2. Two-worlds models

2.1 EAST-WEST MODELS: LOGARITHMIC UTILITY

As announced, Chapters 2 to 4 inclusive will deal with a number of macro-models to be used to estimate the optimal amounts of military and non-military expenditure, of security aid and of development aid. Military expenditures and security aid are mainly a problem of W1 and W2, whereas development aid is mainly a problem of W1 and W3. Since the opinion has been expressed that the two problems are not connected, in the present chapter they will be treated separately. In Chapters 3 and 4 they will be integrated in order to find out whether such integration changes the outcomes.

Two models of the East-West problem will be considered: (W1W2, 1) where only one type of armament is considered, and (W1W2, 2) where two types are considered. The variables of the models are:

n_1, n_2 : population of W1 and W2 ($n_1 = 1$)
x_1, x_2 : national product of W1 and W2 (considered given)
y_1, y_2 : non-military expenditures of W1 and W2
a_1, a_2 : military expenditures (on offensive arms if defensive arms are also considered)
b_1, b_2 : military expenditures on defensive arms (not considered in Model (W1W2, 1))
v_{12} : non-military security assistance from W1 to W2.

The models' equations consist, first of all, of the restrictions. These are the balance equations:

$$x_1 = y_1 + a_1 + b_1 + v_{12} \quad \text{(R1)}$$
$$x_2 = y_2 + a_2 + b_2 - v_{12} \quad \text{(R2)}$$

where the terms b_1 and b_2 only appear in Model (W1W2, 2).

The other equations express that the unknown variables maximize welfare-in-security. Two problems will be considered, of which Problem I in two versions (I_1 and I_2). In the latter, W1 and W2 are supposed to be

sovereign. In that situation v_{12} will be assumed to be negligible ($v_{12} = 0$), and W1's welfare maximized under R1.

As the welfare function we first consider the logarithmic function; later the parabolic function will be taken up. So in Problem I_1 we maximize:

$$\omega_1 = \ln (y_1 + 1) + \alpha_{11} \ln (a_1 + 1) + \lambda_1 (x_1 - y_1 - a_1) \qquad (2.11)$$

where λ_1 is a Lagrange multiplier and in parentheses what remains of R1 if $b_1 = v_{12} = 0$: then R1 reduces to $x_1 - y_1 - a_1 = 0$ (R1')
The additional equations are maximum conditions for the two unknown variables y_1 and a_1:

$$1/(y_1 + 1) - \lambda_1 = 0 \qquad (2.12)$$
$$\alpha_{11}/(a_1 + 1) - \lambda_1 = 0 \qquad (2.13)$$

From (2.12) and (2.13) we derive:

$$a_1 + 1 = \alpha_{11} (y_1 + 1) \qquad (2.14)$$

Elimination of y_1 with the aid of (R1') yields:

$$a_1 (1 + \alpha_{11}) = \alpha_{11} x_1 + \alpha_{11} - 1 \qquad (2.15)$$

The smallness of $\alpha_{11} - 1$ in comparison to $a_1 (1 + \alpha_{11})$ enables us to simplify (2.15) into

$$a_1 = \alpha_{11} x_1 / (1 + \alpha_{11}) \qquad (2.16)$$

Since a_1 and x_1 are observed, (2.16) permits us to estimate α_{11}. Using Leger Sivard's figures for 1982, in billion US $ we have for NATO: $x_1 = 6156$, $a_1 = 311$ and $\alpha_{11} = 0.053$; and for the USA: $x_1 = 3057$, $a_1 = 196$ and $\alpha_{11} = 0.069$.

In the same way we can solve Problem I_2 and find for the Warsaw Pact: $x_2 = 2084$, $a_2 = 187$ and $\alpha_{22} = 0.099$. For the Soviet Union: $x_2 = 1563$, $a_2 = 107$ and $\alpha_{22} = 0.122$. Here α_{11} stands for the portion of national income spent on armament in the USA and NATO, and α_{22} applies to the Warsaw Pact and the Soviet Union. They are close to the observed figures.

Turning now to Problem II: in order to find the optimal level of armament in a Treaty between East and West, we use (R1) and (R2), putting $b_1 = b_2 = 0$ and maximize

$$\Omega = \omega_1 + \omega_2 = \ln (y_1 + 1) + \alpha'_{11} \ln (a_1 + 1) - \alpha'_{12} \ln (a_2 + 1) +$$
$$0.65 \ln (y_2 + 0.65) + 0.65\, \alpha'_{22} \ln (a_2 + 0.65) - 0.65\, \alpha'_{21} \ln (a_1 + 0.65)$$
$$+\lambda_1 (x_1 - y_1 - a_1 - v_{12}) + \lambda_2 (x_2 - y_2 - a_2 + v_{12})$$

