Optimisation of connections to a fibre network

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1 Introduction

In the Netherlands the last couple of years the monopoly situation of having one Telecom Service Provider (KPN Telecom) has changed to a competitive situation. Other companies have penetrated into the telecom market offering a wide range of telecommunication services.
Such a competitive situation together with the development of new technologies have forced the Telecom Service Providers (TSP's) to watch more than before the costs of investment into those new technologies and the resulting pricing of services to the customers.
Therefore TSP's are looking for sophisticated optimisation methods to reduce the costs of their communication services especially for new areas such as the application of fibre technology.
There are several reasons why one should make the decision to apply fibre for telecommunication purposes. Fibre is being considered as the transmission medium of the future because fibre deadens the signals much less than the traditional media such as copper and coax. This means among other things that less amplifiers are needed; a lot of data can be transmitted at the same time and there are only a few failures. Another advantage is that fibre cables are thin and light so that they can be put into the ground rather easily.
The TSP's have the objective to minimise the costs of constructing and managing a fibre network.
Therefore essential decisions have to be made about the design of the network.
This article describes how the management of a TSP can be supported by mathematical modelling in making optimal decisions about the design and use of a fibre network.
The optimisation models are based on the practical situation at Enertel being one of the new TSP's. For Enertel a national backbone was already realised. The main problem to be solved concerned the optimisation of the access to the fibre network.

2 Decisions in perspective.

Although the main subject of this article is the optimisation of the access to the fibre ring, the decisions about optimal access are as a matter of fact part of other decisions at other levels.
In figure 1 a review of the decision levels is given mentioning the main aspects at each level.
On the highest level the choice of technology should be made. Relevant options are : the use of fibre, the use of leased lines and the use of microwave. The latter one might be useful as a temporary solution until the market for fibre has reached such a size that fibre will become economically feasible.
Once the decision has been made to use fibre the optimisation of the fibre network will be the next subject.
Decisions should be made about the design of the fibre network that is to say about number and position of fibre rings and of the number and positions of the Points of Presence (PoP). A PoP connects fibre rings along which the customers are situated with the fibre backbone. A PoP can be considered as a kind of distribution point.
Then we come to the decision level to which this article refers : the access optimisation.

Access optimisation implies the determination of the number and position of the so-called flexibility points being the connections of the customers to the fibre ring so that the total costs of connecting the customers are minimised.
The last kind of decisions concern the determination of the cost price and of the tariffs which are as usual based on the value of the cost price, but also and commercial insight into the reaction of the market on the proposed tariffs compared to tariffs of other telecom operators.
It will be obvious that decisions on a certain level will be influenced by feedback of decisons at lower levels. For instance if it is not possible to offer attractive tariffs to the market a relatively small number of customers is willing to be connected to a fibre ring.

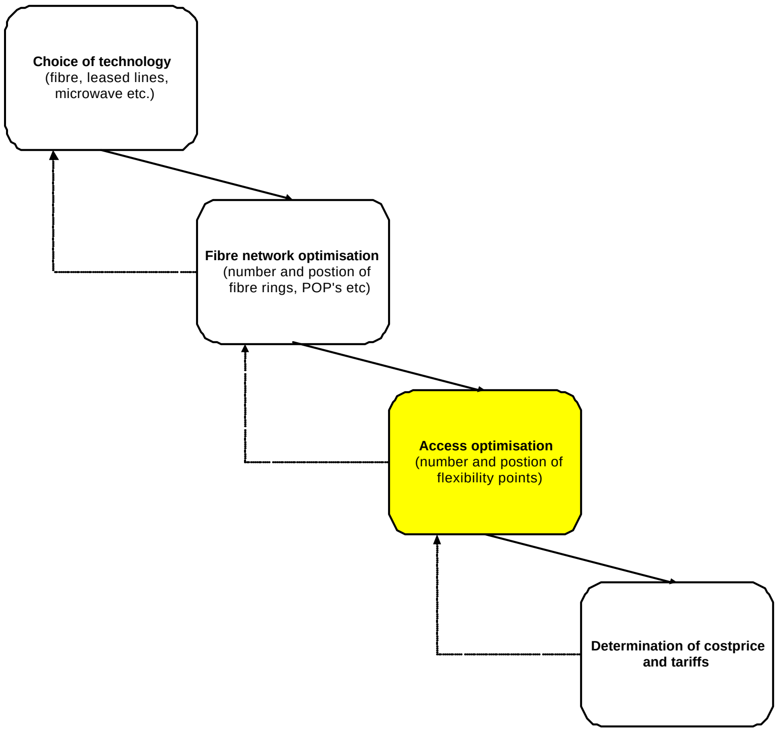


Figure 1 - Review of decisions

3 Overview of models to optimise fibre networks

Quite a number of models have been developed to optimise fibre networks. Cortés et al. [6] have formulated a quantitative model evaluating the global topological design of an optical fibre network over synchronous digital hierarchy (SDH; see also [10]). This model describes the economic aspects The model includes the economic aspects involved in the project, the civil works, the capacity evaluation costs and the introduction of reliability conditions.
The output of the problem must be the location of transmission nodes (hubs), the locations of transmission links and their capacities, the paths to transport the origin-destination pair demand and the costs associated to the planning and dimensioning process in the network.
The same authors [7] have described in another paper an operations research application to the design of an optic fibre network based on B-ISDN for the Andalusian region. The economical appraisal is the main consideration in order to take the appropriate decisions: hub location, region sizes and selection of the urban

