Essays on Consumer Search, Dynamic Competition and Regulation

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Essays on Consumer Search, Dynamic Competition and Regulation

Essays over Zoekgedrag van Consumenten, Dynamische Competitie en Regulering

Thesis

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Contents

List of Figures

Chapter 1

Introduction

1.1 Motivation

The last several decades have seen two important changes in the world around us. First, the role of information in the modern world is by far more important than in previous days. It is hard to imagine a day when one does not use products of the informational technology industry. Entire industries nowadays work with information, and the company with the highest stock value (Microsoft) in the world is an icon of the informational technology industry. These changes affected consumers as well. The product universe became much richer and consumers are each day bombarded by thousands of advertising messages from different firms. Second, the world is becoming more fast and dynamic, time and intertemporal considerations play a prominent role nowadays. A large part of recent inventions are focused on ways to use the available time more efficiently: easier ways of communication, faster transportation, easier housing equipment. This thesis touches upon certain aspects of these two phenomena.

The first part of the thesis focuses on the role of information in consumer markets. Information about prices, product characteristics, etc. is costly to obtain for consumers. Therefore, consumers have to optimize the process of obtaining this information. Firms, in turn, adjust their behaviour in a particular way in order to profit form informational imperfections in the market. The first part of the thesis studies various issues of optimal customer behaviour as well as optimal behaviour of firms in the case when there are some informational frictions in the market.

The second part of the thesis considers the role of time and dynamics for the functioning of different markets. In markets with free competition, intertemporal relations can be important for a firm's decision whom to compete against in order to influence the balance of powers in the industry. In service markets with regulated prices, different processing times can play a crucial role in an agent's decisions to accept or reject customers or be a reason for a moral hazard problem. These issues are covered in the second part of the thesis.

1.2 Consumer Search

The first part of the thesis deals with various issues of consumer search. The main focus of the consumer search literature is how market outcomes are affected by informational frictions, i.e. by the fact that consumers have to incur some costs to get information about products, prices or qualities. The consumer search literature started with the seminal paper "Economics of information" by Stigler (1961). In the seventies and eighties the main focus of the consumer search literature was on consumer (or, more generally, search) behavior. Many important papers emerged during that time, like Kohn and Shavell (1974), Karni and Schwartz (1977), Weitzman (1979) and Morgan and Manning (1985). However, all the aforementioned literature focuses exclusively on the optimal search behaviour. Chapters 2 and 5 further develop this approach.

The impact of informational frictions on market outcomes was not analyzed in detail till the late 70s or early 80s. Classical economic theory predicts that in an ideal world all goods should be sold at the same price. This is the so-called "Law of one price". Since this phenomenon is rarely observed in reality, economists tried to model price dispersion in various ways. Reinganum (1979) presented one model of price dispersion. Price dispersion in this model arises from two sources: (i) consumers are uninformed and have to search for prices and (ii) there is a heterogeneity in firms cost levels. Varian (1980) analysed how heterogeneity in the degree of consumers' information can influence the market outcome. The main result of the paper is the explanation of the phenomenon of the price dispersion. The crucial difference with the model by Reinganum (1979) is that price dispersion is observed even in the case when firms are completely symmetric. However, in Varian's model consumers are not able to search for price quotations. The first author who incorporated sequential consumer search in a model of oligopolistic competition was Stahl (1989). This celebrated paper has had a large impact on the consumer search literature in general and on this thesis in particular. Chapter 3 studies the implication of costly second visits on oligopolistic market outcomes using an approach similar to Stahl (1989). Chapter 4 provides an analysis of minimum price guarantees in a similar framework.

Chapter 2 is based on Janssen and Parakhonyak (2008a). This chapter studies the implication of the introduction of a costly second visits (costly recall) assumption on the optimal consumer search rules. Most of the consumer search literature makes the explicit or implicit assumption that consumers can costlessly return to the firms already sampled. We consider this assumption to be in contradiction with the philosophy of the consumer search literature, which has informational frictions at its core. Consider yourself buying, say, shoes. You have found a particular model which satisfies you, but you think that the price is a bit too high and decide to go to another store. In that store you might find exactly the same shoes, but at a price which is slightly higher than in the first store. Is it optimal for you to return? If the first store is nearby, probably yes, but if it involves quite a trip, you would probably pay a slightly higher price to avoid the trip back.

