Testing for causality in variance in the presence of breaks*

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Abstract

We examine the size properties of tests for causality in variance in the presence of structural breaks in volatility. Extensive Monte Carlo simulations demonstrate that these tests suffer from severe size distortions when such breaks are not taken into account. Pre-testing the series for structural changes in volatility is shown to largely remedy the problem.

Keywords: volatility; causality tests; structural change.
JEL Classification Codes: C12; C22.

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1 Introduction

During the last two decades, a great deal of attention has been paid to modelling the dynamic properties of volatility. An important strand of this research has analysed volatility spillovers, particularly across financial series. Morgenstern (1959) first investigated if financial market crises spill over to other countries. The modelling of spillover effects has subsequently been developed primarily through correlation analysis based on GARCH models, including King and Wadhwani (1990), Lin, Engle and Ito (1994), Susmel and Engle (1994), Ng (2000) and Billio and Pelizzon (2003). These papers estimate parametric models to examine specific formulations for the spillover effects, while Cheung and Ng (1996) and Hong (2001) develop general causality-in-variance tests within this framework.

Through a distinct strand of literature, there has recently been increasing awareness that many time series experience occasional structural breaks in (unconditional) volatility; see Andreou and Ghysels (2002), Lamoureux and Lastrapes (1990), McConnell and Perez-Quiros (2000), Stock and Watson (2003) and Sensier and van Dijk (2004), among others. In the light of the evidence for such structural breaks, it is important to investigate whether they affect the appearance of volatility spillover effects. The present paper examines this question by analyzing the impact of volatility breaks on the causality-in-variance tests of Cheung and Ng (1996) and Hong (2001).

Based on an extensive set of Monte Carlo simulations, we demonstrate that these tests suffer from severe size distortions when such breaks are neglected. However, we also show that pre-testing for volatility breaks provides an effective solution to the size problem.

2 Tests for causality in variance

Let $y_t = (y_{1t}, y_{2t})'$ be a bivariate series of interest. In Cheung and Ng (1996), y_{2t} is said to cause y_{1t} in variance if

$$\mathsf{E}[(y_{1t} - \mu_{1t})^2 | \Omega_{t-1}] \neq \mathsf{E}[(y_{1t} - \mu_{1t})^2 | \Omega_{1t-1}] \tag{1}$$

where Ω_{it} is the information set defined by $\Omega_{it} = \{y_{it-j}; j \geq 0\}$, $i = 1, 2$, $\Omega_t = \Omega_{1t} \cup \Omega_{2t}$, and μ_{it} is the mean of y_{it} conditional on Ω_{it}. Let $\varepsilon_{it} = y_{it} - \mu_{it}$, $i = 1, 2$, and assume that $\varepsilon_{it} = \xi_{it}\sqrt{h_{it}}$, where h_{it} is a positive, time-varying function measurable with respect to Ω_{it-1}, and ξ_{it} is an innovation process with $\mathsf{E}[\xi_{it}|\Omega_{it-1}] = 0$ and $\mathsf{E}[\xi_{it}^2|\Omega_{it-1}] = 1$. Hence h_{it} is the (univariate) conditional variance of ε_{it} by construction, that is $h_{it} = \mathsf{V}[\varepsilon_{it}|\Omega_{it-1}]$. The null hypothesis that y_{2t} does not cause y_{1t} in variance can now be formulated as

$$\mathsf{H}_0: \ \mathsf{V}[\xi_{1t}|\Omega_{t-1}] = \mathsf{V}[\xi_{1t}|\Omega_{1t-1}]. \tag{2}$$

Define the squared standardized residuals

$$\hat{u}_t = (y_{1t} - \hat{\mu}_{1t})^2/\hat{h}_{1t} \quad \text{and} \quad \hat{v}_t = (y_{2t} - \hat{\mu}_{2t})^2/\hat{h}_{2t} \tag{3}$$

where hats indicate suitable estimates of the corresponding quantities, and the sample cross-correlation at lag k,

$$r_{uv}(k) = c_{uv}(k)/\sqrt{c_{uu}(0)c_{vv}(0)}, \tag{4}$$

where $c_{uv}(k)$ is the sample cross-covariance

$$c_{uv}(k) = \begin{cases} \frac{1}{T}\sum_{t=k+1}^{T}(u_t - \overline{u})(v_{t-k} - \overline{v}), & \text{if } k \geq 0 \\ \frac{1}{T}\sum_{t=1}^{T-k}(u_t - \overline{u})(v_{t-k} - \overline{v}), & \text{if } k < 0, \end{cases}$$

with T denoting sample size, $\overline{u}$ and $\overline{v}$ the sample means of u_t and v_t, respectively, and $c_{xx}(0) = 1/T\sum_{t=1}^{T}(x_t - \overline{x})^2$ for $x = u, v$.