nodes that will recieve telecommunications contents. A decision support system with a graphic interface taht allows interactive analysis of different scenarios is presented.
Several computing techniques can be found dealing with the real time network operation. Diverse authors [9,15] have shown the use of neural approaches to solve the real time traffic routing problem. Aboelela and Douligeris [1] present a fuzzy multiobjective optimization model to develop a routing algorithm to guarantee the various quality of service characteristics requested by the wide range of applications supported by B-ISDN. Chou and Wu [5] have developed a hybrid neural-genetic procedure to deal with the bandwidth allocation of virtual paths in ATM networks. However not many references appear with respect to the planning and dimensioning stage. In spite of it, the global telecommunication networks topological design has been dealt with in the bibliography extensively with the use of traditional operational research approaches. Chang and Gavish [3,4] have developed communication models among hubs solved by means of Lagrangian relaxation techniques. Cox et al. [8] have implemented an exhaustive cost evaluation model for the US WEST in the Colorado area solved with the commercial optimisation software CPLEX. More recently, Yoon et al. [16] have developed with success an important model for hierarchical networks attending to hubs and terminal nodes broached with a dual ascent procedure.
Holmberg and Yuan [12] propose a Lagrangean heuristic as a common approach to several fixed charge network design problems, capacitated or uncapacitated, directed or undirected, possibly with staircase costs. Klincewicz et al. [13] describe a heuristic approach for designing tributary networks bases on self-healing rings (SHRs) . A common architecture for a telecommunications network is considered consisting of several tributary (often called access) networks, which connect locations to hubs and a backbone network, which interconnects the hubs.
The tributary network consists of multiple ring families, and each of those is comprised of one or more SHRs, called "stacked" rings. The SHRs in a given ring family are routed over the same cycle of optical fibre cables, but each SHR serves only a subset of locations along the cycle. The tributary ring network design is viewed as a complex version of a vehicle routing problem with a single-depot and multiple vehicles.
Belvaux et al. [2] have studied the problem of optimal placement of add/drop multiplexers to the telecommunication networks at France Telecom. Given a set of centres in a city or conglomeration linked together on a ring architecture, given the expected demand between the centres and an essentially unlimited availability of rings of fixed capacity on the network the demand pairs and corresponding add/drop multiplexers should be assigned to the rings so as to satisfy the demands and minimise the number of 'costly' multiplexers installed.
None of the above mentioned models describes how to connect individual customers (buildings) to a fibre ring by using flexibility points in such a way that the access costs will be minimised. This will be the subject of the following chapters.

4 The access situation and its cost elements

To connect the customers to a fibre ring a so-called tail between the customer and the fibre ring will be constructed consisting of a hand hole with a socket and an individual fibre cable having the hand hole and the building of the customer as endpoints. The hand hole with the socket will be called a flexibility point. In the socket (a kind of box) the individual fibre cable and the fibre ring cable will be connected to each other physically.

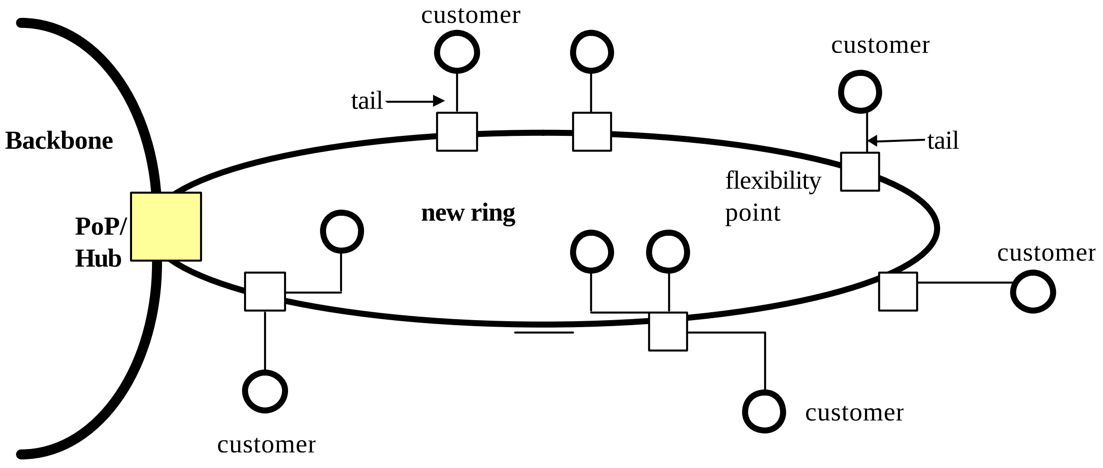


Figure 2: Construction of a new fibre ring

In practice more than one customer (building) can be connected to one flexibility point as illustrated by figure 3. Once the ring has been constructed it is rather easy to connect new customers to the same flexibility point by just opening the hand hole and adapting the contents of the socket. This explains as a matter of fact the use of the word flexibility point

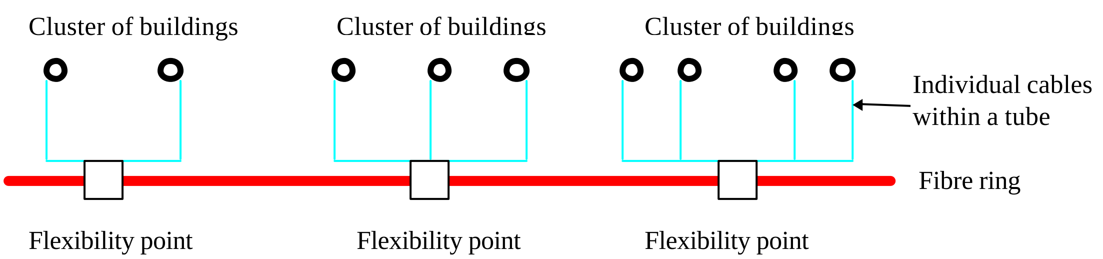


Figure 3: Ways of connecting buildings to flexibility points

The cost elements which are relevant for the determination of the optimal access are:

k_{fl} = costs of constructing a flexibility point
k_{las} = costs of welding an individual cable to a socket
k_{opp} = costs to break the surface and of recovery per meter
k_{gr} = costs of digging per meter
k_{mb} = initial expense and costs of placing of a tube per meter
k_{kb} = initial expense of an individual fibre cable per meter

k_{bl} = costs of blowing an individual cable into a tube per meter
k_{cr} = costs of making a crossing per meter (in case a road is between a building and a flexibility point)

These costs elements will be used in the connecting optimisation models The most comprehensive model describes the situation at which buildings are positioned at different sides of the fibre ring and a road is separating the fibre ring from the buildings at one side of the fibre ring. The latter aspect requires that in order to connect these buildings to the fibre ring the road should be crossed at one or more points. The corresponding model will be discussed in chapter 7.
The results should be the optimal number and positions of flexibility points and crossings and the optimal assignment of the buildings to those flexibility points and crossings in order to get minimal costs of connecting the buildings to the fibre ring.

