Perceptual mapping of multiple variable batteries by plotting supplementary variables in correspondence analysis of rating data

Anna Torres[*] & Michel van de Velden[†]

February, 2005

Econometric Institute Report EI 2005-14

Abstract

In this paper we consider the use of correspondence analysis (CA) of rating data. CA of rating data allows a joint representation of the rated items (e.g. attributes or products) and individuals. However, as the number of individuals increases, the interpretation of the CA map becomes difficult. To overcome this problem, we propose a method that allows the depiction of additional variables, for example, background characteristics that may be of interest in identifying consumer segments, in the CA map. The idea we use is based on the representation of supplementary variables in ordinary CA. However, as the format of the additional variables is typically different from the rating data, a recoding is required. We illustrate our new method by means of an application to data of a product perception study for five cream soups.

Key words: Perceptual mapping, correspondence analysis, rating data, multiple variable batteries, consumer preferences, individual differences.

[*] Departamento de Economía de la Empresa, Universidad Carlos III de Madrid, C/Madrid, 126, 28903 Getafe (Madrid), Spain, email: atorres@emp.uc3m.es

[†] Econometric Institute, Erasmus University Rotterdam, P.O. Box 1738, 3000 DR Rotterdam, the Netherlands, tel: +31104088692, fax: +31104089162, email: vandevelden@few.eur.nl

1. **Introduction**

An important issue in marketing research concerns the construction of perceptual maps (Moskowitz, 2000 and 2002; Arditti, 1997; Monteleone, Frewer, Walkeling and Mela, 1998; Hough and Sanchez, 1998). In a perceptual map, products (or brands) are represented graphically in a space spanned by attributes. For this purpose, consumers are often asked to indicate their preference on a, usually predefined, rating scale. A multivariate analysis method, for example, principal component analysis, discriminant analysis, multidimensional scaling or canonical correlation analysis, is applied either directly to the ratings or after some data preparation steps. Discussion concerning the best multivariate technique to analyze rating data has been extensive (e.g. Hauser and Koppelman, 1979; Dillon, Frederick and Tangpanichdee, 1985; Huber and Holbrook, 1979; Holbrook and Moore, 1982; Pouplard, Qannari and Simon, 1997; Wedel, 1996) but inconclusive.

Instead of mapping products (or brands) in a space spanned by attributes, one may also be interested in mapping attributes in low-dimensional space. This situation could occur, for example, in product test analysis, where a consumer panel evaluates a certain product on a number of attributes. Applying a perceptual mapping technique to such data yields a map that best reflects the relationships among attributes according to the subjects' preferences. In addition to the relationships among the attributes, the position of subjects with respect to the attributes is often of interest as well. In particular, marketing researchers are often interested to see if segments of individuals can be distinguished. For example, in a product test analysis, the researcher is interested in identifying the set of dimensions, constructed from a list of attributes that apply to the product, for a set of subjects (Moskowitz, Jacobs and Lazar, 1985). Then, by incorporating additional, subject specific information, clusters of subjects showing similar hedonic responses to stimuli, may be identified in the perceptual map spanned by the attributes. These clusters can be of strategic importance to the researcher (Monteleone, Frewer, Wakeling and Mela, 1998). Hence, the perceptual map should not only depict the attributes but also subject related variables.

DeSarbo and Wu (2001) recognized the importance of multivariate techniques that enable a joint representation of data in which, in addition to the subjects’ attribute ratings, information concerning the subjects is available through, for example, demographic variables. They refer to such data as multiple variable batteries. DeSarbo and Wu (2001) propose a latent structure multidimensional scaling procedure to jointly represent the structure in multiple variable batteries. Here we follow a different route and propose the use of correspondence analysis (CA) to depict additional variables containing subject specific information.

Although CA itself is a fairly well-known method, there appear to be no applications in the field of marketing research where subjects and attributes are jointly displayed. Typically (e.g., Best, Rayner and O'Sullivan, 2000; Pouplard, Qannari and Simon, 1997), CA is applied to rating data after transforming the original data to a contingency table. That is, the subject by attributes data matrix is transformed into an attributes by ratings table. The cells of such a table give the number of times that a certain rating is assigned to a certain attribute. CA of rating data in contingency tables presents some limitations. Firstly, by aggregating over the individuals the ratings are disengaged from the subjects. Secondly, the obtained CA scores may not be ordered in the same way as the categories (Pouplard, Qannari and Simon, 1997).

