Evaluating Portfolio Value-At-Risk Using Semi-Parametric GARCH Models

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ERIM REPORT SERIES *RESEARCH IN MANAGEMENT*	
ERIM Report Series reference number	ERS-2004-107-F&A
Publication	January 2009
Number of pages	28
Email address corresponding author	mverbeek@rsm.nl
URL	http://hdl.handle.net/1765/1833
Address	Erasmus Research Institute of Management (ERIM) Rotterdam School of Management / Rotterdam School of Economics Erasmus Universiteit Rotterdam P.O. Box 1738 3000 DR Rotterdam, The Netherlands Phone: + 31 10 408 1182 Fax: + 31 10 408 9640 Email: info@erim.eur.nl Internet: www.erim.eur.nl

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Library of Congress Classification (LCC) LCC Webpage	Mission: HF 5001-6182	
	Programme: HF 4001-4280.7	
	Paper: HG 4529.5	Portfolio management
Journal of Economic Literature (JEL) JEL Webpage	Mission: M	
	Programme : G 3	
	Paper: G 11 C 14	Portfolio choice Semiprarmetric methods
Gemeenschappelijke Onderwerpsontsluiting (GOO)		
Classification GOO	Mission: 85.00	
	Programme: 85.30	
	Paper: 85.30	Financieel management, financiering
Keywords GOO	Mission: Bedrijfskunde / Bedrijfseconomie	
	Programme: Financieel management, besliskunde	
	Paper: Portfolio-analyse, parametrische modellen	
Free keywords	Multivariate GARCH, semi-parametric estimation, Value-at-Risk, asset allocation	

Evaluating Portfolio Value-at-Risk using Semi-Parametric GARCH Models

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January 28, 2009

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Abstract

In this paper we examine the usefulness of multivariate semi-parametric GARCH models for evaluating the Value-at-Risk (VaR) of a portfolio with arbitrary weights. We specify and estimate several alternative multivariate GARCH models for daily returns on the S&P 500 and Nasdaq indexes. Examining the within sample VaRs of a set of given portfolios shows that the semi-parametric model performs uniformly well, while parametric models in several cases have unacceptable failure rates. Interestingly, distributional assumptions appear to have a much larger impact on the performance of the VaR estimates than the particular parametric specification chosen for the GARCH equations.

Keywords: multivariate GARCH, semi-parametric estimation, Value-at-Risk, asset allocation

JEL Classification: C14, C22, C53, G11.

1 Introduction

Value-at-Risk (VaR) defines the maximum expected loss on an investment over a specified horizon at a given confidence level, and is used by many banks and financial institutions as a key measure for market risk (see Jorion (2000) for an extensive introduction to VaR methodology). Estimating the VaR of a portfolio requires a model or, more generally, a set of assumptions relating to the joint (conditional) distribution of underlying asset returns. If returns in the portfolio are (conditionally) normally distributed the calculation of its Value-at-Risk is fairly straightforward, and moreover, the use of VaR as a risk measure is equivalent to using traditional measures like variance or standard deviation.

The normality assumption is restrictive, however, because it implies that investors do not attach particular weight to skewness and kurtosis or to specific quantiles of the distribution, while empirical evidence strongly indicates that asset returns exhibit nonzero skewness and kurtosis, particularly at high frequencies. This is stressed by several recent papers. Harvey and Siddique (2000), for example, argue that conditional skewness is an important factor explaining the cross-section of expected returns, while Barone Adesi, Gagliardini, and Urga (2004) investigate the role of co-skewness for testing asset pricing models.

Several recent papers analyze the risk-return trade off from a VaR perspective, for example Duffie and Pan (1997), Lucas and Klaassen (1998), Gourieroux, Laurent, and Scaillet (2000), Campbell, Huisman, and Koedijk (2001) and Alexander and Baptista (2002). These studies require an appropriate model for the tail behavior of the return distributions and their interdependence. Typically, the simultaneous distribution of the innovations in these models is assumed to be multivariate normal or Student t. This is restrictive, for example in the presence of nonzero third moments or in cases where the tail behavior is different across the portfolio components. Other papers, like Bingham and Kiesel (2002) and Bingham, Kiesel, and Schmidt (2003), apply a multivariate semi-parametric approach to determine the VaR of a portfolio, but do not explicitly allow for time-varying volatilities and correlations. For dynamic risk management using daily returns where volatility clustering and time-varying dependence are important this appears restrictive.

In this paper we investigate the implications on conditional Value-at-Risk (VaR) calculations for a portfolio when returns are described by a multivariate GARCH model, with an unrestricted distribution for the innovations. We investigate the empirical performance of the semi-parametric GARCH model and compare it with that of its parametric counterparts. Because of the multivariate nature of the GARCH model, we can fully take into account the dynamic interrelationships between the portfolio components, while the model underlying the VaR calculations is independent of the portfolio composition. That is, when the portfolio weights are adjusted, the same model can be used to determine the implied Value-at-Risk. In contrast, many existing approaches, including the regime-switching model of Billio and Pelizzon (2000) and the semi-parametric approach of Fan and Gu (2003), only allow one to determine the Value-at-Risk of a given asset or a given portfolio of assets. Accordingly, the multivariate GARCH approach has some potential in an optimal asset allocation framework, although we do not pursue this in this paper.