The maximum conditions are:

$$\begin{aligned} &1/(y_1 + 1) - \lambda_1 = 0 \qquad\qquad 0.65/(y_2 + 0.65) - \lambda_2 = 0 \\ &\qquad\qquad -\lambda_1 + \lambda_2 = 0 \\ &\alpha'_{11}/(a_1 + 1) - 0.65\, \alpha'_{21}/(a_1 + 0.65) - \lambda_1 = 0 \\ &-\alpha'_{12}/(a_2 + 1) + 0.65\, \alpha'_{22}/(a_2 + 0.65) - \lambda_2 = 0 \end{aligned} \tag{2.17}$$

In the expression for Ω, α'_{12} represents the impact of W2's armament on W1 and α'_{21} the impact of W1 armament on W2. The figure 0.65 is the population of W2 in terms of W1, and its appearance in ω_2 is explained in Appendix I. Equation (2.17) results from differentiation of Ω with respect to v_{12}.

For the solution of the system of equations a good approximation will be used for values of $a_1 >> 1$, $a_2 >> 1$:

$$\alpha'_{11}/(a_1 + 1) - 0.65\, \alpha'_{21}/(a_1 + 0.65) \simeq (\alpha'_{11} - 0.65\, \alpha'_{21})/(a_1 + 0.825) \tag{2.18}$$

In the expression for Ω we used the symbols α'_{ij}, $(i, j = 1, 2)$ instead of α_{ij} in order to express that in Problem II, where the two parties are co-operating in some form (for example by a treaty), their need for armament will be smaller than without such co-operation. They need armament to defend themselves against possible aggression by outsiders and against deviation from the treaty by the other party. In all probability this need for arms will be lower than before. Disregarding, to begin with, this effect of co-operation, we stick to the value found for α_{11} and α_{22} and ask ourselves what value α_{12} and α_{21} may have. As far as the weapons are defensive these values may be 0, but since most of them are offensive, they may be of the order of, respectively, α_{22} and α_{11} of which the values were found in the solution of Problems I_1 and I_2. So we think it proper to assume that:

$$0 \leq \alpha_{12} \leq \alpha_{22} \text{ (0.099 for NATO, 0.122 for the US)}$$
$$0 \leq \alpha_{21} \leq \alpha_{11} \text{ (0.053 for the WP, 0.069 for the SU)}$$

But there are additional restrictions, because the maximum conditions

(2.17), after elimination of the λ_1 and λ_2 reduce to:

$$\frac{\alpha'_{11} - 0.65\,\alpha'_{21}}{a_1 + 0.825} = \frac{0.65\,\alpha'_{22} - \alpha'_{12}}{a_2 + 0.825} = \frac{1}{x_1 - a_1 - v_{12} + 1} = \frac{0.65}{x_2 - a_2 + v_{12} + 1} \qquad (2.19)$$

These equations constitute three equations for the three unknowns a_1, a_2 and v_{12}. The numerators of the first two fractions must be positive. For α_{21} this has no implications, but for α_{12} it imposes lower upper limits, namely 0.65 α_{22}, which for the WP means 0.064 and for the SU 0.079. So for α_{12} we have

$$0 \leq \alpha_{12} \leq \begin{cases} 0.064 \text{ (WP)} \\ 0.079 \text{ (SU)} \end{cases}$$

The preceding statements about the lower and upper limits of α_{12} and α_{21} enable us to find lower and upper limits of the numerators of the first two fractions of (2.19). The results are shown in Table 2.11, where we apply the lower and upper values found for the α and the α'.

Table 2.11 Maximum and minimum values of numerators of 1st and 2nd fractions in (2.19)

		NATO/WP	US/US
1st numerator	maximum	0.093	0.069
	minimum	0.019	0.024
2nd numerator	maximum	0.064	0.079
	minimum	0	0
Average of 4 values		0.034	0.043

Finally, it seems appropriate to assume that $a_1 = a_2 = a$ ('equal strength'), which requires equality of the two numerators, for which we take the averages. The values then obtained for a and v_{12} are shown in Table 2.12.

Table 2.12 Optimal values for military expenditure a, compared with 1982 values, and security assistance v_{12} *(billion US$ of 1982)*

	a		v_{12}
	opt.	1982	
NATO/WP	162	311	1197
US/SU	114	196	281

As observed, these figures, 162 and 114, are overestimated, since the α'_{11} and α'_{22} presumably are lower than the α_{11} and α_{22} used in the computations. Interesting research remains to be done, but a reduction by about 50 per cent of military expenditure seems close to an optimum. In addition, large amounts of 'security aid' or non-military security expenditures, 10 or 20 per cent of x_1, are found to be optimal; so large, indeed, as to require explanation. This will be discussed in Section 2.2.

