7. Dynamic models of development co-operation

7.1 INTRODUCTION

So far we only dealt with static models. In static models all variables are supposed to be constant ('stationary') or to be moved by exogenous forces to which the endogenous variables adapt themselves instantaneously. Their values are optimal in one of the senses discussed, that is, either to maximize world welfare or to attain a given lower level of income inequality. A third alternative sense of optimality will be proposed and discussed in this chapter (see Section 7.6). The welfare functions were chosen differently: we used logarithmic welfare functions which assume absence of satiation and two types of parabolic welfare functions in which satiation occurs.

In dynamic models variables are also changing over time when exogenous variables are constant and such movements of endogenous variables are called endogenous movements. Such movements may be 'cyclical', i.e. change direction in successive time periods, or 'one-sided', i.e. either rise or fall over the whole future. These one-sided movements may be directed towards some equilibrium or away from an equilibrium. Accordingly the equilibrium is called stable in the first case and unstable in the second case. Static models make sense only if the constant value constitutes a stable equilibrium, or if the exogenous movements are the result of an immediate adaptation to a stable equilibrium. Optimality, when defined as a maximum of welfare, in a dynamic model may refer to welfare over a finite period or to welfare over an infinitely long period. Welfare over a period will be considered as the sum of welfare in each time unit (e.g. years) of the period. If an infinitely long period is considered this sum is one of an infinite number of terms. It makes sense only if this sum is a finite figure. This requires that the series of terms is converging by a sufficiently strong decrease of the successive terms. This implies that the successive welfare figures are discounted. The question then arises what rate of discount of future welfare must be chosen on the basis of which criteria. Considering welfare over a finite period in fact also constitutes a form of discounting.
welfare in time units after that period are considered to be irrelevant. One argument in favour of such irrelevance may be uncertainty; another argument may be the finite lifetime of the product about which a decision has to be made or of the decision-makers. The problem of discounting constitutes a well-known and difficult problem which has been dealt with by, among many others, T. C. Koopmans (1970) and M. Inagaki (1970).

In this book only a few of the simplest cases will be analysed. The simplest case evidently is the maximization of welfare in one time unit (for which we take one year) following the time unit in which the decision is made, to be indicated by \( t = 1 \) and \( t = 0 \), respectively. This subject will be dealt with theoretically in Section 7.2.

In Section 7.3 concrete figures will be given for all concepts discussed in Section 7.2. In Section 7.4 the time unit chosen will be five or ten years instead of one. In Section 7.5 welfare in more than one time unit will be treated.

In the last section of this chapter an additional, hence a third, alternative criterion of optimality will be discussed that requires a dynamic model and therefore could not be discussed earlier.

7.2. A ONE-YEAR FUTURE: VARIABLES AND RELATIONS CONSIDERED

A dynamic treatment of our problems requires the distinction of values of the variables in different time units. The time unit considered will be indicated by a suffix \( t \); \( x \) meaning ‘national’ income in year \( t \). The world considered (W1 or W3) will now be indicated by an upper index: \( \chi_1^t \) and \( \chi_2^t \) respectively for ‘national’ income of W1 and W3. As before, we shall also consider \( y_i^t \) \((i = 1, 2)\) to be defined a bit more specifically, namely as consumption expenditure. Since trade between W1 and W3 will not be considered, net exports – which should be part of \( y_i^t \) – are neglected. An essential new variable is \( z_i^t \), gross investment of a world’s own resources, hence not of another world’s resources. These are financed, in W1, by that world’s own savings, which are expressed as a portion \( s^1 \) of \( y_1^t \). In W3 there are two sources of financing the investments: (i) that world’s own savings \( s^3 y_3^t \) and (ii) a portion \( d \) of \( x_1^t \), W1 ‘national’ income. In both worlds two relations exist between the variables enumerated. First, a spending equation that for W1 runs:

\[
x_i^t (1 - d) = y_i^t (1 + s^1)
\]  

(7.21)
The left-hand side are the resources available after \( dx^1_t \) has been made available for development assistance to W3. The right-hand side constitutes consumption \( y^1_t \) plus savings \( s^1 y^1_t \).

For W3 the spending equation is:

\[
x^3_t + dx^1_t = y^3_t + z^3_t + dx^1_t
\]  

(7.22)

The left-hand side are the resources available and the right-hand side contains: (i) consumption \( y^2_t \), (ii) investments financed by W3's own savings, and (iii) investments financed out of development assistance.

