Investor Preferences for Oil Spot and Futures
Based on Mean-Variance and Stochastic Dominance

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Abstract

This paper examines investor preferences for oil spot and futures based on mean-variance (MV) and stochastic dominance (SD). The mean-variance criterion cannot distinct the preferences of spot and market whereas SD tests leads to the conclusion that spot dominates futures in the downside risk while futures dominate spot in the upside profit. It is also found that risk-averse investors prefer investing in the spot index, whereas risk seekers are attracted to the futures index to maximize their expected utilities. In addition, the SD results suggest that there is no arbitrage opportunity between these two markets. Market efficiency and market rationality are likely to hold in the oil spot and futures markets.

Keywords: Stochastic dominance, risk averter, risk seeker, futures market, spot market.
JEL classifications: C14, G12, G15.
Introduction

Crude oil is an important commodity for the world economy. With the increasing tension of crude oil price, oil futures have become a popular derivative to hedge against the risk of possible oil price changes. Spot and futures prices of oil have been investigated over an extended period. Substantial research has been undertaken to analyze the relationship between spot and futures prices, and their associated returns. The efficient market hypothesis is crucial for understanding optimal decision making with regard to hedging and speculation, and also for making financial decisions about the optimal allocation of portfolios of assets with regard to their multivariate returns and associated risks.

The literature on the relationships between spot and futures prices of petroleum products has examined issues such as market efficiency and price discovery. Bopp and Sitzer (1987) find that futures prices have a significant positive contribution to describe past price changes, even when crude oil prices, inventory levels, weather, and other important variables are accounted for. Serletis and Banack (1990) use daily data for the spot and two-month futures crude oil prices, and for prices of gasoline and heating oil traded on the New York Stock Exchange (NYMEX), to test for market efficiency. They find evidence in support of the market efficiency hypothesis. Crowder and Hamid (1993) use cointegration analysis to test the simple efficiency hypothesis and the arbitrage condition for crude oil futures. Their results support the simple efficiency hypothesis that the expected returns from futures speculation in the oil futures market are zero.

In the price discovery literature, Quan (1992) examines the price discovery process for the crude oil market using monthly data, and finds that futures prices do not play an important role in this process. Using daily data from NYMEX closing futures prices, Schwartz and Szakmary (1994) find that futures prices strongly dominate in the price discovery process relative to the deliverable spots in all three petroleum markets. Gulen (1999) applies cointegration tests in a series of oil
markets with pairwise comparisons on post-1990 data, and concludes that oil markets have become more unified during the period 1994-1996 as compared with the period 1991-1994. Silvatulle and Moosa (1999) examine the daily spot and futures prices of WTI crude using both linear and non-linear causality testing. They find that linear causality testing reveals that futures prices lead spot prices, whereas non-linear causality testing reveals a bidirectional effect. Bekiros and Diks (2008) test the existence of linear and nonlinear causal lead–lag relationships between spot and futures prices of West Texas Intermediate (WTI) crude oil. They find strong bi-directional Granger causality between spot and futures prices, but the pattern of leads and lags changes over time.

Lin and Tamvakis (2001) investigate information transmission between NYMEX and London’s International Petroleum Exchange. They find that NYMEX is a true leader in the crude oil market. Hammoudeh et al. (2003) also investigate information transmission among NYMEX WTI crude prices, NYMEX gasoline prices, NYMEX heating oil prices, and among international gasoline spot markets, including the Rotterdam and Singapore markets. They conclude that the NYMEX gasoline market is the leader. Furthermore, Hammoudeh and Li (2004) show that the NYMEX gasoline price is the gasoline leader in both pre- and post- Asian crisis periods.

Empirical studies indicate that commodity prices can be extremely volatile at times, and that sudden changes in volatility are quite common in commodity markets. For example, using an iterative cumulative sum-of-squares approach, Wilson et al. (1996) document sudden changes in the unconditional variance in daily returns on one-month through six month oil futures and relate these changes to exogenous shocks such as unusual weather, political conflicts and changes in OPEC oil policies. Fong and See (2002) conclude that regime switching models provide a useful framework in studying factors behind the evolution of volatility and short-term volatility forecasts. Moreover, Fong and See (2003) show that the regime switching model outperforms the standard conditional volatility GARCH model based on standard evaluation criteria for short-term volatility forecasts.
Much of the literature has employed conventional parametric tests, such as the mean-variance (MV) criterion and CAPM statistics. These approaches rely on the normality assumption and the first two moments. However, the presence of non-normality in portfolio stock distributions has been well documented (Beedles, 1979; Schwert, 1990).

The stochastic dominance (SD) approach differs from conventional parametric approaches in that comparing portfolios by using the SD approach is equivalent to the choice of assets by utility maximization. It endorses the minimum assumptions of investor utility functions, and analyses the entire distributions of returns directly. The advantage of SD analysis over parametric tests becomes apparent when the asset returns distributions are non-normal, as the SD approach does not require any assumption about the nature of the distribution, and hence can be used for any type of distribution. In addition, SD rules offer superior criteria on prospects investment decisions as it incorporates information on the entire returns distribution, rather than the first two moments, as in MV and CAPM, or higher moments by the extended MV. The SD approach is widely regarded as one of the most useful tools to rank investment prospects as the ranking of assets has been shown to be equivalent to utility maximization for the preferences of risk averters and risk seekers (Tesfatsion, 1976; Stoyan, 1983; Li and Wong, 1999).