2.2 INTERPRETATION OF THE FINDING OF HIGH FIGURES OF SECURITY AID

The unrealistically high figures found for optimal security aid constitute a problem that we shall face again when development aid is studied (Section 2.5 and beyond). For an appropriate interpretation of our findings it needs careful study. One aspect is that the seriousness of the problems studied (the need for arms reduction and for more development co-operation) is underestimated by many citizens and politicians. Another aspect, however, is that the models used are static models and as a rule the optima found are long-term optima. This aspect may be eliminated by replacing the models by dynamic models and in Chapter 7 this will be done. It deprives us of the simplicity and the clarity of the models, however, and so of their persuasive power. This is why, in the present section, another way of interpreting our results will be offered. The explanation seems to us to be that welfare is assumed to depend on instantaneous expenditure y_1 (mainly consumption) only, and not on any downward change in y_1. A sudden reduction in y_1 as a consequence of security aid has not been taken into account. This implies that we implicitly assumed this reduction to take place in such a way as to avoid a sudden

reduction in y_1. This can be arranged by an annual buildup of security aid from the increase of production. This annual increase was, around 1970, 4.15 per cent for W1's production x_1. For NATO it takes less than four years for the fall in a_1 plus the increase in x_1 (= 6156) to become 1197, the optimum of v_{12}. During these four years only what remains after the initial y_1 and the optimal armament have been paid for is available for security aid. These amounts are shown in Table 2.21.

Table 2.21 *National product* x_1, *armament expenditure* a_1, *non-military expenditure* y_1 *and security aid* v_{12} *of NATO for years 0 to 5 (bnUS$)*

t =	0	1	2	3	4
x_1	6156	6411	6678	6955	7243
a_1	311	162	162	162	162
y_1	5845	5845	5845	5845	5845
v_{12}	0	404	671	948	1236
v_{12} as per cent of x_1	0	6.3	10.0	13.6	17.1

The value of y_1 is kept constant and v_{12} is financed out of the rise in x_1, which in year 1 is 6411 – 6156 = 255; since a_1 is reduced to 162, 311 – 162 = 149 is available from that source, so in total 255 + 149 = 404. Security assistance v_{12} will reach its optimum shortly before year 4.

From it we see that, under the assumptions made, in the years 1 to 4 inclusive the percentage of national income spent on 'security aid' will only gradually rise to 17.1. This interpretation enhances the credibility of our results. This method will be described again in Section 2.5, where development aid is discussed, a subject more often debated than security aid.

Table 2.21 is based on income figures x_1 = 6156, obtained with the aid of official exchange rates for non-USA countries. Since it is better to use purchasing power exchange rates, as done by Kravis *cum suis* we add Table 2.22 where Kravis's figures have been used, for real incomes and expenditures:

Table 2.22 National product x_1, *armament expenditures* a_1, *non-military expenditure* y_1 *and security aid* v_{12} *of NATO for years 0 to 3 (bn 1965 US$)*[1]

t =	0	1	2	3
x_1	2406	2506	2610	2718
a_1	122	71.6	71.6	71.6
y_1	2284	2284	2284	2284
v_{12}	0	150.4	254.4	362.4
v_{12} as % of x_1	0	6.0	9.7	13.3

[1]Calculated the same way as Table 2.21

Of course, a more satisfactory method to deal with this approach is one of a dynamic model (cf. Chapter 7).

These are 1975 dollar figures, however, which explains the large part of the deviation from the figures in Table 2.21. Using the Kravis figures we find optimal $a_1 = 71.6$ and $v_{12} = 204.3$ or 8.49 per cent of NATO $x_1 = 2406$. The optimal armament level is well below the 1975 level of 122. Introducing again a gradual buildup of v_{12} we now need somewhat less than 2 years only.

2.3 EAST-WEST MODELS: TWO TYPES OF ARMS

As announced in Section 2.1, models with two types of armament (W1W2,2) have also been studied. Although not all weapons can be strictly classified into offensive and defensive weapons, the classification plays a predominant role already in the discussion of the Anti-Ballistic Missile (ABM) Treaty of 1972 and, again, in the discussion about the Strategic Defence Initiative (SDI).

