Relative Distress and Return Distribution Characteristics of Japanese Stocks, a Fuzzy-Probabilistic Approach

Willem-Max van den Bergh, Onno Steenbeek and

Jan van den Berg

ERIM Report Series *Research in Management*	
ERIM Report Series reference number	ERS-2002-29-F&A
Publication	March 2002
Number of pages	19
Email address corresponding author	jvandenberg@few.eur.nl
Address	Erasmus Research Institute of Management (ERIM) Rotterdam School of Management / Faculteit Bedrijfskunde Erasmus Universiteit Rotterdam P.O. Box 1738 3000 DR Rotterdam, The Netherlands Phone: + 31 10 408 1182 Fax: + 31 10 408 9640 Email: info@erim.eur.nl Internet: www.erim.eur.nl

Bibliographic data and classifications of all the ERIM reports are also available on the ERIM website:
www.erim.eur.nl

ERASMUS **R**ESEARCH **I**NSTITUTE OF **M**ANAGEMENT

REPORT SERIES
RESEARCH IN MANAGEMENT

BIBLIOGRAPHIC DATA AND CLASSIFICATIONS		
Abstract	In this article, we demonstrate that a direct relation exists between the context of Japanese firms indicating relative distress and conditional return distribution properties. We map cross-sectional vectors with company characteristics on vectors with return feature vectors, using a fuzzy identification technique called Competitive Exception Learning Algorithm (CELA)1. In this study we use company characteristics that follow from capital structure theory and we relate the recognized conditional return properties to this theory. Using the rules identified by this mapping procedure this approach enables us to make conditional predictions regarding the probability of a stock's or a group of stocks' return series for different return distribution classes (actually return indices). Using these findings, one may construct conditional indices that may serve as benchmarks. These would be particularly useful for tracking and portfolio management.	
Library of Congress Classification (LCC)	5001-6182	Business
	4001-4280.7	Finance Management, Business Finance, Corporation Finance
	HG 4028.A84	Asset-backed financing (Financial Management)
Journal of Economic Literature (JEL)	M	Business Administration and Business Economics
	G 3	Corporate Finance and Governance
	G 12	Asset pricing
European Business Schools Library Group (EBSLG)	85 A	Business General
	220 A	Financial Management
	220 P	Portfolio Management
Gemeenschappelijke Onderwerpsontsluiting (GOO)		
Classification GOO	85.00	Bedrijfskunde, Organisatiekunde: algemeen
	85.30	Financieel management, financiering
	85.30	Financieel management, financiering
Keywords GOO	Bedrijfskunde / Bedrijfseconomie	
	Financieel management, bedrijfsfinanciering, besliskunde	
	Portfolio-analyse, Fuzzy Set Theory, Waarschijnlijkheidsleer	
Free keywords	capital structure, asset pricing, conditional return distribution, fuzzy systems, heuristic learning.	

Relative Distress and Return Distribution Characteristics of Japanese stocks, a Fuzzy–Probabilistic Approach

Willem-Max van den Bergh(a) Onno Steenbeek(a,b)

Jan van den Berg(a)

(a) Erasmus University Rotterdam, Faculty of Economics

(b) ABP Investments, Schiphol

Dept. of Finance, room H14-14, P.O. Box 1738, 3000 DR, Rotterdam, the Netherlands

tel: +31-10-408 1286, +31-10-408 1494, +31-10-408 1342, fax: +31-10-408 1834

e-mail: vandenbergh@few.eur.nl, steenbeek@few.eur.nl, jvandenberg@few.eur.nl

Abstract

In this article, we demonstrate that a direct relation exists between the context of Japanese firms indicating relative distress and conditional return distribution properties. We map cross-sectional vectors with company characteristics on vectors with return feature vectors, using a fuzzy identification technique called Competitive Exception Learning Algorithm (CELA)[1]. In this study we use company characteristics that follow from capital structure theory and we relate the recognized conditional return properties to this theory. Using the rules identified by this mapping procedure this approach enables us to make conditional predictions regarding the probability of a stock's or a group of stocks' return series for different return distribution classes (actually *return indices*). Using these findings, one may construct conditional indices that may serve as benchmarks. These would be particularly useful for tracking and portfolio management.

Keywords: capital structure, asset pricing, conditional return distribution, fuzzy systems, heuristic learning.

Acknowledgements: We are grateful to Stephen Brown and Casper de Vries for their comments on an earlier version. We would like to thank Danny Vlasblom for excellent research assistance.

[1] After having discovered the precise mathematical background of Competitive Exception Learning [14], it may be better to rename CELA into PFELA (Probabilistic Fuzzy Exception Learning Algorithm), but here we stick to the original name.

1 Predicting distribution characteristics in stock returns

In this article, we demonstrate that a direct relation exists between the context of Japanese firms indicating relative distress and conditional return distribution properties. A clear understanding of what drives returns of a portfolio containing low-investment grade assets is of course vital for all those whose economic welfare is, involuntarily or on purpose, dependent on it. This is especially true for Japan, because we expect that our methodology is able to make a better distinction between good and bad companies. Since Japan has experienced an 11-year period of economic downturn, with numerous bankruptcies and many more to follow, we expect the results to be more significant. With the CELA methodology, explained later, we are able to determine the factors that predict the stock price distribution of Japanese companies. This is particularly useful for index tracking, active portfolio management, pricing of contingent claims and for financial risk management in general.

A majority of studies focuses on the explanation of *expected* returns as a linear combination from one or more factors reflecting risk. Usually the Arbitrage Pricing Model gives the theoretical justification for these studies. The 3-factor Fama and French [4] model is undeniably the best-known empirical article in this field. The equity value book-to-market ratio (often considered a good proxy for relative distress) emerges as the most important ex-post explanatory state variable in conjunction with the market risk. However, the discussion on an explicit theoretical interpretation of these results is still open.

The motivation of our study is to get a better insight in conditional price formation and clearer view of the book-to-market puzzle. We examine endogenously determined classes of return properties. This approach was inspired by Brown *et al.* [3] who look after similarities in raw return series. However, we do not use raw return patterns, but examine similarities in return distribution properties like market beta and first and higher order moments. Such procedure greatly reduces the dimensionality of the return space although we realize that it is at a cost of information loss. We look for similarities of equity return distributions conditional on the capital structure.

In our approach, company characteristics related to capital structure are mapped on a set of statistical properties of the return series using a methodology termed Competitive Exception Learning (CELA) [16, 12, 15, 14]. The CELA method developed observes average behavior of system outputs (here the statistical properties of the return series) and tracks deviations from this average behavior. These deviations are then correlated to regions within the system's input space (here the company characteristics). The result is a set of fuzzy rules [9] that describe the specific company characteristics which lead to 'exceptional' return series. Since CELA concentrates explicitly on the discovery of exceptions and constructs a fuzzy rule base, the rules of which can be expressed in linguistic terms interpretable by experts in the field, we think CELA is an excellent tool for solving our portfolio management problem.

The structure of the rest of this paper is as follows. In the next section, we describe the background theory for solving our problem and take a closer look to the dissipation of news in capital markets. In section 3, we describe CELA in mathematical terms. In section 4, we illuminate the experimental setup and in section 5 we describe and analyze the mapping of relative distress to stock distribution characteristics. We finalize with a discussion and outlook.

2 Dissipation of News in Capital Markets

Asset prices move when news reaches the market. By definition news is unpredictable but once news arrives, the reaction of specific asset prices may be systematic in such way that they show any kind of co-movement. Thus, for a given flow of news events during some period of time, return series may show local similarities. Such local similarities may potentially be identified endogenously. In their 1997 working paper Brown *et al.* [3] attempted to identify return driving factors using cluster techniques. Monthly stock return series over two consecutive years were examined in a 24-dimensional Euclidian space (each single sample point representing the series of one stock). With an iterative replacing algorithm 10 'similarity' clusters (i.e. stock portfolios) were identified by minimizing the Euclidian distance between the 24-dimensional vectors in each cluster. Thus similarity refers to neighboring sample points which tend to display similar behavior of the return series. Subsequently the portfolios are related to a classification according to industry or size. This procedure yields evidence for both a size factor and factors associated with certain industries.