One reason for the lack of applications of CA of rating data may be the fact that the application of CA to rating data is not straightforward. In fact, we can distinguish at least two different approaches for applying CA to rating data. The first approach is due to Nishisato (1980) and fits in the framework of dual scaling analysis. The second approach, which will be the topic of this paper and to which we will refer as CA of rating data, was presented in Benzécri (1973) and reintroduced (in English) by Greenacre (1984). Recent papers by Van de Velden (2000), Torres and Greenacre (2002), and Van de Velden (2004), deal with theoretical issues concerning the two different approaches for the special case of rank order data.

In this paper, we consider CA of rating data from an applied point of view. Moreover, we introduce a new method that allows the depiction of additional, subject specific information in the CA map. The new method is based on the representation of supplementary points in ordinary CA. It allows the representation of background characteristics into the attribute map in such a way that individuals corresponding to certain characteristics are close to attributes for which they indicated high preferences. Hence, our proposal facilitates the graphical representation of multiple variable batteries. By applying the method to a product perception (taste) study, we illustrate what a powerful tool CA can be.

2. Methodology

2.1 Correspondence analysis of ratings

Correspondence analysis is an exploratory multivariate technique that converts a matrix of nonnegative data into a graphical display in which the rows and columns of the matrix are depicted as points (Greenacre and Hastie, 1987).

Greenacre (1984) describes a particular way to code rating data to be displayed in correspondence analysis. First, as the geometry of correspondence analysis was elaborated to treat frequencies, the original ratings are transformed in such a way that the lowest rating is zero. This can usually be achieved by simply subtracting one from the data. Next, for each rating we add an additional rating on a reversed scale. This can be achieved by subtracting the original ratings from the highest possible rating. This procedure is denominated as "doubling". As we will explain below, doubling allows us to recover the "mean" as well as the "variation" for the evaluated attributes.

After doubling of the data, the evaluations of subjects are described by a rating on the original scale and a rating on the reversed scale. Consequently, we obtain a data matrix where the number of rows is the total number of subjects, and the number of columns is two times the number of attributes. We refer to the original attributes as "positive attributes" and to the doubled set as "negative attributes".

We illustrate the doubling procedure by means of a small example. In Table 1, the first three columns correspond to ratings for three attributes on a scale from 0 to 4 for 3 subjects (S1, S2 and S3). The last three columns are the values obtained after doubling.

Table 1: Example data

	Attribute 1+	Attribute 2+	Attribute 3+	Attribute 1-	Attribute 2-	Attribute 3-
S1	4	3	2	0	1	2
S2	3	2	2	1	2	2
S3	3	2	3	1	2	1

CA of rating data simply amounts to applying the usual CA algorithm, which is based on a so-called generalized singular value decomposition (for computational details see, e.g., Greenacre, 1984), to the doubled data matrix. The CA solution makes it possible to graphically represent the rows (i.e. the subjects) and columns (i.e. the negative and positive attributes) of the table. Some important CA formulae are given in Appendix A.

2.2 Interpretation of the CA of ratings solution

In the CA plot of the attributes, the negative and positive attributes are represented as separate points. However, each pair can be connected by drawing a straight line that runs through the origin. This line between the two points represents the rating scale used by the subjects. For each attribute, the rating scale can be indicated on the line by assigning the lowest rating that was used by the subjects to the point corresponding to the negative attribute, and the highest rating that was used by the subjects to the positive attribute point. Then, as the origin in CA corresponds to the average rating, the mean rating for each attribute can be read from the map. If a positive attribute is closer to the origin than the negative attribute, the average rating for that attribute is high, and vice versa. In addition, the distance between the positive and the negative pole is a measure of the variance within an attribute. Some authors like Best, Rayner and O'Sullivan (2000) recognize that the dispersion effect can indicate market segmentation of niche markets.

2.3 Representing additional variables in CA of rating

In CA of rating data, subjects (rows) and positive and negative attributes (columns) can be depicted simultaneously in such a way that a close position between a subject and a positive attribute, indicates a relatively high level of preference. However, as the number of subjects increases, it becomes difficult to interpret maps where each subject is depicted as an individual point. One solution to this problem is to calculate average ratings for sub-groups of the individuals. Then, the sub-groups and attributes can be plotted using CA of the average ratings. For example, Thiessen and Blasius (2000) analyse average ratings for subgroups of individuals based on their background profiles. However, by analysing average ratings, subject specific information is lost and the positions of the attributes are no longer based on the variation in all n observations. Moreover, taking averages becomes more troublesome if more background variables are of interest. Therefore, we propose a method that does not require the calculation of averages for sub-groups. Instead, we introduce a way of coding the data that lets us to recover, in a more comprehensive way, subject related information. The idea is based on adding supplementary columns (e.g., Benzécri, 1973; Greenacre, 1984), that contain relevant information in the form of variables that may affect preferences.