Cheung and Ng (1996) suggest testing H_0 using the statistic

$$S = T\sum_{k=1}^{K} r_{uv}^2(k), \tag{5}$$

which has an asymptotic chi-square distribution with K degrees of freedom. The reverse hypothesis that y_{1t} does not cause y_{2t} in variance can be tested analogously, summing the squared cross-correlations $r_{uv}^2(k)$ from $k = -K$ to -1.

Hong (2001) modifies the Cheung-Ng test in two ways. First, μ_{it}, $i = 1, 2$, is defined to be the mean of y_{it} conditional on the complete information set Ω_{t-1}. Using such a "bivariate" conditional mean definition ensures that any causality-in-mean is filtered out when testing for causality-in-variance.[1] Pantelidis and Pittis (2004) show that neglected causality-in-mean leads to severe (positive) size distortions for the Cheung-Ng test, but this is easily remedied by obtaining $\hat{\mu}_{it}$ from a bivariate conditional mean specification. In this paper, we abstract from conditional mean issues by using a data generating process with no causality in mean, so that we can use the corresponding sample mean for $\hat{\mu}_{it}$. Second, Hong (2001) suggests weighting the cross-correlations to obtain more powerful tests. In particular, he defines the statistic

$$Q = \frac{T\sum_{k=1}^{T-1} w^2(k;K) r_{uv}^2(k) - C(w)}{\sqrt{2D(w)}} \overset{\text{asy}}{\sim} N(0,1), \tag{6}$$

where $w(k;K)$ is a weight function, and

$$C(w) = \sum_{k=1}^{T-1}(1-k/T)w^2(k;K),$$
$$D(w) = \sum_{k=1}^{T-1}(1-k/T)(1-(k+1)/T)w^4(k;K),$$

are approximately the mean and variance, respectively, of $T\sum_{k=1}^{T-1} w^2(k/K)r_{uv}^2(k)$. There is considerable freedom in the choice of weight function $w(k;K)$, see Hong (2001) for examples. Here we use the Bartlett kernel[2]

$$w(k;K) = \begin{cases} 1-|k/(K+1)| & \text{if } k/(K+1) \leq 1 \\ 0 & \text{otherwise.} \end{cases}.$$

3 Monte Carlo design and size results

We examine the size of causality-in-variance tests for a data generating process with $y_t = (y_{1t}, y_{2t})' \sim NID(0, \Sigma_t)$, where $\Sigma_t = \begin{pmatrix} \sigma_{1t}^2 & \sigma_{1t}\sigma_{2t}\rho \\ \sigma_{1t}\sigma_{2t}\rho & \sigma_{2t}^2 \end{pmatrix}$. The effects of neglected changes in volatility are examined through the following five experiments:[3]

[1] See also Hong (2001, footnote 5) for discussion.

[2] Results for other weight functions are qualitatively similar and are available upon request.

[3] Analogously to experiments D and E, we also examined increases in volatility in σ_{1t}^2 and σ_{2t}^2 from 1 to 2. Results for these are the mirror images of the corresponding cases shown. These results are available on request.

A. Shift in volatility of y_{2t} only: $\sigma_{1t}^2 = 1$ for all t, while $\sigma_{2t}^2 = \sigma_b^2$ for $t \leq T/2$ and $\sigma_{2t}^2 = \sigma_a^2$ for $t > T/2$;.

B. Simultaneous and identical changes in volatility: $\sigma_{1t}^2 = \sigma_{2t}^2 = \sigma_b^2$ for $t \leq T/2$ and $\sigma_{1t}^2 = \sigma_{2t}^2 = \sigma_a^2$ for $t > T/2$;

C. Simultaneous but opposite changes in volatility: $\sigma_{1t}^2 = \sigma_a^2, \sigma_{2t}^2 = \sigma_b^2$ for $t \leq T/2$, and $\sigma_{1t}^2 = \sigma_b^2, \sigma_{2t}^2 = \sigma_a^2$ for $t > T/2$;

D. Simultaneous decline in volatility: σ_{2t}^2 and σ_{1t}^2 change from 1 to 0.5 at $t = \pi T$;

E. Declines at different times: σ_{1t}^2 changes at $t = T/2$ while σ_{2t}^2 changes at $t = \pi T$, both from 1 to 0.5.

Experiments A to C use $\sigma_b^2 = 1$ and σ_a^2 such that $\frac{\sigma_a^2}{\sigma_b^2} = 0.1, 0.2, \ldots 0.9, 1.0, \frac{1}{0.9}, ..., \frac{1}{0.1}$. Experiments D and E use $\pi = 0.10, 0.15, \ldots, 0.90$ with $\pi = 0.45, 0.46, \ldots, 0.55$ also examined for E).