Less complicated models describe the following situations:
- all buildings are positioned at the same side of the fibre ring and have an equal distance to the fibre ring (chapter 5).
- the buildings are positioned at different sides of the fibre ring and at unequally distances to the fibre ring (chapter 6).

5 Connection optimisation model for buildings at one side of and at equal distance to the fibre ring

The optimisation model to connect buildings being positioned at one side of the fibre ring an with equal distances to that fibre ring has to give a solution for the following questions:

1. How may flexibility points are needed to connect all buildings by minimal costs?
2. To which flexibility point should a building be assigned?
3. Which are the optimal positions of the required flexibility points?

Figure 4 illustrates the situation to be optimised.

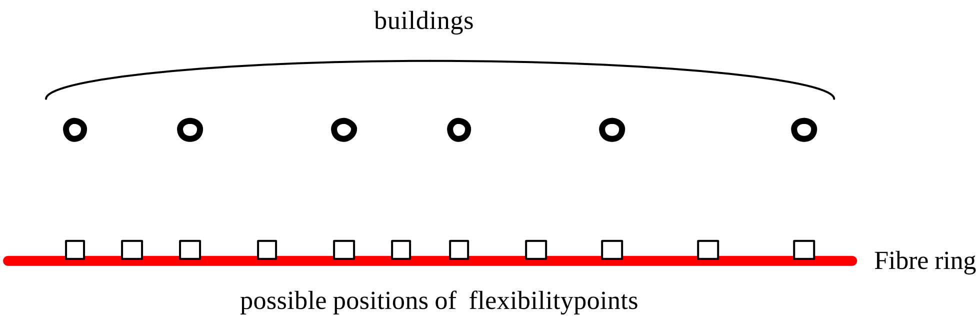


Figure 4: Situation to be optimised

5.1 Formulation of the facility location problem

The problem above can be considered as the so-called *facility location problem on one line* as described by Wagelmans[14] and Hillier and Lieberman[11].

To solve the problem the following variables have been defined:

n = number of buildings

m = maximum number of buildings to be connected to one flexibility point

x_{ij} = decision variable having the value 1 if building i (i = 1,…,n) will be connected to the flexibility point at position j (j = 1,…,2n-1) and having the value 0 otherwise

y_j = decision variable having the value 1 if the flexibility point at position j (j = 1,….,2n-1) will be used and having the value 0 otherwise

f_j = costs of the flexibility point at position j (j = 1,…,2n-1)

c_{ij} = costs to connect building i (i = 1,…,n) to the flexibility point at position j (j = 1,…,2n-1)

The facility location problem has been modelled as follows :

$$MIN \qquad \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} + \sum_{j=1}^{m} f_j y_j \qquad (1)$$

Subject to

$$\sum_{j=1}^{2n-1} x_{ij} = 1 \qquad i = 1,..,n \qquad (2)$$

$$\sum_{i=1}^{n} x_{ij} \le m \qquad j = 1,..,2n-1 \qquad (3)$$

$$x_{ij} \le y_j \qquad i = 1,..,n \quad j = 1,..,2n-1 \qquad (4)$$

$$x_{ij}, y_j = 0 \;,\; 1 \qquad i = 1,..,n \quad j = 1,..,2n-1 \qquad (5)$$

(1) *objective function consisting of the total costs to connect the buildings and to construct the required flexibility points; the coefficient c_{ij} is equal to k_{las}+ h*(k_{opp} + k_{gr} + 2*k_{mb} + k_{kb} + k_{bl}) + d*(2*k_{mb} + k_{kb} + k_{bl}), where h is the shortest distance from the building to the ring and d is the length of the tube along the ring to connect the building to the flexibility point; the coefficient f_j is equal to k_{fl}*

(2) *restriction which describes that a building can be connected to one flexibility point only*

(3) *restriction which describes that no more than M buildings can be connected to one flexibility point*

(4) *restriction which describes that a building can not be connected to a flexibility point which is not being used*

(5) *the decision variables x_{ij} en y_j are binary variables (value 0 or 1)*

For a practical application of the above mentioned model the following issues have been studied into more detail:

- the break-even point for the distance between two adjacent buildings to be connected to one and the same flexibility point or to two separate ones
- dividing the total number of buildings into one or more clusters
- the determination of the possible positions of the flexibility points

5.2 Connection of two buildings

Figures 5a and 5b show two ways of connecting two buildings to a fibre ring. At figure 5a each building has its own flexibility point, while in figure 5b both buildings have use the same flexibility point

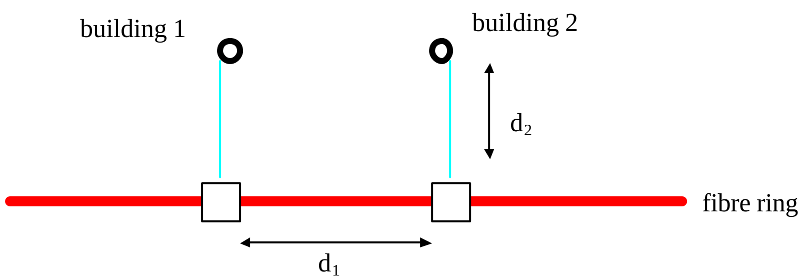


Figure 5a: Connection of two buildings by two flexibility points

To connect the two buildings in the situation of figure 5a the following costs will be made:

$$K_1 = 2*k_{fl} + 2*k_{las} + 2*d_2*(k_{opp} + k_{gr} + 2*k_{mb} + k_{kb12} + k_{bl})$$