CA has the option to display additional (or supplementary) variables in such a way that they do not play a role in the determination of the map. The levels of the additional variables are merely projected into the CA map. Based on the positions of the levels of the additional variables we may be able to establish segments related with the obtained dimensions. However, in order to meaningfully represent additional variables in the CA map, it is necessary that they are coded in a similar format as the original rating data. Therefore, unless the additional variables are ratings, a transformation is required that enables the projection of the columns into the original map.

The columns in CA of rating data give a distribution of the assigned ratings over the subjects. By dividing the ratings through the column totals we obtain a profile. The profiles are depicted in the CA map in such a way that profiles that are similar are close to each other and profiles that are dissimilar are far away from each other. Now, suppose that for each individual we have data concerning background characteristics or variables coded by means of a so-called dummy or indicator matrix. In an indicator matrix, rows represent subjects and columns represent levels or categories. The appropriate category for an individual is coded as a one, whereas the remaining categories are coded by zeros. We want to plot the characteristics in the CA map in such a way that their positions represent the preference structures for the individuals associated with the characteristics. In particular, the background characteristics should be plotted in the CA map in such a way that a characteristic is close to the positive or negative attribute(s) that were highly evaluated by the subjects with the characteristic. To achieve this, we replace the ones in the indicator matrix by the highest rating that was used by an individual in his/her evaluation of the (both positive and negative) attributes. Moreover, to minimize the role played by the subjects that do not correspond to a characteristic or category of the additional variable, we replace the zero values in the columns of the indicator matrix by the subjects' mean rating. In that way, the sum of squared differences between the value for the additional variable and the ratings for the attributes is minimized.

Let **F** denote the n x $2p$ doubled data matrix. Hence, $f_{i(p+j)} = r - f_{ij}$, where r is the highest possible rating on the scale with as lowest value zero. The additional variables can be collected in an n x q matrix **Z**, whose columns correspond to the categories of the additional variables, with as elements

$$z_{ik} = \begin{cases} \max_j (f_{ij}), & \text{if subject } i \text{ corresponds to category } k \\ \dfrac{\sum_{j=1}^{2p} f_{ij}}{2p} = \dfrac{pr}{2p} = \dfrac{1}{2} r, & \text{else} \end{cases}$$

Note that, by using the doubled data, the mean rating value is in fact equal to the mid-point of the scale. Using formula (A.3) from Appendix A, the profiles corresponding to the categories of the additional variables (i.e. the columns of **Z** divided by their totals) can be calculated as $\mathbf{H}^* = \mathbf{D}_z^{-1}\mathbf{Z}'\mathbf{X}$, where, $\mathbf{D}_z^{-1}$ is a diagonal matrix with as diagonal elements the column sums of **Z,** and **X** is the matrix of standard coordinates for the subjects (see the Appendix for a more detailed description of these concepts). The supplementary point is projected into the map using the full, n-dimensional profile. However, for the subjects that do not correspond to the jth category, the n-dimensional profile contains n - n_j (where n_j denotes the number of subjects corresponding to the j-th level of the additional variable) average ratings. As the variation for these subjects is zero by construction, the variance of the n-dimensional profile point underestimates the variance based on the n_j observations. To correct for this underestimation, we assign a mass of $\sqrt{\dfrac{n}{n_j}}$ to the j-th level of the supplementary variable so that the sample variance becomes: $S^2\left(\sqrt{\dfrac{n}{n_j}} z_j\right) = \dfrac{1}{n}\sum_{i=1}^{n}\dfrac{n}{n_j}(z_i - \bar{z})^2 = \dfrac{1}{n_j}\sum_{i=1}^{n_j}(z_i - \bar{z})^2$. Algebraically, we can express this procedure by introducing a diagonal matrix $\mathbf{D}_n$ with as diagonal elements $\sqrt{\dfrac{n}{n_j}}$, for $j=1\ldots p$. The coordinates for the supplementary points can thus be calculated using: $\mathbf{H}^* = \mathbf{D}_n\mathbf{D}_z^{-1}\mathbf{Z}'\mathbf{X}$.