We consider sample sizes of $T = 160$ and 480, corresponding to 40 years of quarterly and monthly data, respectively; 10000 replications are used throughout. The test statistics S in (5) and Q in (6) for testing causality-in-variance from y_{2t} to y_{1t} and *vice versa* are computed for $K = 1, 2, \ldots, 10$. The tests are applied to y_t, replacing $\hat{\mu}_{it}$ and $\hat{h}_{it}$ in (3) by the sample mean and variance, respectively. Rejection frequencies at the nominal 5% significance level[4] are shown graphically, with empirical rejection frequencies for sample sizes of 160 and 480 shown in panels (a) and (b) respectively of Figures 1-5. The results for the Cheung-Ng and Hong test statistics are very similar; hence, to save space, only results for the former test are shown.

Figure 1 shows that neglecting structural breaks in volatility has only minor effects when just one of the series experiences a volatility change, although tests using $|K| > 1$ are slightly oversized when the volatility shift is relatively large. In contrast, simultaneous

[4]Results for other nominal significance levels are qualitatively similar and are available upon request.

changes in volatility lead to substantially larger size distortions, irrespective of whether the volatility change is identical or opposite (Figures 2 and 3). In addition, these plots demonstrate that the size distortion is symmetric in the ratio $\frac{\sigma_a^2}{\sigma_b^2}$, and becomes larger as the sample size T increases. Figure 4 makes clear that the timing of the volatility change matters: the size distortions are largest when the simultaneous volatility decline occurs at one-third of the sample period. Finally, from Figure 5 it appears that simultaneous volatility changes lead to the largest size distortions, with the distortion declining as the time interval between breaks increases.

Since volatility changes have been shown to occur across a wide range of observed economic and financial time series, the severity of the size distortions revealed in our results appears to indicate that these tests may, in practical applications, provide unreliable inference about (the non-existence of) causality in variance. Size problems arise particularly when both series exhibit volatility changes in close temporal proximity, in which case the tests frequently and incorrectly attribute this occurrence to an underlying causality.

4 Solving the problem: pre-testing for volatility breaks

The Monte Carlo results in the previous section convincingly demonstrate that the tests for causality-in-variance suffer from considerable size distortions when breaks in volatility are neglected. In this section we explore the usefulness of pre-testing for structural changes in volatility to remedy this problem.

As in McConnell and Perez-Quiros (2000) and Sensier and van Dijk (2004), we test for a break in volatility using the mean of the absolute value of the demeaned series. Let $W_T(\tau_v)$ denote the Wald test of the null hypothesis $H_0 : \delta_{i1} = \delta_{i2}$ in the regression

$$\sqrt{\frac{\pi}{2}}|y_{it} - \hat{\mu}_i| = \delta_{i1}(1 - \mathtt{I}(t > \tau_i)) + \delta_{i2}\mathtt{I}(t > \tau_i) + \varepsilon_{it}, \qquad t = 1, \ldots, T \qquad (7)$$

where, in our context, $\hat{\mu}_i$ is again the sample mean of y_{it}, τ_i is the specified break date and $\mathtt{I}(A)$ is an indicator function for the event A[5]. We treat the break date τ_i as unknown

[5]If y_{it} is iid and follows a normal distribution with mean μ_i and variance σ_i^2, $\sqrt{\frac{\pi}{2}}|y_{it} - \hat{\mu}_i|$ is a unbiased

and use the sup-Wald statistic developed by Andrews (1993), given by[6]

$$\text{SupW} = \sup_{\tau_{\min} \leq \tau_i \leq \tau_{\max}} W_T(\tau_i). \tag{8}$$

Both pre- and post-break periods are required to contain at least 10 % of the available observations, that is we set $\tau_{\min} = [\pi T]$ and $\tau_{\max} = [(1 - \pi)T] + 1$ with $\pi = 0.10$, where $[\cdot]$ denotes integer part. We use the method of Hansen (1997) to obtain approximate asymptotic p-values.

This volatility break test is applied prior to the causality-in-variance test. If, using (8), the null hypothesis of no volatility change in y_{it} is rejected at a 5% significance level, we take the estimated volatility break at $\hat{\tau}_i$ (the time period that minimizes the sum of squared residuals in (7)) into account when standardizing the series. This is achieved by replacing $\hat{h}_{it}$ in (3) by the sample variance before (after) $\hat{\tau}_i$ for all $t \leq (>)\ \hat{\tau}_i$.

