7

Actual, feasible and optimal income distribution

7.1. Complete models for the Netherlands around 1962

The utility and production functions discussed in Chapters 4 and 5 enable us to build a more complete model than the partial one used in Chapter 6 to illustrate a major dynamic feature of our income distribution theory. This *more complete model* (Section 7.2) will contain all five types of labour introduced in Chapter 5 and enable us to study the effect of some *changes in data* on frequencies and incomes (and hence their distribution) of all five groups. To begin with, in Section 7.3 we will vary the numbers of persons with first-, second- and third-level education; and if these are feasible, the model will show the consequences for incomes. In Section 7.4, the effects of changes in *taxes* and in *technology*, the latter as treated in Chapter 6, will be studied. Next, in Section 7.5, we will postulate a social welfare function, specifying it as the sum of all personal welfare, and inquire as to what the *optimal* income distribution looks like. Whereas in the first exercise *taxes* will be considered given, in the second problem they will be considered unknown and will result from the optimization process. Finally, in Section 7.6, we return to our 21-group Dutch job–education groups and to our American data of Chapter 4, in order to illustrate our concept of optimality in income distribution in some more detail (cf. also [68]).

In the present chapter education *required* will be indicated by h (taking values of 1, 2 or 3) and education *supplied* by h' (also taking these three values).

The utility functions used have been described in Chapter 4, but using other units for education levels. Adapted to our units they become for the Netherlands around 1962,

$$\omega_{hh'} = \ln \{x_{hh'} - 0.90h + c_1 h' - 0.64(h - h')^2\}, \tag{7.1}$$

where we will take $c_1 = 0$ for reasons indicated in Section 4.2.

7.2. The complete analytical model

As set out in Chapter 5, equation (5.4), we have chosen a Cobb–Douglas-like production function,

$$y = 15(\phi_{11} + \pi_{21}\phi_{21})^{0.648}(\phi_{22} + \pi_{32}\phi_{32})^{0.088}\phi_{33}^{0.064}, \tag{7.2}$$

with y being expressed in thousands of guilders (of around 1962) per capita of the labour force and a capital income equal to 20 per cent of total income. The exponents represent the *shares* in total income of the labour force with, respectively, levels 1, 2 and 3 of actual education; therefore the exponents add up to 0.80. The expressions in parentheses represent these components of the labour force, taking into account that the *productivity* of an individual with education level 1 on a job requiring level 2 will be higher by a factor π_{21} than an individual (1, 1)'s productivity; and similarly for the labour force (3, 2). While in a given situation of society as a whole, individuals will consider π_{21} and π_{32} as given and constant while making their choices, the values of the π are linked to those of the ϕ, according to the following rules:

$$\pi_{21} = 1 + \alpha_1\phi_{21} \qquad \text{as long as} \quad l_{21} < l_{22}, \tag{7.3}$$

$$\pi_{32} = 1 + \alpha_2\phi_{32} \qquad \text{as long as} \quad l_{32} < l_{33}, \tag{7.4}$$

where $\alpha_1 = 2.3$ and $\alpha_2 = 5.0$ and the l's represent the marginal productivities (and hence *primary* incomes) of the groups concerned. If l_{21} reaches the level l_{22}, relation (7.3) will no longer be valid, and will be replaced by

$$l_{21} = l_{22}, \tag{7.3'}$$

similarly for (7.4),

$$l_{32} = l_{33}. \tag{7.4'}$$

The values for α_1 and α_2 have been derived from the statistical material available. *Marginal productivities* will be, as usual for a Cobb–Douglas production function,

$$l_{11} = \frac{0.648y}{\phi_{11} + \pi_{21}\phi_{21}}, \tag{7.5}$$

$$l_{21} = \frac{0.648\pi_{21}y}{\phi_{11} + \pi_{21}\phi_{21}}, \tag{7.6}$$

$$l_{22} = \frac{0.088y}{\phi_{22} + \pi_{32}\phi_{32}}, \tag{7.7}$$

$$l_{32} = \frac{0.088\pi_{32}y}{\phi_{22} + \pi_{32}\phi_{32}}, \tag{7.8}$$

$$l_{33} = \frac{0.064y}{\phi_{33}}. \tag{7.9}$$

These equations represent the *demand* equations for abilities (11), (21), (22), (32) and (33). With the aid of (7.3) and (7.4) they can also be used to express the ϕ explicitly in terms of the l.