The second relation expresses that the increase of production is the result of investments. It equals the product of these investments (the capital input \( z^3_{t-1} \) and the output-capital ratio \( b \) (which is the inverse of the capital-output ratio \( 1/b \)) and the effect of technological development \( cx^1_{t-1} \). The gestation period is assumed to be one year. So for W1 we have:

\[
x^1_t - x^1_t = b^1 z^1_{t-1} + cx^1_{t-1} = b^1 s^1 y^1_{t-1} + cx^1_{t-1}
\]

which can be written:

\[
x^1_t = \left\{ (1 + c) + b^1 s^1 (1 - d)/(1 + s^1) \right\} x^1_{t-1} = B x^1_{t-1}
\]  

(7.23)

For W3 we have:

\[
x^3_t = x^3_{t-1} + b^3 \left\{ s^3/(1 + s^3) + c \right\} x^3_{t-1} + b^3 dx^1_{t-1} = (1 + s^3) (dx^3_{t-1} + Ex^1_{t-1})
\]  

(7.24)

For the welfare functions we need the \( y^i_t \) as arguments. They are simple functions of the \( x^i_t \) \((i = 1, 2)\):

\[
y^1_t = x^1_t (1 - d)/(1 + s^1) = C x^1_t
\]  

(7.25)

\[
y^2_t = x^3_t/(1 + s^3) = D x^3_t + Ex^1_{t-1}
\]  

(7.26)

For the reader's convenience we list the capital-letter symbols introduced:

\[
B = 1 + c + b^t s^1 (1 - d)/(1 + s^1)
\]  

(7.27)

\[
C = (1 - d)/(1 + s^1)
\]  

(7.28)

\[
D = (1 + c)/(1 + s^3) + s^3 b^t s^1/(1 + s^3)^2
\]  

(7.29)
\[ E = b^t d/(1 + s^d) \]  

(7.210)

The relations enumerated enable us to derive from the initial values \(x_0^1\) and \(x_0^2\) the development over time of all variables, if we know the numerical value of all coefficients \(b^t, b^s, s^d\) and \(d\). In addition we may solve our main problem, where targets for \(X_2^1\) or \(Y_2^2\) are set and \(d\) is the unknown rate of development assistance.

7.3 ESTIMATES OF THE COEFFICIENTS

INTRODUCED AND THE INITIAL INCOMES

In this section estimates will be presented of the values of the coefficients introduced in Section 7.2. We assume that these coefficients tend to a constant value which we want to know for 1970 as our ‘initial year’, the year in which the Pearson Commission and the UN Development Planning Commission reported and the UN General Assembly discussed the Second Development Decade (DD II). In order to eliminate random fluctuations in the coefficients’ values we considered the period 1960–79 for which important sources were available to us, to be mentioned below.

From the World Bank’s *World Development Report 1981* (pp. 136–7) we took the average annual production growth percentages shown below:

<table>
<thead>
<tr>
<th></th>
<th>1960–70</th>
<th>1970–9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low income countries</td>
<td>4.5</td>
<td>4.7</td>
</tr>
<tr>
<td>Middle income countries</td>
<td>6.1</td>
<td>5.5</td>
</tr>
<tr>
<td>Weighted averages (W3)</td>
<td>5.86</td>
<td>5.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(weights 1 and 5.6, ratio of 1979 incomes)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-comm. industrial countries (W1)</td>
<td>5.1</td>
<td>3.2</td>
</tr>
<tr>
<td>Nor weighted average of 2 periods (W3)</td>
<td>5.62</td>
<td>so growth factor: 1.0562</td>
</tr>
<tr>
<td>(W1)</td>
<td>4.15</td>
<td>1.0415</td>
</tr>
</tbody>
</table>

Alternative source Kravis et al. (1982) yields somewhat rounded growth factors for W3 and W1 of 1.0575 and 1.0430, respectively, meaning a difference between W3 and W1 of 0.0145 as compared to 0.0147 for our
Dynamic Models of Development Co-operation

figures. Technological progress, measured by its impact on the annual growth factor of production, was taken from Denison (1967), p. 281, where this impact was measured as a residual for nine countries. A minimum figure of 0.76 per cent and a maximum of 1.56 are mentioned. We made alternative calculations using \( c = 1.0076 \) and \( c = 1.0150 \).