Instead of depicting all levels of the variable into the CA map one may also choose to depict some of the levels. For example, if there are only few occurrences of a certain category, it may not be worthwhile representing the category in the map. In that case, the corresponding column of **Z** can simply be discarded.

3. Application: Product test of refrigerated cream soups

3.1 The data

We consider data from a product test study of refrigerated cream soups made in Barcelona and Madrid, in 2004. A panel of 380 female household shoppers where asked to evaluate a cream soup on a number of attributes. Five types of cream soups were considered: carrot cream soup, mushroom cream soup, vegetable cream soup, zucchini cream soup, and vichyssoise. Each woman evaluated only one type of cream soup. The attributes with respect to which the soups were evaluated are: “taste”, “intensity of taste”, “authenticity of taste”, “natural taste”, “homemade taste”, “smell”, “thickness”, “creaminess”, “saltiness”, “colour”, “appearance” and “homemade appearance”. Subjects evaluated all attributes on a 5-point hedonic scale. However, the levels for different attributes had different meanings for each attribute. Table 2 (in Appendix B) gives all attributes and their levels. The rating data were collected in a 380 by 9 subjects by attributes matrix.

In addition to the attribute ratings, subject specific information was gathered that could help in linking the preferences to background variables. For each subject the following information was collected: the cream soup that was tested, affinity with cooking, civil status, number of persons in the household, status of the interviewed woman at home, level of studies and job of the interviewed person as well as those of the head of the family (if they are different persons). A description of these variables can be found in Table 3 (in Appendix B).

3.1 Correspondence analysis results

Figure 1 gives a two-dimensional CA plot based on the doubled attribute ratings. Moreover, using the procedure described in Section 2, the five different cream soups have been projected into the attribute map. The two dimensional CA solution accounts for 57% of the inertia: the first dimension accounts for 44% and the second for 13%.

We have connected the positive and negative attributes by straight lines through the origin to simplify the interpretation of the mean and variance of the attributes. Each line represents the rating scale for the attribute. The negative attribute point corresponds to the lowest rating and the positive point to the highest rating. In Table 4, we see that for the attribute “creaminess” the maximum rating was 4 rather than 5. Hence, the positive attribute point for “creaminess” corresponds to a rating 4. All other positive attribute points correspond to the rating 5.

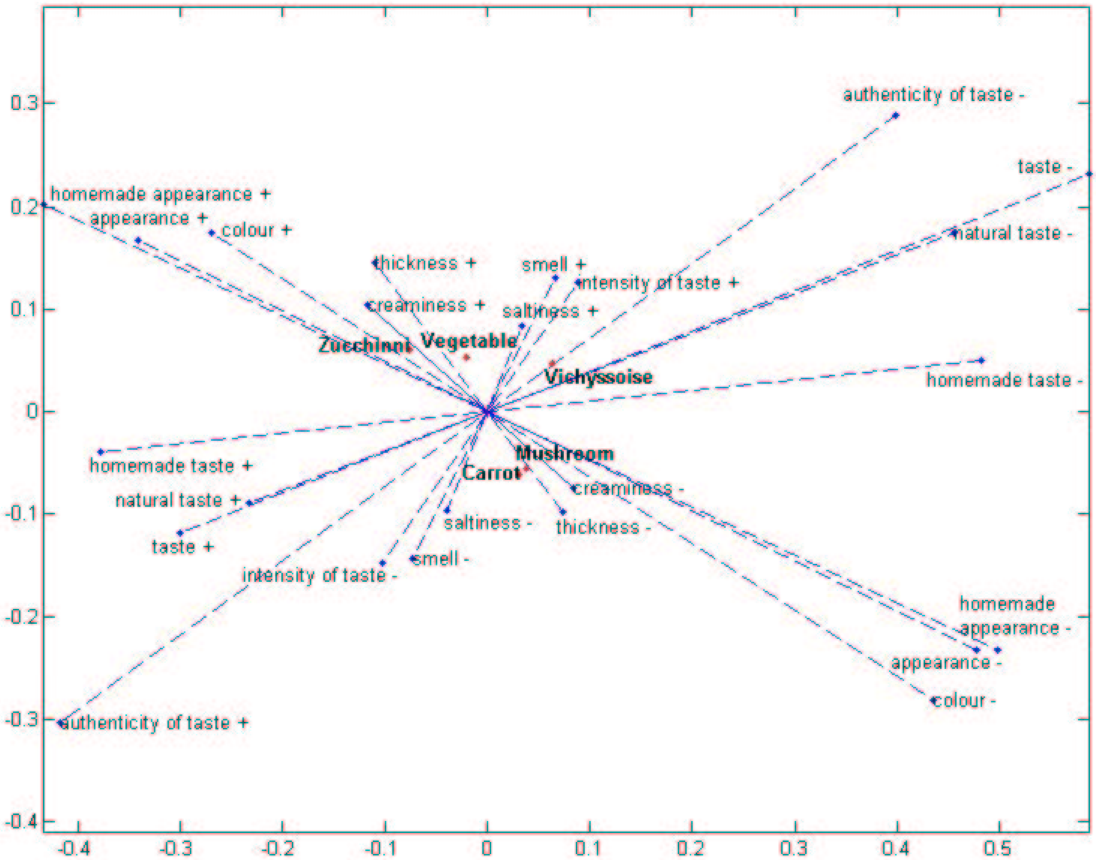
The mean rating for each attribute can be obtained by considering the position of the origin relative to the two endpoints. We see that for most attributes the origin is close to the middle of the scale. Hence, as can also be verified from the numerical results in Table 4, the mean ratings for the attributes are close to the middle rating.