Realizing that company-specific news will affect the value of a company's assets, it doesn't come as a surprise that the value of companies in the same industry or country will react in a more or less similar fashion to a given news event. More generally, when the assets of companies are sensitive to the same sources of risk, the market value of these companies is expected to react in a similar fashion to news about these sources of risk, resulting in a clustering in time.

But one should recognize that the market valuation of firms takes place via the capital structure. In other words, news that directly affects the asset value may have a more complex effect on the value of the various claims on the company. Almost 50 years ago Modigliani and Miller [10] already proposed that all claim holders require a fair part of the asset value, depending on the nature of their claim and equilibrium prices are set accordingly. But if the asset value changes, the 'fair' relative part of each claim holder will *not* necessarily remain the same. Positive news may lead to a higher share price, but the value of debt will as well increase, mainly due to the fact that the probability of bankruptcy has declined. In other words, there is a wealth transfer from shareholders to the holders of fixed debt whenever positive news reaches the market. The opposite occurs in the case of negative news: The value of debt may fall more than proportionally. Of course, this effect will be stronger, the higher the financial leverage of the firm, resulting in a clear non-linear relationship between the nature of the news and a stock's return. Black and Scholes showed in their seminal article [2] that this relationship could well be modelled in terms of an options framework with the important result that the slope between the (contingent) value of debt and the underlying asset value is not constant.

The eventual effect on stock returns depends on e.g.: the location of the company, the industry, the distribution of the claims, the probability of bankruptcy and the estimated bankruptcy costs should a firm go under, and agency costs.

Stock prices of a specific group of firms moving in a similar way also implies that the returns of these stocks should have a more or less similar distribution. Not only the mean return but also standard deviation, skewness, kurtosis etc. may potentially cluster. So if we divide stocks over portfolios based on some criterion that captures the specific reaction to news, we may expect to find conditional distribution behavior.

Expectedly the resulting conditional co-distribution of stock returns is not fully encapsulated by the market beta. Recent studies seem to confirm that conditional higher moments play a significant role, *c.f.* skewness found by Harvey *et al.* [5] among others. Moreover, a classification conditional on higher return moments appears to coincide largely with a classification based on firm size and book-to-market equity value. In our view the Fama & French 3-factor model [4] picks up the specific way stock prices react to news quite satisfactory. But even more complex conditional co-movement than captured by centered higher order moments may occur. If for instance some stocks only move with the market when bad news reaches the market, adding an extra factor that distinguishes such stocks from the others will undoubtedly improve the overall behavior of the model. In a recent working paper Ang *et al.* [1] show that stocks with a relatively high downside correlation factor have higher expected returns (up to 6.5% per annum) than returns that can be explained by the 3-factor Fama and French model. Even after controlling for the size effect and the book-to-market effect such extra factor may pick up some ex-post explanation of returns.

Although these pieces of evidence certainly provide more insight, we know of no theory describing how risk is priced. An important open question is therefore through which channels the arrival of news in the market affects equity returns. This news arrives in the market and affects similar companies in a similar fashion. However, we hypothesize that the specific capital structure will dampen or intensify the effect on the equity returns of an individual company.

Theories of Capital Structure

An important factor explaining the level of downside risk perceived by investors, is bankruptcy risk, which in turn is largely determined by the D/E ratio. We mentioned already an important cause for the observation that stock returns are not normally distributed, namely the fact that the correlation between the D/E ratio and the level of bankruptcy risk is probably not linear: until a certain level of debt, bankruptcy risk will be deemed negligible. Beyond a certain critical point, however, the investors' perception of bankruptcy risk suddenly increases and the required risk premium on debt increases.

A central determinant of the perception of bankruptcy risk is the guarantees a company can give its debtholders. First of all, the larger the company, the more debt will be allowed by investors before they start worrying about bankruptcy risk. The reason is that direct bankruptcy costs form a smaller share of the value of a company when that value is higher. Moreover, larger companies are usually more diversified and hence less risky. In addition, the cost of issuing new shares is relatively less costly for larger companies, leading to a larger fraction of debt.

In general, management has more information regarding the current value and future opportunities of a company than shareholders or bondholders. This leads to a pecking order of the various ways a company's activities are financed: first choice is for internal funds, next debt and finally equity (Myers, [11]). The underlying explanation is that the costs of attracting internal funds are less dependent on the correct estimation of the company's value, more when attracting debt and still more when issuing equity. The difficulty to assess the value of the assets will limit the amount of additional debt that is acceptable for debtholders. In other words: the higher the level of intangible assets, the sooner the effects of bankruptcy risk described above will kick in. In case managers are not or only partial owners of the firm, the costs of the consumption of perquisites are not or partially born by the managers. As a result, managers can reduce effort or use company funds to their own benefits (Jensen and Meckling, [7]). By financing more with debt, it will be more difficult for the manager to finance these value-destroying activities internally. In other words, the more free cash is available internally, the more debt is required in order to mitigate these conflicts. This implies a positive relationship between the amount of free cash and the debt-equity. The required return on equity depends on the extent to which agency costs can be reduced by introducing more debt in the capital structure and bankruptcy risks described above. A high dividend yield would have the same effect. Note however that the pecking order theory suggests an opposite relationship: when there are a sufficient number of profitable investment opportunities, a larger amount of internal funds will limit the need to attract additional debt, resulting in a lower debt-equity ratio.

Hypotheses

The central factor in the test is the D/E ratio and the amount of debt acceptable for debtholders and shareholders without introducing bankruptcy risk. The smaller the company, the higher the volatility of profits and the higher the level of intangible assets, the lower the D/E ratio is allowed to be. In general, once bankruptcy risk becomes important, the return distribution should become more skewed to the left. This motivates us to examine a direct but non-linear relation between the capital structure and cross-sectional return distribution characteristics. Factors related to relative distress as well as size should be visible in return distribution characteristics, such as skewness and kurtosis, in a non-linear way. Financing a company with debt should lead to a certain level of skewness in equity returns, even if the return on assets is distributed normally. This asymmetry may lead to a higher required equity return.

A challenging question is to what extent such effects are related to industry. The clearly inadequate industrial SIC classification of Japanese firms interferes with a viable study. At the end of our experiments we will take a short look to this.

3 Fuzzy Exception Learning

In this section we first sketch CELA's background, we present a mathematical framework of probabilities and statistics of fuzzy sets, and then, by using this framework, describe the various steps of the CELA-algorithm. For more information on CELA, we refer to [12, 15, 14].

3.1 Background of CELA

CELA constructs a stochastic mapping from a M-dimensional input sample space X to an N-dimensional output sample space Y. The corresponding stochastic variables are denoted as $\underline{\mathbf{x}} = (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_M)$ and $\underline{\mathbf{y}} = (\underline{y}_1, \underline{y}_2, \ldots, \underline{y}_N)$ respectively. A representative set of sample vector values $(\mathbf{x}_p; \mathbf{y}_p), (p = 1, 2, \ldots)$ is supposed to be available with $\mathbf{x}_p = (x_{p1}, x_{p2}, \ldots, x_{pM})$ and $\mathbf{y}_p = (y_{p1}, y_{p2}, \ldots, x_{pN})$. The data set has an unknown joint probability density function (p.d.f.) described by $f(\mathbf{x}, \mathbf{y})$.

