Forecasting aggregates using panels of nonlinear time series*

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Abstract

Macroeconomic time series such as total unemployment or total industrial production concern data which are aggregated across regions, sectors, or age categories. In this paper we examine if forecasts for these aggregates can be improved by considering panel models for the disaggregate series. As many macroeconomic variables have nonlinear properties, we specifically focus on panels of nonlinear time series. We discuss the representation of such models, parameter estimation and a method to generate forecasts. We illustrate the usefulness of our approach for simulated data and for the US coincident index, making use of state-specific component series.

Keywords: Data aggregation, Forecasting, Panel of time series, Business cycle, Nonlinearity, Multi-level models
JEL Classification Codes: C23, C51, C53, E32

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1 Introduction

Macroeconomic time series such as total unemployment or total industrial production concern data which are aggregated across regions, sectors, or age categories. In this paper we examine if it can be beneficial to forecast these aggregates using models for the disaggregate series when available. Often, macroeconomic variables display nonlinear features, regime-switching behaviour in particular. If the disaggregate series show such nonlinearity it may then be unclear what the dynamic properties of the aggregated series are, see Granger and Lee (1999). Hence, aggregating forecasts for regions or sectors to a forecast of the macro series may lead to more accurate forecasts than when a model for the aggregate is considered. This is the key issue examined in this paper.

The analysis is motivated by an empirical application to forecasting the aggregate US coincident index, making use of the underlying state-specific series recently constructed by Crone and Clayton-Matthews (2004). These measures of economic activity often display regime-switching behaviour, with different dynamics in business cycle recessions and expansions. To capture this nonlinear feature we employ models of the two-regime smooth transition autoregressive [STAR] type. While US states obviously are closely related, they may differ with respect to timing and the duration of recessions, see Owyang *et al.* (2004) for example. Hence we put forward a panel version of the STAR model, allowing the parameters that govern the regime-switching to differ across states. To achieve parsimony and to facilitate interpretation of the model parameters, we impose some structure on these parameters. In particular, we assume that these parameters can partly be explained by characteristics of the particular states. As such, our model contains two levels and hence we call it a multi-level panel STAR model. A basic version of this model has been proposed in Fok *et al.* (2005), but here we extend it to allow for multiple variables indicating the regime. Furthermore, we explicitly focus on forecasting aggregate data. For that purpose, we supplement our empirical evidence with simulation experiments, confirming that using disaggregate data might indeed by beneficial.

The outline of our paper is as follows. In Section 2 we discuss the components of the multi-level panel STAR model. In the same section we also outline the estimation method based on Simulated Maximum Likelihood. In Section 3 we elaborate on the way forecasts can be generated from our model. Given that our empirical application concerns economic activity at the national level and at the disaggregated state level, we consider the construction of forecasts (i) in case a model is constructed for the aggregate growth rate and forecasts are generated from this model, (ii) in case state-specific models are considered, from which forecasts are created for state-specific growth that are then aggregated to an aggregate growth forecast, and (iii) in case we rely on our panel model for the state-level growth rates for generating forecasts for aggregate growth. Naturally, the advantage of the first approach is simplicity, although information contained in the disaggregate series is not taken into account. Case (ii) seems a natural way to go, but it may lack efficiency as it ignores any linkages across states. In addition, it may be that STAR type models cannot be fitted easily to all disaggregate series. To alleviate this drawback, our proposal (iii) is to introduce a second level in a panel model, where this level contains a description of the parameters in the regime-switching mechanism, as these parameters are notoriously difficult to estimate. Before we turn to our empirical illustration, we perform simulation experiments and report on their outcomes in Section 4. In Section 5 we then consider the state-level coincident indexes, and show that forecasts for aggregate economic activity are improved by employing disaggregate data. Section 6 concludes this paper with some suggestions for further work.

2 A multi-level panel STAR model with multiple leading indicators

In this section we present the multi-level panel smooth transition autoregressive [STAR] model. First, we discuss the univariate model for an individual series, see also Granger and Teräsvirta (1993), Teräsvirta (1994), Franses and van Dijk (2000), and van Dijk, Teräsvirta and Franses (2002). The use of multiple business cycle indicators is not standard in the STAR model, hence we pay special attention to this feature of the model. Next, we discuss the panel version of the model. The presentation of the model is geared towards our empirical application to state-level output growth rates, but obviously it can be applied in different contexts as well.

2.1 Univariate STAR model with multiple indicators

Let $Y_{i,t}$ denote the level of economic activity for state $i = 1, \dots, N$ at time $t = 1, \dots, T$, such that the (one-period) growth rate can be defined as $y_{i,t} = \log Y_{i,t} - \log Y_{i,t-1}$. The basic STAR model assumes the existence of two regimes in the series $y_{i,t}$. Within each regime, the dynamics of the time series can be adequately described by means of a linear AR model. The autoregressive coefficients are allowed to differ across regimes though. In the context of output growth rates, the two model regimes usually are intended to correspond with the main business cycle phases, recessions and expansions. Transitions between these two states are governed by a continuous switching function, denoted by $G(\boldsymbol{z}_t; \boldsymbol{\pi}_i, \gamma_i, \tau_i)$, taking on values between 0 and 1. The value of $G()$ depends on a vector of observable leading indicator variables $\boldsymbol{z}_t = (z_{t,1}, \dots, z_{t,K})'$ and on the parameters $\boldsymbol{\pi}_i, \gamma_i$ and τ_i. We discuss the parameters of the switching function in detail below. Note that, contrary to the typical STAR model, we allow the switching function to depend on $K > 1$ indicators.