The length of the lines between the negative and positive points for an attribute, give an indication of the variance for that attribute. We see that the attributes “saltiness”, “creaminess”, “thickness”, “smell” and “intensity of taste”, have smaller variances than the other attributes. Table 4 confirms this result.

If we only consider the positive attributes in Figure 2, we can roughly distinguish 4 clusters of attributes. The first cluster located in the lower left quarter of the plot, consists of attributes pertaining to general aspects of taste: “authenticity of taste”, “homemade taste”, “taste” and “natural taste”. In the upper left quarter we find the attributes “homemade appearance”, “appearance” and “colour”, which are all related to the appearance of the product. Also related to appearance but located closer to the origin with smaller variances, are the attributes “thickness” and “creaminess”. The difference in position may be explained by the different scales used for these attributes. They range from “not creamy” and “not thick enough” towards “too creamy” and “too thick”. Hence, the middle point is the most positive evaluation for these attributes. On the positive side of the first dimension we find the fourth cluster containing the attributes “smell”, “intensity of taste” and “saltiness”. Again, these attributes are measured on a scale in which both endpoints have negative connotations. In addition, these attributes are related to more specific taste characteristics than the taste characteristics of the first cluster.

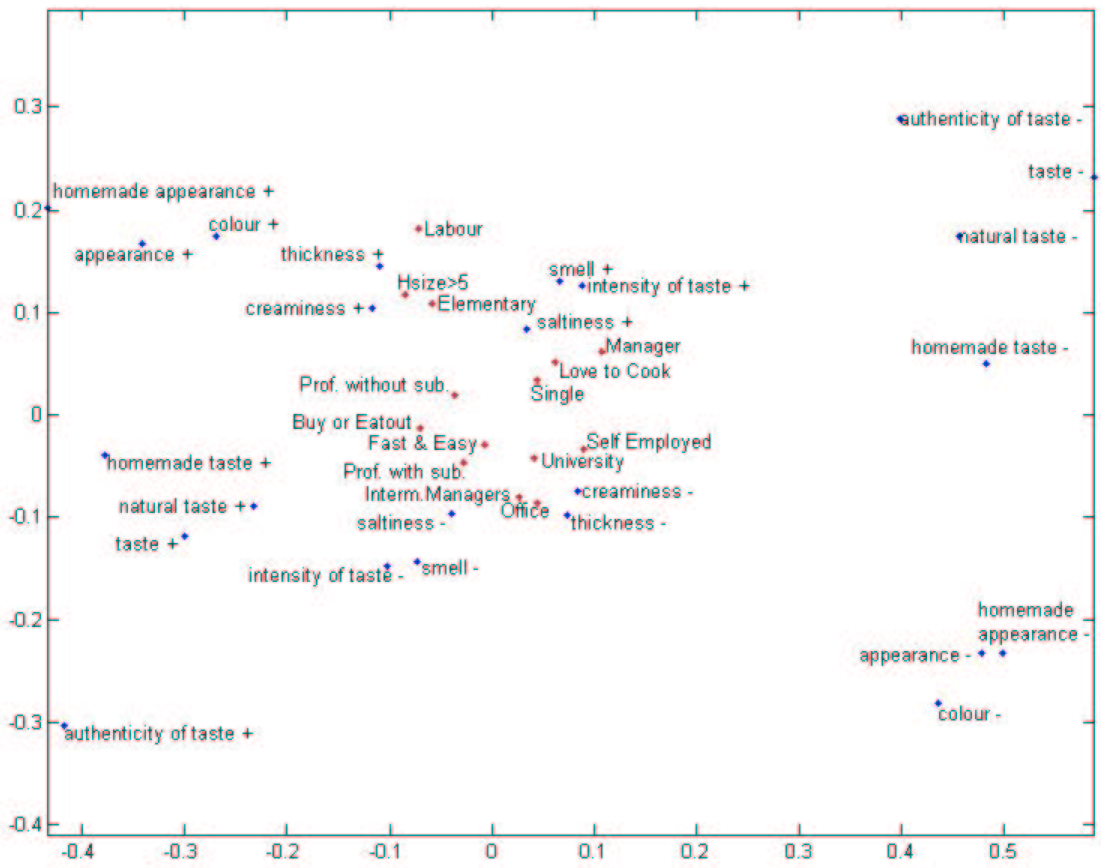
Comparing the positions of the soups with those of the attributes we obtain an idea about the evaluation of the individual soups. We see that the point corresponding to the “zucchini” cream soup is located relatively close to the points in the appearance cluster. This means that the women evaluating the zucchini cream soup gave, on average, high ratings for these attributes. The mushroom and carrot cream soups are located close to each other indicating that the evaluations for these soups were similar. Their position is close to the negative poles of the attributes “creaminess” and “thickness”. Hence, the women in the test panel found these soups much less creamy and thick than they preferred. These soups also scored low with respect to the other appearance attributes. The vegetable soup was perceived as being too creamy and too thick but with respect to the appearance attributes the soup score relatively high. Finally, the vichyssoise cream soup’s taste was perceived as neither natural nor authentic. Moreover, it is described as too salty, too intense and with a strong smell.

Figure 1: Correspondence analysis map of the attributes, with soups as supplementary points