An *analytical* model for us is that version of the model most appropriate for solving the analytical problem, that is, expressing the target or aim variables in terms of instrument variables, or the variables within the control of government. Among the latter are, to begin with, the tax rates and, up to a point, the distribution of the levels of education 1, 2 and 3 over the active population. This distribution is reflected in the values of F_1, F_2 and F_3, where

$$F_1 = \phi_{11} + \phi_{21}, \tag{7.10}$$

$$F_2 = \phi_{22} + \phi_{32}, \tag{7.11}$$

$$F_3 = \phi_{33}. \tag{7.12}$$

The control of government over these data is limited, however, by the innate qualities of the population which may set upper limits to F_2 and F_3. In this book we will not study these limitations, but only show the consequences of a few different values to be given to the F.

The analytical version of our model appears to have *four subversions:* (i) one in which $\phi_{21} = \phi_{32} = 0$, (ii) or (iii) in which either ϕ_{21} or ϕ_{32} equals zero, and (iv) one where both are $\neq 0$. In all subversions equations (7.2) and (7.5) through (7.12) must be fulfilled. In case (iv) we have, in addition, deduced from (7.1),

$$x_{11} - 0.9 = x_{21} - 1.8 - 0.64, \tag{7.13}$$

$$x_{22} - 1.8 = x_{32} - 2.7 - 0.64. \tag{7.14}$$

For subversion (i) the latter two equations become inequalities,

$$x_{11} - 0.9 > x_{21} - 1.8 - 0.64, \tag{7.13'}$$

and

$$x_{22} - 1.8 > x_{32} - 2.7 - 0.64. \tag{7.14'}$$

In subversion (ii) equations (7.13′) and (7.14) and in subversion (iii) (7.13) and (7.14′) must be satisfied. Inequality (7.13′) expresses that for individuals with education 1 it is preferable to have a job 1 than to have a job 2 and that hence $\phi_{21} = 0$; similarly for (7.14′) and $\phi_{32} = 0$.

Finally in the analytical version of the model the x will be linked to the l by the relations

$$x_{11} - x_{21} = 0.85\,(l_{11} - l_{21}), \tag{7.15}$$

and

$$x_{22} - x_{32} = 0.75\,(l_{22} - l_{32}), \tag{7.16}$$

expressing that a *marginal tax rate* of 0.15 applies to the interval $l_{11} - l_{21}$ and one of 0.25 to the interval $l_{22} - l_{32}$. Clearly these are approximations only, since the marginal tax rates will vary with the levels of l. Upon inspection the approximation is very close, however.

Table 7.I
Analytical model; equations and unknowns of each of the four subversions.

Subversion		Equations	Unknowns[b]
(i) 11[a]	$\phi_{21} = \phi_{32} = 0$	(7.2) (7.3) (7.4) (7.5) (7.7) (7.9) (7.10) (7.11) (7.12)	y, 5ϕ, 2π, $3l$
(ii) 13[a]	$\phi_{21} = 0$	(7.2) (7.3) (7.4) (7.5) (7.7) (7.8) (7.9) (7.10) (7.11) (7.12) (7.14) (7.16)	y, 5ϕ, 2π, $4l$, $1\Delta x$
(iii) 13[a]	$\phi_{32} = 0$	(7.2) (7.3) (7.4) (7.5) (7.6) (7.7) (7.9) (7.10) (7.11) (7.12) (7.13) (7.15)	y, 5ϕ, 2π, $4l$, $1\Delta x$
(iv) 15[a]		(7.2) (7.3) (7.4) (7.5) (7.6) (7.7) (7.8) (7.9) (7.10) (7.11) (7.12) (7.13) (7.14) (7.15) (7.16)	y, 5ϕ, 2π, $5l$, $2\Delta x$

[a] This figure indicates the number of unknowns and equations.
[b] 5ϕ means five different ϕ, etc.; Δx means $x_{11} - x_{21}$ or $x_{22} - x_{32}$.