Because Value-at-Risk depends upon the joint tail behavior of the conditional distribution of asset returns, we expect that the parametric specifications only perform well in particular cases and at particular confidence levels, while the semi-parametric approach is expected to be robust against distributional misspecifications. Given the empirical evidence of asymmetries and – most importantly – excess kurtosis in the (conditional) distribution of stock returns, this is a potentially important advantage.

A wide range of multivariate GARCH models has been proposed to model time-varying variances and covariances, see Bauwens, Laurent, and Rombouts (2006) for a survey. In the empirical section, we estimate several popular semi-parametric multivariate GARCH models for the returns on two broad stock market indexes (S&P 500 and Nasdaq). The GARCH parameters are estimated without making restrictive assumptions about the distributions of the innovations, while the latter are estimated non-parametrically using a technique proposed by Hafner and Rombouts (2007). This way we obtain a model where we specify the first two conditional moments of the returns jointly in a parametric way while the rest of the return distribution is determined non-parametrically. The advantage of the multivariate approach is that the Value-at-Risk of any portfolio of assets can be determined from the GARCH estimates and the corresponding non-parametric estimate of the multivariate distribution of the innovations. Because our interest is in comparing alternative specifications for the multivariate GARCH models and in comparing parametric versus non-parametric distributions for the innovation, we limit the empirical application to only two dimensions.

The rest of this paper is organized as follows. Section 2 describes a number of alternative multivariate GARCH (MGARCH) specifications, and explains how the VaR of a portfolio can be calculated on the basis of these models. Section 3 describes the data and reports the estimation results for the MGARCH models for the S&P 500 and Nasdaq indexes over the period January 1988 – August 2006. Section 4 focuses on the VaR calculations and summarizes the results, by means of failure rates, for the different MGARCH models. Finally, Section 5 concludes.

2 Multivariate GARCH models

In this section, we describe several alternative semi-parametric multivariate GARCH models and link them to the conditional Value-at-Risk of a portfolio constructed from the different asset categories. The GARCH model describes the conditional distribution of a vector of returns, from which quantiles of the distribution of portfolio returns can be derived. Let r_t denote the N-dimensional vector of stationary returns. The model can be written as follows

$$\begin{aligned} r_t &= \mu_t(\theta) + \epsilon_t \\ \epsilon_t &= H_t^{1/2}(\theta)\xi_t \qquad t = 1, \ldots T, \end{aligned} \tag{1}$$

where $\mu_t(\theta)$ is an N-dimensional vector of conditional mean returns, ξ_t is an *i.i.d.* vector white noise process with identity covariance matrix and density $g(\cdot)$, and the symmetric $N \times N$ matrix $H_t(\theta)$ denotes the conditional covariance matrix of r_t. Unknown parameters are collected in the vector θ. Both the mean and covariance matrix are conditional upon the information set I_{t-1}, containing at least the entire history of r_t until $t-1$. Returns are thus assumed to be generated by a parameterized time varying location scale model. The conditional density of r_t is given by

$$f_{r_t|I_{t-1}}(r) = \mid H_t(\theta) \mid^{-1/2} g\left(H_t^{-1/2}(\theta)\ (r - \mu_t(\theta))\right). \tag{2}$$

The expression in (2) shows how the conditional distribution of the returns varies over time and allows one to estimate the time-varying quantiles of the return vector by replacing the parameter vector θ and the unknown density $g(\cdot)$ by their estimates, $\hat{\theta}$ and $\hat{g}(\cdot)$, respectively. We consider three multivariate GARCH models, that specify different functional forms for the conditional covariance matrix $H_t(\theta)$ and how it depends upon the information set I_{t-1}. Further, we combine these specifications with alternative assumptions about the density $g(\cdot)$ of ξ_t. The MGARCH models we consider are the diagonal VEC (DVEC) model and the dynamic conditional correlation (DCC) models of Tse and Tsui (2002) and Engle (2002). The specific assumptions of these three models are given in Definitions 1 to 3 below. We consider these alternative MGARCH models to make sure that our results are not specific to one particular, perhaps inappropriate, specification. Moreover, it allows us to analyze how sensitive the VaR estimates are with respect to the choice of the multivariate GARCH model.

Definition 1 *The* $DVEC(1,1)$ *model is defined as:*

$$h_t = c + A\,\eta_{t-1} + G\,h_{t-1}, \tag{3}$$

where

$$h_t = vech(H_t) \tag{4}$$

$$\eta_t = vech(\epsilon_t \epsilon_t'), \tag{5}$$

and $vech(.)$ *denotes the operator that stacks the lower triangular portion of a* $N \times N$ *matrix as a* $N(N+1)/2 \times 1$ *vector.* A *and* G *are diagonal parameter matrices of order* $(N+1)N/2$ *and* c *is a* $(N+1)N/2 \times 1$ *parameter vector.*

Definition 2 *The DCC model of Tse and Tsui (2002) or $DCC_T(M)$ is defined as:*

$$H_t = D_t R_t D_t, \tag{6}$$

where $D_t = diag(h_{11t}^{1/2} \dots h_{NNt}^{1/2})$, h_{iit} can be defined as any univariate GARCH model, and

$$R_t = (1 - \theta_1 - \theta_2)R + \theta_1 \Psi_{t-1} + \theta_2 R_{t-1}. \tag{7}$$

