AN ANALYSIS OF OCCUPATIONAL PENSION PROVISION
FROM EVALUATION TO REDESIGN

Around the turn of the 21st century, the “perfect storm” implied by low interest rates, poor stock market returns and an ageing society led collective pension plans into under-funded situations and caused considerable concerns over their financial sustainability. This thesis analyzes the prevailing collective pension plans in the Netherlands and makes suggestions on improving the occupational pension provision in changing demographic, financial and regulatory environments. Chapter 2 probes the pension investment performance at the overall plan level. We find that pension plans on average do not outperform their pre-selected benchmarks, reflecting that trustees fail to select superior asset management strategies. Comparatively, however, large plans outperform their smaller peers. Chapter 3 investigates the strategic asset allocation of pension plans under market consistent valuation. We find that a slight change in the model specification of asset return dynamics can have a significant impact on the optimal mix. In chapter 4, we propose a new generational plan design and find that it provides higher value and welfare to participants when compared with the current collective plan design. This is due to the fact that it allows for risk sharing via time diversification of long-term investments and prevents a-priori value transfers. To allow for intergenerational risk sharing, in Chapter 5 we introduce further design improvements by having generations trade contingent claims among them. Our estimates show that the guarantees are affordable and the surplus call option has substantial value. The option prices also give an indication of value transfers in traditional collective pension designs.
An Analysis of Occupational Pension Provision: From Evaluation to Redesign

Xiaohong Huang
An Analysis of Occupational Pension Provision:
From Evaluation to Redesign

Een analyse van bedrijfstakpensioenfondsen:
van evaluatie tot herontwikkeling

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Voorwoord (Preface)

Before I joined this PhD project, I have never thought of engaging myself in the pension research, as it is often regarded as a domain for the elderly people. But now I feel very fortunate to have chosen this area, because it is a great pleasure of being involved in tackling one of the most concerned issues in a modern welfare state.

This thesis would not be complete without my sincere gratitude for those who have guided and helped me to the current stage.

I thank Kees Koedijk and Ronald Mahieu for enrolling me in this fascinating pension project. I am mostly indebted to my daily supervisor Ronald, who is ALWAYS there for my questions either in research or life. From him I have learned a lot from reading tables to thinking critically, from explaining results to teaching classes. I just regret that after the PhD study I do not have the privilege any more to receive his instruction by percept and example, but I am looking forward to continuing our academic cooperation in future.

I thank Marno Verbeek who has taken over the role of my promotor near the end of my study. Though we worked together for a limited period of time, I greatly benefit from his rigorous scholarship. I would also like to thank the other members of my doctoral committee. Discussions with them have undoubtedly improved my thesis and brought many new research ideas. Thank Casper de Vries for his fresh perspective as an economist and his statistical savvy. Thank Willem Verschoor for his constructive advice on writing the summary chapter, and for challenging my views on research results. Thank Eduard Ponds for his very valuable expert opinions and critical comments on the pension design. I am looking forward to continuing our discussions in future research.

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Last but not least, I thank my parent and my sister for their unceasing and unconditional love. I am blessed to have grown up in this happy family. Bangjun, my workaholic-like husband, thank you for all the inspiring, comforting, irritating, and exciting discussions. I am glad to find you as my lifetime companion. Tiantian, my eternal schatje, you make my life complete and I dedicate this book to you!

Xiaohong Huang
Rotterdam, The Netherlands
April, 2010
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Chapter 1

Introduction

1.1 Basic concepts

A pension is an arrangement to provide people with an income when they retire. Pension or retirement income is provided by two sources. According to a taxonomy of the Organization for Economic Co-operation and Development (OECD) shown in Figure 1.1, they are public pension and private pension plans. The public plans refer to social security schemes where the government administers the payment of pension benefits. Such a plan is often financed by tax revenues and often referred to as "pay-as-you-go" (PAYG) system. All the other types of pension provision are classified as private plans, which typically consist of occupational and personal plans.

![Pension plan classification diagram](image)

Figure 1.1: Pension plan classification

Occupational pension arrangements are linked to employment. The employers or groups of
employers set up the plan and the plan sponsor makes contributions. Personal plans are not linked to an employment relationship, and individuals make payments and can independently purchase a pension product from the market.

Defined Benefit (DB) and Defined Contribution (DC) are two general designs of a private pension plan. They refer to the accumulation and distribution mode of a pension plan. A pension plan accumulates assets from (1) the contributions by working participants and plan sponsors and (2) investment returns, and distributes benefits to retired participants. In a DB plan, contributions are variable and benefits are based on a formula linked to salary and years of employment. The investment risk therefore is born by the plan sponsors and working participants. In a DC plan, contributions are fixed and benefits vary with the investment performance of the plan. In that case the participants assume all the investment risk.

1.2 Pension provision around the world

The public pension system is financed by tax revenues from working people to pay pensions to retirees. But the demographic change of an increasing number of retirees relative to working people put this system into funding troubles. The resulting deficiency in the public pension system is compensated by the private pension system. The last decade has seen a rapid expansion in the private pension system. The OECD private pension assets have increased from 64.8% of Gross Domestic Products (GDP) in 2001 to 111% of GDP in 2007 according to OECD Private Pensions Outlook 2008.

After years of development the source of pension provision still varies considerably across countries. Figure 1.2 displays the size of private pension assets in the economy and the role of the public pension system in providing retirement income in OECD countries in 2007. The x-axis shows the size of private pension assets as a percentage of GDP. The vertical dashed line shows the average assets-to-GDP ratio. The gross replacement rate is the ratio between gross pension benefits and gross pre-retirement earnings. The y-axis shows this rate provided by the public pension system. The horizon dashed line shows the average replacement rate from public pensions. In the bottom right quadrant, Switzerland, Netherlands and Iceland show the highest assets-to-GDP ratio and the lowest replacement rate from the public system. In these countries, the private system plays an important role in the economy and also provides the majority of the retirement income. In the bottom left quadrant including Mexico, Slovak Republic, and Poland, their public pension systems are found to provide little retirement income, and their private systems have little assets, reflecting a shortage of pension provision. In the top
The adequacy of pension income can be measured by the gross replacement rate. Figure 1.3 shows that Greece, Iceland and Netherlands provide generous retirement income, while the UK, Japan and Ireland offer relatively less generous pensions compared with their pre-retirement earnings.

Private pension plans in almost all cases are funded plans and financed by contributions and investment returns. The performance of private plans is largely determined by the investment returns of their portfolios. Because the private pension arrangement varies considerably across countries in terms of (1) occupational or personal, (2) voluntary or mandatory, (3) DB or DC, the investment regulations differ accordingly. Some countries, such as Mexico and Czech Republic, impose limits on the allocation to certain asset classes; some countries, such as Chile and Poland, require a minimum return. The peculiarity of each country makes it difficult to compare investment performance of private pension plans across countries as discussed in Tapia (2008). But the operating cost of these plans is a meaningful indicator of the efficiency in running private pension plans. Figure 1.4 shows the operating expenses as a percentage of the total assets in 2007.
Figure 1.3: Gross replacement rate

Source: OECD Global Pension Statistics 2007
of private pension plans in the OECD countries. The expenses comprise all costs arising from
the general administration of the plan such as administrative costs and investment management
costs. Netherlands is among the lowest cost group.

1.3 The pension system in the Netherlands

The Netherlands has a multi-pillar pension system. The first pillar refers to General Old Age
pensions (AOW), or the public pension plan in OECD taxonomy. It regulates that people at age
65 are eligible for a flat rate old age benefit, regardless of their employment history. The second
The pension system in the Netherlands and the third pillar respectively refer to occupational pension plans and personal plans. The occupational pension plan consists of three types and their respective sizes are shown in Table 1.1. The industry-wide pension funds are set up for employees working in the same industry. According to the Dutch Central Bank at end of 2005, there are only 103 out of 784 funds in this category, but they take up 70% of the total pension assets. The company pension funds are for employees working for the same company and consist of 707 funds. The vocational pension funds are organized for certain professions, such as pension fund for family doctors. The majority of the occupational plans are mandatory, thus covering 90% of the working population. The personal plans are voluntary, where people can buy extra pensions from insurance companies if they perceive their pensions from the first and second pillar are not sufficient.

<table>
<thead>
<tr>
<th>Number</th>
<th>Assets (%)</th>
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<tr>
<td>Industry-wide funds</td>
<td>103</td>
</tr>
<tr>
<td>Company pension funds</td>
<td>669</td>
</tr>
<tr>
<td>Vocational Pension funds</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td>784</td>
</tr>
</tbody>
</table>

Source: Dutch Central Bank

The three pillars respectively provide 50%, 40% and 10% of the retirement income. The first pillar is important in eliminating poverty during retirement. The second pillar, often linked with employment, is also necessary as it encourages labor participation. Compared with other countries, shown in Table 1.2, the second pillar in the Netherlands plays a prominent role in the Dutch pension provision, and this thesis focuses on this branch of the system.

<table>
<thead>
<tr>
<th>First Pillar</th>
<th>Second Pillar</th>
<th>Third Pillar</th>
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<tbody>
<tr>
<td>Netherlands</td>
<td>50</td>
<td>42</td>
</tr>
<tr>
<td>Switzerland</td>
<td>42</td>
<td>32</td>
</tr>
<tr>
<td>UK</td>
<td>65</td>
<td>26</td>
</tr>
<tr>
<td>US</td>
<td>45</td>
<td>10</td>
</tr>
<tr>
<td>France</td>
<td>79</td>
<td>26</td>
</tr>
<tr>
<td>Germany</td>
<td>85</td>
<td>32</td>
</tr>
<tr>
<td>Spain</td>
<td>92</td>
<td>26</td>
</tr>
<tr>
<td>Italy</td>
<td>74</td>
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1.4 Challenges to the current pension plans

The beginning of the new millennium witnessed the “perfect storm” in the pension world.¹ The stock market went down, interest rates dropped to record lows and baby-boomers started to retire. The combination of these factors resulted in severe underfunding of DB plans. This has attracted extensive attention from accounting and financial regulators and politicians, and come with it several new accounting rules and new regulations.

In order to improve the transparency of pension fund accounting, International Financial Reporting Standards (IFRS) requires fair value accounting for liabilities. Since 2005 the mismatch between assets and liabilities should be stated in the main body of the annual reports of listed companies rather than in the footnote of the annual reports. In addition, the value of liabilities should be marked to market. As a result the funding ratio becomes more volatile. As this funding status is directly reflected in the balance sheet, companies are more concerned with the mismatch risk of their pension funds than before. New investment strategies such as alternative investments and liability driven investment can be employed to manage the risk. In addition, changing the designs of the pension plans can also be a solution, such as from a defined benefits based on final pay to a one based on average pay, and from a DB to a DC scheme (see Ponds & Van Riel (2007)).

Coinciding with the accounting rule, the Dutch pension supervisor introduced a new supervision framework, ”Financieel Toetsingskader” (FTK), effective from January 2007. This financial assessment framework is also based on the mark-to-market evaluation of pension funds. Specifically both the value of assets and liabilities should be marked to market. The framework requires pension funds to perform three tests/analyses to ensure their short- and long-term solvency. The minimum test requires that the present value of assets is at least equal to the present value of liabilities, namely the founding ratio should be at least equal to 1. The solvency test requires that the underfunding probability within one year should be less than 2.5%. The continuity analysis asks the fund to show that the fund is able to meet its obligations over the long term. This new regulation again put strong emphasis on managing the mismatch risks over both short and long terms.

The international developments in pension provision in other countries provide another stimulus to the Dutch pension reform. Chile pioneered individual pension accounts in the 1980s to replace its bankrupt public system. This institutional arrangement has been adopted by many

¹“Perfect storm” is a buzz word often used around this period to describe the hostile environment in which the pension plans are operated. The phrase originally refers to the simultaneous occurrence of weather events which, taken individually, would be far less powerful than the storm resulting of their chance combination.
other countries like the US, Australia and Hong Kong. The individual component in this DC type scheme has invoked a rethinking of the Dutch pension design. The following provides a brief history of occupational pension provision in the past few decades.

1.4.1 Dominant DB plans

The occupational pension plan was primarily a collective DB plan till the end of last century. In such a plan, the promised retirement benefits to participants are defined by a formula linked to years of working and salary path, and are not related to the investment performance. It was seen as a delayed salary and meant to attract loyal employees (Bodie (1990)). Any deficiency in the pension assets was made up for jointly by the plan sponsors, the current and the future participants. As a result the current retirees do not bear investment risk and longevity risk.

Around the turn of the Millennium, however, such a DB arrangement came under increasing pressures. Poor stock market performance in the early 21st century squeezed the value of assets, and falling interest rates inflated the market value of pension liabilities. The resulting underfunding in many DB plans puts high funding pressures on the sponsors and the contributing participants. Due to aging the pool of contributing participants is shrinking while the pool of retirees is increasing. In addition, the longevity phenomenon (that people live longer than expected) also demands more assets for benefit payment. Consequently a large increase of contribution rate are required. This, however, is not accepted by working participants and sponsors. Solely raising the contribution rate is no longer feasible and effective to turn around the underfunding situation. In addition, the new IFRS accounting rules make a company’s balance sheet vulnerable to the changes in the funding status of its pension plan. All these new situations compel the pension sponsors to curtail the influence from their pension plans.

1.4.2 Individual plans for a change

In the US, in addition to the above reason, increased global competition in product markets has also made it expensive for companies to sponsor traditional DB plans. Increased flexibility and mobility in labor markets also ask for a change of pension plans (Viceira (2009)). The collective DB is then replaced by 401(k) plans (Munnell & Perun (2006)), an individual DC plan. In such an arrangement, the employer and participants put aside an amount of money in an individual account, then the participants decide their own investment portfolios and at retirement they can annuitize their life-time savings on the 401(k) accounts to pension benefits to insur against longevity risk. The funding risk that the value of assets is less than the value of liabil-
ities is mostly transferred to individuals. The advantage of individual plans is that individuals can optimize their saving and investment decisions to satisfy their individual preferences. The implementation of the optimal strategies, however, suffers a series of problems. Firstly, individuals suffer from borrowing constraints at their early working years when they should borrow to invest in equity market. Secondly, not every participant is financially sophisticated enough to make the optimal dynamic decisions on saving, investing and annuitizing. Thirdly, people suffer from behavioral constraints such as irrationality and lack of self-discipline to save sufficiently (Benartzi & Thaler (2007)). Fourthly, the cost of running individual pension plan is much higher than the cost of running a collective pension plan, as shown in Bikker & de Dreu (2009). Fifthly, the annuitization exposes individuals to annuity risks that they have to annuitize their pension accumulation at a certain point in time, especially when the market is unfavorable at their retirement dates. In addition, buying annuity deprives participants of the ability to take risks after retirement. The problems together lead to a serious shortage of retirement income provision and sub-optimal life-time consumption. More theoretical and empirical details can be found in Bovenberg, Koijen, Nijman & Teulings (2007), Munnell & Sunden (2006), and Koijen, Nijman & Werker (2006). In the UK, personal pension schemes are replacing the DB schemes to relieve the financing pressure on the government and the sponsors. But the UK pension system suffers from the similar problems as in the US, as discussed in Blake (2006).

1.4.3 Redesigned collective plans

The problems encountered in the US and the UK and the highly embraced value of social solidarity in the Dutch society render individual plans not a solution to the underfunding situation among Dutch pension plans. Social solidarity generally can be understood as members of society should be prepared to pay for each other (Aalbers, Dietvorst, Gier, Janssens & Kok (2004)). Survey results in van Rooij, Kool & Prasta (2007) show that Dutch participants prefer a collective plan with a guarantee on their pensions. As a result, the traditional DB plans in the Netherlands have mostly shifted to a hybrid plan, called the conditional DB plans. In this plan the indexation of benefits to inflation is conditional on the funding status. When the fund is underfunded, the indexation to both working participants’ pension rights and retirees’ benefits could be cut. Hence the mismatch risk is also born by the retirees in addition to sponsors and contributing participants. There also emerges another type of hybrids, a collective DC plan, where contributions from sponsors and participants are fixed and benefits are determined by investment results.\(^2\) In this plan the funding risk are totally born by participants including both contributing participants

\(^2\)Such as Akzo Nobel pension fund, DSM pension fund and SNS Reaal pension fund.
and retirees. Both designs mitigate the underfunding situation to some extent because the mis-
mismatch risk can also be managed by changing the value of liabilities via indexation adjustment. But new problems arise due to the conditional arrangement.

The conditional plan exposes all its participants to volatile pension rights due to the con-
ditional indexation. With only one composite return from a plan’s common investment pool, questions arise as to how risks and returns should be allocated fairly among its participants. The current conditional plan does not specify the rules or establish a direct link between risk and return. This can lead to three issues. Firstly, when an overfunding or especially an underfunding situation occurs, it is very costly for various participants with differing interests to reach a consen-
sus in handling the situation. Secondly, among heterogenous groups of participants, any pre-set policies are very vulnerable to changes. Such changes of policies are undesirable as they could lead to substantial value transfer among different groups (Hoevenaars & Ponds (2008)). Thirdly, maturing plans with aging participants tend to make more conservative investments. This is to the disadvantage of young participants since they can not exploit the risk premium on the one hand and have to pay a higher contribution rate on the other hand (Ponds & Van Riel (2007)). The resulting differential treatment of different generations could endanger the sustainability of the mandatory feature of collective plans.

A collective plan has its advantages in intergenerational risk sharing and economies of scale. Any shock can be smoothed over multiple and even infinite generations. The collective plans charge much lower administrative costs than insurance companies who sell individual pension products as pointed out in Bikker & de Dreu (2009). A more in-depth discussion of the costs and benefits of collective plans can be found in Steenbeek & van der Lecq (2007).

The credit crisis in 2008 and the consequent economic recession again eroded the pension assets significantly. How to provide a proper pension through the volatile financial environment and varying demographic environment has become a central issue. This thesis is written under such backdrop in hope of finding feasible solutions to improve pension provision in the Netherlands.

1.5 Outline of the thesis and contributions

The poor stock market performance early this century and in the last year presented an imme-
diate probe to Dutch pension fund investments. The thesis starts with an overall evaluation of the investment performance of Dutch pension plan portfolios in Chapter 2. When little attention is paid to the total pension portfolio performance, we exploit a unique dataset of $z$-scores for
Dutch pension funds to examine the investment performance of pension fund portfolios. Our results show a close-to-benchmark performance using a Dutch sample, and detect no performance persistence. Cross-sectionally, we show that large plans persistently outperform their smaller peers.

This chapter contributes to the literature on pension fund performance evaluation in two aspects. Firstly, it is one of only a handful papers in the literature that focus on the multi-asset pension plan portfolio instead of focusing on the performance of individual asset classes. Investigation at this level reflects the overall efficiency of pension funds in managing investments. Secondly, the \textit{z-score}, the unique dataset for Dutch pension funds, has some merits that allows us to show conclusions on the abnormal performance. The \textit{z-score} uses a benchmark portfolio in adjusting raw returns. In this benchmark portfolio, the chosen indices are plan-specific and the weights to various asset classes are time-varying with the plan portfolio composition. This makes the \textit{z-score} a more appropriate measure than some stylized risk factors in evaluating a multi-asset portfolio.

The market valuation of liabilities introduced by IFRS in 2005 and the FTK regulation in 2007 point to a market-based perspective in managing the mismatch risk of pension funds. Under this backdrop, the model to describe the financial market becomes a essential choice. Chapter 3 pays an exclusive attention on the dynamics of the funding ratio over different time horizons and under various models for asset return dynamics. In addition, we also investigate the model impact on the strategic asset allocation. We consider two varieties of vector autoregressive models for return dynamics and find small changes in modeling asset returns can have a significant impact on the term structure of the optimal strategic asset allocation. Such an impact should not be overlooked or underestimated.

This chapter contributes to the literature in two aspects. Firstly we give an explicit delineation of both nominal and real funding ratios over various time horizons under market valuation. With a constant asset mix, a stylized fund has an increasing mean nominal funding ratio and a decreasing underfunding probability over longer horizons, but a decreasing mean real funding ratio and an increasing underfunding probability over longer horizons, reflecting the difficulty in matching real liabilities. Secondly, previous research often takes the first order vector autoregressive model (VAR) for granted to describe return dynamics, but we challenge this practice by using a restricted VAR (2) with a better statistical fit. We find this alternative model improves the dynamics of both nominal and real funding ratio due to its prediction of a lower liability return. Different from what a benchmark model suggests, this alternative model prescribes an increasing bond allocation with time horizon for the nominal liabilities. For the real liabilities, though
both models show similar dynamics of the optimal mix, comparatively, the alternative model prescribes a higher bond allocation and a lower stock allocation for long horizons. This model impact is robust irrespective of conditional information and risk attitudes of pension funds. Our analysis helps deepen fund managers’ understanding of model uncertainty. 

To solve underfunding problems, we propose to rethink the whole set-up of the pension contract. In Chapter 4 we redesign the pension contract and propose a generational plan where pensions are organized at a generational level rather than at an individual level in the individual DC plan or at a universal level in the traditional DB plan. This new plan can on the one hand accommodate an individual’s needs (though at a generational level) in planning his life time consumption, and on the other hand allow for intra-generational risk sharing. We compare this new design with the current DB plan and find that it provides a higher net present value and a higher welfare to participants. This result reflects that there are a-priori value transfers from new participants to other generation in the current collective plans.

This chapter contributes to the current discussion on the optimal pension design for all stakeholders. We propose an innovative but feasible design. Our numerical simulation allows for several practical considerations. Firstly, our modeling of liabilities allows for its time-varying feature induced by the yield curve movement. Secondly we consider the policy variables with their realistic specifications. Thirdly, for comparison purposes we take a representative collective plan to reflect the current liability structure. Our results provide new insights to pension providers in exploring new pension products.

As a follow-up study of Chapter 4, Chapter 5 explores possibilities of intergenerational risk sharing in a generational plan, and we use contingent claim valuation to measure intergenerational risk sharing and transfers. A benefit guarantee ensures a minimum benefits against uncertainty from investment, labor income and longevity risks. It can be seen as compound put options with guaranteed liabilities as the strike. The uncertainty also produces an upside potential that a fund can sell to avoid under-consumption. Using the setup of a generational conditional DB fund, we evaluate the prices of these benefit guarantees and the call option in a complete market.

This chapter contributes to the burgeoning literature on identifying and pricing implicit options in current collective pension plans. The generational fund provides a clean setting that facilitates a transparent pricing of various options and that makes the intergenerational risk sharing in a collective plan transparent and quantifiable. By presenting the generational account of a pension deal, we are able to identify the mechanism of intergenerational risk sharing and value transfers ex ante and ex post in a generational plan, which can serve as a reference to the risk sharing or transfers in the current collective plan.
Chapter 2

Performance Persistence of Dutch Pension Plans*

2.1 Introduction

The aggregated market value of Dutch pension plan investment portfolios is enormous. At the end of 2006 the total asset size of Dutch pension funds at year end 2006 was around €691 billion, while the assets from sources other than pension plans and insurance company, managed by collective investment schemes such as mutual funds and hedge funds are only about €117 billion for the same period.¹ Most of these assets are associated to mandatory industry-wide pension plans (€470 billion). The sheer size of the pension plan asset management sector warrants a careful investigation of the performance of their investment portfolios.

In the Netherlands a mandatory industry-wide pension plan is a multi-sponsor pension plan providing defined benefit pension services to all employees of the companies affiliated to a particular industry. Employees of these companies are obliged to participate in these schemes. The mandatory feature of this pension sector leads to a legal requirement of such pension plans to report a z-score, a risk-adjusted measure of their investment returns. Using this unique data set, we are able to provide a cross-sectional and longitudinal description of the investment performance of Dutch mandatory industry-wide pension plans at the level of total portfolio. Our study adds to the current limited literature on pension plan performance and provides an evidence that pension plan does not add value in implementing investments. Our study also shows the variation in performance across plans of different sizes, revealing that big plans persistently outperform small plans. It contributes to the discussion of the optimal size a pension plan should be as far as the

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*This chapter is based on the article by Huang & Mahieu (2008).
¹According to statistics on the website of the Dutch central bank (DNB).
quality of investment implementation is concerned.

A pension portfolio consists of various asset classes, and the study of its investment performance can be performed both on the level of the individual asset class portfolio and on the level of overall pension plan investment portfolio. Previous empirical studies focus on the first level to investigate the ability of individual asset managers for pension assets. This type of performance evaluation can aid pension plan trustees in their decisions of hiring and firing asset managers.\(^2\) However, the beneficiaries of a plan are more concerned with the overall investment performance of the plan’s entire asset pool, as it will directly influence the premium they have to pay and the benefits they can receive after retirement. The investment return is determined by the strategic asset allocation and the implementation of the allocation. The strategic allocation is set prudently by the trustees with the help of consultants and investment advisers. The implementation of the strategic portfolio is delegated by trustees to asset managers with different specializations.\(^3\) Our paper focuses on the second role of trustees in the delegation task. The success of the delegation is measured by the plan’s overall portfolio performance in excess of pre-agreed benchmark portfolio.

The lack of empirical studies on the total pension plan investment performance is related to the fact that there is little detailed information available on asset allocation and returns for individual components of the investment portfolio of pension plans. In the Netherlands, we suffer from the similar problems till recently Dutch industry-wide pension funds are required to report a \textit{z-score}. The \textit{z-score} has an advantage that it adjusts returns with fund-specific benchmarks, allowing us to examine and compare the quality of investment implementation across different plans.

Compared with retail investors, pension fund trustees are more resourceful in selecting asset managers, for the fact that they can receive more extensive help from advisers and consultants, gain more information before making the decision, and can establish procedures to better monitor the investment progress. We could reasonably expect that trustees should be able to select a superior group of asset managers or establish effective investment procedures to encourage asset managers to beat the benchmarks. However our study finds that over time an average pension plan cannot earn above-benchmark returns. This reflects that the rich information and the exclusive professional help do not win pension funds an edge in implementing the investment strategy to earn excess returns.

\(^2\)A trustee, often a board of directors, is the custodian appointed by plan sponsors and participants to hold and manage the assets in trust for the benefit of plan participants.

\(^3\)A new trend is that an investment house acts as a fiduciary asset manager delegated by pension trustees to look after the whole investment process from strategic asset allocation to individual asset manager selection.
The abnormal performance of mutual fund managers is largely unpredictable by using past performance (Gruber (1996), Carhart (1997), Bollen & Busse (2001)), and this lack of persistence is due to the competitive capital supply in the mutual fund industry (Berk & Green (2004)). But in other industries such as hedge funds liquidity restrictions prevent money to move freely, which may result in short-run persistence. In the context of pension funds, though in principal pension fund trustees can fire the asset managers within a short notice such as one day, in practice a mandate often has a contract life of two or more years. The inconvenience of moving a large amount of pension assets across different asset managers may also predict some type of performance persistence. But our study does not detect any significant performance persistence over a one-year horizon. Cross-sectionally, however, big plans are able to persistently outperform their smaller peers.

2.2 Literature review

In this part we review previous papers on pension fund performance evaluation. Lakonishok, Shleifer & Vishny (1992) and Coggin, Fabozzi & Rahman (1993) look at the US pension plan equity portfolios. The first paper takes a sample, as large as it could be, to examine the double-agency structure and its relation with underperformance. The second paper makes a random selection of equity pension funds to investigate the ability of fund managers. Busse, Goyal & Wahal (2006) extend to fixed income portfolio and use a even larger dataset to estimate the abnormal returns. Tonks (2005) studies the UK samples and investigates the ability of investment houses rather than individual fund managers in managing pension assets. Bauer, Frehen, Lum & Otten (2007) study the aggregate equity portfolio at the plan level, and focus on the comparison with the equity portfolio in mutual funds. Brinson, Hood & Beebower (1986) and Ippolito & Turner (1987) pioneered the performance evaluation at the total plan level, and they both use a benchmark portfolio including at most three asset classes, which is no longer proper for the current complex pension portfolio. Blake, Lehmann & Timmermann (1999) study the UK pension plans and focus on the return attribution to strategic asset allocation and managerial skills. The conclusions on outperformance from pervious studies are mixed and hard to compare, because they focus on different sampling period and sampling region, use different benchmarks, and differ in fee consideration and subject focus such as asset managers or investment houses.

Lakonishok et al. (1992) investigate the performance of 769 US equity pension fund portfolios for the period 1983Q1 through 1989Q4. Using quarterly gross return (before management fees) they find on average the equity portfolios under perform S&P 500, and conclude that active
management does not add value. They propose the reason for the underperformance is the cost of double-agency structure existing in the pension fund industry where the investment is delegated from corporate management to corporate treasurer and then to portfolio managers. As persistence is concerned, the equity portfolio as a whole does not show any performance persistence for a one-year horizon. This result holds for the growth and the yield portfolio, but the value portfolio shows some persistence.

Coggin et al. (1993) shed light directly on the performance of individual asset managers for pension funds. They study the investment performance of US equity pension fund portfolios by randomly sampling 71 managers for the period January 1983 through December 1990. They distinguish monthly returns attributed to managers’ stock selection and timing ability. By using both a general equity market index and a style-specific index, they find the equity asset managers for pension funds on average have positive selection ability but negative timing ability. They do not consider fees.

In addition to the equity portfolios, Busse et al. (2006) extends the analysis to other asset classes including 1,683 fixed income portfolios and 1,196 international equity portfolios held by the US pension plans over the period of 1991-2004. Using quarterly returns, they find a positive abnormal return after adjusting for the relevant risk factors for all three asset classes, even after controlling for costs. As persistence is concerned, only winner portfolios show performance persistence for a one-year horizon.

Rather than looking at the individual asset managers and their portfolios, Tonks (2005) turns his attention to fund management houses for the UK pension plans. His data are quarterly returns on the equity portfolios of 2,175 U.K. pension plans managed by 191 fund management houses from 1983Q1 to 1997Q4. Using a variety of risk adjustment, such as single factor Capital Asset Pricing Model, the Fama-French three-factor model, and a Carhart four-factor model, and averaging the abnormal returns of all portfolios under one management house, he finds a positive abnormal return and persistent performance of fund management houses over a one-year horizon. His results do not take costs into account.

In addition to the performance evaluation at the portfolio level and fund management house level, Bauer et al. (2007) study the performance of the aggregate equity portfolio for the US pension plans (including 716 Defined Benefit and 238 Defined Contribution plans) between 1992 and 2004 at the plan level. Deducting the gross annual returns by the returns to fund-specific benchmark and the costs, they find close-to-benchmark performance. This result also holds when the net returns are adjusted by the Fama-French risk factors. They argue that this is because pension funds are less exposed to agency costs than mutual funds for their monitoring capacity.
and negotiation power. They detect no persistence of the equity portfolios at the plan level for one year horizon.

Studies on the overall pension plan portfolio performance are even scarce. Among the earliest is Brinson et al. (1986). They study 91 large US pension plans over a sample period of 1974-1983. Attributing returns to investment policy returns and active returns due to market timing and security selection, they document negative active returns, meaning underperformance with respect to the benchmark portfolio which includes only coarse categories of stocks, bonds and cash. Our paper in essence is similar to this paper in that our data separate active returns from policy returns, and the policy returns are represented by the benchmark return which is more accurate with a finer asset classification to reflect the current complex investment practice.

Rather than using a plan-specific benchmark portfolio, Ippolito & Turner (1987) use the same benchmark indices such as S&P500 for all pension plans. Based on the net-of-fee returns of 1,526 US pension plans they find that the investment performance is very sensitive to the choice of benchmark whether it is a stock index or a bond-stock mix index. They also find that larger pension plans outperformed smaller plans substantially. Their results are not very informative for the current practice. Their choice of a broad stock or a stock-bond mix index as a benchmark is not appropriate in the current performance evaluation of pension plans, as the current pension plan investment is no longer limited to general stocks or bonds as in the paper's sample period back to 1977-1983, but is much more complicated in multiple asset classes and styles.

A more recent paper on the pension plan performance is on the UK sample by Blake et al. (1999). They investigate the monthly return of the overall pension portfolios of over 300 UK pension plans for the period of 1986-1994. Besides documenting underperformance with respect to the benchmark portfolio, their paper has a focus on decomposing the returns into a component attributed to strategic asset allocation and a component attributed to management skills. They adjust the raw returns with a benchmark portfolio where the weight to each asset class is either the time-average of realized allocation or an estimated weight by associating it to time. In addition, they apply the same external indices for all pension funds regardless of the difference of investment style within a certain asset class.

Concluding the previous papers we find there are two ways to construct a benchmark portfolio in computing abnormal returns. One way is using risk factors (such as a equity market index, a bond index or even detailed size factor, distress factor, etc.) for each asset class and estimate the loading on these factors, such as Coggin et al. (1993), Tonks (2005), and Busse et al. (2006). The benchmark portfolio is then composed by the loadings and the returns to the risk factors. This approach can be used when there is a large dataset and a proper estimation can be made. A
second way is to use the holding information or asset allocation of a portfolio, such as Brinson et al. (1986), Ippolito & Turner (1987), and Blake et al. (1999). The benchmark portfolio is composed of the weights and the returns to the respective asset class. This approach is more accurate in accounting for the risks but requires the information on holdings.

Our study distinguishes itself from most of the previous studies in three major aspects. Firstly, we focus on the multi-asset pension plan portfolio rather than a delegated portfolio in some asset class, as the performance of the entire portfolio is more relevant to plan sponsors and beneficiaries. Secondly, the data we use in this study provides the returns after accounting for a holding-based benchmark. Such benchmark is more appropriate for a multi-asset portfolio than a benchmark based on some stylized risk factors. Thirdly, in the choice of benchmark portfolio, our data improves in two aspects compared to that used in Blake et al. (1999). Firstly the precise fund-specific and period-specific weights for each asset class are used in the benchmark portfolio. Secondly the indices for the benchmark portfolio are also fund-specific. For example, some fund may use S&P500 for an equity allocation, but some other fund may use a MSCI index. Therefore we have the excess return that is appropriately adjusted by the fund-specific benchmark portfolio, and it represents the value added by any deviation from the benchmark portfolio, due to either stock selection or marketing timing strategy of the selected group of asset managers.

2.3 Investment process and performance

Before we address the performance, we provide a brief description of the investment process of a defined benefit Dutch pension plan. Though it is a Dutch practice, it is not very different from the rest of the world. In brief trustees set out the investment policy with the help of consultants. The execution of this policy is delegated to asset managers selected by trustees.

The investment policy is often motivated by an Asset Liability Management (ALM) study, which is an integral risk management study of the fund, taking into account the short-term and long-term objectives of the fund. The investment policy is represented by the strategic asset allocation. This is a portfolio based on the fund’s (subjective) view of expected returns and risks of each asset class and an estimation of the plan’s liabilities from a long-term perspective. The strategic asset allocation is often reviewed every 3 or 5 years to reflect major changes in the underlying assumptions regarding the assets and the liabilities. From the strategic asset allocation trustees define an investment plan that can be implemented by asset managers, often on an annual basis. It reflects a short-term view on the risk-return profile of each asset class and tries to exploit forecasting skills. The plan typically consists of weights that differ from the weights implied
from the strategic allocation. According to the annual investment plan, trustees assign mandates for each asset class to a selected group of asset managers. These managers can be either in-house or external, one or multiple, of passive or active style. The reason for delegation is mainly related to the expertise of an asset manager in a particular asset class. Other reasons can be economies-of-scale in trading and record keeping (Sharpe (1981)). The performance of asset managers is monitored at a regular frequency (often a quarter) based on the agreement of service level signed between trustees and asset managers.