Consider an expected-utility-maximizing investor who holds a portfolio of two assets, namely oil spot and oil futures. The objective of the investor is to rank the preferences of these two assets to maximize expected utility. In this paper, we use the SD test proposed by Davidson and Duclos (2000) (hereafter DD) to examine the behavior of both risk averters and risk seekers with regard to oil futures and spot prices. We apply the DD test to investigate the characteristics of the entire distribution for oil futures and spot returns, instead of the commonly-used mean-variance criterion, which only examine their respective means and standard deviations.
This paper contributes to the energy economics and finance literature in three ways. First, the paper discusses oil prices from the investor perspective by the SD approach. Second, a more robust decision tool is used for investment decision making under uncertainty for the oil spot and futures markets. Third, more useful information and inferences regarding investor behaviour can be made using the DD statistics.

Data and Methodology

We examine the performance of Brent Crude oil spot and futures for the period January 1, 1989 to June 30, 2008. The daily closing prices for Brent Crude oil spot and futures are collected from Datastream. The daily log returns, \( R_{i,t} \), for the oil spot and futures prices are defined to be \( R_{i,t} = \ln \left( \frac{P_{i,t}}{P_{i,t-1}} \right) \), where \( P_{i,t} \) is the daily price at day \( t \) for asset \( i \), with \( i = S \) (Spot) and \( F \) (Futures), respectively. We examine the effect of the Asian Financial Crisis on oil prices by examining two sub-periods: the first sub-period is the pre-Asian Financial Crisis (pre-AFC) period and the second sub-period is the period after the Asian Financial Crisis (post-AFC), using July 1, 1997 as the cut-off point. For computing the CAPM statistics, we use the 3-month U.S. T-bill rate and the Morgan Stanley Capital International index returns (MSCI) as proxies for the risk-free rate and the global market index, respectively.

Mean-Variance criterion and CAPM statistics

For purposes of comparison, we calculate the MV and CAPM statistics. The MV model developed by Markowitz (1952) and Tobin (1958), and the CAPM statistics developed by Sharpe (1964), Treynor (1965) and Jensen (1969), are commonly used to compare investment prospects.¹

¹ We note that Bai, et al. (2009a,b) have developed a new bootstrap-corrected estimator of the optimal return for the Markowitz mean-variance optimization, whereas Leung and Wong (2008) have developed a multivariate Sharpe ratio statistic to test the hypothesis of the equality of multiple Sharpe ratios (refer to Egozcue and Wong (2010) for the theory of portfolio diversification).
For any two investment prospects with returns $Y_i$ and $Y_j$, with means $\mu_i$ and $\mu_j$ and standard deviations $\sigma_i$ and $\sigma_j$, respectively, $Y_j$ is said to dominate $Y_i$ by the MV rule if $\mu_j \geq \mu_i$ and $\sigma_j \leq \sigma_i$ significantly (Markowitz, 1952; Tobin, 1958). CAPM statistics include the beta, Sharpe ratio, Treynor’s index and Jensen (alpha) index to measure performance\(^2\).

**Stochastic Dominance Theory and Tests**

SD theory, developed by Hadar and Russell (1969), Hanoch and Levy (1969) and Rothschild and Stiglitz (1970), is one of the most useful tools in investment decision-making under uncertainty to rank investment prospects. Let $F$ and $G$ be the cumulative distribution functions (CDFs), and $f$ and $g$ be the corresponding probability density functions (PDFs), of two investments, $X$ and $Y$, respectively, with common support of $[a,b]$, where $a < b$. Define\(^3\)

$$
H^A_j = H^D_j = h, \quad H^A_j(x) = \int_a^x H^A_{j-1}(t) dt \quad \text{and} \quad H^D_j(x) = \int_x^b H^D_{j-1}(t) dt
$$

for $h = f, g$; $H = F, G$; and $j = 1, 2, 3$.

We call the integral $H^A_j$ the $j^{th}$ order ascending cumulative distribution function (ACDF), and the integral $H^D_j$ the $j^{th}$ order descending cumulative distribution function (DCDF), for $j = 1, 2$ and $3$ and for $H = F$ and $G$.

**SD for Risk Averters**

\(^2\) Refer to Sharpe (1964), Treynor (1965) and Jensen (1969) for details regarding the definitions of these indices and statistics, Leung and Wong (2008) for the test statistic of the Sharpe ratios, Morey and Morey (2000) for the test statistic of the Treynor index, and Cumby and Glen (1990) for the test statistic of the Jensen index.

\(^3\) See Wong and Li (1999), Li and Wong (1999), and Sriboonchita, et al. (2009) for further discussion.
The most commonly-used SD rules corresponding to three broadly defined utility functions are the first-, second- and third-order Ascending SD (ASD)\textsuperscript{4} for risk averters, denoted as FASD, SASD and TASD, respectively. All investors are assumed to have non-satiation (more is preferred to less) under FASD, non-satiation and risk aversion under SASD; and non-satiation, risk aversion and decreasing absolute risk aversion (DARA) under TASD. The ASD rules are defined as follows (see Quirk and Saposnik, 1962; Fishburn, 1964; Hanoch and Levy, 1969):

\[ X \text{ dominates } Y \text{ by FASD (SASD, TASD), denoted by } X \succ_1 Y \ (X \succ_2 Y, \ X \succ_3 Y) \text{ if and only if } \]
\[ F_1^A(x) \leq G_1^A(x) \ (F_2^A(x) \leq G_2^A(x), \ F_3^A(x) \leq G_3^A(x)) \text{ for all possible returns } x, \text{ and the strict inequality holds for at least one value of } x. \]

The theory of SD is important as it relates to utility maximization (see Quirk and Saposnik 1962, Hanoch and Levy 1969, Li and Wong 1999). The existence of ASD implies that risk-averse investors always obtain higher expected utilities when holding the dominant asset than when holding the dominated asset, such that the dominated asset would not be chosen. We note that hierarchical relationship exists in ASD: FASD implies SASD which, in turn, implies TASD. However, the converse is not true: the existence of SASD does not imply the existence of FASD. Likewise, a finding of the existence of TASD does not imply the existence of SASD or FASD. Thus, only the lowest dominance order of ASD is reported.