A main goal of CELA is to find the $\mathbf{x}$-clusters (in time series applications often termed 'regimes') for which the conditional p.d.f.'s $f(\mathbf{y}|\mathbf{x})$ deviate 'exceptionally' from the marginal p.d.f. $f(\mathbf{y})$. By setting up a fuzzy rule base, the exceptional behavior can be expressed in linguistic terms hereby increasing transparency for human beings. The fuzzy rule base can be used to validate the system by putting it to experts in the field and asking for (in)correctness. Furthermore, a non-linear regression model of $\mathbf{y}$ on $\mathbf{x}$ can be assessed using fuzzy interpolation. The underlying regression model can be formulated as

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) + \epsilon(\mathbf{x}), \tag{1}$$

where $\epsilon(\mathbf{x})$ is a random vector with a probability distribution dependent on $\mathbf{x}$. To deal with both probabilities and fuzziness, we need a well-founded mathematical framework.

3.2 Probabilistic fuzzy framework

We start by presenting a probability theory on fuzzy sets, then change to statistics where it is shown how probabilities on fuzzy sets and fuzzy regression lines can be estimated.

3.2.1 Well-defined fuzzy sample spaces

We here confine our presentation to the continuous case. For the discrete analogon we refer to [14]. Let $f(\mathbf{x})$ be a p.d.f. defined on a M-dimensional continuous sample space X. The probability of a multi-dimensional *fuzzy event* P, defined on X by means of membership $\mu_P(\mathbf{x})$, is given by expression

$$\Pr(P) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \mu_P(x_1, x_2, \ldots, x_M) f(x_1, x_2, \ldots, x_M) dx_1 dx_2 \ldots dx_M. \tag{2}$$

More compactly, we write this expression as

$$\Pr(P) = \int_{-\infty}^{\infty} \mu_P(\mathbf{x}) f(\mathbf{x}) d\mathbf{x} = E(\mu_P(\underline{\mathbf{x}})), \tag{3}$$

where $\int$ represents the M-fold integral of (2). Note from (3) that the probability of a fuzzy event equals the mathematical expectation $E(\mu_P(\underline{\mathbf{x}}))$ of the membership function describing the fuzzy event.

Theorem 3.1 *Let a set of continuous fuzzy events $P_1, P_2, .., P_b, \ldots$ form a fuzzy partition in a continuous sample space implying that*

$$\forall \mathbf{x} : \sum_{P_b} \mu_{P_b}(\mathbf{x}) = 1, \tag{4}$$

then the sum of all probabilities of fuzzy events P_b equals one, or

$$\sum_{P_b} \Pr(P_b) = 1. \tag{5}$$

The (straightforward) proof can be found in [14]. Since the theorem guarantees that the sum of probabilities equals one, the corresponding sample space is termed *well-defined*.

3.2.2 Assessing fuzzy probabilities

Having a finite set of representative sample data $\mathbf{x}_p$, mathematical statistics can be used to assess probability distributions. In the fuzzy case, the domain of X is fuzzily partitioned in a set of fuzzy classes $P_b, (b = 1, \ldots, B)$ such that condition (5) holds. This guarantees that X is well-defined. It also implies that a fuzzy sample $\mathbf{x}_p$ belongs to each fuzzy class to a certain degree. Let $\tilde{f}_{P_b}$ denote the relative frequency and f_{P_b} the absolute frequency of the contributions of the fuzzy samples $\mathbf{x}_p$ to the fuzzy class P_b, then the probability of fuzzy class (fuzzy event) P_b can be assessed conform

$$\Pr(P_b) \approx \tilde{f}_{P_b} = \frac{f_{P_b}}{P} = \frac{1}{P} \sum_{\mathbf{x}_p} \mu_{P_b}(\mathbf{x}_p) = \hat{\mu}_{P_b}. \tag{6}$$

Note that the vector $(\tilde{f}_{P_1}, \tilde{f}_{P_2}, \ldots)$ describes an assessment of the probability distribution over all fuzzy classes P_b. In line with the above-given presentation, conditional probabilities on fuzzy sets can be dealt with. E.g., the conditional probability of fuzzy event (class) P_b, given fuzzy event (class) $P_{b'}$, is given by

$$\Pr(P_b | P_{b'}) = \frac{\Pr(P_b \bigcap P_{b'})}{\Pr(P_{b'})} \approx \frac{\sum_{\mathbf{x}_p} \mu_{P_b}(\mathbf{x}_p)\mu_{P_{b'}}(\mathbf{x}_p)}{\sum_{\mathbf{x}_p} \mu_{P_{b'}}(\mathbf{x}_p)}. \tag{7}$$

3.2.3 Assessing fuzzy expectations

The above-given theory demands that membership functions μ_{P_b} describing the fuzzy classes P_b in X should (a) be defined locally to enable interpretability and (b) meet condition (4) to guarantee that the fuzzy sample space X is well-defined. These requirements motivated us to define them according to

$$\mu_{P_b}(\mathbf{x}) = \frac{d_b^{-q}(\mathbf{x})}{\sum_{k=1}^{B} d_k^{-q}(\mathbf{x})}. \tag{8}$$

The letter 'd' represents a distance measure where $d_b(\mathbf{x})$ represents the (e.g., Euclidean) distance between vector point $\mathbf{x}$ and class centroïd $\bar{\mathbf{x}}_b = (\bar{x}_{b1}, \bar{x}_{b2}, \ldots, \bar{x}_{bM})$ in X. For the power q used in (8), we normally choose the value 2. Note that by this definition $\mu_{P_b}(\mathbf{x})$ is locally defined around $\bar{\mathbf{x}}_b$ with

$$\mu_{P_b}(\mathbf{x}) \begin{cases} = 1 & \text{if } \mathbf{x} = \bar{\mathbf{x}}_b \\ < 1 & \text{if } \mathbf{x} \neq \bar{\mathbf{x}}_b. \end{cases} \tag{9}$$

In addition, we note that

$$\mu_{P_b}(\bar{\mathbf{x}}_b) = 1 \Rightarrow \forall b' \neq b : \mu_{P_{b'}}(\bar{\mathbf{x}}_b) = 0. \tag{10}$$

Let $\bar{\mathbf{x}}_b$ represent the class centroïd of fuzzy class P_b in X, then we can estimate the mathematical means $E(\underline{x}_i), (i = 1, \ldots, M)$ by calculating the fuzzy sample means m_{x_i} using

$$E(\underline{x}_i) = \int_{-\infty}^{\infty} x_i f(\mathbf{x}) d\mathbf{x} \approx \sum_b \bar{x}_{bi} Pr(P_b) \approx \sum_b \bar{x}_{bi} \hat{\mu}_{P_b} = m_{x_i}. \tag{11}$$

3.2.4 Assessing fuzzy regression hyperplanes

Let, in addition to the fuzzy partitioning of X in B fuzzy classes P_b , space Y be partitioned in C fuzzy classes F_c with fuzzy centroïds $\bar{\mathbf{y}}_c = (\bar{y}_{c1}, \bar{y}_{c2}, \ldots, \bar{y}_{cN})$ and locally defined fuzzy membership functions μ_{F_c} defined according to

$$\mu_{F_c}(\mathbf{y}) = \frac{d_c^{-q}(\mathbf{y})}{\sum_{k=1}^{C} d_k^{-q}(\mathbf{y})}. \tag{12}$$

Classical regression [8] defines the regression hyperplanes of y_j on $\mathbf{x}$ as the location of all mathematical expectations $E(\underline{y}_j|\mathbf{x})$ defined by

$$E(\underline{y}_j|\mathbf{x}) = \int_{-\infty}^{\infty} y_j f(y_j|\mathbf{x}) dy_j, \ (j = 1, 2, \ldots, N). \tag{13}$$