In (7), θ_1 and θ_2 are non-negative parameters satisfying $\theta_1 + \theta_2 < 1$, R is a symmetric $N \times N$ positive definite correlation matrix with diagonal elements $\rho_{ii} = 1$, and Ψ_{t-1} is the $N \times N$ sample correlation matrix of ϵ_τ for $\tau = t - M, t - M + 1, \dots, t - 1$. Its i,j-th element is given by:

$$\psi_{ij,t-1} = \frac{\sum_{m=1}^{M} u_{i,t-m} u_{j,t-m}}{\sqrt{(\sum_{m=1}^{M} u_{i,t-m}^2)(\sum_{h=1}^{M} u_{j,t-h}^2)}}, \tag{8}$$

where $u_{it} = \epsilon_{it}/\sqrt{h_{iit}}$. The matrix Ψ_{t-1} can be expressed as:

$$\Psi_{t-1} = B_{t-1}^{-1} L_{t-1} L_{t-1}' B_{t-1}^{-1}, \tag{9}$$

with B_{t-1} a $N \times N$ diagonal matrix with i-th diagonal element being $(\sum_{h=1}^{M} u_{i,t-h}^2)^{1/2}$ and $L_{t-1} = (u_{t-1}, \dots, u_{t-M})$, an $N \times M$ matrix.

Definition 3 *The DCC model of Engle (2002) or $DCC_E(S, L)$ is defined as:*

$$H_t = D_t R_t D_t \tag{10}$$

where $D_t = diag(h_{11t}^{1/2} \dots h_{NNt}^{1/2})$, h_{iit} can be defined as any univariate GARCH model, and

$$R_t = (diag\, Q_t)^{-1/2} Q_t (diag\, Q_t)^{-1/2}. \tag{11}$$

where the $N \times N$ symmetric positive definite matrix Q_t is given by:

$$Q_t = (1 - \alpha - \beta)\overline{Q} + \alpha u_{t-1} u_{t-1}' + \beta Q_{t-1}, \tag{12}$$

where $u_{it} = \epsilon_{it}/\sqrt{h_{iit}}$, $\overline{Q}$ is the $N \times N$ unconditional variance matrix of u_t, and α (≥ 0) and β (≥ 0) are scalar parameters satisfying $\alpha + \beta < 1$.

From the joint return distribution we can calculate the quantiles of the marginal distributions $r_{it} \mid I_{t-1}, \quad i = 1, \ldots, N$. These marginal densities are given by

$$f_{r_{it}|I_{t-1}}(r_i) = \int_{R^{N-1}} f_{r_t|I_{t-1}}(r_i, \bar{r}_{-i}) d\bar{r}_{-i}, \tag{13}$$

where $\bar{r}_{-i}$ indicates everything in r except r_i. The main interest, however, lies in the distribution of a linear combination of the vector of returns, $w_t' r_t$ or a portfolio, which depends upon the salient dependencies between the different returns. Because $g(\cdot)$ is left unspecified the distribution of a linear combination of r_t can be calculated by the following well known result. Consider a random vector $(X, Y) \sim f_{X,Y}$ and $(U, V) = (R(X, Y), S(X, Y))$ a new random vector as a function of the previous vector. Suppose that R and S are functions such that we can calculate $(X, Y) = (L(U, V), T(U, V))$. Then we have

$$f_{U,V}(u, v) = \mid \det J \mid \cdot f_{X,Y} \left(L(u, v), T(u, v) \right) \tag{14}$$

where

$$J = \begin{pmatrix} \frac{\partial X}{\partial U} & \frac{\partial X}{\partial V} \\ \frac{\partial Y}{\partial U} & \frac{\partial Y}{\partial V} \end{pmatrix}. \tag{15}$$

We are interested in a single linear combination, corresponding to an asset portfolio, so we can take $Y = V = T(U, V)$ and integrate this part out of the multivariate density. In the bivariate case, for example, the density at time t of the return on a portfolio with weights $w_{1t} \neq 0$ and $w_{2t} = 1 - w_{1t}$ is

$$f_{w_{1t}r_{1t}+w_{2t}r_{2t}|I_{t-1}}(r_p) = \frac{1}{w_{1t}} \left| H_t(\theta) \right|^{-1/2} \int g \left(H_t^{-1/2}(\theta) \left[\begin{pmatrix} \frac{r_p - w_{2t} v}{w_{1t}} \\ v \end{pmatrix} - \mu_t(\theta) \right] \right) \mathrm{d}v. \tag{16}$$

Because in our framework $g(\cdot)$ is unknown, numerical integration techniques will be used to obtain the distribution of the portfolio return. See for example Bauwens, Lubrano, and Richard (1999) for details on numerical integration. At a given confidence level $1 - \alpha$, the Value-at-Risk (VaR) of a portfolio with weights w_t is defined as follows.

Definition 4 *The VaR at level α is the solution to*

$$P(w_t' r_t < VaR_\alpha) = \alpha \tag{17}$$

or

$$\alpha = \int_{-\infty}^{VaR_\alpha} f_{w_t' r_t | I_{t-1}}(r_p) dr_p. \tag{18}$$

The VaR is a measure of the market risk of the portfolio and measures the loss that it could generate (over a given time horizon) with a given degree of confidence. Above, we have expressed the VaR in relative terms as the quantile at level α of the distribution of portfolio returns. With probability $1 - \alpha$, the losses on the portfolio will be smaller than VaR_α. The VaR is widely adopted by banks and financial institutions to measure and manage market risk, as it reflects downside risk of a given portfolio or investment. In general, the VaR is a function of the confidence level α, the density $g(\cdot)$, the portfolio weights w_t, the functional form of the mean vector μ_t and of the covariance matrix H_t, where the latter three are time dependent. In the case where $g(\cdot)$ is the multivariate normal density the definition of the VaR reduces to the well known formula $VaR_\alpha = w_t'\mu_t + (w_t' H_t w_t)^{1/2} z_\alpha$ where z_α is the α-th quantile of the univariate standard normal distribution.