Results are shown in panels (c) and (d) of Figures 1-5. In all cases this pre-testing procedure yields empirical rejection frequencies close to the nominal significance level of 5%. For $K = \pm 1$, the procedure tends to yield some under-sizing, with some evidence of over-sizing for larger K. Nevertheless, these distortions are relative modest, with (for example) the empirical size almost always being between 4% and 8% when $T = 160$. Therefore, pre-testing for breaks in volatility provides an effective solution to the size problem of the causality-in-variance tests.

5 Conclusions

In this paper we have examined the effects of volatility breaks on the tests for causality-in-variance developed by Cheung and Ng (1996) and Hong (2001). Based on an extensive set of Monte Carlo simulations, we demonstrate that these tests suffer from severe size

estimator of σ_i.

[6] Average and exponential statistics, computed as $\text{AveW} = \frac{1}{\tau_{\max} - \tau_{\min} + 1} \sum_{\tau_i = \tau_{\min}}^{\tau_{\max}} W_T(\tau_i)$ and $\text{ExpW} = \ln\left(\frac{1}{\tau_{\max} - \tau_{\min} + 1} \sum_{\tau_i = \tau_{\min}}^{\tau_{\max}} \exp\left(\frac{1}{2} W_T(\tau_i)\right)\right)$, respectively, render qualitatively similar results. The same holds for Lagrange Multiplier and Likelihood Ratio based tests. Details are available upon request.

distortions when such breaks are neglected. Pre-testing for structural changes in volatility is shown to remedy this problem. Therefore, we recommend that these causality-in-variance tests should be applied only after such pre-testing for breaks in volatility.

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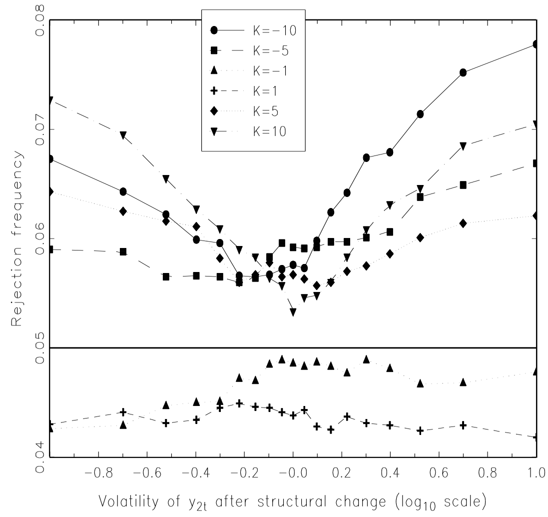
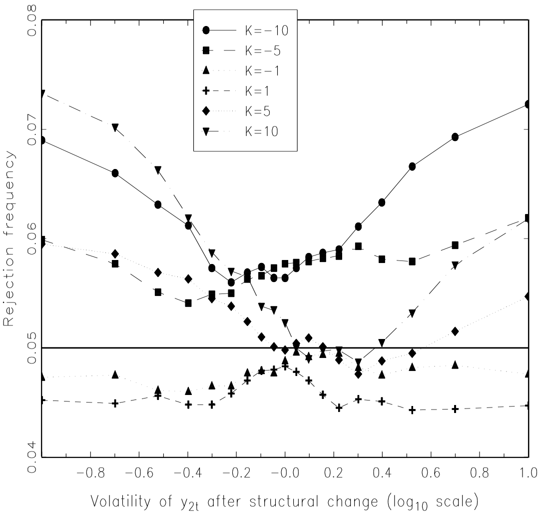


(a) Raw series, $T = 160$



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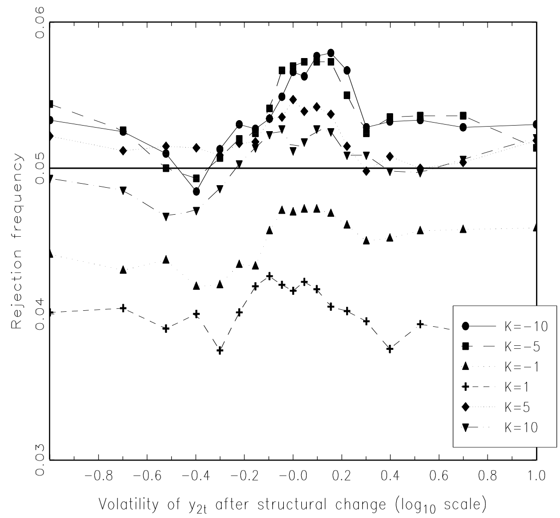