To find approximations of regression hyperplanes (13), we start estimating the conditional mathematical expectations $E(\underline{y}_j|\bar{\mathbf{x}}_b)$. Using property (10) it is rational to choose as assessment

$$E(\underline{y}_j|\bar{\mathbf{x}}_b) = \sum_c \bar{y}_{cj} \Pr(F_c|P_b) \approx \sum_c \bar{y}_{cj} \frac{\sum_{\mathbf{x}_p} \mu_{F_c}(\mathbf{y}_p) \mu_{P_b}(\mathbf{x}_p)}{\sum_{\mathbf{x}_p} \mu_{P_b}(\mathbf{x}_p)} = m_{y_j|\bar{\mathbf{x}}_b}. \tag{14}$$

Note that we applied equations (11) and (7) here. The conditional sample means $m_{y_j|\bar{\mathbf{x}}_b}, (b = 1, 2, \ldots, B)$ are used as points of support in a fuzzy interpolation approach for estimating the complete regression planes according to

$$E(\underline{y}_j|\mathbf{x}) \approx \sum_b \frac{\phi_{P_b}(\mathbf{x})}{\sum_{b'} \phi_{P_{b'}}(\mathbf{x})} m_{y_j|\bar{\mathbf{x}}_b}. \tag{15}$$

Here, each $\phi_{P_b}(\mathbf{x}) / \sum_{b'} \phi_{P_{b'}}(\mathbf{x}), (b = 1, 2, \ldots, B)$ is a normalized weighted fuzzy membership function[2] with

$$\phi_{P_b}(\mathbf{x}) = \Pr(P_b)\, \mu_{P_b}(\mathbf{x}). \tag{16}$$

By combining equations (15), (16), (6), and (14), the fuzzy regression hyperplanes $E(\underline{y}_j|\mathbf{x})$, $(j = 1, 2, \ldots, N)$ can be estimated conform

$$E(\underline{y}_j|\mathbf{x}) = \sum_b \frac{\phi_{P_b}(\mathbf{x})}{\sum_{b'} \phi_{P_{b'}}(\mathbf{x})} E(\underline{y}_j|\bar{\mathbf{x}}_b) \approx \sum_b \frac{\hat{\mu}_{P_b} \mu_{P_b}(\mathbf{x})}{\sum_{b'} \hat{\mu}_{P_{b'}} \mu_{P_{b'}}(\mathbf{x})} m_{y_j|\bar{\mathbf{x}}_b} = m_{y_j|\mathbf{x}}. \tag{17}$$

3.3 The CELA-algorithm

We here formulate CELA within the general framework as introduced in the previous section. As mentioned in section 3.1, CELA constructs a stochastic mapping from an M-dimensional (fuzzily partitioned) input sample space X to an N-dimensional (fuzzily partitioned) output sample space Y. We use fuzzy classes having as parameters the above-introduced fuzzy centroïds $\bar{\mathbf{x}}_b$ and $\bar{\mathbf{y}}_c$, the values of which should be fixed. The idea behind fixing the output classes F_c is to facilitate a compact, close assessment of the unconditional probability distribution in Y describing average statistical behavior. The fixation of the input classes concerns the discovery of the above-mentioned regimes.

[2]Fuzzy interpolation approach (15) is quite similar to the Takagi-Sugeno fuzzy inference scheme [9]. However, instead of just using normalized membership functions, we here also apply probabilistic fuzzy rule weights $\Pr(P_b)$. For further discussion on this topic, we refer to [6, 13].

3.3.1 Algorithmic steps of CELA

Step 1: Fixing the output classes

First, the output classes F_c are fixed by calculating appropriate locations of all fuzzy class centers in Y. This is done using a fuzzy clustering heuristic. Actually, we identify each fuzzy class (event) F_c with a fuzzy cluster and, in line with that, each class center $\bar{\mathbf{y}}_c$ with a cluster centroïd. In the original approach, competitive learning was applied but other fuzzy clustering algorithms might be used as well. The result of the clustering is a fuzzy partitioning [9] of Y such that each class center $\bar{\mathbf{y}}_c$ of the fuzzy class F_c is situated in the center of a 'cloud' of fuzzy sample points.

Step 2: Assessing unconditional output probabilities

Since the sample space Y is well-defined, the probability of each fuzzy class F_c can be assessed by summing up the membership values $\mu_{F_c}(\mathbf{y}_p)$ conform equation (6) with $\mathbf{x}_p$ replaced by $\mathbf{y}_p$. This yields the probability vector $(\tilde{f}_{A_1}, \tilde{f}_{A_2}, \ldots) = (\hat{\mu}_{F_1}, \hat{\mu}_{F_2}, \ldots)$ with

$$\forall F_c : \tilde{f}_{F_c} = \frac{1}{P} \sum_p \mu_{F_c}(\mathbf{y}_p) = \hat{\mu}_{F_c} \approx \Pr(F_c). \tag{18}$$

The probability vector (18) characterizes the unconditional behavior in Y and has been termed the Unconditional Output cluster membership Distribution (UOD).

Step 3: Fixing the input classes and assessing the conditional output probabilities

For each b-th regime, i.e., for each fuzzy input class P_b in X $(b = 1, 2, \ldots, B)$, a Conditional Output cluster membership Distribution (COD) $(\Pr(F_1|P_b, \Pr(F_2|P_b, \ldots, \Pr(F_C|P_b)$ in Y can be calculated conform the theory of section 3.2.2. As mentioned, we are interested in exceptional behaviour in Y due to special regimes in X. In order to quantify the degree of exceptionality, we use the exception fitness function $EF()$ defined by

$$EF(P_1, P_2, \ldots, P_b) = \sum_{b=1}^{B} \sum_{c=1}^{C} \left(\Pr(F_c|F_b) - \Pr(F_c) \right)^2. \tag{19}$$

By changing the fuzzy events P_b, i.e., by changing the locations of the centroïds $\bar{\mathbf{x}}_b$ of the membership functions $\mu_{P_b}(\mathbf{x})$, the fitness function $EF()$ can be optimized.

Step 4: Deriving a fuzzy rule base

By comparing the various CODs found to the UOD as found in step 2, we can determine the most exceptional relationships, i.e., the regimes for which the deviations from the UOD are most exceptional. Then we can express these most exceptional fuzzy relationships in a fuzzy rule base. For each regime, the deviations from the UOD can be expressed in linguistic terms. It is of interest to put the results to experts working in the domain at stake and to verify whether the rules found are consonant with their experience. For an example on this, we refer to [15].

Step 5: Assessing the regression hyperplane

We can use (17) to assess the N regression hyperplanes of y_j on $\mathbf{x}$.

3.4 Data Correction

It may occur that output values $\mathbf{y}_p$ are structurally biased for known reasons while we are actually interested in other effects. E.g., when analyzing return characteristics from companies, we might observe a country effect or an industry effect while we only want to understand the effect due to different capital structures of companies. In these cases, it is highly desirable to have a procedure to correct the $\mathbf{y}_p$-values for the known effects. Suppose we know that the variable $\underline{x}_i$ causes a structural deviation such that for each fixed value x_i, all values y_j are displaced with the same amount d_{ji} given by

$$d_{ji} = E(\underline{y}_j | x_i) - E(\underline{y}_j). \tag{20}$$

Then, first order data correction can be applied by adjusting all sample component values y_{pj} of the sample vector values $\mathbf{y}_p$ according to

$$y_{pj}^{\text{new}} = y_{pj}^{\text{old}} - d_{ji} \approx y_{pj}^{\text{old}} - (m_{y_j|x_i} - m_{y_j}), \ (j = 1, 2, \ldots, N). \tag{21}$$

where the $m_{y_j|x_i}$'s and m_{y_j}'s are calculated using equations (17) and (11) respectively.