The way that pension plan investments are carried out reveals that the investment returns are generated from three major sources. One is from the strategic asset allocation allocation using the portfolio weights and returns per asset class specified in the investment policy. The second source is from executing the annual investment plan, but measured against the benchmark returns. This part of the returns measures the added value from over- or under-weighing the strategic benchmark, reflecting the timing skills of trustees when they draw up the investment plan. The last source comes from the actual execution of the investment plan delegated to a group of asset managers. It reflects the timing and selection skills of the asset managers as a group. Good (1984) and Brinson et al. (1986) have a similar return attribution except that they distinguish the returns from timing and selection skills. This paper focuses on the returns from the third source, which tell us whether the selected group of asset managers of a pension plan can deliver the performance as specified in an annual investment plan.

To obtain the difference between the actual returns and the returns attainable from the strict adherence to an annual investment plan, we need to define the benchmark portfolio that represents the annual investment plan. This benchmark portfolio is a hypothetical portfolio, which is "structurally identical to the investment strategy without whatever active management takes place” as defined in Logue & Rader (1998) (p168) or a "passive mix with the same style” as in Sharpe (1992).

The benchmark portfolio has the same investment style/category as the pension plan’s annual investment plan and uses the index in the respective style/category as the return benchmark. The overall return from the benchmark portfolio represents the return that can be obtained from a passive management of the pension plan portfolio. One example can be found in Table 2.1. The benchmark portfolio has a twofold purpose. First, the index for each component portfolio is used by trustees to evaluate the performance of individual asset managers for a particular asset class. Second, the overall return from the benchmark portfolio serves as a return target. In our

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4There is a trend that trustees delegate this manager selection job to a fiduciary asset manager in order to avoid contacts with too many asset managers.

5It is also called "norm portfolio" in the Netherlands.
study we use the benchmark portfolio for its second purpose to evaluate the quality of investment implementation by the asset managers.

2.4 The z-score

Since 1998 every Dutch mandatory industry-wide pension plans must compute a so-called z-score to reflect their investment performance. In this section, we introduce this unique measure and discuss its advantages and disadvantages in the performance evaluation.

The z-score is the difference between the actual return and the return on a predefined benchmark portfolio, net of expenses, and normalized by the riskiness of the portfolio, as in the following equation:

$$z_{i,t} = \frac{(R_{p,i,t} - c_{p,i,t}) - (R_{b,i,t} - c_{b,i,t})}{E_{i,t}}$$

where $R_{p,i,t}$ and $c_{p,i,t}$ are the gross investment return and internal investment cost of pension plan $i$ at time $t$ respectively. The internal investment cost also includes the fees paid to the external asset managers and investment related custodian and administrative cost. $R_{b,i,t}$ is the plan $i$'s benchmark portfolio return using market indices in the respective asset categories at time $t$. The fine asset classification in the benchmark portfolio as shown in an example in Table 2.1 reveals that the benchmark portfolio can adjust the risks in the plan portfolio in a more accurate manner than the stylized risk factors often used in other studies. $c_{b,i,t}$ is the associated investment cost of the benchmark portfolio which depends on the percentage of equity in the portfolio.\(^6\) The benchmark portfolio is determined by trustees at the beginning of each year and fixed for one year. Specifically, the weights in the benchmark portfolio are defined a priori, and the chosen index should represent the asset class, be investable and objectively measurable.\(^7\) $E_{i,t}$ is the risk of the plan portfolio as a function of the asset mix. The asset mix for this purpose contains only two major categories: equity and fixed income (including cash). The risk percentages assigned for equity and fixed income are fixed by law at 2.6% and 0.6% respectively.\(^8\) For example, if a plan has an asset mix of 60% equity and 40% fixed income, then $E_{i,t} = 0.6 \times 2.6\% + 0.4 \times 0.6\%$. The reported z-score is checked by external accountants.

The way the z-score is constructed reveals that it is not a measure to evaluate the effectiveness of the investment policy, but a measure of the quality of the implementation of the investment policy.

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\(^6\)This cost is presented in Bpf (2000), and range from 0.10% to 0.22%. It varies to the equity proportion of the pension portfolio.

\(^7\)See Article 5.3 in Bpf (2000).

\(^8\)According to Bpf (2000), the riskiness of equity and fixed income, namely 2.6% and 0.6%, is fixed across plans and over time.
plan. A positive (negative) z-score means that the selected group of asset managers can collectively beat (be beaten by) the benchmarks set by trustees.

The intention to create the $z$-score is to have a standardized normal distribution of the returns so that the regulator can examine whether the fund’s investment performance falls to the lower 10% of its distribution. If an industry-wide plan falls to its 10% percentile and does not deliver a satisfactory investment performance, the participating companies are no longer obliged to join this plan, and can have the option to leave the plan. A statistical test (performance test in Dutch) is used to support this decision. The test statistic is based on the five-year geometric average of the $z$-scores:

$$P_{3 \text{ year}} = \frac{\sum_{t=1}^{5} Z_{i,t}}{\sqrt{5}}.$$ 

The critical value of the test is -1.28, which corresponds to a confidence level of 90% for a standardized normal distribution. If the five-year geometric average of $z$-score is less than -1.28, a sponsor can choose to fire the trustees by opting out of this industry-wide pension plan. Subsequently they can either form a new pension plan or they may join another industry-wide pension plan.

The $z$-score is intended to compare the investment performance across different pension plans. Yet it receives some criticisms on its use. We elaborate on these criticisms in the following to show that they do not pose a major obstacle to its use as a performance measurement. The first criticism is that the $z$-score does not reflect investment performance properly. This is true as investment performance is largely determined by the strategic asset mix and only marginally determined by the execution of the strategic investment plan. The $z$-score can only evaluate the quality of the implementation. This is exactly our purpose in this paper. We do not investigate which plan has a better investment policy, but want to judge whether pension funds earn an excess return over the a priori benchmark and whether plan trustees are able to employ the right asset managers to achieve that goal.

A second criticism is that the benchmark portfolio is a static benchmark, in which the weights to different asset style are fixed for one year. So the intertemporal changes in the investment plan during the year cannot be captured by the benchmark portfolio used in the $z$-score calculation, but do change the return of the actual benchmark portfolio. This can hamper a fair evaluation of asset managers, because part of the deviations is due to the change of the benchmark portfolio and has nothing to do with the implementation ability of the asset managers. This staleness problem of the benchmark portfolio is more of a conceptual problem than a practical problem. The current practices help mitigate such problems. Firstly, though fixed weights as a general
rule, the benchmark portfolio is allowed to be changed once when there is a considerable change in the liability structure or the old investment plan is obviously no longer appropriate for the plan. Secondly, changing the investment plan during the year is more of a practice per Jan 1, 2007 when the regulation on financial assessment is implemented, which requires the investment plan to match the market value of liabilities. Thus during our sample period we do not expect material changes in the investment plan during the year. Therefore the static benchmark portfolio is still a reasonable reflection of the investment plan as a blueprint for asset managers to carry out the investments.

A third criticism concerns the risk adjustment in the denominator of the z-score, where riskiness of equity and fixed income are the same for all plans and for all time. This may lead to an unfair evaluation of both an average plan’s ability to beat the benchmarks and the cross-sectional comparison among pension plans. If an average plan takes more risks than what is assumed in the respective benchmark, the z-score of the whole pension industry is upward-biased. Bauer et al. (2007), however, shows that for US pension fund equity portfolios, benchmark-adjusted returns have marginal loadings on risk factors. Blake, Lehmann & Timmermann (2002) show UK pension fund portfolios have betas clustering around 1 for the most important asset classes like equity, bonds and real estate. This means that within an asset class the pension plans on average do not take excessive risks than what is assumed in the benchmark. So we conjecture that an average pension plan at the overall pension portfolio level does not suffer much from this fixed risk adjustment problem. Even when the average plan takes more risks than the benchmarks, if our sample period covers both up and down market conditions, such higher risk taking won’t give an advantage in the z-score calculation. Therefore the fixed risk adjustment, if any, will only have limited impact on an average plan’s z-score over time. Cross-sectionally, a plan investing in more risky securities in an asset class than its peers can be better off in calculating its z-score by using the fixed risk adjustment. But it will also be worse off in the down market conditions. Unless a plan can consistently time the market by taking higher risks in up market and lower risks in down market than its peers, we won’t expect the fixed risk adjustment can impact the cross-sectional comparison among plans in a significant way.

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9See Article 5.4 in Bpf(2000).
10To support our conjecture, we make a numerical example here. Suppose the benchmark portfolio only defines 40% in equities and 60% in fixed income. External index for equity index is S&P500. Suppose a small cap investment can earn an extra annual return of 3% above S&P500. If a plan wants to exploit the fixed risk adjustment, it has to invest 98% of its equity portfolio in such small caps in order to increase its z-score by 1 standard deviation of 0.84. This example shows that a plan has to take considerable more risks than the benchmark in this asset class in order to have a material impact on its z-score. But such practice is highly unlikely.
11Because the z-score is upward biased in the up market, but downward biased in the down market. Over time the impact can be negligible.
In the following of this paper we will use the $z$-score to evaluate the quality of investment implantation in the pension plans to see whether pension plans as a group add value by beating benchmarks. As the $z$-score is normalized by the respective riskiness linked to a plan’s asset mix, the comparison of the $z$-scores across pension plans allows us to identify plans with performance persistence.

2.5 Data

We use the publications of the Dutch industry-wide pension plan association.\textsuperscript{12} In addition we obtained data from pensioninfo which collects and composes aggregate financial information of companies and organizations including pension plans.\textsuperscript{13} We merged and verified data from both sources. When there is a discrepancy between the $z$-scores from the two sources, we used the $z$-score reported in a plan’s annual report.

Our sample runs from 1998 through 2006 and covers the entire history of $z$-scores and the entire population of mandatory industry-wide pension plans.\textsuperscript{14} Over this sample period, the number of plans varies between 59-65 for a number of reasons. Some plans either started to exist or become mandatory after 1998, one plan merged, and two plans bought the insurance of guaranteed returns. In the end, we have a sample of 57 plans that contain a complete data set on $z$-scores, and this sample will be used for the persistence test. No plan becomes non-compulsory or ceases during our sample period and thus our sample does not suffer from survivorship bias. A graphic overview of the $z$-scores can be found in Figure 2.1.

Since there is no considerable change in the relative sizes of the pension plans in our sample, we use the amount of invested assets in 2006 as a proxy for size. The data is obtained from all pension plans’ 2006 annual reports and shown in Figure 2.2. The smallest plan in the sample is €1.47 million, the largest is €208.9 billion, and the median and mean are €426 million and €7.2 billion respectively. This reflects a large size spread among Dutch pension plans. Most plans are small- and medium-sized within €10 billion except for four multi-billion plans.

\textsuperscript{12}In Dutch it is called the Vereniging van Bedrijfstakpensioenfondsen (VB). See their website at www.vb.nl.
\textsuperscript{13}See their website at http://www.iqinfo.com.
\textsuperscript{14}According to DNB 2007 there is 71 mandatory industry-wide pension plans including 7 pre-pension plans which provide pensions for early retirement. Only mandatory funds are required to report $z$-scores.
2.6 Empirical results

The $z$-score is based on the plan-specific benchmark, the benchmark portfolio. As a result the performance analysis in our paper focuses on a plan’s ability to beat its own benchmark. Descriptive statistics in Table 2.2 show that through the sample period the average $z$-score varies around 0. For a one-year horizon, except in the year 2002 and 2004, the average plan outperforms its benchmark. In total, an average pension plan does slightly better than its benchmark.

We perform a $t$-test to examine the statistic significance of the above results. During the buoyant period of 1998 through 2000 and the recovering period of 2005 and 2006, the $z$-scores are positive at 5% significance level, while in 2002 and 2004 the $z$-scores are negative at a 5% significance level. When pooled together, the $z$-score is not significantly different from 0. Considering the possible correlation of the $z$-scores for one plan over time, we also calculate the equally-weighted $z$-score across plans and test its time average. We find the average $z$-score is 0.04, and $t$-statistic is 0.28. In sum we can not reject the hypothesis that industry-wide pension plans as a group over time are not able to out-/under- perform their own benchmarks, namely that they deliver a close-to-benchmark performance. This result agrees with the implication of non-superior selection ability in Goyal & Wahal (2008) that plan trustees cannot time the decisions of hiring and firing asset managers successfully.

2.6.1 Performance persistence

The descriptive statistics show that the average pension plan is not able to beat its benchmark over time. In this section we focus on the performance persistence of the pension plans in our sample. Most studies suggest that there is no performance persistence within mutual funds.\footnote{Among others there are Gruber (1996), Carhart (1997), Bollen & Busse (2001). Some recent studies though point out short-run persistence when using daily and monthly returns and certain performance measures such as Bollen & Busse (2005) and Huij & Verbeek (2007).} Within the rational market framework, this is due to the free movement of competitive capital discussed in (Berk & Green (2004)). In the pension fund industry, however, mandates stay with one asset manager often for more than two years. There is no competitive supply of capital to pension asset managers, so we should expect some persistence here. To this end, we present the results from three methods.

Following the methodology of Fama & MacBeth (1973), we first run a cross-sectional regression of the future $z$-score on the past $z$-score on a yearly basis as in

$$z_{i,t} = a_t + b_t z_{i,t-1} + \epsilon_{i,t},$$
for every year during the period 1999-2006. Using standard OLS we obtain a time series of coefficient estimates ($\hat{a}_t$ and $\hat{b}_t$). Then we perform a $t$-test on the average estimated coefficients, shown in Panel A of Table 2.3, which gives a slightly positive correlation ($\tilde{b}_t = 0.06$). It says the past $z$-score positively predicts the future $z$-score, but not statistically significant at a 5% significance level. The pension plans as a group does not show persistence in their investment performance.

We also apply a Spearman rank correlation test for persistence, which does not require a normal distribution from the underlying data. In this test we only use the plans with a complete set of $z$-scores in all 9 years, namely a sample of 57 plans. Each year we give a rank to each plan based on its $z$-score. The Spearman rank correlation coefficient for two consecutive years is then computed as

$$\rho_{t,t-1} = 1 - \frac{6 \sum_{i=1}^{N} d_{i,t,t-1}^2}{N(N^2-1)},$$

where $\sum_{i=1}^{N} d_{i,t,t-1}^2$ is the sum of squared differences of ranks over two consecutive years for all funds. $N$ is the number of funds (or ranks), i.e. $N=57$ in our case.

For our 9-year sample, we obtain a time series of correlation coefficients for 8 years. As in the previous regression test, we apply a $t$-test using the average and the standard deviation of the time series, shown in Panel B of Table 2.3, and find the average coefficient (0.26) is not significantly different from zero. This is consistent with our earlier results.

There might be concerns that the above results may be subject to noise from a few individual plans and the tests are not reliable based on a short time span. Therefore we construct 3 (and 5) portfolios based on their past performance to show their future performance year by year. Every year, 3 (and 5) portfolios are formed based on the previous year’s $z$-score. For each individual portfolio the average $z$-score is computed in every year. Repeating this for each year, we obtain a times series of $z$-scores for the 3 (and 5) portfolios in Table 2.4. If performance is persistent, the best-performing portfolios should provide the best performance in the subsequent years again. However, our results show that in some years the best performing portfolio from the past year provides the worst performance in this year. The paired sample $t$-tests among the 3 (and 5) portfolios reported in Table 2.5 shows none of the test statistics is statistically different from zero. This again confirms no persistence in plan performance over time.

In order to understand the no-persistence better, we look further into the composition of the performance portfolios over time, by applying the methodology of Fama & French (2007). In this analysis we only use the three-portfolio division. Each column in Table 2.6 reports the percentages of plans in the current portfolio that originated from the previous year’s top, mid and
bottom portfolio, respectively. We find plans move dramatically among the top, mid and bottom portfolios. For example, of the current top portfolio 30% are plans that were in the previous year’s bottom portfolio, and another 30% come from the mid portfolio of the previous year. Of the current bottom portfolio 31% and 41% are the plans from the top and mid portfolio in the previous year respectively. We test the hypothesis of random migration of plans among the three portfolios. The null hypothesis is that the migration probabilities should be all equal to 1/3. The test statistics show that we cannot reject the hypothesis at 5% significance level. This random movement of plans among the three performance portfolios underlines the lack of persistence that we found earlier.

In addition to the examination of how plans migrate between performance portfolios over time, we also investigate the contributions to the current \( z\)-score made by the migrating plans. Results are presented in Table 2.7. In 1999, a large part \((-0.39)\) of the bottom portfolio’s \( z\)-score \((-0.65)\) is contributed by the plans that used to be in the top portfolio in the past, while the top portfolio obtains a large chunk of its \( z\)-score \((0.49\) out of \(1.23)\) from the plans in the previous bottom portfolio. Similar patterns can be found in year 2001 and 2005, where the current bottom portfolio’s negative \( z\)-score is mostly contributed by the plans in the past top portfolio. In the years 2000 and 2003 the current top portfolio obtains a big portion of its \( z\)-score from the plans that was in the past bottom portfolio. Such dramatic changes of performance attribution between years again confirm our previous results that past performance does not tell us much about future performance.

### 2.6.2 Performance and plan size

The previous analysis shows that as a group the pension plans do not show any out- or under-performance with respect to their benchmarks. It is interesting, however, to investigate the cross-sectional difference among plans. Ambachtsheer, Capelle & Scheibelhut (1998) investigate 80 US and Canadian pension plans for the period 1993-1996 and find that large plan size is an important driver for good pension performance, measured by risk-adjusted net value added by asset mix decision and implementation. Reasons are that large size brings economy of scale in operating cost and enables plans to support a full-time professional management team. Following this lead we test whether pension plan size is relevant for explaining the different performance across Dutch pension plans. We perform a regression of the time-average \( z\)-score on the plan’s size.

The test is done on a sample of 57 plans with complete \( z\)-scores over the sample period. Table 2.8 shows that size indeed matters. Size alone explains almost 28% of the variation in a
plan’s average \textit{z-score}. The larger plans have a higher average \textit{z-score} than the smaller plans. This finding says that asset managers selected by the larger plans can implement the investment better than those selected by the smaller plans. This result is also supported by two other papers. Goyal & Wahal (2008) study the sponsors’ decision to hire and fire asset manager. They find that plan size can explain the post-hiring excess returns, and suggest that large size allows sponsors to develop expertise in selecting asset managers. Bauer et al. (2007) study the mandate size of delegated portfolio. They find size is not a factor driving the benchmark adjusted net return, but size does bring economy of scale in reducing cost of external managers. Both these reasonings support our findings on size, but we cannot distinguish which is exactly at work.

We also compute the \textit{z-scores} on equally weighted portfolios sorted on size, and the results are presented in Panel A and B of Table 2.9. For tertile portfolios, the size effect is not obvious, but in the quintile portfolios we see a clear difference in \textit{z-score} between the largest and the smallest plan. In the range of middle-sized portfolio, there seems no clear difference in their \textit{z-scores}. This says that the top 20% of the plans ranked on size persistently outperform the bottom 20%. As Figure 2 suggests that most plans are small and medium-sized except a few huge plans, we thus form 4 size portfolios in Panel C into the categories of "smaller than €0.1 billion", "between €0.1 billion and €1 billion", "between €1 billion and €10 billion", and "larger than €10 billion".\textsuperscript{16} Respectively they contains 8, 27, 18 and 4 plans. We apply paired sample \textit{t}-tests to the largest and the smallest portfolio in Table 2.10. We find that the difference in the \textit{z-scores} between the largest and the smallest size portfolios is statistically significant no matter how many portfolios are formed. To relieve the concern over the power of the \textit{t}-test in this small sample, we also perform the Wilcoxon signed ranks test that is a nonparametric test suitable for small samples. The test results maintain the original conclusion. Our results are consistent with the findings on US and Canadian pension plans that size is a driver for performance. Large plans can better beat the benchmark than the small plans. Due to the data limitation, we cannot explain what causes this better performance. Possible explanation could be negotiation power in lower costs, reputation effect, better monitoring of asset managers, or more expertise in selecting superior asset managers.

\textsuperscript{16}Our sample of pension plans contains two very large plans, ABP and PGGM with asset sizes of €208 billion and €81 billion respectively in 2006.
2.7 Conclusions

Pension plan trustees make decisions on asset allocation and on selecting asset managers to manage their investments. The first decision determines the majority part of the investment return, and the second decision determines whether the selected asset managers can deliver or even beat the benchmark performance. Our paper focuses on the effectiveness of the second decision. Dutch mandatory industry-wide pension plans are obliged to publish a $z$-score to show their net-of-fees investment performance relative to a priori self-selected benchmarks. These scores reflect the implementation quality of the asset allocation. After a study of the $z$-scores on a comprehensive set of industry-wide pension plans in the Netherlands, we find no outperformance and no performance persistence. We conclude that pension plans on average cannot generate investment return above their benchmarks, and trustees as a whole do not deliver a superior performance in implementing the investment strategy by either selecting superior asset managers or establishing proper investment procedures. Cross-sectionally, however, we do find that large plans are better able to beat their benchmarks than smaller plans. Yet we are unable to explain the driving forces of this better performance.
2.8 Tables and figures

Table 2.1: An example of a benchmark portfolio
This is a reproduction of a benchmark portfolio. It specifies the weight and the indices used for different investment styles. The range specifies the bound within which an active asset manager must control the weight. Source: 2006 annual report of the Agriculture and Food Supply Pension Plan, which can be found via www.iqinfo.com.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Weight</th>
<th>Range</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed income</td>
<td>75%</td>
<td>65%-85%</td>
<td></td>
</tr>
<tr>
<td>Governments</td>
<td>70%</td>
<td>60%-80%</td>
<td>Citigroup Gov Bond Index</td>
</tr>
<tr>
<td>Corporates</td>
<td>15%</td>
<td>10%-20%</td>
<td>Citigroup non-EGBI EMU index</td>
</tr>
<tr>
<td>Private Loans</td>
<td>15%</td>
<td>10%-20%</td>
<td>Customized Private Loan Index</td>
</tr>
<tr>
<td>Equity</td>
<td>15%</td>
<td>5%-25%</td>
<td></td>
</tr>
<tr>
<td>Europe</td>
<td>40%</td>
<td>30%-50%</td>
<td>MSCI Europe</td>
</tr>
<tr>
<td>USA</td>
<td>20%</td>
<td>10%-30%</td>
<td>MSCI North America</td>
</tr>
<tr>
<td>Pacific</td>
<td>15%</td>
<td>5%-25%</td>
<td>MSCI Pacific</td>
</tr>
<tr>
<td>EM Global</td>
<td>25%</td>
<td>15%-35%</td>
<td>MSCI EM Global</td>
</tr>
<tr>
<td>Real estate</td>
<td>5.0%</td>
<td>0%-10%</td>
<td></td>
</tr>
<tr>
<td>Residential</td>
<td>50%</td>
<td>25%-75%</td>
<td>ROZ- IPD Woningen</td>
</tr>
<tr>
<td>Shops</td>
<td>50%</td>
<td>25%-75%</td>
<td>ROZ- IPD Winkels</td>
</tr>
<tr>
<td>Alternatives</td>
<td>5.0%</td>
<td>0%-10%</td>
<td></td>
</tr>
<tr>
<td>Commodities</td>
<td>50%</td>
<td>0%-100%</td>
<td>DJ-AIG Commodity Index</td>
</tr>
<tr>
<td>Hedge Fund</td>
<td>50%</td>
<td>0%-100%</td>
<td>Euro 7-day Libid</td>
</tr>
</tbody>
</table>

Table 2.2: Descriptive statistics of the $z$-scores
Descriptive statistics for the $z$-scores of Dutch industry-wide pension plans over the period of 1998-2006. (*), (**), and (***) indicate a significance at the 10%, 5% and 1% levels. T-statistics on the bottom line tests whether the mean $z$-score for each year and for the pooled sample is different from 0.

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>Pooled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.26*</td>
<td>0.27*</td>
<td>0.29***</td>
<td>0.08</td>
<td>-0.89***</td>
<td>0.14</td>
<td>-0.39***</td>
<td>0.30***</td>
<td>0.30***</td>
<td>0.03</td>
</tr>
<tr>
<td>Med.</td>
<td>0.14</td>
<td>0.19</td>
<td>0.28</td>
<td>-0.08</td>
<td>-1.00</td>
<td>0.04</td>
<td>-0.39</td>
<td>0.25</td>
<td>0.14</td>
<td>0.02</td>
</tr>
<tr>
<td>Maxi.</td>
<td>2.25</td>
<td>3.43</td>
<td>3.44</td>
<td>3.84</td>
<td>0.80</td>
<td>1.74</td>
<td>1.34</td>
<td>2.30</td>
<td>2.27</td>
<td>3.84</td>
</tr>
<tr>
<td>Mini.</td>
<td>-3.07</td>
<td>-1.22</td>
<td>-1.59</td>
<td>-2.25</td>
<td>-2.91</td>
<td>-1.14</td>
<td>-1.79</td>
<td>-0.87</td>
<td>-0.58</td>
<td>-3.07</td>
</tr>
<tr>
<td>Std.</td>
<td>0.89</td>
<td>0.92</td>
<td>0.81</td>
<td>0.90</td>
<td>0.80</td>
<td>0.56</td>
<td>0.56</td>
<td>0.62</td>
<td>0.56</td>
<td>0.84</td>
</tr>
<tr>
<td>Skew</td>
<td>-0.38</td>
<td>0.84</td>
<td>0.59</td>
<td>0.98</td>
<td>-0.26</td>
<td>0.67</td>
<td>0.16</td>
<td>1.15</td>
<td>1.03</td>
<td>0.22</td>
</tr>
<tr>
<td>Kurt</td>
<td>5.65</td>
<td>4.20</td>
<td>6.17</td>
<td>6.98</td>
<td>3.31</td>
<td>4.23</td>
<td>3.79</td>
<td>4.85</td>
<td>4.41</td>
<td>5.29</td>
</tr>
<tr>
<td>Obs.</td>
<td>59</td>
<td>59</td>
<td>60</td>
<td>61</td>
<td>62</td>
<td>63</td>
<td>65</td>
<td>64</td>
<td>62</td>
<td>555</td>
</tr>
<tr>
<td>t-stat</td>
<td>2.21</td>
<td>2.25</td>
<td>2.75</td>
<td>0.69</td>
<td>-8.83</td>
<td>1.98</td>
<td>-5.67</td>
<td>3.90</td>
<td>4.26</td>
<td>0.97</td>
</tr>
</tbody>
</table>
Table 2.3: Persistence tests based on regression and ranking
Panel A reports the average coefficients from the cross-sectional Fama-MacBeth regression $z_{i,t} = a_t + b_t z_{it-1} + \epsilon_{i,t}$, and $\bar{a}_t$ and $\bar{b}_t$ are the time-average values of the estimated coefficients $\hat{a}_t$ and $\hat{b}_t$ from the Fama-MacBeth regressions. $R^2$ is the time average of the explanation power of the Fama-MacBeth regressions. Panel B reports the Spearman rank correlation coefficient over time, and a $t$-test on the average coefficients. $t$-statistics are within brackets.

<table>
<thead>
<tr>
<th>Year</th>
<th>'98-'99</th>
<th>'99-'00</th>
<th>'00-'01</th>
<th>'01-'02</th>
<th>'02-'03</th>
<th>'03-'04</th>
<th>'04-'05</th>
<th>'05-'06</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{t,t-1}$</td>
<td>-0.19</td>
<td>-0.28</td>
<td>-0.02</td>
<td>0.22</td>
<td>-0.29</td>
<td>0.15</td>
<td>0.15</td>
<td>0.44</td>
</tr>
<tr>
<td>$\bar{a}_t$</td>
<td>0.01 (0.05)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{b}_t$</td>
<td>0.06 (0.55)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.4: Persistence test based on pension plan portfolios
This table reports the $z$-score in each year of a portfolio formed on the previous year’s $z$-score. A sample of 57 plans with a complete set of $z$-scores are used. Panels A and B show 3- and 5-portfolio divisions respectively.

<table>
<thead>
<tr>
<th>Year</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Best past performer)</td>
<td>-0.08</td>
<td>0.18</td>
<td>0.09</td>
<td>-0.78</td>
<td>0.11</td>
<td>-0.30</td>
<td>0.60</td>
<td>0.80</td>
</tr>
<tr>
<td>2</td>
<td>0.57</td>
<td>0.21</td>
<td>0.05</td>
<td>-1.02</td>
<td>-0.04</td>
<td>-0.35</td>
<td>0.22</td>
<td>-0.01</td>
</tr>
<tr>
<td>3 (Worst past performer)</td>
<td>0.32</td>
<td>0.46</td>
<td>-0.09</td>
<td>-0.96</td>
<td>0.31</td>
<td>-0.40</td>
<td>0.14</td>
<td>0.09</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Best past performer)</td>
<td>-0.10</td>
<td>-0.03</td>
<td>-0.17</td>
<td>-0.63</td>
<td>0.16</td>
<td>-0.11</td>
<td>0.64</td>
<td>0.87</td>
</tr>
<tr>
<td>2</td>
<td>0.02</td>
<td>0.31</td>
<td>0.35</td>
<td>-1.01</td>
<td>-0.05</td>
<td>-0.52</td>
<td>0.43</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>0.59</td>
<td>0.48</td>
<td>0.05</td>
<td>-0.87</td>
<td>-0.03</td>
<td>-0.22</td>
<td>0.10</td>
<td>-0.07</td>
</tr>
<tr>
<td>4</td>
<td>0.23</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-1.01</td>
<td>0.11</td>
<td>-0.34</td>
<td>0.19</td>
<td>-0.01</td>
</tr>
<tr>
<td>5 (Worst past performer)</td>
<td>0.52</td>
<td>0.63</td>
<td>-0.09</td>
<td>-0.99</td>
<td>0.42</td>
<td>-0.53</td>
<td>0.25</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Table 2.5: Paired sample $t$-tests on pension plan portfolios

The table reports the paired sample $t$-test for the mean differences of $z$-scores of various portfolios from Table 2.4. Panels A and B show 3- and 5-portfolio division, respectively.

<table>
<thead>
<tr>
<th>Panel A:</th>
<th>Mean of paired difference</th>
<th>Std. Deviation</th>
<th>$t$-test</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top - Mid</td>
<td>0.12</td>
<td>0.41</td>
<td>0.85</td>
<td>7</td>
<td>0.42</td>
</tr>
<tr>
<td>Mid - Bottom</td>
<td>-0.03</td>
<td>0.20</td>
<td>-0.42</td>
<td>7</td>
<td>0.69</td>
</tr>
<tr>
<td>Top - Bottom</td>
<td>0.09</td>
<td>0.38</td>
<td>0.70</td>
<td>7</td>
<td>0.51</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B:</th>
<th>Mean of paired difference</th>
<th>Std. Deviation</th>
<th>$t$-test</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top1 - Top2</td>
<td>0.08</td>
<td>0.36</td>
<td>0.59</td>
<td>7</td>
<td>0.57</td>
</tr>
<tr>
<td>Top1 - Mid3</td>
<td>0.08</td>
<td>0.54</td>
<td>0.39</td>
<td>7</td>
<td>0.70</td>
</tr>
<tr>
<td>Top1 - Bottom4</td>
<td>0.19</td>
<td>0.38</td>
<td>1.41</td>
<td>7</td>
<td>0.20</td>
</tr>
<tr>
<td>Top1 - Bottom5</td>
<td>0.03</td>
<td>0.50</td>
<td>0.14</td>
<td>7</td>
<td>0.89</td>
</tr>
<tr>
<td>Top2 - Mid3</td>
<td>0.00</td>
<td>0.38</td>
<td>0.00</td>
<td>7</td>
<td>1.00</td>
</tr>
<tr>
<td>Top2 - Bottom4</td>
<td>0.12</td>
<td>0.29</td>
<td>1.14</td>
<td>7</td>
<td>0.29</td>
</tr>
<tr>
<td>Top2 - Bottom5</td>
<td>-0.05</td>
<td>0.35</td>
<td>-0.40</td>
<td>7</td>
<td>0.70</td>
</tr>
<tr>
<td>Mid3 - Bottom4</td>
<td>0.12</td>
<td>0.23</td>
<td>1.44</td>
<td>7</td>
<td>0.19</td>
</tr>
<tr>
<td>Mid3 - Bottom5</td>
<td>-0.05</td>
<td>0.25</td>
<td>-0.56</td>
<td>7</td>
<td>0.59</td>
</tr>
<tr>
<td>Bottom4 - Bottom5</td>
<td>-0.17</td>
<td>0.27</td>
<td>-1.76</td>
<td>7</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 2.6: Migration statistics

This table reports plan migrations among portfolios sorting on performance. Every year portfolio is formed into top, middle and bottom portfolio according to their $z$-scores in that year. The column shows the composition of the current portfolio that comes from the past top, mid or bottom portfolio respectively. In brackets are the $t$-statistics testing whether the percentage is equal to $1/3$. With a degrees of freedom equal to 7, critical values of 10%, 5%, 1% significance level are 1.42, 1.90, and 3 respectively. (*) indicates a significant level of 10%.

<table>
<thead>
<tr>
<th>Portfolio based on current performance</th>
<th>Top</th>
<th>Mid</th>
<th>Bottom</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio based on past performance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top</td>
<td>40% (0.95)</td>
<td>29% (-1.27)</td>
<td>31% (-0.43)</td>
</tr>
<tr>
<td>Mid</td>
<td>30% (-1.00)</td>
<td>29% (-1.04)</td>
<td>41% (2.01*)</td>
</tr>
<tr>
<td>Bottom</td>
<td>30% (-0.46)</td>
<td>42% (1.84*)</td>
<td>28% (-1.36)</td>
</tr>
<tr>
<td>Total</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 2.7: Decomposition of portfolio z-scores over time
Portfolios are formed on the z-scores in the same way as in Table 2.6. Each column decomposes the total z-score of the current portfolio for each year between 1999-2006 into the z-score contributed by the previous year’s portfolios. The portfolio z-score is averaged by the number of plans in the portfolio.

<table>
<thead>
<tr>
<th></th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top</td>
<td>Mid</td>
<td>Bottom</td>
<td>Top</td>
<td>Mid</td>
<td>Bottom</td>
<td>Top</td>
<td>Mid</td>
</tr>
<tr>
<td>Total</td>
<td>1.27</td>
<td>0.20</td>
<td>-0.65</td>
<td>1.06</td>
<td>0.28</td>
<td>-0.51</td>
<td>0.90</td>
<td>-0.12</td>
</tr>
<tr>
<td>Top</td>
<td>0.25</td>
<td>0.05</td>
<td>-0.39</td>
<td>0.21</td>
<td>0.11</td>
<td>-0.15</td>
<td>0.45</td>
<td>-0.03</td>
</tr>
<tr>
<td>Mid</td>
<td>0.39</td>
<td>0.13</td>
<td>-0.05</td>
<td>0.32</td>
<td>0.10</td>
<td>-0.21</td>
<td>0.34</td>
<td>-0.04</td>
</tr>
<tr>
<td>Bottom</td>
<td>0.63</td>
<td>0.02</td>
<td>-0.20</td>
<td>0.53</td>
<td>0.08</td>
<td>-0.14</td>
<td>0.12</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Table 2.8: Pension plan performance regressions and size
The dependent variable is the time-average z-score for each plan. The independent variable is the logarithm of a plan’s invested assets in 2006. (***) indicates a significant level of 1%.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-1.20</td>
<td>0.26</td>
<td>-4.59***</td>
</tr>
<tr>
<td>Log(assets)</td>
<td>0.14</td>
<td>0.03</td>
<td>4.76***</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj. R-squared</td>
<td>0.28</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.9: Average $z$-scores of size portfolios over time

Panel A and B are 3 and 5 equally weighted portfolios formed on the plan’s size in 2006. Panel C are 4 portfolios based on some specific size breakpoints, the number of plans are indicated in brackets. The tables reports the equally-weighted $z$-score for each portfolio over time. The sample includes 57 plans that have complete $z$-scores over the whole sample period 1998-2006.

