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Surveys in Game Theory and Related Topics

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The book is dedicated to Stef Tijs, professor of OR at the mathematical department of Nijmegen University. In some way the book demonstrates the development of game theory in the Netherlands which is mainly the outcome of Tijs' influence. His school, a generation of the 80s, has gained a broad international reputation during the last years.

The authors attempt to give surveys on their specific fields of game theory. In the non-cooperative fields, the surveys give easy and valuable access to important recent developments. The articles on cooperative game theory cover a narrower subfield. Nevertheless, the book is suitable not only for readers familiar with game theory, but also for those who would like to deepen their introductory knowledge of game theory. It motivates own research, and helps to find the basic texts in the areas reported on.

The contents of the book are split into three parts. The first part surveys non-cooperative game theory in the light of three recent developments, which are the most important ones since about the year 1970:

(1) The development of the model and the technique to deal with incomplete information, initiated 1966–1968 by Harsanyi, and Aumann and Maschler. Our knowledge on this subject is based on research by Kohlberg, Mertens, Zamir and Sorin. It is surveyed in Chapter 3, written by P. Borm.

(2) The development of refinements of the non-cooperative solution concept 'equilibrium', that allows a discussion of the difficulties with this concept in both extensive and normal form games. This excellent survey opens the book; after this first chapter by E. van Damme the following one clarifies the situation for (finite) two-person normal form games. This second chapter is written by M. Jansen and contains a concise and clear introduction to the techniques used. Both authors give valuable examples for the crucial points of their demonstrations.

(3) The elaboration of the concept of 'stochastic games' (stochastic processes with multi-agent control) and attempts to 'solve' them; the roots of this development reach to Shapley's article from 1951. Chapters 4 and 5 (by K. Vrieze and by F. Thuijsman) survey the field of stochastic games.

The cooperative part of game theory is represented by more specialized chapters. The main body on solution concepts deals with the core of a game. The introductory Chapter 7 (Th. Driessen) is followed by a survey on Tijs' (concept of) value. Chapters 10 and 11 (by I. Curiel and by Jos Potters) can serve as a gold-mine for (applied) examples of cooperative games for the reader. This mine of material can be seen as evidence of the importance of the cooperative theory but it should give rise to the question: What else (tools other than 'core') the cooperative theory can add to the analysis, classification and solution of these games? Chapter 9 (J. Dierks) contains some theoretical work on the structure of the space of all games: the author attempts to construct the extreme points of the class of superadditive games (games without losses in cooperation).

From the articles on related topics one, namely Chapter 6 (by P. Wakker), may refresh the reader's mood. A typical illness of our growing over-productive scientific community is the lack of information and understanding between the highly differentiated disciplines: the author gives some examples of translation, that can serve to rediscover theorems of other fields in your own field.

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Vollstochastische Kaufverhaltensmodelle: Ihr Beitrag zur Analyse realer Märkte

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Stochastic models

Stochastic Models for brand choice have a prominent place in the early quantitative approaches in marketing. The basic idea is that a consumer has a probability vector over the several brands that are available in the market:

$$P_{it} = (p_{i1t}, \; p_{i2t}, \ldots, P_{int}),$$

where p_{ijt} = probability that consumer i chooses brand j in period t.

Important questions are then:

– What is the distribution of the probabilities in the population? (With as an important dichotomy: *homogeneity* i.e. all consumers have identical probability vectors ($P_{it} = P_{ht}$ for two consumers i and h) versus *heterogeneity* i.e. different consumers have different probability vectors).

– What is the development of the probability vectors over time? (i.e. the relationship over time between P_{it} and marketing instruments such as advertising, prices and other factors).

– What is the dependence of the purchase probabilities on the purchase history i.e. is the probability to choose brand j in periode t dependent on the brand purchased in period $(t - 1)$? This effect is referred to as 'purchase event feedback'. A well-known model in OR-circles, the Markov model became quite popular in specifying the dependence of current purchase probabilities on the earlier brand choices.

In these 'brand choice models' the probability of choosing the individual brands are specified under the assumption that a purchase in the given product category is actually taking place. However stochastic models can also be used to model the interval times between subsequent purchases in the given product category, or—equivalently—the number of purchases made in time periods of a specific length. This class of stochastic models became known as 'purchase incidence models'.

The data used to estimate the model parameters and to test the models are purchase histories of individual consumers, collected by consumer panels which—especially for fast moving consumer goods—are widely available in most countries. A representative picture of the earlier work done in this area is: Massy, Montgomery and Morrison (1970).

Since the middle of the seventies the stochastic model approach has become a less dominant topic in marketing, but nevertheless remained a substantive research area, with ongoing contributions, see e.g. Jeuland et al. (1980); Hauser and Wisniewski (1982) and Bass et al. (1984). Most notably further progress has been made on topics such as insight in the order of the process (how many preceding brands are important for the current brand choice probabilities?), the combination of brand choice and purchase incidence models, and with respect to the incorporation of the effects of marketing instruments in stochastic choice models.