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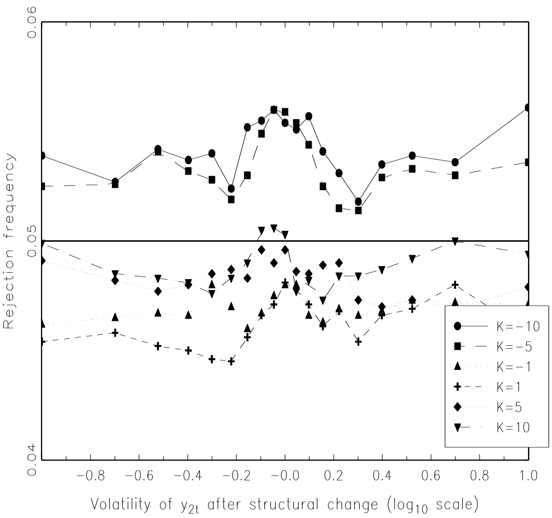
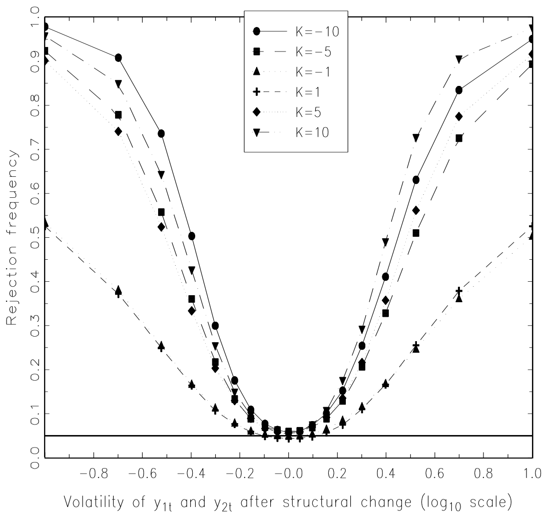


(d) Pre-tested series, $T = 480$

Figure 1: Rejection frequencies of the Cheung-Ng test (5) at 5% nominal significance level. Positive (negative) values of K indicate the null hypothesis is no causality-in-variance from y_{2t} to y_{1t} (from y_{1t} to y_{2t}). Experiment A: shift in volatility of y_{2t} only.



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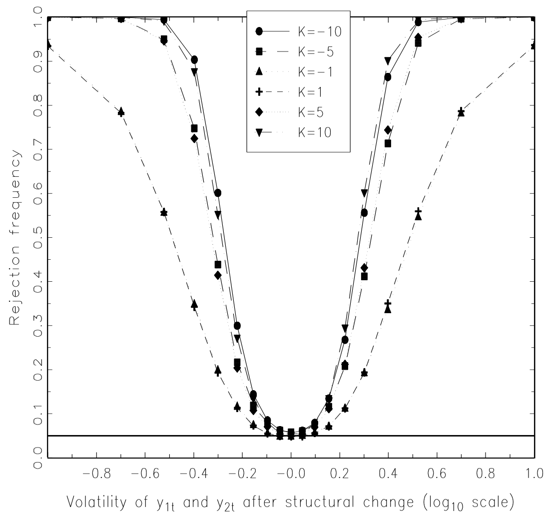

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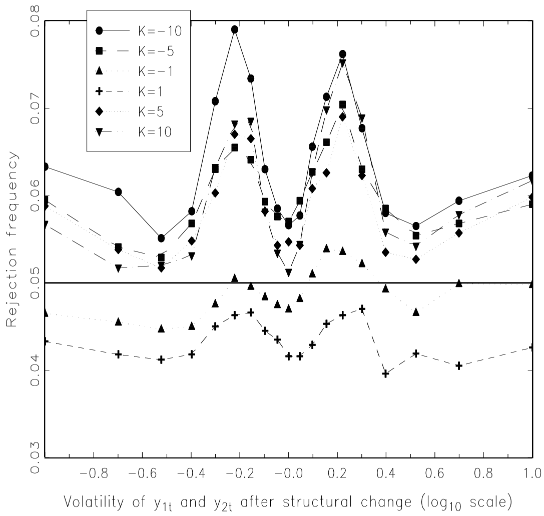