The parameter vector θ is estimated by quasi maximum likelihood (QML) which implies that during estimation we suppose that $g(x) \propto \exp(-\frac{x'x}{2})$. The relevant part of the loglikelihood function for a sample $t = 1, ..., T$ then becomes

$$-\sum_{t=1}^{T} \left(\ln |H_t(\theta)| + (y_t - \mu_t(\theta))' \; H_t^{-1}(\theta) \; (y_t - \mu_t(\theta)) \right), \tag{19}$$

conditional on some starting value for μ_0 and H_0. Equation (19) can be maximized with respect to θ using a numerical algorithm, which results in a consistent and asymptotically normally distributed estimator, provided that $\mu_t(\cdot)$ and $H_t(\cdot)$ are correctly specified (Bollerslev and Wooldridge (1992)). Alternatively, it is possible to make other parametric assumptions about the distribution $g(.)$, for instance the multivariate t-distribution with arbitrary degrees of freedom ν. For more information on the estimation of MGARCH models we refer to Bauwens, Laurent, and Rombouts (2006).

The density $g(\cdot)$ in (2) is estimated by a kernel density estimator. A general multivariate kernel density estimator with bandwidth matrix H and multivariate kernel $\mathcal{K}$ can be written as

$$\hat{g}_H(x) = \frac{1}{T|H|}\sum_{t=1}^{T}\mathcal{K}(H^{-1}(\xi_t - x)).$$

Since the variance of the innovations should be the same in all directions, it is reasonable to use a scalar bandwidth, $H = hI_N$, with $h > 0$ and I_N the N dimensional identity matrix. It is well known that by requiring $Th^N \to \infty$ and $h \to 0$ as $T \to \infty$, the multivariate kernel density estimates are consistent and asymptotically normally distributed. The MSE-optimal rate for the bandwidth is $T^{-1/(4+N)}$ which is a rule of thumb bandwidth proposed by Silverman (1986). Furthermore, we use a product kernel $\mathcal{K}(x) = \prod_{i=1}^{N} K(x_i)$ and some univariate kernel function K such as Gaussian, quartic or Epanechnikov. Our density estimate becomes

$$\hat{g}_h(x) = \frac{1}{Th^N}\sum_{t=1}^{T}\prod_{i=1}^{N}K(\frac{\xi_{i,t} - x_i}{h}).$$

More details on multivariate kernel density estimation can be found in Scott (1992). In our application we will use a Gaussian kernel.

For high dimensions, nonparametric estimation of the innovation density becomes unreliable because of the curse of dimensionality problem. To circumvent this problem, the innovation density itself could be modelled semiparametrically, by the use of copulas for example. Copulas allow one to decompose the information captured in the joint distribution into information concerning the marginal distributions and information about the dependence structure. To keep the innovation distribution flexible one could estimate the univariate marginal distributions nonparametrically and fit a parametric copula.

Table 1: Summary statistics

	04/01/1988–30/08/2006	
	$T = 4708$	
	Nasdaq	**S&P 500**
Mean (%)	0.0396	0.0346
Standard Deviation (%)	1.4434	0.9970
Maximum	13.255	5.5732
Minimum	−10.168	−7.1127
Skewness	−0.0253	−0.2242
Kurtosis	9.5747	7.4123

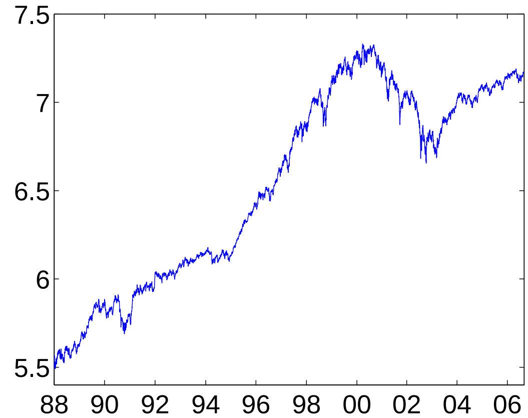
Daily Nasdaq and S&P 500 index returns descriptive statistics.

The estimated correlation coefficient is 0.818.

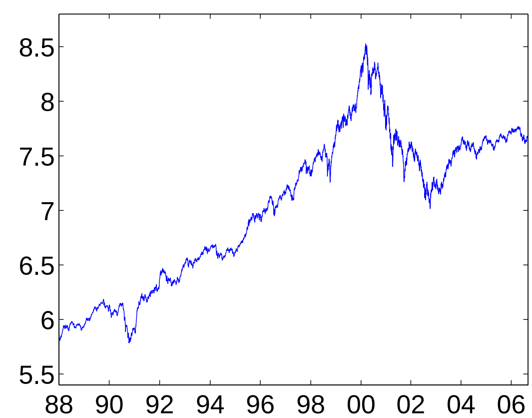
3 Data and estimation of the MGARCH models

We consider daily returns on two stock market indexes, namely the Standard & Poor's 500 (S&P 500) index and the Nasdaq index, covering the period 04/01/1988 to 30/08/2006 (4708 daily observations). Both daily log-prices and returns are plotted in Figure 1 and descriptive statistics are given in Table 1. There is a clear presence of fat tails in the return distributions. The kurtosis of the S&P 500 index and the Nasdaq index are 7.41 and 9.57, respectively. Even after estimation of a multivariate GARCH model to these data, we may expect that the nonparametrically estimated innovation density still features quite some pronounced departures from normality. The estimated unconditional correlation coefficient is 0.818.