4 Experimental Setup

4.1 Data

We use data from the largest companies in Japan. We take the company specific information on 31 December 1999 and relate that to the return distribution characteristics over the period 1 July 1999 until 30 June 2000. We only included the companies of which all information was retrieved. This resulted in a total number of 196 companies. Stock price information is obtained from Datastream. Balance sheet information is provided by Worldscope.

From the daily return series we calculated the following distribution characteristics: expected return, standard deviation, skewness, kurtosis and market beta to arrive at a 5-dimensional return distribution space (Y-space).

The firm's characteristic vectors in the 7-dimensional X-space embody the following features:

Book to market ratio: The book value of the equity divided by the market value.

Company size: Proxied by the total yearly turnover.

Collateral: Material fixed assets + inventories. Division by total assets.

Free cashflow: Operational income - interest costs - taxes + depreciation. Division by total assets.

Dividend payout: The dividend per share divided by the net profit per share.

Convertibility: The amount of convertible capital divided by total capital.

D/E ratio: The book value of the debt divided by the book value of the equity.

4.2 Filtering 'zero returns' effects from the data

A common problem in return series is caused by zero returns. A zero return will occur when the stock price did not change, either because no transaction took place or when the the sampling at t and $t+1$

incidentally captured the same last transaction price. But zero returns may also stem from missing data because the data feed was interrupted for some period. It is as well possible that the series starts or ends with zero returns because the company was not listed before or after a certain moment in time. The latter may be the result of default, merger or take-over. Clearly these zero returns may bias the return distribution characteristics in a complicated way. As explained earlier the CEL Algorithm enables a method to filter such unwanted zero returns effects from the data.

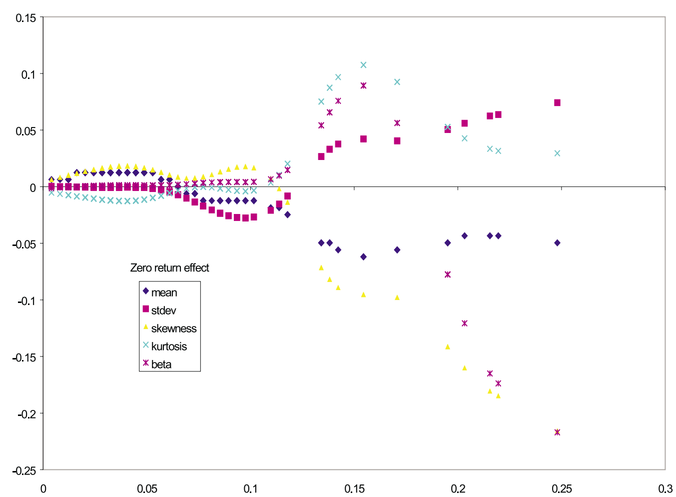

Figure 1: *Distribution effect from zero returns. The relative number of zero returns is on the horizontal axis. We only considered return series with less than 25% zero returns. On the vertical axis we measure the relative deviation from the unconditional mean value for the 5 distribution features. Each deviation is normalized to enable plotting the values for all distribution characteristics in one figure.*

For this purpose we map the fraction of zero returns in the series (an 1-dimensional X-space) on the return distribution space. Once we have estimated the regression hyperplanes of y_j on x we can use this to correct the original return distribution features.

The estimated zero returns effects are summarized in figure 1. We note that the effects become more important when the fraction of zero returns rises but we must realize that the frequency of companies is very low at the right side of the figure. The most important effect appears to arise for the beta and the skewness. The beta tends to be higher for a high fraction of zero returns, but for extreme high fractions (above 20%) it tends to be lower. The skewness indicates a tendency to more negative values (or less positive values) for higher fraction of zero returns. As to be expected, the standard deviation gets lower when there are more zero returns, but for a few companies with an extreme amount of zero returns the standard deviation tends to be higher.

5 Results

5.1 Mapping Company Characteristics to Return distribution regimes

We consider five clusters in the Y-space. Each cluster represents a (fuzzy) regime F_c regarding the return distribution properties.

	F_1	F_2	F_0	F_4	F_3
Prob	0.416	0.329	0.126	0.080	0.050
average	-1.314	0.063	0.063	-1.752	2.359
st dev	0.969	-0.616	-0.170	-0.631	2.755
skewness	-1.698	0.967	0.151	-5.491	0.455
kurtosis	1.420	-0.380	-0.588	3.507	-0.032
beta	0.677	0.357	-0.975	-1.279	1.884

Table 1: *Probability and Location of Y-clusters*

Table 1 shows the probability and the relative centroid location of the optimized[3] clusters. The columns are sorted according to the cluster probability. The centroid values are expressed as normalized deviations from the overall fuzzy mean m_{y_i} for output dimension i following equation (11). Cluster F_0 for instance has a probability of 12.6% while the expected return in the centroid (denoted as *average*) is 0.063 standard deviations higher than the overall average, the standard deviation is 0.17 standard deviations lower etc.

	Prob	D/E	BK/MKT	Div Pay	Free CF	Turnov	Collat	Convert
P_0	0.289	-0.418	-1.170	-1.345	-1.061	0.002	-0.765	-1.143
P_4	0.235	-1.758	0.391	1.394	3.618	-0.002	-0.536	0.359
P_2	0.139	-0.820	1.113	1.599	-1.048	0.000	-0.500	-0.728
P_1	0.067	2.642	1.005	0.766	-1.501	-0.007	-0.768	0.542
P_5	0.063	-0.787	0.108	0.219	-0.754	0.001	0.091	0.215
P_9	0.047	1.743	-2.733	-0.506	0.459	-0.001	-0.908	2.480
P_6	0.047	2.191	2.246	0.367	2.039	-0.001	3.363	0.191
P_8	0.042	2.500	-2.033	-1.200	3.736	-0.006	2.759	0.878
P_3	0.040	0.707	3.631	2.218	3.300	0.000	1.521	0.487
P_7	0.032	2.358	1.545	-1.030	1.376	0.000	1.748	2.588

Table 2: *Probability and Location of X-clusters*

Table 2 details the relative location of the cluster centroids in the X-space resulting from the CELA mapping procedure. We arbitrarily chose a number of ten clusters. Each cluster represents a (fuzzy) regime P_b regarding the company characteristics. The centroids can be seen as 'hot spots' indicating

[3] See the sections on the CELA algorithm for details.

non–average behavior of the return distribution. Note that the table is sorted following the probability of occurrence. Each location is expressed as the normalized deviation from the overall fuzzy mean m_{x_i} for input dimension i. To illustrate, cluster P_0 has a probability of 28.9 % while the centroid value for the D/E ratio is 0.418 standard deviations lower than the average, the Book to Market ratio is 1.17 standard deviations lower etc.

5.2 Fuzzy Rule Base

Table 3 illustrates the fuzzy rule base identified by CELA. Each line represents a rule associated with a cluster centroid. Such rule implies a conditional output distribution (COD) that differs from the unconditional one (UOD). The last but on column shows the (Euclidian) distance between COD and UOD. The rules are sorted following the probability of occurrence. Note that the distance measure, which tells us something about the 'exceptionality' of the rule, differs from the probability of the regime (i.e. the X-cluster) for which the rule applies.