### Panel A: 3 size (tertile) portfolios

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (largest plan)</td>
<td>0.31</td>
<td>0.31</td>
<td>0.27</td>
<td>0.09</td>
<td>-0.46</td>
<td>0.08</td>
<td>-0.20</td>
<td>0.64</td>
<td>0.51</td>
</tr>
<tr>
<td>2</td>
<td>0.48</td>
<td>0.09</td>
<td>0.48</td>
<td>-0.17</td>
<td>-0.83</td>
<td>0.19</td>
<td>-0.37</td>
<td>0.14</td>
<td>0.25</td>
</tr>
<tr>
<td>3 (smallest plan)</td>
<td>0.06</td>
<td>0.43</td>
<td>0.09</td>
<td>0.13</td>
<td>-1.41</td>
<td>0.11</td>
<td>-0.45</td>
<td>0.18</td>
<td>0.09</td>
</tr>
</tbody>
</table>

### Panel B: 5 size (quintile) portfolios

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (largest)</td>
<td>0.49</td>
<td>0.51</td>
<td>0.13</td>
<td>0.10</td>
<td>-0.42</td>
<td>0.19</td>
<td>-0.11</td>
<td>0.68</td>
<td>0.65</td>
</tr>
<tr>
<td>2</td>
<td>0.43</td>
<td>-0.04</td>
<td>0.22</td>
<td>0.02</td>
<td>-0.48</td>
<td>-0.07</td>
<td>-0.38</td>
<td>0.46</td>
<td>0.39</td>
</tr>
<tr>
<td>3</td>
<td>0.22</td>
<td>0.22</td>
<td>0.52</td>
<td>-0.22</td>
<td>-0.96</td>
<td>0.20</td>
<td>-0.36</td>
<td>0.08</td>
<td>0.21</td>
</tr>
<tr>
<td>4</td>
<td>0.15</td>
<td>0.09</td>
<td>0.55</td>
<td>0.17</td>
<td>-1.43</td>
<td>0.24</td>
<td>-0.62</td>
<td>0.10</td>
<td>-0.04</td>
</tr>
<tr>
<td>5 (smallest)</td>
<td>0.10</td>
<td>0.60</td>
<td>-0.01</td>
<td>-0.02</td>
<td>-1.31</td>
<td>0.08</td>
<td>-0.26</td>
<td>0.25</td>
<td>0.17</td>
</tr>
</tbody>
</table>

### Panel C: 4 size (quartile) portfolios

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;€10 bln (4)</td>
<td>0.31</td>
<td>1.45</td>
<td>0.57</td>
<td>-0.34</td>
<td>-0.18</td>
<td>0.49</td>
<td>0.29</td>
<td>0.92</td>
<td>1.28</td>
</tr>
<tr>
<td>€1-€10bln(18)</td>
<td>0.42</td>
<td>-0.03</td>
<td>0.08</td>
<td>0.22</td>
<td>-0.54</td>
<td>-0.02</td>
<td>-0.30</td>
<td>0.49</td>
<td>0.33</td>
</tr>
<tr>
<td>€0.1-€1bln (27)</td>
<td>0.17</td>
<td>0.28</td>
<td>0.43</td>
<td>-0.11</td>
<td>-1.10</td>
<td>0.16</td>
<td>-0.54</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>&lt;€0.1bln (8)</td>
<td>0.37</td>
<td>0.34</td>
<td>0.07</td>
<td>0.15</td>
<td>-1.44</td>
<td>0.17</td>
<td>-0.08</td>
<td>0.22</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 2.10: Paired sample $t$-tests and Wilcoxon signed ranks test on size portfolios

The table reports the paired sample $t$-test and the Wilcoxon signed ranks test for $z$-score difference between the top portfolio and the bottom portfolio in the respective 3 (tertile), 5 (quintile) and 4 (quartile)portfolio divisions. Portfolios are formed on size, which is measured by the investment amount in 2006. With a degrees of freedom equal to 8, critical $t$-values of 10%, 5%, 1% significance level are 1.40, 1.86, and 2.90, respectively. (**)(***) indicates a significant level at 5% and 1%. The last row reports the significance level from the Wilcoxon signed ranks test.

<table>
<thead>
<tr>
<th>Mean of paired difference</th>
<th>3-portfolio division</th>
<th>5-portfolio division</th>
<th>4-portfolio division</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean t-value</td>
<td>2.34**</td>
<td>3.00***</td>
<td>2.76**</td>
</tr>
<tr>
<td>Sig. (2-tailed) from t-test</td>
<td>0.05</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>Sig. (2-tailed) from Wilcoxon test</td>
<td>0.05</td>
<td>0.01</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Figure 2.1: Pension plan \textit{z-scores}: 1998 - 2006
This figure reports the box plots of \textit{z-scores} of all reporting industry-wide pension plans for each of the years in our sample period of 1998-2006. The boxes around the median line represent the interquartile range. The dotted lines extend to the most extreme data values within 1.5 times the interquartile range. ‘+’s denote further outlying observations.
Figure 2.2: Size histogram of 57 pension plans in 2006
This figure draws the histogram of 57 pension plans based on their invested assets in 2006.
Chapter 3

An analysis of the funding ratio in pension fund risk management*

3.1 Introduction

In the beginning of the 21st century, the pension fund industry has gone through a combination of decreasing asset returns and increasing liabilities. This caused substantial underfunding for DB plans and as a consequence regulatory measures were sharpened. As a pension fund’s objective is to meet (future) pension obligations, the funding ratio has become a crucial criterion to assess the financial health of a pension fund by regulators. The funding ratio is defined as the market value of assets divided by the market value of liabilities.\(^1\) The Financial Assessment Framework ("FTK") in the Netherlands discussed in Section 1.4, and the Minimum Funding Requirement (MFR) in the UK\(^2\), for example, impose solvency checks on the funding ratio.

The new accounting rule of the market valuation of liabilities introduced by International Financial Reporting Standards (IFRS) in 2005 subjects pension fund liabilities to financial market volatilities in addition to actuarial factors. This chapter will start with an examination of the projected funding ratio over various time horizons when both the value of assets and the value of liabilities are influenced by market volatilities. In this aspect, Leibowitz & Bader (1994) comes closest to our setup, except that he studies the distribution of the funding ratio only for the next single period.

The model that describes the market value of the assets and the market value of the liabilities becomes an essential choice in the analysis of the funding ratio. There have been numerous

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*This chapter is based on the article by Huang & Mahieu (2009a).

\(^1\)See Head, Adkins, Cairns, Corvesor, Cule, Exley, Johnson, Spain & Wise (2000) for the change from actuarial value to market value.

models proposed for various asset dynamics for long term investment. Vector autoregressive models (VAR) have become the current most popular model used by the academics and the industry because it is parsimonious and reasonably captures the dynamics among various assets. In its application, however, its basic specification is often used without an explicit consideration of its statistical fit to data, or its other variants are indicated to produce similar results. We therefore investigate the impact of an alternative VAR variety with a better statistical fit on the funding ratio development.

In addition to the regulatory attention, the understanding of the funding ratio is also important in making strategic investment decisions for a pension fund. Maximizing a funding ratio objective over a certain strategic asset mix is an important component of an Asset Liability Management (ALM) study. There have been several papers discussing the choice of an optimal asset portfolio of a DB pension fund to maximize utility defined over funding ratios. For example, Nijman & Swinkels (2008) analyze the optimal asset mix when commodities are considered. Hoevenaars et al. (2008) extend the analysis to more asset categories such as real estate and hedge funds. Binsbergen & Brandt (2007) analyze the optimal asset mix when constraints like short sales, value at risk and contribution limits are considered. We will add to this stream of literature by showing the impact of model choice on the optimal strategic asset allocation. This question is economically meaningful. Pension fund managers often follow the popular practice while ignoring its statistical justification. Our study of the model impact can help them gain extra insights on the model selection and be aware of the impact of their choices. Similarly, to pension supervisors they should be alert when judging the solvency of a pension plan under different model descriptions of the financial market.

Different models represent different assumptions on financial market volatility. We consider two models for their comparability and relevance. The first one (VAR1) is the basic Vector Autoregressive (VAR) model with one lag. We chose it as the benchmark model because it is often used for long term investment problems like in Campbell & Viceira (2002), Campbell & Viceira (2005), Binsbergen & Brandt (2007), and Hoevenaars et al. (2008). This model captures the dynamics between various assets and shows a mean reversion of stock returns. An alternative model is chosen among the group of vector autoregressive models for the purpose of comparability. A VAR2 model is selected for its best statistical fit with a minimum number of parameters. It is a

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5In addition to the investment policy optimization, an ALM study is also applied for the optimization of the pension financials as a whole including contribution and indexation policy. It is an important tool for integral risk management.
VAR model with a lag length of 2 where some of the statistically non-significant variables are dropped out of the model. We find the second lags are significant in explaining bond and liability returns, leading to a lower mean liability return, a lower volatility of bond returns and a lower correlation between stock returns and liability returns than what the VAR1 model describes.

In analyzing the distribution of the funding ratio over various horizons, we use a stylized pension fund that invests in a constant mix asset portfolio. The value of its assets in the next period is a function of the investment return of its asset portfolio, and the value of its liabilities in the next period is a function of the yield curve. We find that the VAR2 model predicts a higher future mean nominal funding ratio and a lower underfunding probability than what the VAR1 model predicts. This impact is robust for utilities defined over real funding ratios, and for different conditional information.

In analyzing the strategic asset allocation, we assume fund managers are mean-variance maximizers of funding ratios. Under the VAR2 model, we find that pension fund managers should allocate more in bonds and less in stocks when compared with the case of the VAR1 model. This impact is robust for utilities defined over real funding ratios, for different risk averse funds and for different conditional information.

The rest of this chapter is organized as follows. First, we introduce the construction of nominal and real funding ratios. Section 3.3 specifies the objectives of pension plans in order to derive the optimal asset allocation. Section 3.4 discusses models for return dynamics and motivate our use of the VAR1 and VAR2 model. Describes two models we use for return dynamics. The data, estimation and simulation procedures are presented in Section 3.5. In this section we also discuss the difference in the estimation results for the two models. In Section 3.6 we present the simulation results on both nominal and real funding ratios and discuss the model impact on funding ratios. Section 3.7 continues the discussion of the model impact on the optimal asset mix. Section 3.8 concludes.

### 3.2 Funding ratio determination

This section describes our construction of the nominal and real funding ratio. The value of a pension fund’s assets changes due to contributions from working participants and sponsors, benefits paid to retirees and investment returns. The investment return makes the funding ratio development subject to financial market volatility. In order to focus on the impact of financial market volatility on the funding ratio, we abstract from cash flows incurred by contributions and benefits by assuming that each year inflows from contributions equal outflows for benefit payments. In
this way the change of the asset value is only determined by the investment return. The value of the liabilities is the present value of the future pension obligations. The measurement of future obligations itself is a complicated issue in actuarial science. To focus on the variation in the liabilities due to financial market volatility, we assume that the fund is in a stationary state where the distribution of the age cohorts and their respective nominal pension rights are constant over time. Such an assumption allows us to proxy liabilities with a constant duration-matched bond. Similar assumptions can be also found in Binsbergen & Brandt (2007) and Hoevenaars et al. (2008).

Suppose at time $t$ the value of a fund’s assets is $A_t$, and the fund invests in two major asset categories: stocks and bonds with constant weights. After one period, the value of assets becomes $A_{t+1} = A_t(1 + R_{A,t+1})$. $R_{A,t+1}$ is the investment return of the asset portfolio. Based on the assumptions, the pension fund liabilities are proxied by a duration-matched bond portfolio. For period $t + 1$, the value of nominal liabilities becomes $L_{t+1} = L_t(1 + R_{L,t+1})$. $R_{L,t+1}$ is the liability return, which is assumed to equal the holding period return of a 15-year bond portfolio in period $t + 1$.

The nominal funding ratio is derived as $A_t/L_t$. The relationship between the two consecutive nominal funding ratios ($F_{t+1}$) is

$$F_{t+1} = F_t \frac{1 + R_{A,t+1}}{1 + R_{L,t+1}}$$

(3.1)

In log terms, $f_t = \ln(F_t)$, $r_t = \ln(1 + R_t)$, and the relationship becomes:

$$f_{t+1} = f_t + r_{F,t+1} = f_t + (r_{A,t+1} - r_{L,t+1})$$

where $r_{A,t}$ is the log return of the asset portfolio, $r_{L,t}$ is the log return of the liabilities, and $r_{F,t+1}$ is the funding ratio return, a term coined in Leibowitz & Bader (1994). Accordingly the long-term funding ratio is dependent on the initial funding ratio and the ratio of cumulative asset returns to cumulative liability returns.

If a pension fund aims to provide fully inflation-indexed pension benefits, namely real liabilities, ideally the present value of the real benefits should change by the return of an inflation-indexed and duration-matched bond. The data, however, for such a long-duration inflation-linked bond is scarce. We follow the method in Hoevenaars et al. (2008) and approximate this return

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by adding the inflation rate to the nominal return of the matching bond. Hence the real liability return is \((1 + R_{L,t+1})(1 + \pi_{t+1})\), where \(\pi_{t+1}\) is the inflation rate for period \(t + 1\). The relationship between the two consecutive real funding ratios \((RF_{t+k})\) is

\[
RF_{t+k} = RF_{t+k-1} \frac{(1 + R_{A,t+k})}{(1 + R_{L,t+k})(1 + \pi_{t+k})}
\]

The real funding ratio in multiple periods is then dependent on the initial real funding ratio and the ratio of asset portfolio returns to inflation-enhanced bond returns.

### 3.3 Optimal strategic asset allocation

A pension fund prefers a high and stable funding ratio so that the pension obligations can be made with a stable stream of contributions. To achieve this desired tradeoff between the level of funding ratio and the volatility at targeted time horizons, the investment policy is an important instrument. We follow the practice in Binsbergen & Brandt (2007) and Hoevenaars et al. (2008) that assumes CRRA preferences on the funding ratio for a certain time horizon. The objective of a pension fund is to maximize the utility defined over the future funding ratio at time \(t + k\), namely

\[
\max_{\alpha^{(k)}} \{E_t[F_{t+k}^{(1-\lambda)}]\}
\]

where \(\alpha^{(k)}\) is the strategic asset allocation that a pension fund should hold for the targeted \(k\)-period horizon. \(\lambda\) is the risk aversion coefficient of the pension fund. Assuming a lognormal distribution of the funding ratio returns, the maximization problem is equivalent to

\[
\max_{\alpha^{(k)}} \{E_t[r_{F,t+k}^{(k)}] + \frac{1}{2}(1 - \lambda)Var_t[r_{F,t+k}^{(k)}]\}
\]

where \(r_{F,t+k}^{(k)}\) is the log funding ratio return for a time horizon of \(k\) periods. The resulting optimal asset allocation for a \(k\)-period horizon \((\alpha^{(k)})\) is

\[
\alpha^{(k)} = (\Sigma_{AA} + (\lambda - 1)\Sigma_{AA}^{(k)})^{-1}(\mu^{(k)}_A + \frac{1}{2}\sigma^2_A + (\lambda - 1)\sigma_{AL}^{(k)})
\]

\(\Sigma_{AA}\) and \(\Sigma_{AA}^{(k)}\) are respectively the variance-covariance matrix for the single-period and the \(k\)-period log excess returns of the assets in the asset portfolio. \(\mu^{(k)}_A\) is the annualized expected return of the \(k\)-period log excess returns of the assets. \(\sigma^2_A\) is a vector of the variance of the log
excess returns of the assets. $\sigma_{AL}^{(k)}$ is a vector of the covariance of the log excess returns between the assets and the liabilities at a k-period horizon. All derivations can be found in the appendix.

This portfolio can be decomposed into a speculative and a hedging component. The speculative portfolio is

$$\alpha_S^{(k)} = (\Sigma_{AA} + (\lambda - 1)\Sigma_{AA}^{(k)})^{-1}(\mu_A^{(k)} + \frac{1}{2}\sigma_A^{2})$$ (3.3)

It shows that the asset that has a better risk return trade-off will get more allocation in the speculative portfolio. The hedging portfolio is

$$\alpha_H^{(k)} = (\Sigma_{AA} + (\lambda - 1)\Sigma_{AA}^{(k)})^{-1}(\lambda - 1)\sigma_{AL}^{(k)}$$ (3.4)

It says that the asset that has a higher correlation with liability returns will get more allocations in the hedging portfolio.

If the pension fund objective is defined over the real funding ratio, its optimal allocation for a k-period horizon ($\beta^{(k)}$) is

$$\beta^{(k)} = (\Sigma_{AA} + (\lambda - 1)\Sigma_{AA}^{(k)})^{-1}(\mu_A^{(k)} + \frac{1}{2}\sigma_A^{2} + (\lambda - 1)(\sigma_{AL}^{(k)} + \sigma_{A,tb}^{(k)} - \sigma_{A,rtb}^{(k)}))$$

The speculative component is

$$\beta_S^{(k)} = (\Sigma_{AA} + (\lambda - 1)\Sigma_{AA}^{(k)})^{-1}(\mu_A^{(k)} + \frac{1}{2}\sigma_A^{2})$$ (3.5)

and the hedging component is

$$\beta_H^{(k)} = (\Sigma_{AA} + (\lambda - 1)\Sigma_{AA}^{(k)})^{-1}(\lambda - 1)(\sigma_{AL}^{(k)} + \sigma_{A,tb}^{(k)} - \sigma_{A,rtb}^{(k)})$$ (3.6)

The speculative portfolio is the same for funds targeting nominal and for funds targeting real liabilities. However, the hedging portfolio differs if we compare Equations 3.4 and 3.6. When the funds targets real liabilities, their hedging portfolio will also allocate more to the asset that is also positively correlated with the nominal short rate and negatively correlated with the real short rate, in addition to its correlation with the liabilities.

### 3.4 Modeling asset and liability returns

There have been numerous proposals on modeling the stochastic processes of investment returns for pension plans. There are mainly four approaches as summarized in Hoevenaars, Molenaar & Steenkamp (2003). The first approach is derived from the efficient market hypothesis that
asset returns are unpredictable. Accordingly asset returns are typically described by a normal distribution with an assumed or estimated mean and variance. This approach is simple to implement but lacks correlation dynamics along the time dimension as the long-term correlation may differ from the short-term correlation. The second approach is called a cascade approach. It is rooted in a belief that inflation is the driving force of all returns. Hence any security return can be eventually described by inflation and its lagged values. This approach is represented by the "Wilkie model". This model is proposed by Wilkie (1987) and remains the dominant approach in the actuarial profession particularly in the UK. The Wilkie model, however, receives an enormous amount of criticism for its statistical fit to the data, as discussed in Kitts (1990), Geohegan, Clarkson, Feldman, Green, Kitts, Ross, Smith & Toutounchi (1992), Huber (1995), Wilkie (1995), Whitten & Thomos (1999), Hardy (2004), and Sahin, Cairns, Kleinow & Wilkie (2008). The third approach, proposed by Mulvey (1994), is similar to the second approach, but cast in continuous time. It uses stochastic differential equations to generate scenarios in continuous time. The fourth approach comes from developments in financial economics on the predictability of asset returns discussed recently in Cochrane (2008). The asset returns can be modeled by a VAR model, represented by the work from Campbell & Viceira (2002). A VAR model is a parsimonious system that captures the dynamics of many variables in one system without identifying the causality and exogeneity a priori. This dynamic feature makes it convenient for forecasting and generating scenarios for long-term financial planning like pension funds.

We apply the VAR model approach for return dynamics for its suitability for policy analysis and its popularity in academic research and practice. We describe two of its varieties to show the impact of model uncertainty. One is the fully-fledged first order VAR1 model, which is the model used in many ALM studies. The other one is a restricted version of the second order VAR model, which is obtained after performing statistical tests on the lag length and the parameters.

3.4.1 A fully-fledged first order VAR1 model

A Vector Auto Regressive (VAR) model, introduced by Sims (1980), has been used extensively for forecasting and policy analysis. In particular, the VAR1 with one lag length is often used to describe asset returns in an ALM study, like Hoevenaars et al. (2003), Kouwenberg & Zenios (2006), Boender, Dert, Heemskerk & Hoek (2007), Binsbergen & Brandt (2007), and Hoevenaars et al. (2008).

The stylized pension fund in our analysis has only bonds and stocks in its asset portfolio to reflect two general asset categories that represent two distinctive risk profiles. In the VAR model, following Campbell & Viceira (2005), we describe the excess stock and excess bond
returns by using their own lagged values and the lagged values of the other state variables such as real interest rates, nominal interest rates, dividend yields and yield spreads, which have been acknowledged as predicting variables.\(^7\) We also add another variable - the excess liability return, as to model all interested variables in one system and to capture their interaction. Specifically, the VAR model is written as

\[
\mathbf{z}_t = \mathbf{C}_1 + \mathbf{A}_1 \mathbf{z}_{t-1} + \mathbf{\varepsilon}_{1,t},
\]

where \(\mathbf{z}_t\) is a \(7 \times 1\) vector, including excess log stock returns, excess log bond returns, excess log liability returns, log real short rates, log nominal short rates, log dividend yields and log yield spreads. \(\mathbf{C}_1\) is a \(7 \times 1\) constant vector, \(\mathbf{A}_1\) is a \(7 \times 7\) coefficient matrix, and \(\mathbf{\varepsilon}_{1,t} \sim IIDN(\mathbf{0}, \Omega_1)\). \(\Omega_1\) is the matrix of contemporaneous variance and covariance of the error terms, which is assumed to be time-invariant.

### 3.4.2 An alternative model

The above VAR1 model is used extensively in ALM studies and is seldom questioned for its statistical fit. The vector autoregressive model with an alternative lag length is indicated to produce similar features for asset returns.\(^8\) To investigate this, we choose a vector autoregressive model that has the best statistical fit with the data and examine its impact on funding ratios and optimal asset mixes.

A critical element in the specification of VAR models is the determination of the lag length of the VAR. We use Akaike’s information criterion (AIC) as an explicit statistical criterion to select the lag length. The lowest AIC is produced by the VAR(2) with two lags of each variable.\(^9\) With 8 variables there are in total 112 coefficients to estimate. Too many explanatory variables could lead to multicollinearity and overfitting problems. Therefore we drop out the insignificant variables that do not help in achieving a better AIC. This is done by the following steps: (1) determine one explanatory variable which has the lowest t-statistic; (2) estimate the model after removing the variable selected in step (1), compare the AIC with the previous specification, and choose the model specification model with a lower AIC; (3) now from the model determined in step (2), repeat step (1) and (2) and stop until the AIC does not decrease any more. In the end

\(^{7}\text{To mention a few, Campbell (1987) points out the term structure of interest rate can predict stock and bond returns; Fama & French (1988) study the dividend yield in explaining stock returns; Campbell & Robert (1991) show yield spreads can forecast interest rates.}\)

\(^{8}\text{Campbell & Viceira (2005) use VAR(1) to show the mean reversion of stock returns over long horizons and its impact on asset allocations. They say ”...modifying the approach to include additional lags is straightforward”.}\)

\(^{9}\text{We limit our selection of the lag length within 1 up till 12. We obtain the same result when Hannan-Quinn information criterion is used.}\)
we get a VAR2 model with the lowest AIC, which means the best fit with the smallest number of free parameters. Specifically, the VAR2 model is written as

\[ z_t = C_2 + A_2 z_{t-1} + B z_{t-2} + \varepsilon_{2,t}, \]  

(3.8)

where \( C_2 \) is a \( 7 \times 1 \) constant vector. \( A_2 \) and \( B \) are both a \( 7 \times 7 \) coefficient matrix, but some of the entries are 0 due to the elimination of insignificant variables. \( \varepsilon_{2,t} \sim IIDN(0, \Omega_2) \). \( \Omega_2 \) is the covariance matrix of \( \varepsilon_{2,t} \).

3.5 Data, estimation and simulation methods

3.5.1 Data description

We use quarterly observations for all our variables. Due to the availability of the long term yield data, our sample goes back to 1953Q3 till 2007Q4.

The return on stocks (including dividends) and the dividend yield are for a value-weighted portfolio that includes all stocks traded on the NYSE, NASDAQ, and AMEX. The dividend yield is the ratio between the dividends and the prices. Data are obtained from the Center for Research and Security Prices (CRSP) of the University of Chicago.

The return on bonds is represented by the return of a 5-year T-bond. It is constructed from the yield of a 5-year constant maturity bond according to the loglinear relationship between the log return and the log yield described in Campbell, Lo & MacKinlay (1997).10

The liability return is represented by the return to a 15-year constant maturity T-bond. The liability return is constructed from the loglinear relationship between log return and log yield in the same way as the bond return is composed except that a 15-year T-bond yield is used and \( D_{nt} \) is fixed at 15.

The short term nominal interest rate is measured by the 3-Month T-Bill rate traded in the Secondary Market. The real interest rate is the ex-post real return on 90-day T-bills (i.e. the difference between the yield on T-bills and the inflation rate). Inflation rate is sampled at a quarterly frequency from Consumer Price Index-All Urban Consumers (CPI-U, seasonally adjusted, 1982:Q4=100) from the US Bureau of Labor Statistics.

Yield spread is the difference between the yield on a 15-year T-bond and the yield on a 3-month T-bill. The 15-year yield is the average of 10-year and 20-year yield. When 20-year yield

\[ \ln(1 + R_{n,t+1}) \approx D_{nt}\ln(1 + Y_{n,t}) - (D_{nt} - \frac{1}{2})\ln(1 + Y_{n-1,t+1}), \]  

where \( D_{nt} \) is Macaulay’s duration, calculated as \( (1 - (1 + Y_{n,t})^{-n})/(1 - (1 + Y_{n,t})^{-1}) \). \( Y_{n,t} \) is the annualized yield of a \( n \)-year constant maturity bond at time \( t \) and \( n \) is 5 here.
Data, estimation and simulation methods

is not available, 30-year yield is used to replace it.\textsuperscript{11} All yields and inflation data are directly available from the FRED database.\textsuperscript{12}

Table 3.1 summarizes the sample means and standard deviations for the simple excess returns. Stocks earn the highest excess return of 1.79 percentage points per quarter and also have the highest standard deviation. The quarterly stock return hits as low as -26.41 percentage points. Bonds earn a quarterly average return of 0.26 percentage points and the liabilities modeled by a 15-year bond show a higher return of 0.48 percentage points, and the second highest volatility. This reflects that bond investments in the asset portfolio are not sufficient to match the growth of liabilities. Dividend yields and yield spreads are the most stable and show the lowest standard deviations.

3.5.2 Estimation, derivation and simulation

We assume a constant asset mix of 60\% stocks and 40\% bonds for our pension fund portfolio in order to see the impact on the distribution of future funding ratios over various horizons when different models are used for forecasting.

The VAR models produce the estimates of coefficients and variance-covariance matrix of the error terms. Then we can derive the conditional means and variances of asset returns and liability returns for various horizons using the last observations of our sample.\textsuperscript{13} Accordingly we can also derive the optimal asset allocations.

Given the estimates of the return dynamics and assuming a multivariate normal distribution of the error terms with zero means and the estimated covariance matrices for the two models, we simulate the values for each return variable in the next quarter up till the next 30 years according to Equations (3.7) and (3.8).\textsuperscript{14}

Using simulated future returns for stocks, bonds, liabilities, real and nominal short rates, we can calculate the nominal and real funding ratio according to Equations (3.1) and (3.2) in the next quarter up till the next 30 years. In simulating nominal funding ratios, we assume the initial nominal funding ratio is 1. In simulating real funding ratios, we assume the initial real funding ratio is 1. We repeat the above return generation process and funding ratio calculation for 1000 times. Given 1000 possible future realization, we are able to compute the mean value of the funding ratio, and the probability of underfunding over different time horizons.

\textsuperscript{11}Essentially we are assuming the yield curve is flat between 20 year and 30 year for the period 1987Q1-1993Q3.
\textsuperscript{12}http://research.stlouisfed.org/fred2/
\textsuperscript{13}See derivations in the appendix.
\textsuperscript{14}The average duration of pension plan liabilities is 17 years (Hoevenaars et al. (2008)). The horizon for an ALM study is often longer than this and set at 20 to 30 years. Here we show the results for a horizon of 30 years.
The above procedures are implemented based on the last observation of our sample period, namely 2007Q4 observations. To see the sensitivity of our analysis to the starting values, we choose 2006Q4 observations as another set of starting values to re-derive the conditional means and variances of multi-period asset returns, and re-calculate the funding ratios and the optimal asset allocations. Table 3.6 shows that these two sets of starting values are very different values from each other.

3.5.3 Estimation results

In order to get conditional multi-period means and variances, we use the log excess returns of stocks, bonds and liabilities, the log rates of the other variables. The OLS estimates for the VAR1 model using quarterly observations between 1953Q3 and 2007Q4 are reported in Tables 3.2 and 3.3. In general they confirm the results in Campbell & Viceira (2005) in terms of significant variables and explanation power. The maximum eigenvalue of the coefficient matrix is 0.96, which is less than 1 and indicates our model is stationary.

The first column in Table 3.2 shows the estimated coefficients for the real short rate equation. The real rate is explained by its own lagged value and the lagged nominal rate, and only 28% of its variation is explained by the model. The second column corresponds to the equation for the excess stock return. It is positively related with the lagged dividend yield and negatively related with the lagged nominal rate. The third column corresponds to the equation for the excess bond return, which is explained by the lagged stock return and the lagged yield spread. The fourth, fifth and sixth column respectively correspond to the equations for the nominal short rate, dividend yield and yield spread. These three state variables are highly persistent and their own lagged values can explain the majority of the variations. The excess liability return is only significantly explained by the lagged yield spread, and the majority of its variation remains unexplained.

Table 3.3 reports the standard deviations and the cross correlations of the error terms in the VAR1 model. Unexpected excess stock returns are positively correlated with unexpected excess bond returns, but highly negatively with shocks to the dividend yield. Unexpected excess bond returns are positively correlated with shocks to the yield spread and highly negatively correlated with the nominal short rate. Shocks to expected excess liability returns are very highly correlated with shocks to the excess bond return, and also negatively correlated with shocks to the nominal short rate.

The estimates for the VAR2 model are reported in Table 3.4. Its overall fit is higher than the VAR1 model in that the adjusted $R^2$s for all equations in the VAR2 are not lower than the $R^2$s in the VAR1 model. In particular the explanatory powers in the VAR2 model for the variations in
the real rate, the excess bond return and the excess liability return have improved substantially, respectively from 28% to 40%, from 3% to 12% and from 2% to 13%. Such improvements are mainly due to the inclusion of the second lag of the real rate. In addition, the second lag of the excess stock return is statistically significant in explaining the excess bond and liability return. The second lag of dividend yields becomes also significant in many equations. If we look at the coefficients for its first and the second lag, they are similar in magnitude and opposite in signs. This says the change in dividend yields is significant in explaining the variations in other variables. For the state variables like nominal rates, dividend yields and yield spreads, the VAR2 model does not improve much from the VAR1 model. The standard errors of the estimates in the VAR2 model are in general smaller than the standard errors in the VAR1 model reflected by the higher t-ratios. This says the VAR2 model gives more efficient estimates than the VAR1.

Table 3.5 reports the standard deviations and the cross correlations of the error terms in the VAR2 model. Different from the VAR1 model, the standard deviations of unexpected returns are in general smaller for real rates, nominal rates, excess stock, excess bond and excess liability returns. The correlation between shocks to excess bond returns and shocks to real rates turns negative in the VAR2. Shock correlation between excess bond and excess stock returns, and between excess bond returns and dividend yield turns positive. The other correlations remain similar to the VAR1 model.

Both models show a mean reversion for excess stock returns, reflected by a lower standard deviation in the long run than in the short run. This long-term behavior of excess stock returns is induced by the positive predictability from the lagged dividend yield and the negative correlation between their shocks. The negative correlation between shocks says that a negative shock to dividend yields is accompanied by a positive contemporaneous shock to excess stock returns. At the same time, a negative shock in dividend yields predicts a lower expected future excess stock return due to the positive coefficient. Thereby high excess stock returns are followed by low expected future returns. The annualized standard deviation of excess stock returns declined from 16% for one-quarter horizon to about 9% for a 30-year horizon.

The difference in the estimates between the VAR1 and VAR2 model leads to a different horizon effect on the volatilities of stocks, bonds and liabilities, as shown in Figure 3.1. The VAR2 model forecasts lower standard deviations for the excess stock return, the excess bond return, and the excess liability return than the VAR1 for almost all horizons. This description of generally lower volatilities are due to the inclusion of the second lags in the VAR2 model.

Figure 3.2 displays the horizon effect on the shock correlations between excess stock returns, excess bond returns and excess nominal liability returns implied by the two models. All corre-
lations increase to a certain level and then decline with the time horizon. The VAR2 model in general forecasts lower correlations than the VAR1 model.

Based on an alternative set of starting values, we re-calculate the conditional means and variances of multi-period asset returns. We find the conditional variances and the correlation over different time horizons are insensitive to the starting values while the conditional means are sensitive, especially in the short run as shown in Figure 3.4.

3.6 Analysis of the funding ratio

Figure 3.5 shows the mean and underfunding probability of the nominal funding ratio over time assuming the initial nominal funding ratio is 100%. We observe an increasing mean funding ratio and a declining underfunding probability over time, regardless of the models for return dynamics. Recalling from the construction of the nominal funding ratio, we see the asset portfolio consists of stocks and 5-year bonds and the liability portfolio is represented by 15-year bonds. With the initial nominal funding ratio of 1, Equation (3.1) says that the state of overfunding or underfunding depends on the returns earned by the asset portfolio relative to that earned by the liabilities. Our simulation shows that the asset portfolio on average earns a higher return than the nominal liabilities. This is consistent with the common sense of the distinctive risk-return profile of stocks and bonds that stocks on average earn a higher return than bonds.

In comparison, the mean funding ratio is higher when the VAR2 model is used than when the VAR1 model is used. This is caused by the different conditional means forecasted by the two models. Figure 3.3 shows that the VAR2 model forecasts a considerably lower conditional mean for the liability return. With a higher mean funding ratio, the VAR2 model also predicts a lower underfunding probability.

Under the simulation based on an alternative set of starting values, the model impact on the funding ratio in the short run is reversed. This shows the funding ratio in the short run is sensitive to the choice of starting values as expected. Over the medium and long horizon however, the VAR2 model always predicts a higher mean funding ratio and a lower underfunding probability, irrespective of the starting values, as shown in Figure 3.6. This is mainly caused by the lower mean of the liability return in both models.

With regard to the real funding ratio when pension funds fully index their liabilities to the inflation, Equation (3.2) says that the real funding ratio depends on the portfolio return relative to the inflation-enhanced liability return. Assuming the initial real funding ratio is 100%, the simulation results in Figure 3.7 show that the mean real funding ratio in general decreases over
time, reflecting the difficulty in matching real liabilities. The corresponding underfunding probability of the real funding ratio increases with horizons. The model impact on the real funding ratio distribution is similar to the impact in the nominal case that the VAR2 model predicts a relatively higher mean real funding ratio due to the lower predicted liability return, but the impact is smaller because of a lower magnitude of the real funding ratio than the nominal funding ratio.

3.7 Analysis of the strategic asset allocation

3.7.1 Model impact on allocations targeting nominal liabilities

Figure 3.9 shows the optimal asset allocations in stocks, bonds and cash of a pension fund with a power utility function and a risk aversion coefficient of 5. In general, the optimal investment strategy under either models is to borrow cash to invest a significant percentage in bonds in a range of $[200\% \ 222\%]$ and some in stocks in a range of $[14\% \ 33\%]$ to hedge the liabilities which has a duration of 3 times the bond investment.