The initial (i.e. 1970) value of \( d \) was calculated as
\[
\nu_0^{1.3} / x_0^1 = 15.95 / 2472
\]
where 15.95 is taken from the OECD (1988) p. 198, and constitutes the total net flow of financial resources in milliards ('billions' in American terminology) for 1970 in 1970 prices. Since Kravis's (1978) figure of \( x_0^1 = 2472 \) is measured in 1975 prices, a correction had to be made for inflation between 1970 and 1975. On p. 343 of Kravis, real income per capita of industrial market economies in 1970 is indicated to be 5210 in 1975 prices and 3735 in 1970 prices. The inflation over the five years is 1.395. Multiplying 15.95 by that factor yields an initial value \( d_0 = 0.0090 \). Savings rates in terms of \( x \) were taken from World Bank (1981) pp. 142–3. For 1970 low-income countries saved 23 per cent and middle-income countries 25 per cent; the weighted average is 24.7 per cent. Industrial countries saved 22 per cent. In terms of \( y \) these rates become \( s^2 = 0.33 \) and \( s^1 = 0.28 \).

For W1 we are now able to estimate \( b^1 \), the output–capital ratio, with the aid of equation (7.23); we find \( b^1 = 0.1562 \) if \( c = 0.0076 \); the capital–output ratio follows: \( 1/b^1 = 6.40 \) years. Similarly we derive \( b^2 \) from (7.24) and obtain \( b^2 = 0.1777 \) and \( 1/b^2 = 5.63 \) years. Initial income of W1 was mentioned above to be 2472 milliards of 1975 international $; from the same source (Kravis et al., 1978) we get \( x^1 = 872 \). We conclude this section with some figures on population growth, taken from World Bank (1981).

On pp. 134–5 we find:

<table>
<thead>
<tr>
<th>Population growth per annum, per cent</th>
<th>1960–70</th>
<th>1970–9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-income countries</td>
<td>2.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Middle-income countries</td>
<td>2.5</td>
<td>2.4</td>
</tr>
<tr>
<td>Weighted average, based on population</td>
<td></td>
<td></td>
</tr>
<tr>
<td>in 1979, 2260 and 985 respectively</td>
<td>2.3</td>
<td>2.2</td>
</tr>
<tr>
<td>Industrial market economies</td>
<td>1.0</td>
<td>0.7</td>
</tr>
</tbody>
</table>

For the period 1960–79 the population growth rate is 1.0085 for W1 and 1.0225 for W3.

Now we are able to present the dynamic equations (7.23) to (7.26) inclusive in numerical form with \( d \) as an unknown:
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\[ x_1^1 = (1.0418 - 0.03417 \, d) \, x_{t-1}^1 \]  
\[ x_2^3 = 1.0516 x_{t-1}^3 + 0.1777 d x_{t-1}^1 \]  
\[ y_1^1 = 0.781 (1 - d) x_t^1 \]  
\[ y_2^3 = 0.7907 x_{t-1}^3 + 0.1336 d x_{t-1}^3 \]  

(7.23) \hspace{1cm} \hspace{1cm} (7.24) \hspace{1cm} \hspace{1cm} (7.25) \hspace{1cm} \hspace{1cm} (7.26)

With their aid we can numerically extrapolate the development of our main variables for alternative values of development assistance as a portion \(d\) of W1 income \(x^1\). Our main application of this extrapolation will be discussed in Section 7.6. To begin with, we may check the equations by the substitution of the actual value of \(d = 0.009\). For \(x^1\) and \(x^3\) we find \(x_1^1 x_0^1 = 1.0415\), not different from the observed value 1.0415, and \(x_1^3 = 917 + 4 = 921 = 1.0562 \times 872\), identical to the observed 1.0562. The check appears to be satisfactory.

A second preliminary application is to show the rates of growth \(x^1\) and \(x^3\) of \(x^1\) and \(x^3\) for some arbitrarily chosen values of \(d\):

<table>
<thead>
<tr>
<th>(d)</th>
<th>(x^1)</th>
<th>(x^3)</th>
<th>(x^3 - x^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>1.0411</td>
<td>1.0617</td>
<td>0.0206</td>
</tr>
<tr>
<td>0.05</td>
<td>1.0401</td>
<td>1.0768</td>
<td>0.0367</td>
</tr>
<tr>
<td>0.10</td>
<td>1.0384</td>
<td>1.1020</td>
<td>0.0636</td>
</tr>
</tbody>
</table>