As an example with crude figures only, we choose a model dealing with the two superpowers (US and SU) and with figures rounded to multiples of 25:

$x_1 = 2500$	$x_2 = 1250$
$a_1 = 150$	$a_2 = 150$
$b_1 = 25$	$b_2 = 25$
$y_1 = 2325$	$y_2 = 1075$

These figures reflect the lower national product of the SU, the equal level of armament and the minor role played by defensive weapons. We restrict ourselves to the use of logarithmic welfare-cum-security functions. For Problem I, we choose

$$\omega_1 = \ln (y_1 + 1) + \alpha_{11} \ln (a_1 + 1) + ß_{11} \ln (b_1 + 1)$$

and the restriction: R1: $x_1 = y_1 + a_1 + b_1$
From the optimum conditions we derive:

$$a_1 + 1 = \alpha_{11}(y_1 + 1) \qquad \text{and hence}$$
$$\alpha_{11} = (a_1 + 1)/(y_1 + 1) = 151/2326 = 0.065$$

$$b_1 + 1 = ß_{11}(y_1 + 1) \qquad \text{and}$$
$$ß_{11} = (b_1 + 1)/(y_1 + 1) = 26/2326 = 0.011$$

Similarly for the SU, Problem I_2 yields

$$\alpha_{22} = (a_2 + 1)/(y_2 + 1) = 0.140 \text{ and } ß_{22} = (b_2 + 1)/(y_2 + 1) = 0.024$$

Problem II, where a joint optimum is searched for, requires the introduction of the impact of each power's armament on the welfare-cum-security of the other power and the maximization of:

$$\Omega = \omega'_1 + \omega'_2 = \ln(y_1 + 1) + \alpha'_{11}\ln(a_1 + 1) + ß'_{11}\ln(b_1 + 1) - \alpha'_{12}\ln(a_2 + 1)$$
$$+ ß'_{12}\ln(b_2 + 1) + \ln(y_2 + 1) + \alpha'_{22}\ln(a_2 + 1) + ß'_{22}\ln(b_2 + 1) - \alpha'_{21}\ln(a_1 + 1)$$
$$+ ß'_{21}\ln(b_1 + 1) + \lambda_1(x_1 - y_1 - v_{12} - a_1 - b_1) + \lambda_2(x_2 - y_2 + v_{12} - a_2 - b_2)$$

Here α'_{ij} constitutes the impact of j's armament on i's welfare and similarly for $ß'_{ij}$. Offensive weapons are supposed to have a negative impact on the other power. Moreover an amount of 'security aid' v_{12} may be agreed upon, flowing from 1 to 2, together with the negotiations on armament reductions. We assume that the two nations want to keep their armament in both categories at the same level, $a_1 = a_2 = a$ and $b_1 = b_2 = b$. The ordinary Lagrange procedure after elimination of λ_1 and λ_2 leads to:

$$y_1 = y_2 = y$$
$$2(a + 1) = (\alpha'_{11} + \alpha'_{22} - \alpha'_{12} - \alpha'_{21})(y + 1)$$
$$2(b + 1) = (ß'_{11} + ß'_{22} + ß'_{12} + ß'_{21})(y + 1)$$

Direct observations of the 'mixed' coefficients α'_{12}, α'_{21}, $ß'_{12}$ and $ß'_{21}$ are not available. As long as no direct observations are available we shall use some alternative values satisfying the plausible conditions that $\alpha'_{12} + \alpha'_{21} < \alpha'_{11} + \alpha'_{22} = 0.205$ and $ß'_{12} + ß'_{21}$ are of the same order of magnitude as $ß'_{11} + ß'_{22} = 0.035$. The first assumption is plausible since it leads to positive values of a, and the second constitutes an upper limit to b; a possible lower limit may be $ß'_{12} + ß'_{21} = 0$. The assumptions made and results obtained are shown in Table 2.31. For the reasons mentioned the figures indicate orders of magnitude rather than exact figures. Those orders of magnitude are instructive, and surprisingly differ from what many experts would have expected. This applies to the possible role of security assistance in particular and perhaps to the desirable shift towards defensive arms.

Table 2.31 Coefficients assumed and optimum values of variables obtained

	Case (i)	Case (ii)	Case (iii)
α_{11}	←	0.065	→
α_{22}	←	0.140	→
α_{12}	0.05	0.06	0.04
α_{21}	0.10	0.12	0.08
$ß_{11}$	←	0.011	→
$ß_{22}$	←	0.024	→
$ß_{12}$	0.010	0.008	0.014
$ß_{21}$	0.020	0.015	0.030
y	1771	1758	1735
a	48	67	73
b	57	50	68
v_{12}	←	625	→

The results share with previous results the strong redistribution of civilian expenditure y, mainly obtained with the aid of a high value of v_{12}. The particular topic dealt with here is, however, the distribution over defensive and offensive weapons and the total level of military expenditure. In the cases shown a clear shift from offensive to defensive armament is found, even if the values for b are overestimated. Total military

expenditures $a + b$ are significantly lower than in the observed situation, where sovereign decisions are made.

The very high value of v_{12} reflects the real income differences between the US and the SU. As soon as the optimum is defined by optimal welfare, income redistribution is the answer. In reality this will not be claimed by the SU; rather they will try to raise their own productivity. A much more modest amount of security aid may be a matter for negotiation, as a *quid pro quo* for some concession, and in the form of the supply of grain or of high technology.