From the above it follows that the STAR model for the growth rate in state i

reads

$$y_{i,t} = \boldsymbol{\alpha}_i' \mathbf{x}_{i,t} + \boldsymbol{\beta}_i' \mathbf{x}_{i,t} G(\boldsymbol{z}_t; \boldsymbol{\pi}_i, \gamma_i, \tau_i) + \varepsilon_{i,t}, \tag{1}$$

where $\mathbf{x}_{i,t} = (1, y_{i,t-1}, \ldots, y_{i,t-P_i})'$, $\boldsymbol{\alpha}_i = (\alpha_{i,0}, \alpha_{i,1}, \ldots, \alpha_{i,P_i})'$, $\boldsymbol{\beta}_i$ is similarly defined, and the properties of $\varepsilon_{i,t}$ are discussed in detail below. The AR order for state i is given by P_i. In case $G()$ equals 0 the model implies an AR process for $y_{i,t}$ with parameters $\boldsymbol{\alpha}_i$, for $G()$ equal to 1 we have an AR process with parameters $\boldsymbol{\alpha}_i + \boldsymbol{\beta}_i$.

For the switching function we use the logistic function

$$G(\boldsymbol{z}_t; \boldsymbol{\pi}_i, \gamma_i, \tau_i) = \frac{1}{1 + \exp(-\gamma_i(\boldsymbol{\pi}_i' \boldsymbol{z}_t - \tau_i))}, \tag{2}$$

with $\gamma_i > 0$ and $\boldsymbol{\pi}_i = (\pi_{i,1}, \ldots, \pi_{i,K})'$. The value of the switching function ranges between 0 for very small values of the linear combination of the leading indicators $\boldsymbol{\pi}_i' \boldsymbol{z}_t$ to 1 for very large values of $\boldsymbol{\pi}_i' \boldsymbol{z}_t$, where “small” and “large” are defined relative to the threshold value τ_i. The importance weight of indicator k for state i is given by $\pi_{i,k}$. Through these weights we effectively allow different states to respond to (a combination of) different leading indicators. The speed of transition from one state of the economy to the other is captured by γ_i. For larger values of γ_i, regime-switches occur more rapidly.

For identification and interpretation purposes, we impose the parameter restrictions $\gamma_i > 0$, $\sum_{k=1}^{K} \pi_{i,k} = 1$ and $\pi_{i,k} \geq 0$. By restricting $\pi_{i,k}$ to be positive we require all indicators to have the same qualitative relation with the business cycle, in the sense that large values of each indicator should correspond with the same state of the economy. Note that this of course does not rule out any indicator *a priori*, as one can always take minus one times the indicator instead. For estimation purposes, we reparameterize the indicator weights using a logit transformation,

$$\pi_{i,k} = \frac{\exp(u_{i,k})}{\sum_{j=1}^{K} \exp(u_{i,j})}, \tag{3}$$

where $u_{i,1} = 0$ for identification. The advantage of this specification is that $\mathbf{u}_i = (u_{i,2}, \ldots, u_{i,K})'$ can be left unrestricted, while the restrictions imposed on $\pi_{i,k}$ will be satisfied automatically.

Model (1) can be generalized further by allowing for additional regressors, denoted by $\mathbf{v}_t$, with parameters that are independent of the business cycle, that is,

$$y_{i,t} = \boldsymbol{\lambda}_i' \mathbf{v}_t + \boldsymbol{\alpha}_i' \mathbf{x}_{i,t} + \boldsymbol{\beta}_i' \mathbf{x}_{i,t} G(\boldsymbol{z}_t; \boldsymbol{\pi}_i, \gamma_i, \tau_i) + \varepsilon_{i,t}. \tag{4}$$

Oftentimes, one includes the business cycle indicators as linear explanatory variables, that is, $\mathbf{v}_t = \mathbf{z}_t$. This is also what we will do in the empirical section of this paper.

We assume that the error terms in (4) are martingale difference series, that is, $\mathsf{E}[\varepsilon_{i,t} | \mathbf{v}_t, y_{i,t-1}, y_{i,t-2}, \ldots, y_{i,t-P_i}] = 0$. The conditional variance of the errors is constant over time and equal to σ_i^2. Finally, the errors are independent across states. To be precise, we assume that any correlation that may exist across states can be explained by the common exogenous variables $\mathbf{v}_t$ in (4).