The last column shows the velocity with which production of W3 out-takes W1. So far, we based our figures on the value of \(c = 0.076\), a minimum value. Now we take \(c = 0.015\), a maximum. This changes also the values of \(b^1\) and \(b^3\). These now become \(b^1 = 0.1221\) and \(b^3 = 0.1506\) respectively. The capital–output ratio \(1/b^1 = 8.19\) and \(1/b^3 = 6.64\), admittedly quite high. The numerical forms of our fundamental four equations (7.23) to (7.26) inclusive now become:

\[ x_1^1 = (1.0417 - 0.02671 \, d) \, x_{t-1}^1 \]  
\[ x_2^3 = 1.0524 x_{t-1}^3 + 0.1506 d x_{t-1}^1 \]  
\[ y_1^1 = 0.781 (1 - d) x_t^1 \]  
\[ y_2^3 = 0.9498 x_{t-1}^3 + 0.1132 d x_{t-1}^3 \]  

(7.23') \hspace{1cm} \hspace{1cm} (7.24') \hspace{1cm} \hspace{1cm} (7.25') \hspace{1cm} \hspace{1cm} (7.26')

Substitution of the initial value \(d = 0.009\) transforms equations (7.23') and (7.24') into \(x_{t-1}^1 = 1.0415 \, x_{t-1}^1\) and \(x_{t-1}^3 = 1.0562 \, x_{t-1}^3\) as observed. Substitution of some other values of \(d\) yields:
\begin{tabular}{lccc}
\hline
\textit{d} & \textit{x} & \textit{x'} & \textit{x}^{3} - \textit{x'}^{1} \\
\hline
0.02 & 1.0609 & 1.0412 & 0.0197 \\
0.05 & 1.0737 & 1.0404 & 0.0333 \\
0.10 & 1.0951 & 1.0390 & 0.0912 \\
\hline
\end{tabular}

7.4 \textbf{GESTATION PERIODS OF FIVE AND TEN YEARS}

In the dynamic model so far used the meaning of the time unit (of one year) is that it constitutes the length of the gestation period: investments made in year \(t-1\) raise production in year \(t\) by the amount indicated as the output–capital ratio. (As is well known, the term ratio is misleading: the dimension of the capital–output ratio is time and so far we have referred to it in years. The inverse output–capital ratio has the dimension of one over time.) A capital–output ratio of 6.40 years, as we found for W1, means that for an increase of production by one unit a quantity of capital is needed equal to the production of 6.40 years.

A gestation period of one year is rather low. It is realistic for the production of non-durable products such as grain, vegetables and some fruits. The gestation period of investment in durables is longer, depending on the lifetime of the object. Investment in a ship with a lifetime of twenty years has a gestation period of ten years if no technological development occurs to make the ship obsolete before its technical lifetime. Investment in coffee trees or in cattle also shows a gestation period depending on the economically useful lifetimes of the coffee trees or the cattle.

For these reasons we shall now adapt our dynamic model to average gestation periods of five and ten years. To start with, we take five years. Some of our data will change and some will not. No change will occur in pure ratios such as the savings rates \(r^i\) \((i = 1, 3)\) and the portion \(d\) of W1 income spent on development assistance. Incomes per time unit will change: initial incomes \(x_0^0\) and \(x_0^3\) become five times what they were before: 12,360 and 4,360 milliards of \$ with 1975 buying power. Growth rates per time unit must be raised to the fifth power: \(\hat{x}_3^3 = 1.0415^5 = 1.2255\) and \(\hat{x}_3^3 = 1.0562^5 = 1.3144\). The growth factor \(1 + c\) due to technological development must be changed into \(1.0076^5 \approx 1.0385\), and hence \(c = 0.03858\).

The capital–output ratios, whose dimension is time as we stated before, must be recalculated from equations (7.23) and (7.24), respectively,
applied to \( t = 1 \) and the results are \( 1/b^4 = 1.1597 \) or 5.8 years and \( 1/b^5 = 0.8915 \) or 4.5 years; both somewhat lower than our earlier figures and clearly somewhat more realistic.