Finally, we note that, under certain regularity conditions\textsuperscript{5}, investment \( X \) stochastically dominates investment \( Y \) at first-order, if and only if there is an arbitrage opportunity between \( X \) and \( Y \), such that investors will increase their expected wealth and their expected utility if their investments are shifted from \( Y \) to \( X \) (Bawa, 1978; Jarrow, 1986; Wong et al 2008). In addition,\textsuperscript{4} We call it Ascending SD as its integrals count from the worst return ascending to the best return.\textsuperscript{5} See Jarrow (1986) for the conditions.
if no first-order SD is found between \( X \) and \( Y \), one could infer that market efficiency and market rationality could hold in the markets. Though SD results, in general, cannot be used to accept or reject market efficiency and market rationality, the SD results could be used to draw inferences about market efficiency and market rationality (see Bernard and Seyhun, 1997; Larsen and Resnick, 1999; Sriboonchita, et al., 2009). In addition, it could reveal the existence of arbitrage opportunities, and identify the preferences of risk averters and risk seekers in these markets. When such an opportunity presents itself, investors can increase their expected utility as well as expected wealth to make huge profits by setting up zero dollar portfolios to exploit this opportunity.

**SD for Risk Seekers**

The SD theory for risk seekers is also well established in the literature. Whereas SD for risk averters works with the ACDF, which order the worst to the best returns, SD for risk seekers works with the DCDF, which orders from the best to the worst returns (Stoyan, 1983; Wong and Li, 1999; Levy and Levy 2004; Post and Levy, 2005). Hence, SD for risk seekers is called Descending SD (DSD). DSD is defined as follows (see Hammond, 1974; Meyer, 1977; Wong and Li, 1999; Anderson, 2004):

\[
X \text{ dominates } Y \text{ by FDSD (SDSD, TDSD)) denoted by } X \succ X Y \ (X \succ^2 Y, X \succ^3 Y) \text{ if and only if } F^D_i(x) \geq G^D_i(x) \ (F^D_2(x) \geq G^D_2(x), F^D_3(x) \geq G^D_3(x)) \text{ for all possible returns } x, \text{ the strict inequality holds for at least one value of } x; \text{ where FDSD (SDSD, TDSD) stands for first-order (second-order, third-order) Descending SD.}
\]

All investors are assumed to have non-satiation under FDSD; non-satiation and risk seeking under SDSD; and non-satiation, risk seeking and increasing absolute risk seeking under TDSD. Similarly, the theory of DSD is related to utility maximization for risk seekers (see Stoyan 1983, Li and Wong 1999, Anderson 2004), and a hierarchical relationship also exists for DSD, so that only
the lowest dominance order of DSD is reported.

Typically, risk averters prefer assets that have a smaller probability of loss, especially in downside risk; while risk seekers prefer assets that have a higher probability of gaining, especially in upside profit. In order to make a choice between two assets, $X$ and $Y$, risk averters will compare their corresponding $j^{th}$ order ASD integrals and choose $X$ if $F^A_j$ is smaller. On the other hand, risk seekers will compare their corresponding $j^{th}$ order DSD integrals and choose $X$ if $F^D_j$ is larger (Wong and Chan, 2008).

In the finance literature, when two prospects have been compared, the SD approach examines their distributions of returns directly. If the perceived distribution of return on prospect $X$ stochastically dominates that of prospect $Y$ in a particular manner then we can conclude that the agent has a preference for prospect $X$.

The advantages presented by SD have motivated prior studies which use SD techniques to analyze many financial puzzles. There are two broad classes of SD tests. One is the minimum/maximum statistic, while the other is based on distribution values computed on a set of grid points. McFadden (1989) was the first to develop a SD test using the minimum/maximum statistic, followed by Klecan et al. (1991) and Kaur et al. (1994). Barrett and Donald (2003) develop a Kolmogorov-Smirnov-type test, and Linton et al. (2005) extended their work to relax the iid assumption. On the other hand, the SD tests developed by Anderson (1996, 2004) and Davidson and Duclos (2000) compare the underlying distributions at a finite number of grid points. The SD test developed by DD has been examined to be one of the most powerful approaches, and yet less conservative in size (see Tse and Zhang, 2004; Lean et al., 2008).
**Davidson and Duclos (DD) Test**

Let \{ (f_i, s_i) \} (i = 1,...,n)\(^6\) be pairs of observations drawn from the random variables \( X \) and \( Y \), with distribution functions \( F \) and \( G \), respectively and with their integrals \( F_j^A(x) \) and \( G_j^A(x) \) defined in (1) for \( j=1,2,3 \). For a grid of pre-selected points \( x_1, x_2, ..., x_k \), the \( j^{th} \) order Ascending DD test statistic for risk aveters, \( T_j^A \) is:

\[
T_j^A(x) = \frac{\hat{F}_j^A(x) - \hat{G}_j^A(x)}{\sqrt{\hat{V}_j^A(x)}}
\]

(2)

where \( \hat{V}_j^A(x) = \hat{V}_{Hj}^A(x) + \hat{V}_{Gj}^A(x) - 2\hat{V}_{FGj}^A(x) \);

\[
\hat{H}_j^A(x) = \frac{1}{N(j-1)!} \sum_{i=1}^{N} (x-z_i)_+^{j-1},
\]

\[
\hat{V}_{Hj}^A(x) = \frac{1}{N} \left[ \frac{1}{N((j-1)!)^2} \sum_{i=1}^{N} (x-z_i)_+^{2(j-1)} - \hat{H}_j^A(x)^2 \right], H = F, G; z = f, s;
\]

\[
\hat{V}_{FGj}^A(x) = \frac{1}{N} \left[ \frac{1}{N((j-1)!)^2} \sum_{i=1}^{N} (x-f_i)_+^{j-1} (x-s_i)_+^{j-1} - \hat{F}_j^A(x)\hat{G}_j^A(x) \right].
\]