In this chapter we propose a model which explicitly takes the costs of going back into account. This assumption drastically changes the optimal search rule. The consumer's behaviour is no longer described by a unique number – the reservation price. In contrast, the reservation price here (i) depends on the number of firms left in the sample, and (ii) depends on the search history (the best price found so far). We show that consumers accept higher prices in the latter rounds of search, i.e. when the number of firms that are not visited yet decreases. We also show that the reservation price is a non-decreasing function of the best price searched. Thus, the optimal search rule implies that prices rejected in the first round of search might be acceptable in the subsequent rounds. A notable property of the optimal search rule is that the consumers never come back to a sampled store till they learn all the prices in the market.

Because most of the equilibrium analysis in the consumer search literature (eg. Stahl (1989), Stahl (1996), Guimaraes (1996), Janssen et al. (2004), Janssen and Non (2008)) is entirely based on the assumption of a unique reservation price, the results of this chapter have important consequences for further research.

In Chapter 3 we analyse consequences of costly second visits on equilibrium market outcomes. We use an oligopolistic competition setup introduced by Stahl (1989) and change the assumption about consumer behaviour. In this setup, the assumption of costly second visits has the potential to drastically change the equilibrium outcome. In the original model, firms charge prices up to the reservation price. The reason is that if they were to play a strategy which includes prices above the reservation price, then the profits at the upper bound of the support must equal 0. That is because both informed and uninformed consumers (who continue to search) will find the lower price with probability one. However, the situation changes with the introduction of costly second visits. In this case the firm might get some positive profits at the upper bound if it is the last firm consumers visit. The reason is that consumers might not want to go back because of the costs, i.e. when the difference between the current and previous offers is sufficiently small.

First, we show that the equilibrium when all firms charge prices up to the reservation price with perfect recall is still an equilibrium with costly second visits. This result is not of particular surprise since if consumers stop at the first store, the recall costs should not matter. Second, we show this is a *unique symmetric equilibrium* of the model. This result is rather surprising. The driving force for this result is that the upper bound of the support is anyway bounded from above due to the structure of the profit function. It appears that firms cannot be sufficiently compensated for losses in demand by charging higher prices. This result justifies implementing the perfect recall assumption in the oligopolistic competition model by Stahl (1989), but it might not be the case for other industry setups.

Chapter 4 is based on Janssen and Parakhonyak (2009). This chapter analyses minimum price guarantees (MPGs), an important strategy tool firms use in consumer markets. MPGs nowadays are widely used in various industries and are subject to close study both from scholars and antitrust authorities. In the modern literature, MPGs are considered to be anticompetitive. Salop (1986) argued that MPGs work as trigger strategies to sustain collusive outcomes. Empirical research by Arbatskaya et al. (2004) confirmed that for firms which offer MPGs (price-matching), prices are higher.

In this chapter we analyse the phenomenon of MPGs using a conventional sequential consumer search framework developed by Stahl (1989). We concentrate on the case when MPGs are not preannounced, i.e. the fact whether MPGs are set or not is revealed simultaneously with the price observation. The main part of the chapter focuses on price-matching strategies. We show that due to a free-rider problem, it is not possible that all firms set MPGs for sure. There are two equilibria in the model: when MPGs are never set, and when MPGs are played with a certain positive probability. The latter exists only if there is a sufficiently high level of consumer interaction which translates into a relatively high chance of exercising MPGs. The equilibrium with MPGs is characterized by two notable properties. First, firms which set MPGs set strictly higher prices than firms who do not. Second, even the firms which do not set MPGs in a particular realization of (mixed) equilibrium strategy sell at higher prices than in the equilibrium when MPGs are never set. This result has important implications for empirical research. We also show that price-beating is never optimal due to the free-rider problem. All the results predict similar outcomes as observed by Arbatskaya et al. (2004).