As a consequence the main equations become:

\[
\begin{align*}
x_t^1 &= (1.2272 - 0.18863 \, d) \, x_{t-1}^1 \quad \text{(7.43)} \\
x_t^2 &= (1.3168 + 3.1790 \, d) \, x_{t-1}^2 \quad \text{(7.44)} \\
y_t^1 &= 0.781 \, (1 - d) \, x_t^1 = (0.9584 - 0.1473 \, d) \, (1 - d) \, x_{t-1}^1 \quad \text{(7.45)} \\
y_t^3 &= (0.9901 + 2.3909 \, d) \, x_{t-1}^3 \\end{align*}
\]

With the aid of (7.43) and (7.44) the growth rates of \( x^3 \) and \( x^1 \) and the rates of overtaking over five years may be calculated for some values of \( d \):

<table>
<thead>
<tr>
<th>( d )</th>
<th>( x^3 )</th>
<th>( x^1 )</th>
<th>( x^3 - x^1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>1.3804</td>
<td>1.2234</td>
<td>0.1570</td>
</tr>
<tr>
<td>0.05</td>
<td>1.4758</td>
<td>1.2178</td>
<td>0.2580</td>
</tr>
<tr>
<td>0.10</td>
<td>1.6348</td>
<td>1.2083</td>
<td>0.4265</td>
</tr>
</tbody>
</table>

Subsequently we consider the figures for a gestation period of ten years. Incomes for \( t = 0 \) now become 24720 and 8720 for W1 and W3 respectively. Growth rates now are the tenth power of the annual growth rates (and the squares of those for a five-year gestation period). We find:

\( x_0^1 = 1.5017 \), \( x_0^3 = 1.7277 \) and \( e = 0.07865 \). The output–capital and capital–output 'ratios' now turn out to be \( b^1 = 1.9515 \) and \( 1/b^1 = 0.5124 \), \( b^3 = 2.6396 \) and \( 1/b^3 = 0.3788 \) and so capital–output 'ratios' become respectively 5.1 and 3.8 years, again more realistic than those found for a five-year gestation period. The main equations change into:

\[
\begin{align*}
x_t^1 &= \{1.07865 + 0.4269 \, (1 - d)\} \, x_{t-1}^1 \quad \text{(7.43')}
\\x_t^2 &= (1.7335 + 7.4832d) \, x_{t-1}^2 \quad \text{(7.44')}
\\y_t^1 &= 0.781 \, (1 - d) \, x_t^1 = \{0.8424 + 0.334 \, (1 - d)\} \, (1 - d) \, x_{t-1}^1 \\
y_t^3 &= (1.3034 + 5.6264d) \, x_{t-1}^3 \\end{align*}
\]

With the aid of (7.43') and (7.44') we again illustrate the impact of \( d \) on the growth rates \( x^3 \) and \( x^1 \) and their difference:
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<table>
<thead>
<tr>
<th>$d$</th>
<th>$x^3$</th>
<th>$x$</th>
<th>$x^3 - x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>1.8832</td>
<td>1.4970</td>
<td>0.3862</td>
</tr>
<tr>
<td>0.05</td>
<td>2.1077</td>
<td>1.4842</td>
<td>0.6235</td>
</tr>
<tr>
<td>0.10</td>
<td>2.4818</td>
<td>1.4629</td>
<td>1.0189</td>
</tr>
</tbody>
</table>

These differences illustrate the core of our problem, which is how to reduce the income differences between the Third and the First Worlds, to which a possible alternative answer will be given in Section 7.6 as announced before. First, however, in Section 7.5 we complete our set of relations with those needed to estimate welfare over a period longer than one time unit.

7.5 WELFARE OVER A PERIOD LONGER THAN ONE TIME UNIT

In this section, welfare over more than one time unit (i.e. gestation period) will be calculated. We do so in order to illustrate that, with the aid of the relations elaborated, a more satisfactory theory of optimal development can be achieved, based on our first criterion of optimality, maximal ‘world’ (W1 + W3) welfare. Since only an illustration is aimed at, we choose a period of two time units of five years each, and show numerical results for time units of one year and a decade respectively.

Welfare over two successive time units of five years for $W1 + W3$ equals

$$\ln (y^4_2 + 1) + \ln (y^1_1 + 1) + 3.21 \ln (y^3_0 + 3.21) + 3.21 \ln (y^1_1 + 3.21).$$

In order to obtain its numerical value, we have to express them in terms of $x^1_0$ and $x^3_0$, of which the values are known and equal respectively to $12360$ and $4360$ 'billion' dollars with 1975 buying power in the USA. To that end, equations (7.23)–(7.26) inclusive will be applied with the coefficient values for five years as the time unit. These equations are:

\[
x^1_t = (1.2272 - 0.1886 \, d) \, x^{1}_{t-1} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \ quad
Applying them to $t = 1$ we find the following expressions in bn 1975 US$:

\[
\begin{align*}
    x^1_1 &= 15168 - 2331d \\
    y^1_0 &= 9656 \, (1 - d) \\
    y^1_1 &= (11846 - 1821d) \, (1 - d) \\
    x^2_1 &= 5742 + 10425d \\
    y^2_0 &= 3278 \\
    y^2_1 &= 4317 + 7838d
\end{align*}
\]