It is empirically impossible to test the null hypothesis for the full support of the distributions. Thus, Bishop et al (1992) propose to test the null hypothesis for a pre-designed finite numbers of values \( x \). Specifically, for all \( i = 1,2,...,k \); the following hypotheses are tested:

\[
H_0 : F_j^A(x_i) = G_j^A(x_i) \text{ , for all } x_i;
\]

\[
H_A : F_j^A(x_i) \neq G_j^A(x_i) \text{ for some } x_i;
\]

\[
H_{d1} : F_j^A(x_i) \leq G_j^A(x_i) \text{ for all } x_i, F_j^A(x_i) < G_j^A(x_i) \text{ for some } x_i;
\]

\[
H_{d2} : F_j^A(x_i) \geq G_j^A(x_i) \text{ for all } x_i, F_j^A(x_i) > G_j^A(x_i) \text{ for some } x_i.
\]

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\(^6\) In the context of this paper, \( f \) denotes the returns of futures prices, while \( s \) denotes the returns of spot prices.
We note that, in the above hypotheses, $H_A$ is set to be exclusive of both $H_{A1}$ and $H_{A2}$; this means that, if the test does not reject $H_{A1}$ or $H_{A2}$, it will not be classified as $H_A$. Under the null hypothesis, DD show that $T_j^A$ is asymptotically distributed as the Studentized Maximum Modulus (SMM) distribution (Richmond, 1982) to account for joint test size. In order to implement the DD test, the test statistic, $T_j^A(x)$, at each grid point, $x$, is computed and the null hypothesis, $H_0$, is rejected if there is a grid point $x$ such that the test statistic, $T_j^A(x)$, is significant. The SMM distribution with $k$ and infinite degrees of freedom, denoted by $M^k_{\infty,\alpha}$, is used to control the probability of Type I error, for $j = 1, 2, 3$. The following decision rules are adopted based on the $1-\alpha$ percentile of $M^k_{\infty,\alpha}$ tabulated by Stoline and Ury (1979):

\[ T_j^A(x_i) < M^k_{\infty,\alpha} \quad \text{for} \quad i = 1, \ldots, k, \quad \text{accept} \quad H_0; \]
\[ T_j^A(x_i) < M^k_{\infty,\alpha} \quad \text{for all} \quad i \quad \text{and} \quad -T_j^A(x_i) > M^k_{\infty,\alpha} \quad \text{for some} \quad i, \quad \text{accept} \quad H_{A1}; \]
\[ -T_j^A(x_i) < M^k_{\infty,\alpha} \quad \text{for all} \quad i \quad \text{and} \quad T_j^A(x_i) > M^k_{\infty,\alpha} \quad \text{for some} \quad i, \quad \text{accept} \quad H_{A2}; \]
\[ T_j^A(x_i) > M^k_{\infty,\alpha} \quad \text{for some} \quad i \quad \text{and} \quad -T_j^A(x_i) > M^k_{\infty,\alpha} \quad \text{for some} \quad i, \quad \text{accept} \quad H_A. \]

Accepting (specifically, not rejecting) either $H_0$ or $H_A$ implies non-existence of any SD relationship between $X$ and $Y$, non-existence of any arbitrage opportunity between these two markets, and neither of these markets is preferred to the other. If $H_{A1}$ ($H_{A2}$) of order one is accepted, $X$ ($Y$) stochastically dominates $Y$ ($X$) at first-order. In this situation, and under certain regularity conditions\(^7\), an arbitrage opportunity exists, and any non-satiated investors will be better off if they switch from the dominated asset to the dominant one. On the other hand, if $H_{A1}$ ($H_{A2}$) is accepted for order two (three), a particular market stochastically dominates the other at second-

\(^7\) Refer to Jarrow (1986) for the conditions.
(third-) order. In this situation, an arbitrage opportunity does not exist, and switching from one asset to another will only increase the risk averters’ expected utility, but not their expected wealth (Jarrow, 1986; Falk and Levy, 1989; Wong et al. 2008). These results could be used to infer that market efficiency and market rationality could still hold in these markets (Bernard and Seyhun, 1997; Larsen and Resnick, 1999; Sriboonchita et al., 2009).

The DD test compares distributions at a finite number of grid points. Various studies have examined the choice of grid points. For example, Tse and Zhang (2004) show that an appropriate choice of \( k \), for reasonably large samples, ranges from 6 to 15. Too few grids will miss information of the distributions between any two consecutive grids (Barrett and Donald, 2003), and too many grids will violate the independence assumption required by the SMM distribution (Richmond, 1982). In order to make the comparisons comprehensive without violating the independence assumption, we follow Fong et al. (2005, 2008), Gasbarro et al. (2007) and Lean et al. (2007) to make 10 major partitions, with 10 minor partitions within any two consecutive major partitions in each comparison, and base statistical inference on the SMM distribution for \( k=10 \) and infinite degrees of freedom\(^8\). This allows the consistency of both the magnitude and sign of the DD statistics between any two consecutive major partitions to be examined.