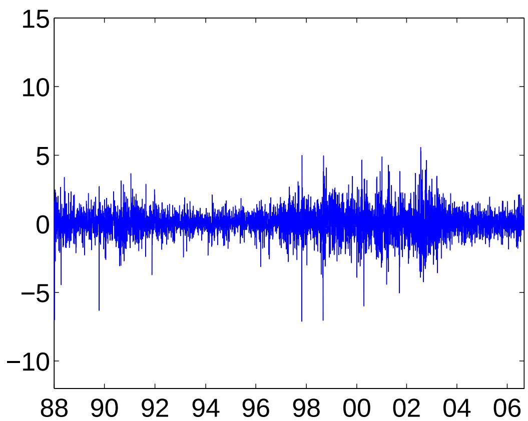
We estimate the DVEC, DCC Tse and DCC Engle models, described in Section 2, over the whole sample period by QML. For daily horizons expected returns can safely be assumed to be almost zero. Accordingly, we do not model time-variation in the conditional means and let $\mu_t(\theta) = \mu$. Imposing $\mu = 0$ has negligible consequences for our results.



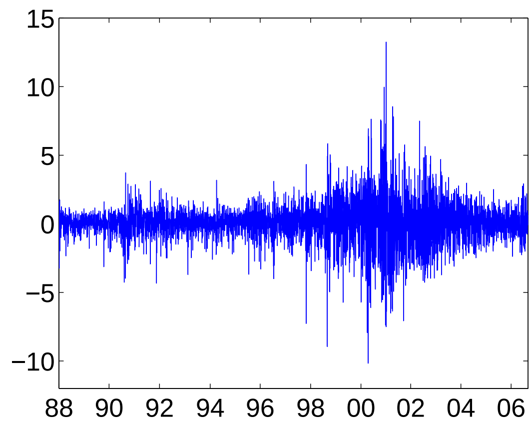
(a) daily Standard & Poor's 500 index log-prices



(b) daily Nasdaq index log-prices



(c) daily Standard & Poor's 500 index returns



(d) daily Nasdaq index returns

Figure 1: Sample period: 04/01/1988−30/08/2006 or 4708 observations. The returns are measured by their log-differences.

The parameter estimates and corresponding robust standard errors and t-statistics for the DVEC model are given in Table 2. Note that we are close to the covariance stationarity bound because the maximum eigenvalue of $\hat{A} + \hat{G}$ is equal to 0.998. Generally speaking, the QML standard errors are small. There is also quite some persistence, measured by $\hat{\alpha}_{22} + \hat{\beta}_{22}$, in the conditional covariance process.

The DCC models are estimated in one step. We first estimate the DCC model of Tse, where we choose GARCH(1,1) models for the conditional variances of both series and and we set $M = 2$ in the correlation specification (8). The estimation results are given in Table 3. As already mentioned for the DVEC model, there is a high persistence in the conditional variance series. The same still holds for the conditional correlation series. We remark that the parameter estimates related to the conditional variances change only marginally between the DVEC and the DCC model. This is not surprising given the identical functional forms for the variances in both models.

The third model is the DCC model of Engle. The elements of $\overline{Q}$ in (12) are set to their empirical counterparts to render the estimation simpler because there are less parameters to estimate. The estimation results are given in Table 4. Comparing the estimates with the corresponding estimates of the DVEC and the DCC Tse model, we only observe small differences. The correlation model parameter estimates of the DCC models are slightly different. The next two sections investigate whether these small differences have consequences for VaR computations. The estimated conditional correlation series is plotted in Figure 3. While the estimated unconditional correlation coefficient equals 0.818, there is substantial time variation in the conditional correlations. A test for constant conditional correlations, that is $\theta_1 = \theta_2 = 0$, would easily be rejected. Because the three MGARCH specifications are non-nested, a direct comparison based on statistical tests is complicated and we shall evaluate them on the basis of their performance regarding the implied Value-at-Risk.

Finally, we estimate the bivariate density $g(\cdot)$ of the innovations ξ_t, for each of the three MGARCH specifications, on the basis of the estimated standardized residuals. These can be computed from (1) as $\hat{H}_t^{-1/2}(r_t - \hat{\mu}_t)$ where the hats indicate that the unknown parameters in θ are replaced by their estimates, and the square root of a matrix is computed here using the usual spectral decomposition. As mentioned in Section 2 we use a Gaussian product kernel for the nonparametric estimation of the innovation density. The bandwidth is obtained by the conventional rule of thumb and in our application equals 0.24426. The estimated innovation density for the DCC model of Engle is displayed in Figure 2. We do not show the estimated densities for the DVEC and the DCC Tse model because there are no marked differences. Figure 2 exhibits clear departures from normality. The skewness for the S&P 500 and Nasdaq residuals are -0.45 and -0.41, respectively, while the kurtosis are 4.30 and 7.94, respectively. The Jarque Bera normality tests for the marginal distributions reject at any significance level. These deviations from normality may have an important impact upon the Value-at-Risk that is implied by the distribution of portfolio returns, and suggest that the normal distribution may provide inaccurate VaR estimates. Below we shall determine and evaluate the VaRs on the basis of the semi-parametric distribution, as well as the bivariate normal and t-distributions.