2.4 EAST-WEST MODELS: PARABOLIC WELFARE

We are now going to introduce the parabolic utility function, in order to present the possibility of satiation or even oversatiation. This utility function for one determinant y_1 (non-military expenditure of World 1) takes the form:

$$\omega_1 = \omega_o - \frac{\omega_o}{y_{o1}^2}(y_1 - y_{o1})^2 \tag{2.41}$$

where ω_o is the maximum level of satisfaction, attained for $y_1 = y_{o1}$, the level of expenditure where satiation has been attained. The first and the second derivatives are:

$$d\omega_1/dy_1 = -\frac{2\omega_o}{y_{o1}^2}(y_1 - y_{o1}) \text{ and}$$

$$d^2\omega_1/dy_1^2 = -\frac{2\omega_o}{y_{o1}^2}$$

From these expressions we see that indeed for $y_1 = y_{o1}$ this utility function has a horizontal tangent and that that point constitutes a maximum of ω_1.

Total welfare will also depend on armament expenditures of the two worlds under consideration (NATO and WP or, occasionally, the US and the SU). As in Section 2.1, Problems I_1, I_2 and II will be formulated and solved. The new parameters y_{o1}, y_{o2}, a_{o1}, a_{o2} are supposed to be given, but have not, to my knowledge, actually been measured so far. Their estimation from public-opinion polls constitutes an interesting research programme. An additional assumption we shall make is that per person the satiation values of a_1 and a_2 are the same for both parties. We assume that each variable has one satiation value. The main purpose of the present

section is to find out what effect changes in satiation values and differences of the satiation values of y_1 and y_2 etc. have on the optimal values.

The data of our problems will first be listed, for NATO and WP (cf. Appendices)

Population: $n_2 = 0.65\, n_1$
'National' product $x_1 = 0.62 \times 3880 = 2406$; $x_2 = 1252$ (1975 US$bn)
Military expenditures $a_1 = (311/6156) \times 2406 = 122$; $a_2 = (127/2084) \times 1252 = 112$
Satiation values (where numerical assumptions are made): $y_{o1} = 6000$, meaning that NATO citizens don't want more non-military expenditures than 1975 US $6 trillion:

$$y_{o2} = 0.65 \times 6000 = 3900;\ a_{o1} = 400;\ a_{o2} = 0.65 \times 400 = 260$$

Population figures are for 1982, from Leger Sivard (1985).
Ratio 0.62 of NATO and all Developed countries' GNP from ibid.
Absolute figures 3880 and 1252: from Kravis (1982) (last figure given for WP).
Ratios a/x : from Leger Sivard (1985) in 1982.

Problem I_1 assumes that NATO maximizes

$$\omega_1 = \omega_o - \frac{\omega_0}{y_{0\,1}^2}(y_1 - y_{01})^2 + \alpha_{11}\,\omega_0 - \frac{\alpha_{11}\,\omega_o}{a_{01}^2}(a_1 - a_{01})^2$$

subject to restriction (R1) : $x_1 = y_1 + a_1$
This requires $(y_1 - y_{o1})/y_{o1}^2 = \alpha_{11}\,(a_1 - a_{o1})/a_{01}^2$
Substituting the data, we get $\alpha_{11} = 0.059$
Similarly the formulation of Problem I_2 yields: $\alpha_{22} = 0.043$.
Problem II – optimizing welfare of NATO and WP together – requires the maximization of:

$$\Omega = v - \frac{\omega_o}{y_{o\,1}^2}(y_1 - y_{o1})^2 - \frac{\alpha'_{11}\,\omega_o}{a_{o1}^2}(a_1 - a_{o1})^2$$

$$+ \frac{\alpha'_{12}\,\omega_o}{a_{o2}^2}(a_2 - a_{o2})^2 -$$

$$0.65\left\{\frac{\omega_o}{y_{o\,2}^2}(y_2 - y_{o2})^2 - \frac{\alpha'_{22}\,\omega_o}{a_{o2}^2}(a_2 - a_{o2})^2 + \right.$$

$$\left. \frac{\alpha'_{21}\,\omega_o}{a_{o1}^2}(a_1 - a_{o1})^2\right\} + \tag{2.42}$$

$$+ \lambda_1\,(2406 - y_1 - a_1 - v_{12}) + \lambda_2\,(1252 - y_2 - a_2 + v_{12})$$

where v_{12} constitutes the 'security aid' transferred by NATO to WP.