In order to test SD for risk seekers, the DD statistic for risk averters is modified to be the Descending DD test statistic, \( T_j^D \), such that:

\[
T_j^D(x) = \frac{\hat{F}_j^D(x) - \hat{G}_j^D(x)}{\sqrt{\hat{V}_j^D(x)}},
\]

where

\[
\hat{V}_j^D(x) = \hat{V}_{F_j}^D(x) + \hat{V}_{G_j}^D(x) - 2\hat{V}_{FG_j}^D(x);
\]

\[
\hat{V}_{FG_j}^D(x) = \frac{1}{N(j-1)!} \sum_{i=1}^{N} (z_i - x)_+^{j-1},
\]

\(^8\) Refer to Lean et al. (2008) for further explanation.
where the integrals $F_j^D(x)$ and $G_j^D(x)$ are defined in (1) for $j = 1, 2, 3$. For $i = 1, 2, ..., k$, the following hypotheses are tested for risk seekers:

- $H_0: F_j^D(x_i) = G_j^D(x_i)$, for all $x_i$;
- $H_D: F_j^D(x_i) \neq G_j^D(x_i)$ for some $x_i$;
- $H_{D1}: F_j^D(x_i) \geq G_j^D(x_i)$ for all $x_i$, $F_j^D(x_i) > G_j^D(x_i)$ for some $x_i$;
- $H_{D2}: F_j^D(x_i) \leq G_j^D(x_i)$ for all $x_i$, $F_j^D(x_i) < G_j^D(x_i)$ for some $x_i$.

and the following decision rules are adopted for risk seekers:

- If $|T_j^D(x_i)| > M_{x,a}^k$ for $i = 1, ..., k$, accept $H_0$;
- if $-T_j^D(x_i) < M_{x,a}^k$ for all $i$ and $T_j^D(x_i) > M_{x,a}^k$ for some $i$, accept $H_{D1}$;
- if $T_j^D(x_i) < M_{x,a}^k$ for all $i$ and $-T_j^D(x_i) > M_{x,a}^k$ for some $i$, accept $H_{D2}$; and
- if $T_j^D(x_i) > M_{x,a}^k$ for some $i$ and $-T_j^D(x_i) > M_{x,a}^k$ for some $i$, accept $H_D$.

As in the case for risk averters, accepting either $H_0$ or $H_D$ implies non-existence of any SD relationship between $X$ and $Y$, non-existence of any arbitrage opportunity between these two markets, and neither of the assets is preferred to the other. If $H_{D1}$ ($H_{D2}$) of order one is accepted, asset $X$ ($Y$) stochastically dominates $Y$ ($X$) at first-order. In this situation, an arbitrage opportunity exists and the non-satiated investors will be better off if they switch their investments from the dominated asset to the dominant one. On the other hand, if $H_{D1}$ or $H_{D2}$ is accepted at order two (three), a particular asset stochastically dominates the other at second- (third-) order. In this
situation, an arbitrage opportunity does not exist, and switching from one asset to another will only increase the risk seekers’ expected utility, but not expected wealth.

**Empirical Results and Discussion**

[Table 1 here]

Table 1 provides the descriptive statistics for the daily returns of oil spot prices and oil futures prices for the entire sample period. The means of their daily returns are about 0.04%, significant at 10% for oil spot but not significant for oil futures. From the unreported paired t-test, the mean return of oil spot is insignificantly higher than that of futures whereas, as expected, its standard deviation is not significantly smaller than that of futures. As the means and standard deviations are not significantly different for the two returns, the MV criterion is unable to indicate any preference between these two assets.

For the CAPM measures, the beta (absolute value) of oil spot return is smaller than that of futures; both being negative and less than one. Both returns have similar Sharpe ratios, Treynor and Jensen indices, with no significant difference between the returns for each statistic. Thus, the information drawn from the CAPM statistics cannot lead to any preference between the spot and futures prices. In addition, the highly significant Kolmogorov-Smirnov (K-S) and Jarque-Bera (J-B) statistics shown in Table 1 indicate that both returns are non-normal. The results of other normality tests, such as Shapiro-Wilk, lead to the same conclusion. The results are available on request.

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9 The results of other normality tests, such as Shapiro-Wilk, lead to the same conclusion. The results are available on request.
SD Analysis for Risk Averters

We first depict the CDFs of the returns for both oil spot and futures prices and their corresponding first three orders of the Ascending DD statistics, $T_j^A$, for risk averters in Figure 1. If oil futures dominate spot in the sense of FASD, then the CDF of futures returns should lie significantly below that of spot for the entire range. However, Figure 1 shows that the CDF of spot lies below that of futures in the downside risk, while the CDF of futures lies below that of spot on the upside profit. This indicates that there could be no FASD between the two returns and that spot could dominate futures on the downside risk, while futures could dominate spot on the upside profit range. In order to verify this finding formally, we employ the first three orders of the Ascending DD statistics, $T_j^A$ ($j = 1, 2, 3$), for the two series, with the results reported in Table 2. DD states that the null hypothesis can be rejected if any of the test statistics $T_j^A$ is significant with the wrong sign.

The values of $T_1^A$ depicted in Figure 1 move from positive to negative along the distribution of returns, together with the percentage of significant values reported in Table 2, show that 5% of $T_1^A$ is significantly positive, whereas 6% of it is significantly negative. Thus, the hypotheses that futures stochastically dominate spot, or vice-versa, at first-order are rejected, implying that no arbitrage opportunity exists between these two series. We can, however, state that oil spot dominates futures marginally in the downside returns, while oil futures dominate spot marginally in the upside profit.
The SD criterion enables us to compare utility interpretations in terms of investors’ risk aversion and decreasing absolute risk aversion, respectively, by examining the higher order SD relationships. The Ascending DD statistics, $T_2^A$ and $T_3^A$, depicted in Figure 1 are positive in the entire range of the returns distribution, with 7% of $T_2^A$ (5% of $T_3^A$) being significantly positive and no $T_2^A$ ($T_3^A$) being significantly negative. This implies that oil spot marginally SASD (TASD) dominates futures, and hence risk-averse investors would prefer investing in oil spot than futures to maximize their expected utility.