Under the market described by the VAR1 model, the fund decreases its bond allocation and increases its stock allocation for longer horizons, consistent with the results in Hoevenaars et al. (2008). This is because for longer horizons stocks become less risky and bonds become less correlated with liabilities for longer horizons. Under the VAR2 model, however, the optimal mix exhibits a different dynamics. The fund increases both its stock and bond allocation for longer horizons.

To see the driving factors behind the different dynamics of the optimal mix, Figure 3.10 decomposes the optimal mix into a speculative and a hedging component according to Equations 3.3 and 3.4. In the long run the bond investment for the speculative purpose in the VAR2 model is higher than in the VAR1 model because bonds under the VAR2 model show a lower volatility than under the VAR1 model, shown in Figure 3.1. The speculative stock allocations are similar under the two models because under the VAR2 model the lower volatility in stocks is also accompanied by a lower mean. Regarding the hedging portfolio, under the VAR2 model stocks show a much lower correlation with liabilities than under the VAR1 model, thus the VAR2 model prescribes less stock allocation. On the other hand, the bonds show similar correlation with liabilities across the two models. In comparison, bonds are more attractive relative to stocks in hedging liabilities under the VAR2 than the bonds under the VAR1, thus leading a higher bond allocation in the VAR2 model. Overall the VAR2 model prescribes a higher bond and a lower stock allocation than the VAR1 model.

Figures 3.11 and 3.12 show the optimal mix and its decomposition when alternative starting
values are used. The difference only lies in the very short run of the speculative portfolio and the total allocation. Yet the model impact on the allocation dynamics remains.

When the pension fund is more risk averse with an risk aversion coefficient of 20, the fund invests more in bonds and less in stocks than the less risk averse case. Shown in Figure 3.13, the model impact has the same pattern as in the less risk averse case.

### 3.7.2 Model impact on allocations targeting real liabilities

We also examine the model impact on the optimal asset mix when the fund aims to match the real liabilities, which is shown in Figure 3.15 for a pension fund with a risk aversion coefficient of 5. The impact is different from the nominal case because the inflation hedging ability also plays a role in determining the optimal asset mix. Both models prescribes a declining trend in the bond allocation with longer horizons. The VAR1 model prescribes a slightly increasing stock allocation while the VAR2 model prescribes a slightly decreasing stock allocation with horizons.

Figure 3.17 decomposes the optimal real asset allocation mix into a speculative and a hedging component according to Equations 3.5 and 3.6. The speculative component is the same across the nominal and the real case. The hedging portfolio in the real case needs to consider the covariance between the assets and the nominal rates and between the assets and the real rates. Equation 3.6 says that the higher covariance with nominal rates and the lower covariance with real rates, the higher allocation should be given to this asset. Figure 3.16 shows the covariances between assets and the short rates. The declining covariance between bonds and nominal rates dominates the declining trend between bonds and real rates, thus it leads to a declining allocation to bonds for longer horizons. The VAR1 model describes a steeper decline of the covariance than the VAR2 model, thus prescribes a lower bond allocation for the hedging purpose than the VAR2 model. Incorporating the bond allocation for the speculative purpose, both models prescribes a declining bond allocation with horizon, and the VAR1 model prescribes a lower bond allocation than the VAR2 model.

Regarding the stock allocation for the hedging purpose, similar to the bond allocation, the declining covariance between stocks and nominal rates dominates the declining covariance between stocks and real rate, and leads to a further lower stock allocation for the hedging purpose, when compared with the nominal case. Both models prescribe a declining stock allocation with horizons. Due to the poor hedging ability of the stocks to the nominal liabilities, the VAR2 model prescribes a relatively lower stock allocation than the VAR1 model, as in the nominal case.

When the risk aversion efficient increases to 20, shown in Figure 3.19, and when alternative conditional information is used to derive the risk profiles of the assets, shown in 3.18, the model
impact on the dynamics of optimal asset allocation targeting real liabilities remains robust.

## 3.8 Conclusions

In this paper we study the funding ratio dynamics over various time horizons by considering the market valuation of both the assets and the liabilities of a DB pension fund. When the asset portfolio is composed of 60% stocks and 40% bonds and the liabilities are proxied by a duration-matched bond return, the mean nominal funding ratio increases, and the underfunding probability decreases with time horizons. The real funding ratio, showing an opposite pattern, declines in its means and increases in its underfunding probability with time horizons, reflecting that pure investments in bonds and stocks are not sufficient in beating inflation-linked liabilities.

We put our second focus on the impact of model choices that differ in describing asset and liability returns. We consider a VAR1 model, often used by academics and practitioners, where all variables are explained by their own lagged values of order one and also the lagged values of the other variables. We consider a VAR2 model with a better statistical fit to the data. The VAR2 shows that the second lag of real rates and stock returns are significant in explaining bond and liability returns.

The inclusion of the second lags in the VAR2 model decreases the mean liability returns. This leads to higher means of both nominal and real funding ratios and lower underfunding probabilities in the VAR2 model than in the VAR1 model. This model impact in the short run is sensitive to the starting values of asset returns in our simulation, but very robust for the medium and long horizons irrespective of the starting values.

Pension funds are assumed mean-variance maximizers in funding ratios and their optimal asset allocations are also influenced by the choice of return models. When the nominal liabilities are targeted, both models prescribe an increasing stock allocation with horizon due to stocks’ mean reversion features. The bond allocation, on the other hand, shows a declining trend with horizon in the VAR1 model due to the declining correlation between bond and liability returns. In the VAR2 model, however, this decreasing allocation is dominated by a higher bond demand to hedging liabilities due to a even worse hedging ability of stocks. Thus the VAR2 model prescribes an increasing bond allocation with horizon. Relatively, the VAR2 model still prescribes a higher bond and a lower stock allocation than the VAR1 model in the long run. This model impact is robust for different risk-averse pension funds and for different conditional information.

When pension funds target real liabilities, the hedging ability of the assets to the real liabilities should be considered in the optimal allocation. Due to the declining covariance between
bonds and nominal rates over longer horizons, both models prescribe a declining bond allocation with horizon. Relatively, however, for the same reason in the nominal case, the VAR2 model prescribes a higher bond allocation than the VAR1 model. Due to the declining covariance between stocks and short rates, both models prescribe a decreasing stock allocation for long horizons. Relatively, however, the VAR2 model predicts a lower stock allocation than the VAR1 model due to its description of a worse nominal liability hedging ability of stocks in the long run. This model impact is robust for different risk averse pension funds and for different conditional information.

The investigation of the model choice shows that its impact on funding ratio distributions and optimal asset mixes is robust for various risk aversion, both nominal and real objectives and different conditional information. Pension fund managers should be aware of this non-negligible impact when drafting their investment plans. At a 30-year horizon, the difference in the bond allocation amounts to around 10%. For a pension industry with assets more than the size of the country’s GDP, this 10% difference could also have a significant impact on the capital market.
3.9 Appendix

3.9.1 Notation of the mean and variance of asset returns

The excess logarithmic return of the liabilities is $x_{L,t} = r_{L,t} - r_{tb,t}$. The excess logarithmic return of the assets is $x_{A,t} = r_{A,t} - r_{tb,t}$. They are collected in a vector $x_t$:

$$x_t = \begin{pmatrix} x_{A,t} \\ x_{L,t} \end{pmatrix}$$

From the model describing the asset returns, we can estimate the expected mean of the excess log returns for the next single period based on the information at time $t$, and they are denoted as

$$\mu_t = E_t[x_{t+1}] = \begin{pmatrix} \mu_{A,t} \\ \mu_{L,t} \end{pmatrix}$$

$$\Sigma_t = Var_t[x_{t+1}] = \begin{pmatrix} \Sigma_{AA} & \sigma_{AL} \\ \sigma_{AL}' & \sigma_L^2 \end{pmatrix}$$

And the annualized values of their expected mean ($E_t[x_{t+k}]$) and variance ($Var_t[x_{t+k}]$) for the next $k$-period excess logarithmic returns are denoted as

$$\mu_t^{(k)} = \frac{1}{k} E_t[x_{t+k}] = \begin{pmatrix} \mu_{A,t}^{(k)} \\ \mu_{L,t}^{(k)} \end{pmatrix}$$

$$\Sigma_t^{(k)} = \frac{1}{k} Var_t[x_{t+k}] = \begin{pmatrix} \Sigma_{AA}^{(k)} & \sigma_{AL}^{(k)} \\ \sigma_{AL}'^{(k)} & \sigma_L^{(k)2} \end{pmatrix}$$

$\Sigma_{AA}$ is the variance-covariance matrix of the excess log returns of stocks and bonds for the next single period. Its diagonal elements are collected in the vector $\sigma_A^2$. $\sigma_{AL}$ is a vector of the covariance between the excess log returns of various assets and the liabilities. $\sigma_L^2$ is the variance of the excess log liability return for the next period. Their annualized multiple-period equivalents are respectively $\Sigma_{AA}^{(k)}$, $\sigma_A^{(k)2}$, $\sigma_{AL}^{(k)}$, and $\sigma_L^{(k)2}$.

3.9.2 Derivation of nominal funding ratio returns and their moments

The asset portfolio invests in stocks and bonds with a allocation vector of $\alpha_t$ at time $t$, and in T-bills with $1 - \mathbf{1}'\alpha_t$. The log-linear approximation of the asset portfolio return (Campbell & Viceira (2001a)) is

$$r_{A,t+1} = r_{tb,t+1} + \alpha_t'(x_{A,t+1} + \frac{1}{2} \sigma_A^2) - \frac{1}{2} (\alpha_t \Sigma_{AA} \alpha_t)$$
\( r_{A,t+1} \) is the logarithmic return of the asset portfolio, \( r_{tb,t+1} \) is the logarithmic nominal short rate, used to calculate excess returns.

The excess log return of nominal liabilities is \( r_{L,t+1} = r_{tb,t+1} + x_{L,t+1} \).

The log nominal funding ratio return for the next period is

\[
 r_{F,t+1} = r_{A,t+1} - r_{L,t+1} = \alpha_t'(x_{A,t+1} + \frac{1}{2} \sigma_A^2) - \frac{1}{2} \alpha_t \Sigma_{AA} \alpha_t - x_{L,t+1}
\]

For a multiple period setting, suppose the pension fund rebalances its asset portfolio at the end of each year to its strategic asset allocation \( \alpha_t(k) \), which is defined at time \( t \) for a time horizon of \( k \) periods. The log nominal funding ratio return of the next \( k \) periods is

\[
 r_{F,t+k} = r_{A,t+k} - r_{L,t+k} = \alpha_t(k)'(x_{A,t+k} + \frac{k}{2} \sigma_A^2) - \frac{k}{2} \alpha_t(k) \Sigma_{AA} \alpha_t(k) - x_{L,t+k}
\]

The mean and the variance of the \( k \)-period nominal funding ratio return are

\[
 E_t[r_{F,t+k}] = k(\alpha_t(k)'(\mu_{A,t} + \frac{1}{2} \sigma_A^2) - \frac{1}{2} \alpha_t(k) \Sigma_{AA} \alpha_t(k) - \mu_{L,t}) \quad (3.9)
\]

\[
 Var_t[r_{F,t+k}] = k(\sigma_L^2(k) - 2\alpha_t(k)'\sigma_{AL} + \alpha_t(k)'\Sigma_{AA} \alpha_t(k)) \quad (3.10)
\]

### 3.9.3 Derivation of the optimal allocation targeting a nominal objective

Suppose pension funds are CRRA on the funding ratio, their objective is to

\[
 \max_{\alpha_t} \left\{ E_t \left[ \frac{F_{t+1}^{1-\lambda}}{1-\lambda} \right] \right\}
\]

subject to

\[
 F_{t+1} = (1 + R_{F,t+1})F_t
\]

Assuming \( 1 + R_F \) is lognormal,

\[
 logE_t[1 + R_F] = E_t[r_F + \frac{1}{2} \sigma_F^2]
\]

using which the maximization problem is equivalent to

\[
 \max E_t[r_F + \frac{1}{2}(1 - \lambda)\sigma_F^2]
\]
Similarly a k-period optimization problem becomes

$$\max E_t[r_{E,t+k}^{(k)}] + \frac{1}{2}(1 - \lambda)\Var_t[r_{E,t+k}^{(k)}]$$

For notational convenience, we use $\alpha$ for $\alpha_t^{(k)}$, and removes conditional time $t$ for all notations. With the mean and variance inputs in Equation (3.11) and (3.12), the problem becomes maximizing the following over $\alpha$:

$$\alpha' \left( \mu_A^{(k)} + \frac{1}{2} \sigma_A^2 \right) - \frac{1}{2} \alpha' \Sigma_{AA} \alpha - \frac{1}{2} (1 - \lambda) (\sigma_L^{(k)})^2 - 2 \alpha' \sigma_{AL}^{(k)} + \alpha' \Sigma_{AA}^{(k)} \alpha$$

Put the first derivative with respective to $\alpha$ to 0, we have

$$\left( \mu_A^{(k)} + \frac{1}{2} \sigma_A^2 \right) - \Sigma_{AA} \alpha - (1 - \lambda) (\Sigma_{AA} \alpha - \sigma_{AL}^{(k)}) = 0$$

Accordingly, the optimal asset allocation for a horizon of $k$ periods is

$$\alpha^{(k)} = (\Sigma_{AA} + (\lambda - 1) \Sigma_{AA}^{(k)})^{-1} \left( \mu_A^{(k)} + \frac{1}{2} \sigma_A^2 + (\lambda - 1) \sigma_{AL}^{(k)} \right)$$

This portfolio can be decomposed into a speculative portfolio

$$\alpha_s^{(k)} = (\Sigma_{AA} + (\lambda - 1) \Sigma_{AA}^{(k)})^{-1} \left( \mu_A^{(k)} + \frac{1}{2} \sigma_A^2 \right)$$

and a hedging portfolio

$$\alpha_h^{(k)} = (\Sigma_{AA} + (\lambda - 1) \Sigma_{AA}^{(k)})^{-1} (\lambda - 1) \sigma_{AL}^{(k)}$$

$$= (1 - \frac{1}{\lambda}) \left( \frac{1}{\lambda} \Sigma_{AA}^{(k)} + (1 - \frac{1}{\lambda}) \Sigma_{AA} \right)^{-1} \sigma_{AL}^{(k)}$$

When $\lambda$ goes to infinitive, the complete liability hedging portfolio is

$$\alpha_h^{(k)} = (\Sigma_{AA})^{-1} \sigma_{AL}^{(k)}$$

### 3.9.4 Derivation of real funding ratio returns and the optimal allocation

The excess log return of real liabilities is $r_{L,t+1} = r_{tb,t+1} + x_{L,t+1} + r_{tb,t+1} - r_{rtb,t+1}$, in which $r_{tb,t+1} + x_{L,t+1}$ is the return of nominal liabilities, and $r_{tb,t+1} - r_{rtb,t+1}$ is the inflation rate, the difference between the nominal T-bill rate and the real rate.
Following the same procedure in the previous section, the log real funding ratio return for the next $k$ periods is

$$r_{F,t+k}^{(k)} = \beta^{(k)}(x_{A,t+k}^{(k)} + \frac{k}{2}\sigma_{A}^{2}) - \frac{k}{2}\beta^{(k)}\Sigma_{AA}\beta^{(k)} - x_{L,t+k}^{(k)} + r_{tb,t+k}^{(k)} + r_{rtb,t+k}^{(k)}$$

where $\beta^{(k)}$ is the optimal allocation to maximize the real funding ratio for a time horizon $k$.

The mean and the variance of the $k$-period real funding ratio return are

$$E_{t}[r_{F,t+k}^{(k)}] = k(\beta^{(k)}(\mu_{A,t}^{(k)} + \frac{1}{2}\sigma_{A}^{2}) - \frac{1}{2}\beta^{(k)}\Sigma_{AA}\beta^{(k)} - \mu_{L,t}^{(k)} - \mu_{tb,t}^{(k)} + \mu_{rtb,t}^{(k)})$$

$$Var_{t}[r_{F,t+k}^{(k)}] = k(\beta^{(k)}\Sigma_{AA}\beta^{(k)} + \sigma_{L}^{2} + \sigma_{tb}^{2} + \sigma_{rtb}^{2}$$

$$-2\beta^{(k)}\sigma_{AL}^{(k)} - 2\beta^{(k)}\sigma_{A,tb}^{(k)} + 2\beta^{(k)}\sigma_{A,rtb}^{(k)}$$

$$+2\sigma_{L,tb}^{(k)} - 2\sigma_{L,rtb}^{(k)} - 2\sigma_{tb,rtb}^{(k)})$$

Bring the above to

$$\max_{\beta^{(k)}} E_{t}[r_{F,t+k}^{(k)}] + \frac{1}{2}(1 - \lambda)Var_{t}[r_{F,t+k}^{(k)}]$$

and equal the first derivative with respective to $\beta^{(k)}$ to 0, we obtain

$$\beta^{(k)} = (\Sigma_{AA} + (\lambda - 1)\Sigma_{AA}^{(k)})^{-1}(\mu_{A}^{(k)} + \frac{1}{2}\sigma_{A}^{2}) + (\lambda - 1)(\sigma_{AL}^{(k)} + \sigma_{A,tb}^{(k)} - \sigma_{A,rtb})$$

The speculative component is

$$\beta_{S}^{(k)} = (\Sigma_{AA} + (\lambda - 1)\Sigma_{AA}^{(k)})^{-1}(\mu_{A}^{(k)} + \frac{1}{2}\sigma_{A}^{2})$$

and the hedging portfolio is

$$\beta_{H}^{(k)} = (\Sigma_{AA} + (\lambda - 1)\Sigma_{AA}^{(k)})^{-1}(\lambda - 1)(\sigma_{AL}^{(k)} + \sigma_{A,tb}^{(k)} - \sigma_{A,rtb})$$

### 3.9.5 Derivation of conditional moments from VAR models for multiple periods

The single-period returns are described by

$$z_{t+1} = C_{1} + A_{1}z_{t} + \varepsilon_{1,t+1}$$

$$z_{t+2} = C_{1} + A_{1}z_{t+1} + \varepsilon_{1,t+2}$$

$$= C_{1} + A_{1}C_{1} + A_{1}\varepsilon_{1,t+1} + \varepsilon_{1,t+2}$$

$$...$$

$$z_{t+k} = C_{1} + A_{1}C_{1} + A_{1}^{2}C_{1} + ... + A_{1}^{k-1}C_{1} + A_{1}^{k}z_{t} + A_{1}^{k-1}\varepsilon_{1,t+1} + A_{1}^{k-2}\varepsilon_{1,t+2} + ... + A_{1}\varepsilon_{1,t+k-1} + \varepsilon_{1,t+k-1}$$
After forward recursion of the above, we can get the cumulative returns \((cz_{t+k})\) for multiple periods:

\[
(cz_{t+k}) = z_{t+1} + z_{t+2} + ... z_{t+k}
\]

\[
= [k + (k - 1)A_1 + (k - 2)A_1^2 + ... + A_1^{k-1}]C_1 + (A_1^k + A_1^{k-1} + ... + A_1)z_t \\
+ (1 + A_1 + ... + A_1^{k-1})\varepsilon_{1,t+1} + (1 + A_1 + ... + A_1^{k-2})\varepsilon_{1,t+2} + ... + (1 + A_1)\varepsilon_{1,t+k}
\]

Its conditional mean is

\[
E_t[cz_{t+k}] = z_{t+1} + z_{t+2} + ... z_{t+k}
\]

\[
= [k + (k - 1)A_1 + (k - 2)A_1^2 + ... + A_1^{k-1}]C_1 + (A_1^k + A_1^{k-1} + ... + A_1)z_t
\]

Its conditional variance is

\[
Var_t[cz_{t+k}] = \Omega + (1 + A_1)\Omega(1 + A_1)' + (1 + A_1 + A_1^2)\Omega(1 + A_1 + A_1^2)' + ... + (1 + A_1 + ... + A_1^{k-1})\Omega(1 + A_1 + ... + A_1^{k-1})'
\]

where \(\Omega\) is the estimated variance matrix of the error terms in the VAR model.

The VAR2 model \(z_{t+1} = C_2 + A_2 z_t + B z_{t-1} + \varepsilon_{2,t+1}\) can be written in the form of a VAR(1) model

\[
\begin{bmatrix}
  z_{t+1} \\
  z_t
\end{bmatrix} = \begin{bmatrix}
  C_2 \\
  0
\end{bmatrix} + \begin{bmatrix}
  A_2 & B^T
\end{bmatrix} \begin{bmatrix}
  z_t \\
  z_{t-1}
\end{bmatrix} + \begin{bmatrix}
  \varepsilon_{2,t+1} \\
  0
\end{bmatrix}
\]

Its conditional mean and variance for multiple periods can be derived in the same way as described above.
### 3.10 Tables and figures

Table 3.1: Sample descriptive statistics

Descriptive statistics for the entire sample of 218 quarterly observations between 1953Q3 and 2007Q4. All numbers are quarterly rates and in percentage. Variables are the real rate of the 3-month T-bill, excess returns of the CRSP stock, excess returns of the 10-year T-bond return, the nominal rate of the 3-month T-bill, dividend yields of the CRSP stock index, yield spreads between the 15-year T bond and 3-month T-bill, excess returns of pension fund liabilities.

<table>
<thead>
<tr>
<th></th>
<th>Real rates</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Nominal rates</th>
<th>Div. yields</th>
<th>Yield spr.</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.32</td>
<td>1.79</td>
<td>0.26</td>
<td>1.28</td>
<td>0.79</td>
<td>0.36</td>
<td>0.48</td>
</tr>
<tr>
<td>Median</td>
<td>0.35</td>
<td>2.93</td>
<td>-0.08</td>
<td>1.23</td>
<td>0.79</td>
<td>0.34</td>
<td>-0.43</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.61</td>
<td>23.07</td>
<td>13.03</td>
<td>3.76</td>
<td>1.44</td>
<td>1.05</td>
<td>49.61</td>
</tr>
<tr>
<td>Minimum</td>
<td>-1.58</td>
<td>-26.41</td>
<td>-8.77</td>
<td>0.18</td>
<td>0.28</td>
<td>-0.65</td>
<td>-18.50</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.63</td>
<td>8.21</td>
<td>2.61</td>
<td>0.70</td>
<td>0.27</td>
<td>0.32</td>
<td>7.31</td>
</tr>
</tbody>
</table>
Table 3.2: Coefficient estimates of the VAR1 model

The table reports full sample OLS parameter estimates of the VAR1 model. Regressors are the lagged value of the log real T-bill rate, excess log stock return, excess log bond return, log nominal T-bill rate, log dividend yield, log yield spread, and excess log liability return. The log nominal rate is used to calculate excess returns. Each column represents one equation. T ratios are in parentheses. The last row is the $R^2$ for the respective equations.

<table>
<thead>
<tr>
<th>Real rate (-1)</th>
<th>0.44</th>
<th>1.83</th>
<th>0.50</th>
<th>-0.03</th>
<th>-0.01</th>
<th>0.01</th>
<th>1.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6.82)</td>
<td>(1.88)</td>
<td>(1.61)</td>
<td>(-1.40)</td>
<td>(-1.90)</td>
<td>(0.61)</td>
<td>(1.49)</td>
<td></td>
</tr>
<tr>
<td>ex Stock (-1)</td>
<td>0.01</td>
<td>-0.01</td>
<td>-0.05</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.10</td>
</tr>
<tr>
<td>(1.41)</td>
<td>(-0.19)</td>
<td>(-2.45)</td>
<td>(3.21)</td>
<td>(0.00)</td>
<td>(-2.78)</td>
<td>(-1.64)</td>
<td></td>
</tr>
<tr>
<td>ex Bond (-1)</td>
<td>0.04</td>
<td>-0.10</td>
<td>-0.12</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>-0.06</td>
</tr>
<tr>
<td>(0.84)</td>
<td>(-0.15)</td>
<td>(-0.58)</td>
<td>(-0.02)</td>
<td>(-0.31)</td>
<td>(2.11)</td>
<td>(-0.10)</td>
<td></td>
</tr>
<tr>
<td>Nominal rate (-1)</td>
<td>0.20</td>
<td>-2.82</td>
<td>0.32</td>
<td>0.97</td>
<td>0.02</td>
<td>0.01</td>
<td>0.78</td>
</tr>
<tr>
<td>(2.69)</td>
<td>(-2.52)</td>
<td>(0.89)</td>
<td>(34.71)</td>
<td>(2.23)</td>
<td>(0.39)</td>
<td>(0.81)</td>
<td></td>
</tr>
<tr>
<td>Div. yield (-1)</td>
<td>-0.04</td>
<td>8.31</td>
<td>-0.11</td>
<td>0.01</td>
<td>0.93</td>
<td>-0.01</td>
<td>-0.22</td>
</tr>
<tr>
<td>(-0.27)</td>
<td>(3.69)</td>
<td>(-0.16)</td>
<td>(0.15)</td>
<td>(52.03)</td>
<td>(-0.29)</td>
<td>(-0.11)</td>
<td></td>
</tr>
<tr>
<td>Yield spr. (-1)</td>
<td>0.11</td>
<td>2.86</td>
<td>1.50</td>
<td>0.00</td>
<td>-0.02</td>
<td>0.92</td>
<td>4.89</td>
</tr>
<tr>
<td>(0.57)</td>
<td>(1.02)</td>
<td>(1.67)</td>
<td>(0.01)</td>
<td>(-0.73)</td>
<td>(26.79)</td>
<td>(2.01)</td>
<td></td>
</tr>
<tr>
<td>ex Liability(-1)</td>
<td>0.00</td>
<td>0.20</td>
<td>0.08</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.13</td>
</tr>
<tr>
<td>(-0.23)</td>
<td>(0.81)</td>
<td>(1.02)</td>
<td>(-0.52)</td>
<td>(-0.45)</td>
<td>(-1.16)</td>
<td>(0.62)</td>
<td></td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.28</td>
<td>0.10</td>
<td>0.03</td>
<td>0.92</td>
<td>0.94</td>
<td>0.84</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 3.3: Estimates of the standard deviations and correlations for the VAR1 error terms

The table reports the standard deviations and correlations for the error terms of the VAR1 model. Diagonal entries are standard deviations; off-diagonal entries are correlations.

| Real rate | 0.53 | 0.14 | 0.02 | 0.08 | -0.17 | -0.22 | 0.11 |
| ex Stock  | 7.95 | -0.02| -0.03| -0.91| 0.01  | 0.04  |     |
| ex Bond   | 2.55 | -0.74| -0.06| 0.32 | 0.04  | 0.04  |     |
| Nominal rate | 0.20 | 0.08 | -0.74| -0.61|     |     |     |
| Div. yield| 0.06 | -0.05| -0.09|     |     |     |     |
| Yield spr. | 0.10 | 0.12 |     |     |     |     |     |
| ex Liability | 6.89 |     |     |     |     |     |     |
Table 3.4: Coefficient estimates of the VAR2 model

This table reports the parameter estimates of the VAR2 model for real rates, excess stock returns, excess bond returns, nominal rates, dividend yields, yield spreads and excess liability returns. The VAR2 model is selected by choosing the best lag length of 2 based on AIC and dropping the insignificant explanatory variables based on AIC. Each column represents one equation. T statistics are reported in brackets. The last row reports the adjusted $R^2$ of the corresponding equation.

<table>
<thead>
<tr>
<th></th>
<th>Real rate ex Stock ex Bond Nom. rate Div. yield Yield spr. ex Liab.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real rate (-1)</td>
<td>0.29 2.40</td>
</tr>
<tr>
<td></td>
<td>(4.51) (2.31)</td>
</tr>
<tr>
<td>Real rate (-2)</td>
<td>0.36 -2.09 1.35 -0.09 0.03 3.68</td>
</tr>
<tr>
<td></td>
<td>(5.64) (-2.01) (4.84) (-3.83) (2.30) (4.92)</td>
</tr>
<tr>
<td>ex Stock (-1)</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(-2.86)</td>
</tr>
<tr>
<td>ex Stock (-2)</td>
<td>-0.06 0.01 -0.00 -0.18</td>
</tr>
<tr>
<td></td>
<td>(-2.95) (3.11) (-2.93) (-3.25)</td>
</tr>
<tr>
<td>ex Bond(-1)</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>(2.14)</td>
</tr>
<tr>
<td>ex Bond(-2)</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(-3.84)</td>
</tr>
<tr>
<td>Nominal rate (-1)</td>
<td>0.13 -2.60 0.99 0.01</td>
</tr>
<tr>
<td></td>
<td>(1.82) (-2.64) (36.94) (1.92)</td>
</tr>
<tr>
<td>Nominal rate(-2)</td>
<td></td>
</tr>
<tr>
<td>Div. yield (-1)</td>
<td>-4.82 7.63 6.98 -0.67 0.94 0.26 15.17</td>
</tr>
<tr>
<td></td>
<td>(-3.79) (3.33) (2.69) (-3.25) (36.94) (2.45) (2.19)</td>
</tr>
<tr>
<td>Div. yield (-2)</td>
<td>4.87 -6.80 0.67 -0.26 -14.53</td>
</tr>
<tr>
<td></td>
<td>(3.84) (-2.68) (3.36) (-2.54) (-2.14)</td>
</tr>
<tr>
<td>Yield spr. (-1)</td>
<td>0.18 0.00 0.94</td>
</tr>
<tr>
<td></td>
<td>(1.09) (-0.02) (32.80)</td>
</tr>
<tr>
<td>Yield spr. (-2)</td>
<td>1.87 -0.01 6.48</td>
</tr>
<tr>
<td></td>
<td>(2.57) (-0.06) (3.33)</td>
</tr>
<tr>
<td>ex Liability (-1)</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(-2.49)</td>
</tr>
<tr>
<td>ex Liability (-2)</td>
<td>-0.01 0.19</td>
</tr>
<tr>
<td></td>
<td>(-2.29) (2.31)</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.40 0.13 0.12 0.92 0.95 0.84 0.13</td>
</tr>
</tbody>
</table>
Table 3.5: Estimates of the standard deviations and correlations for the VAR2 error terms
The table reports the standard deviations and correlations for the error terms of the VAR2 model. Diagonal entries are standard deviations; off-diagonal entries are correlations.

<table>
<thead>
<tr>
<th></th>
<th>Real rate</th>
<th>ex Stock</th>
<th>ex Bond</th>
<th>Nominal rate</th>
<th>Div. yield</th>
<th>Yield spr.</th>
<th>ex Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.47</td>
<td>0.21</td>
<td>-0.05</td>
<td>0.15</td>
<td>-0.22</td>
<td>-0.28</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>7.71</td>
<td>0.00</td>
<td>-0.06</td>
<td>-0.90</td>
<td>0.01</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.41</td>
<td>-0.72</td>
<td>0.10</td>
<td>0.30</td>
<td>0.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.19</td>
<td>0.09</td>
<td>-0.72</td>
<td>-0.58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.06</td>
<td>-0.04</td>
<td>-0.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.44</td>
</tr>
</tbody>
</table>

Table 3.6: Two sets of starting values
The table displays two sets of starting values that we use to calculate the conditional means and variances of multi-period asset returns, the funding ratios and the optimal asset allocation. One set is the last two observations of our sample period, and the other set is the last two observations of a year ago.

<table>
<thead>
<tr>
<th>Time</th>
<th>Real rate</th>
<th>ex Stock</th>
<th>ex Bond</th>
<th>Nom. Rate</th>
<th>Div. yield</th>
<th>yield spr.</th>
<th>ex Liab.</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-09-2007</td>
<td>0.33</td>
<td>0.98</td>
<td>3.05</td>
<td>0.97</td>
<td>0.45</td>
<td>0.15</td>
<td>6.24</td>
</tr>
<tr>
<td>31-12-2007</td>
<td>-0.97</td>
<td>-3.65</td>
<td>5.52</td>
<td>0.69</td>
<td>0.47</td>
<td>0.30</td>
<td>9.42</td>
</tr>
<tr>
<td>An alternative set of starting values</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-09-2006</td>
<td>1.71</td>
<td>3.04</td>
<td>1.46</td>
<td>1.22</td>
<td>0.45</td>
<td>-0.01</td>
<td>4.73</td>
</tr>
<tr>
<td>31-12-2006</td>
<td>0.43</td>
<td>5.89</td>
<td>-0.34</td>
<td>1.24</td>
<td>0.44</td>
<td>-0.02</td>
<td>-0.34</td>
</tr>
</tbody>
</table>
Figure 3.1: Annualized standard deviation
The upper, middle and bottom graphs respectively show the estimated annualized standard deviation over different time horizons for the excess log stock return, the excess log bond return and the excess log liability return. The solid line represents the estimation from the VAR1 model and the dotted line represents the estimation from the VAR2 model.
Figure 3.2: Correlation over horizons

The upper, middle and bottom graphs respectively show the estimated correlation between the stock return and the liability return, between the bond return and the liability return, between the stock return and the bond return. The solid line represents the estimation from the VAR1 model and the dotted line represents the estimation from the VAR2 model.
Figure 3.3: Annualized mean excess log returns for various horizon

The upper, middle and bottom graphs respectively show the estimated annualized mean excess log returns over different time horizons for stocks, bonds and liabilities. The solid line represents the estimation from the VAR1 model and the dotted line represents the estimation from the VAR2 model.
Figure 3.4: Annualized mean excess log returns under alternative starting values
This figure shows the derivation results when the 2006Q4 and 2006Q3 observations in Table 3.6 are used as the starting values. The upper, middle and bottom graphs respectively show the estimated annualized mean excess log returns over different time horizons for stocks, bonds and liabilities. The solid line represents the estimation from the VAR1 model and the dotted line represents the estimation from the VAR2 model.
Figure 3.5: Nominal funding ratio dynamics over time
This figure shows the simulation results when the 2007Q4 and 2007Q3 observations in Table 3.6 are used as the starting values. The upper graph shows the mean nominal funding ratio over time. The lower graph shows the underfunding probability. The solid line represents the predictions from the VAR1 model. The dotted line represents the predictions from the VAR2 model. All results are based on the initial nominal funding ratio of 1 and an investment portfolio fixed at 60% stocks and 40% bonds.
Figure 3.6: Nominal funding ratio dynamics under alternative starting values
This figure shows the simulation results when the 2006Q4 and 2006Q3 observations in Table 3.6
are used as the starting values. The upper graph shows the mean nominal funding ratio over time.
The lower graph shows the underfunding probability. The solid line represents the predictions
from the VAR1 model. The dotted line represents the predictions from the VAR2 model. All
results are based on the initial nominal funding ratio of 1 and an investment portfolio fixed at
60% stocks and 40% bonds.
Figure 3.7: Real funding ratio dynamics over time
This figure shows the simulation results for real funding ratios using 2007Q4 and 2007Q3 observations in Table 3.6 as the starting values. The upper graph shows the mean real funding ratio over time. The lower graph shows the underfunding probability. The solid line represents the predictions from the VAR1 model. The dotted line represents the predictions from the VAR2 model. All results are based on the initial real funding ratio of 1 and an investment portfolio fixed at 60% stocks and 40% bonds.
Figure 3.8: Real funding ratio dynamics under alternative starting values
This figure shows the simulation results when the 2006Q4 and 2006Q3 observations in Table 3.6 are used as the starting values. The upper graph shows the mean real funding ratio over time. The lower graph shows the corresponding underfunding probability. The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2. All results are based on the initial real funding ratio of 1 and an investment portfolio fixed at 60% stocks and 40% bonds.
The upper graph shows the optimal bond allocation when the pension funds targets a nominal funding ratio when its risk aversion coefficient is 5. The lower graph shows the optimal stock allocation. The allocation to cash is $100\% - \text{bonds}\% - \text{stocks}\%$. The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2.