In Figure 2, we introduce as supplementary points the classificatory variables. Four background variables were used: Profession of the head of the family, level of education of the head of the family, number of persons in the household and civil status. In addition, the respondents' affinity with cooking was plotted as well. An explanation of the additional variables and their levels can be found in Table 3. Plotting the additional variables into the attribute map yields a configuration in which several category points where extremely close to the origin. Hence, for those profiles, the distribution did not differ much from the average attribute profile. These points have been removed from the plot to avoid a big clutter of points at the origin. They are indicated in the last column of Table 3.

Figure 2: Correspondence analysis map of attributes with classificatory variables as supplementary points.



On the top left side of the plot, exhibiting similar profiles as the (positive) appearance attributes, are "labour" families. The education level of this group is low (up to elementary) and the households tend to be high (>5). In the same direction but less far from the average, are “professionals without subordinates” (e.g. electricians, carpenters, lawyers, architects).

Families where the head is a “professional worker with subordinates” an “office worker” or an “intermediate manager”, appear more towards the negative side of the plot. The education level of this group tends to be higher (university) and they show similar profiles as the negative attributes “saltiness”, “thickness” and “creaminess”. Thus, for these groups, the soups were often considered to be not creamy, salty, and thick enough. Families where the head is “self-employed”, also show similar profiles to the negative “creaminess” attribute. In addition, they appear to be more in the direction of the negative appearance attributes. Finally, towards the right top side we find the managers. This group appears to be more critical especially with respect to the taste attributes.

If we look at the positions of the respondents’ affinity with cooking we see that the women who “love to cook” appear to me more critical with respect to the taste attributes. The profiles for the women who “buy or eat-out” or who are interested in preparing “fast and easy” meals are quite similar. The “buy or eat-out” group is more positive with respect to the (general) taste attributes “homemade taste” and “natural taste”, whereas the “fast and easy” group finds that the soups lack in taste intensity, smell and salt.

4. Conclusions

In this paper we proposed a method for displaying additional variables in correspondence analysis of rating data. By coding the additional variables in such a way that subjects corresponding to a category are located close to the attributes that received high ratings, a relationship between background variables and attributes may become apparent. Note that in the proposed method the configuration is solely based on the rating data. The additional points are projected in the existing map. It is also possible to apply correspondence analysis to the rating data and the supplementary variables. However, given the different nature of the columns corresponding to the attributes and those corresponding to the additional variables, we chose not do so.