With their aid we obtain

\[
\Omega = \ln \left\{ 9656 \, (1 - d) + 1 \right\} + \ln \left\{ (11846 - 1821d)\, (1 - d) + 1 \right\} + 3.21 \left\{ \ln (3278 + 3) + \ln (4317 + 7838d + 3) \right\}
\]

In order to find the value of $d$ which maximises $\Omega$ we have to put

\[
\partial \Omega / \partial d = 0
\]

Elaboration and simplifying the fractions occurring in the result we find:

\[
\frac{-1}{1 - d} + \frac{-1 + 0.2665d}{0.8668 - d + 0.13d^2} + \frac{3.21}{0.5512 + d} = 0
\]

This equation is satisfied by $d = 0.37$.

In the same way the other two cases for time units $= 1$ and $10$ years have been calculated. In addition, three cases were dealt with where periods $t = 0$ and $1$ were replaced by periods $t = 1$ and $2$ (for W3 only) since the effects of development co-operation by W1 in period $t$ occur in period $t + 1$. In Table 7.51 the results for all cases described so far are shown, and for comparison purposes those for maximizing $\Omega$ over only one time unit are added.

A discussion of the results may start with summing up some of the features of the set of values found for $d$.

1. In the right-hand part, referring to a horizon of one time unit only, the two figures are very close. In the left-hand part they are much more spread and they are spread around the values in the right-hand part.

2. Values of $d$ are higher, the longer the gestation period.
Dynamic Models of Development Co-operation

Table 7.51 Values of d which maximize welfare in W1 + W3 for the periods specified. Logarithmic welfare functions used

<table>
<thead>
<tr>
<th>Gestation period</th>
<th>Horizon 2 gestation periods</th>
<th>1 gestation period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>For W1</td>
<td>For W3</td>
</tr>
<tr>
<td>One year</td>
<td>t = 0.1</td>
<td>t = 0.1</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Five years</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Ten years</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>1.2</td>
</tr>
</tbody>
</table>

(3) In the cases of identical horizons, printed in italics, longer horizons show lower values of d, but in the cases of non-identical horizons, longer horizons show higher values of d.

Next, some possible reasons for differences found between d values may be discussed. Two forces at work seem to be clear:

(1') If development co-operation investments take a short time (short gestation period) less aid (lower d) is needed.

(2') If these investments operate more 'intensively' (high output-capital ratio), less aid (lower d) is also needed. Since we found that, expressed in the same time units, b² is 0.1777, 0.2243 and 0.2640 respectively for a gestation period of 1, 5 and 10 years the 'intensity' of investment rises with the gestation period. The two forces work in the same direction and are (part of) an explanation of the two effects mentioned.

Thus, features (1) and (2) may be explained. It is less clear what may be the explanation of feature (3). On the one hand, identical horizons seem to be more natural, but, on the other hand, the effect of an investment financed in time unit t takes place in time unit t+1.

In order to complete the survey of our results, a comparison with those obtained from static models is presented in Table 7.52.

How must we interpret these results? Why are the d values found from the dynamic models, especially those assuming low gestation periods, so much lower than the values found for the higher gestation periods?
Table 7.52  Values for development assistance as a percentage of donor countries' income (100d) maximizing welfare of W1 + W3 found from static and dynamic models

<table>
<thead>
<tr>
<th>Utility functions</th>
<th>Static models with China in W2</th>
<th>Static models with China in W3</th>
<th>Dyn. models with gestat. period as W4</th>
<th>Dyn. models with gestat. period 1 yr</th>
<th>Dyn. models with gestat. period 5 yrs</th>
<th>Dyn. models with gestat. period 10 yrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logarithmic</td>
<td>50.6</td>
<td>76.4</td>
<td>66.7</td>
<td>Horizon one gestat. period</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parabolic I</td>
<td>50.1</td>
<td>75.6</td>
<td>60.5</td>
<td>25.5</td>
<td>64.0</td>
<td>69.5</td>
</tr>
<tr>
<td>Parabolic II</td>
<td>50.5</td>
<td>67.0</td>
<td>66.5</td>
<td>21.0</td>
<td>56.5</td>
<td>64.0</td>
</tr>
<tr>
<td>Source: Tables</td>
<td>3.21</td>
<td>3.41</td>
<td>3.22</td>
<td>3.42</td>
<td>4.41</td>
<td>7.51</td>
</tr>
</tbody>
</table>

Probably because the target they aim at is maximal welfare for a period in the initial years, when the total production ($x^1, x^2$) is low. Such modest aims evidently require relatively modest investments.