Figure 3.9: Optimal allocation for $\lambda = 5$ in nominal cases
Figure 3.10: Allocation decomposition for $\lambda = 5$ in nominal cases
The upper two graphs decompose the optimal bond allocation into the speculative component (upper left) and the hedging component (upper right) when the pension funds targets a nominal funding ratio when its risk aversion coefficient is 5. The lower two graphs decomposes the optimal stock allocation. The allocation to cash is $100\% - \text{bonds}\% - \text{stocks}\%$. The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2.
Figure 3.11: Optimal allocation for $\lambda = 5$ in nominal cases under alternative starting values. This figure shows the results when 2006Q4 and 2006Q3 observations in Table 3.6 are used as the starting values. The upper graph shows the optimal bond allocation when the pension funds targets a nominal funding ratio when its risk aversion coefficient is 5. The lower graph shows the optimal stock allocation. The allocation to cash is $100\% - \text{bonds}\% - \text{stocks}\%$. The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2.
Figure 3.12: Allocation decomposition in nominal cases under alternative starting values
This figure shows the results when 2006Q4 and 2006Q3 observations in Table 3.6 are used as the starting values. The upper two graphs decompose the optimal bond allocation into the speculative component (upper left) and the hedging component (upper right) when the pension funds targets a nominal funding ratio when its risk aversion coefficient is 5. The lower two graphs decomposes the optimal stock allocation. The allocation to cash is $100\% - \text{bonds}\% - \text{stocks}\%$. The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2.
Figure 3.13: Optimal allocation for $\lambda = 20$ in nominal cases

The upper graph shows the optimal bond allocation when the pension funds targets a nominal funding ratio when its risk aversion coefficient is 20. The lower graph shows the optimal stock allocation. The allocation to cash is $100\% - bonds\% - stocks\%$. The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2.
Figure 3.14: Allocation decomposition for $\lambda = 20$ in nominal cases
The upper two graphs decompose the optimal bond allocation into the speculative component (upper left) and the hedging component (upper right) when the pension funds targets a nominal funding ratio when its risk aversion coefficient is 20. The lower two graphs decomposes the optimal stock allocation. The allocation to cash is $100\% - \text{bonds}\% - \text{stocks}\%$. The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2.
Figure 3.15: Optimal allocation for $\lambda = 5$ in real cases
The upper graph shows the optimal bond allocation when the pension funds targets a nominal funding ratio when its risk aversion coefficient is 5. The lower graph shows the optimal stock allocation. The allocation to cash is $100\% - \text{bonds}\% - \text{stocks}\%$. The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2.
The upper two graphs show the covariance between bond returns and nominal rates (upper left), and real rates (upper right). The lower two graphs show the covariance between stock returns and nominal rates (lower left), and real rates (lower right). The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2.

Figure 3.16: Covariance between asset returns and short rates
Figure 3.17: Allocation decomposition for $\lambda = 5$ in real cases
The upper two graphs decompose the optimal bond allocation into the speculative component (upper left) and the hedging component (upper right) when the pension fund targets a nominal funding ratio when its risk aversion coefficient is 5. The lower two graphs decomposes the optimal stock allocation. The allocation to cash is $100\% - \text{bonds}\% - \text{stocks}\%$. The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2.
Figure 3.18: Optimal allocation for $\lambda = 5$ in real cases under alternative conditional information

This figure shows the results when 2006Q4 and 2006Q3 observations in Table 3.6 are used as the starting values. The upper graph shows the optimal bond allocation when the pension funds targets a nominal funding ratio when its risk aversion coefficient is 5. The lower graph shows the optimal stock allocation. The allocation to cash is $100\% - bonds\% - stocks\%$. The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2.
Figure 3.19: Optimal allocation for $\lambda = 20$ in real cases

The upper graph shows the optimal bond allocation when the pension funds targets a nominal funding ratio when its risk aversion coefficient is 20. The lower graph shows the optimal stock allocation. The allocation to cash is $100\% - bonds\% - stocks\%$. The solid line represents predictions from the VAR1 model. The dotted line represents predictions from the VAR2.
Chapter 4

Generational Pension Plan Designs*

4.1 Introduction

Pension fund providers in many countries have experienced some rough times in the last decade. The hazard of underfunding and volatility of their balance sheets during the beginning of 21st century has provoked the rethinking of occupational pension contract design. A burgeoning literature on pension design has developed over the recent years. They are mostly design proposals from a central planner’s perspective like Gollier (2008) and Beetsma, Bovenberg & Romp (2009) in order to produce the best risk sharing among generations. However, given the current collective pension system in many countries, like the Netherlands, UK and Belgium, it is more feasible and effective to implement a gradual reform at the industry level than to redesign the entire pension system. Therefore this paper aims to propose such a design, called a generational pension plan, that can greatly improve the welfare for the major stakeholders of the pension plans.

As its name suggests, a generational pension plan clearlydifferentiates pension provisions among generations. It contains multiple sub-funds, called generational funds, which each serve a particular generation. It differs from the traditional collective plan in that each generational fund has its own policies regarding investment, contribution and indexation. The contribution policy influences current consumption of a generation. The investment and indexation policy mainly adjust their future consumption. The combination of these policies can be customized to the preferences of a particular generation. Our generational plan differs from the individual pension plan in that it is still organized at an aggregate level, namely a generational level. This allows for economies of scale in investment and operation costs. In addition, the separation to different generations in a generational plan saves the negotiation costs that are often embedded in the collective plan in the event of an asset-and-liability mismatch. This setup also help avoiding

*This chapter is based on the article by Huang & Mahieu (2009b).
a change of the pre-agreed policies, which can cause unfair value transfers across generations as discussed in Hoevenaars & Ponds (2008).

The generational plan shares a similar spirit as the life-cycle funds thriving in the US (Viceira (2009)), where age prescribes investment policies for two reasons. Firstly, different ages imply different investment horizons and consequently different investment opportunities. Secondly, different ages imply different decompositions of total wealth into financial wealth and human capital. Especially, the correlation between human capital and returns on financial markets has an impact on the optimal investment policy. Compared with the life-cycle funds heavily promoted in the US now, the generational plan has two additional merits. First, it is industry-specific or company-specific, which allows for a better accommodation of risk preferences of its participants than the life-cycle plans. Second, besides the investment policy, generational plans can also adjust contribution and indexation policies to further improve participants’ welfare while life-cycle funds only exploit the flexibility in the investment policy.

The idea of a generational plan originated from Teulings & de Vries (2006) who suggest creating generational accounts in a collective plan. In their paper they focus on the theoretical optimal portfolio choices for the different generational accounts and calculate the welfare loss for the various deviations from the optimal choices. Our paper develops their concept of a generational account, and transfers it into a fully fledged generational pension plan, in which not only portfolio choice but also contribution and indexation policies can be employed to accommodate individual generations.

Cui, de Jong & Ponds (2009) evaluate different pension designs and conclude that a collective plan provides a higher welfare than an individual plan due to its ability to smooth consumption provided by intergenerational risk sharing. We extend their evaluation further to compare the collective plan with our generational plan. Moreover we model the collective plan in a more realistic way in two aspects. Firstly we allow for time-varying liabilities caused by the volatility of discount rate and the yearly indexation to reflect the current accounting practice of market valuation of the liabilities. Secondly, we use an existing pension plan from the practice in our collective modeling to reflect the current demographic characteristics, which are often ignored in other studies like Cui et al. (2009) and Gollier (2008).

When modeling the financial markets, we do not assume i.i.d for the return dynamics of risky assets, but take the long-term characteristics of pension fund investment into account. For this purpose, we apply Vector AutoRegression to capture mean reversion of the risky assets. This feature helps to explain that the generational plan can smooth consumption shocks by allowing for time diversification in its investment strategy.
Through numerical simulations, we examine the performance of our new generational plans in terms of the funding ratio, contribution rate, indexation rate, accrued rights, net present value and welfare. We compare its performance with the performance of the current conditional DB plans which are regarded as the most welfare-improving in Cui et al. (2009). Our main finding shows that our new design provides a higher net present value and a higher certainty equivalent consumption to participants. These results are driven by the nature of the generational plan setup that rules out ex ante value transfer across generations. We conclude that a generational plan is a very promising and sustainable design choice for future occupational pension arrangements.

This rest of this chapter is structured as follows. First, we introduce our new design and compare it with the collective plan in Section 4.2. Section 4.3 presents the models for the pension designs that we use in our numerical analysis. In this section the new generational design is also introduced in more detail. Section 4.4 presents our data and the setup of the simulation analysis. In Section 4.5 the results are presented, and some concluding remarks are presented in Section 4.6.

### 4.2 A generational pension plan

The proposed generational pension plan, by construction, avoids the problems caused by the nature of heterogenous participants in the collective plan.\(^1\) We make a reasonable assumption that people of the same generation in an industry show similar risk preferences and characteristics such as age, salary path and retirement schedule, which are very important to saving and investment decisions over their life cycle.

In the generational plan people of the same generation are pooled in one fund. As a result, there are multiple generational funds. For each generational fund, the preference of a particular generation can be satisfied by making generation-specific investment, contribution and indexation choices, which are almost impossible in a collective plan.\(^2\) A generational fund ceases after all its participants die. When a new generation starts, a new generational fund is created. A generational fund can adopt a DC scheme that it has a fixed contribution rate and a variable benefit payment. It can adopt a DB scheme that requires a variable contribution rate and pays a fixed benefit. In this case the fund will need to buy some benefit guarantee from other funds which we will discuss in the next chapter. The fund can also have a hybrid scheme that both contribution rate and benefits can be varied. Whatever scheme is chosen, it reflects the particular preferences

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\(^1\)Consult Section 1.4 for the challenges to the current collective plan.

\(^2\)Though Ponds & Van Riel (2007) proposed age-dependent indexation policy, such differentiation is only limited to indexation and at most to investment to some degree.
A generational pension plan of a generation.

4.2.1 Comparison with the collective DB plan

The starting points of the generational plan and the collective plan to a new entrant are different. A generational fund always starts with a zero asset position. After the first period when contributions flow in, the fund builds up assets and participants accrue pension rights. The assets of a generational fund grow with contribution collections and investment returns. The liabilities are the discounted present value of accrued rights. Such a setup causes a high starting funding ratio that alleviates the fund from funding ratio constraints from pension supervisors. The high starting funding ratio also provides a high buffer that enables risk taking and the ability to smooth shocks. A collective plan, on the other hand, is already in operation and holds a certain funding status. A participant joining a collective plan has to bear along with the funding situation at the time of entrance. Even when the funding ratio is positive, a new entrant could face an unfavorable collective plan that has a high percentage of retirees and makes conservative investments. This means that when a negative shock hits, young participants would probably be charged a high contribution rate so to pay the huge bulk of retirement benefits.

A collective plan has an important feature in intergenerational risk sharing (IRS). It is realized by adjusting contribution and indexation policies, which can dissolve the shocks over multiple generations. In a generational plan, contribution and indexation policies can also be adjusted but they are born by the same generation. Therefore risk sharing occurs within one generation.³

A collective plan lacks individualization in its saving and investment strategy, as pointed out by Bovenberg et al. (2007) and Steenkamp (2004). For example, a collective plan has only one single asset allocation geared to the average participant of the plan. This can be problematic for the participants who deviate from the representative participant. A generational plan, on the other hand, can customize its investment policy to the preference of its participants, such as adopting a life cycle investment strategy pursued by life-cycle funds (Viceira (2009)).

Aging is an acknowledged fact. It means that in the future there will be less young people than old people. This fact does not influence the generational plan, as less people mean less assets as well as less liabilities. But this has a considerable impact on a collective plan. It has a direct negative influence on the risk sharing capacity of the younger generation that contribution rate will be a less effective tool to manage mismatch risk. It also has an indirect influence on the investment policy. An increasing proportion of retirees in a pension plan will operate a

³By trading options with other generations, a generational fund can also share risks with other generations. This will be discussed in Chapter 5.
more conservative investment policy (Ponds & Van Riel (2007)). This in return means a lower investment return and consequently a higher contribution rate and a lower indexation for the working participants.

A collective plan is troubled by conflict of interests, especially when the mismatch between assets and liabilities occurs. For example, when an external shock occurs, both contribution and indexation rate can be used to manage the funding ratio. Contribution rate involves only the working participants, while the indexation rate matters more to the retirees than to the working participants as it is immediately realized in the benefit payment. Accordingly the degree of adjustment to contribution or indexation becomes a power play and can lead to time-consuming negotiation process between the working participants, the retirees and the sponsors. A generational plan, on the other hand, can avoid this by construction. Due to a lower degree of heterogeneity among its participants a generational plan incurs less negotiation costs and is in a better position to adjust to changes than a collective plan.

4.3 Design of collective and generational pension plans

Before we simulate the plan development under certain market conditions, we first introduce the modeling for the two plans and related assumptions. We take the prevailing conditional DB plan in the Netherlands as a prototype for the collective plan. The generational plan is created from scratch. In principal, participants in the generational plan can choose any policies they prefer. For the sake of comparison with the collective plan, we fix the contribution rate and investment policy over time in the plan design. Participants in the generational plan accrue pension rights in the same way as in the DB plan that pension rights are defined according to the salary, years of service and replacement rate, and the accrued rights are variable depending on the funding status. Because the generational plan is self financed, there is no guarantee to the accrued rights. The pension plan will pay the accrued benefits when its assets are sufficient, otherwise it will pay all it has and then pay nothing for future. This design is not the optimal design of the generational plan, but it is used to make the comparison with the collective plan to show that even this suboptimal design outperform the current collective plan.

4.3.1 General assumptions

We make the following assumptions in order to focus on the comparison between the two designs. Firstly, we abstract from the role of sponsors. Though in practice the sponsors is the major party to pay contributions, in this paper we make participants the only contributors. But the
impact of contribution policy on sponsors can be inferred similarly as that on participants.

Secondly, both plans are of a DB-DC hybrid scheme. They have a feature of DB average wage where pension rights are based on the life-time average salary and years of service, and no-negative indexation is allowed, which ensures a nominal guarantee of accrued rights. The DC feature refers that accrued rights vary with the financial status of the fund as indexation is conditional on the funding status. The generational plan will pay the accrued unless the fund runs out of assets.

Thirdly, we assume a stationary state only for the collective plan, where the distribution of the age cohorts and their respective nominal pension rights are constant over time. So the market value of liabilities only varies with the movement of the yield curve and the conditional indexation, and not with the demographics.

### 4.3.2 Modeling a collective conditional DB plan

In a collective conditional DB plan the contributions collected from all its participants are put in one monetary pool and invested as a unity in various assets. At the same time the plan pays the indexed benefits to the current retirees. To model the development of assets and liabilities, we need the initial values for assets \(A_0\) and liabilities \(L_0\), the premium base/pensionable salary \(P_{base}\) and the benefits payout \(B_{base}\). These initial values describe the starting status of the collective plan under study. In our later simulation, the starting values for these variables are taken from an existing pension fund in practice.

Each year the premium base increases with the salary growth rate \(s\), and the pension benefits payout increases with the cumulative indexation \(cumind_{ct}\). All of the cash flows are assumed to take place at the end of each year. Therefore the total assets of the collective plan change positively with the investment return \(r_{A,t}\) and the premium collection \(p_{ct}\): contribution rate), and negatively with the benefit payout:

\[
A_{ct}^e = A_{c,t-1}(1 + r_{A,t}) + p_{ct} \times P_{base}(1 + s)^t - cumind_{ct} \times B_{base}
\]  

This asset dynamics is similar to Cui et al. (2009) except that we are modeling in discrete time.

Liabilities are marked to market, defined as the discounted value of the future expected benefits payout. This is different from Cui et al. (2009) where the liabilities are time-invariant. The change of the liabilities comes from three sources: actuarial factors, inflation and yield curve.
factors, discussed in Bauer, Hoevenaars & Steenkamp (2005). The assumption of a stationary plan allows us to ignore the impact from actuarial factors. Each year the expected cash flow of benefit payout increases with the granted indexation \((ind_t^c)\) in that year. The influence from the yield curve is approximated by the return of a bond whose duration matches the duration of the plan liabilities \((m)\).\(^6\) We call this duration-matched bond return the liability return \((r_{L,m,t})\), reflecting the impact from the yield curve changes. Thereby, the value of liabilities \((L_t^c)\) each year increases with the liability return and the indexation as follows:

\[
L_t^c = L_{t-1}^c (1 + r_{L,m,t})(1 + ind_t^c)
\] (4.2)

At the end of each year, the funding ratio \((A_t^c/L_t^c)\) is computed to determine the contribution rate \((p_{t+1}^c)\) and indexation rate \((ind_{t+1}^c)\) for the coming year according to the policy ladder\(^7\).

### 4.3.3 Modeling a generational plan

A generational plan can have multiple generational sub-plans, and each sub-plan serves the people of one generation with similar characteristics such as age, retirement date, salary path, and risk preference. We only model one generation, and they start working at time \(T_0\), retire at time \(T_r\), and die at time \(T_d\).

During the working years the plan collects contributions and makes investments. The change of the assets comes from two sources, premium collections \((p_t^g)\) and investment returns \((r_{A,t})\).

\[
A_t^g = A_{t-1}^g (1 + r_{A,t}) + p_t^g S_0 (1 + s)^t
\] (4.3)

where \(S_0\) is the initial salary level of the generation.

During the working years the change of liabilities comes from the change in accrued rights and the yield curve. In year 1 the accrued rights are \(AR_1 = NAR(1 + ind_1)\). From year 2 onwards until retirement the accrued rights are given by \(AR_{t+1} = (AR_t + NAR)(1 + ind_{t+1})\). \(NAR\) is the newly accrued rights for one year of service. It is defined as the product of lifetime average salary and the replacement rate divided by the total working years. \(AR_t\) refers to the nominal amount of money a participant will receive each year after retirement if he leaves the plan after working for \(t - T_0\) years. Discounting these expected annual benefits with the

\(^6\)By this approximation, only small parallel shift in the yield curve is captured. This could be a limitation of the modeling of liabilities.

\(^7\)A policy ladder is a set of rules specifying how indexation and contribution rate are dependent on the funding ratio. More details can be found in Section 4.4.3.
corresponding interest rates, we can compute the value of the liabilities.

\[ L_t^g = \sum_{n=0}^{T_d - T_r - 1} \frac{AR_t}{(1 + y_{T_r - t + n, t})^{T_r - t + n}} \]  

(4.4)

where \( y_{T_r - t + n, t} \) is the yield of maturity \( T_r - t + n \) at time \( t \) and \( T_d - T_r - 1 \) is the number of years in which participants receive pension benefits.

As of retirement \((T_r)\), while continuing its investments the plan starts to pay out benefits as defined in \( AR_t \). The assets on the one hand increase with the investment returns, while on the other hand decrease with the benefits payout. There are two cases here. When the level of assets is sufficient to pay the accrued rights, it will follow

\[ A_t^g = A_{t-1}^g (1 + r_{A,t}) - AR_t \]  

(4.5)

As no new rights are accrued as of retirement, \( NAR = 0 \) and the total accrued rights only increase with indexation \( AR_{t+1} = AR_t (1 + ind_{t+1}) \). The value of the plan liabilities is the discounted value of the benefit payments with a decreasing number of years.

\[ L_t^g = \sum_{n=0}^{T_d - t - 1} \frac{AR_t}{(1 + y_{n, t})^n} \]  

(4.6)

When the level of assets is insufficient to pay the accrued rights, the fund will pay all it has till that moment, namely \( A_{t-1}^g (1 + r_{A,t}) \), and the assets for the next period is

\[ A_t^g = 0 \]  

(4.7)

The liabilities also become 0, and there will be no indexation and no accrued rights for the rest of the years.

The generational fund will end in either a zero or a surplus position. The surplus will be left behind to the parent plan, and very probably will be transferred to other generational funds. In the next chapter we will discuss how to deal with this surplus. In the current comparison between the two plan designs, we do not account for this surplus, which will definitely add value to a generational plan design.

### 4.3.4 Indicators of fund performance

A good plan caters for the interests of major stakeholders like the participants and the sponsors. Following the industry practice, we present six aspects to describe the fund performance. The
following indicators are also used in academic papers like Bauer et al. (2005), Ponds & Van Riel (2007) and Hoevenaars & Ponds (2008).

a. Funding ratio
The funding ratio is defined as the value of assets divided by the value of liabilities. It is the first important parameter that plan trustees care about, as it reflects the overall financial health of a plan, over which solvency requirement is imposed. The funding ratio is also important to participants, as it decides how much contribution rate should be paid and how much indexation should be granted for the coming year. Underfunding probability measures the expected probability that the fund has less assets than its liabilities over a given period, and it is an important indicator monitored by pension supervisors. Mismatch risk measures how volatile the funding ratio over time. It is calculated as the standard deviation of the funding ratio over the lifetime of a generation.

b. Contribution rate
The contribution rate measures the percentage of their pensionable salaries that participants pay into the pension plan. In a collective plan, the contributions collected from the working participants are used to pay benefits to the retirees and the rest are used for investments. In a generational plan, the contributions is solely used for investments. Upon retirement, the contribution rate in the generational plan becomes 0 and the plan counts on the accumulated assets and the ongoing investments to pay retirement benefits. The contribution rate can be contingent on funding ratio. In this paper we fix the contribution rate in both plans in order not to have too many free parameters.

c. Indexation ratio
Each year contingent on the funding ratio, indexation is granted to protect the accrued rights from inflation. An indexation ratio of 100% means the current accrued rights are fully indexed to the price inflation in that year.

d. Accrued rights
Accrued rights represent the amount of money that a participant will expect to receive annually as of retirement if he decides to leave the plan at a certain moment. Accrued rights over time are determined by the sum of previously accrued rights and the newly accrued rights, multiplied by the indexation rate, thereby reflecting the cumulative effect of all the previously granted indexation.

e. Net present value
This is a composite indicator that a newly entering participant should consider when choosing a plan. Participants pay premiums at a rate of $p_t$ into the plan during the working period and
receive benefits as defined in $AR_t$ from the plan as of retirement. The net present value from participating a pension plan is computed as the discounted cash inflow of benefits minus the discounted cash outflow of contributions. In our example, the net present value is computed as the sum of $T_d - T_r - 1$ discounted benefits receipt minus the sum of $T_r - T_0$ discounted premium payments. We use the historical nominal short rate as the discount rate. In computing the net present value a generational plan offers, we do not include the terminal assets left in the plan.

f. Welfare analysis
To take the risk attitudes of the participants into account we apply a welfare analysis. We use a CRRA power utility function as in Cui et al. (2009), and the welfare $U$ of joining a plan is computed as

$$U = \sum_{0}^{T_d-T_0} e^{-\delta t} \frac{W_t^{1-\lambda} - 1}{1-\lambda}$$

where $\delta$ is the parameter for time preference, $\lambda$ is the risk aversion coefficient, and $W_t$ is the wealth for consumption, calculated as

$$W_t = \begin{cases} S_0(1 + s)^t (1 - p_{t+T_0}), & \text{for } 0 < t \leq T_r - T_0 \\ AR_{t+T_0}, & \text{for } T_r - T_0 < t \leq T_d - T_0 \end{cases}$$

For the ease of interpretation, we translate this welfare into the certainty equivalent consumption ($CEC$), derived from the following equation:

$$\sum_{0}^{T_d-T_0} e^{-\delta t} \frac{CEC^{1-\lambda} - 1}{1-\lambda} = E(U)$$

### 4.4 Data, estimation and simulation

To simulate the assets and liabilities of a pension plan, the exogenous financial markets are described by the following asset return dynamics.

#### 4.4.1 Asset returns
It is assumed that pension funds will only invest in stocks and bonds. In order to capture the long term features of returns, we model stock and bond returns by a one lag Vector Autoregressive (VAR(1)) as in Campbell & Viceira (2005), where the stock and bond returns are described by their own lagged values and the lagged values of real interest rates, nominal interest rates, dividend yields and yield spreads as in Equation (4.8).

$$z_{t+1} = A + B z_t + \varepsilon_{t+1}$$

(4.8)
where $z_t$ is a vector of stock returns, bond returns, the real interest rate, the nominal interest rate, the dividend yield and the yield spread. $A$ is a $6 \times 1$ constant vector, $B$ is a $6 \times 6$ coefficient matrix for $z_t$, and $\varepsilon_{t+1} \sim N(0, \Omega)$. In our robustness analysis, we also examine the scenario when the error term follows a student’s t-distribution.

In order for the sample period to be non-selective, we use the sample period as long as possibly available to estimate return dynamics. We use the data provided by Robert Shiller.\footnote{Available on his homepage "http://www.econ.yale.edu/shiller/data.htm". Campbell & Viceira (2002) use the same dataset.} The data covers the period between year 1872 and 2007, including yearly observations on the $S&\ P$ stock index, the one-year interest rate computed from the US 6-month commercial paper rate, the 10-year US T bond yield and the US consumption price index.

The descriptive statistics, the coefficient estimates and the correlation matrix are shown in Tables 4.1 and 4.2. As recognized in many studies, our estimates also show that the lag dividend yield positively forecasts (0.07) stock returns but shocks to dividend yields and shocks to stocks are negatively related (−0.1). Accordingly, stock returns have a declining volatility over longer horizon.

### 4.4.2 The yield curve

The yield curve data is constructed from the monthly euro swap rate sampled during the period between Jan 1999 and June 2007 for maturities running from 1 year up till 10 year, and 12-, 15-, 20-, 25-, 30-year.\footnote{We use euro swap rate because it is the rate required by Dutch Central Bank for Dutch pension funds to use for the discount rate of their liabilities.} The data is obtained from DataStream. We use a dynamic Nielson-Siegel with AR(1) factor model proposed by Diebold & Li (2006) to model the yield curve. We use this model rather than the affine equilibrium model because this model fits yield curve data well. In addition, it is popular in practice as many central banks use it.

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1 - \exp(-\lambda \tau)}{\lambda \tau}\right) + \beta_{3t}\left(\frac{1 - \exp(-\lambda \tau)}{\lambda \tau} - \exp(-\lambda \tau)\right)$$  \hspace{1cm} (4.9)

where $y_t(\tau)$ refers to the interest rate for a maturity of $\tau$-years. Factors $\beta_{1t}$, $\beta_{2t}$ and $\beta_{3t}$ respectively govern the level, slope and curvature of the yield curve. A graph of loadings on $\beta_{3t}$ against maturity shows that this loading reaches its maximum at mid-term maturity and it is regarded as a mid-term factor. In accordance with the common practice, we pick 30 months for $\tau$, and choose a $\lambda$ that maximizes the loading on $\beta_{3t}$. So the value for $\lambda$ is 0.0747. In other words, at this value the maximum of the loading on $\beta_{3t}$ occurs at 30 months. Every year, we regress
yields of different maturities (namely, 1-10, 12, 15, 20, 25 and 30 years) on factor loadings of 
$\left(1 - \exp\left(-\frac{\lambda t}{\tau}\right)\right)$, and $\frac{1 - \exp(-\lambda \tau)}{\lambda \tau} - \exp(-\lambda \tau)$, we get OLS estimates for $\beta_{1t}$, $\beta_{2t}$ and $\beta_{3t}$. With the times series of the three $\beta$s, and using the following AR process we estimate the coefficients to describe the $\beta$s.

$$\beta_{i,t+1} = c_i + \gamma_i \beta_{i,t} + \epsilon_{i,t+1}, i = 1, 2, 3$$ (4.10)

Using the estimates of coefficients and variance of the residuals of Equation (4.10), we can forecast future $\beta$s, then plug these forecasts in Equation (4.9) to generate the future yield curve.

### 4.4.3 Numerical simulation

We use a numerical simulation of two pension plans as it is often done in Asset Liability Management (ALM) studies, see also Hoevenaars & Ponds (2008).

We make some assumptions of a new participant on his entering age, retirement age, death age, salary path and replacement rate. This new entrant can choose to enter a current existing collective plan or a generational plan. The initial status of a collective plan is taken from a real-life pension plan. A generational plan starts from scratch, created by the new entering generation. The two plans collect contributions from their participants and make investments only in stocks and bonds. The fixed contribution rate is determined with the real interest rate as the discount rate, aiming to save for the inflation-protected retirement benefits. Indexation policy is specified in the following policy ladder. The investment policy is fixed at 56% stocks and 44% bonds, which is the aggregate asset allocation of the Dutch pension funds at the end of year 2006 according to the Dutch Central Bank’s statistics. Due to the time cost of generating a yield curve, we make 500 simulation in our comparison.

### General plan characteristics

In the collective plan we assume a stationary liability structure and the impact of yield curve changes on the liabilities is approximated by a 15-year bond return, which is inferred from the simulated 15-year yield in our paper. The initial values of the assets, of the liabilities, the premium base and the benefit payment are assumed to be 200, 160, 32.5 and 6.1 billion respectively to represent a current collective plan. It essentially means the starting funding ratio is 125%.

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10Numerical simulation is often used in ALM studies because the pension plan involves multiple objectives and quite a few independent time-varying variables, over which the optimization is notoriously complicated and even not possible. In addition the simulation result allows for a straightforward representation of pension dynamics.

11Values are taken from a major Dutch pension fund ABP results in 2006 as an example. (See their 2006 annual report). However, note that absolute values do not matter and only relative proportions matter here.
Then the assets and liabilities develop as described in Section 4.3.2.

In the generational plan, the development of only one generational fund is simulated. The initial funding ratio is set to 0. The assets and liabilities develop as described in Section 4.3.3.

Policy ladders

Policy ladders specify how the contribution rate and indexation rate are adjusted according to the funding ratio. In general, the contribution rate consists of two components: the base rate and an adjustment. The base rate is determined in an actuarially fair way such that the sum of contribution payment equals to the sum of nominal benefits given a certain replacement rate. Thereby the discount rate, the salary growth rate and the replacement rate determine the value of the actuarially fair base contribution rate. We take the historical mean real rate (2.77%) as the discount rate as the plan aims for full indexation. The replacement rate is set at 70%. Then a 0% salary growth prescribes a 11.22% base contribution rate. In our comparison we fix the contribution rate and make it the same in both plans, so we remove the adjustment element.

The indexation policy is also made the same in both plans. The indexation ratio ($\alpha$) is contingent on the funding ratio. When the funding ratio is lower than 80%, the indexation ratio is -0.2. When the funding ratio is 100%, the indexation ratio is 0. When the funding ratio is 130%, the indexation ratio is 1. When the funding ratio is 150% or above, the indexation ratio is 2. A linear rule applies when the funding ratio is in between. The indexation rate is then calculated as $\alpha \times \max(\text{inflation}, 0)$. A graphic presentation of the indexation rate can be found in Figure 5.1.

Participant characteristics

The two plans are compared from a new entrant’s perspective. This participant starts working at the age of 25, retires at 65, and earns a flat annual salary of €30,000. At the end of each year he pays contributions to a plan, either a collective or a generational one, at a rate of $p$ out of his salary to build his pension rights $AR_t$. For every year of service, he accrues additional new rights (NAR)\(^{12}\). Together with the previously accrued rights, the total accrued rights increase with yearly granted indexation. Due to the conditional indexation the pension benefits continue to change each year after retirement. In sum, he makes the first contribution at age of 26, for 40 year till age 65. Then he starts to receive benefits at age 66, for 14 times till age 79, and dies at age 80. In the generational plan the benefits specified in the accrued rights will only be made when the assets are sufficient. In other cases the whole amount of the assets will be paid and nothing is paid for later periods.

\(^{12}\)Following the above example, $NAR = \frac{1}{40} \times 30000 \times 70\% = 525$, where 70% is the assumed replacement rate.
Description of six scenarios

Six scenarios are designed to see how two plans perform under various situations including a baseline scenario, an alternative contribution rate, a $t$-distribution of asset returns, a declining equity premium, a conservative investment in the collective plan induced by its aging participants, and the case of longevity. Table 4.3 summarizes the specifics of the six setups and the values of exogenous parameters.

Scenario 1 studies the baseline situation. Contribution, investment and indexation policies are the same for two plans. The contribution rate is constant at 11.22%, based on the historical mean real short rate. The investment is 56% stocks and 44% bonds, held constant over time. Their returns are simulated from the model estimated in Section 4.4.1. The indexation policy is defined in the policy ladder above. The risk free rate used for net present value calculation is 4.79%, the historical mean nominal rate. Time preference parameter $\delta$ is 0.04. The risk aversion coefficient $\lambda$ is 2.

In Scenario 2, rather than using the real short rate to determine the contribution rate, we use the real investment return.13 This means a 9.77% contribution rate, for both plans.

In Scenario 3, rather than using a normal distribution assumption of the asset returns, we apply a student’s $t$-distribution to reflect a more practical view of the financial market that the frequency of extreme values is higher than predicted by a normal distribution.

In Scenario 4, we decrease the simulated equity premium by 2%. In the generational plan all accumulations are in principal consumed by that generation. But in the collective plan, affluent accumulation can be put in reserve and saved for future generations. Therefore a favorable external environment might inherently benefit the generational plan. Therefore we design this scenario to show how the two plans perform under adverse market situations.

In Scenario 5, a conservative asset allocation of 40% stocks and 60% bonds is applied to the collective plan to see the indirect consequence of aging when retirees are the dominant decision makers in the plan. The generational plan still applies the original constant 56%-44% allocation. Other conditions are the same as in Scenario 1.

Scenario 6 is designed to show the impact of longevity risks. In this Scenario, participants’ life is extended for one more year. Because this information is unexpected until the last year, the contribution policy are kept intact as in the baseline scenario. There is no investment for this additional year, but the indexation policy still applies for the year.

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13Real return of an investment of 56% stocks and 44% bonds is $10.44\% \times 0.56 + 4.98\% \times 0.44 = 3.25\%$. $10.44\%$ and $4.98\%$ are respectively historical mean stock and bond return.
4.5 Simulation results and analysis

With the estimates of Equation (4.8) for the asset returns and Equation (4.9) and (4.10) for the yield curve, we generate 500 paths for stock and bond returns for a period of 54 years (the assumed lifetime of a generational fund). The average annual stock and 10-year bond return simulated for each year and the simulated average yield curve are shown in Figure 4.2. The overall averages for stocks and bonds are 9.44% and 5.09% per year respectively.

4.5.1 Baseline scenario results

The dynamics of funding ratio, indexation ratio and accrued benefits are presented in Figure 4.3. The solid line represents the generational plan and the dotted line represents the collective plan.

Funding ratio and underfunding probability

The top row displays the mean simulated funding ratio and the underfunding probability at different ages of the fund participants. The average funding ratio in the collective plan shows a rising trend over time. The underfunding probability is declining. This reflects that the growth of assets from investment returns and contribution collections are faster than the increases in liabilities due to the indexation and the yield curve changes.

In the generational plan the funding ratio starts high because the present value of the accrued rights in the early phase is very low. As the accrued rights and the liabilities accumulate, the funding ratio declines. At the time of retirement (age 65) the funding ratio hits the lowest and also at this point the accrued rights reaches its highest. After retirement the funding ratio climbs up for two reasons. One is the declining liabilities, as part of the liabilities are fulfilled by benefit payments. The other is the growing assets, as the assets not yet used for benefit payments are still earning investment returns. The explosion of the funding ratio in the last few years indicates that on average people have accrued more assets than they need from a lifetime perspective. The underfunding probability is virtually 0 in the beginning due to the low discounted accrued rights, but it increases as more rights are accrued. After retirement, the underfunding probability further increases as the there are more cases of deficit in assets in the later periods.