**Will Risk Seekers Have Different Preferences?**

So far, if we apply the existing ASD tests, we could only draw conclusions regarding the preference of risk-averse investors, but not of risk seekers. Nonetheless, the result also shows that futures dominate spot for the upside profit. However, applying the ASD test alone could not yield any inference based on this information. Thus, an extension of the SD test for risk seekers is necessary, as discussed in previous sections. Subsequent discussions illustrate the applicability of the DSD test for risk seekers in this section.

It is well known that investors could be risk-seeking (see, for example, Markowitz, 1952; Kahneman and Tversky, 1979; Tversky and Kahneman, 1992; Levy and Levy, 2004; Post and Levy, 2005). In order to examine the risk-seeking behavior, DSD theory for risk-seeking has been developed. In this paper, we put the theory into practice by extending the DD test for risk seekers, namely Descending DD statistics, $T_j^D$ ($j = 1, 2$ and $3$), of the first three orders for risk seekers, with the correspondence statistics as discussed in the previous section.
Figure 2 shows the descending cumulative density functions (DCDFs) for the daily returns of both oil spot and futures prices over the entire distribution range for the whole sample period. The cross of the two DCDFs suggests that there is no FDSD between futures and spot returns. The DCDF of the futures lies above that of spot for the upside profit, while the DCDF of the spot lies above that of futures for the downside risk. This indicates that futures could be preferred to spot for upside profit, while spot could be preferred to futures for downside risk.

[Table 3 here]

In order to test this phenomenon formally, we plot the Descending DD statistics, $T_j^D$, of the first three orders in Figure 2, and report the percentages of their significant positive and negative portions in Table 3. Figure 2 shows that $T_1^D$ is positive in the upside profit range and negative in the downside risk range, whereas Table 3 shows that 6% (5%) of the positive (negative) values of $T_1^D$ is significant. This indicates that there is no FDSD relationship between the two series for the entire period.

As there is no FDSD, we examine the $T_j^D$ for the second and third orders. Both $T_2^D$ and $T_3^D$ depicted in Figure 2 are positive for the entire range, implying that risk-seeking investors could prefer futures to spot. In order to verify this statement statistically, we use the results in Table 3 that 7% (9%) of $T_2^D$ ($T_3^D$) are significantly positive, while no $T_2^D$ ($T_3^D$) is significantly negative. This leads us to conclude statistically that the oil futures SDSD and TDSD the oil spot and consequently, risk-seeking investors, prefer oil futures to spot to maximize their utility.

In addition, neither FASD nor FDSD leads us to conclude that market efficiency or market rationality could hold in the oil spot and futures markets. The preferences of risk-averse and risk-seeking investors towards spot and futures do not violate market inefficiency, unless the oil
market has only one type of investors. Our results are consistent with existing results in the literature, for example, Fong et al. (2005), who examine momentum profits in stocks markets.

**The Impact of Oil Crises**

The oil market is very sensitive, not only to news, but also to the expectation of news (Maslyuk and Smyth, 2008). For example, when the OPEC countries agreed to reduce the combined production of crude oil in 1999, oil prices increased further. Similarly, the Iraq War (that is, the Second Gulf War) occurred in March 2003. This caused oil futures prices to increase further due to the fear that Iraq’s oil fields and pipelines might be destroyed during the war. We employ regression analysis, with the cut-off points of the crises being stated in the previous section, as dummies and find that the dummies affect both spot and futures in the Iraq war crisis, but not in the OPEC crisis. This indicates that the war’s impact is greater for both spot and futures markets. On the other hand, it is of interest to examine the effects of these oil crises while comparing the performances of oil spot and futures markets and the investors’ preferences in these markets. To this end, we employ the SD tests to analyse the return series for the pre- and OPEC, and pre- and Iraq-War, sub-periods.

Tables 4A and 4B provide descriptive statistics of the daily returns of oil spot and futures prices for the OPEC and Iraq War sub-periods. As most of the results of the MV criterion and CAPM statistics for the sub-periods are similar to those for the entire full sample period, we will only discuss those results that are different from the full sample period. However, compared with the pre-OPEC sub-period, the means for both spot and futures returns in the OPEC sub-period dramatically increased five-fold. On the other hand, compared with the pre-Iraq-War sub-period, both spot and futures returns in the Iraq-War sub-period were reduced by 90%. Nonetheless, the

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10 We do not report these results, which are available on request.
difference between the means of spot and futures in each sub-period is still not significant. In addition, the standard deviations for the returns of spot and futures are also not significantly different in each of the sub-periods. Thus, similar to the inference for the entire sample, both the MV criterion and the CAPM statistics are unable to indicate any dominance between the spot and futures markets.

We turn to the SD tests to conduct the analysis. From the DD test, we find that all values of $T_{ij}^A$ and $T_{ij}^D$ ($j = 1, 2$ and $3$) for both risk averters and risk seekers are not significant at the 5% level for the first three orders in the pre-OPEC sub-period. Therefore, there is no arbitrage opportunity in these markets, and both risk averters and risk seekers are indifferent between these two indices in the pre-OPEC sub-period. However, in the OPEC sub-period, Table 2 shows that 17% (16%) of $T_{2}^A$ ($T_{2}^D$) are significantly positive, and none of the $T_{2}^A$ ($T_{2}^D$) is significantly negative, while Table 3 reveals that 22% (30%) of $T_{2}^D$ ($T_{3}^D$) are significantly positive and none of the $T_{2}^D$ ($T_{3}^D$) is significantly negative at the 5% level. Similar inferences can be drawn for the Iraq War sub-period. Hence, we conclude that, compared with the full sample period, the risk-averse investors prefer the spot index more, and risk seekers are attracted to the futures index more to maximize their expected utility, but not their expected wealth, in both the OPEC and Iraq War sub-periods.