Table 7.I summarizes the equations and the unknowns of each of the four subversions.

7.3. Variation of numbers of persons with education levels 1, 2 and 3

We will not illustrate each of the possible situations, but only subversions (i) and (iv), with some numerical results obtained for the situation originally observed in the Netherlands and for some alternative values of the data F_1, F_2 and F_3. These figures are shown in Table 7.II.

Table 7.II
Changes in incomes and frequencies as a consequence of changes in the total manpower with education levels 1, 2 and 3.

Case Subversion	A (iv)	B (iv)	C (i)	D (i)
F_1	0.91	0.88	0.85	0.81
F_2	0.059	0.08	0.09	0.11
F_3	0.030	0.04	0.06	0.08
ϕ_{21}	0.120	0.12	0.00	.
ϕ_{32}	0.030	0.045	0.00	.
l_{11}	6.2	6.6	6.9	7.35
l_{21}	7.9	8.4	.	.
l_{22}	12.5	9.11	8.9	7.35
l_{32}	14.4	11.15	.	.
l_{33}	21.4	14.90	9.7	7.35
y	9.05	9.31	9.10	9.15

Case D has been chosen on purpose so as to make $F_1 : F_2 : F_3 = 0.648 : 0.088 : 0.064$, that is proportional to the exponents in the production function. As a consequence, $l_{11} = l_{22} = l_{33}$; this will imply tax rates different from the prevailing ones or the ones assumed in equations (7.15) and (7.16). In fact, the *tax equations are not included* in subversion (i); and other tax equations can be added without affecting the figures of cases C and D. Case D has mainly been chosen in order to remind the reader of the fact that a complete equalization of primary incomes does not require all manpower to be of equal qualification, but only to be available in the numbers 'needed' by the production process.

The interesting feature of Table 7.II is that the intermediary cases B and C already show a *substantial decrease in inequality* compared with case A.

7.4. Changes in taxes and technology

The other data – apart from numbers of active persons with the three levels of schooling – appearing in our model are taxes [in subversions (ii), (iii) and (iv)] and the exponents of the production function. As announced in Section 7.1, we will now have a look at the impacts of changes in taxes and in technology on the incomes and income distribution in our model.

To begin with, the *impact of taxes:* this problem is of interest for at least two reasons. First of all, governments have increasingly used taxes in order to arrive at an income distribution different from the one produced by *laissez-faire.* In Chapter 2 extensive statistical information has been presented showing the important differences between (i) primary income distribution, that is, income distribution before taxes are paid, (ii) distribution after taxes and (iii) distribution after complete redistribution, taking into account also the effect of services by the community supplied to consumers at prices below their cost. In this section the incomes after taxes, indicated by the variables x, will be studied in their relation to incomes before taxes, l, especially in order to discover how taxes affect *income distribution.* The second reason for analysing this problem is the importance attached by fiscal experts to the so-called problem of *tax shifting.* The central problem here is whether, and if so to what extent, some groups of the population can avoid the burden of taxation by asking for a higher primary income and actually obtaining it.

The nature of our model permits us to formulate some *general statements* which, however, because of the semi-macro character of our model, do not completely cover the problem.

In addition, the model enables us to make *numerical estimates* of the changes in incomes, primary as well as after tax, caused by given changes in tax rates.

One general statement was already made at the end of the preceding section, where the reader was reminded of the non-occurrence in subversion (i) of the model of any tax variables. The implication is that cases C and D in Table 7.II constitute examples of situations in which *primary incomes are not affected by tax rates.* A transparent situation is in particular the one of D, showing that, if the numbers of active persons with schooling of levels 1, 2 and 3 are proportional to the exponents ρ_1, ρ_2 and ρ_3, primary incomes of the three categories will become equal. Whether this situation can be attained in reality clearly depends on the numbers of students who are able to absorb a secondary and a third-level education. Most developed countries are in the process of experimenting with this possibility: the numbers of students have increased very considerably during the last two decades. As illustrated by the estimates given in Chapter 6, with a time lag of a few decades this process will affect the composition of the labour force and may well reduce primary income inequalities in the coming decades further, as it did during several decades after 1900.