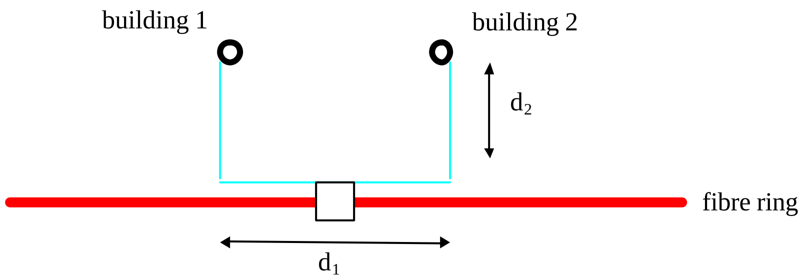


Figure 5b: *Connection of two buildings by one flexibility point*

To connect the two buildings in the situation of figure 5b the following costs will be made:

$$K_2 = k_{fl} + 2*k_{las} + 2*d_2*(k_{opp} + k_{gr} + 2*k_{mb} + k_{kb} + k_{bl}) + 2*½*d_1*(2*k_{mb} + k_{kb} + k_{bl})$$

To find the right choice between the two situations we put K_1 equal to K_2 which gives:

$$d_1 = \frac{k_{fl}}{2*\;\; k_{mb} + k_{kb} + k_{bl}} \qquad (1)$$

as the break-even distance between the two buildings.

It can easily be proven that it does not matter where the flexibility point in figure 5b is situated provided it is between the positions of the two flexibility points in figure 5a:

Suppose the flexibility point in figure 5b is positioned at a distance p at the right from the left flexibility point in figure 5a. The remaining distance to the flexibility point at the right is equal to d-p.
The relevant costs as far as the cost of connection are concerned are:

$$p*(2*k_{mb} + k_{kb} + k_{bl}) + (d - p)*(2*k_{mb} + k_{kb} + k_{bl}) = d*(2*k_{mb} + k_{kb} + k_{bl})$$

Thus the value of p does not matter.

5.3 Connection of n buildings

The results of the previous section provide the basis for the following theorems

Theorem 1

If n buildings should be connected to a fibre ring and the distance between two adjacent buildings is larger than d_1 (in (1)), then these two buildings do not belong to the same cluster of buildings, which means they will not be connected to the same flexibility point.

Proof

Let a and b be two adjacent buildings and let d be the distance ($d > d_1$) between these two buildings (the distance is measured over the ring). See figure 6.

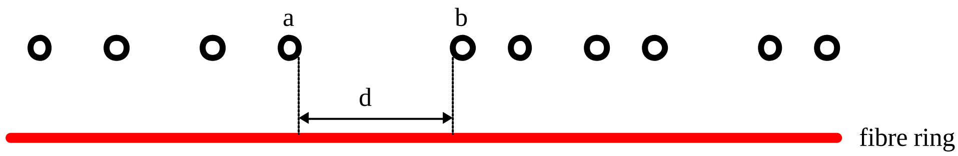


Figure 6: Buildings a and b with a mutual distance of d

If *a* and *b* would be connected to a flexibility point to the left of *a* respectively to the right of *b* (along the ring) the distance (over the ring) between *b* and the flexibility point respectively between *a* and the flexibility point is larger than d_1 (see (1)).
Then it is cheaper to connect *b* respectively *a* to a separate flexibility point.
If *a* and *b* would be connected to a flexibility point between *a* and *b* (along the ring) we have the situation as described by figure 5b. as we have to bridge over a distance d which is larger than d_1 (see(1)) it is cheaper to give *a* and *b* an 'own' flexibility point.

Theorem 2

If we consider a cluster of n buildings numbered from 1 to n, the optimal position of the flexibility point is at the median of the n buildings.

Proof

Let M be the point with n_1 buildings at the left and n_2 (= $n - n_1$) buildings at the right.

When we change the position of M by p to the right the total costs of connection change by:

$$(n_1-n_2)*p*(2*k_{mb} + k_{kb} + k_{bl}) = (2n_1-n)*p*(2*k_{mb} + k_{kb} + k_{bl})$$

When we change the position of M by p to the left the total costs of connection change by:

$$(n_2-n_1)*p*(2*k_{mb} + k_{kb} + k_{bl}) = (n-2n_1)*p*(2*k_{mb} + k_{kb} + k_{bl})$$

From the results above it is easily to be seen that we can reduce the costs of connection by moving M to the right if $n_1 < n_2$ respectively to the left if $n_1 > n_2$. Only for $n_1 = n_2$ no further reduction of the costs are possible. This means that M should be the middle one of all n buildings if n is odd and between the two middle ones if n is even.

5.4 Optimisation method

The optimisation method consists of the following steps.

1. We divide the total number of buildings into clusters according to theorem 1. This means each time we have two adjacent buildings with a mutual distance larger than d_1 we get a new cluster.
2. We determine the possible positions of the flexibility points within each of the clusters separately using theorem 2, which says that a flexibility point will either be positioned straight under a building (see figure 5a) or everywhere between the two positions in figure 5a.. Figure 7 illustrates this step.

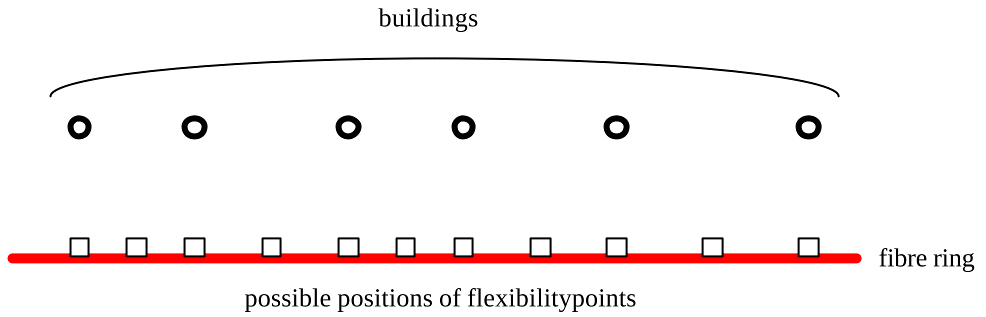


Figure 7: Possible postiions of flexibility points within one cluster of buildings

3. The optimal number and positions of the flexibility points will be determined for each cluster separately by using the assignment model from paragraph 3.1, where the possible positions of the flexibility points as the result of the previous step.