	F_0	F_1	F_2	F_3	F_4	dist UOD	Prob
P_0	0.187	-0.032	-0.054	0.044	0.070	0.033	0.289
P_4	-0.097	0.016	0.048	-0.052	-0.094	0.022	0.235
P_2	-0.092	0.052	0.002	-0.084	-0.081	0.026	0.139
P_1	-0.097	0.006	0.018	-0.029	0.063	0.015	0.067
P_5	-0.037	-0.008	-0.007	0.064	0.088	0.010	0.063
P_9	-0.031	-0.015	0.022	0.055	0.003	0.011	0.047
P_6	-0.055	-0.014	0.038	0.046	-0.023	0.016	0.047
P_8	-0.082	0.004	0.030	-0.003	-0.013	0.014	0.042
P_3	-0.026	0.006	-0.021	0.072	0.050	0.010	0.040
P_7	-0.034	-0.018	0.004	0.058	0.091	0.012	0.032

Table 3: *The Fuzzy Rule Base*

The others cells in the table are relative deviations of their unconditional pendant. To exemplify: Rule 0 (P_0) entails an 18.7% higher probability to arrive in regime 0 (F_0) than the unconditional probability of 0.126 (see first table), i.e. $1.187 \times 0.126 = 0.150$.

5.3 Intrapolation for Individual Firms

With CELA we assess a multiple mapping (regression hyperplanes) from the firm's characteristics data space (7-dimensional) to the return data space (5-dimensional). As explained earlier, we can assess conditional estimates of the return feature vector for a specific firm's characteristics vector. To be able to plot the return features in one single graph, we calculate per estimated feature the distance from it's (unconditional) mean and normalize that by dividing by the standard deviation of the feature in the total sample.

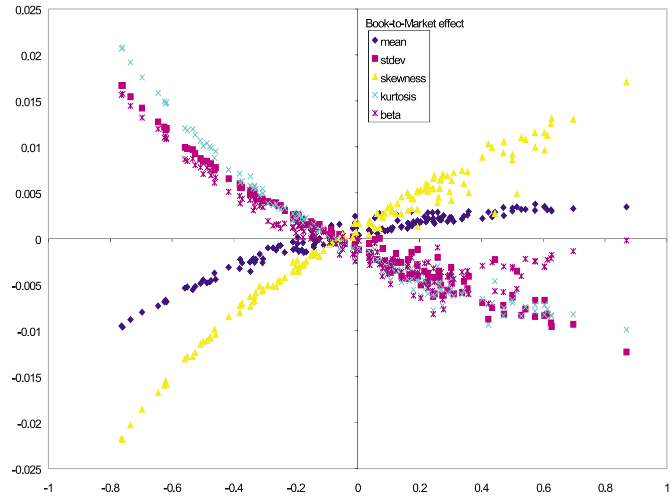

Figure 2: *Book to Market effect on return distribution.*

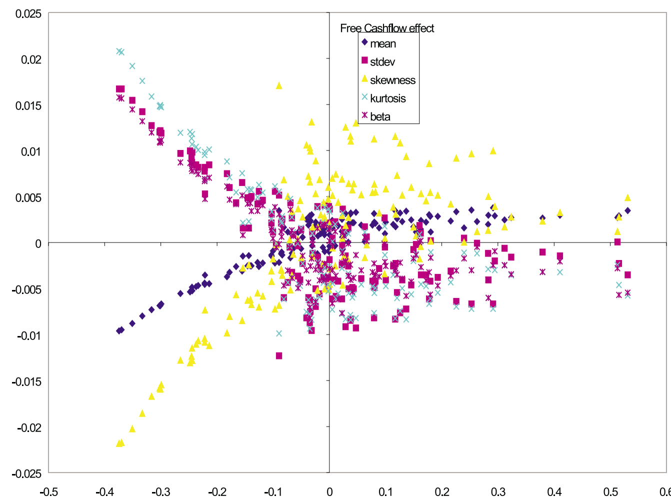

Figure 3: *Free Cash Flow effect on return distribution.*

In the next 7 charts (figure 2–8), we have scattered these return features against each of the firm's characteristics. Note that these relations are not partial but just one component plane of a multiple relation. From these charts we draw the following conclusions. There appears to be a distinct effect of the firm's characteristics on the return distribution. However, the model does only account for about 2 percent of the total variation in the return characteristics. For some characteristics the effect is less clear, for instance the D/E ratio measured in book values. For other characteristics, the relation with return features is less clear for lower than average values as compared with higher than average values, notably for free cash flow. We observe also that in all cases the market beta behaves clearly in an opposite direction (although sometimes only for lower than, resp. higher than average values).

The direction of the size effect and the D/E effect (although less clear) is mirrored when compared to the other effects. Thus above average sized firms seem to have a similar effect on the return distribution as firms with relatively low free cash flow, book-to-market ratio's, low collateral, low convertibility etc.

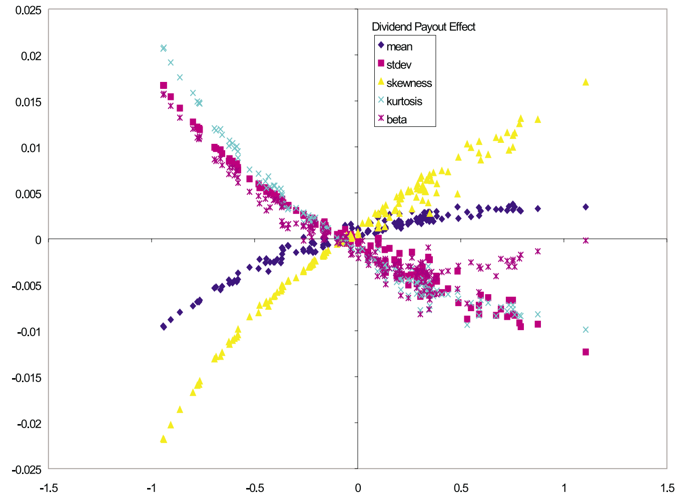

Figure 4: *Dividend Payout effect on return distribution.*

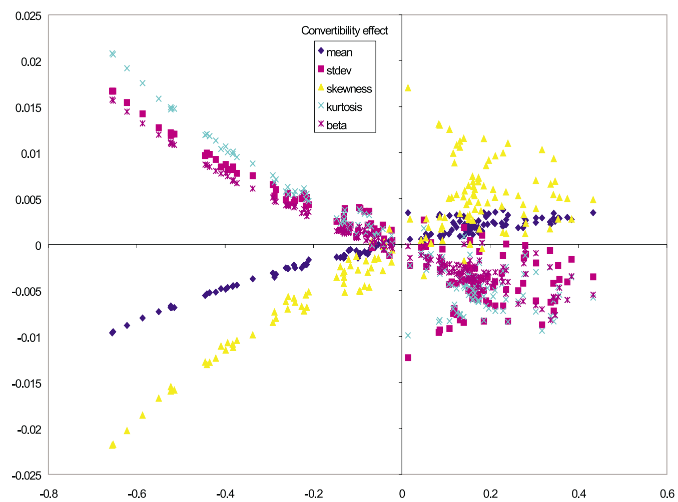

Figure 5: *Convertibility effect on return distribution.*

but the opposite (i.e. for relatively small firms and high free cash flow etc. firms) is less clear. We think that this justifies the claim that priced factors exist (apart from the market beta) in an asset pricing model, one that is related to size and one that is related to free cash flow or book to market, collateral, etc.

5.4 Industry Effects

In figure 9 we have plotted the relative frequency of companies in the 2-digit industry classes.