2.2 Multi-level panel STAR model

If one opts to estimate STAR models for each individual state separately, (2) and (4) specify the complete model. However, as mentioned before, estimating state-specific STAR models may be difficult in practice due to outliers or a small number of observations in one of the regimes. Furthermore, it seems plausible that similar states will show similar business cycle patterns, that is, similar switching parameters. The use of this information may lead to improved forecasting performance. To incorporate this, we introduce a second-level model relating the switching parameters to observable state characteristics (or other exogenous regressors), that is,

$$\begin{pmatrix} \log(\gamma_i) \\ \tau_i \\ \mathbf{u}_i \end{pmatrix} = \boldsymbol{\delta}' \mathbf{w}_i + \boldsymbol{\eta}_i, \qquad \boldsymbol{\eta}_i \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\eta), \tag{5}$$

with $\mathbf{w}_i$ a $(Q \times 1)$ vector consisting of a constant and Q characteristics of state i, $\boldsymbol{\delta}$ a $((1+Q) \times (1+K))$ matrix of unknown coefficients, and $\boldsymbol{\eta}_i$ a vector of random effects. Note that by modeling $\log(\gamma_i)$ we naturally obtain that $\gamma_i > 0$.

The panel STAR model given by (2), (4) and (5) is somewhere in between a fully pooled model and a fully heterogeneous model, where the switching parameters are estimated separately for each state. Both alternative specifications can be seen as extreme cases of (5). The pooled model is obtained by setting $\mathbf{w}_i = 1$ and $\boldsymbol{\Sigma}_{\boldsymbol{\eta}} = 0$, the other extreme corresponds to including state dummies in $\mathbf{w}_i$.

2.3 Parameter estimation

Parameter estimation of the panel model in (4) and (5) is a straightforward extension of the method outlined in Fok *et al.* (2005). The extension to multiple indicators does not change the estimation procedure to a large extent. For completeness we briefly present the estimation procedure here, for a more detailed discussion we refer to Fok *et al.* (2005).

The complete model for state $i = 1, \ldots, I$ reads

$$\begin{aligned} y_{i,t} &= \boldsymbol{\lambda}_i'\mathbf{v}_t + \boldsymbol{\alpha}_i'\mathbf{x}_{i,t} + \boldsymbol{\beta}_i'\mathbf{x}_{i,t}G(\mathbf{z}_t;\gamma_i,\tau_i,\mathbf{u}_i) + \varepsilon_{i,t}, \\ \begin{pmatrix}\log(\gamma_i)\\ \tau_i \\ \mathbf{u}_i\end{pmatrix} &= \boldsymbol{\delta}'\mathbf{w}_i + \boldsymbol{\eta}_i, \qquad \boldsymbol{\eta}_i \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\eta), \qquad \varepsilon_{i,t} \sim N(0,\sigma_i^2). \end{aligned} \tag{6}$$

The likelihood function for this model equals

$$\mathcal{L} = \prod_{i=1}^{N} \mathcal{L}_i = \prod_i \int_{\boldsymbol{\eta}_i} \prod_{t=1}^{T} \phi(e_{i,t}(\boldsymbol{\lambda}_i, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i, \boldsymbol{\delta}'\mathbf{w}_i + \boldsymbol{\eta}_i); 0, \sigma_i^2)\phi(\boldsymbol{\eta}_i; \mathbf{0}, \boldsymbol{\Sigma}_\eta)d\boldsymbol{\eta}_i, \tag{7}$$

where we (implicitly) condition on initial observations $(y_{i,1-P_i}, \ldots, y_{i,0})$. Furthermore, $\phi(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes the $(K+1)$-variate normal density function with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ evaluated at $\mathbf{x}$ and

$$e_{i,t}(\boldsymbol{\lambda}_i, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i, \boldsymbol{\theta}_i) = y_{i,t} - \boldsymbol{\lambda}_i'\mathbf{v}_t - \boldsymbol{\alpha}_i'\mathbf{x}_{i,t} - \boldsymbol{\beta}_i'\mathbf{x}_{i,t}G(\mathbf{z}_t;\gamma_i,\tau_i,\mathbf{u}_i) \tag{8}$$

gives the error for sector i and period t, given the parameters $\boldsymbol{\lambda}_i$, $\boldsymbol{\alpha}_i$ and $\boldsymbol{\beta}_i$, and the switching parameters $\boldsymbol{\theta}_i = (\log(\gamma_i), \tau_i, \mathbf{u}_i')'$.

Parameter estimation is done through concentrated simulated maximum likelihood. We use simulation to calculate the likelihood (7) and concentrate it with respect to the parameters in the first level model $(\boldsymbol{\lambda}_i, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i, \sigma_i^2,\ i = 1, \ldots, I)$. The simulated likelihood contribution of state i is

$$\begin{aligned}\tilde{\mathcal{L}}_i &= \frac{1}{L}\sum_{l=1}^{L}\prod_t \phi(e_{i,t}(\boldsymbol{\lambda}_i, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i, \boldsymbol{\delta}'\mathbf{w}_i + \boldsymbol{\Sigma}_\eta^{1/2}\tilde{\boldsymbol{\eta}}_{i,l}); 0, \sigma_i^2)\\ &= \frac{1}{L}\sum_{l=1}^{L}\prod_t \phi(e_{i,t,l}; 0, \sigma_i^2),\end{aligned} \tag{9}$$