Conclusions

This paper offered a robust decision tool for investment decisions with uncertainty to the oil markets. The SD tests enabled us to reveal the existence of arbitrage opportunities, identify the preferences for both risk averters and risk seekers over different investment prospects, and enable us to make inference on market rationality and market efficiency. We developed the SD tests of DD for risk seekers, and applied the DD tests to examine the behavior of both risk averters and risk
seekers with regard to oil spot and futures markets, and compared the performance between these two markets.

Our results showed conclusively that oil spot dominates oil futures on the downside risk, whereas the futures dominate spot on the upside profit range. We concluded that there is neither arbitrage opportunity nor preference being prevalent between these two indices for both risk-averse and risk-seeking investors in the pre-AFC sub-period. However, risk-averse investors prefer the oil spot, while risk seekers are attracted to the oil future in order to maximize their expected utility in the post-AFC sub-period.

We note that some authors have proposed to use higher order (higher than three) SD in empirical applications. For example, Vinod (2004) recommends employing 4th order SD to make the choice among investment prospects, with an illustration of 1281 mutual funds. We also note that the most commonly-used orders in SD for empirical analyses, regardless of whether they are simple or complicated, are the first three, and one could easily extend the theory developed in this paper to any order.

It should be noted that many studies have claimed that if the normality assumption fails, the results drawn using the MV criterion and CAPM statistics can be misleading. We point out that, unlike the SD approach that is consistent with utility maximisation, the dominance findings using the MV and CAPM measures may only be consistent with utility maximization, if the asset returns are not normally distributed, under very specific conditions. For example, Meyer (1977), Wong (2006, 2007) and Wong and Ma (2008) show that, if the returns of two assets follow the same location-scale family, then an MV domination could infer preferences by risk averters on the dominant fund to the dominated one.

Finally, if all of the regularity conditions are satisfied (for example, assets follow the normality assumption), the MV and CAPM measures be consistent if asset returns possess the second order SD preference characteristic. However, even if all of the regularity conditions are satisfied, the MV and CAPM measures cannot identify the situations in which one fund dominates
another at first or third order SD. Thus, the SD approach allows more accurate and useful assessments for financial assets, regardless of whether those returns are normally or non-normally distributed.
Table 1: Descriptive Statistics of Oil Spot and Futures Returns for 1989 – 2008

<table>
<thead>
<tr>
<th>Variable</th>
<th>Oil Spot Returns</th>
<th>Oil Futures Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>0.04354*</td>
<td>0.04323</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.01864</td>
<td>0.02193</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.9201***</td>
<td>-1.6782***</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>12.9542***</td>
<td>32.0111***</td>
</tr>
<tr>
<td>Jarque-Bera (J-B)</td>
<td>21711.86***</td>
<td>180710.47***</td>
</tr>
<tr>
<td>Kolmogorov-Smirnov (K-S)</td>
<td>0.06536***</td>
<td>0.07046***</td>
</tr>
<tr>
<td>Beta</td>
<td>-0.0153</td>
<td>-0.1617</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>3.68</td>
<td>3.04</td>
</tr>
<tr>
<td>Treynor Index</td>
<td>-0.96252</td>
<td>-0.08788</td>
</tr>
<tr>
<td>Jensen Index</td>
<td>0.014768</td>
<td>0.014404</td>
</tr>
<tr>
<td>F Statistics</td>
<td>0.7221</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>5085</td>
<td>5085</td>
</tr>
</tbody>
</table>

Note: *** significant at 1%, ** significant at 5%, * significant at 10%. F Statistic is for testing the equality of variances. Refer to footnote 4 for the formula of Sharpe Ratio, Treynor Index, and Jensen Index, and more information about these statistics. The values of Sharpe Ratio, Treynor Index and Jensen Index are annualized.
Table 2: Results of DD Test for Risk Averters

<table>
<thead>
<tr>
<th>Sample</th>
<th>FASD</th>
<th>SASD</th>
<th>TASD</th>
<th>Whole Period</th>
<th>Pre-OPEC</th>
<th>OPEC</th>
<th>Pre-Iraq</th>
<th>Iraq War</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% $T_A^1 &gt; 0$</td>
<td>% $T_A^1 &lt; 0$</td>
<td>% $T_A^2 &gt; 0$</td>
<td>% $T_A^2 &lt; 0$</td>
<td>% $T_A^3 &gt; 0$</td>
<td>% $T_A^3 &lt; 0$</td>
<td>% $T_A^3 &gt; 0$</td>
<td>% $T_A^3 &lt; 0$</td>
</tr>
<tr>
<td>Whole Period</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pre-OPEC</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>OPEC</td>
<td>14</td>
<td>14</td>
<td>17</td>
<td>0</td>
<td>16</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pre-Iraq</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Iraq War</td>
<td>3</td>
<td>16</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: DD test statistics are computed over a grid of 100 daily oil spot and futures returns. This table reports the percentage of DD statistics, which are significantly negative or positive at the 5% level, based on the asymptotic critical value of 3.254 of the studentized maximum modulus (SMM) distribution. Refer to equation in (2) for the definition of $T_A^j$ for $j = 1, 2$ and $3$ where $F_A^j$ and $G_A^j$ represent the $j^{th}$ ACDFs for the returns of futures and spot, respectively.