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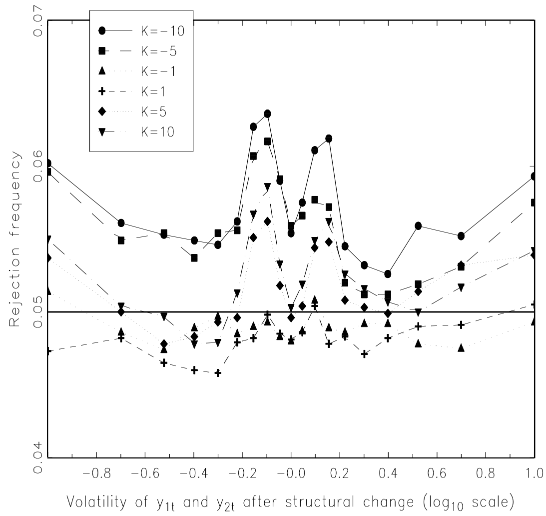
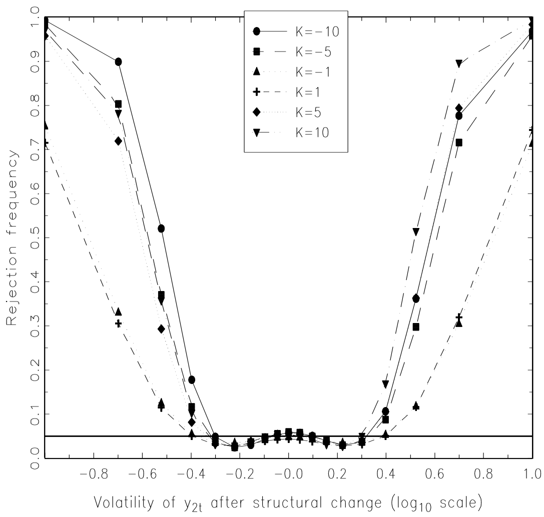


(d) Pre-tested series, $T = 480$

Figure 2: Rejection frequencies of the Cheung-Ng test (5) at 5% nominal significance level. Positive (negative) values of K indicate the null hypothesis is no causality-in-variance from y_{2t} to y_{1t} (from y_{1t} to y_{2t}). Experiment B: simultaneous and identical shift in volatility of y_{1t} and y_{2t}.



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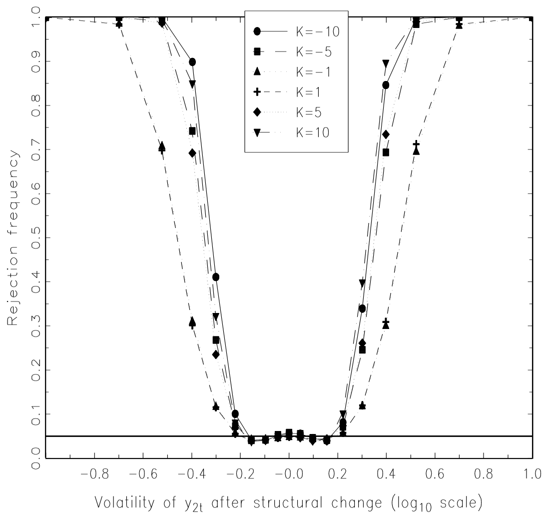

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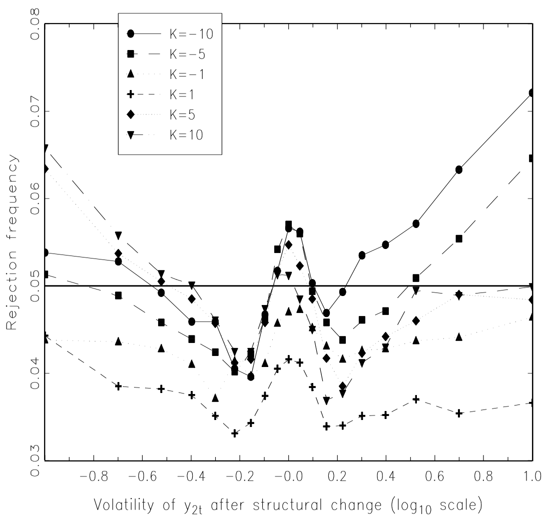