where we use the shorthand notation $e_{i,t,l}$ to denote the residual for state i at time t conditional on the l-th draw of the random effects, $\tilde{\boldsymbol{\eta}}_{i,l} \sim N(0, \mathbf{I})$, $l = 1, \ldots, L$. For the concentration step we need to solve $\max_{\boldsymbol{\lambda}_i, \boldsymbol{\alpha}_i, \boldsymbol{\beta}_i, \sigma_i^2} \tilde{\mathcal{L}}_i$. One can show that the first order conditions for $\max \tilde{\mathcal{L}}_i$ are

$$\begin{aligned}\frac{1}{L}\sum_l\sum_t w_{i,l}\frac{e_{i,t,l}}{\sigma_i^2}\mathbf{x}_{i,t,l} &= \mathbf{0} \text{ and}\\ \frac{1}{L}\sum_l\sum_t \frac{w_{i,l}}{2\sigma_i^2}(\frac{e_{i,t,l}^2}{\sigma_i^2} - 1) &= 0,\end{aligned} \tag{10}$$

where $\mathbf{0}$ denotes a vector of zeros and

$$\begin{aligned}\mathbf{x}_{i,t,l} &= (\mathbf{v}_t', [1, G(\mathbf{z}_t; \boldsymbol{\delta}'\mathbf{w}_i + \boldsymbol{\Sigma}_\eta^{1/2}\tilde{\boldsymbol{\eta}}_{i,l})] \otimes \mathbf{x}_{i,t}')',\\ w_{i,l} &= \prod_t \phi(e_{i,t,l}; 0, \sigma_i^2)\end{aligned} \tag{11}$$

The expressions in (10) bear close resemblance to weighted least squares [WLS], although here a complication arises due to the fact that the weights $w_{i,l}$ depend on the parameter values. Following this observation, (10) can be solved by iterating between WLS and updating the weights. Denoting $\mathbf{x}_{i,l} = (\mathbf{x}_{i,1,l}, \ldots, \mathbf{x}_{i,T,l})'$ and $\mathbf{y}_i = (y_{i,1}, \ldots, y_{i,T})'$, we alternate between

$$\begin{pmatrix}\hat{\boldsymbol{\lambda}}_i\\ \hat{\boldsymbol{\alpha}}_i\\ \hat{\boldsymbol{\beta}}_i\end{pmatrix} = \left(\frac{1}{L}\sum_{l=1}^{L} w_{i,l}\mathbf{x}_{i,l}'\mathbf{x}_{i,l}\right)^{-1}\left(\frac{1}{L}\sum_{l=1}^{L} w_{i,l}\mathbf{x}_{i,l}'\mathbf{y}_i\right) \tag{12}$$

$$\hat{\sigma}_i^2 = \frac{\frac{1}{L}\sum_l\sum_t w_{i,l}e_{i,t,l}}{\frac{T}{L}\sum_l w_{i,l}}. \tag{13}$$

and updating the weights according to (11). After convergence we have the optimal AR parameters conditional on the switching parameters. Using these optimal parameters we calculate the (log) concentrated likelihood function which is in turn (numerically) optimized to obtain estimates of the switching parameters.

The resulting Simulated Maximum Likelihood estimator is consistent for $N \to \infty$ and $L \to \infty$, see Hajivassiliou and Ruud (1994). The asymptotic covariance matrix can be estimated through the Hessian of the log *concentrated* likelihood, see Davidson and MacKinnon (1993). The estimated covariance matrix equals

$$\widehat{\mathrm{Var}(\vartheta)} = \left(-\frac{\partial^2 \log \mathcal{L}^c}{\partial\vartheta\partial\vartheta'}\right)^{-1} \tag{14}$$

where ϑ contains the parameters in $\boldsymbol{\delta}$ and $\boldsymbol{\Sigma}_\eta$ and where $\mathcal{L}^c$ denotes the concentrated likelihood function.

3 Forecasting

In this section we discuss forecasting growth rates and levels using the multi-level panel STAR model. First, we consider forecasts for a single state. In Section 3.2 we present forecasts for the aggregate series.

3.1 Forecasting using panel STAR model

In the panel STAR model it is not possible to obtain forecasts of the state-level growth rates or the level of the underlying series directly from the estimated parameters. The complicating factor is that one needs to calculate the expected value of $y_{i,t}$ over the random terms $\varepsilon_{i,t}$ and $\boldsymbol{\eta}_i$, where this expectation is usually calculated conditional on the observed series.