The optimum conditions will be derived, first of all, from the assumption of a horizontal-tangent maximum, that is, by assuming that the first derivatives of Ω with regard to the unknown variables y_1, a_1, y_2, a_2 and v_{12} are zero. After elimination of the λ_1 and λ_2 and omitting the common factor ω_o, the conditions are:

$$\frac{2\,(y_1 - y_{o1})}{y_{o2}^2} = \frac{1.3\,(y_2 - y_{o2})}{y_{o2}^2} \tag{2.43}$$

$$\frac{2\,(y_1 - y_{o1})}{y_{o2}^2} = \frac{2\alpha'_{11}\,(b_1 - b_{o1})}{y_{o1}^2} - \frac{1.3\,\alpha'_{21}\,(a_1 - a_{o1})}{y_{o1}^2} \tag{2.44}$$

$$\frac{1.3\,(y_2 - y_{o2})}{y_{o2}^2} = \frac{1.3\alpha'_{22}\,(a_2 - a_{o2})}{y_{o2}^2} - \frac{2\alpha'_{12}\,(a_2 - a_{o2})}{y_{o2}^2} \tag{2.45}$$

For convenience's sake we introduce the positive variables:

$$z_1 = y_{o1} - y_1 \qquad z_2 = y_{o2} - y_2$$
$$b_1 = a_{o1} - a_1 \qquad b_2 = a_{o2} - a_2$$

This transforms equations (2.43), (2.44) and (2.45) into:

$$2\,z_1/y_{o1}^2 = 1.3\,z_2/y_{o2}^2 \tag{2.43'}$$

$$2\,z_1/y_{o1}^2 = \alpha_1\,b_1/a_{o1}^2 \text{ where } \alpha_1 = 2\alpha'_{11} - 1.3\alpha'_{21}$$
$$= 0.7\alpha'_{11} \tag{2.44'}$$

$$1.3\,z_2/y_{o2}^2 = \alpha_2\,b_2/a_{o2}^2 \text{ where } \alpha_2 = 1.3\,\alpha'_{22} - 2\,\alpha'_{22} \tag{2.45'}$$

In the last two equations we assumed that $\alpha'_{21} = \alpha'_{11}$ and $\alpha'_{12} = \alpha'_{22}$, which seems plausible for offensive weapons, but should be a matter for empirical research. In order to have the simplest set of solvable equations we add up the two restrictions

$$2406 = y_1 + a_1 + v_{12} \tag{R1}$$
$$1252 = y_2 + a_2 - v_{12} \tag{R2}$$

and obtain $3658 = y_1 + y_2 + a_1 + a_2$ (R12)

which leaves us with 4 unknowns y_1, y_2, a_1 and a_2 and 4 equations (2.43), (2.44), (2.45) and (R12). As a numerical illustration we choose the

satiation values $y_{o1} = 0.65 \times 6000 = 3900$; $a_{o1} = 400$; $a_{o2} = 0.65 \times 400 = 260$.

The numerical solutions are (rounded):

$$z_1 = 3965 \quad z_2 = 2587 \qquad b_1 = 855 \qquad b_2 = 495$$

with which correspond:

$$y_1 = 2035 \quad y_2 = 1393 \qquad a_1 = -455 \qquad a_2 = 755$$

For each unknown an 'admissable' interval exists . Assuming rational behaviour, we must require the y and a to be smaller than or equal to their satiation values, and larger than the observed values for y_1 and y_2.

For the armament figures in the future we may assume figures down to zero (disarmament). So:

$$2284 \leq y_1 \leq 6000; \quad 1140 \leq y_2 \leq 3900; \qquad 0 \leq a_1 \leq 400$$
$$\text{and } 0 \leq a_2 \leq 260.$$

Consequently the solutions for y_1, a_1 and a_2 are inadmissable. The assumptions that the maxima with regard to the a_i ($i = 1, 2$) are horizontal ($\partial\Omega/\partial a_i = 0$; $i = 1, 2$) don't apply. Assuming instead that $a_1 = 0$ and maintaining $\partial\Omega/\partial a_2 = 0$ yields $a_2 = -272$, which leads us to the view that $a_1 = a_2 = 0$ yields the admissable maximum Ω. With the satiation values mentioned *complete disarmament* yields maximum welfare-cum-security.

The solutions for the other variables appear to be

$$y_1 = 2217; \quad y_2 = 1441 \qquad \text{and } v_{12} = 189$$

This means that a high security aid be paid by NATO to WP, constituting 7.86 per cent of NATO income 2406. This is partly compensated for by the reduction 122 of armament expenditure. Apart from the increased security a financial loss remains of 189 – 122 = 67 or 2.8 per cent. For WP countries a financial gain of 189 + 112 = 301 alongside higher security is the result.

More relevant is, of course, the welfare-cum-security gain. This can be calculated from our utility functions and appears to be 0.08351, with a gain for the WP of 0.10471 and a loss for NATO of 0.02120. Here again the overall optimum appears to imply a gain for WP versus a loss for NATO. The explanation is that, as a consequence of its lower real income *per capita* the WP citizens' marginal utility is higher than NATO's. Our remark at the end of Section 2.3 also applies here.