An additional general statement can be made with regard to the other subversions of our model. The only equations needed to calculate primary incomes l are (7.15) and (7.16), and these equations *do not contain full information about the tax system.* On the contrary, *only tax differentials* between groups 11 and 12 and 22 and 32, respectively, enter into these equations, and we can state, therefore, that primary incomes are independent of all other features of the tax system. Primary incomes of people with third-level income are not, therefore, dependent on the tax rates they have to pay themselves. This implies that they cannot shift their taxes. The statement is correct only, of course, under the assumptions on which the model is built. The most important among these assumptions would seem to be:

(i) Third-level educated manpower is scarce.

(ii) The demand elasticity of substitution between this type of manpower and the other types is unity.
(iii) There is free competition among the organizers of production.

It should be noted that assumption (ii) has been verified statistically and that assumption (iii) does not imply whether or not there is competition *on the supply side* of this compartment of the labour market.

So much about the general statements. Let us now turn to some *numerical estimates* made with the aid of the model. A number of variants were made of case B, characterizing a situation where slightly more people are available with a second- and a third-level education than were present in the Netherlands in 1962. This situation is close to reality and provides realism to our variations as well. The results have been collected in Table 7.III.

Before commenting on the results, some more information about the variations chosen is fitting. Case B′ has only been calculated in order to *correct* case B for the small shift in differential tax rates needed in order to take into account the changes in incomes l in comparison to case A (Table 7.II). In view of the very minor differences between B and B′, the latter was not used for tax variation purposes. Cases B″, B‴ and B⁗ all constitute *attempts to reduce inequality* in incomes x *after tax*. First, taxes t were assumed to be a *quadratic function* Q of incomes l, as specified in the table. Parameters of the tax schedules have been chosen so as to attain a given total revenue of 1.38 per active person, as collected in 1962. Since these functions were chosen in such a way that the tax *differentials* chosen were approximately in accordance with them, they somewhat miss the point, in that, especially for B″ and B‴, the *rates* for the lowest income group become higher than for B. The only equalizing effect they have is to reduce income after tax for group 33. Then a broken linear tax function BL was chosen, which indeed brings less inequality for group 11. With a broken linear system one has to see to it that the ranking of incomes after tax remains the same as that of incomes before tax. With regard to groups 32 and 33 this was introduced explicitly. Figures for total product y are averages for all active persons and are in accordance, for case A, with actual 1962 figures.

Table 7.III
Incomes before (l) and after (x) taxes (t) of the five types of labour considered, assuming that 8 per cent of the labour force has secondary and 4 per cent higher education; under various tax systems.

		Case symbols				
		B	B′	B″	B‴	B⁗
Differential tax rates	educ. 1	0.15	0.16	0.10	0.00	0.39
	educ. 2	0.25	0.20	0.25	0.25	0.25
Tax scale[a]		Q_1	n.s.	Q_2	Q_3	BL
Average income y[b]		9.31	9.30	9.29	9.24	9.24
Primary incomes 1000 Dfl.	l_{11}	6.60	6.58	6.63	6.63	6.63
	l_{21}	8.40	8.38	8.30	8.15	8.15
	l_{22}	9.11	9.30	9.10	8.98	8.98
	l_{32}	11.15	11.20	11.18	11.03	11.03
	l_{33}	14.90	14.90	14.86	14.75	14.75
Tax amounts 1000 Dfl.	t_{11}	1.22	.	1.24	1.34	0.975
	t_{21}	1.51	.	1.30	1.34	1.56
	t_{22}	1.65	.	1.44	1.46	2.07
	t_{32}	2.00	.	1.97	2.20	2.58
	t_{33}	2.95	.	3.49	3.88	5.80
Income after taxes 1000 Dfl.	x_{11}	5.4	.	5.39	5.29	5.66
	x_{21}	6.9	.	7.00	6.81	6.59
	x_{22}	7.5	.	7.66	7.52	6.91
	x_{32}	9.0	.	9.21	8.83	8.45
	x_{33}	12.0	.	11.37	10.87	8.95
'Mixed groups'	ϕ_{21}	0.12	0.12	0.11	0.10	0.10
	ϕ_{32}	0.045	0.041	0.045	0.0456	0.456