What we want, however, is maximum welfare over longer periods, and for a gestation period of 5 years and 10 years figures of some 60 per cent for $d$ are needed. This brings us in the neighbourhood of the figures found with the aid of statistical models, as shown by Table 7.52. These figures, as we already argued in Chapters 2, 3 and 4, are completely unrealistic. The aim set evidently cannot be attained in the course of the next decade. Before we make an alternative choice some more figures about the impact of $d$ are shown in Table 7.53. This table shows that growth rates of W3 production surpass those of W1 production, but only modestly for low $d$ values. For high values of $d$, W3 catches up with W1 much more quickly. For values of $d$ up to 0.10 there is a maximum at the 5 years gestation period. The figures for $\hat{x}^1$ and $\hat{x}^2$ differ from those given in Section 7.4, because the time units used there are 5 and 10 years, whereas in Table 7.53 time units are 1 year.

A compromise has to be found between the donor countries' willingness to make available more development assistance $dx^1$ and the interest of both W3 and W1 in reducing the differences in income and hence welfare to an extent that creates a sizeable perspective for W3 citizens. It is such a compromise that will be proposed and discussed in Section 7.6.
Table 7.53  Annual rates of growth $x^3$ and $x^1$ of national product of W3 and W1 for gestation periods of 1, 5 and 10 years and various values of $d$, the portion of $x^1$ made available for development co-operation

<table>
<thead>
<tr>
<th>$d$</th>
<th>$x^3$</th>
<th>$x^1$</th>
<th>$x^3 - x^1$</th>
<th>$x^3$</th>
<th>$x^1$</th>
<th>$x^3 - x^1$</th>
<th>$x^3$</th>
<th>$x^1$</th>
<th>$x^3 - x^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>1.0617</td>
<td>1.0411</td>
<td>0.0206</td>
<td>1.0412</td>
<td>0.0279</td>
<td>1.0652</td>
<td>1.0412</td>
<td>0.0241</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>1.0768</td>
<td>1.0401</td>
<td>0.0367</td>
<td>1.0402</td>
<td>0.0407</td>
<td>1.0774</td>
<td>1.0403</td>
<td>0.0371</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>1.1020</td>
<td>1.0384</td>
<td>0.0636</td>
<td>1.1011</td>
<td>0.0715</td>
<td>1.1092</td>
<td>1.0388</td>
<td>0.0664</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>1.1524</td>
<td>1.0530</td>
<td>0.1174</td>
<td>1.1432</td>
<td>0.1079</td>
<td>1.1244</td>
<td>1.0557</td>
<td>0.0887</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>1.3085</td>
<td>1.0247</td>
<td>0.2788</td>
<td>1.2379</td>
<td>0.1253</td>
<td>1.1853</td>
<td>1.0260</td>
<td>0.1593</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>1.4295</td>
<td>1.0162</td>
<td>0.4133</td>
<td>1.2992</td>
<td>0.1016</td>
<td>1.2207</td>
<td>1.0169</td>
<td>0.2038</td>
<td></td>
</tr>
</tbody>
</table>

7.6  A CONCRETE THIRD CRITERION OF OPTIMALITY OF DEVELOPMENT CO-OPERATION

The concepts of welfare and the question of its measurability are perhaps somewhat too abstract for political discussions, negotiation and decision-making. From the material collected and the calculations made we may derive some less abstract suggestions for another, our third, alternative. As set out earlier, the development during the three decades 1950–80 has been characterized by a virtually immobile world income distribution. From an article by Summers et al. (1984) we may derive the simplest evidence about world income distribution by taking the unweighted averages of the Gini coefficients calculated by these authors on the basis of alternative assumptions. These averages are shown in Table 7.61.