We observe that some classes are heavily overweighted, especially 28 (Chemicals And Allied Products), 35 (Industrial And Commercial Machinery And Computer Equipment), 36 (Electronic And Other Electrical Equipment And Components, Except Computer Equipment) and 37 (Transportation Equipment). In addition we have 5 to 10 observations in 15 (Building Construction General Contractors

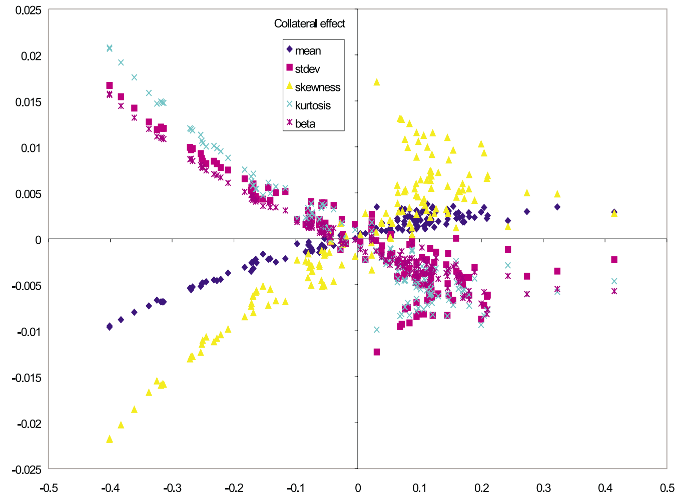

Figure 6: *Collateral effect on return distribution.*

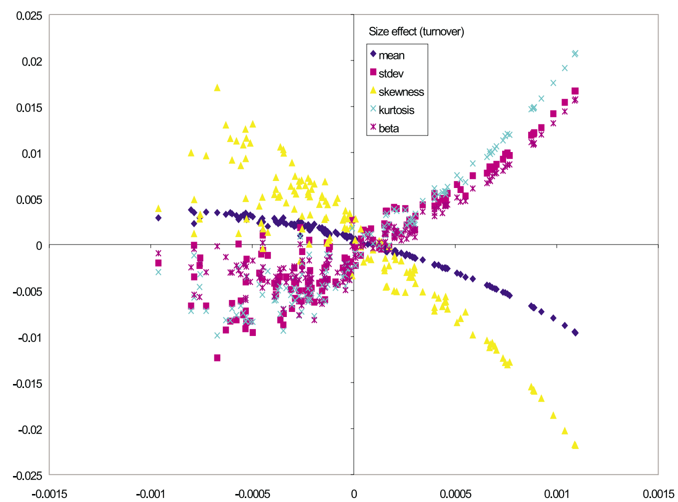

Figure 7: *Size (turnover) effect on return distribution.*

And Operative Builders), 20 (Food And Kindred Products), 38 (Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks) and 53 (General Merchandise Stores). Some 20 other classes have very few observations (mostly 1) and many other classes do not have observations at all. Clearly this uneven distribution may influency our results in the sense that they do not apply to the average Japanese company. However, since our sample represents the most heavily traded stocks on the Tokyo Sock Exchange, our results may be considered representative for the average Japanese investment portfolio.

But are the effects of firm's characteristics on the return distribution are (partly or entirely) related to industry effects? The characteristics we use are roughly the same as the proxies for the factors in the Fama en French. This model appears to remain valid even after correction for industry classes, although we know that average beta, book-to market ratio and size will vary among industry classes. But somehow a more or less complex relationship between beta, book-to market ratio and size compensates for the

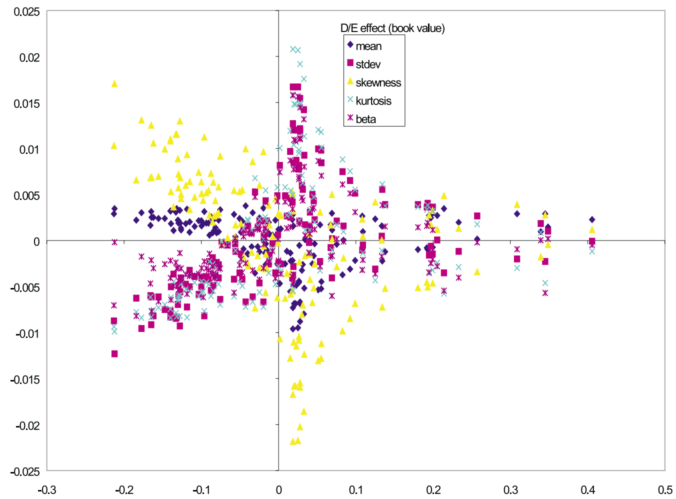

Figure 8: *Debt/Equity (book value) effect on return distribution.*

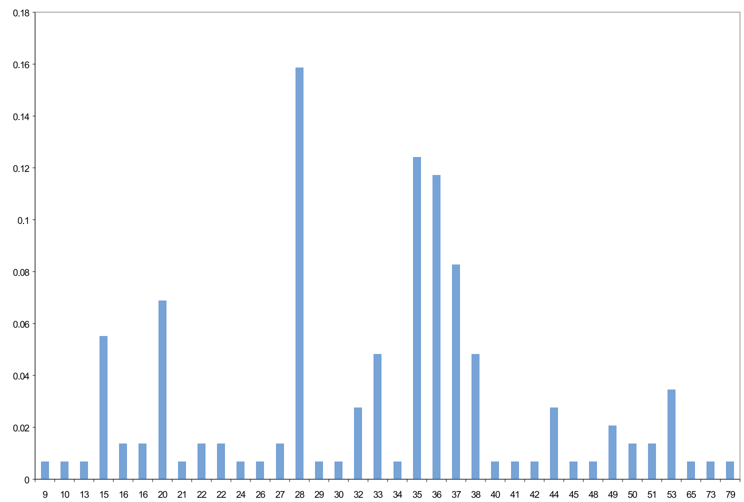

Figure 9: *Frequency Distribution of Industries*

industry effect. Anyhow, from these results we may hypothesize that the return distribution effects we identified will not disappear after correction for industry. The CELA method is well suited to assess a fuzzy classification of return characteristics that is conditional on the industry. Based on such mapping we may filter the industry effects from our return data and then again examine the relation between firm's characteristics and return distribution features.

The same filter procedure was already used for the zero-return effect set out earlier. Figure 10 shows the normalized deviation of the conditional estimates from their mean value. All distribution features seem to co-vary in a similar manner with the industry with the kurtosis as a noteworthy exception. The effects seem to be the strongest around 35 (material and equipment), 45 (transportation) and 52 (trade). This said we must note that the estimations of the conditional industry effect are far from perfect because of the uneven distribution of our data over the industry classes. However, it appears that our earlier results for the firm's characteristics remain nearly unchanged after correction for the industry effect,

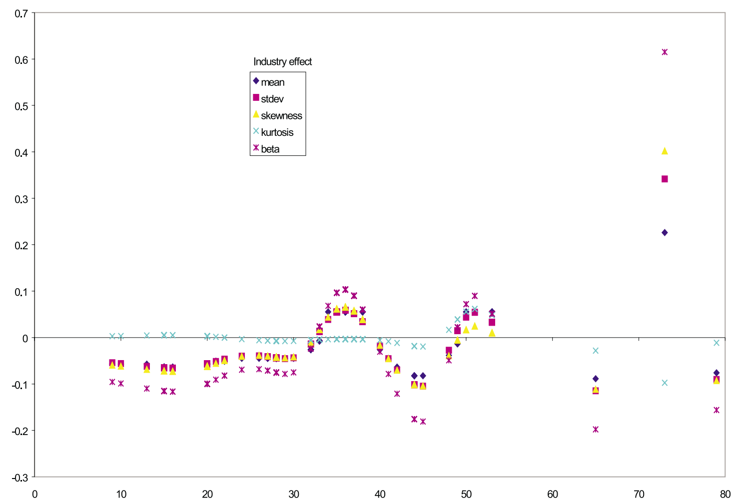

Figure 10: *Industry effect on return distribution.*

which is in line with the Fama and French finding.

6 Conclusions

In this article we construct fuzzy classes of Japanese stocks with similar return distribution patterns over time and map company characteristics on these classes, using the Competitive Exception Learning Algorithm (CELA).