6 Connection optimisation model for buildings at two sides of the fibre ring and at unequal distances to the fibre ring

In the previous description it has been assumed that all buildings are along the same straight line, whereby the distance between each building and the fibre ring is the same.
In practice we have different distances between buildings and the fibre ring and buildings may be located at different sides of the ring. Figure 8 illustrates such a situation.

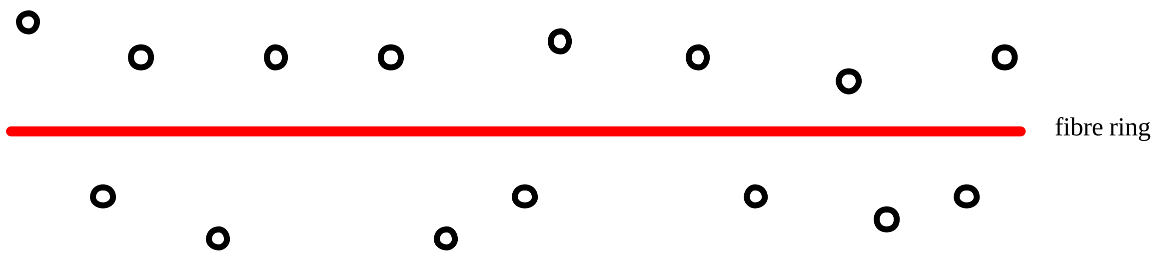


Figure 8: Buildings at both sides of a road

To find the optimal connections in the situation at figure 8 we start with the optimal connection considering all buildings at the same site of the fibre ring and with the same distance to it.
See the figure below.

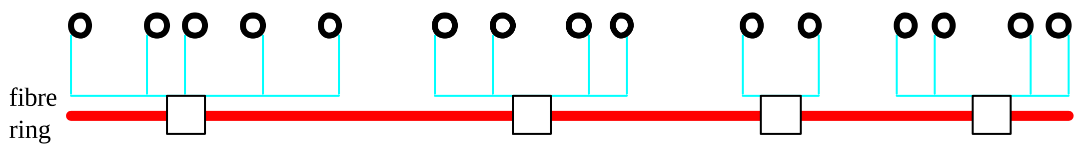


Figure 9: Optimal connection of buildings situated along one straight line

Moving the buildings back to their original position does not change anything to the optimal connection (see figure 10).

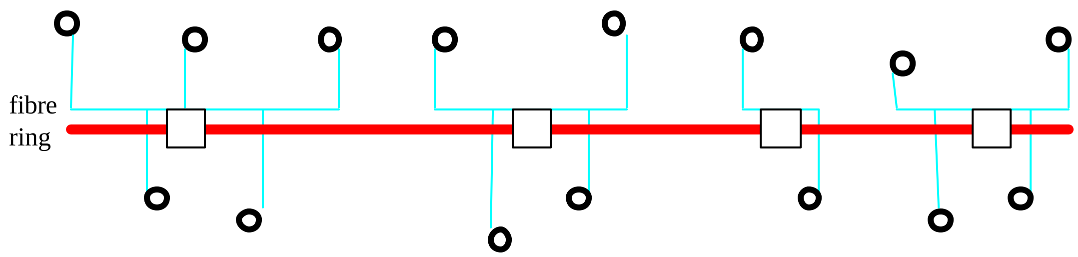


Figure 10: Optimal connection of buildings at two sides of the fibre ring

The reason for it is that the distance from the building to the ring does not influence the optimal connection, because we always dig from the building to the ring via the shortest way.
The optimal connection will depend on the distances between two adjacent buildings (along the ring), but these distances do not change.

7 Connection optimisation model for buildings at two sides of the fibre ring and an obstacle

In the most realistic situation we have different distances between buildings and the fibre ring and moreover an obstacle for instance a road or a canal might be situated between the buildings and the fibre ring.
Figure 11 illustrates such a situation.

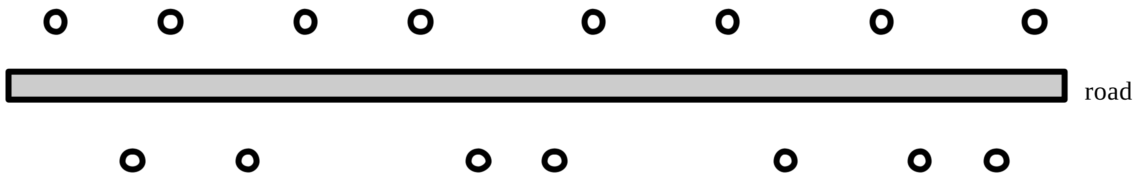


Figure 11: Buildings at both sides of a road

7.1 Optimisation when each building has its own crossing

To find the optimal connections in the situation at figure 8 we start with the optimal connection as given by figure 10 in chapter 6..

Now we add the road to figure 10 (see figure 12). We assume that for each building at the non-ring side a crossing should be done by a pressure technique under the road.

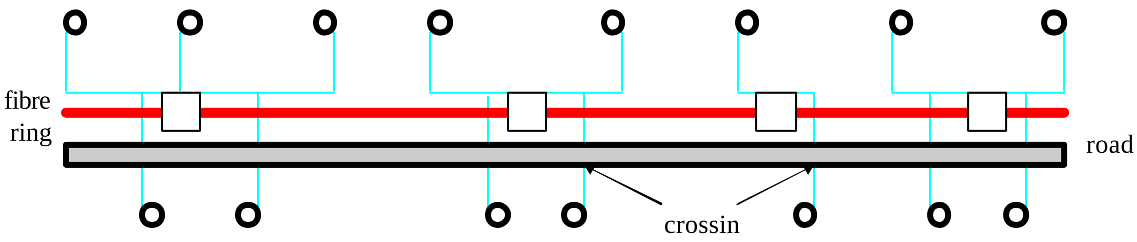


Figure 12: Optimal connection having a road as an obstacle

As each building at the non-ring side has its own crossing the addition of the road to the whole has no effect on the optimal connection; the number of flexibility points, their position and the assignment of buildings do not change at all.

7.2 One crossing for all buildings with the same flexibility point

However we have to do with another optimisation problem: to use one crossing for all buildings at the non-ring side (see figure 13), having the same flexibility point.