Denoting the relevant information set by Ω_{t-1}, to obtain (one-step ahead) forecasts of the level $Y_{i,t}$ we have to calculate

$$\begin{aligned}\hat{Y}_{i,t} &= \mathsf{E}[\exp(y_{i,t})Y_{i,t-1}|\Omega_{t-1}] = \mathsf{E}_{\boldsymbol{\eta_i}}[\mathsf{E}_{\varepsilon_{i,t}}[\exp(y_{i,t})|\boldsymbol{\eta_i}, \Omega_{t-1}]|\Omega_{t-1}]Y_{i,t-1} \\ &= \mathsf{E}_{\boldsymbol{\eta_i}}[\exp(\boldsymbol{\lambda}_i\mathbf{v}_t + \boldsymbol{\alpha}_i\mathbf{x}_{i,t} + \boldsymbol{\beta}_i\mathbf{x}_{i,t}G(\mathbf{z}_t;\gamma_i,\tau_i,\mathbf{u}_i) + \tfrac{1}{2}\sigma_i^2)|\Omega_{t-1}]Y_{i,t-1}.\end{aligned} \tag{15}$$

For forecasts of the growth rates we need

$$\begin{aligned}\hat{y}_{i,t} &= \mathsf{E}[y_{i,t}|\Omega_{t-1}] = \mathsf{E}_{\boldsymbol{\eta_i}}[\mathsf{E}_{\varepsilon_{i,t}}[y_{i,t}|\boldsymbol{\eta_i}, \Omega_{t-1}]|\Omega_{t-1}] \\ &= \boldsymbol{\lambda}_i \mathbf{v}_t + \boldsymbol{\alpha}_i \mathbf{x}_{i,t} + \boldsymbol{\beta}_i \mathbf{x}_{i,t} \mathsf{E}_{\boldsymbol{\eta_i}}[G(\mathbf{z}_t; \gamma_i, \tau_i, \mathbf{u}_i)|\Omega_{t-1}].\end{aligned} \tag{16}$$

Both forecasts require computing the (conditional) expectation of a complex function of the random effects ($\boldsymbol{\eta}_i$). The expression for such an expectation conditional on the complete series $\mathbf{y}_i$ can be found in Fok *et al.* (2005), and is restated here

$$\begin{aligned}\mathsf{E}_{\boldsymbol{\eta}_i}[f(\boldsymbol{\eta}_i)|\mathbf{y}_i] &= \int_{\boldsymbol{\eta}_i} f(\boldsymbol{\eta}_i) g(\boldsymbol{\eta}_i|\mathbf{y}_i) d\boldsymbol{\eta}_i \\ &= \frac{\int_{\boldsymbol{\eta}_i} f(\boldsymbol{\eta}_i) g(\mathbf{y}_i|\boldsymbol{\eta}_i)\phi(\boldsymbol{\eta}_i; \mathbf{0}, \boldsymbol{\Sigma}_\eta) d\boldsymbol{\eta}_i}{\int_{\boldsymbol{\eta}_i} g(\mathbf{y}_i|\boldsymbol{\eta}_i)\phi(\boldsymbol{\eta}_i; \mathbf{0}, \boldsymbol{\Sigma}_\eta) d\boldsymbol{\eta}_i} \\ &= \frac{\frac{1}{L}\sum_l f(\tilde{\boldsymbol{\eta}}_{i,l}) w_{i,l}}{\frac{1}{L}\sum_l w_{i,l}},\end{aligned} \tag{17}$$

where $f()$ denotes a function of $\boldsymbol{\eta}_i$, and, as before, $\tilde{\boldsymbol{\eta}}_{i,l} \sim N(0, \mathbf{I})$ and the weights $w_{i,l}$ as defined in (11), $g(\mathbf{x}|\mathbf{z})$ denotes the density function of $\mathbf{x}$ given $\mathbf{z}$.

3.2 Forecasting aggregate growth

Denote the aggregate, national measure of economic activity as $\tilde{Y}_t = \sum_i w_i Y_{i,t}$ where w_i denotes the constant and exogenous weight of state i in the total economy. The growth rate of aggregate output is given by $\tilde{y}_t = \log \tilde{Y}_t - \log \tilde{Y}_{t-1}$.

Forecasts of $\tilde{Y}_t$ are easily obtained from forecasts of the state-level series, that is, $\widehat{\tilde{Y}}_t = \mathsf{E}[\tilde{Y}_t|\Omega_{t-1}] = \sum_i w_i \mathsf{E}[Y_{i,t}|\Omega_{t-1}] = \sum_i w_i \widehat{Y}_{i,t}$. However, usually forecasts of the growth rate $\tilde{y}_t$ are desired. In case models for the disaggregate growth rates are considered it is not straightforward to convert the resulting state-level growth forecasts into forecasts of the aggregate growth rate. To see this, consider the one-step ahead forecast

$$\begin{aligned}\mathsf{E}[\tilde{y}_t|\Omega_{t-1}] &= \mathsf{E}[\log \tilde{Y}_t|\Omega_{t-1}] - \log \tilde{Y}_{t-1} = \mathsf{E}[\log \sum_{i=1}^{N} w_i Y_{i,t}|\Omega_{t-1}] - \log \tilde{Y}_{t-1} \\ &= \mathsf{E}[\log \sum_{i=1}^{N} w_i Y_{i,t-1} \exp(y_{i,t})|\Omega_{t-1}] - \log \tilde{Y}_{t-1}\end{aligned} \tag{18}$$