The optimum as here defined also implies an element of redistribution between East and West. This constitutes an element not so far discussed by either East or West. The higher real income *per capita* in Western compared with Eastern developed countries is due to a variety of causes. Partly it may be due to better natural resources available in the West, partly

to a more productive social system. The reforms now under way in the East may raise the productivity.

Depending on some of the data of our example, solutions are also possible which fall inside the admissible intervals of the variables. This will now be shown with the aid of another example.

Instead of the values for the coefficients α_1 and α_2 in equations (2.24') and (2.25') we assume $\alpha_1 = \alpha_2 = 0.1$. Whether such values are realistic can only be judged after a closer study has been made of α_{12} and α_{21}, the coefficients indicating the impact of one side's armament on the other side's security. For defensive weapons this impact may be very different from the impact of offensive weapons. Assuming further $y_{o1} = 2300$, $y_{o2} = 1200$, $a_{01} = a_{02} = 125$, the four equations for y_1, y_2, a_1 and a_2 become, similar to (2.43), (2.44), (2.45) and R12:

$$y_1 - 2300 = 2.3878\ (y_2 - 1200) \qquad (2.43')$$
$$y_1 - 2300 = 0.1\ (a_1 - 125) \qquad (2.44')$$
$$y_2 - 1200 = 0.1\ (a_2 - 125) \qquad (2.45')$$
$$y_1 + y_2 + a_1 + a_2 = 3658 \qquad (R12)$$

The solution of the optimum problem now runs:

$$y_1 = 22941.1; \qquad y_2 = 1197.5; \quad a_1 = 66.0 \text{ and } a_2 = 100.3$$

where the values for y_1 and y_2 are within the admissible intervals mentioned and those of the a_i ($i = 1,2$) positive and below the initial figures 122 and 112. From these figures the amount of 'disarmament aid' $v_{12} = 45.9$ follows, a much more realistic figure (1.9 per cent of NATO's national income).

2.5 NORTH–SOUTH MODELS: LOGARITHMIC WELFARE

North–South models are in a way simpler than East–West models. Only one type of good needs to be considered and the main problem is whether a flow of development aid should be directed from North to South, that is, in our notation, from W1 to W3. In some other, economic and non-economic, respects, the problems are more complicated, since the distribution over industries and the sociological differences are much larger. Thanks to the pathbreaking work done by Kravis and his collaborators some of the main difficulties have been reduced. They consist of the large

price differences between developed and underdeveloped countries. These enlarge the difference between official parities of currencies and their buying parities. For a comparison of welfare the latter are much more appropriate.

The simplest form of the problem of the optimal level of development aid will be discussed with the aid of the following model, where, first, we use logarithmic utility functions. We consider two 'worlds', W1 and W3 which together are the non-communist world. The data used are Kravis's figures for these worlds' real incomes, expressed in 'milliards' (10^9 called 'billions' by Americans) of US dollars with 1975 buying power: $x_1 = 3880$ and $x_3 = 1999$. Unknowns of the problem are the 'development aid' or 'financial transfers' from W1 to W3, v_{13}, and the total expenditure for these two worlds own consumption and investment, indicated as y_1 and y_3 respectively. The optimum values of the unknowns must satisfy the restrictions

$$x_1 = y_1 + v_{13} \qquad \text{(R1)}$$

and

$$x_3 = y_3 - v_{13} \qquad \text{(R3)}$$

and maximize:

$$\Omega = \ln(y_1 + 1) + 2.85 \ln(y_3 + 2.85) + \lambda_1(x_1 - y_1 - v_{13}) + \lambda_2(x_3 - y_3 + v_{13})$$

where 2.85 is the population ratio of W3 to W1. Applying the Lagrange method and eliminating the Lagrangian multipliers λ_1 and λ_2 we are left with:

(2.51)

$$2.85\,(y_1 + 1) = y_3 + 2.85 \text{ or } 2.85\, y_1 = y_3 \qquad (2.52)$$

Equations (R1), (R3) and (2.52) yield the unknowns:

$$y_1 = 1527; \quad y_3 = 4352; \quad v_{13} = 2353 \text{ (or 60.6 per cent of } x_1)$$

Here we are faced with the very high figures which we also found, in Section 2.2, for security aid. In fact, the value found here for optimal development aid is even much higher still. Apart from reflecting the enormous welfare gap between W1 and W3 the figure also illustrates the long-term character of the elimination of that gap. Figures due to Summers *et al.* (1984) show that world income inequality hardly changed between 1950 and 1980. Between 1960 and 1979 total income x_1 showed an annual

increase of 4.15 per cent and x_3 one of 5.76 per cent. In order to fulfil the optimum condition (2.52) x_1/x_3 should become 0.351. Today it is 1.94. It would take 112 years. This is another way of saying that even the next generation of inhabitants of developing countries will experience hardly any improvement of their living conditions in comparison to those in developed countries.