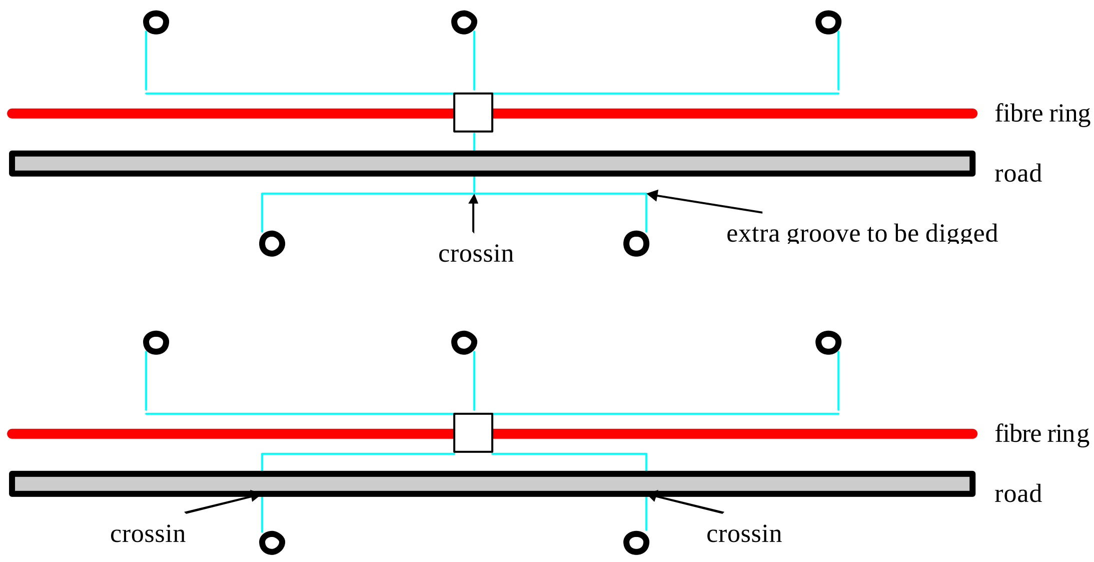


Figure 13: Connection of buildings with one or two crossings

We consider the situation with n buildings at the ring side and m at the non-ring side.
The obstacle (road) has a width of d_o. Th total length of the extra groove to be digged at the non-ring side is equal to d_g.
We now have the following costs of crossing:

For m crossings:

$$m{*}d_o{*}k_{cr}$$

For one crossing:

$$d_o{*}k_{cr}+ d_g{*}(k_{opp}+ k_{gr})$$

The break-even point can be found from:

$$d_g = \frac{(m-1) * d_o * k_{cr}}{k_{opp} + k_{gr}} \tag{2}$$

Formula (2) learns that it is cheaper to use only one crossing if the length of the extra groove to be digged is smaller than the value from (2). Otherwise it is better to have a crossing for each building.
However both situations (a separate crossing for each building at the non-ring side respectively one crossing for all) might be not optimal, but a mix could be. That is to say that we have to find the optimal number of crossings.
Figure 14 illustrates this.

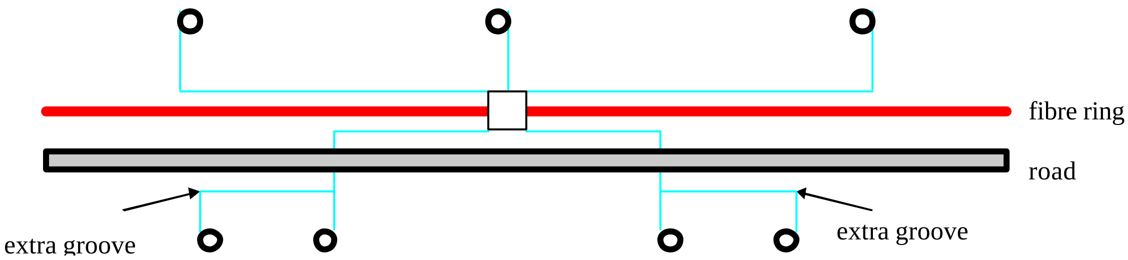


Figure 14: Connection with more common crossings

We have to make a choice between a connection with let's say m_1 crossings and a connection with m_2 crossings. We assume that m_2 is smaller than m_1, which means that the total length d_{g1} of the extra groove to be digged for m_1 crossings is smaller than d_{g2} for m_2 crossings.

$$d_{g_2} - d_{g_1} = \frac{(m_1 - m_2) * d_o * k_{cr}}{k_{opp} + k_{gr}} \tag{3}$$

This means that we should prefer m_2 crossings above m_1 if the length of the extra groove (which can exists of a number of subsequent grooves) is smaller than the value of formula (3); otherwise we have to use m_1 crossings.

The only question which is left is to determine the optimal position of a crossing. It is easily to be seen that the crossing should be done as close as possible to the appropriate flexibility point to prevent unnecessary placing of tubes, as illustrated by figure 14. In this figure we have an unnecessary surplus of d_1 .

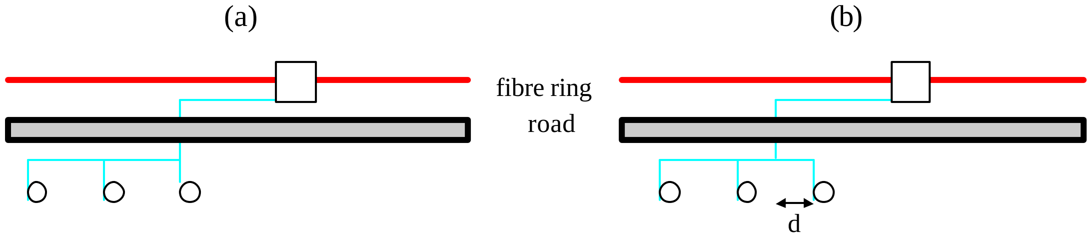


Figure 15: An optimal position for the crossing (a) and a non optimal one (b).