Chapter 5 is based on Parakhonyak (2009). This chapter does not consider equilibrium behaviour but as Chapter 2 concentrates on developing the optimal search rule for a searcher (consumer). The novelty of this model is that it concentrates on investigating a particular object's properties and their interdependence. Consider yourself buying a sophisticated product: a laptop, a camera, etc. Most of the time a consumer spends is on figuring out different characteristics of the product, not just the price. Moreover, the price is usually easily accessible, but other characteristics might be hidden quite deeply and are costly to uncover. This justifies, for example, the existence of various product reviewing magazines. The same idea holds for investment decisions, when the investments are roughly known but different parameters of the final product (quality of oil and reserves, demand for new products and costs) are unknown, but can be fairly easily estimated based on (costly) research. We show that objects which are ex ante identical in terms of expected utility and risk might be evaluated very differently given the possibility of investigating the attributes. Asymmetry in the probability distribution over the characteristics, which is seemingly irrelevant, plays an important role in consumer behaviour. The optimal search rule is such that the searcher first observes an attribute which is least likely to take an acceptable value. We show that sometimes a sufficiently high level of asymmetry can induce search, which is not observed when the attributes are (almost) symmetric. The model is first developed for the investigation of one object and then generalized for the case of multiple objects.

1.3 Dynamic Competition and Regulation

Time plays a crucial role in our life and affects most of our decisions. However, time and dynamics have not attained as much attention in microeconomics as for example in macro. This part of the thesis focuses on two problems with dynamics at their core.

Chapter 6 is based on Dubovik and Parakhonyak (2009). We consider a dynamic (differential) game with three players competing against each other. In each period, each player can allocate his resources so as to direct his competition towards particular rivals, i.e. "target" them. This differentiates our model from the classical models of oligopolistic competition, such as Bertrand and Cournot. In these models firms compete against all their rivals equally. In this sense, this chapter has similarities with colonel Blotto games (see, e.g., Roberson, 2006) and truel games (Kilgour (1971)). Our setting can be applied to a wide variety of cases: competition between firms, competition between political parties, warfare.

The key feature of targeted competition is that the power of the players changes with time based upon their actions. This brings an important strategic considerations into the game: players might want to target particular opponents and reduce the level of competition with others in order to influence the balance of powers. We show that if the players are myopic, the weaker players eventually lose the game to their strongest rival. Vice versa, if the players value their future payoffs high enough, each player concentrates more on fighting his strongest opponent. Consequently, the weaker players grow stronger, the strongest player grows weaker and eventually all the players converge and remain in the game.

Chapter 7 is based on Janssen and Parakhonyak (2008b). In this chapter we analyse markets which possess the following crucial properties. First, consumers arrive sequentially to demand some service and come in different treatment times or "complexities". Second, the fare structure (how price depends on treatment time or "complexity") is fixed by a regulating authority or central company management. Third, agents who actually provide the service can either accept or reject customers based on the comparison of benefits and costs. Examples of markets which have the aforementioned features are: taxi markets, doctors' services, some repair markets. These markets quite often are characterized either by selection, or by demand inducement. For example, a taxi driver can reject a customer for a short trip (selection) or take a longer route to the destination (demand inducement). In a similar way, medical service markets are quite often characterized by demand inducement (see, for example, McGuire and Pauly (1991)).

We argue that these phenomena exist due to the disparity between monetary payments (net of material costs) and treatment times of the clients. We show that for a large class of price structures, some group of customers is refused the service. Equilibria with selection are welfare inferior to equilibria without selection. We also characterize the class of price structures for which selection does not arise. As the number of customers increases or agents become more patient, the class of selection-free price structures shrinks and in the limit it is unique. We show that this unique fare structure not only avoids the selection problem, but also eliminates demand inducement. The results of this chapter can be directly applied to policy-making decisions in regulated markets.

Part I

Consumer Search

Chapter 2

Consumer search with costly second visits

2.1 Introduction

The main focus of consumer search theory is to analyze how market outcomes are affected if the cost consumers have to make to get information about the prices and/or qualities firms offer is explicitly taken into account. One of the basic results of the extensive literature is that firms have some market power that they can exploit even if there are many firms in the market and that price dispersion emerges as a consequence of the fact that some firms aim at selling to many consumers at low prices, while others make higher margins over fewer customers (see, e.g., Stigler (1961) and Reinganum (1979)).