[a] Q_1 $t = 0.61 + 0.042\, l + 0.0077\, l^2$. Total tax receipts 1.38.
Q_2 $t = 2.09 - 0.336\, l + 0.029\, l^2$. Total tax receipts 1.38.
Q_3 $t = 3.92 - 0.71\, l + 0.048\, l^2$. Total tax receipts 1.38.
BL For $l < 7.5$: $t = 0.975$; for $7.5 < l < 8.5$: $t = 1.56$; for $8.5 < l < 12.5$: $t = 0.25(l - 8.5) + 1.95$; for $12.5 < l$: $t = 0.80\ (l - 12.5) + 4.00$.
n.s. Not specified.
[b] y Average income per active person in 1000 Dfl.

The quantitative variations shown in Table 7.III clearly support the extension of our theoretical statement for cases C and D made earlier: *Tax changes have a very slight impact on primary incomes for all five labour categories.* This implies that the impossibility of shifting, found for group 33, also applies in our model to the other 'mixed' groups where education required does not always coincide with actual education. Looked at from the opposite side this implies that redistribution by direct taxes *does constitute a means to reduce inequality*, provided it is applied to a sufficient extent. In the next section it will be shown, however, that *optimal* taxes are taxes which do not depend on income; they depend on capabilities and similar data of the problem, rather than on results of the process of choice of occupation such as income.

The impact of *changes in technology* can be ascertained by repeating the calculations for different values of the exponents ρ. This implies, as already discussed in Chapter 6, that the portions of national income imputed to the three categories of manpower with first-, second- and third-level education change accordingly. As discussed in Chapter 6, technological development tends to raise ρ_3. It seems likely that ρ_1 will fall, whereas the change in ρ_2 remains uncertain. Generally, technological development will tend to widen income differences, *unless a deliberate effort* be made to *direct technological innovation* to more labour-intensive methods in the sense of their requiring more *labour of low education level.* While there seem to be such possibilities, it is outside the scope of this study to enter into more detail here.

7.5. The complete optimalization model

In addition to the analytical model just described a policy model will also be used, in particular, a model for determining the *optimal income distribution.* This model will be set up in the tradition of welfare economics, as a model for *maximizing social welfare* under various restrictions. The social welfare function will be taken equal to the weighted average of individual welfare functions (7.1), the weights being the $\phi_{hh'}$. The restrictions will be (a) the production function (7.2), into which equations (7.3) and (7.4) will be substituted from the start, and the balance equations for (b) labour and for (c) the product.

Two subversions will be considered with regard to labour; subversion (i) will consider as given each of the F_1, F_2 and F_3 separately, whereas subversion (ii) will consider as given only their total, implying that any number of people can absorb education of all types. The balance equations for the product will only express that total product y equals the total of expenditures $x_{hh'}$ by each labour group h, h'. We do not specify whether these are expenditures on consumer or on investment goods, since we are not interested here in development, but rather in income structure. Using Lagrangian multipliers, the problem can be formulated as the determination of the maximum of (i),

$$\begin{aligned}
&\phi_{11}\ln(x_{11}-0.90)+\phi_{21}\ln(x_{21}-2.44)\\
&\quad+\phi_{22}\ln(x_{22}-1.80)+\phi_{32}\ln(x_{32}-3.34)\\
&\quad+\phi_{33}\ln(x_{33}-2.70)+\lambda\{y-15(F_1+2.3\phi_{21}^2)^{0.648}\\
&\qquad\times(F_2+5\phi_{32}^2)^{0.088}F_3^{0.064}\}+\mu(F_1-\phi_{11}-\phi_{21})\\
&\quad+\nu(F_2-\phi_{22}-\phi_{32})+\pi(F_3-\phi_{33})\\
&\quad+\tau(\phi_{11}x_{11}+\phi_{21}x_{21}+\phi_{22}x_{22}+\phi_{32}x_{32}\\
&\qquad+\phi_{33}x_{33}-y),
\end{aligned}\tag{7.17}$$