7.3 Overall optimisation for the flexibility points and the crossings

In the previous paragraphs we have assumed that the optimal number and positions of the flexibility points will not be effected by the number and positions of the crossings.
For instance when we would compare two buildings at the non-ring side we first look at formula (1) to decide whether we should have two flexibility points (figure 16a) with each one crossing or one flexibility point (figure 16b) with two crossings. Only in the latter case we use formula (2) with m = 2 to decide whether we should have one crossing (figure 16c) instead of two ones (figure 16b).

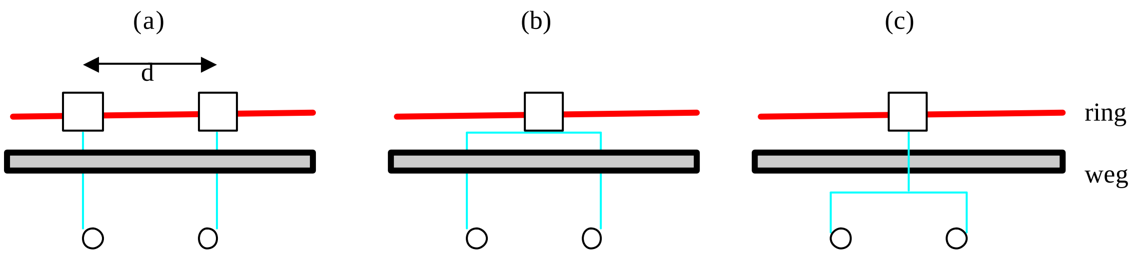


Figure 16: Possible connections and crossing for two buildings at the non-ring side

This means until now we have not taken into account that the costs in situation (c) could be lower than the costs in situation (a), although the costs in situation (b) are the higher than the costs in (a) and (c)..
This might be the case, when the cost per crossing becomes rather high.
For instance considering the costs in the three situations above we have:

Situation (a) : $2k_{fl} + 2d_0.k_{cr} + 2k_{las}$

Situation (b) : $k_{fl} + 2d_0.k_{cr} + 2k_{las} + d(2k_{mb} + k_{kb} + k_{bl})$

Situation (c) : $k_{fl} + d_0.k_{cr} + 2k_{las} + d(2k_{mb} + k_{kb} + k_{bl}) + d(k_{opp} + k_{gr})$

Situation (a) is cheaper than situation (b) if $k_{fl} - d(2k_{mb} + k_{kb} + k_{bl}) < 0$
Situation (c) is cheaper than situation (b) if $d_0.k_{cr} - d(k_{opp} + k_{gr}) > 0$
Thus when the first difference is smaller than the second one, or when $d_0.k_{cr}$ being the costs of a crossing is large enough, situation c) is the cheapest one. This means we should have one flexibility point when we have crossings instead of two flexibility points without crossings.

Practical values of the cost factors show that a relative high value of the costs of a crossings not very likely, but still a theoretical model has been developed to optimise the number and positions of both the flexibility points and the crossings at the same time.

The appropriate model has been described below and is based on the possible positions of the flexibility points and the crossings as illustrated at figure 17.

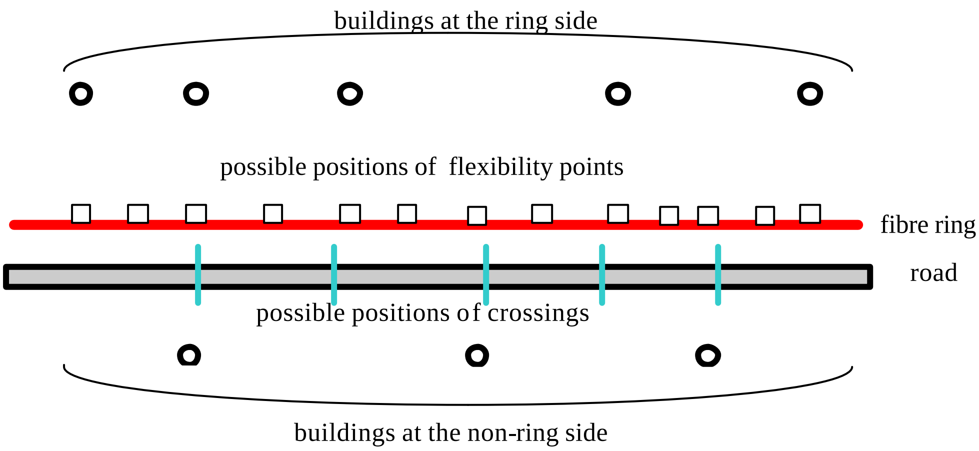


Figure 17: Situation to be optimised for both flexibility points and crossings

To solve the problem the following variables have been defined:

n = number of buildings

s = number of buildings at the non-ring side ($s \leq n$), numbered from 1 to s

m = maximum number of buildings to be connected to one flexibility point

x_{ijk} = decision variable having the value 1 if building i (i = 1,…,s) will be connected to the flexibility point at position j (j = 1,…,2n-1) using crossing k(k = 1,…,2s-1) and having the value 0 otherwise

x_{ij0} = decision variable having the value 1 if building i (i = s+1,…,n) will be connected to the flexibility point at position j (j = 1,…,2n-1) and having the value 0 otherwise

y_j = decision variable having the value 1 if the flexibility point at position j (j = 1,….,2n-1) will be used and having the value 0 otherwise

z_k = decision variable having the value 1 if the crossing at position k (k = 1,….,2s-1) will be used and having the value 0 otherwise

f_j = costs of the flexibility point at position j (j = 1,…,2n-1)

g_k = costs of the crossing at position k (k = 1,…,2s-1)

c_{ijk} = costs to connect building i (i = 1,…,s) to the flexibility point at position j (j = 1,…,2n-1) using crossing k (k = 1,…,2s-1)

c_{ij0} = costs to connect building i (i = s+1,…,n) to the flexibility point at position j (j = 1,…,2n-1)

The model is:

$$MIN \qquad \sum_{i=1}^{s}\sum_{j=1}^{2n-1}\sum_{k=1}^{2s-1} c_{ijk}x_{ijk} + \sum_{i=s+1}^{n}\sum_{j=1}^{2n-1} c_{ij0}x_{ij0} + \sum_{j=1}^{2n-1} f_j y_j + \sum_{k=1}^{2s-1} g_k z_k \qquad (1)$$