In (18) only the $y_{i,t}$ variables are unknown. However, the transformation of forecasts of $y_{i,t}$ to the forecast of the aggregate growth rate is nonlinear. To appropriately evaluate this expectation one would again have to rely on simulation, that is,

$$\mathsf{E}[\tilde{y}_t|\Omega_{t-1}] = \lim_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}\left(\log\sum_{i=1}^{N} w_i Y_{i,t-1}\exp(y_{i,t,l})\right) - \log\tilde{Y}_{t-1}, \tag{19}$$

where $y_{i,t,l}$ denotes a simulated value from the distribution of $y_{i,t}$ conditional on Ω_{t-1}. A forecast of the aggregate growth rate may then be obtained by dropping the limit in (19) and setting L to a relatively large number. Using the same arguments as in (17) one can show that forecasts can be obtained as

$$\widehat{\tilde{y}}_t = \frac{\frac{1}{L}\sum_l \left(\log\sum_{i=1}^{N} w_i Y_{i,t-1}\exp(y_{i,t,l})\right)\prod_{i=1}^{N} w_{i,l}}{\frac{1}{L}\sum_l \prod_{i=1}^{N} w_{i,l}} - \log\tilde{Y}_{t-1}, \tag{20}$$

where $y_{i,t,l}$ now equals $\boldsymbol{\lambda}_i'\mathbf{v}_t + \boldsymbol{\alpha}_i'\mathbf{x}_{i,t} + \boldsymbol{\beta}_i'\mathbf{x}_{i,t}G(\mathbf{z}_t;\boldsymbol{\delta}'\mathbf{w}_i + \boldsymbol{\Sigma}_\eta^{1/2}\tilde{\boldsymbol{\eta}}_{i,l})$.

It is important to note that the simulation weights $w_{i,l}$ enter (20) in a multiplicative way. Simulation noise is therefore amplified. It turns out that an excessive amount of simulations L is required to obtain relatively noise-free forecasts. Furthermore, in practice it may be that the gain of the simulation is very small. We suggest that instead one considers forecasting the aggregate growth rate by transforming forecasted levels, that is,

$$\widehat{\tilde{y}}_t = \log\widehat{\tilde{Y}}_t - \log\tilde{Y}_{t-1}. \tag{21}$$

4 Simulation experiment

In this section we discuss a limited simulation experiment, which is meant to illustrate the potential benefits from considering a panel of nonlinear time series when the main interest is in obtaining forecasts of the aggregate. We generate 25 panel data sets, estimate the various possible models and forecast individual growth rates, aggregate growth rates and the aggregate level. We choose a setting similar to our empirical application, that is, $N = 50$, $T = 264$ and we leave out 50 observations for the out-of-sample forecast comparison. The estimation of our panel STAR model is very time consuming, therefore we only use a relatively small number of replications.

For each replication, we randomly generate the model parameters. The data generating process [DGP] is given by (6), where

$$
\begin{aligned}
\alpha_i &= |0.5 + 0.25\xi_i|, \quad \xi_i \sim N(0,1) \\
\beta_i &= -|1.5 + 0.25\nu_i|, \quad \nu_i \sim N(0,1) \\
\sigma_i^2 &= |3 + \zeta_i|, \quad \zeta_i \sim N(0,1) \\
\delta &= (1,0)' \\
\Sigma_\eta &= \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix},
\end{aligned}
\tag{22}
$$

where the parameters refer to (6). As leading indicator we use the most recent observations of the term spread (see Section 5), standardized to have mean 0 and standard deviation 2. To be specific, for the in-sample period we use the term spread from 1977:11 - 1999:10 and for the out-of-sample period we use 1999:11-2003:12. We use a standard deviation of 2 for the indicator to allow for a wide range of values of the threshold τ. Note that for a proper STAR model the value of τ better not be close to the minimum or maximum values of the leading indicator. We have selected the DGP in such a way that values of the switching function $G()$ close to 1 correspond to periods of a recession. In these periods the growth rate of 'state' i equals $\alpha_i + \beta_i$, while in expansion periods the growth rate equals α_i. The DGP further implies that, on average, the negative growth rate in recessions is (in absolute value) larger than the growth rate in expansions.

Results are presented in Table 1. To forecast individual growth rates, the use of separate univariate STAR models for each series gives the best results. This was to be expected as the individual growth rates were also generated as independent STAR models. As long as the simulated series are informative enough to estimate the model parameters, reasonably accurate forecasts will result from independent STAR models. Note, however, that the quality of the forecasts generated by the panel STAR model is very similar to that of the forecasts made with the independent models. In case individual series do not supply adequate information, for example in case of short time series or outliers, the panel STAR model will outperform forecasts generated with individual models. By also using cross-sectional information one will be able to obtain more accurate estimates for individual series.

We are interested in forecasting the aggregate series using the individual components. Depending on the particular application one can either be interested in forecasting growth rates or in forecasting the level of the aggregate series. In the simulation experiment we compare the performance of the proposed methods on both, see Table 1 again. Qualitatively, the results are the same. Forecasts based on individual STAR models perform best, followed by forecasts generated by the panel STAR model. Forecasts obtained by directly estimating a STAR model for the aggregate growth rates perform worst. The difference in performance of this last model with the other approaches is strikingly large.