4 Value-at-Risk with given portfolio weights

This section explores the Value-at-Risk measures corresponding to the models estimated above. We investigate the Value-at-Risk at the 1%, 2.5% and 5% levels, denoted $\text{VaR}_{0.01}$, $\text{VaR}_{0.025}$ and $\text{VaR}_{0.05}$, respectively, for all the 4708 trading days of the sample. We consider the three different MGARCH models defined in Section 2 and estimated by QML, and three time invariant portfolios with weights $w^1 = (0.25, 0.75)'$, $w^2 = (0.5, 0.5)'$ and $w^3 = (0.75, 0.25)'$. Furthermore, we distinguish between different innovation densities: the Gaussian, the student t and the nonparametric density. The degrees of freedom of the bivariate student t distribution are estimated by maximum likelihood as 9.13 with a standard error of 0.89.

To compare the VaR levels we calculate failure rates for the different specifications. The failure rate (FR) is defined as the proportion of r_t's smaller than the VaR. For a correctly specified model, the empirical failure rate should be close to the specified VaR level α. We compare the empirical failure rate to its theoretical value by means of the Kupiec likelihood

Table 2: Parameter estimates for the DVEC model

	Coefficient	Std error	t-statistic
ω_{11}	0.003712	(0.00127)	2.916
ω_{21}	0.003037	(0.00095)	3.202
ω_{22}	0.003506	(0.00108)	3.236
α_{11}	0.048096	(0.00549)	8.764
α_{22}	0.042731	(0.00506)	8.453
α_{33}	0.044369	(0.00639)	6.948
β_{11}	0.950353	(0.00569)	166.9
β_{22}	0.954787	(0.00546)	174.8
β_{33}	0.952820	(0.00694)	137.4

QML estimates for the DVEC model. QML standard errors in the Std error column. Sample of 4708 observations (04/01/1988−30/08/2006).

Table 3: Parameter estimates for the DCC Tse model

	Coefficient	Std error	t-statistic
ω_{11}	0.005121	(0.00153)	3.341
ω_{22}	0.005284	(0.00144)	3.678
α_{11}	0.054506	(0.00698)	7.812
α_{22}	0.043599	(0.00733)	5.946
β_{11}	0.941310	(0.00733)	128.4
β_{22}	0.949852	(0.00813)	116.9
ρ	0.903246	(0.02549)	35.44
θ_1	0.016844	(0.00275)	6.116
θ_2	0.960823	(0.00782)	122.9

QML estimates for the DCC Tse model. QML standard errors in the Std error column. Sample of 4708 observations (04/01/1988−30/08/2006).

Table 4: Parameter estimates for the DCC Engle model

	Coefficient	Std error	t-statistic
ω_{11}	0.004285	(0.00180)	2.377
ω_{21}	0.004542	(0.00161)	2.824
α_{11}	0.055361	(0.00795)	6.962
α_{22}	0.046606	(0.00829)	5.622
β_{11}	0.943050	(0.00846)	111.5
β_{22}	0.949329	(0.00927)	102.4
θ_1	0.019041	(0.00740)	2.573
θ_2	0.978697	(0.00854)	114.6

QML estimates for the DCC Engle model. QML standard errors in the Std error column. Sample of 4708 observations (04/01/1988−30/08/2006).

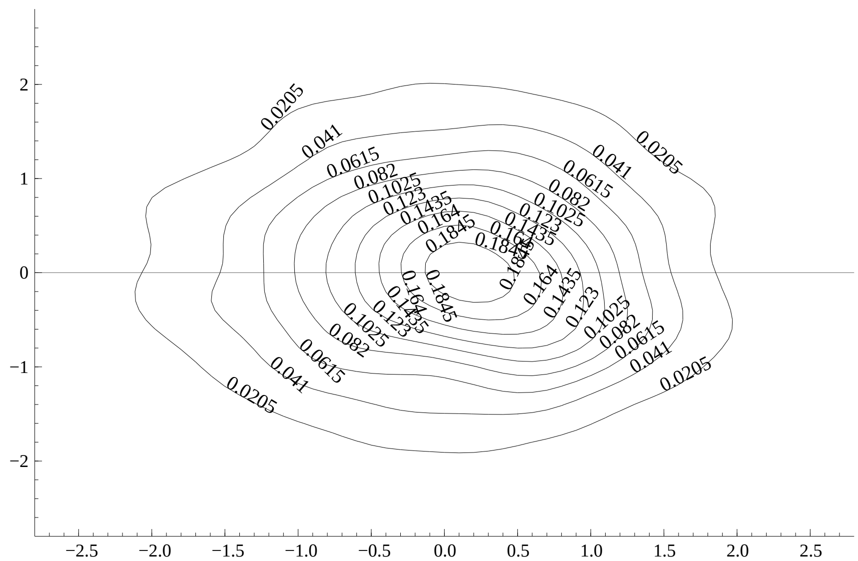


Figure 2: Contour plot of $\hat{g}(\cdot)$, the estimated innovation density of ξ_t implied by the DCC model of Engle. The skewness for the first (Nasdaq) and second (S&P 500) component are -0.45 and -0.41 respectively and the kurtosis is 4.30 and 7.94 respectively.