Most, if not all, of the consumer literature makes implicitly or explicitly the assumption of perfect or free recall: consumers can always come back to previously sampled firms without making a cost.[1] One of the important consequences of this assumption is that consumer search behavior is characterized by one reservation price that is constant over time (Kohn and Shavell (1974)): for any observed price sequence, consumers stop searching and buy at the firm from which they received

[1]See, e.g., Reinganum (1979), Morgan and Manning (1985) , Stahl (1989) and Stahl (1996) for early papers and Janssen et al. (2005), Tse (2006) and Waldeck (2008) for more recent papers explicitly using the perfect recall assumption.

a price quote if that price is not larger than this reservation price; otherwise they continue searching.[1]

The assumption of perfect recall is, so we argue, at odds with the general philosophy of the consumer search literature which has search frictions at its core. If consumers have to make a cost to go to a shop in the first place, then in almost any natural environment it also costly (in terms of time, effort, or money) to go back to that shop. Even while searching on the internet, where the costs of search are arguably lower than in nonelectronic markets, it takes some mouse clicks and time to go back to previously visited websites. In other words, in consumer search it is not only important to remember the offers previously received, but one also has to make a cost to activate these offers again.

In this chapter we replace the perfect recall assumption by the more natural assumption of costly second visits, where the cost of going back to stores previously sampled is explicitly modelled. Under costly second visits, we show that consumer search is no longer characterized by a reservation price that is constant over time. Instead, the reservation price at any moment in time depends on (*i*) the number of firms that are not yet sampled and (*ii*) the lowest price sampled so far. In particular, for a given lowest price in the sample the reservation price is (weakly) decreasing in the number of firms that are not yet sampled (increasing over time) and increasing in the minimum price in the sample if this minimum price is not too large. Of course, if no prices are sampled yet, the reservation price is just a constant (depending on the number of firms that quote prices). Only when there are infinitely many prices to sample, stationarity re-appears and the reservation price in that case coincides with the reservation price under perfect recall.

These two differences in the characterization of reservation prices have important consequences for the actual search behaviour of consumers. Under costly second visits it may very well happen that if consumers observe as part of a price sequence two prices p_t and p_{t+1}, with $p_t < p_{t+1}$, they will rationally decide to accept to buy at p_{t+1} and not at p_t. This behaviour is not possible under perfect

[1]An alternative setting is studied by Weitzman (1979). He considers the interesting case where alternatives differ in the cost of inspection as well as in the distribution of revenues and he asks the question in which order the alternatives should be explored.

recall and rational consumer behaviour. The main reason for the fact that different behaviours are possible is that under costly second visits, no matter how small the cost of retrieving previously sampled information, the search process is no longer stationary. In addition, the fewer the number of firms not yet sampled, the worse the chance of observing a low price if one continues searching. Together, this implies that the class of search behaviours that are consistent with rational behaviour on the part of consumers becomes much richer. Obviously, this has important consequences for the literature studying firm behaviour when consumers search sequentially as this literature is entirely based on the idea of a constant reservation price that is represented as a fixed number.[1]

In contrast to the assumption of perfect recall commonly employed in the literature on consumer search, many papers in the literature on job search assume that only current offers can be accepted as previous offers that are not accepted are foregone. Karni and Schwartz (1977) have interpreted these two applications of search theory as making specific assumptions on the probability with which past observations can be successfully retrieved: in consumer search, the probability of successful retrieval is one, in job market search, this probability is zero. They then go on to study situations with "uncertain recall", where the probability that past observations can be successfully retrieved is less than one but greater than zero (see also Landsberger and Peleg (1977)). We interpret the difference between consumer search and job market search differently, namely in terms of the cost one has to make to retrieve information. This cost is either zero or prohibitively high. We study the intermediate case where the cost is positive, but not too high to make it uninteresting to consider the option of going back to previously sampled firms.[2]

[1] An extensive overview of this literature has recently been given by Baye et al. (2006).