This stagnating large income inequality constitutes not only an ethically unacceptable situation, but also a threat to world political stability. Policy-makers are insufficiently aware of the accumulation of tensions as a consequence of this inequality. The only remedy is a change in development co-operation policy that clearly opens up a perspective of reduction of that inequality. During a sufficiently short period for the majority of today's world population to be still alive after its completion the policy target must be a sizeable improvement in incomes of the Third World compared with those of the First World. Its main, although not only, instrument must be development assistance. More concretely we propose
Table 7.61 Unweighted averages of eight estimates of world personal income distribution Gini coefficients

<table>
<thead>
<tr>
<th>Year</th>
<th>1950</th>
<th>1960</th>
<th>1970</th>
<th>1980</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>0.6205</td>
<td>0.6063</td>
<td>0.6176</td>
<td>0.6131</td>
</tr>
</tbody>
</table>

Source: Summers et al. (1984)

that in twenty years the relative incomes of W3 in comparison to W1—now about one third—should be doubled. Since doubling in twenty years of any variable requires an average annual increase at a rate of 3.52 per cent, we need, according to Table 7.53, a value of $d$ of about 5 per cent. More precise figures can be found in Table 7.62.

Table 7.62 Values of development assistance as a percentage of W1 income (100 d) required for doubling $x_i/x$, in 15, 20 or 30 years

<table>
<thead>
<tr>
<th>Doubling period (years):</th>
<th>15</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gestation period:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 year</td>
<td>7.3</td>
<td>5.0</td>
<td>4.1</td>
</tr>
<tr>
<td>5 years</td>
<td>6.7</td>
<td>4.2</td>
<td>1.8</td>
</tr>
<tr>
<td>10 years</td>
<td>8.0</td>
<td>4.9</td>
<td>2.0</td>
</tr>
</tbody>
</table>

The general conclusion that can be drawn from Table 7.62 is that development assistance of 4 to 5 per cent of national product of W1 countries is needed to produce the perspective mentioned. Even if we in W1 venture to propose thirty years of patience, we still have to treble the 0.7 target. In addition we may have to revise our protective trade policies in order to warrant the marketing of increased production.

The target chosen was a doubling of total incomes compared with total incomes of W1, not income per capita. This implies that the responsibility for population growth is left to the peoples and the governments of W3, which we think to be proper.
7.7 EQUITY VIS-À-VIS FUTURE GENERATIONS

Another aspect of an equitable distribution of welfare is distribution over present and future generations. Our responsibility for future generations does not stop at our children and grandchildren. All—an infinity of—generations will wish to have a happy family life with at least two children. But there is only a finite quantity of natural resources. If we take too much for present generations, the time may come where resources are so scarce as to press consumption and welfare down to poverty levels again. Is it possible to find a policy to prevent this? In the short-term future, further exploration may add as yet unknown resources to those already discovered. But the fact remains that the total stock of resources is a finite figure. In order to provide, with its help, an infinitely long flow of consumer goods the quantity of resources needed per unit of consumer goods must continually fall. This is conceivable only if technology continues to supply increasing productivity in terms of resources. The possibility of such a solution can best be illustrated by the fact that an infinite geometrical series has a finite sum if it is a falling series. Let the quantity of resources per unit of consumer goods be \( z \) and let \( z \) fall 4 percent annually. If at time \( t = 0 \) the value of \( z \) was \( z_0 \), then for some future time \( t \) we have

\[
z_t = z_0 \times 0.96^t
\]

and the total of resources needed over all future will be:

\[
\sum_{t=0}^\infty z_0 \times 0.96^t = 25 z_0
\]

This production programme will be possible if (i) the resources available are equal to 25 times the production in year \( t = 0 \), and (ii) the technological development can continue forever. For this to happen an active policy of research in the widest sense will be needed. It cannot be certain that the rate of technological development will be maintained.

A constant level of consumer good production implies limitations with regard to both the level of consumption per capita and the size of the population. If the world population can be kept constant, a constant level of consumption will be possible, but not, as is currently happening, a rising level of per capita consumption. If the population goes on growing, either a higher rate of growth of productivity will be needed or a gradual reduction in consumption per capita. A fall in population would enable
consumption *per capita* to rise accordingly. Finally, on top of all this we have the task of making resources available to the developing countries, and in greater quantities than so far. An additional rate of growth of some 2 per cent per annum of total world production will be required to meet that aim. Again, it means either a reduction in consumption by the developed countries, or a higher rate of growth in productivity in terms of resource input.

From these examples it will be clear that a satisfactory solution of the problem will not be easy to attain. All we can say is that there are solutions, but the conditions to be fulfilled are heavy. The problem is hardly understood by the majority of the world population. Moreover, the figures we need on available resources are largely non-existent. It would be wise if an international group of experts were created by the Secretary-General of the United Nations to report on this fundamental long-term planning problem.

REFERENCES