where for subversion (ii),

$$\mu=\nu=\pi. \tag{7.18}$$

Differentiation with regard to the eleven variables ϕ, x and y yields

$$\ln(x_{11}-0.90)-\lambda\frac{0.648y}{F_1+2.3\phi_{21}^2}-\mu+\tau x_{11}=0, \tag{7.19}$$

$$\ln(x_{21} - 2.44) - \lambda \frac{0.648(1 + 2.3\phi_{21})y}{F_1 + 2.3\phi_{21}^2} - \mu + \tau x_{21} = 0, \quad (7.20)$$

$$\ln(x_{22} - 1.80) - \lambda \frac{0.088y}{F_2 + 5\phi_{32}^2} - \nu + \tau x_{22} = 0, \quad (7.21)$$

$$\ln(x_{32} - 3.34) - \lambda \frac{0.088(1 + 5\phi_{32})y}{F_2 + 5\phi_{32}^2} - \nu + \tau x_{32} = 0, \quad (7.22)$$

$$\ln(x_{33} - 2.70) - \lambda \frac{0.064y}{F_3} - \pi + \tau x_{33} = 0, \quad (7.23)$$

$$\frac{\phi_{11}}{x_{11} - 0.90} + \tau\phi_{11} = 0, \quad (7.24)$$

$$\frac{\phi_{21}}{x_{21} - 2.44} + \tau\phi_{21} = 0, \quad (7.25)$$

$$\frac{\phi_{22}}{x_{22} - 1.80} + \tau\phi_{22} = 0, \quad (7.26)$$

$$\frac{\phi_{32}}{x_{32} - 3.34} + \tau\phi_{32} = 0, \quad (7.27)$$

$$\frac{\phi_{33}}{x_{33} - 2.70} + \tau\phi_{33} = 0, \quad (7.28)$$

$$\lambda - \tau = 0. \quad (7.29)$$

The solution can be started by the elimination of τ with the aid of (7.29). Equations (7.24) through (7.28) can be satisfied by either (I),

$$x_{11} - 0.90 = x_{21} - 2.44 = x_{22} - 1.80 = x_{32} - 3.34 = x_{33} - 2.70, \quad (7.30)$$

or (II),

$$x_{11} - 0.90 = x_{22} - 1.80 = x_{33} - 2.70, \tag{7.31}$$

together with

$$\phi_{21} = \phi_{32} = 0. \tag{7.32}$$

Pursuing case (I) we can eliminate μ and v from (7.19) and (7.20), respectively, from (7.21) and (7.22); using (7.30) we then find

$$\phi_{21} = \frac{1.54(F_1 + 2.3\phi_{21}^2)}{0.648 \times 2.3y}, \tag{7.33}$$

$$\phi_{32} = \frac{1.54(F_2 + 5\phi_{21}^2)}{0.088 \times 5y}. \tag{7.34}$$

Together with (7.2) we now have three equations in y, ϕ_{21} and ϕ_{32}. It appears possible to solve these non-linear equations numerically. With the initial values $F_1 = 0.911$, $F_2 = 0.059$ and $F_3 = 0.030$ we find the values given in Table 7.IV.

Table 7.IV
Optimal values of variables relevant to income distribution for given values of manpower with first-, second- and third-level education.

Group[a]	l	$l^{i\,b}$	ϕ	$\phi^{i\,b}$	x	t^c	y	$y^{i\,b}$
11	6.1	6.2	0.80	0.79	8.6	−2.5	8.9	9.05
21	7.7	7.9	0.11	0.12	10.1	−2.4		
22	12.6	12.5	0.035	0.029	9.5	3.1		
32	14.1	14.4	0.024	0.03	10.8	3.3		
33	19.0	21.4	0.03	0.03	10.4	8.6		

[a] First figure indicates job level; second figure, education level.
[b] Initial values observed for the Netherlands around 1962.
[c] Taxes calculated as $l - x$.