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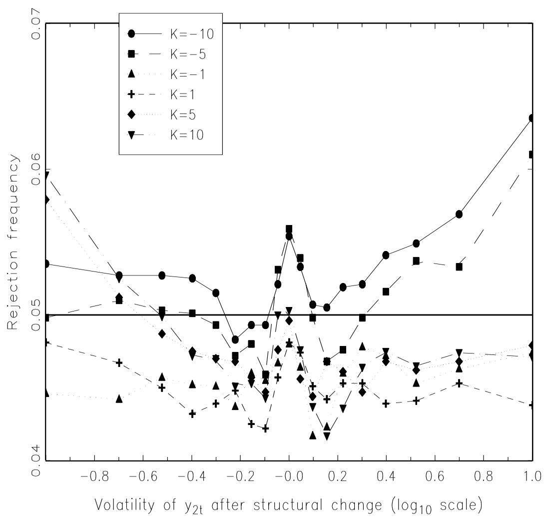
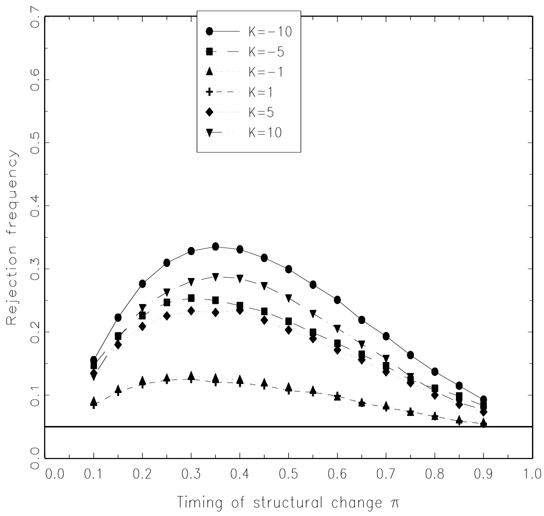


(d) Pre-tested series, $T = 480$

Figure 3: Rejection frequencies of the Cheung-Ng test (5) at 5% nominal significance level. Positive (negative) values of K indicate the null hypothesis is no causality-in-variance from y_{2t} to y_{1t} (from y_{1t} to y_{2t}). Experiment C: simultaneous but opposite shift in volatility of y_{1t} and y_{2t}.



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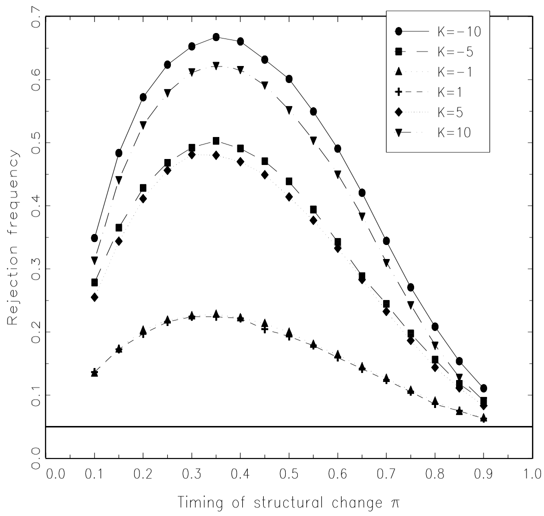

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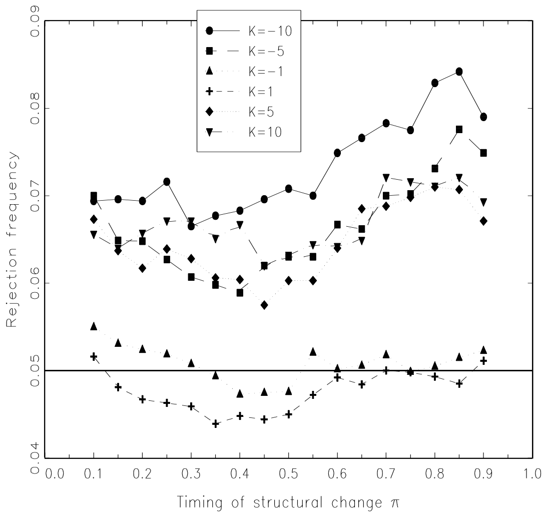

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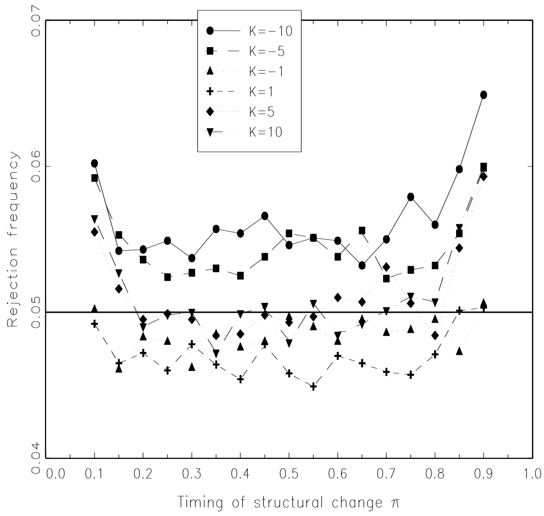
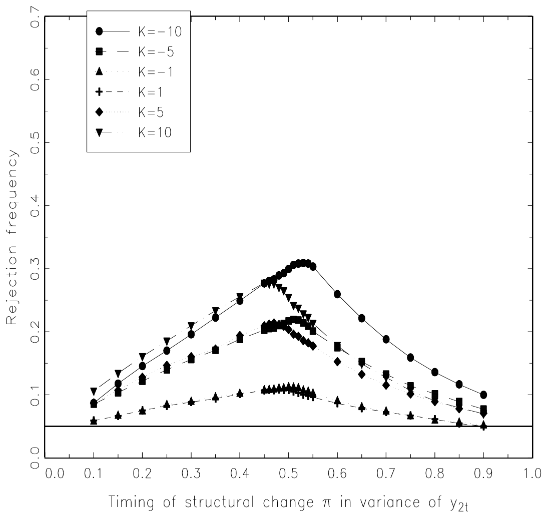