Indexation ratio

The middle row shows the mean and standard deviation of the simulated indexation ratio over time, as a percentage of the inflation rate. In the collective plan, there is a sharp drop of indexation
ratio for the second year because we impose a full indexation for the first year and then the indexation depends on the funding ratio. Due to an increasing mean funding ratio, the mean simulated indexation ratio also increases. The higher magnitude of indexation ratio also leads to a slightly increasing volatility.

In the generational plan, due to the high starting funding ratios, the indexation ratio is almost at its maximum in the beginning, meaning 2 times the inflation rate. Thus we also see a sharp jump for the second year as only full indexation is imposed for the first year. Then the mean indexation ratio follows the trend of mean funding ratio that it declines to its lowest at the retirement and then climbs up. The volatility of indexation is zero in the beginning, then increases as funding ratio drops. During the second half of working period (between age 45 and 65), with a stabilizing funding ratio, the volatility of indexation also declines. After retirement, the volatility rebounds with a more volatile funding ratio.

**Accrued rights**

The accrued rights shown in the bottom row reflect the impact of cumulative indexation. This indicator determines the annual nominal amount of benefits a participant can receive as of retirement. Compared to the goal of fully indexed annual benefits represented by the star line, the collective and generational plan respectively offer 77.3% and 102.1% on average as a percentage of the inflation-protected benefits. The generational plan has a higher cumulative indexation. This is mainly due to the high granted indexation in the beginning and the end of the life cycle when the funding ratio is very high. The accrued rights show a rising standard deviation mainly due to the increasing magnitude of benefits over time.

**Comparison**

Table 4.4 shows the comparison between the two plans over different dimensions.

The generational plan achieves a higher life-time average funding ratio than the collective plan in either the best, the worst, or the average simulated scenario. This is due to the very high funding ratio in the beginning that pushes up the average, accordingly the volatility (mismatch risk) is also higher in the generational plan. The average underfunding probability over the life time is lower in a generational plan (12.53% < 12.76%) due to the high funding ratios during working phase. But it becomes higher during the payout phase, because in the generational plan the funding ratio can only be managed by adjusting indexation ratio, while the collective plan additionally has constant cash inflows from contributions that can steer the funding ratio. However, the generational plan can be improved in terms of its funding ratio volatility by adopting a
dynamic investment strategy.

The life-time average indexation ratio is much higher in the generational plan than the collective plan in either the best, the worst, or the average simulated scenario, due to the higher funding ratio, but again it also suffers from a higher volatility.

During the retirement phase, the mean annual benefits received by participants in the generational plan are higher in both the average and best simulated scenario than in the collective plan. Yet in the worse scenario, it is lower due to the positive possibility that a generational plan has to pay 0 when it runs out of assets.

We calculate the net present value (NPV) of entering a pension plan and perform a paired sample *t*-test to compare the means of the NPV across all simulated scenarios. We find that the generational plan provides a higher NPV than the collective plan in a statistically significant way. In addition, there is a possibility of a surplus in the generational plan. Figure 4.4 shows the box plot of the assets left with the generational plan after its closure. The calculation of the net present value does not take this terminal assets into account. This possibility of terminal surplus will certainly add value to a generational plan.

From a welfare perspective when the risk attitude is considered, the generational plan generates a certainty equivalent consumption stream of €28,252, more than that of a collective plan (€27,941).

In this baseline scenario simulation, we assume away human capital and longevity risk, people can choose to invest in a financial market with only two risky assets, namely stocks with an annual return of 10.44% and a standard deviation of 17.62% and bonds with an annual return of 4.98% and a standard deviation of 5.90%. There are two pension plans available and they adopt the same investment and the same indexation policy. They also adopt the same contribution policy. Basically the two plans are the same except that the collective plan asks for pooling with the other generations while the generational plan does not. It turns out that a participant will get more expected net present value and a higher welfare in a generational plan than the collective plan. This is due to the following two factors.

Firstly, a key feature of the collective plan is that it can smooth shocks over and beyond the lifetime of a single generation via adjustment in contribution and indexation policies. A generational plan also provides such risk sharing due to the fact that benefits are only paid out after retirement. The shocks are smoothed during the working phase and partially during the retirement phase. This smoothing, however, is carried out only within one generation’s time. If the shocks do not last more than one generation’s time, the generational plan is not worse than the collective plan in risk sharing. Taking the whole history of the stock market in our sample period
between 1872 and 2007 for an example, we find the annualized return of any 40-year horizon is within the range of [5.36%, 12.07%]. Compared with the mean stock return of 10.44% and the standard deviation of 17.63%, this range is within half of the standard deviation around the mean, saying that historically there is no major negative or positive shocks that last more than a generation’s time, or 40 years in this study. However, if there is any shock that lasts beyond the lifetime of a single generation, the generational plan can still smooth this shock by trading a shortfall option or a surplus option with other generations.\footnote{We explore these possibilities in the following chapter.} Nevertheless, the generational plan is not worse than the collective plan in handling the investment risk given the past financial market.

Secondly, the generational plan is better in that there is no value transferred to other generations, which is the case in the current collective plan. With the same contributions, a participant gets more from a generational plan than from a collective plan, as reflected by the higher indexation, net present value and certainty equivalent consumption. This implies that in the collective plan the inputs of the new entrants are also used for the current retirees or future entrants rather than for the contributors alone in the generational plan. Therefore we conclude that an a-priori value transfers from the current new entrants to other generations are in place in the current collective plan and this makes the current collective plan inferior to a generational plan. When we simulate a collective plan with an initial funding ratio of nearly 250%, then the two plans provide the similar NPV and the same welfare. This says that the financial health of a collective plan at the time of entering is very important for a new entrant to get a fair deal.

In sum, the generational plan avoids a-priori value transfers while still allowing for risk sharing via time diversification. Such advantages make it an economically and politically favorable choice for new participants.

### 4.5.2 Other scenarios

In Scenario 2, we use an alternative contribution rate of 9.77% determined by the real investment return as the discount rate, reported in Table 4.5. For a lower contribution rate, the indexation becomes lower and the retirement benefits become lower accordingly. In a collective plan, the consequence of a lower cash inflow is born by all participants, so this lower contribution rate from the new entrants do not decrease the benefits as much as it does in a generational plan. Therefore we see an improved NPV and welfare in the collective plan when compared with the baseline scenario. In a generational plan a lower contribution leads to a lower NPV, but it also leads to a lower indexation that smooths the benefit level. Thus the welfare of a generational plan also

In sum, the generational plan avoids a-priori value transfers while still allowing for risk sharing via time diversification. Such advantages make it an economically and politically favorable choice for new participants.
improves upon its baseline scenario. In comparison the generational plan still performs better than the collective plan, saying that with a lower contribution rate there still exists a value transfer in the collective plan. This result will be reversed if the contribution rate of the new participants is set very low. In that case there will be a significant positive transfer to the new participants but at a cost of future generations. This happened in the past years when the contribution holidays were granted.

In Scenario 3, we use a $t$-distribution to simulate asset returns to examine the impact of more frequent occurrence of very favorable and very poor returns. Table 4.6 shows that it has increased the volatility of funding ratio and indexation ratio, leading to a lower welfare in both plans. In comparison the generational plan still provides a higher NPV and a higher welfare than the collective plan. In scenario 4, we lower the equity premium by 2% and the performances of both plans, shown in Table 4.7, become considerably worse than the baseline scenario. However, the generational fund still appears more attractive than the collective plan in terms of its NPV and welfare. Scenario 3 and 4 simulate a volatile and unfavorable investment environment, in which the collective should exhibit its advantages in smoothing the shocks. But the generational plan still provides a higher welfare. It reflects that the new participants suffer from considerable value destruction by entering the current collective plan, where they get less pension benefits.

In Scenario 5 we show the indirect impact of an aging collective plan that it has to adopt a conservative investment strategy. Table 4.8 shows that this lowers the funding ratio in the collective plan, consequently a lower indexation. The collective plan becomes more unattractive.

In Scenario 6, participants live unexpectedly for one more year. This has a different impact on the underfunding probabilities in the two plans. In the collective plan it means one additional year of contribution collection and benefits payment, while in the generational plan it only means one additional year of benefits payout. As the funding ratio of the collective plan shows a steady rising trend in the baseline scenario, adding one more year extends such rising trend and accordingly the funding ratio and underfunding probability have both improved. In the generational plan, however, one additional year of payout could turn some cases of just 100% funding into underfunding. Therefore the underfunding probability goes up. The average life time funding ratio slightly improves from 1.806 to 1.809 because in most of the cases the plan are overfunded near the end. One more year of life means one more benefit payment, therefore the welfare of participating either plan is improved, and the superiority of the generational plan is still maintained. It could be possible that when participants live unexpected for more years, the generational plan may become worse than the collective plan.
4.6 Conclusions

The conflicts of interests in a collective plan and the lack of risk sharing and high operating costs in an individual plan motivate us to consider a hybrid plan for the occupational pension provision. We investigate a new design, called a generational plan, where people of the same generation are pooled in a generational sub-plan. Each generational sub-plan can set its own policies regarding investments, indexation and contributions. For the sake of comparison, we make these policies the same in the generational and collective plan. Our modeling of the pension plans allows for time-varying liabilities. Our modeling of asset return dynamics captures mean reversion for long term investment. Our simulation for the collective plan use a realistic demographics by taking a template from a real-life pension plan.

The simulation results show that the generational plan is preferable to the collective one under a normal economic condition, a $t$-distribution of asset returns, and a lower equity premium environment. Its higher average funding ratio grants higher indexation, and thus distributes higher pension benefits. Overall the generational plan provides a higher net present value and a higher welfare to a new entering generation than the current collective plan. This better performance is driven by the fact that the generational plan prevents a-priori value transfer and still allows for risk sharing via time diversification of long-term investment. In addition, the collective plan can be more unattractive due to its aging demographics where retirees will command more conservative investment strategies. When longevity risk is considered (in our simulation only longer life expectancy is considered), the generational plan suffers a worsening underfunding probability, but relatively still provides a higher welfare than the collective plan.

Based on our simulation results, we believe a generational plan is more beneficial to new participants than the current collective plan with the current indexation policy. It avoids the implicit a-prior value transfer among generations, which are the innate weakness of the collective plan. In essence the generational plan realizes the implementation of individual plans at an aggregate level and a cost effective way. Regarding the intergenerational risk sharing of undiversifiable risks like labor income and longevity risk, the generational plan can buy insurance for this, to be discussed in the next chapter.

As far as the implementation of a generational plan is concerned, it is not very different from that of the existing collective plan except that accounts for multiple generational funds need to be created and respective policy ladders need to be defined per generational fund. Different generations have their own pension funds, except that the execution of fund activities and the knowledge are consolidated at the aggregate level to achieve economies of scale in operation cost.
and investments. Operational efforts regarding contribution collections and benefits payment are still the same except that the amount varies according to which generational sub-plan/fund a participant belongs. Investment is still centralized and then delegated to asset managers, only that the overall asset allocation is the aggregation of the individual generational sub-plan’s asset allocation. Investment results are allocated pro rata. As a result a generational plan can still exploit diversification opportunities. In addition the cost of investment will not increase a priori.

Another operational issue that deserves consideration is the amount of generational funds/sub-plans needed to be created. The choice of age cut points to determine a generation is important to a successful implementation. This will vary per plan and need further research. At this point it is important to realize that any differentiation of a collective plan towards generational accounts is fruitful.
### Table 4.1: Sample descriptive statistics

Descriptive statistics for the entire sample of 136 yearly observations from 1872 to 2007. All numbers are annual rates in percentages. Variables are the real rate of US 6M commercial paper, S&P stock return including dividends, returns on the 10Y US T-bond, nominal rate of US 6M commercial paper, and yield spreads between 10Y T-bond and 6M commercial paper.

<table>
<thead>
<tr>
<th></th>
<th>Real short rate</th>
<th>Stock</th>
<th>Bond</th>
<th>Nominal short rate</th>
<th>Dividend yield</th>
<th>Yield spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>2.77</td>
<td>10.44</td>
<td>4.98</td>
<td>4.79</td>
<td>4.47</td>
<td>-0.11</td>
</tr>
<tr>
<td>Median</td>
<td>2.09</td>
<td>10.60</td>
<td>3.77</td>
<td>4.63</td>
<td>4.37</td>
<td>-0.03</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>6.64</td>
<td>17.62</td>
<td>5.90</td>
<td>2.76</td>
<td>1.65</td>
<td>1.53</td>
</tr>
</tbody>
</table>
Table 4.2: VAR estimates for asset returns

Panel A reports full sample OLS parameter estimates of VAR $z_{t+1} = A + B z_t + \epsilon_{t+1}$, regressors are one-lag values of log real rate ($r$), log excess stock return ($s$), log excess bond return ($b$), log nominal rate ($n$), log dividend yield ($d-p$) and log yield spread ($spr$). T ratios are in parentheses. The last column is the $R^2$ for the respective equations. Panel B reports the standard deviations and correlation matrix for the residuals. Diagonal entries are the standard deviations; off-diagonal entries are the correlations.

### Panel A

<table>
<thead>
<tr>
<th></th>
<th>$r(-1)$</th>
<th>$s(-1)$</th>
<th>$b(-1)$</th>
<th>$n(-1)$</th>
<th>$(d-p)(-1)$</th>
<th>$spr(-1)$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>0.156</td>
<td>-0.061</td>
<td>0.128</td>
<td>0.321</td>
<td>0.004</td>
<td>-1.060</td>
<td>0.214</td>
</tr>
<tr>
<td></td>
<td>(1.78)</td>
<td>(-1.90)</td>
<td>(1.23)</td>
<td>(1.33)</td>
<td>(0.31)</td>
<td>(-2.19)</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>0.209</td>
<td>0.045</td>
<td>-0.204</td>
<td>-0.183</td>
<td>0.071</td>
<td>1.989</td>
<td>0.052</td>
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<tr>
<td></td>
<td>(0.82)</td>
<td>(0.48)</td>
<td>(-0.68)</td>
<td>(-0.26)</td>
<td>(1.87)</td>
<td>(1.42)</td>
<td></td>
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<tr>
<td>$b$</td>
<td>0.327</td>
<td>0.017</td>
<td>-0.293</td>
<td>0.346</td>
<td>0.007</td>
<td>2.875</td>
<td>0.339</td>
</tr>
<tr>
<td></td>
<td>(4.59)</td>
<td>(0.64)</td>
<td>(-3.46)</td>
<td>(1.76)</td>
<td>(0.64)</td>
<td>(7.31)</td>
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</tr>
<tr>
<td>$n$</td>
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<td>0.008</td>
<td>0.018</td>
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<td></td>
<td>(-4.34)</td>
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<td>(0.73)</td>
<td>(15.93)</td>
<td>(-0.47)</td>
<td>(-1.04)</td>
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<tr>
<td>$(d-p)$</td>
<td>-0.652</td>
<td>-0.963</td>
<td>0.034</td>
<td>-0.682</td>
<td>0.924</td>
<td>-1.417</td>
<td>0.927</td>
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<tr>
<td></td>
<td>(-3.82)</td>
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<td>(-1.45)</td>
<td>(36.15)</td>
<td>(-1.51)</td>
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<tr>
<td>$spr$</td>
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<td>-0.010</td>
<td>0.006</td>
<td>0.044</td>
<td>0.000</td>
<td>0.779</td>
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<tr>
<td></td>
<td>(2.59)</td>
<td>(-1.70)</td>
<td>(0.31)</td>
<td>(0.97)</td>
<td>(0.10)</td>
<td>(8.60)</td>
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### Panel B

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<tr>
<th></th>
<th>$r$</th>
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<th>$spr$</th>
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<td>5.73</td>
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<td>-0.06</td>
<td>0.29</td>
<td>-0.05</td>
<td>-0.33</td>
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<tr>
<td>$s$</td>
<td>16.65</td>
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<td>-0.10</td>
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<td></td>
</tr>
<tr>
<td>$b$</td>
<td>4.66</td>
<td>-0.70</td>
<td>-0.02</td>
<td>0.35</td>
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</tr>
<tr>
<td>$n$</td>
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<td>0.00</td>
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<tr>
<td>$(d-p)$</td>
<td>11.16</td>
<td>-11.25</td>
<td>-0.07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$spr$</td>
<td>1.08</td>
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### Table 4.3: Specification of 6 scenarios and parameters

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simulated financial markets. Allocation of 56% stocks-44% bonds are held constant in both plans. Contribution rate is constant at 11.22% in both plans, determined by using real rate 2.77% as the discount rate. Indexation policy, specified in Figure 4.1 is the same in both plans.</td>
</tr>
<tr>
<td>2</td>
<td>Contribution rate is set at 9.77% determined by using real investment return 3.25% as the discount rate. Others are the same as in Scenario 1.</td>
</tr>
<tr>
<td>3</td>
<td>The asset returns follow a student’s t-distribution rather than a normal distribution in the baseline scenario. Others are the same as in Scenario 1.</td>
</tr>
<tr>
<td>4</td>
<td>Equity premium is 2% lower than the simulated. Others are the same as in Scenario 1.</td>
</tr>
<tr>
<td>5</td>
<td>The collective plan adopts a relatively conservative investment of 40% in stocks and 60% in bonds to reflect the dominant force of retirees due to aging. Others are the same as in Scenario 1.</td>
</tr>
<tr>
<td>6</td>
<td>Participants’ life is extended for one more year, and this is unexpected until the last year. Others are the same as in Scenario 1.</td>
</tr>
</tbody>
</table>

**Discount rate**: 4.79%, historical mean short rate, used when calculating net present values.

**Time preference ($\delta$)**: 0.04

**Risk aversion ($\lambda$)**: 2

**Salary growth rate**: 0

**Replacement rate**: 70%

**Real short rate**: 2.77%, historical mean real rate, used when calculating actuarially fair base contribution rate.
Table 4.4: Comparison between collective and generational plans in Scenario 1

We calculated the statistics for the lifetime average funding ratio, standard deviation of the funding ratio over its lifetime (mismatch risk), underfunding probability over the lifetime and during the retirement phase, lifetime average indexation ratio and its standard deviation, average accrued rights, the net present value and the certainty equivalent consumption out of simulations in Scenario 1. The specifics on Scenario 1 and the assumptions on simulation can be found in Table 4.3. Annual salary is constant at €30,000. The initial funding ratio is set at 125% in the collective plan. The indexation ratio is expressed as a percentage of full indexation. Accrued rights, net present value and certainty equivalent consumption are expressed in euros.

<table>
<thead>
<tr>
<th></th>
<th>Collective</th>
<th>Generational</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funding ratio averaged over 54 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.28</td>
<td>1.806</td>
</tr>
<tr>
<td>Max</td>
<td>1.83</td>
<td>2.60</td>
</tr>
<tr>
<td>Min</td>
<td>0.58</td>
<td>1.10</td>
</tr>
<tr>
<td>Mismatch risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.23</td>
<td>0.96</td>
</tr>
<tr>
<td>Max</td>
<td>0.61</td>
<td>2.40</td>
</tr>
<tr>
<td>Min</td>
<td>0.12</td>
<td>0.40</td>
</tr>
<tr>
<td>Underfunding probability over 54 yrs</td>
<td>12.76%</td>
<td>12.53%</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Underfunding probability during retirement phase</td>
<td>10.43%</td>
<td>23.34%</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indexation ratio (as a % of full indexation) averaged over 54 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>71.4%</td>
<td>114.2%</td>
</tr>
<tr>
<td>Max</td>
<td>162.7%</td>
<td>196.1%</td>
</tr>
<tr>
<td>Min</td>
<td>-12.6%</td>
<td>52.6%</td>
</tr>
<tr>
<td>Time variation of indexation rate over 54 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>63.1%</td>
<td>82.0%</td>
</tr>
<tr>
<td>Accrued rights averaged over 14 yrs as of retirement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>72,670</td>
<td>96,305</td>
</tr>
<tr>
<td>Max</td>
<td>673,961</td>
<td>860,573</td>
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<tr>
<td>Min</td>
<td>18,008</td>
<td>12,811</td>
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<tr>
<td>Net present value (in €) averaged over 54 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>49,029</td>
<td>83,635</td>
</tr>
<tr>
<td>Paired sample t test collective-generational t-value</td>
<td>-19</td>
<td></td>
</tr>
<tr>
<td>p value</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Certainty equivalent consumption (in €)</td>
<td>27,941</td>
<td>28,252</td>
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</tbody>
</table>
Table 4.5: Comparison under an alternative contribution rate in Scenario 2
We calculated the statistics for the lifetime average funding ratio, standard deviation of the funding ratio over its lifetime (mismatch risk), underfunding probability over the lifetime and during the retirement phase, lifetime average indexation ratio and its standard deviation, average accrued rights, the net present value and the certainty equivalent consumption out of simulations in Scenario 2, specified in Table 4.3. This scenario uses 9.77% as the constant contribution rate. Annual salary is constant at €30,000. The initial funding ratio is set at 125% in the collective plan. The indexation ratio is expressed as a percentage of full indexation. Accrued rights, net present value and certainty equivalent consumption are expressed in euros.

<table>
<thead>
<tr>
<th></th>
<th>Collective</th>
<th>Generational</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funding ratio averaged over 54 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.27</td>
<td>1.66</td>
</tr>
<tr>
<td>Max</td>
<td>1.94</td>
<td>2.36</td>
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<tr>
<td>Min</td>
<td>0.49</td>
<td>1.03</td>
</tr>
<tr>
<td>Mismatch risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.23</td>
<td>0.83</td>
</tr>
<tr>
<td>Max</td>
<td>0.62</td>
<td>2.15</td>
</tr>
<tr>
<td>Min</td>
<td>0.11</td>
<td>0.34</td>
</tr>
<tr>
<td>Underfunding probability over 54 yrs</td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
<td>13.88%</td>
<td>15.01%</td>
</tr>
<tr>
<td>Underfunding probability during retirement phase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>11.23%</td>
<td>25.54%</td>
</tr>
<tr>
<td>Indexation ratio (as a % of full indexation) averaged over 54 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>69.5%</td>
<td>105.8%</td>
</tr>
<tr>
<td>Max</td>
<td>160.3%</td>
<td>185.3%</td>
</tr>
<tr>
<td>Min</td>
<td>-12.6%</td>
<td>45.6%</td>
</tr>
<tr>
<td>Time variation of indexation rate over 54 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>62.7%</td>
<td>82.8%</td>
</tr>
<tr>
<td>Accrued rights averaged over 14 yrs as of retirement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>70,842</td>
<td>83,968</td>
</tr>
<tr>
<td>Max</td>
<td>672,751</td>
<td>749,387</td>
</tr>
<tr>
<td>Min</td>
<td>18,080</td>
<td>11,175</td>
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<tr>
<td>Net present value (in €) averaged over 54 years</td>
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<td></td>
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<tr>
<td>Mean</td>
<td>54,012</td>
<td>73,482</td>
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<tr>
<td>t-value</td>
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<td></td>
</tr>
<tr>
<td>p value</td>
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<td></td>
</tr>
<tr>
<td>Certainty equivalent consumption (in €)</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>28,329</td>
<td>28,512</td>
</tr>
</tbody>
</table>
Table 4.6: Comparison under a $t$-distribution assumption in Scenario 3

We calculated the statistics for the lifetime average funding ratio, standard deviation of the funding ratio over its lifetime (mismatch risk), underfunding probability over the lifetime and during the retirement phase, lifetime average indexation ratio and its standard deviation, average accrued rights, the net present value and the certainty equivalent consumption out of simulations in Scenario 3, specified in Table 4.3. This scenario simulates asset returns with a $t$-distribution rather than a normal distribution. Annual salary is constant at €30,000. The initial funding ratio is set at 125% in the collective plan. The indexation ratio is expressed as a percentage of full indexation. Accrued rights, net present value and certainty equivalent consumption are expressed in euros.

<table>
<thead>
<tr>
<th></th>
<th>Collective</th>
<th>Generational</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funding ratio averaged over 54 years</td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.28</td>
<td>1.80</td>
</tr>
<tr>
<td>Max</td>
<td>1.86</td>
<td>2.95</td>
</tr>
<tr>
<td>Min</td>
<td>0.54</td>
<td>1.02</td>
</tr>
<tr>
<td>Mismatch risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.23</td>
<td>0.97</td>
</tr>
<tr>
<td>Max</td>
<td>1.02</td>
<td>2.44</td>
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<tr>
<td>Min</td>
<td>0.11</td>
<td>0.42</td>
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<tr>
<td>Underfunding probability over 54 yrs</td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
<td>12.56%</td>
<td>12.73%</td>
</tr>
<tr>
<td>Underfunding probability during retirement phase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>11.11%</td>
<td>24.77%</td>
</tr>
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<td>Indexation ratio (as a % of full indexation) averaged over 54 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>70.8%</td>
<td>113.1%</td>
</tr>
<tr>
<td>Max</td>
<td>154.1%</td>
<td>183.7%</td>
</tr>
<tr>
<td>Min</td>
<td>-14.0%</td>
<td>46.1%</td>
</tr>
<tr>
<td>Time variation of indexation rate over 54 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>63.3%</td>
<td>82.4%</td>
</tr>
<tr>
<td>Accrued rights averaged over 14 yrs as of retirement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>69,578</td>
<td>90,951</td>
</tr>
<tr>
<td>Max</td>
<td>346,902</td>
<td>452,045</td>
</tr>
<tr>
<td>Min</td>
<td>18,217</td>
<td>11,348</td>
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<tr>
<td>Net present value (in €) averaged over 54 years</td>
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<tr>
<td>Mean</td>
<td>44,445</td>
<td>76,278</td>
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<td></td>
</tr>
<tr>
<td>t-value</td>
<td>-22</td>
<td></td>
</tr>
<tr>
<td>p value</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Certainty equivalent consumption (in €)</td>
<td>27,862</td>
<td>28,116</td>
</tr>
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</table>
Table 4.7: Comparison under a lower equity premium environment in Scenario 4

We calculated the statistics for the lifetime average funding ratio, standard deviation of the funding ratio over its lifetime (mismatch risk), underfunding probability over the lifetime and during the retirement phase, lifetime average indexation ratio and its standard deviation, average accrued rights, the net present value and the certainty equivalent consumption out of simulations in Scenario 4, specified in Table 4.3. This scenario features a financial market with a lower equity premium. Annual salary is constant at €30,000. The initial funding ratio is set at 125% in the collective plan. The indexation ratio is expressed as a percentage of full indexation. Accrued rights, net present value and certainty equivalent consumption are expressed in euros.

<table>
<thead>
<tr>
<th></th>
<th>Collective</th>
<th>Generational</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funding ratio averaged over 54 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.17</td>
<td>1.64</td>
</tr>
<tr>
<td>Max</td>
<td>1.66</td>
<td>2.41</td>
</tr>
<tr>
<td>Min</td>
<td>0.40</td>
<td>0.90</td>
</tr>
<tr>
<td>Mismatch risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.22</td>
<td>0.95</td>
</tr>
<tr>
<td>Max</td>
<td>0.52</td>
<td>2.29</td>
</tr>
<tr>
<td>Min</td>
<td>0.10</td>
<td>0.38</td>
</tr>
<tr>
<td>Underfunding probability over 54 yrs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>24.15%</td>
<td>21.51%</td>
</tr>
<tr>
<td>Underfunding probability during retirement phase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>23.88%</td>
<td>41.85%</td>
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<td>Indexation ratio (as a % of full indexation) averaged over 54 years</td>
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<tr>
<td>Mean</td>
<td>49.6%</td>
<td>96.0%</td>
</tr>
<tr>
<td>Max</td>
<td>150.5%</td>
<td>161.2%</td>
</tr>
<tr>
<td>Min</td>
<td>-15.5%</td>
<td>39.8%</td>
</tr>
<tr>
<td>Time variation of indexation rate over 54 years</td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
<td>55.1%</td>
<td>86.0%</td>
</tr>
<tr>
<td>Accrued rights averaged over 14 yrs as of retirement</td>
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<tr>
<td>Mean</td>
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<td>68,715</td>
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<tr>
<td>Max</td>
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<td>602,510</td>
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<tr>
<td>Min</td>
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<td>9,566</td>
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<td>Net present value (in €) averaged over 54 years</td>
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<td>Mean</td>
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<td>44,786</td>
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<td>t-value</td>
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<td></td>
</tr>
<tr>
<td>p value</td>
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<td></td>
</tr>
<tr>
<td>Certainty equivalent consumption (in €)</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>27,491</td>
<td>27,530</td>
</tr>
</tbody>
</table>
Table 4.8: Comparison with a conservative investment in the collective plan in Scenario 5

We calculated the statistics for the lifetime average funding ratio, standard deviation of the funding ratio over its lifetime (mismatch risk), underfunding probability over the lifetime and during the retirement phase, lifetime average indexation ratio and its standard deviation, average accrued rights, the net present value and the certainty equivalent consumption out of simulations in Scenario 5, specified in Table 4.3. The collective plan adopts a conservative investment strategy due to its aging participants. Annual salary is constant at €30,000. The initial funding ratio is set at 125% in the collective plan. The indexation ratio is expressed as a percentage of full indexation. Accrued rights, net present value and certainty equivalent consumption are expressed in euros.

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<tr>
<th>Statistic</th>
<th>Collective</th>
<th>Generational</th>
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<tbody>
<tr>
<td>Funding ratio averaged over 54 years</td>
<td>1.27</td>
<td>1.81</td>
</tr>
<tr>
<td>Mean</td>
<td>1.75</td>
<td>2.60</td>
</tr>
<tr>
<td>Max</td>
<td>0.57</td>
<td>1.10</td>
</tr>
<tr>
<td>Min</td>
<td>0.22</td>
<td>0.96</td>
</tr>
<tr>
<td>Mismatch risk</td>
<td>0.57</td>
<td>2.40</td>
</tr>
<tr>
<td>Mean</td>
<td>0.11</td>
<td>0.40</td>
</tr>
<tr>
<td>Underfunding probability over 54 yrs</td>
<td>12.49%</td>
<td>12.53%</td>
</tr>
<tr>
<td>Mean</td>
<td>10.86%</td>
<td>23.34%</td>
</tr>
<tr>
<td>Underfunding probability during retirement phase</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>69.2%</td>
<td>114.2%</td>
</tr>
<tr>
<td>Indexation ratio (as a % of full indexation) averaged over 54 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>161.7%</td>
<td>196.1%</td>
</tr>
<tr>
<td>Max</td>
<td>-11.8%</td>
<td>52.6%</td>
</tr>
<tr>
<td>Time variation of indexation rate over 54 years</td>
<td>60.9%</td>
<td>82.0%</td>
</tr>
<tr>
<td>Mean</td>
<td>68,775</td>
<td>96,067</td>
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<tr>
<td>Max</td>
<td>729,804</td>
<td>860,573</td>
</tr>
<tr>
<td>Min</td>
<td>18,020</td>
<td>12,811</td>
</tr>
<tr>
<td>Accrued rights averaged over 14 yrs as of retirement</td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
<td>43,325</td>
<td>83,635</td>
</tr>
<tr>
<td>Paired sample t test t-value</td>
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<td></td>
</tr>
<tr>
<td>p value</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Net present value (in €) averaged over 54 years</td>
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<td></td>
</tr>
<tr>
<td>Mean</td>
<td>27,880</td>
<td>28,252</td>
</tr>
</tbody>
</table>
Table 4.9: Comparison under longevity risk in Scenario 6
We calculated the statistics for the lifetime average funding ratio, standard deviation of the fund-
ing ratio over its lifetime (mismatch risk), underfunding probability over the lifetime and during
the retirement phase, lifetime average indexation ratio and its standard deviation, average ac-
crued rights, the net present value and the certainty equivalent consumption out of simulations in
Scenario 6, specified in Table 4.3. This scenario features the impact when the life expectancy is
extended for one additional year unexpectedly. Annual salary is constant at €30,000. The initial
funding ratio is set at 125% in the collective plan. The indexation ratio is expressed as a percent-
age of full indexation. Accrued rights, net present value and certainty equivalent consumption
are expressed in euros.

<table>
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<th>Funding ratio averaged over 55 years</th>
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<th>Generational</th>
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</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.28</td>
<td>1.809</td>
</tr>
<tr>
<td>Max</td>
<td>1.86</td>
<td>2.69</td>
</tr>
<tr>
<td>Min</td>
<td>0.58</td>
<td>1.08</td>
</tr>
<tr>
<td>Mismatch risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.23</td>
<td>0.98</td>
</tr>
<tr>
<td>Max</td>
<td>0.61</td>
<td>2.71</td>
</tr>
<tr>
<td>Min</td>
<td>0.12</td>
<td>0.47</td>
</tr>
</tbody>
</table>

| Underfunding probability over 55 yrs |           |              |
| Mean                                | 12.68%    | 13.01%       |

| Underfunding probability during retirement phase |           |              |
| Mean | 10.27% | 24.43% |

<table>
<thead>
<tr>
<th>Indexation ratio (as a % of full indexation) averaged over 55 years</th>
<th>Collective</th>
<th>Generational</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>71.7%</td>
<td>114.0%</td>
</tr>
<tr>
<td>Max</td>
<td>163.4%</td>
<td>196.2%</td>
</tr>
<tr>
<td>Min</td>
<td>-12.7%</td>
<td>51.6%</td>
</tr>
</tbody>
</table>

| Time variation of indexation rate over 55 years |           |              |
| Mean | 63.4% | 82.0% |

<table>
<thead>
<tr>
<th>Accrued rights averaged over 15 yrs as of retirement</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>74,468</td>
<td>98,990</td>
</tr>
<tr>
<td>Max</td>
<td>746,054</td>
<td>955,142</td>
</tr>
<tr>
<td>Min</td>
<td>17,891</td>
<td>11,957</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Net present value (in €) averaged over 55 years</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>56,629</td>
<td>94,307</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Paired sample t test</th>
<th>collective-generational</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-value</td>
<td>-16</td>
</tr>
<tr>
<td>p value</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Certainty equivalent consumption (in €)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>28,022</td>
<td>28,330</td>
</tr>
</tbody>
</table>
The indexation ratio ($\alpha$) depends on the funding ratio. -20% indexation to the inflation is granted when the funding ratio is below 80%; no indexation for 100% funding ratio; full indexation for the funding ratio of 130%; and 200% indexation for funding ratio equal or above 150%. A linear rule applies when the funding ratio is between.
Figure 4.2: Simulated average annual stock and bond returns and an average yield curve With Equation (4.8) for the asset returns dynamics, we simulate 500 scenarios for stock and 10-year bond returns over a period of 54 years. The upper graph shows the average annual stock and bond return for each year. Using Equations (4.9) and (4.10) we simulate 500 scenarios for the yield curve. The lower graph shows the average simulated yield curve.
Figure 4.3: Fund dynamics in collective and generational plans over time
The top row of graphs draws the dynamics of funding ratio and underfunding probability of the pension plan over time, indicated by the age of the fund participant. The middle row shows the dynamics of means of the simulated indexation rate and its standard deviation. The bottom row shows the dynamics of means of the simulated accrued rights and its standard deviation. The solid line represents the generational plan and the dotted line represents the collective plan. The star line shows the fully indexed accrued rights.
Figure 4.4: Box plots of the present values of terminal assets in the generational plan

When a generational plan is liquidated after all its participants die there is a possibility of an asset surplus. The figure shows the box plots of the present values of such a surplus left in the respective six scenarios.
Chapter 5

 Guaranteeing benefits in generational pension plans* 

5.1 Introduction

In the previous chapter a generational pension plan design was proposed, in which some degree of individualization can be realized at the level of a single generation, while at the same time advantages of economies of scale are present. The economies of scale can be achieved by pooling investment and operational strategies from the different generations. Compared with a pure collective defined benefit plan without any generational discretion, the new generational design provides no intergenerational risk sharing. As intergenerational risk sharing is an important feature of a pension design we explore in this chapter whether risk sharing can be improved by introducing explicit contingent claims contracts.