It will be observed that the changes in *primary* incomes – in comparison with the initial situation – are slight only, except for the *highest* income group. The main change takes place in the *redistribution* system. This is seen at once from the tax figures which show two new features. First, negative taxes are required for the two lower groups, those with education level 1. Secondly, taxes do not depend on the job level, but only on the *level of abilities*. Evidently the *feasibility* of the optimum depends on the feasibility of such a tax system, which represents an example of a *lump-sum* tax: the tax rate does not influence the marginal income connected with a change in job and the corresponding income. For the time being a capability tax seems impossible to administer; this would require a refinement in psycho-technical testing which may take a few decades. The need for lump-sum taxes has been understood for a long time [62]. Until they become feasible only second-best solutions to the problem of how to establish an optimum regime, such as income and wealth taxes, can be used.

Let us now take up case (II), where *no limits to education* are assumed to exist. The solution now becomes as shown in Table 7.V.

Table 7.V
Optimal values of variables relevant to income distribution, assuming unlimited capabilities to absorb more education.

Group[a]	l	l^{i} [b]	ϕ	ϕ^{i} [b]	x	t[c]	y	y^{i} [b]
11	7.1	6.2	0.835	0.79	8.9	− 1.8		
21	.	7.9	0.00	0.12	.	.		
22	8.0	12.5	0.10	0.029	9.8	− 1.8	9.15	9.05
32	.	14.4	0.00	0.03	.	.		
33	9.0	21.4	0.065	0.03	10.7	− 1.7		

[a] First figure indicates job level; second figure, education level.
[b] Initial values observed for the Netherlands around 1962.
[c] Taxes calculated as $l - x$.

As could be expected, a considerable reduction in income inequality results. Two features of the solution, however, are surprising at first sight. One is that the *number of persons with third-level education* required in this case is *not so large* – it is about double the number prevailing in the initial situation. Also the number required with secondary education is not large at all. With the information now available – that is about ten years after the initial period – the figures for ϕ_{22} and ϕ_{33} do not seem illusory. Before discussing the question why the actual income distribution did not at all change as much as suggested by Table 7.V, we first want to draw attention to the fact that all taxes are now negative. What is the source of these taxes in the model used? It appears to be capital income, which represents, as already observed, 20 per cent of y, which equals 1.8. In the optimum situation capital income is distributed proportionately over the working population.

This feature of our model is less essential, however, than the feature of lump-sum taxes. We could have changed the balance equation for product use by introducing into it public consumption of a given portion of total product, say ψy; this would have changed (7.29) into

$$\lambda - \tau(1 - \psi) = 0, \tag{7.29'}$$

and accordingly reduced the negative taxes by a factor $1 - \psi$.

Returning to the question why in 1962–1973 the income distribution has not changed so much as the figures of Table 7.V suggest, two points seem to be relevant. One is that only some recent student generations have doubled in comparison to the composition of the labour force; it will take some decades before the composition of the entire labour force will be as indicated in Table 7.V. In addition, there may also have been changes on the side of demand for qualified manpower, showing up as possible changes in the parameters of the production function. In fact, these questions have already been dealt with in Chapter 6.

7.6. Optimality of income distribution: some more details

In Section 7.5 we applied the method proposed to define optimal income distribution to the crude five-compartment material for the Netherlands. The somewhat more detailed data presented in Chapter 4 for the Netherlands and for some American states enable us to go into some more detail by applying the same method to the larger number of groups considered there – 21 for the Netherlands and about 15 for each of the American states considered.

For the utility function proposed in Chapter 4, optimality implies that, according to equation (7.30), utilities of the groups considered *must be equal.* Using the notation of Chapter 4, where s characterized jobs by indicating the years of schooling needed in multiples of 3 years and v was education actually completed in the same units, Table 7.VI shows, alongside the actual incomes x after tax, the optimal values in parentheses calculated with the aid of the numerical specification,

$$x = +0.45s + 0.32(s - v)^2 + 5.7. \qquad (7.35)$$

The constant has been chosen so as to obtain the average labour income after tax for the two corresponding combinations ($s = 2, v = 3$) and ($s = v = 2$) as it actually was in 1962.