5 Forecasting aggregate US economic activity

Aggregate US output probably is the most popular macroeconomic variable when it comes to applications of nonlinear time series models. Numerous attempts have been made at describing its presumably different dynamics in business cycle expansions and recessions, see Hamilton (1989), Teräsvirta (1995) and Pesaran and Potter (1997), among many others. While some doubt has been cast on the usefulness of nonlinear models for this purpose, see Engel *et al.* (2004), it may still be the case that such models render more accurate forecasts of growth rates and business cycle turning points. The evidence here is mixed, see Chauvet and Piger (2003), van Dijk and Franses (2003) and Camacho (2004) for recent accounts. In this section we examine whether the use of our nonlinear panel STAR model for state-level output series results in improved forecasts for the aggregate.

Unfortunately, the most suitable measure of state-level output, Gross State Product (GSP), is available only at the annual frequency and with a delay of two years. For that reason, we employ the monthly state-level coincident indexes developed in Crone and Clayton-Matthews (2004). These indexes measure economic activity in general and are extracted from a dynamic factor model for nonagricultural employment, the unemployment rate, average hours worked in manufacturing, and real wage and salary disbursements. Although indexes are available for all 50 states, we exclude Alaska and Hawaii from the analysis, focusing on the 48 contiguous states. The sample period for which information for all states is available covers July 1979-October 2003. The panel STAR model is specified using observations up to December 2001, while the final two years are saved for out-of-sample forecasting. We assume that the business cycle regimes for all states can be related to (a linear combination of) the four components of the Conference Board's Composite Index of Leading Indicators (CLI). These are initial claims for unemployment insurance, new orders of consumer goods and materials, stock prices, and the interest rate spread.[1] These variables, after transformation to month-to-month changes or monthly growth rates, enter the model as $\mathbf{z}_t$ in the logistic transition function, while in addition they are included as regressors $\mathbf{v}_t$. Note that we take the negative of the average initial claims given that this variable is counter-cyclical, such that low values of all leading indicators correspond with recessions. Finally, we employ several industrial, demographic and tax variables to explain differences in timing and duration of recessions through the second-level model. In particular, we include states' employment shares in manufacturing, in construction and mining, and in finance, insurance, and real estate (FIRE), and the shares of a state's population aged 25 and older with a school diploma (but no college degree), and the population share with a bachelor's degree,

[1]We started with the complete set of 10 components of the CLI, including average weekly hours in manufacturing, vendor performance, new orders of nondefense capital goods, building permits, money supply, and the index of consumer expectations. Preliminary estimates suggested that the four selected series suffice.

the state's population share that is of prime working age (between 18 and 44), and finally, the maximum marginal tax rates on wages and salaries and on capital gains.

A preliminary analysis of the series shows that there are few differences in the speed of transition (γ) across the states. Furthermore, the results are rather insensitive to the exact value of γ. Therefore we choose to fix the value of γ to 25. Next we determine the autoregressive lag orders P_i by means of Schwarz' BIC in univariate ARX models for monthly growth rates of the state-level coincident indexes, including the four leading indicator variables as exogenous variables. The selected AR orders, which are fixed for the remainder of the analysis, generally are fairly low, equal to 2 or 3 for most states. Next, we estimate the three models of interest, (i) univariate STAR models for the state-level growth rates, (ii) a univariate STAR model for the monthly growth rate in the aggregate coincident index, and (iii) our two-level panel STAR model. The aggregate coincident index is constructed from the state-level indexes, using the average share of GSP over the sample period as weights. We determine the appropriate delay of each of the four business cycle indicators by means of grid search, allowing for a maximum delay of six months. This procedure results in delays of a single month for average initial claims, new orders and the interest rate spread, and of three months for stock prices. Next, only the employment share in manufacturing, the maximum marginal tax rate on wages and salaries, and the population share with college education (bachelor's degree) are retained as state characteristics in the final specification, in addition to a geographical dummy variable, where states in the Plains, Rocky Mountains or the Southwest are coded with a 0 and the other states with a 1.

Given our focus on out-of-sample forecasting, in-sample estimation results are not discussed extensively here. Table 2 shows the parameter estimates for the second-level model, which are of most interest. Full details are available upon request, but now we highlight some of the findings. The negative intercept for the threshold indicates that states tend to be longer in expansions than in recessions. States with high marginal wage tax rates tend to have shorter recessions. For states with many people with a college degree the opposite holds. Across all states, the interest spread turns out to be the most important indicator, which is reflected by the large estimated intercept for the corresponding weight. The least important indicator is the stock price. However, in states with a high marginal tax rate on wages, states that heavily rely on manufacturing or states with a large percentage of the population with a college degree, the weight of the stock price is significantly higher. Furthermore, the estimated standard deviation of the random effect associated with this weight is rather large. This indicates that next to explained differences in the weight of this indicator, there are also large differences that cannot be explained.

Inspecting the estimated transition functions $G()$, we observe a wide variety of patterns. For some, notably the larger states, the model regimes correspond quite closely with the nation-wide business cycle expansions and recessions, as dated by the NBER. Several of the smaller states appear to exhibit more idiosyncratic regime-switches in addition.