[2] As far as we are aware, there is no paper studying this most relevant case. Kohn and Shavell (1974) say that some of their results continue to hold if there is no possibility of recall, but they also do not analyze the situation of costly recall. Some of the results of Landsberger and Peleg (1977) are similar in nature to ours. Most notably that for every search there is a time-dependent reservation price and that this price is constant in case of perfect recall and infinitely many firms. In the operations research literature Kang (1999) studies an optimal stopping problem where the cost of a second visit is a percentage of the utility derived from previous observations and arrives at a technical analysis that resembles our analysis of the

The structure of the rest of this chapter is as follows. Section 2 presents the basic framework of analysis. Section 3 analyzes the optimal search behavior of consumers and Section 4 presents an example illustrating the nonstationarity and the the fact that rational consumers may decide to buy later at higher prices. Section 5 concludes.

2.2 Framework of Analysis

Consumers are confronted with the following situation. There are N firms selling some product. Each firm makes a specific price-quality offering that can be ranked according to some one-dimensional criterion, denoted by p. For simplicity, one may think of this ranking in terms of price: consumers prefer to buy the good with the lowest price. Each firm chooses a p according to some continuous mixed strategy distribution $F(p)$ with support $[\underline{p}, \overline{p}]$. Each consumer has a unit demand and valuation v for the product. Consumers have search costs c – the price they are paying for visiting a store. Costly second visits are modeled by saying that consumers have a cost b of returning back to a store already visited, with $0 \leq b \leq c$. Consumers sequentially sample the prices chosen by firms. Consumers first have to decide whether or not to search, and after the first and each subsequent price offer, whether they want to obtain one more price quote or whether to stop searching, and if they decide to stop searching whether to buy at all and if so whether to buy at the current price or at previously sampled prices. The main issue we are interested in is how the presence of costly second visits ($b > 0$) affects the optimal search rule.[1]

optimal search rule. See Section 3 for a more detailed comparison.

[1]In later research (see Chapter 3) we intend to investigate this search rule in the context of a specific search model where also the behaviour of firms is explicitly modeled. We do not do that in the context of the present chapter as we do not want to mix the very general context in which we analyze the optimal search rule with a specific model of price setting in the market.

2.3 Optimal Consumer Search

We start the analysis by considering the optimal stopping rule for consumers. Before searching once, consumers compare the benefits and cost of a first price search, and if the expected benefits exceed cost, which is, if

$$v - \int_{\underline{p}}^{v} p dF(p) - c \geq 0 \tag{2.3.1}$$

consumers will search (at least) once. This is a sufficient condition for searching once. Integrating by parts, this (first-step) condition can be rewritten as follows:

$$\int_{\underline{p}}^{v} F(p) dp \geq c. \tag{2.3.2}$$

If $F(p)$ satisfies this first-step condition (2.3.2)[1] we can analyze whether the consumer decides to continue searching or not after having observed a first price.

Since the expected value of continuing to search depends on future period expected values we use backward induction to analyze the optimal stopping rule. To this end, define p^s_{k-1} as the smallest price in a sample of $k-1$ prices previously sampled. We will argue that for each value of p^s_{k-1} there is a unique value of p_k such that an individual consumer is indifferent between buying at p_k and either going back to one of the previously sampled firms and buying there or continue searching. We denote this price by $\rho_k(p^s_{k-1})$. If $p_k \leq \rho_k(p^s_{k-1})$, the consumer decides to buy at p_k. Otherwise, he either buys at p^s_{k-1} (if this price is relatively small) or continues to search.

The proof is by induction starting at the last firm. The following lemma introduces the base of induction.

Lemma 2.3.1. *Let $F(p)$ be a distribution of prices. Then for $k = N-1$ the reservation price ρ_{N-1} is uniquely defined as a function of $p^s_{N-2} \in [\underline{p}, \overline{p}]$ by*

[1] Alternatively, we may follow Stahl (1989) and assume that the first price quotation is given for free.

$$\rho_{N-1}(p^s_{N-2}) = \min\left(p^s_{N-2}+b,\ c+p^s_{N-2}+b-\int_{\underline{p}}^{p^s_{N-2}+b} F(p)dp,\ p^*_{N-1}\right)$$

where p^*_{N-1} satisfies the equation

$$p^*_{N-1} = c + E(p_N|p_N < p^*_{N-1}+b)F(p^*_{N-1}+b) + (1-F(p^*_{N-1}+b))(p^*_{N-1}+b).$$