The properties of the return distribution appears to be clearly related to the factors following from the theory. Apart from beta, important explanatory factors appear to be size and relative distress. The debt-equity ratio, when measured in book values, does not show any strong relationship with unexpected events and thus the market apparently has other sources of publicly available information, as the effect is priced in.

Our approach enables us to make conditional predictions regarding the probability of a stock's or a group of stocks' return series for different return distribution classes (actually *return indices*). Using these findings, one may construct conditional indices that may serve as benchmarks. These would be particularly useful for tracking and portfolio management. In fact, there is another powerful motivation for this research. In many contexts researchers and practitioners form industry portfolios for a variety of purposes. To do this, they rely on the classifications provided by the government and industry research organizations that typically measure the major line of business of the corporation. This procedure is clearly inadequate, especially for Japan.

From the conditional probability distribution we can even intrapolate and estimated return properties for individual stocks. This offers great potential for conditional option pricing and risk management techniques, like VAR, relying on it.

References

[1] Andrew Ang, Joseph Cheng, and Yuhang Xing. Downside risks and the momentum effect. Technical report, National Bureau of Economic Research, 2001. NBER working paper 8643.

[2] F. Black and M. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, pages 637–657, May/June 1973.

[3] Stephen J. Brown, W.N. Goetzmann, and M. Grinblatt. Positive portfolio factors. Technical report, Yale School of Management, 1997. working paper F-57.

[4] E.F. Fama and K.R. French. Multifactor explanations of asset pricing anomalies. *Journal of Finance*, LI(1):457–482, 1996.

[5] Campbell R. Harvey and Akhtar Siddique. Conditional skewness in asset pricing tests. *Journal of Finance*, LV(3):1263–1295, 2000.

[6] H. Ishibuchi and T. Nakashima. Effect of rule weights in fuzzy rule-based classification systems. *IEEE Transactions on Fuzzy Systems*, 9(4):506–515, August 2001.

[7] M.C. Jensen and W. Meckling. Theory of the firm: Managerial behavior, agency costs and capital structure. *Journal of Financial Economics*, 3:305–360, 1976.

[8] V. Kecman. *Learning and Soft Computing*. MIT Press, Cambridge, MA, 2001.

[9] G. J. Klir and B. Yuan. *Fuzzy Sets and Fuzzy Logic: theory and applications*. Prentice Hall, Upper Saddle River, 1995.

[10] F. Modigliani and M. Miller. The cost of capital, corporation finance, and the theory of investment. *The American Economic Review*, page 268, June 1958.

[11] Stewart C. Myers. The capital structure puzzle. *Journal of Finance*, 39:6575–592, 1984.

[12] J. van den Berg and W.M. van den Bergh. On competitive exception learning. In *Proceedings of the 19th NAFIPS International Conference*, pages 34–38, Atlanta, USA, July 2000.

[13] Jan van den Berg, Uzay Kaymak, and Willem Max van den Bergh. Fuzzy classification using probability-based rule weighting. In *Proceedings of the Eleventh IEEE International Conference on Fuzzy Systems*, Honolulu, Hawai, May 2002. to appear.

[14] Jan van den Berg, Willem Max van den Bergh, and Uzay Kaymak. Probabilistic and statistical fuzzy set foundations of competitive exception learning. In *Proceedings of the Tenth IEEE International Conference on Fuzzy Systems*, volume 2, Melbourne, Australia, December 2001.

[15] Willem Max van den Bergh, Jan van den Berg, and Uzay Kaymak. Detecting noise trading using fuzzy exception learning. In *Proceedings of Joint 9th IFSA World Congress and 20th NAFIPS International Conference*, pages 946–951, Vancouver, Canada, July 2001.

[16] W.M. van den Bergh and J. van den Berg. Competitive exception learning using fuzzy frequency distributions. *Neural Network world*, 10(1–2):59–71, 2000.

Publications in the Report Series Research* in Management

ERIM Research Program: "Finance and Accounting"

2002

A Stochastic Dominance Approach to Spanning
Thierry Post
ERS-2002-01-F&A

Testing for Third-Order Stochastic Dominance with Diversification Possibilities
Thierry Post
ERS-2002-02-F&A

Towards a Transaction Cost Theory of Management Control
Roland F. Speklé
ERS-2002-06-F&A

Modeling the Conditional Covariance between Stock and Bond Returns: A Multivariate GARCH Approach
Peter De Goeij & Wessel Marquering
ERS-2002-11-F&A

An Empirical Comparison of Default Swap Pricing Models
Patrick Houweling & Ton Vorst
ERS-2002-23-F&A

Relative Distress and Return Distribution Characteristics of Japanese stocks, a Fuzzy-Probabilistic Approach
Willem-Max van den Bergh, Onno Steenbeek & Jan van den Berg
ERS-2002-29-F&A

2001

Non-Parametric Tests for Firm Efficiency in Case of Errors-In-Variables: Efficiency Depth
Timo Kuosmanen & Thierry Post
ERS-2001-06-F&A

Testing for Productive Efficiency with Errors-in-Variables. With an Application to the Dutch Electricity Sector
Timo Kuosmanen, Thierry Post & Stefan Scholtes
ERS-2001-22-F&A

Nonparametric Efficiency Estimation in Stochastic Environments (II): Noise-to-Signal Estimation, Finite Sample Performance and Hypothesis Testing
Laurens Cherchye & Thierry Post
ERS-2001-26-F&A

Unpredictable after all? A short note on exchange rate predictability
Gerard A. Moerman
ERS-2001-29-F&A

---

* A complete overview of the ERIM Report Series Research in Management:
http://www.ers.erim.eur.nl

ERIM Research Programs:
LIS Business Processes, Logistics and Information Systems
ORG Organizing for Performance
MKT Marketing
F&A Finance and Accounting
STR Strategy and Entrepreneurship

Testing for Stochastic Dominance with Diversification Possibilities
Thierry Post
ERS-2001-38-F&A

Regime Jumps in Electricity Prices
Ronald Huisman & Ronald Mahieu
ERS-2001-48-F&A

Methodological Advances in DEA: A Survey and an Application for the Dutch Electricity Sector
Laurens Cherchye & Thierry Post
ERS-2001-52-F&A

Informed Option Trading Strategies: The Impact on the Underlying Price Process
ERS-2001-55-F&A
Cyriel de Jong

Spanning and Intersection: A Stochastic Dominance Approach
ERS-2001-63-F&A
Thierry Post

LP tests for MV efficiency
ERS-2001-66-F&A
Thierry Post

The Economic Value of Predicting Stock Index Returns and Volatility
Wessel Marquering & Marno Verbeek
ERS-2001-75-F&A

Investment and Internal Finance: Asymmetric Information or Managerial Discretion?
Hans Degryse & Abe de Jong
ERS-2001-86-F&A

The Role of Self-Regulation in Corporate Governance: Evidence from The Netherlands
Abe de Jong, Douglas V. DeJong, Gerard Mertens & Charles Wasley
ERS-2001-87-F&A

The Dividend and Share Repurchase Policies of Canadian Firms: Empirical Evidence based on a New Research Design
Abe de Jong, Ronald van Dijk & Chris Veld
ERS-2001-88-F&A

The Impact of Institutional Differences on Derivatives Usage: A Comparative Study of US and Dutch Firms
Gordon M. Bodnar, Abe de Jong & Victor Macrae
ERS-2001-89-F&A

2000

Evaluating Style Analysis
Frans A. De Roon, Theo E. Nijman & Jenke R. Ter Horst
ERS-2000-11-F&A

From Skews to a Skewed-t: Modelling option-implied returns by a skewed Student-t
Cyriel de Jong & Ronald Huisman
ERS-2000-12-F&A

Currency Hedging for International Stock Portfolios: A General Approach
Frans A. de Roon, Theo E. Nijman & Bas J.M. Werker
ERS-2000-21-F&A