Table 3 summarizes the out-of-sample forecasting performance of the three different models. First, note that the panel STAR model renders more accurate out-of-sample forecasts for the state-level growth rates than the individual univariate STAR models. This is probably due to the fact that for several states estimating a univariate STAR model proves to be difficult, due to the presence of some aberrant observations (although the individual models provide a slightly better in-sample fit than the panel STAR model).

Turning to the forecasts for the aggregate coincident index, we find that the panel STAR model produces the smallest mean squared prediction error, when forecasting the level as well as the growth rate. Corresponding with our simulation experiment, the univariate STAR model for the aggregate growth rate shows the worst forecasting performance.

From these results. we conclude that it indeed appears to be useful to consider nonlinear models for disaggregate series, and to combine these into a panel framework in order to exploit cross-sectional linkages, even when the ultimate interest is in forecasting the aggregate.

6 Conclusion

In this paper we examined if forecasts for aggregates like total output or total unemployment could be improved by considering panel models for the disaggregated series, where these series show nonlinear properties. Based on simulated results and on comparing total output forecasts with forecasts obtained from a panel model covering 48 states, we conclude that such gains can indeed be achieved.

We believe that our model class opens ways to improve forecasting aggregates. These days many disaggregate data are available, and somehow these contain information that could benefit aggregate forecasts. Unrestricted panel models may be useful, but they may also contain difficult to estimate or interpret parameters. Hence, we believe that multi-level panels are perhaps more useful. We hope to see more applications of this approach to various other situations, although we must admit that parameter estimation is not straightforward. Hence, we also would welcome more research in improved methods for estimation.

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Table 1: Monte Carlo forecasting results

	Forecasting model		
	Panel STAR	Aggregate STAR	Individual STAR
Forecasting individual growth rates			
Average MSPE	3.103	–	3.102
No. best forecast	605	–	645
Forecasting aggregate level			
Average MSPE	348.75	415.93	338.14
No. best forecast	7	2	16
No. best forecast (excl. indiv. STAR)	22	3	–
Forecasting aggregate growth			
Average MSPE	0.176	0.203	0.170
No. best forecast	8	2	15
No. best forecast (excl. indiv. STAR)	20	5	–

Note: The table reports results the simulation experiment where panels of N=50 series of length $T = 264$ are generated according to the multi-level panel STAR model (6), with parameterizations given in (22). In addition to the panel STAR model, forecasts are obtained from univariate STAR models for the individual series and a univariate STAR model for the aggregate. Results are based on 25 replications.

Table 2: Empirical estimation results

	Intercept	MANUF	WTAX	CEDU	BEA	σ_η
Speed of transition γ_i	25					
	-	-	-	-	-	-
Threshold τ_i	-0.409	-0.093	-0.461	0.294	0.294	0.939
	(0.162)	(0.044)	(0.041)	(0.035)	(0.144)	(0.053)
Economic indicators (π_i)						
New orders of consumer goods and materials	2.449	0.010	0.762	0.980	0.005	0.242
	(0.403)	(0.146)	(0.230)	(0.275)	(0.444)	(0.067)
Interest rate spread	4.634	-0.060	0.882	1.228	-0.038	1.679
	(0.713)	(0.186)	(0.230)	(0.276)	(0.706)	(0.195)
Stock prices	-1.193	1.607	0.554	2.155	-2.567	3.605
	(0.743)	(0.623)	(0.265)	(0.572)	(1.030)	(0.657)

Note: The table shows estimates of the parameters $\boldsymbol{\delta}$ and σ_η in the second-level model in the panel STAR model (6) applied to monthly growth rates of the coincident index of the 48 contiguous US states, using 4 economic indicators and 4 state characteristics, over the period July 1979-December 2001. Standard errors are given in parentheses. Initial claims for unemployment insurance is used as baseline indicator. MANUF=the employment share in manufacturing, WTAX=the maximum marginal tax rate on wages and salaries, CEDU=the population share with college education (bachelor's degree), and BEA=a geographical dummy (states in the Plains (MN,MO,KS,NE,IA,SD,ND), Rocky Mountains (MT,ID,WY,UT,CO) and the South West (TX,OK,NM,AR) are coded with a 0, other states with a 1)

Table 3: Empirical forecasting results

	Forecasting model		
	Panel STAR	Aggregate STAR	Individual STAR
Forecasting individual growth rates (average MSPE across states)			
In sample	4.829	–	4.748
Out-of-sample	4.705	–	4.935
Forecasting aggregate level			
In sample	9.004	9.884	8.983
Out-of-sample	27.005	29.106	28.178
Forecasting aggregate growth			
In sample	0.610	0.661	0.615
Out-of-sample	0.853	0.920	0.890

Note: The table shows the MSPE of one-step ahead forecasts for monthly growth rates and levels of US state-level and aggregate coincident indexes, over the in-sample period July 1979-December 2001 and the out-of-sample period January 2002-October 2003. In addition to the panel STAR model, forecasts are obtained from univariate STAR models for the individual series and a univariate STAR model for the aggregate index.