In our method, we used average attribute ratings for subjects that do not correspond to a certain category. An alternative approach would be to code the ratings for those subjects as zeros or as the lowest assigned rating. By coding the data in that way, the positions of the supplementary points become weighted averages of subject points. However, the interpretation of the zeros in the columns for the additional variables and the zeros in the columns corresponding to attributes differs significantly. A zero for an additional variable indicates that the subject does not correspond to a certain category whereas a zero for the attribute rating indicates that the subject assigned a low (in fact: the lowest) rating to the attribute.

Lawrence (2000) proposed a method in the context of dual scaling analysis of rank-order data, in which the subjects are coded as the end-points of the scale. In our treatment that would amount to using the original rating values (on a scale from 1 to the maximum value), and assigning a zero to the subjects that do not correspond to a category and the maximum rating value plus one to the other subjects. It is not trivial how to interpret and justify such a procedure for the type of analysis discussed in this paper.

With an application we showed how CA of rating data can serve as a useful tool in obtaining a perceptual map based on rating data. Moreover, by projecting additional variables in the CA map, we are able to relate the preference structure to background variables. This may be of great value to practitioners in the field of product design or market research.

Appendix A

Here we give a brief summary of some important CA formulae. As there already exist several excellent expositions of CA (e.g. Greenacre, 1984 and 1993; Lebart et al., 1984), we limit ourselves to a presentation of some formulae that are important in the context of this paper. For a complete treatment of CA we refer to the extant CA literature.

Let **F** denote an n x p data matrix where each element has been divided by the sum of all elements so that $\sum_{i,j} f_{ij} = 1$. We define $\mathbf{D}_r$ as a diagonal matrix with as diagonal elements the row sums of **F**, and $\mathbf{D}_c$ as a diagonal matrix with as diagonal elements the column sums of **F**. Vectors of row and column sums of **F** are denoted by **r** and **c** respectively. The goal of CA is to approximate, in a least-squares sense, the p-dimensional row profiles (i.e. the rows of **F** divided by the corresponding row sums) and the n-dimensional column profiles (i.e. the columns of $\mathbf{F}$ divided by the corresponding column sums) in low dimensional space.

The CA solution can be obtained by considering the singular value decomposition

$$\tilde{\mathbf{F}} = \mathbf{U}\boldsymbol{\Lambda}\boldsymbol{\Lambda}', \tag{A.1}$$

where $\tilde{\mathbf{F}} = \mathbf{D}_r^{-\frac{1}{2}}(\mathbf{F} - \mathbf{rc}')\mathbf{D}_c^{-\frac{1}{2}}$, $\boldsymbol{\Lambda}$ is a diagonal matrix of singular values (in descending order of magnitude), and $\mathbf{U}$ and $\mathbf{V}$ are orthonormal matrices of singular vectors. So-called matrices of k-dimensional *principal* coordinates for the rows and columns of $\mathbf{F}$ can be obtained by considering the first k columns of $\mathbf{G} = \mathbf{D}_r^{-\frac{1}{2}}\mathbf{U}\Lambda$, and $\mathbf{H} = \mathbf{D}_c^{-\frac{1}{2}}\mathbf{V}\Lambda$ respectively. The principal coordinates are standardized in such a way that $\mathbf{G}'\mathbf{D}_r\mathbf{G} = \mathbf{H}'\mathbf{D}_c\mathbf{H} = \Lambda^2$. The points in principal coordinates are crucial in CA. Distances between points of one mode (i.e. rows or columns) are so-called chi-squared distances.

In addition to the principal coordinates it is often useful to consider so-called *standard* coordinates. Standard coordinates can be obtained by considering the first k columns of $\mathbf{X} = \mathbf{D}_r^{-\frac{1}{2}}\mathbf{U}$ and $\mathbf{Y} = \mathbf{D}_c^{-\frac{1}{2}}\mathbf{V}$ respectively. Hence, the standard coordinates are standardized in such a way that $\mathbf{X}'\mathbf{D}_r\mathbf{X} = \mathbf{Y}'\mathbf{D}_c\mathbf{Y} = \mathbf{I}$.