A generational pension plan consists of multiple generational funds. Each generational fund serves one particular generation regarding its contribution, indexation and investment policies. A generational fund is a self-financed fund in the sense that no transfers are possible. When the assets of a particular fund are lower than its liabilities, fund participants run the risk of having to accept lower benefits during retirement. On the contrary, when the fund has a surplus after having paid all promised benefits, it has to forego this terminal surplus. These make two possible consequences of the absence of risk sharing among generations. In order to make risk sharing possible and explicit we introduce contingent claims that individual generations can trade with each other. By buying a benefit guarantee or put options on future expected benefit payments participants can insure themselves against downside risks that cannot be diversified.

*This chapter is based on the article by Huang & Mahieu (2010).
1For example, the fund gets a windfall in its last period investment or its participants die earlier than expected.
within a generation. More specifically the pension put protects the participants against less-than-promised pension payments. Secondly, by writing a call option on the surplus of its terminal assets, after all pension payments are made, a particular generation can sell its surplus assets to other generations. In this chapter we use the setup of a generational conditional DB fund to evaluate both these contingent claims.

Allowing for contingent claims to be traded among generational funds greatly improves the possibilities for intergenerational risk sharing. In a way the identification of these claims makes the collection of individual generational funds resemble a traditional collective defined benefit plan. An important difference between the generational plan and the traditional collective plan is that in the latter plan all generations adopt the same contribution, investment and indexation policies, while in the generational plan these policies still differ across generations. From this perspective a traditional collective plan can be seen as a special case of a generational plan.

To facilitate intergenerational risk sharing via contingent claims, the contracts need to be traded among generational funds within an umbrella parent plan. The claims cannot be traded with counterparties outside the parent plan due to moral hazard problem. We assume a complete market in the sense that all types of payoffs, especially the liabilities, can be replicated within this internal market.² Within a complete market we can apply traditional valuation methods for the contingent claims.

The benefit guarantees in our setup differ from the guarantees discussed in Pennacchi (1999), Feldstein & Ranguelova (2000), and Lachance, Mitchell & Smetters (2003) in two aspects. The benefit guarantee is a compound option that has payoffs in each of the retirement years. Secondly, both the underlying asset and the strike price of the guarantees are time-varying and depend on the particular contribution, indexation and investment policies of the generational fund. Therefore we use a Monte Carlo simulation to value the options. In addition, we analyze three types of benefit guarantees that respectively provide nominal, accrued rights and real benefits as a minimum.

The rest of the chapter is organized as follows. First, in Section 5.2 we briefly describe a generational defined benefit pension plan, and introduce the design of the contingent claims. We clarify how intergenerational risk sharing can be achieved in our setup and how value transfers occur. Section 5.3 presents the specifics of a generational DB fund and the resulting payoff structures for the claims. Section 5.4 presents our empirical setup and discusses the approach for valuation. In Section 5.5 results are presented and discussed. Section 5.6 concludes this chapter.

²Our assumption of a complete market is somewhat restrictive. In particular, labor income and longevity risks are difficult to hedge within the composite generational plan. However, in this chapter we abstract from an incomplete market setting.
5.2 Generational pension plans and risk sharing

5.2.1 A generational pension plan

In our setup a generational pension plan consists of a number of generational accounts. Each account serves to provide pension services to one specific generation. When a person enters the plan, he/she will enter the appropriate generational account and will stay in this account until death. A distinctive property of each generational account is that contribution, indexation and investment policies can be set independently. This distinguishes the generational plan from the traditional collective plan where participants across all living generations share uniform policies and are highly susceptible to differential treatment. The umbrella parent plan only specifies the level of annual nominal accrued rights.

In order to focus the discussion we assume that a defined benefit plan can be divided into \( N \) generational accounts. Each account \( n = 1, \ldots, N \) is responsible for its investment, contribution and indexation policies. A time index \( t = T_0, \ldots, T_d \) indicates the age of the members in the account. At age \( T_0 \) the first members enter the account and at age \( T_d \) the last member dies. A fixed retirement date is set at \( T_r \), with \( T_0 \leq T_r \leq T_d \). The nominal benefit is set by the parent plan in the form of an annual nominal pension benefit accrual \( AR_{nt} \), which we assume to be representative for all people in generation \( n \). Contributions are accumulated into an asset account \( A_{nt} \), which needs to be invested.

An important feature is that each generation can make its own decisions regarding the level of annual contributions and indexation. In order to prevent opportunistic behavior each generation has to fix its strategy by writing a policy ladder at the inception (\( t = T_0 \)) of the account. These strategies are typically conditional on the funding ratio of the generation: \( A_{nt}/PV(\sum AR_{nt}) \). Intuitively, the higher this ratio, the more indexation can be granted, and the lower the contributions can be. The policy ladders of the generation need to be approved by the parent plan. Furthermore, each generation can decide upon rules on how to allocate the investments accumulated in the generational account. The guideline regarding investment decision should also be made at the inception of the account. For example, the fund can draft guidelines whether they adopt age-dependent investment policy or funding ratio-dependent policy. These pre-set rules on the levels of contributions and indexation are fixed at the inception of the account.

Footnotes:
3. A person can leave the account only when she leaves the organization that provides the pension arrangement (company/government), and transfers the accrued pension rights to another pension plan. We do not consider these transfers in this paper.
4. In the simulations later in this chapter we set \( T_0 = 25 \), \( T_r = 65 \), and \( T_d = 80 \).
5. This may be very restricted. To allow varying investment policy will increase the complication of valuing fund-related options. In this paper we aim to give a first approximate of the option value, therefore we restrain ourselves from injecting too many complications.
investment policy are necessary in valuing latent options.

In general the financial situation of the $n$-th generational account at time $t$ from the perspective of participants can be represented by the following

$$\sum AR_{nt} \mid \sum (C + R)_{nt}$$

$\sum AR_{nt}$ represents the pension benefits received by participants, and $\sum (C + R)_{nt}$ represents the amount that participants have accumulated by contributions and investments. The difference between these two is the transfer ($X_{nt}$) that this account gives away to other generations. $X_{nt}$ can be positive or negative, depending on the pension deal.

### 5.2.2 Option designs

For a generational fund that adopts a DC deal, its participants receive any pension benefits out of the fund assets they have accumulated. The nominal benefits suffer from uncertainty mainly caused by three sources: investment risk, mortality risk and labor income risk. The real pension benefits bear additionally the inflation risk. For a generational fund choosing a DB deal, it has to handle these risks to make the promised benefit payments. The fund can adjust its investment policy to reduce investment risk. However, there are pan-generation risks such as war outbreak or long-running economic depression that may not be diversified away within the generation itself. Regarding the mortality risk rising from uncertain death time of individuals, the idiosyncratic part of this risk can be diversified away within a generation. For example, some individuals die earlier while others die later than expected. The benefit payouts saved from the early death can supplement the payment to the late death. This is also the so-called intra-generational risk sharing. However, the systematic part of this risk, also referred as longevity risk, can not be diversified away within a generation. Longevity risk refers to the uncertainty in the life expectancy of a generation. The life expectancy is an important assumption in setting the contribution rate. If the life expectancy of a generation is longer than the assumption, a generational fund will run short of assets to pay for the extra living years. The fund may buy a longevity linked bond in the open market but such securities are scarcely available currently\(^6\). The labor income risk refers to the mismatch risk between assets and liabilities that arises from the stochastic future income. The stochastic income influences the cash inflow of contributions and is also an important factor in determining the defined benefits for a given replacement rate. Unfortunately the current market does not provide income-linked securities. Thus the hedge against this risk also needs to be provided by other generations.

\(^6\)See Blake, Cairns & Dowd (2006) for a discussion of a limited range of current longevity-linked securities.
To protect the benefits from the mentioned risks, a generational fund that prefers a DB deal can purchase a benefit guarantee from other generational funds. The guarantee functions like a series of put options that generate payoffs each time when assets are not sufficient to pay for the retirement benefits.

Benefit guarantee provides a channel to eliminate the downside risk of pension assets. On the other hand, the mentioned risks can also lead to a surplus to a generational DB fund. This upside risk can also be sold. For example, there is a possibility that a generational fund has experienced favorable economic conditions during its lifetime, or the cohort lives shorter than expected, or the average lifetime income rises higher than expected. In these case, after paying all its obligations, the fund will end in surplus. A generational fund is designed to serve one birth cohort. Once all its participants die, the fund will be automatically dissolved. This means the fund has to give up its terminal assets (possibly to the plan). This is a waste to the fund because it is giving away assets without any compensation. Thus a generational DB fund can sell a call option on the surplus of its terminal assets to other generational funds. Incorporating the guarantee, a DB deal is formed by buying a put and selling a call on top of a DC deal, which corresponds with Blake (1998) who showed one type of pension scheme can be modeled by another pension scheme and contingent claims.

Trading benefit guarantees and call options between generational funds is essentially a realization of intergenerational risk sharing. This is beneficial to both sides as long as different generations have a different appetite for risks. Option buyers can pay a price to eliminate unwanted risks and option writers can earn a premium for taking the risks. As each generational fund is financially independent, pricing the guarantee and the surplus option of terminal asset in a fair way is essential in executing the intended risk sharing. If the price is fair, we assume there always exists a counterparty for option trading, either the living generation or the future generation. A suitable fair price is the market-consistent price where the value of options are derived from the asset prices used by the financial market. This chapter provides a framework to price these options.

Any design of an insurance-like contract has to handle moral hazard and adverse selection problems. The potential moral hazard problem makes it difficult for generational funds to buy benefit guarantees in the open market. For example, a generational fund after purchasing a benefit guarantee, may take excessive risks to maximize the upside potential while ignoring the downside losses, which is at the expense of the guarantee provider. Solutions could be guarantee-

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7To enable this arrangement and contracting between non-overlapping funds that prefer a DB deal, the parent plan can serve as a financial intermediary.
ing standardized portfolio or taxing excess return or subsidize the shortfalls of non-standardized portfolios as proposed by Smetters (2002). In our case, the best solution is to have this guarantee provided by the other generational funds under the same umbrella plan. Within the same plan all generational funds are operated centrally. All the information and activities are administered and regulated at the plan level by a group of representative trustees from each generational funds. The motive to take advantage of each other is thus minimal and the moral hazard problem can be greatly mitigated.

The best time to sign the contract on guarantee provision is at the establishment of a generational fund as soon as it has decided its contribution, indexation and investment policies. If a fund can choose when to buy guarantees, they may want to time the market and this can lead to the collapse of risk sharing. For example, if a fund has experienced a period of good investment returns, the benefit guarantee for this fund will be cheaper due to less possibility that the guarantee will be effected. The fund also finds it is easy to buy a guarantee as it has accumulated much assets. Such funds that are least in need of guarantee can either easily buy the guarantees or may not buy them at all. On the contrary, if a fund has experienced a period of disappointing investment returns, the benefit guarantee on this fund will become expensive as there is a higher chance that the guarantee will be effected. The fund also finds it is difficult to buy a guarantee as its assets are less than satisfactory. Such funds that are most in need of guarantees may not be able to buy the guarantees. In equilibrium, the fund that can afford the guarantees finds it unnecessary to buy, and the fund that needs the guarantees are not able to afford them. Then trading of guarantees will not occur. Therefore to facilitate this risk sharing mechanism, the decision and the contract to purchase the guarantees should be determined and signed at the fund establishment.

5.2.3 Intergenerational risk sharing

To elaborate the intergenerational risk sharing via contingent claims, we follow the spirit in Ponds (2003) and present the generational account of a DB fund at two dates from the perspective of participants. The first is at the time of the fund closure. By then everything is realized and known. The account at this time shows the ex post actual transfers between this generation and the rest. The second is at the contracting time when a generational fund decides its deal concerning contribution, indexation and investment policies, and accordingly the value of options related to this fund are also determined. The value at this moment is derived from the economic value of the items at the time of fund closure. We apply the value-based ALM in Hoevenaars & Ponds (2008) that the value of assets and liabilities of a pension fund should account for their respective
risks. We use a "V" operator in the sequel to show the economic value of the items adjusted by their risks. Readers can think of this value as the present value. The account expressed in economic values at the fund establishment shows whether there exist a prior transfers and can signify whether the fund strikes a fair deal for its participants.

For a generational DB fund with a DB deal, its generational account at the fund disclosure looks like

\[
\sum (AR) | \sum (C + R)
\]

Pension benefits \(\sum (AR)\) are defined according to a function of an accrual rate, a life-time average salary and years of employment, and also represent what participants actually receive. Contributions and investment returns \(\sum (C + R)\) are the actual amount that this generation has accumulated. Because the adjustment of contributions is only effective during the working phase. To obtain the specified benefit payment, participants of a DB fund actually own a contingent claim, which generates payoffs \((PO)\) when the accumulated contributions and investment returns are lower than the defined benefits. With the benefits specified by a formula independent from investment results, there is also a possibility of positive assets left \((POC)\) at the fund closure, and such assets will be automatically transferred to other funds. Essentially participants are writing a call option on this terminal surplus. Therefore the above account can be decomposed into the following two states:

Generational account (1) at fund closure in case of an asset deficit

\[
\sum (AR^{self}) | \sum (C + R) - PO
\]

or

Generational account (2) at fund closure in case of an asset surplus

\[
\sum (AR) - \sum (C + R)^{self} - POC
\]

\(\sum (AR^{self})\) and \(\sum (C + R)^{self}\) are respectively the affordable pension benefits and the necessary contributions supposing the fund is self financed. \(PO\) is the payoff of the implicit benefit guarantees when the accumulated assets are insufficient to pay the defined benefits. \(POC\) is the payoff of the implicit surplus call when the accumulated assets are more than necessary in paying the defined benefits. The ex post actual net transfer \((X)\) is calculated as \(\sum (AR^{self}) + PO - \sum (C + R)\) in case of an asset deficit or \(\sum (AR) - \sum (C + R)^{self} - POC\) in case of an asset surplus.

At the time of fund establishment before guarantees or surplus options are realized, the generational account looks like:
Generational account (3) at the fund establishment

\[
\begin{array}{c|cc}
V(AR) & V(C + R) \\
V(PO) & V(PO)C
\end{array}
\]

At the contracting time, the economic value of contributions and investments equals the economic value of the accrued rights, \( V(AR) = V(C + R) \), as the contribution rate is determined in such a way that the defined benefits can be fully financed (even though ex post they are not often matched). The economic value of the option payoffs is simply their respective market prices, \( V(PO) = P_{gua} \) and \( V(POC) = P_{call} \). Therefore we can see that the net transfer ex ante is

\[
V(X) = V(AR) + V(PO) - V(C + R) - V(POC) = P_{gua} - P_{call}
\]

The price of the guarantee \( (P_{gua}) \) and the price of the call \( (P_{call}) \) are derived from the pension deal once the fund has set its contribution, indexation and investment policy. If the price of the guarantee \( (P_{gua}) \) is larger than the price of the call \( (P_{call}) \), then the fund has a positive net transfer, meaning the fund is a net receiver. If the value of the guarantee is smaller than the value of the call, then the fund has a negative net transfer, meaning the fund is a net giver. Whether the net transfer is 0 determines whether this DB deal is a fair contract. This also highlights the importance of explicitly pricing the options in designing a fair plan, as put forward by Kocken (2006). In this paper we try to price this guarantee and surplus call so that a generational DB fund can explicitly trade them and strike a fair deal for its participants. The existence of the guarantee and the call reveals the mechanism for intergenerational risk sharing in a generational plan, because the deficiency in assets is made up by guarantee providers and the surplus of terminal assets is left to other generations. The magnitude of this risk sharing is \( P_{call} + P_{gua} \).

The above account is only for one generational fund. At the pension plan level, the consolidated account is simply the aggregation of all individual accounts for each generational fund. Since options are traded within the pension plan at their fair prices or market consistent prices, the total net transfer \( (X) \) will be 0, making a generational plan with option arrangement a sustainable system. The zero-sum makes the generational plan resembles the collective plan that they are both a self-contained system. What distinguishes a generational plan from a collective one is that the transfers between generations are explicitly priced in the generational plan. Such prices quantify intergenerational risk sharing and contribute to the sustainability of a pension plan. In a collective plan there exist transfers between generations, but they are implicit and can lead to differential treatment to different generations. As a consequent some generations may opt out of the plan.

Our analysis of the generational account combines ideas from previous studies, but also distinguishes itself in the following aspects. Kocken (2006) detects implicit options embedded in
pension contracts but does not assign them to different generations. Hoevenaars & Ponds (2008) introduce the generational account to study the transfers, but their focus is the change of the value of an generational account due to a change of the pension deal. In addition, they study the value changes for a generational account for a time window of 20 years, while we study the value transfers over the entire life of a generation. Cui et al. (2009) discuss the value transfers between generations within a collective plan in the context that all generations apply the uniform policies in contribution, investment, and indexation. We show the value transfer from one particular generation, and this transfer varies to the particular pension deal chosen by the generation.

5.3 Option payoff structures

This section describes the development of the assets and liabilities of a generational conditional DB fund. We explain how the values of the contingent claims are determined. Guarantees to pension benefits can take two forms: minimum rate of return guarantees and minimum benefit guarantees as summarized in Lachance & Mitchell (2003). In our setup, the guarantees are of the second form. We allow for three types of guarantees: (1) guaranteeing nominal pension benefits, based on life-time average salaries and a replacement rate; (2) guaranteeing the accrued pension rights including previously granted conditional indexation rights; and (3) guaranteeing unconditional real pension benefits that are fully indexed to the inflation rates.

5.3.1 A conditional DB generational fund

We use a representative individual for a generation to model the development of a generational fund. Assume a participant enters the labor market at $T_0$, retires at $T_r$, and dies at $T_d$. He earns a flat salary ($S_0$).

The assets ($A_t$) grow due to contribution collections ($p_t S_0$) and investment returns ($r_{A,t}$).

$$A_t = A_{t-1}(1 + r_{A,t}) + p_t S_0 \quad (5.1)$$

During the working years the liabilities change due to the change in accrued rights ($AR_t$) and the corresponding yield curve.

$$L_t = \sum_{n=0}^{T_d-T_r} \frac{AR_t}{(1 + y_{T_r-t+n,t})^T_r-t+n} \quad (5.2)$$

where $y_{T_r-t+n,t}$ is the yield of maturity $T_r - t + n$ at time $t$. 
AR\_t refers to accrued rights at time t, it is the future annual benefits that is entitled to participants since their retirement. In year 1 the accrued rights are \( AR\_1 = NAR(1 + \text{ind}_1) \). From year 2 onwards until retirement the accrued rights are given by \( AR\_{t+1} = (AR\_t + NAR)(1 + \text{ind}_{t+1}) \).

\text{ind}_t is the indexation rate. \( NAR \) is the newly accrued rights for one year of service and time independent. If \( RR \) is the replacement rate, then for every year of service \( NAR \) is \( RR/(T_r - T_0) * S_0 \). The indexation rate in this study is bounded within \([0,100\%]\), so the accrued rights is always between the nominal rights and real rights.

Upon retirement (\( t = T_r \)), the fund starts to pay out benefits as defined in \( AR\_t \) and still invests the rest. So the assets on the one hand decrease with the benefit payments, while on the other hand increase with the investment returns, and they follow

\[
A_t = A_{t-1}(1 + r_{A,t}) - AR_t
\] (5.3)

No new rights are accrued as of retirement, \( NAR = 0 \). The total accrued rights only increase with indexation \( AR_{t+1} = AR\_t(1 + \text{ind}_{t+1}) \). The value of the fund liabilities is the discounted value of the benefit payments with a declining amount of years.

\[
L_t = \sum_{n=0}^{T_d-t} \frac{AR_t}{(1 + y_{n,t})^n}
\] (5.4)

At the end of each year, the funding ratio (\( FR_t \)) is computed as \( A_t/L_t \). Every year the contribution rate \( p_t \) and the indexation rate \( \text{ind}_t \) are determined according to a policy ladder, which stipulates the values for \( p_t \) and \( \text{ind}_t \) for the next year according to current funding ratio. This policy ladder is often set by the fund at its establishment and fixed for the whole life of the fund. The dependency of indexation rate on the funding status reflects the conditional feature of this DB fund.

### 5.3.2 Provision of three types of benefit guarantees

This section presents the payoff structure of various benefit guarantees for a conditional DB generational fund. A benefit guarantee can be seen as a series of put options, which are exercised at the time of each benefit payment. The underlying asset of the options is the pension assets. The strike is the value of guaranteed benefits.

We still use a representative individual to model the fund. During the working phase of its participants, the fund’s assets and liabilities follow the same path as described in Equations (5.1) and (5.2). Upon retirement, the value of assets determines whether a put option is exercised, then whether an option is excised determines the starting value of assets for the next period.
At the time before each benefit payment, if the assets accumulated till that moment is lower than the guarantee, the guarantee put option is exercised. The fund receives the difference between the assets available and the guarantee. This amount is also the payoff of the guarantee option. Then the asset value for the next period becomes 0. If the asset value is not lower than the guarantee, the fund assets will continue as specified in Equation (5.3). The following lists the payoff of the guarantees, the development of the asset value and the accrued rights during the payout phase.

When a nominal pension benefit guarantee is provided, the minimum benefits participants receive are the nominal benefits \( (AR_{\text{nom}}) \). Depending on the assets at the time relative to the nominal benefits, the payoff of each put option entailed by the nominal guarantee \( (PO_{\text{nom}}^t) \) is given by\(^8\)

\[
PO_{\text{nom}}^t = \begin{cases} 
0, & \text{for } A_t(1 + r_{A,t}) \geq AR_t \\
0, & \text{for } AR_{\text{nom}} \leq A_t(1 + r_{A,t}) < AR_t \\
AR_{\text{nom}} - A_t(1 + r_{A,t}), & \text{for } A_t(1 + r_{A,t}) < AR_t 
\end{cases}
\]

With the existence of guarantees, the actual benefits received by participants depends on the value of assets. Because the indexation rate is bounded within \([0,100\%]\), the accrued rights are between nominal benefits and real benefits. When the value of assets is lower than the nominal benefits, nominal guarantee is effected and participants receive the nominal benefits. When the value of assets is higher than the nominal benefits, but lower than the accrued rights, guarantee is not effected, participants receive all the assets. When the value of assets is higher than the accrued rights, nominal guarantee is effected and participants receive the accrued rights. Specifically they are

\[
AR_{\text{updated}}^t = \begin{cases} 
AR_t, & \text{for } A_t(1 + r_{A,t}) \geq AR_t \\
A_t(1 + r_{A,t}), & \text{for } AR_{\text{nom}} \leq A_t(1 + r_{A,t}) < AR_t \\
AR_{\text{nom}}, & \text{for } A_t(1 + r_{A,t}) < AR_{\text{nom}} 
\end{cases}
\]

Accordingly the value of assets for the next period depends on how much benefits are actually paid. When the assets are less than the nominal benefits, the fund pays the nominal benefits by using the money received from the guarantee. The assets for the next period is 0. In sum they are

\(^8\)Benefits are paid at the end of the period, so at the time of option exercise the assets have already earned the investment return.
given by

\[ A_{t+1} = A_t(1 + r_{A,t}) - AR_{t}^{\text{updated}} = \begin{cases} 
A_t(1 + r_{A,t}) - AR_t, & \text{for } A_t(1 + r_{A,t}) \geq AR_t \\
0, & \text{for } AR_{t}^{\text{nom}} \leq A_t(1 + r_{A,t}) < AR_t \\
0, & \text{for } A_t(1 + r_{A,t}) < AR_{t}^{\text{nom}}
\end{cases} \]

When an accrued pension rights guarantee is provided, the indexation won’t be negative and the accrued pension rights \((AR_t)\) are protected from any current and future cutdown. Participants receive what they have accrued, namely \(AR_{t}^{\text{updated}} = AR_t\). The payoff of this guarantee is

\[ PO_t^{AR} = \begin{cases} 
0, & \text{for } A_t(1 + r_{A,t}) \geq AR_t \\
AR_t - A_t(1 + r_{A,t}), & \text{for } A_t(1 + r_{A,t}) < AR_t
\end{cases} \]

The assets for the next period becomes

\[ A_{t+1} = A_t(1 + r_{A,t}) - AR_{t}^{\text{updated}} = \begin{cases} 
A_t(1 + r_{A,t}) - AR_t, & \text{for } A_t(1 + r_{A,t}) \geq AR_t \\
0, & \text{for } A_t(1 + r_{A,t}) < AR_t
\end{cases} \]

When a real pension benefit guarantee is provided, participants always receive the real benefits, \(AR_{t}^{\text{updated}} = AR_{t}^{\text{real}}\), as the real benefits are the highest possible benefits participants can receive\(^9\). The real benefits are fully indexed to all the inflation rates over time, \(AR_{t+1}^{\text{real}} = (AR_{t}^{\text{real}} + NAR)(1 + \pi_{t+1})\) and \(\pi_t\) is the inflation rate. The payoff of this guarantee only depends on the comparison between the assets and the real benefits. It is given as

\[ PO_t^{\text{real}} = \begin{cases} 
0, & \text{for } A_t(1 + r_{A,t}) \geq AR_{t}^{\text{real}} \\
AR_{t}^{\text{real}} - A_t(1 + r_{A,t}), & \text{for } A_t(1 + r_{A,t}) < AR_{t}^{\text{real}}
\end{cases} \]

The assets for the next period is

\[ A_{t+1} = A_t(1 + r_{A,t}) - AR_{t}^{\text{updated}} = \begin{cases} 
A_t(1 + r_{A,t}) - AR_{t}^{\text{real}}, & \text{for } A_t(1 + r_{A,t}) \geq AR_{t}^{\text{real}} \\
0, & \text{for } A_t(1 + r_{A,t}) < AR_{t}^{\text{real}}
\end{cases} \]

We give a numerical example. Suppose now the fund is at the point to pay out its first benefits. The assets accumulated till this moment is €40,000. The nominal and real pension rights are respectively €21,000 and €80,000, and the accrued rights till this moment is €60,000. At this point, the first put option contained in a guarantee is to be expired. If the fund holds a nominal guarantee, because the value of assets is larger than the minimum guarantee of nominal benefits, this first put option is not exercised, the payoff of the nominal guarantee at this moment is 0. Participants should get the accrued rights, but the fund assets are not enough. Participants then

\(^9\text{As indexation rate is not larger than 100\%, } AR_t \text{ is not higher than } AR_{t}^{\text{real}}.\)
receive what the fund can afford, which is €40,000. The starting value of assets for the next period is 0. If the fund holds an accrued rights guarantee, because the value of assets is smaller than the accrued rights, the first put option contained in this guarantee is exercised, payoff of the accrued rights guarantee at this moment is €(60,000 − 40,000). Participants receive accrued rights, €60,000. The starting value of assets for the next period is 0. If the fund has a real benefit guarantee, because the value of assets is smaller than the real rights, the first put option contained in this guarantee is exercised, payoff of the real guarantee at this moment is €(80,000 − 40,000). Participants receive real benefits, €80,000. The starting value of assets for the next period is 0.

The value of liabilities is calculated in the same way as described in Equation (5.4).

5.3.3 Selling the fund surplus

As mentioned, the generational fund is an independent fund. At the fund disclosure when all participants die, the fund is dissolved automatically. If there are any assets left, then they would be automatically transferred to the other funds or to the plan level for central planning. From the perspective of the generational fund, this potential surplus is given away without compensation. To avoid wasting resources, the fund can sell a call option on its terminal surplus. This call option entitles the buyer any positive assets left with the fund at its closure. Using the previous setup the payoff of this call option (POC) is

\[ POC = \max(0, A_{T_d}(1 + r_{A,T_d}) - A_{R_{T_d}}^{updated}) \]

As the value of assets varies to the guarantee arrangement, the value of the call also differs for various guarantee arrangements.

5.4 Empirical methodology

Benefit guarantees and surplus options are traded among generational funds within a common parent plan, which forms an internal market. We assume a complete market that all states of liabilities can be replicated within this internal market. We apply the risk neutral technique introduced by Cox & Ross (1976). It says that in the absence of arbitrage opportunities, there exists a risk neutral probability such that

\[ V(t, T_r, T_d) = e^{-r_f(t-T)} \tilde{E} \left[ \sum_{T=T_r}^{T=T_d} P_O(T) \right] \]

(5.5)

The left hand side is the value of the guarantees at time t and the guarantees are effective from the retirement date \((T_r)\) until the death date \((T_d)\). The guarantee functions like a number of \(T_d - T_r\)
put options. \( rf_t \) is the risk free rate at \( t \). \( \tilde{E}_t \) is the expectation taken at time \( t \) and under risk-neutral measure. In its bracket is the sum of options which each has a payoff at the time of each benefit payment. These payoffs are specified in the previous section. In the same way, the value of the surplus call at time \( t \) and expires at \( T_d \) is

\[
VC(t, T_d) = e^{-rf(T_d-t)} \tilde{E}_t[POC(T_d)]
\]

(5.6)

The payoffs are generated in a risk neutral world when all investments are earning a risk free rate. We specify the following stochastic process for stock returns, bond returns, yield curve and inflation in the risk-neutral world where risk premium is 0. We use the Vasicek model (1977) to model the risk free rate. We choose this approach for yield curve modeling because it is based on the no-arbitrage principle and suitable for pricing purpose. As the Vasicek model does not preclude a negative rate, we apply this model to the real risk free rate. Then according to Fisher’s hypothesis, the nominal short rate is the sum of the real rate and expected inflation rate. This two-factor approach for the nominal rate is also applied by Campbell & Viceira (2001b). Specifically, the real rate \( (r_t) \) and expected inflation \( (\pi_t) \) respectively follow:

\[
\begin{align*}
    r_{t+1} &= \bar{r} + \phi(r_t - \bar{r}) + \epsilon_{r,t+1} \\
    \pi_{t+1} &= \bar{\pi} + \kappa(\pi_t - \bar{\pi}) + \epsilon_{\pi,t+1}
\end{align*}
\]

\( \bar{r} \) and \( \bar{\pi} \) are the long term means of the real rate and expected inflation. \( \phi \) and \( \kappa \) are mean reversion coefficients of the real rate and expected inflation. The risk free rate is \( rf_t = r_t + \pi_t \).

According to the expectation theory, a long term rate is the average of expected future short rates. Therefore we have the yield for \( m \) periods at \( t \) as

\[
y_{m,t} = \bar{r} + \bar{\pi} + \frac{1 - \phi^m}{m(1 - \phi)}(r_t - \bar{r}) + \frac{1 - \kappa^m}{m(1 - \kappa)}(\pi_t - \bar{\pi})
\]

Accordingly the return of a zero-coupon bond with \( m \) years to maturity follows

\[
\text{bond}_{m,t} = \frac{\text{price}_{m-1,t+1}}{\text{price}_{m,t}} - 1 = \exp(my_{m,t} - (m - 1)y_{m-1,t+1}) - 1
\]

We assume pension funds invest in 10-year zero bonds. Assuming stock prices follow a lognormal distribution, then the annual stock return follows

\[
stock_t = \exp(rf_t - \sigma^2/2 + \sigma \epsilon_t^s)
\]

\( \epsilon_{r,t+1}, \epsilon_{\pi,t+1}, \epsilon_t^s \) are shocks to the real rate, expected inflation and stock return respectively. Each follows a standard normal distribution and is independent from each other.
Because the payoffs of the guarantees and the surplus call option are path-dependent on the development of assets and liabilities, which are mediated by the contribution, indexation and investment polices set in the pre-specified policy ladder, it is hard to derive a closed-form solution for the values. We make a numeric valuation with a Monte Carlo simulation. Such simulation approach has been applied in various pension contexts in different countries\textsuperscript{10}.

Specifically, we simulate 1000 paths for stock returns and yield curves through the lifetime of a generational fund. From the simulated yield curves we derive bond returns, together with the simulated stock returns, we get the value for assets. Discounting accrued rights with the simulated yield curves, we get the value for liabilities. Accordingly we obtain 1000 scenarios of option payoffs. Discounting the payoffs at the risk free rates we get the present value of the options.

The values of all options will be determined at the time of its establishment as soon as the fund has announced its contribution, indexation and investment policy. A fund can only trade such options at this time by signing a contract with counter parties. However, they can fulfill the payment during the whole working phase. Of course the price will be higher the later they buy due to the time value of money.

We collect all the data in an annual frequency over the period between year 1954 and 2007\textsuperscript{11}. We take the 3-month US T-bill rate for the short nominal rate. The annual inflation rate is calculated from US Consumer Price Index-All Urban Consumers. This realized inflation rate is used to back out the expected inflation assuming an AR(1) process. Then the real rate is obtained as the difference between the nominal rate and the expected inflation rate according to the Fisher hypothesis. From the Center for Research and Security Prices (CRSP) of the University of Chicago we get the stock return (including dividends), which is for a value-weighted portfolio including all stocks traded on the NYSE, NASDAQ, and AMEX. The parameter estimates are shown in Table 5.1.

## 5.5 Prices of guarantees and the call option

The options are designed to handle the investment risk, longevity risks, and labor income risks that lead to the mismatch between assets and liabilities. Firstly we will present two sets of prices in a baseline case when only diversifiable financial risks are considered. One set of prices is


\textsuperscript{11}This is the period when all the data are available.
under the assumption of a deterministic labor income, and the other is under a stochastic labor income. Then starting from the base prices under deterministic labor income, we go on to show the sensitivities of these prices when there is a generation-long shock to the financial market reflected by a higher market volatility of the stock market; and the sensitivity when there is a longevity risk reflected by a stochastic life expectancy.

### 5.5.1 Base prices under a deterministic labor income

In the baseline case, the uncertainty of the assets and liabilities of a generational fund comes only from the financial market risks, which are represented by the yield curve and the stock market described in the previous section. The assumptions on the fund and participants are specified in Table 5.2. The entire life of a generational DB fund is 55 years. The first 40 years are the working phase when the fund collects contributions, and the later 15 years are the retirement phase where the fund pays out retirement benefits. The average of the participants earn a flat salary \( (S_0) \) over the working years at €30,000. All cash flows occur at the end of each year, namely participants get salary, pay contributions and receive pension benefits all at the end of the year. The policy ladder concerning contribution and indexation policy is defined in Figure 5.1. The contribution rate is adjusted within a range of \([-5\%, 5\%]\) around the base rate according to the funding ratio. In the beginning when the funding ratio is above 6, the contribution rate is 5% lower than the base rate, and is 5% higher when the funding ratio is lower than 1. Over time the upper bound of the funding ratio linearly declines to 2 at the time of retirement. The upper bound is set initially high at 6 in the consideration of the mechanically low starting liabilities which otherwise lead to no contribution requirement. Investment policy is set constant at 50% stocks and 50% bonds during the working and retirement phase. The base contribution rate is determined in an actuarially fair way. It is solved from \[ \sum_{t=1}^{T_d} p S_0 (1 + r_t)^{-t} = \sum_{t=T_r}^{T_d} S_0 RR (1 + r_t)^{-t}, \] which says the lifetime individual pension entitlements equals to lifetime individual pension contributions. Based on the historical mean real rate of 1.27% as the discount rate \((r_t)\) and 70% replacement rate \((RR)\), the base contribution rate \((p)\) is set at 18.38%. The dynamics of the fund over time is shown in Figure 5.2 depicting the average simulated contribution rate, indexation rate and various types of pension rights.