Subject to

$$\sum_{j=1}^{2n-1}\sum_{k=1}^{2s-1} x_{ijk} = 1 \qquad i = 1,..,s \qquad (2)$$

$$\sum_{j=1}^{2n-1} x_{ij0} = 1 \qquad i = s+1,..,n \qquad (3)$$

$$\sum_{i=1}^{s}\sum_{k=1}^{2s-1} x_{ijk} + \sum_{i=s+1}^{n} x_{ij0} \le m \qquad j = 1,..,2n-1 \qquad (4)$$

$$x_{ijk} \le y_j \qquad i = 1,..,n \quad j = 1,..,2n-1 \quad k = 0,...,2s-1 \qquad (5)$$

$$x_{ijk} \le z_k \qquad i = s+1,..,n \quad j = 1,..,2n-1 \quad k = 0,...,2s-1 \qquad (6)$$

$$x_{ijk}, y_j, z_k = 0,1 \qquad i = 1,..,n \quad j = 1,..,2n-1 \quad k = 0,...,2s-1 \qquad (7)$$

(1) objective function consisting of the total costs to connect the buildings and to construct the required flexibility points;
the coefficient c_{ijk} is equal to
*k_{las}+ $(h – d_0)$ *$(k_{opp} + k_{gr})$+ $(h + d + d_{cf})$*$(2*k_{mb} + k_{kb} + k_{bl}) + d_n*(k_{opp} + k_{gr})$,*
where h is the shortest distance from the building to the ring, d_0 is the length of the crossing, d is the lenght of the groove (parallel to the road) from the building to the crossing, is the distance between the crossing and the flexibility point and d_n is the distance (parallel to the road) between the building and the next building at the non-ring side or if it is smaller to the crossing;
the coefficient c_{ij0} is equal to $k_{las}+ h*(k_{opp} + k_{gr} + 2*k_{mb} + k_{kb} + k_{bl}) + d*(2*k_{mb} + k_{kb} + k_{bl})$, where h is the shortest distance from the building to the ring and d is the length of the tube along the ring to connect the building to the flexibility point;
the coefficient f_j is equal to k_{fl};
the coefficient g_k is equal to $d_0.k_{cr}$ where d_0 is the length of the crossing.

(2) restriction which describes that a building at the non-ring side can be connected to one flexibility point and one crossing only

(3) restriction which describes that a building at the ring side can be connected to one flexibility point only

(4) restriction which describes that no more than m buildings can be connected to one flexibility point

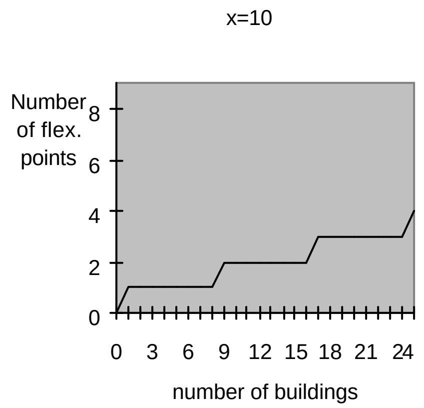
(5) restriction which describes that a building can not be connected to a flexibility point which is not being used

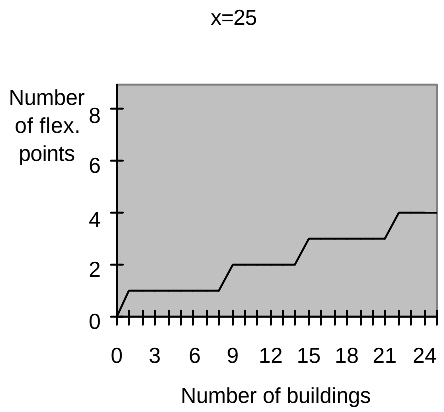
(6) restriction which describes that a building at the non-ring side can not be connected to a crossing which is not being used

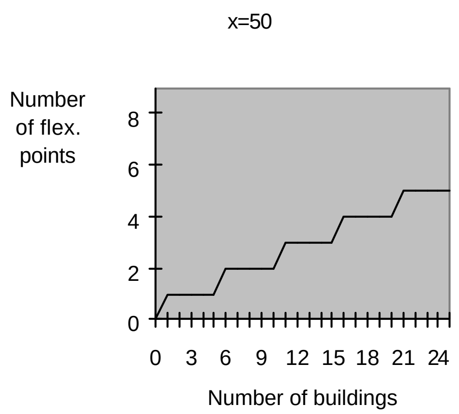
(7) the decision variables x_{ijk}, y_j and z_k are binary variables (value 0 or 1)

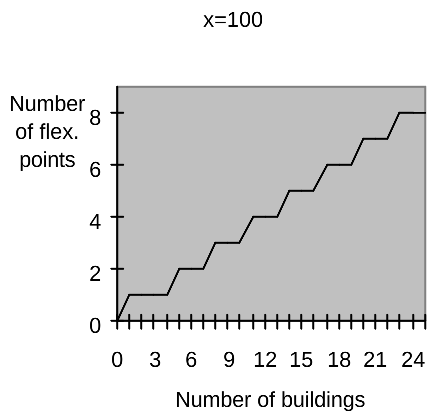
8 Some numerical results

In the figures below the optimal number of flexibility points are shown as a function of the cluster size (number of buildings) and of the mutual distance x between the buildings within one cluster.









9 Conclusions

The investigation in the field of connecting buildings to fibre rings has shown that the use of modelling techniques and a mathematical approach can be an efficient way of supporting the management of telecom service providers in their access policies.

Once having the formulas and procedures as mentioned in this article one is able to do not only optimisations for an existing number of buildings, but one is also able to calculate the expected results of adding future buildings to the same fibre ring. Therefore the same model can be used by running it twice: one time for the existing buildings and one time for all buildings including the future buildings.

Taking into account estimations of the probability a future building will be built at which place and at which month, models as above will be an efficient tool to support investments into fibre rings, whereby also the risks of the investments can be made visibly and thus responsible decisions can be taken.

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