From these definitions and the singular value decomposition (A.1) we get

$$\mathbf{D}_r^{\frac{1}{2}}\mathbf{G}\mathbf{Y}'\mathbf{D}_c^{\frac{1}{2}} = \mathbf{D}_r^{\frac{1}{2}}\mathbf{X}\mathbf{H}'\mathbf{D}_c^{\frac{1}{2}} = \mathbf{U}_k\boldsymbol{\Lambda}_k\mathbf{V}_k', \tag{A.2}$$

where $\mathbf{U}_k$ and $\mathbf{V}_k$ are matrices with the first k columns of $\mathbf{U}$ and $\mathbf{V}$ respectively, and $\boldsymbol{\Lambda}_k$ is the corresponding k x k matrix of singular values. From Eckart and Young's theorem (1936), it follows that (A.2) gives a least-squares approximation of the matrix $\tilde{\mathbf{F}}$. Moreover, (A.2) shows that the joint representation of principal coordinates for one mode and standard coordinates for the other mode constitutes a so-called biplot (Gabriel, 1971). This biplot representation has various interesting properties. For a detailed description of the CA biplot see Greenacre (1993).

An important property of the CA solution is that the coordinates for one mode (e.g. the columns) are closely related to the coordinates for the other mode. That is,

$$\mathbf{H} = \mathbf{D}_c^{-1}\mathbf{F}'\mathbf{X}, \tag{A.3}$$

and

$$\mathbf{G} = \mathbf{D}_r^{-1}\mathbf{F}\mathbf{Y}. \tag{A.4}$$

Formulae (A.3) and (A.4) are referred to as transition formulae. They play a crucial role in the calculation of coordinates for supplementary variables.

Appendix B

Table 2: Descriptor list of sensory analysis

Descriptors	Scales
Taste	I do not like it at all; I do not like it; I like it a bit; I quit like it; I like it a lot.
Intensity of taste	Much too weak.; A bit too weak; Just as I like it; A bit too strong; Much too strong (-than I would like)
Authenticity of taste	Not similar at all; Not similar; A bit similar; Quite similar; Very similar (-to a homemade product).
Natural taste	Very artificial; Artificial; Not really natural; Quite natural; Very natural.
Homemade taste	No homemade taste at all; No homemade taste; Some homemade taste; Quite homemade taste; A very homemade taste.
Smell	Much too weak.; A bit too weak; Just as I like it; A bit too strong; Much too strong (-than I would like).
Thickness	Much too thin; A bit too thin; Just as I like it; A bit too thick; Much too thick (-than I would like).
Creaminess	Not at all creamy enough; Not creamy enough; Just as I like it; A bit too creamy; Much too creamy (-than I would like).
Saltiness	Not enough salt at all; Not enough salt; Just as I like it; A bit too salty; Much too salty (-than I would like).
Colour	Very artificial; Artificial; Not really natural; Natural; Very natural;
Appearance	Not appetizing at all; Not appetizing; Appetizing; Quite appetizing; Very appetizing.
Homemade appearance	No homemade appearance at all; No homemade appearance; Homemade appearance; Quite a homemade appearance; A really homemade appearance.

Table 3: Classificatory variables

Variables	Levels	Comments
Afinity with cooking	Love to Cook	
	Fast and Easy (cooking)	
	Buy or Eat-out	
Civil status	Single	
	Married	Close to the origin, removed from the plot
	Divorced	Close to the origin, removed from the plot
Nº persons at home	Hsize=1	Close to the origin, removed from the plot
	Hsize=2	Close to the origin, removed from the plot
	Hsize=3-5	Close to the origin, removed from the plot
	Hsize>5	
Level of education (head of family)	Elementary	
	Highschool	Close to the origin, removed from the plot
	University	
Job (head of family)	Not working	Close to the origin, removed from the plot
	Labour	
	Professional without Subordinates	
	Professional with Subordinates	
	Office work	
	Intermediate Manager	
	Self Employed	
	Manager	

Table 4: Descriptive statistics of the attributes

Attributes	N	Min.	Max	Mean	Std. Deviation
Taste	380	1	5	3.65	1.06
Intensity of taste	380	1	5	3.15	0.77
Authenticity of taste	380	1	5	2.95	1.20
Natural taste	380	1	5	3.65	0.86
Homemade taste	380	1	5	3.24	1.03
Smell	380	1	5	3.10	0.75
Thickness	380	1	5	2.60	0.71
Creaminess	380	1	4	2.66	0.65
Saltiness	380	1	5	3.15	0.60
Colour	380	1	5	3.47	1.01
Appearance	380	1	5	3.33	1.05
Homemade appearance	380	1	5	3.14	1.13

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