(d) Pre-tested series, $T = 480$

Figure 4: Rejection frequencies of the Cheung-Ng test (5) at 5% nominal significance level. Positive (negative) values of K indicate the null hypothesis is no causality-in-variance from y_{2t} to y_{1t} (from y_{1t} to y_{2t}). Experiment D: simultaneous and identical decline in volatility of y_{1t} and y_{2t}.



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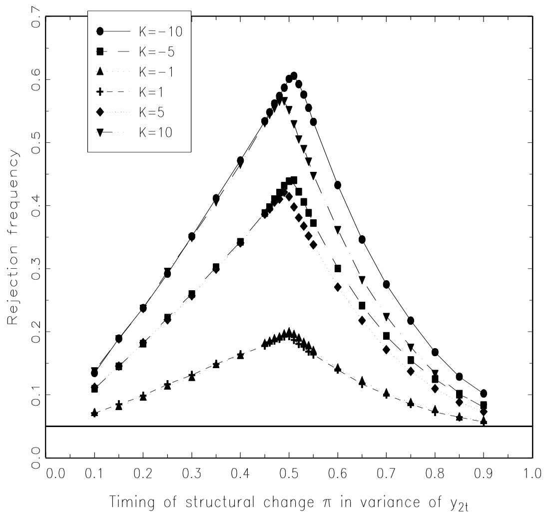

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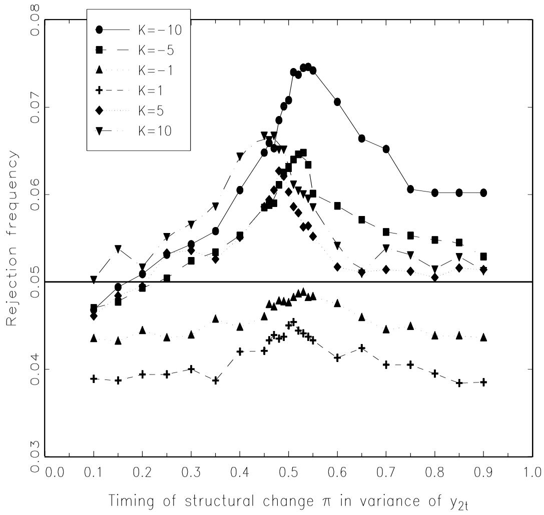

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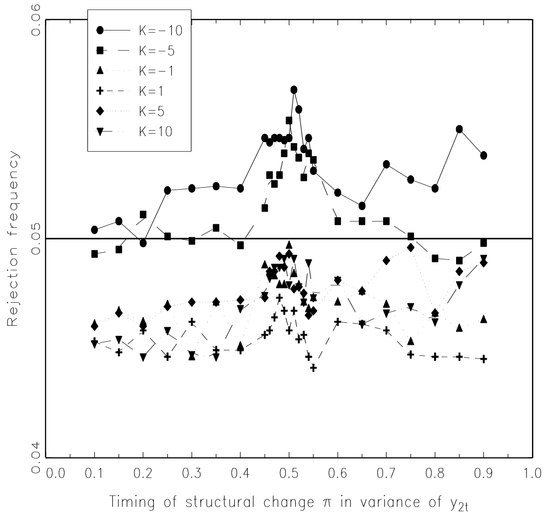

(d) Pre-tested series, $T = 480$

Figure 5: Rejection frequencies of the Cheung-Ng test (5) at 5% nominal significance level. Positive (negative) values of K indicate the null hypothesis is no causality-in-variance from y_{2t} to y_{1t} (from y_{1t} to y_{2t}). Experiment F: identical decline in volatility of y_{1t} and y_{2t} with different timing.