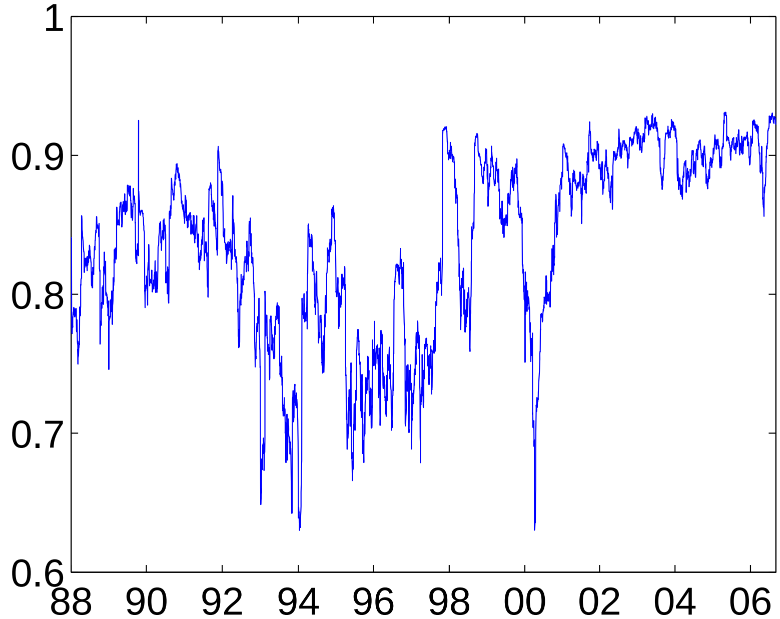


Figure 3: Plot of conditional correlations implied by the DCC model of Engle.

ratio test, see Kupiec (1995). The failures rates and the p-values for the Kupiec test are displayed in Tables 5, 6 and 7 for 5%, 2.5% and 1% VaR level respectively.

From the failure rates and the p-values in Table 5, we observe that the normal distribution performs reasonably well for the 5 percent VaR level. The failure rates that are calculated on the basis of the student distribution are too low, while the semi-parametric procedure works well with p-values for the Kupiec test always larger than 0.1. Notice that the results for the DVEC and the DCC Engle model are very similar, while the DCC Tse model produces slightly different results. Table 6 presents the results for the 2.5 percent VaR level. The normal distribution performs less well in some situations while the student t distribution has no predictive power here; the associated p-values for the Kupiec test are all very close to zero. In contrast, our semi-parametric approach performs very well in each of the situations. Table 7 displays the results for the 1 percent VaR level. In this case, the normal distribution has a difficult job in providing failure rates close to the VaR level. Overall, the empirical failure rates are too high, which means that we overestimate the first percentile of the distribution of the portfolio return. The Kupiec likelihood ratio test consistently rejects the normal distribution with p-values very close to zero. The student distribution generates failure rates that are consistently too low, although reasonably close to the theoretical values. Contrary to the five percent VaR level case, this suggests that the degrees of freedom are correctly estimated. Again, the semi-parametric procedure works well, it is able to pin down almost exactly the correct quantile of the return distribution.

The above results show that the semi-parametric procedure proposed in this paper is a promising tool for risk management analysis. Firstly, the procedure is based on a natural idea. We do not impose a specific functional form on the innovation distribution when we calculate the VaR. Secondly, we do not have to worry which innovation distribution to use for which specific VaR level. The fact that semi-parametric estimation of the VaR dominates parametric approaches is also demonstrated by Fan and Gu (2003) in the univariate case. Obviously, one can always impose other parametric distributional assumptions for the innovations that are more flexible than the normal and student t distributions. For example, Mittnik and Paolella (2000) and Giot and Laurent (2003) work with a skewed t

distribution, but there are always chances of severe misspecifications.

Admittedly, compared to parametric approaches our multivariate semi-parametric procedure is computationally more complex and demanding. However, we do not feel that this shortcoming is essential. Software for estimating multivariate GARCH models using quasi-maximum likelihood is readily available, while the calculation of Value-at-Risk using our non-parameteric procedure is relatively straightforward. On a typical desktop PC, implementation currently requires only about one or two minutes for the bivariate case. For larger dimensions, say $N > 4$, computational costs are much higher, and we advise the use of semi-parametric techniques for modelling the joint distribution of the innovations; see, for example, Bingham and Kiesel (2002) and Bingham, Kiesel, and Schmidt (2003). Overall, we feel that the advantages of using a semi-parametric multivariate GARCH model for establishing a portfolio's Value-at-Risk in many circumstances more than makes up for the fact that its implementation is more involved.