Table 7.VI
Actual and optimal income distribution under the assumption set out in text (optimal in parentheses); 1962, in thousands of Dfl.

s	v				
	2	3	4	5	6
6	.	.	14.0	14.0	14.0
			(9.7)	(8.7)	(8.4)
5	.	11.3	11.3	11.3	11.3
		(9.2)	(8.3)	(8.0)	(8.3)
4	8.3	8.3	8.3	8.3	8.3
	(8.8)	(7.8)	(7.5)	(7.8)	(8.8)
3	9.9	10.2	10.5	10.8	.
	(7.4)	(7.1)	(7.4)	(7.2)	
2[a]	4.9	7.7	.	.	.
	(6.6)	(6.9)			

[a] Weighted average of two groups for $v = 2$ mentioned in Table 7.I.

The reader should be reminded that our assumptions in Chapter 4 are that v and W constitute parameters in the strict sense, hence cannot be changed by some learning process. To the extent that v or W or both can be obtained by efforts constituting a sacrifice, our formula (7.35) should obtain terms in v and W or both, expressing the corresponding sacrifices. These terms cannot surpass the additional scarcity incomes at present enjoyed by those endowed with higher values of v or W. If v and W are parameters in the strict sense, then the optimal income distribution can be reached by measures counteracting the scarcity incomes, without killing the stimuli for the better endowed individuals. This problem will be taken up in further detail in Chapter 8.

For the state of Illinois we found, in the notation used in Section 4.5, the utility function's argument to be $x = -0.156h - 0.06(h - h')^2$; and equality of utility would therefore mean that this expression is the same for all groups considered. Its value was chosen equal to the value for the most numerous group, represented by $h = 6$ and $h' = 8$, which happens to be a group near the median as well. Accordingly optimal incomes were estimated with the aid of (7.36),

$$x = 0.156h + 0.06(h - h')^2 + 3.4. \tag{7.36}$$

Table 7.VII
Actual and (in parentheses) optimal income for groups with different occupations h and education h'; Illinois, 1959, in thousands of US $.

h	h'							
	0	3	6	8	10	12	14	18
0	3.1 (3.4)	3.3 (3.9)						
3		3.4 (3.9)	3.7 (4.4)					
6			4.2 (4.3)	4.6 (4.6)				
8				4.6 (4.7)	5.1 (4.9)			
10					5.7 (5.0)	6.2 (5.2)		
12						6.7 (5.3)		
14						7.2 (5.8)	7.8 (5.6)	
18							8.4 (7.1)	9.5 (6.2)

Table 7.VII shows the actual and (in parentheses) the optimal incomes for all groups, assuming the very provisional conditions specified.

The interesting feature of both Table 7.VI and Table 7.VII is the reduction in inequality they require in order to arrive at an optimal income distribution as here estimated. The difference between highest and lowest incomes after taxes for the groups considered should be reduced by 66 per cent for the Netherlands in 1962 and by 56 per cent for Illinois in 1959.

In Table 7.VIII some more information on the same subject is given in a more summarized form for the other American states discussed (cf. Section 4.6).

Table 7.VIII
Actual and optimal income differences for labour with different schooling, derived from the first attempt to measure utility functions; 1959, in thousands of US $.

State	Differences between incomes of labour without schooling and			
	(I) Labour with 18 years schooling		(II) Labour with 12 years schooling	
	Actual	Optimal	Actual	Optimal
Cal	6.3	4.4 (2.2)	3.4	2.9 (1.4)
Ill	6.4	2.8 (2.7)	3.6	1.8 (1.8)
NY	6.1	4.5 (2.2)	3.2	3.0 (1.5)
Mich	7.9	3.1 (2.0)	3.4	2.1 (1.3)
SoCa	5.7	1.7 (1.7)	2.9	1.1 (1.2)
Tex	6.0	1.9 (2.1)	3.4	1.3 (1.4)
Wis	5.8	1.9 (1.8)	2.9	1.3 (1.2)

Optimal figures have been estimated with the aid of first regression values of c_0 (Table 4.VII) from the quadratic tension method and (in parentheses) values of c_0 obtained from the linear tension method. They apply to groups for which $h = h'$ (that is, on the diagonal in Table 7.VII).