Panel A in Table 5.3 shows the prices of the three types of benefit guarantees and the surplus call under the three types of guarantees at the time of the first contribution collection. When

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12Pointed out in Queisser & Whitehouse (2006), the discount rate is a central and contentious issue in setting contribution rate. In general there are three possible choices for the discount rate. They are market rate of return, riskless interest rate and fiscally sustainable rate. Here we choose the real riskfree rate to make a conservative calculation of contribution rate.
only the nominal pension benefit is guaranteed, namely participants can get at least 70% of the annual salary after retirement, the guarantee costs for one time 4.17% of the yearly salary when purchased at age 26, the time of the first contribution payment. If paid in annual installment over the whole working years, it is only 0.1%\(^{13}\) of the salary. Compared to the base contribution rate of 18%, this nominal guarantee is only a marginal addition to the pension cost. The average paid benefits are €65,510, far over the nominal guarantee of €21,000. It reflects the fact that the nominal guarantee is not exercised most of the time. This low cost is expected because a real rate 1.27% is used as the discount rate to set the base contribution rate to afford the real benefits.

When the accrued pension rights are guaranteed, namely, participants at least maintain the level of the previous year’s benefits, this guarantee costs for one time 12.11% of the yearly salary. Such guarantee can provide an average annual benefit of €68,219, about 86% of the real benefits.

When the real pension benefit is guaranteed, where participants get inflation-proof benefits, it costs considerably 60% of the yearly salary. If paid in annual installment over 40 years, this guarantee costs an annual amount of 1.5% of the salary.

Selling a call option on the surplus of the terminal assets, a generational fund can avoid wasting the upside potential of its assets to generate an extra income to pay for the benefit guarantees. In general, the value of this call option on the fund surplus goes in a different direction from the value of the benefit guarantee. When the fund becomes affluent, the value of the guarantees decreases while the value of the call increases.

When a fund is provided with a nominal guarantee, the call option it can issue at the fund establishment is worthy of 49.31% of the annual salary. For a fund provided with an accrued rights guarantee, the call option is worth the same value. This is because the payoff of a call option only counts the upside potential, and this upside potential is the same for a fund with a nominal guarantee and a fund with an accrued rights guarantee. The conditional DB plan defines that participants get a high indexation when the value of assets are high. Therefore when the assets develop to its up state, participants receive the same granted indexation when the fund is provided with either a nominal or an accrued rights guarantee. The assets accordingly follow the same path in the up states, leading to the same value of the surplus call option.

For a fund provided with a real guarantee, the call option is worth 47.31%, not much lower than the call under the other two guarantee types. This is because the base contribution rate is set to aim for real benefits, which most of the time enables the fund to grant a high indexation. This leads to little difference between the surplus under a real guarantee and the surplus under the other guarantees.

\(^{13}\)≈4.17%/40.
5.5.2 Base prices under a stochastic labor income

The previous case assumes a deterministic labor income. Now we incorporate an exogenous labor income risk\textsuperscript{14}. The shocks in the labor income can influence the contributions and the asset accumulation, accordingly will influence the accrued rights and liabilities.

We apply a simple process for the labor income as $S_t = E(S) \times (1 + s + \xi_t)^{15}$. $S_t$ is the income flow at year $t$. $s$ is the expected income growth rate, and $\xi_t$ follows $NIID(0, \sigma_\xi^2)$. For the simulation and comparison purpose we set $S_0 = 30,000$, $s = 0$ and $\sigma_\xi = 0.1$\textsuperscript{16}.

The second set of prices in Panel A of Table 5.3 shows that the prices do not change much from the base case with a deterministic labor income. This is because the average income is the same in both cases and after 40 years of accumulation the distribution of assets and liabilities are comparable at the time of retirement in both cases. In addition, the labor income risk does not influence the fund dynamics after retirement when the guarantee starts to be effective. Therefore a random income shock from a standard normal distribution has negligible impact on the option prices.

5.5.3 Sensitivities to non-diversifiable investment and longevity risks

The base prices of the guarantees and the call are based on the assumptions summarized in Table 5.2. This section relaxes some of the assumptions and considers two non-diversifiable risks that the generation has to share with other generations, namely the uncertainty on the financial market and the life expectancy.

There is a possibility that one generation could suffer from a life-long shock from the financial market so that the generation cannot diversify such risk away within itself. We apply an alternative parameter for the stock market volatility to reflect this risk. We calculate the volatility of stock returns within a 20-year window during our sample period and find the highest volatility is 18.33%, which is a 22% increase from 15.06% for the base case. The investment risk directly influences the volatility of assets both during the working phase and the post-retirement phase. This causes considerable increases in the prices of the guarantees and the call, ranging from a 14% to a 43% increase. Figure 5.3 compares the payoffs of three types of guarantees under this

\textsuperscript{14}Here we abstract from the endogeneity that labor supply can vary with labor income shocks and the correlation between the labor income shock and the investment return shock.

\textsuperscript{15}Often used in literature is that labor income follows log normal distribution such as in Carroll (1997) and Viceira (2001) that $S_{t+1} = S_t \exp(s + \xi_{t+1})$. But in order to make it comparable to the baseline case we assume labor income is normally distributed rather than being lognormally distributed, because the later assumption will lead to an increase in the expected labor income even when the error term has a mean of 0.

\textsuperscript{16}This is taken from Viceira (2001).
scenario with the payoffs under the base case. There is an increase in the payoffs irrespective of the guarantee type. Relatively the percentage increase in the price of the nominal benefit guarantee is the highest, and the percentage increase in the price of the real benefit guarantee is the lowest.

The average annual pension benefits received by the participants under the macro-investment risk, seen in the last two columns of Table 5.3, are lower than the base case because of the no catch-up indexation in our pension design. More volatile assets under the investment risk will miss some indexation which will not be made up later even when the value of assets picks up.

The base contribution rate is set based on the life expectancy of the generation. The uncertainty in the life expectancy is a non-negligible risk. We incorporate this risk by making the life expectancy a random variable from a normal distribution with a mean of 80 years and a standard deviation of 2 years.

When the generation has a lower life expectancy than the expected, the extra surplus assets are transferred to other funds who bought a surplus call from this fund. When the generation has a longer life expectancy than the expected, the fund policy is set as follows. For the years after the expected life expectancy, no investment is made and no indexation is given. Under the nominal guarantee, the fund aims to pay what it can afford, with the order of firstly the accrued rights, secondly the less indexed rights, and lastly the nominal benefits. Under the accrued rights guarantee, the fund always pay the accrued rights as participants have accrued till the last year of their expected life expectancy. Under the real guarantee, the fund still pays inflation-indexed benefits, including the inflation rate for these extra years.

Panel B in Table 5.3 shows that this longevity risk increases the prices of the guarantees considerably. Figure 5.4 compares the average payoffs of three types of guarantees under this scenario with the payoffs under the base case. In the base case, the payoffs stop at the expected death age of 80. In the longevity case, the payoffs for short-lived years are on average lower than the payoffs for long-lived years. Therefore the guarantee often pays more and accordingly more valuable when there exists uncertainty on the life expectancy.

The impact of the longevity risk on the call option is negligible. This is because the call only concerns the terminal assets when obligations to all participants are filled. The possibility of both out-living and under-living the expected age leads to a non-significant change in the average value of the terminal assets.
5.5.4 Net Transfers

A generational fund accommodates the needs of a particular generation. Buying a benefit guarantee protects its participants from non-diversifiable generation-specific shocks. Selling a call option on the fund surplus avoids under-consumption. Recalling the generational account of a DB deal at the fund establishment, the net transfer a priori is determined by the difference between the value of the guarantee and the value of the call. A conditional DB deal in our numerical example is a net giver when it is provided with either a nominal or an accrued rights guarantee, meaning this fund transfers net positive values to other funds because the call it sells has a higher value than the guarantee it receives. For a conditional DB deal with a real benefit guarantee, the fund is a net receiver, as the real guarantee it receives is more valuable than the call it sells. The above results remains when the additional investment risk and longevity risk are considered.

To make the generational DB deal a fair deal, we should equalize the value of the guarantee and the surplus call. This can be done via a change in the contribution, indexation or investment policy, or simply pay or receive the difference between the price of the guarantee and the price of the surplus call. Here we give an example of doing so via a change in the contribution policy. Table 5.4 reports the break-even base contribution rate that equalizes the value of the guarantee and the value of the surplus call when the underlying fund has the characteristics defined in Table 5.2. With this base contribution rate, a generational fund with a conditional DB deal does not have to pay a cash outflow explicitly for the guarantee or receive cash inflow explicitly for the call. The break-even base contribution under the nominal guarantee is 10.16%. It means if a generational fund whose characteristics are defined in the assumptions in Table 5.2 adopts a conditional DB deal with a nominal guarantee, then this DB deal is a fair deal if it sets its base contribution rate at 10.16%. It will be net giver(receiver) if its base contribution rate is higher(lower) than 10.16%. For a fund with an accrued rights and a real benefit guarantee, the break-even base contribution rate is respectively 13.09% and 19.28%.

Comparing the value of the guarantee or the call under three types of guarantees, we find the value is highest in the case of a real guarantee provision. This reveals that the risk sharing among generational funds is the highest when a real benefit guarantee is traded, and is the lowest when only nominal benefit guarantee is traded.

5.6 Conclusion and discussion

A generational pension fund enables customized fund management in contribution, indexation and investment policies, facilitates risk management of a pension fund geared to the preference
of a particular generation, and frees pension sponsors from unwanted risks. To further improve the welfare of participants, guaranteeing pension benefits can be desirable. As a generational fund is financially independent, the upside potential of its terminal assets should also be sold to avoid under-consumption. The arrangement of such options provides a mechanism for intergenerational risk sharing, and this chapter tries to quantify this risk sharing by pricing the options. In the collective plan, such options are implicitly embedded in the contract and can hardly be quantified. Our design of option trading and their valuation in the setup of a generational plan makes intergenerational risk sharing more explicit and transparent, and can be used as a reference to the risk sharing in the current collective plan.

Applying risk-neutral and simulation techniques, we show that with a base contribution rate determined by using a real rate as the discount rate the nominal and accrued rights guarantees are relatively cheap. The one-time cost of the nominal benefits guarantee is 4%, the accrued rights guarantee is 12% and the real rights guarantee is 60% of the yearly salary. A call option written on the surplus of the terminal assets is worth about 50% of the yearly salary for all types of the benefit guarantee. We consider another base case with a stochastic labor income that has the same expected annual salary as in the deterministic labor income case. This has negligible impact on the option prices as labor income shocks only influence the fund assets till the retirement.

We also show the sensitivity of the option prices when some of the non-diversifiable risks are incorporated. We reflect the generation-long shock in investment by increasing the volatility of the stock returns to the historic highest in our sample period. This investment risk directly influences the volatility of the fund asset accumulation both during the working phase and the retirement phase. This causes considerable increases in the prices of all guarantees and the surplus call. We reflect the longevity risk by making the life expectancy a random variable from a normal distribution. The uncertainty of the life expectancy leads to only a marginal change in the value of the surplus call due to the two-side possibilities of realized life-time. However it increases the prices of all benefit guarantees considerably.

The explicit pricing of the contingent claims help to identify whether a deal is a fair deal ex ante. To make a fair deal in a generational plan, a change can be made in the fund policies concerning contribution, indexation, and investment, or a money difference between the prices of the guarantee and the surplus call can be paid. Regarding the DB deal in our example, we find that respectively at a break-even base contribution rate of 10.16%, 13.09% and 19.28%, a generational fund featured in our base case with a deterministic labor income does not have to make explicit cash flows for the options under nominal, accrued rights and real guarantee provisions. At these rates, a conditional DB deal is a fair deal. The break-even values of the
guarantees or the call also reflect that the intergenerational risk sharing is the highest when a real benefit guarantee is traded among generational funds.

In our evaluation, we actually assume a partial equilibrium that a generation only buy the guarantee and sell a surplus call from and to a young or possibly an unborn generation. Yet in a complete equilibrium, this generation can also be the seller of the guarantee and the buyer of the surplus call to and from the current elder generations. In this case we should apply an overlapping generation model with three generations, where each have their own deal concerning investment, contribution and indexation policies.\textsuperscript{17} Nevertheless, this chapter provides a framework how the contingent claims can be priced explicitly to make a fair deal.

The valuation of the options traded among generational funds is done under the complete market assumption. This is a very restrictive assumption as replicating the option payoffs and a market for continuous trading of the contingent claims can be hard to implement in practice. Hence our current estimation only provides the first proximate of their market values. The future research can be in the direction of the valuation of such options under an incomplete market.

\textsuperscript{17}Then there will be a concern over the credibility of the guarantee provider. Our primary solution is having the parent plan acting as the intermediary to execute the trading and making the payoff on behalf of the generational funds.
5.7  Tables and figures

Table 5.1: Parameter estimates

<table>
<thead>
<tr>
<th></th>
<th>Real world</th>
<th>Risk neutral world</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real rate</td>
<td>( \bar{r} = 1.27% )</td>
<td>( \bar{r} = 1.27% )</td>
<td>1.50%</td>
</tr>
<tr>
<td>( \phi = 0.56 )</td>
<td>( \phi = 0.56 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_T = 1.43% )</td>
<td>( r_T = 1.43% )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected inflation</td>
<td>( \bar{\pi} = 4.16% )</td>
<td>( \bar{\pi} = 4.16% )</td>
<td>1.83%</td>
</tr>
<tr>
<td>( \kappa = 0.78 )</td>
<td>( \kappa = 0.78 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi_T = 4.12% )</td>
<td>( \pi_T = 4.12% )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock return</td>
<td>10.81%</td>
<td>5.43%</td>
<td>15.06%</td>
</tr>
</tbody>
</table>

\( \bar{r} \) and \( \bar{\pi} \) are the long term means, \( \phi \) and \( \kappa \) are the mean reversion coefficients of real rate and expected inflation.

Table 5.2: Assumptions for baseline scenarios

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of labor entry</td>
<td>25</td>
</tr>
<tr>
<td>Retirement age</td>
<td>65</td>
</tr>
<tr>
<td>Death age</td>
<td>80</td>
</tr>
<tr>
<td>Starting annual salary</td>
<td>€30,000</td>
</tr>
<tr>
<td>Annual salary growth rate</td>
<td>0</td>
</tr>
<tr>
<td>Replacement rate</td>
<td>70%</td>
</tr>
<tr>
<td>Investment policy</td>
<td>constant 50% in stocks and 50% in bonds</td>
</tr>
<tr>
<td>Risk free rate</td>
<td>5.43%</td>
</tr>
<tr>
<td>Base contribution rate</td>
<td>18.38%</td>
</tr>
</tbody>
</table>
### Table 5.3: Value of the guarantees and the call across different scenarios

<table>
<thead>
<tr>
<th>Gua. type</th>
<th>Guarantee</th>
<th>Call</th>
<th>Ann. B.</th>
<th>% of real B.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base case with a deterministic labor income</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal guarantee</td>
<td>4.17</td>
<td>49.31</td>
<td>65,510</td>
<td>83.45</td>
</tr>
<tr>
<td>Accursed rights gua.</td>
<td>12.11</td>
<td>49.31</td>
<td>68,219</td>
<td>85.84</td>
</tr>
<tr>
<td>Real guarantee</td>
<td>59.34</td>
<td>47.31</td>
<td>83,652</td>
<td>100</td>
</tr>
<tr>
<td>Base case with a stochastic labor income</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal guarantee</td>
<td>4.17</td>
<td>49.39</td>
<td>65,517</td>
<td>83.45</td>
</tr>
<tr>
<td>Accursed rights gua.</td>
<td>12.10</td>
<td>49.39</td>
<td>68,225</td>
<td>85.84</td>
</tr>
<tr>
<td>Real guarantee</td>
<td>59.32</td>
<td>47.32</td>
<td>83,652</td>
<td>100</td>
</tr>
<tr>
<td><strong>Panel B</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sensitivity to investment risk</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal guarantee</td>
<td>5.98</td>
<td>60.70</td>
<td>63,709</td>
<td>81.32</td>
</tr>
<tr>
<td>Accursed rights gua.</td>
<td>15.99</td>
<td>60.70</td>
<td>67,028</td>
<td>84.35</td>
</tr>
<tr>
<td>Real guarantee</td>
<td>67.69</td>
<td>57.99</td>
<td>83,652</td>
<td>100</td>
</tr>
<tr>
<td>Sensitivity to longevity risk</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nominal guarantee</td>
<td>5.42</td>
<td>49.64</td>
<td>63,981</td>
<td>81.69</td>
</tr>
<tr>
<td>Accursed rights gua.</td>
<td>17.65</td>
<td>49.64</td>
<td>67,170</td>
<td>84.4</td>
</tr>
<tr>
<td>Real guarantee</td>
<td>66.92</td>
<td>44.49</td>
<td>84,423</td>
<td>100</td>
</tr>
</tbody>
</table>

The table reports the prices of the nominal benefit, accrued rights, and real benefit guarantees and the surplus call option by simulating a generational conditional DB fund. Panel A reports the prices for the base cases respectively with a deterministic and a stochastic labor income. The assumptions for the base case are specified in Table 5.2 and Figure 5.1. Panel B reports the prices when the uncertainties in the volatility of the stock market and the life expectancy are considered. Specifically, in considering the investment risk we increase the volatility of the stock market to the highest 20-year volatility in the sample period. In considering the longevity risk, we make the life expectancy a random variable with a nominal distribution that has a mean of 80 years and a standard deviation of 2 years. "Guarantee" and "Call" are the one-time price of the benefit guarantee and the call respectively at the time of first contribution collection, expressed as a percentage of the average annual salary. "Ann. B." displays the average annual benefits expressed in that participants receive since their retirements, and its average percentage of the fully indexed benefits/real benefits is shown in the last column. Numbers in brackets are the change in value when compared with the base case with a deterministic labor income.
Table 5.4: Break-even base contribution rate

<table>
<thead>
<tr>
<th></th>
<th>Contribution rate</th>
<th>Value of guarantee</th>
<th>Value of call</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal guarantee</td>
<td>10.16%</td>
<td>10.87%</td>
<td>10.86%</td>
</tr>
<tr>
<td>Accrued rights gua.</td>
<td>13.09%</td>
<td>19.97%</td>
<td>19.98%</td>
</tr>
<tr>
<td>Real guarantee</td>
<td>19.28%</td>
<td>54.41%</td>
<td>54.43%</td>
</tr>
</tbody>
</table>

The table reports the break-even base contribution rate that equalizes the value the guarantee and the value of the call under DB deal with a nominal benefit, accrued rights, and real benefit guarantee respectively. The last two columns report the resulting values of the guarantee and the call, which should be the same.
The contribution rate is adjusted between [-5%, 5%] above the base rate depending on the funding ratio of [100%, upper bound (t)]. This upper bound is time varying running from 600% at $T_0$ to 200% at $T_r$. The right graph shows the indexation ratio. No indexation to the inflation is granted when the funding ratio is below 100%, and a full indexation is granted when the funding ratio is higher than 110%. A linear rule applies when the funding ratio is between 100% and 110%.

Figure 5.1: Contribution and indexation policy in the generational plan
Figure 5.2: Fund dynamics in the base case with a deterministic labor income
The Figure shows the average simulated contribution rate, indexation ratio and pension rights over the life time of a generational fund. In the lowest graph, the solid line shows the path of the nominal pension rights. The dotted line shows the path of the accrued pension rights. The starred line shows the path of the real pension rights.
Figure 5.3: Payoff comparison between the base case and the investment risk case. The lowest set shows the average payoffs in euros of the nominal benefit guarantee. The middle set of the lines shows the payoffs in euros of the accrued rights guarantee. The upper set of the lines shows the payoffs in euros of the real benefit guarantee. The solid line represents the base case, and the dotted line represents the case incorporating additional investment risk.
The lowest set shows the average payoffs in euros of the nominal benefit guarantee. The middle set of the lines shows the payoffs in euros of the accrued rights guarantee. The upper set of the lines shows the payoffs in euros of the real benefit guarantee. The solid line represents the base case, and the dotted line represents the case incorporating longevity risk.
Chapter 6

Summary and conclusions

The Dutch occupational pension plans have excelled their peers in other countries by providing almost half of the retirement income for their retirees. The changing demographic, financial and regulatory environments, however, have confronted Dutch pension funds with new problems and challenges. This thesis draws its relevance from the current underfunding situation, and aims to evaluate some of the problems and propose some solutions.

Chapter 2 evaluates the investment performance of the Dutch industry-wide pension funds by using a unique annual data *z-score* that mandatory industry-wide pension funds are obliged to report. The *z-score* we collect covers its entire history (at the time of writing) between 1998-2006 for 65 funds out of the total 71 mandatory industry-wide funds. The *z-score* is a net-of-fee investment performance measure in which risks are adjusted by the fund- and period-specific pre-select benchmark portfolio. The resulting excess return reflects the implementation quality of pension fund investments. Our results show that the Dutch pension plans as a group cannot beat their self-selected benchmarks, but deliver a close-to-benchmark performance. In addition we find no persistence in performance, reflecting that pension plan trustees are not able to select a superior group of asset managers or establish effective investment procedures to encourage asset managers to beat the benchmarks. Cross-sectionally large plans are persistently outperforming their smaller peers.

Chapter 3 studies the distribution of future funding ratios and the strategic asset allocation over different time horizons as a result of the market valuation of liabilities, and shows their sensitivity to model uncertainty. Different assumptions on the asset and liability returns are featured in two models. Vector autoregression model with one lag length (VAR1) is often used by academics and practitioners, where all variables are explained by their own lagged values of order one and also the lagged values of the other variables. An alternative model is a restricted VAR2 model with a better statistical fit to the data. It is determined by performing statistical
tests on the optimal lag length and on the significance of the coefficients. The inclusion of the second lags in the VAR2 model decreases the mean liability return, and this leads to a higher mean funding ratio and a lower underfunding probability over medium and long horizons in both nominal and real cases. The inclusion of the second lags in the VAR2 model significantly decreases the risk of bonds over long time horizons, and lowers the correlation between stock returns and liability returns, when compared with the VAR1 model. This leads to a higher bond and a lower stock allocation under the VAR2 model than under the VAR1 model. Such a model impact on the funding ratio dynamics and the term structure of strategic asset allocations is robust for either nominal or real pension objectives, for any risk averse pension funds, and for different conditioning information.

In Chapter 4 we redesign the pension contract and propose a generational plan where pensions are organized at a generational level. The generational plan differs from the prevailing collective or individual plans in that it contains multiple generational funds and allows each fund to set their own contribution, indexation, and investment policies independently from another generational fund. The new design avoids conflict of interests between generations, and allows for customization and economy of scale in operation. We use vector autoregression model to describe asset returns and Nielsen-Siegel model to describe yield curves. We numerically simulate the development of a generational plan and a collective plan with the same investment, contribution and indexation policies for the lifetime of a representative generation. We find that the generational plan provides a higher net present value and welfare to its participants. These advantages come from the unique setup of the generational plan that avoids implicit value transfers among generations while still allowing for risk sharing via time diversification of its long-term investment.

Chapter 5 further develops the generational plan by introducing contingent claims that can be traded among generational funds to enable inter-generational risk sharing. A generational fund can buy a benefit guarantee from and sell a surplus call to other generational funds. Benefit guarantees protect participants from undiversifiable downside generational-specific risks. A surplus option can avoid ending in surplus without consuming it. We analyze three types of benefit guarantees that respectively provide nominal, accrued rights and real benefits as a minimum. We use risk-neutral valuation and Monte Carlo simulation to value the guarantees and the call option. We show that a generational fund can obtain a guarantee to their nominal and accrued benefits by paying a one-time amount as low as 4% and 12% of the yearly salary respectively. A real benefit guarantee costs substantially more at 60%. However, it can be partially covered by the proceeds of 50% from the sale of a surplus call option on the terminal assets. The labor income
risk, undiversifiable investment risk, longevity risk and are shown to have varying impacts on the values of the guarantees and the call option. A stochastic labor income but with the same expected income has nearly no impact on the prices. An undiversifiable investment risk reflected by an unexpected shock of a high stock volatility can significantly increase the value of both the guarantees and the surplus option. The longevity risk reflected by a stochastic life expectancy increases the value of guarantees considerably, but hardly any influence on the value of the surplus call.

This thesis can further be extended with future research on the following:

(1) Chapter 2 finds that pension funds on average cannot beat the passive benchmark. However, large funds persistently perform better than small funds. This size effect is worthy of a thorough investigation so that suggestions regarding pension fund merge and acquisition can be provided. Another branch of future research following this chapter is to compare this collective pension fund performance with the individual DC account performance to contribute to the discussion of added value of pension funds in the collective investment.

(2) Chapter 3 studies the impact of return models on funding ratios and strategic asset allocations based on a consideration of a pension portfolio including only stocks and bonds. The further research can continue in two aspects. Firstly the portfolio analysis can be extended to other asset classes, so to make a direct comparison with previous research. Secondly other categories of return models can be included for comparison, especially the models used by various ALM consulting firms. Analyzing the model choice by the consultants, fund managers can get further insight of the advice on the optimal asset mix and thus make a better decision.

(3) Chapter 4 recommends a generational plan design that adopts the same policies as the collective plan design. Future research can be the comparison of the two when each has adopted their optimal policies under various demographic and financial environments.

(4) Chapter 5 prices different options to be traded among generational funds under separate considerations of various risks. Future study can be made more realistic by integrating the models for various risk types. In addition, the valuation is done in a complete market where there exists a replicating portfolio for any realization of liabilities. It would be more realistic to relax such an assumption to arrive at an incomplete market price. Another extension is to use an overlapping generation model to price the options in a complete equilibrium.

(5) Chapter 4 and 5 are aimed to propose a new feasible occupation pension design. In order to achieve this, in addition to the issues discussed in these two chapters, many other issues like pension portability, industry-wide income risk need to be considered. Therefore I look forward to cooperations with multiple pension stakeholders like sponsors, participants, pension
administrators, supervisors, to make this design a better choice for future pension provision.

Though plenty needs to be done with more data after this thesis, some results and suggestions can be derived from this thesis to improve the pension provisions:

(1) *Dutch pension funds on average do not outperform their investment benchmarks. Large funds, however, deliver a better investment performance than small funds.* Pension funds as an institutional investor should be able to select superior asset managers to beat the passive benchmarks. However, evidences from the US show underperformance, and this is due to the double agency structure within the pension investment industry, pointed out by Lakonishok et al. (1992). At the same time, Bauer et al. (2007) challenge this view by showing a close-to-benchmark performance due to the close monitoring mechanism in pension investments. I believe both mechanisms are at work. However, for large pension funds with more in-house expertise, the monitoring effect dominates the agency effect and thus they show a better performance than small funds. My suggestion for small funds would be to enhance the investment monitoring or merge with other funds to allow for building in-house expertise.

(2) *Small changes in modeling asset returns can have a significant impact on the term structure of the optimal strategic asset allocation for pension funds.* The sensitivity of the optimal mix to model choice is mainly caused by the description of liability returns and its correlation with other asset classes. To immune from this uncertainty, pension funds can choose liability driven investment that one portfolio is exclusively used to hedge the liabilities and another portfolio is to maximize the risk adjusted return. The liability matching portfolio can adopt investment vehicles such as interest rate swap and inflation swap whose performance does not depend on the modeling of liability returns.

(3) *Pension contract can be redesigned around generations and contingent claims can be introduced to effectively settling the conflict between freedom of choices and social solidarity.* Though the generation plan design is far from a mature design for immediate implementation, this idea can be borrowed by the current collective pension plan in setting their policies regarding contribution, indexation and investment. Using the setup of a generational plan, the break-even values of these policy variables can be derived such that new participants are indifferent between the generational and the collective plan so to avoid ex ante transfers.
Nederlandse samenvatting (Summary in Dutch)

Rond het begin van de 21ste eeuw zorgde een combinatie van lage rentevoeten, slecht presterende effectenbeurzen en een ouder wordende samenleving voor aanzienlijke zorgen over de financiële duurzaamheid van collectieve pensioenvoorzieningen. Dit proefschrift analyseert de financiële situatie van de huidige collectieve pensioenplannen in Nederland en maakt suggesties voor verbetering.

Hoofdstuk 2 analyseert de prestaties van de beleggingsportefeuilles van bedrijfstakpensioenfondsen. Om de beleggings prestaties te analyseren gebruiken wij de z-score, die pensioenfondsen verplicht zijn te rapporteren. De z-score verzamelen wij voor een steekproef van 65 fondsen over de periode 1998-2006. De z-score meet de prestaties van de beleggingsportefeuille van pensioen fondsen, waarin gecontroleerd wordt voor het risico en tevens de resultaten van een benchmark portefeuille. Het resulterende rendement reflecteerd de implementatie kwaliteit van de beleggings beslissingen van de pensioen fonds. Wij vinden dat gemiddeld genomen pensioenfondsen niet beter presteren dan hun zelfgekozen benchmarks. Pensioen fonds beheerders zijn niet in staat om superieure activamangers te selecteren of om efficiënte investeringsprocedures te bepalen om de benchmarks te verslaan. Grotere fondsen lijken echter beter in staat tot betere beleggingsprestaties.

Hoofdstuk 3 onderzoekt de gevolgen van het op marktwaarde waarderen van pensioenfondsverplichtingen en de relatie die dit heeft met het kiezen van modellen die de langtermijn rendementen van beleggingen weergeven. Het vector autoregression model met één vertragingslengte (VAR1) wordt vaak gebruikt door academics en vaklieden, waarin alle variabelen door hun eigen achtergebleven waarden van orde één en ook achtergebleven waarden van de andere variabelen worden verklaard. Een alternatief model is een beperkt VAR2 model met een betere statistische fit. Het wordt bepaald door statistische tests aangaande de optimale vertragingslengte en de significantie van coëfficiënten. De tweede vertragingen in het VAR2 model vermindert het gemiddelde rendement van pensioen verplichtingen, en dit leidt tot een hogere gemiddelde fi-
Lagere financieringsverhouding en een lagere underfunding waarschijnlijkheid over middelgrote en lange horizonen zowel voor nominale als reële waardes.

De opneming van de tweede vertragingen in VAR2 model verminderd aanzienlijk het risico van obligaties voor lange investerings horizonnen, en vermindert de correlatie tussen aandelen en pensioen verplichtingen, wanneer vergeleken met het VAR1 model. Dit leidt tot een hogere allocatie voor obligaties en een lagere allocatie voor aandelen. Model keuzes hebben dus een effect op de financieringsverhouding en de strategische beleggings beslissingen.

In hoofdstuk 4 introduceren wij een nieuw ontwerp voor pensioenfondsen dat gebaseerd is op verschillende generaties. Het generationalplan verschilt van de heersende collectieve of individuele plannen in zoverre dat het generatie plan elk fonds toe staat om de eigen bijdrage, indexatie, en investeringsbeleid aan te passen aan de wensen van elke generatie onafhankelijk van een ander generationalfonds te plaatsen. Het nieuwe ontwerp vermijdt de belangen conflicten tussen generaties en biedt de schaal voordelen.

Wij simuleren de ontwikkeling van een generationalplan en collective plan met dezelfde investeringen, bijdrage en indexatie beleid voor het leven van een representatieve generatie. Wij vinden dat het generational plan een hoger netto contante waarde en een hoger welzijn voor deelnemers biedt. Dit is toe te schrijven aan het feit dat a priori geen transfer plaatsvindt van waarde tussen generaties. Bovendien staat een generationeel ontwerp toe dat meer geprofiteerd kan worden van tijdsdiversificatie.

In Hoofdstuk 5 voegen wij aan het generatie plan ontwerp een aantal garantie-contracten toe, die het mogelijk maken dat specifieke risico’s voor generaties afgekocht kunnen worden. Een generatie fonds kan garanties kopen van huidige gepensioneerde of deze verkopen aan andere fondsen. De garanties beschermen deelnemers tegen niet diversificeerbaar risico, en vermijden het niet benutten van een surplus.

Wij analyseren drie soorten garanties; nominaal garanties, groeidien rechten en reële garanties. Onze resultaten tonen aan dat deze garanties betaalbaar zijn en leiden tot de gewenste risico spreiding in een pensioenfonds dat opgedeeld in verschillende generaties. Nochtans, kan dit door het arbeidsinkomen, beleggings resultaat en levensduur worden beïnvloed.


Munnell, A.H. & A. Sunden (2006), ‘401(k) plans are still coming up short’, *Issue in Brief* **43**.


Biography

Xiaohong Huang was born in Jingzhou on December 17, 1977. She obtained her B.A. degree from Xi’an Jiaotong University (Xi’an, China) in 1999 with a major in English for Science and Technology. In 2000 she came to the Netherlands and joined the first class of the International Business Administration (IBA) program at Erasmus University. In 2003 she received her B.S. degree in IBA. Then she went on for a Master study in Finance at the same university. In 2005 after she successfully finished the exchange program to the Wharton School for five months, she received her M.S. degree in Finance & Investment with appellation *cum laude*. In the same year, she joined the Department of Finance of RSM Erasmus University as a PhD Candidate. Her PhD trajectory was supported by the Erasmus Research Institute of Management (ERIM). Her research focuses on pension finance including pension fund performance evaluation, asset and liability management, and pension plan design. Her work has been presented at various conferences of *European Financial Management Association (EFMA)*, *Network for Studies on Pensions, Aging and Retirement (NETSPAR)*, and the *Research Conference on Risk Sharing in DC Pension Schemes* at the University of Exeter. At Erasmus she has been involved in the teaching and theses supervision at both undergraduate and graduate levels.


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Srour, F.J., Dissecting Drayage: An Examination of Structure, Information, and Control in Drayage Operations, Promotor: Prof.dr. S.L. van de Velde, EPS-2010-186-LIS, http://hdl.handle.net/1765/1


AN ANALYSIS OF OCCUPATIONAL PENSION PROVISION
FROM EVALUATION TO REDESIGN

Around the turn of the 21st century, the “perfect storm” implied by low interest rates, poor stock market returns and an ageing society led collective pension plans into under-funded situations and caused considerable concerns over their financial sustainability. This thesis analyzes the prevailing collective pension plans in the Netherlands and makes suggestions on improving the occupational pension provision in changing demographic, financial and regulatory environments. Chapter 2 probes the pension investment performance at the overall plan level. We find that pension plans on average do not outperform their pre-selected benchmarks, reflecting that trustees fail to select superior asset management strategies. Comparatively, however, large plans outperform their smaller peers. Chapter 3 investigates the strategic asset allocation of pension plans under market consistent valuation. We find that a slight change in the model specification of asset return dynamics can have a significant impact on the optimal mix. In chapter 4, we propose a new generational plan design and find that it provides higher value and welfare to participants when compared with the current collective plan design. This is due to the fact that it allows for risk sharing via time diversification of long-term investments and prevents a-priori value transfers. To allow for intergenerational risk sharing, in Chapter 5 we introduce further design improvements by having generations trade contingent claims among them. Our estimates show that the guarantees are affordable and the surplus call option has substantial value. The option prices also give an indication of value transfers in traditional collective pension designs.

ERIM

The Erasmus Research Institute of Management (ERIM) is the Research School (Onderzoekschool) in the field of management of the Erasmus University Rotterdam. The founding participants of ERIM are Rotterdam School of Management (Economie), and the Erasmus School of Economics (ESE). ERIM was founded in 1999 and is officially accredited by the Royal Netherlands Academy of Arts and Sciences (KNAW). The research undertaken by ERIM is focused on the management of the firm in its environment, its intra- and interfirm relations, and its business processes in their interdependent connections.

The objectives of ERIM is to carry out first rate research in management, and to offer an advanced doctoral programme in Research in Management. Within ERIM, over three hundred senior researchers and PhD candidates are active in the different research programmes. From a variety of academic backgrounds and expertises, the ERIM community is united in striving for excellence and working at the forefront of creating new business knowledge.