Applying the method set out in Section 2.2 we may also illustrate our point as follows. In order that development aid be given without reducing the standard of living of the donor countries we can make available the increase in national product which is 4.15 per cent per annum minus the rate of increase of population in the developed countries which over the last decades (1960–83) amounted to 0.91 per cent. The percentage of national income annually available for development aid will then be 3.24 per cent, or, in the first year, 126. This amount will increase by 3.24 per cent and reach the optimal amount of 60.6 per cent in 60.6/4.15 = 14.6 years. The figures in Table 2.51 give some more details.

Table 2.51 Increasing development aid while maintaining the standard of living of the developed countries

Year		0	1	2	3	4	...	14
Production of W1 (x_1)		3880	4041	4209	4383	4518	...	6856
Needed by W1		3866	3901	3937	3973	4009	...	4389
Available for develop. aid	absolute	14	105	199	297	399	...	2467
	% of 3880	0.36	3.61	7.01	10.6	13.1	...	63.6

Judging these figures against the background of today's power distribution, most readers will be unable to accept them as realistic. But power relations will not remain what they are now. Moreover, the problem we are studying is the problem of what income distribution will maximize welfare and, as observed before, so eliminate some of the tensions that would develop if inequality of world income distribution remains what it is today.

2.6 NORTH–SOUTH MODELS: PARABOLIC WELFARE

Now we take up the problem of optimum development aid using parabolic utility functions. The latter introduce the satiation levels of y_1 and y_3, y_{o1} and y_{o3}, which introduce an additional problem, namely the possibility to introduce the latter and, in particular, y_{o1} as an instrument of policy instead of an independent variable. We propose to use an 'integrated approach', meaning that we combine the original problem with the use of this new instrument. This 'integrated approach' should not be mixed up with other forms of integration, such as integration of economic and security policy or integrated territories (such as the European Community).

Our problem contains four unknowns, y_1, y_3, y_{01}, y_{03} and after having calculated these we are able to determine development aid v_{13}. The data we use are the population ratio $p_3/p_1 = 2.85$ and real products $x_1 = 3880$ and $x_3 = 1999$.

We want to maximize

$$\Omega = -\frac{(y_{o1} - y_1)^2}{y_{o1}^2} - 2.85\,\frac{(y_{o3} - y_3)^2}{y_{o3}^2} \tag{2.61}$$

under the restriction (R12) $y_1 + y_3 = x_1 + x_3 = 5879$ (2.62)

Clearly Ω will be a maximum when

$$y_{o1} = y_1 \tag{2.63}$$

and

$$y_{o3} = y_3 \tag{2.64}$$

Since we have for the four unknowns only three equations, we have one degree of freedom. This may be used in different ways. One way consists of maximizing instantaneous welfare of the whole 'world' considered, that is W1 + W3,and assuming the same utility function for W1 and W3. It brings us back to equal *per capita* spendable income:

$$y_3/y_1 = 2.85 \tag{2.65}$$

with the same result as found in Section 2.5 and the same interpretation.

The degree of freedom may also be used to apply an alternative criterion, for instance, that the ratio between y_3 and y_1 or of the *per capita*

y_3 and y_1 which is now about constant, double in a period of twenty years. Such doubling requires an annual increase by 3.5 per cent or

$$y_3/y_1 = (1999/3880) \times 1.035 = 0.5332 \tag{2.66}$$

This yields $y_1 = 3834.5$; $y_3 = 2044.5$ and $v_{13} = 45.5$ or 1.17 per cent of x_1. Also here, as in Table 2.6, the continuation of the process requires rising amounts of development aid up to the level of 60 per cent of the initial x_1; the only consolation being that, as a percentage of x_1, after 20 years this is 27 per cent.

2.7 THE NEED FOR INTEGRATED MODELS

In this chapter, the problems of disarmament and of development have been treated separately, as if there were no interrelation – an opinion adhered to by the American government around 1986. These separate analyses of the two problems had the advantage that relatively simple models could be used, especially because of the macro-character of these models. We introduced security aid alongside development aid as a useful concept and we introduced two alternative utility functions. We also introduced two types of armament, offensive and defensive. Notwithstanding their pluriform attempts to show the main features of macro-reality, our studies showed some common characteristics worth being highlighted. The most striking result is that the optimal values we found for both security aid and development aid are so much higher than the amounts usually discussed among politicians.

It appeared possible to explain this divergency to a considerable extent. But explaining it does not mean that the values currently discussed are acceptable. They are not. If the world income distribution remains as unequal as it is today, the migration of citizens of the Third World to the First World will drastically increase and cause increasing tensions. We must get accustomed to higher, much higher, figures of development aid if we want to avoid these tensions.

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