.

I applied a similar procedure to the cross-section material collected mainly by Dougherty for the 28 most populous American states.

The relative employment figure now used was the per millage of effective employment in the experienced labour force in 1959 (equivalent males) with third-level education; the relative income figure was the ratio of third-level mean income of males aged 25–64 in the experienced labour force to median income. As the additional supply factor I introduced the median years of schooling S (instead of L_2 in Bowles' case), taken from the US Summary of the 1960 Census of Population; as the additional demand factor v I used the percentage of the active population employed in transportation, etc., finance, professional services and public administration, from Table 128 of the State Volumes of the 1960 Census.

The results of the regression analysis applied to these data are given in Table 5.IV, where the upper half again gives the estimates obtained by least squares applied to the supply and demand equation and the lower half those obtained from reduced-form estimation. In the upper half for the demand equation also the coefficients have been added which were obtained when price ratios are considered the dependent variable.

Table 5.IV
Values found for coefficients for US states, *c*, standard deviations * and elasticities ε.

	a_1	a_2	a_3	$\bar{R}_{sup}$	b_1	b_2	b_3	$\bar{R}_{dem}$
c	24.0	23.8	−186.6	0.81	−17.9	3.58	69.1	0.72
*	(9.9)	(4.0)			(7.0)	(0.86)		
ε	0.54				−0.40			
c					−94	2.42	28.3	0.45
ε					−2.1			
c	11.7	5.35			−3.78	3.30		
ε	2.64			$R_p = 0.80$	−0.85			$R_q = 0.85$

Note: R_p, R_q, cf. Table 5.III.

From the table we see that this time the multiple correlation coefficients obtained for both the price and the quantity equations are rather satisfactory. The demand elasticity obtained from the second demand equation (where price ratios were considered dependent) is −2.1 which comes closer to the Ullman figure of −2.5; but the reduced-form estimate remains (as an absolute figure) below 1. The conclusion seems warranted that the generalized Cobb–Douglas function used in my earlier estimations gives a realistic picture for the substitution elasticity between third-level educated and all other manpower.