This book

The study by Wagner is in the stochastic models tradition and deals with the further developments just referred to.

The study begins with a discussion of existing models and the theoretical and empirical work that has been carried out. Subsequently a comprehensive treatment is given of statistical methods that can be used to discriminate between alternative models on the basis of consumer panel data. Most notably attention is paid to the determination of the order of the process: does the brand choice process or the purchase incidence process have a memory, i.e. does the purchase history contain information with regard to the brand a consumer will choose at his next purchase occasion or the number of purchases he will make in the next period?

An important point here is the interaction between consumer heterogeneity and the 'observed' order of the process. By sampling from a heterogeneous population a zero order brand choice process can produce transition probabilities that seem to point to a higher order process. So all discrimination procedures have to take into account this heterogeneity aspect explicitly.

Then follows a description of the data used throughout the study: purchase data of 5891 households with respect to household cleaners in West Germany over the period 1974–1976. For most of the analyses only the purchases of a subset of 886 households could be used: the households that made more than 12 purchases in the time period studied. All the analyses were

carried out with the consideration of 5 different brands (which together formed 85% of the market).

The first empirical part then deals mainly with tests on the order of both the brand choice and the purchase incidence processes. Five discrimination tests are used: multinomial run tests, asymptotic multinomial run tests, χ^2-test, likelihood ratio test and a Bayesian method. Unequivocally all tests indicated that both processes are predominantly zero order (i.e. processes without memory). This is consistent with the recent findings elsewhere in the literature.

Then follows another theoretical part in which the author develops his own contribution. Three different stochastic models are presented. The first is a dynamic heterogeneous brand choice model. The model is a multinomial Dirichlet model (generalization of a beta-binomial model). In this model it is a possible to relate at an aggregate level quantities such as market shares and 'switching constants' to marketing, trend and seasonal variables.

The second model adds the timing of purchases (or equivalently the numbers of product class purchases in a period) to the brand choice model just described. The number of product class purchases is described by a beta-binomial model across households. Independence is assumed between brand choice and purchase timing. With this model it is possible to predict penetration phenomena.

Whereas in the first two models distributions are specified for the important parameters (e.g. brand choice) from which aggregate quantitatives are derived (such as switching probabilities) which are compared with corresponding aggregate empirical quantities in the data, the third model is specified at the individual level as far as the product class purchases are concerned. This leads to a so-called Polyaprocess.

The last substantial chapter gives estimation and testing results for all three models. Estimation methods are of the econometric type OLS, GLS, the so-called Attraction Model, seemingly unrelated regressions, iterative frequency fitting methods and maximum likelihood methods (Model 3).

Generally the fit of the models to the data is very good. Various computational methods are compared. In a number of cases the (theoretically) superior methods do not show better results in terms of fit and prediction: e.g. OLS as compared to the more robust Attraction Models. Also the considerably higher computational effort for Model 3 is not rewarded by a more accurate reproduction of the observed switching matrices.

Marketing insights obtained are of the following nature: a clear impact of prices on market shares; non significant advertising effects and the information provided by the marginal purchase probability distribution for each brand. Some brands have probability distribution with an inverse J-form: concentration of the purchase probability at $p = 0$; other brands have a U-form. The latter means that there is a market segment with p near one which implies a high repurchase probability and loyalty, i.e. low vulnerability for competing marketing efforts.

Discussion

The book is a sophisticated treatment of consumer purchasing behavior from a statistical and probability theoretic view. The models are developed in a rigorous and logical way, although the discussion is very compact. Many elements are found in the work of earlier authors, but the book offers a satisfactory degree of originality. The empirical analyses are carried out in a painstaking way. The total amount of empirical work must have been enormous. Unfortunately generalizations from the empirical results are difficult, since throughout the study only one data base referring to one product (household cleaners) has been used.

At the same time this concentration on the statistical analysis of observed purchasing behavior is also an important limitation. The consumer is from the outset completely treated as a 'black box' (see page 5 of the book).

Over the last 15 years the development in (micro) marketing models has been much more in the direction of models that relate brand choice to consumer decision processes and to variables that are 'in the minds' of consumers during these decision processes: brand perceptions, preferences, attitudes, etc. The models developed from that perspective, for example multidimensional perception and preference models are much richer in their explanatory power.

The tremendous research efforts in consumer behavior over the last decades have provided us with invaluable insights in the working of the consumer's 'black box', although this knowledge

still is far from perfect. For marketing policy it is not enough to just know that some marketing effort (e.g. a specific advertising campaign) affects brand choice (e.g. observed as a change in brand switching probabilities). It is also important to know more about the cause and effects chain. A major step forward would be the merger of stochastic choice models with the multidimensional perception and choice models.

The book by Wagner does not give a perspective in this direction yet, but as a work on stochastic models in consumer research it is a very valuable and up-to-date contribution to the literature.

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