Table 3: Results of DD Test for Risk Seekers

<table>
<thead>
<tr>
<th>Sample</th>
<th>FDSD</th>
<th>SDSD</th>
<th>TDSD</th>
<th>Whole Period</th>
<th>Pre-OPEC</th>
<th>OPEC</th>
<th>Pre-Iraq</th>
<th>Iraq War</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% $T_D^1 &gt; 0$</td>
<td>% $T_D^1 &lt; 0$</td>
<td>% $T_D^2 &gt; 0$</td>
<td>% $T_D^2 &lt; 0$</td>
<td>% $T_D^3 &gt; 0$</td>
<td>% $T_D^3 &lt; 0$</td>
<td>% $T_D^3 &gt; 0$</td>
<td>% $T_D^3 &lt; 0$</td>
</tr>
<tr>
<td>Whole Period</td>
<td>6</td>
<td>5</td>
<td>7</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pre-OPEC</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>OPEC</td>
<td>14</td>
<td>14</td>
<td>22</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pre-Iraq</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Iraq War</td>
<td>16</td>
<td>3</td>
<td>21</td>
<td>0</td>
<td>26</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: DD test statistics are computed over a grid of 100 daily oil spot and futures returns. This table reports the percentage of DD statistics, which are significantly negative or positive at the 5% level, based on the asymptotic critical value of 3.254 of the studentized maximum modulus (SMM) distribution. Refer to equation in (3) for the definition of $T_D^j$ for $j = 1, 2$ and $3$ where $F_D^j$ and $G_D^j$ represent the $j^{th}$ DCDFs for the returns of futures and spot, respectively.
Table 4A: Descriptive Statistics of Oil Spot Prices and Oil Futures Prices for Sub-Periods

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pre-OPEC</th>
<th>OPEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>0.01287 0.01185</td>
<td>0.08185** 0.08242*</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.01969 0.02240</td>
<td>0.01723 0.02134</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.08807*** -2.6108***</td>
<td>-0.5726 -0.3245</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>17.4315*** 51.5032***</td>
<td>2.5760 2.1657</td>
</tr>
<tr>
<td>J-B</td>
<td>25063.69* 280027.11***</td>
<td>140.526 105.264</td>
</tr>
<tr>
<td>K-S</td>
<td>0.08918* 0.1069***</td>
<td>0.05249*** 0.03683***</td>
</tr>
<tr>
<td>Beta</td>
<td>0.01124 -0.3738</td>
<td>-0.03372 -0.00047</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>-0.8875 -1.0375</td>
<td>10.35 8.45</td>
</tr>
<tr>
<td>Treynor Index</td>
<td>-0.33592 0.01326</td>
<td>-1.1232 -81.9728</td>
</tr>
<tr>
<td>Jensen Index</td>
<td>-4108 -0.00203</td>
<td>0.037648 0.038168</td>
</tr>
<tr>
<td>F Statistics</td>
<td>0.7726 0.6523</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>2824 2824</td>
<td>2261 2261</td>
</tr>
</tbody>
</table>

Note: *** significant at 1%, ** significant at 5%, * significant at 10%. F Statistic is for testing the equality of variances. Refer to footnote 4 for the formula of Sharpe Ratio, Treynor Index, and Jensen Index, and more information about these statistics. The values of Sharpe Ratio, Treynor Index and Jensen Index are annualized.

Table 4B: Descriptive Statistics of Oil Spot Prices and Oil Futures Prices for Sub-Periods

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pre-Iraq War</th>
<th>Iraq War</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>0.01566 0.01339</td>
<td>0.001184*** 0.001200**</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.01956 0.02284</td>
<td>0.01586 0.01932</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.01998*** -2.03499***</td>
<td>-0.2882*** 0.02237</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>14.2659*** 37.07080***</td>
<td>1.4916*** 0.7288***</td>
</tr>
<tr>
<td>J-B</td>
<td>20252.50*** 181905.92***</td>
<td>149.716*** 296.282***</td>
</tr>
<tr>
<td>K-S</td>
<td>0.07501*** 0.09179***</td>
<td>0.04717*** 0.02983***</td>
</tr>
<tr>
<td>Beta</td>
<td>0.02377 0.1861</td>
<td>-0.1645 -0.07809</td>
</tr>
<tr>
<td>Sharpe Ratio (annualize)</td>
<td>0.3443    -0.65</td>
<td>17.24 14.35</td>
</tr>
<tr>
<td>Treynor Index</td>
<td>0.001172 0.0003267</td>
<td>-0.006794 -0.01452</td>
</tr>
<tr>
<td>Jensen Index</td>
<td>-2.66<em>10^-3 -7.09</em>10^-3</td>
<td>0.001155 0.001152</td>
</tr>
<tr>
<td>F Statistics</td>
<td>0.7338 0.67425</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>3708 3708</td>
<td>1378 1378</td>
</tr>
</tbody>
</table>

Note: *** significant at 1%, ** significant at 5%, * significant at 10%. F Statistic is for testing the equality of variances. Refer to footnote 4 for the formula of Sharpe Ratio, Treynor Index, and Jensen Index, and more information about these statistics.
Figure 1: Distribution of Returns and DD Statistics for Risk Averters - Whole Period

Notes: ASD1 (ASD2, ASD3) refers to the first (second, third)-order ascending DD statistics, $T_j^A$, for $j = 1, 2$ and 3.

Readers may refer to equation (2) for the definition of $T_j^A$. The right-hand side Y-axis is used for the ascending CDF of the spot and futures returns whereas the left-hand side Y-axis is used for $T_j^A$ for $j = 1, 2$ and 3.
Figure 2: Descending Distribution of Returns and DD Statistics for Risk Seekers - Whole Period

Notes: Refer to the right hand side Y-axis for the descending CDF of the spot and futures returns. DSD1 refers to the first-order descending DD statistics; DSD2 refers to the second-order descending DD statistics; and DSD3 refers to the third-order descending DD statistics. DSD1 (DSD2, DSD3) refers to the first (second, third)-order descending DD statistics, \( T_j^D \), for \( j = 1, 2 \) and 3. Readers may refer to equation (3) for the definition of \( T_j^D \). The right-hand side Y-axis is used for the descending CDF of the spot and futures returns whereas the left-hand side Y-axis is used for \( T_j^D \) for \( j = 1, 2 \) and 3.
References


Quan, J., 1992. Two step testing procedure for price discovery role of futures prices. Journal of Futures Markets 12, 139-149.