Table 5: $\text{VaR}_{0.05}$ results

		w^1		w^2		w^3	
		FR	p-value	FR	p-value	FR	p-value
DVEC	Normal	0.0528	(0.367)	0.0493	(0.820)	0.0533	(0.302)
	$t_{\hat{\nu}}$	0.0380	(0.000)	0.0359	(0.000)	0.0378	(0.000)
	semi-parametric	0.0469	(0.331)	0.0448	(0.100)	0.0480	(0.527)
DCC_T	Normal	0.0565	(0.045)	0.0520	(0.524)	0.0567	(0.038)
	$t_{\hat{\nu}}$	0.0395	(0.001)	0.0374	(0.000)	0.0395	(0.001)
	semi-parametric	0.0489	(0.717)	0.0478	(0.484)	0.0491	(0.768)
DCC_E	Normal	0.0540	(0.219)	0.0486	(0.667)	0.0548	(0.136)
	$t_{\hat{\nu}}$	0.0380	(0.000)	0.0361	(0.000)	0.0378	(0.000)
	semi-parametric	0.0469	(0.331)	0.0452	(0.128)	0.0480	(0.527)

This table presents failure rates (FR) and p-values for the Kupiec LR test. We report this for the DVEC, the DCC Tse (DCC_T) and the DCC Engle (DCC_E) model. We distinguish between the normal, the student and the nonparametric innovation density, for three portfolio weights $w^1 = (0.25, 0.75)'$, $w^2 = (0.5, 0.5)'$ and $w^3 = (0.75, 0.25)'$.

Table 6: $\text{VaR}_{0.025}$ results

		w^1		w^2		w^3	
		FR	p-value	FR	p-value	FR	p-value
DVEC	Normal	0.0289	(0.095)	0.0295	(0.053)	0.0293	(0.065)
	$t_{\hat{\nu}}$	0.0155	(0.000)	0.0168	(0.000)	0.0176	(0.001)
	semi-parametric	0.0219	(0.161)	0.0234	(0.467)	0.0236	(0.528)
DCC_{T}	Normal	0.0297	(0.043)	0.0306	(0.017)	0.0314	(0.007)
	$t_{\hat{\nu}}$	0.0170	(0.000)	0.0185	(0.000)	0.0185	(0.003)
	semi-parametric	0.0232	(0.411)	0.0244	(0.800)	0.0240	(0.659)
DCC_{E}	Normal	0.0289	(0.095)	0.0291	(0.079)	0.0295	(0.053)
	$t_{\hat{\nu}}$	0.0159	(0.000)	0.0163	(0.000)	0.0174	(0.000)
	semi-parametric	0.0217	(0.134)	0.0232	(0.411)	0.0236	(0.528)

This table presents failure rates (FR) and p-values for the Kupiec LR test. We report this for the DVEC, the DCC Tse (DCC_{T}) and the DCC Engle (DCC_{E}) model. We distinguish between the normal, the student and the nonparametric innovation density, for three portfolio weights $w^1 = (0.25, 0.75)'$, $w^2 = (0.5, 0.5)'$ and $w^3 = (0.75, 0.25)'$.

Table 7: $\text{VaR}_{0.01}$ results

		w^1		w^2		w^3	
		FR	p-value	FR	p-value	FR	p-value
DVEC	Normal	0.0149	(0.002)	0.0138	(0.013)	0.0151	(0.001)
	$t_{\hat{\nu}}$	0.0081	(0.169)	0.0068	(0.020)	0.0079	(0.123)
	semi-parametric	0.0101	(0.893)	0.0089	(0.449)	0.0099	(0.991)
DCC_{T}	Normal	0.0151	(0.001)	0.0159	(0.000)	0.0168	(0.000)
	$t_{\hat{\nu}}$	0.0083	(0.223)	0.0070	(0.029)	0.0081	(0.168)
	semi-parametric	0.0099	(0.991)	0.0096	(0.759)	0.0130	(0.197)
DCC_{E}	Normal	0.0144	(0.004)	0.0142	(0.006)	0.0157	(0.000)
	$t_{\hat{\nu}}$	0.0076	(0.090)	0.0068	(0.019)	0.0079	(0.125)
	semi-parametric	0.0099	(0.991)	0.0087	(0.363)	0.0099	(0.991)

This table presents failure rates (FR) and p-values for the Kupiec LR test. We report this for the DVEC, the DCC Tse (DCC_{T}) and the DCC Engle (DCC_{E}) model. We distinguish between the normal, the student and the nonparametric innovation density, for three portfolio weights $w^1 = (0.25, 0.75)'$, $w^2 = (0.5, 0.5)'$ and $w^3 = (0.75, 0.25)'$.

5 Conclusion

Analyzing the Value-at-Risk of a portfolio of assets with arbitrary holdings requires information about the (conditional) joint distribution of returns. In this paper we explored the usefulness of semi-parametric multivariate GARCH models for asset returns for evaluating the Value-at-Risk of a portfolio. While parametric multivariate GARCH models impose strong distributional assumptions about the joint distribution of the innovations, the semi-parametric approach allows us to estimate the joint distribution without making restrictive assumptions. While this is theoretically superior, its performance in finite samples, and taking into account realistic conditions, is not necessarily optimal.

We examined the usefulness by considering the joint distribution of the returns on the S&P 500 and Nasdaq indexes. Our analyses of the 1%, 2.5% and 5% Value-at-Risk for a set of three different portfolio holdings show that the semi-parametric multivariate GARCH models perform well and consistently over the different models and significance levels. This is a promising result. Interestingly, the sensitivity of the failure rates with respect to the distributional assumptions is larger than that with respect to the parametric specification that was chosen for the conditional covariance matrix (diagonal VEC model and two variants of dynamic conditional correlations models). While the normal distribution and the t distribution are rejected in specified cases, the semi-parametric approach passes the Kupiec likelihood ratio test in all situations.

At the more general level, our analysis emphasises the sensitivity of risk management techniques to distributional and other assumptions. An important lesson from the current financial crisis is that standard risk management methods may be inadequate and that improved measures are required. In light of this it may be wise, if anything, to be overcautious and more prudent than current risk management techniques have been.

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