Moreover, if the consumer decides to continue searching, the continuation cost of search, defined as the additional net expected cost of continuing to search conditional on optimal behaviour after the search is made, is given by

$$C_{N-1}(p^s_{N-1}) = c + p^s_{N-1} + b - \int_{\underline{p}}^{p^s_{N-1}+b} F(p)dp.$$

Proof. We consider the situation where $N-2$ firms have been sampled and the consumer has decided to make one more search. In this case, the consumer has three options: to buy now at the newly observed price p_{N-1}, to buy now at lowest price among the previously sampled prices p^s_{N-2}, or to continue searching. Knowing the value of $\min(p_{N-1}, p^s_{N-2})$, the last option gives an expected value of

$$\begin{aligned} v - c - E(p_N|p_N &< \min((p_{N-1}, p^s_{N-2}) + b)F(\min(p_{N-1}, p^s_{N-2}) + b) - \\ &\quad (1 - F(\min(p_{N-1}, p^s_{N-2}) + b))(\min(p_{N-1}, p^s_{N-2}) + b). \end{aligned}$$

Let us first concentrate on the case where $p_{N-1} \geq p^s_{N-2}$. In this case the pay-off of continuing to search does not depend on p_{N-1} so that the reservation price is given by the point where the consumer is either (i) indifferent between buying now at p_{N-1} or buying at p^s_{N-2} (and paying the additional cost of going back b) or (ii) indifferent between buying now at p_{N-1} and continue searching. In the first case $\rho_{N-1}(p^s_{N-2}) = p^s_{N-2} + b$; in the second case

$$\begin{aligned} \rho_{N-1}(p^s_{N-2}) &= c + E(p_N|p_N < p^s_{N-2}+b)F(p^s_{N-2}+b) + \\ &+ (1-F(p^s_{N-2}+b))(p^s_{N-2}+b) = \\ &= c + \int_{\underline{p}}^{p^s_{N-2}+b} pdF(p) + (1-F(p^s_{N-2}+b))(p^s_{N-2}+b) = \\ &= c + p^s_{N-2} + b - \int_{\underline{p}}^{p^s_{N-2}+b} F(p)dp. \end{aligned}$$

It is easily seen that the first-order derivative of this expression w.r.t. p^s_{N-2} is positive and strictly smaller than 1. Moreover, it is easily seen that at $p^s_{N-2} = \underline{p}$, this expression equals $p^s_{N-2} + c > p^s_{N-2} + b$. Hence, by continuity, for small values of p^s_{N-2} the reservation price is given by $\rho_{N-1}(p^s_{N-2}) = p^s_{N-2} + b$. For larger values of p^s_{N-2} it is $\rho_{N-1}(p^s_{N-2}) = c + p^s_{N-2} + b - \int_{\underline{p}}^{p^s_{N-2}+b} F(p)dp$, at least when $\rho_{N-1}(p^s_{N-2})$ is still larger than p^s_{N-2}.

Let us next concentrate on the case where $p_{N-1} \leq p^s_{N-2}$. In this case the consumer will never go back to previously sampled prices and thus the reservation price is implicitly characterized by the price that solves

$$p_{N-1} = c + E(p_N | p_N < p_{N-1} + b)F(p_{N-1} + b) + (1 - F(p_{N-1} + b))(p_{N-1} + b).$$

Because of continuity at $p_{N-1} = p^s_{N-2}$, the fact that when $p^s_{N-2} < \rho_{N-1}(p^s_{N-2}) < p^s_{N-2} + b$, the derivative of the reservation price is strictly smaller than 1, and the fact that left differentiability holds at $p_{N-1} = p^s_{N-2}$, we should have that there is exactly one p_{N-1} that solves the above equation. This implies that in the region where $p_{N-1} \leq p^s_{N-2}$, $\rho_{N-1}(p^s_{N-2})$ is independent of p^s_{N-2}. Thus, also in this case $\rho_{N-1}(p^s_{N-2})$ is uniquely defined and non-decreasing in p^s_{N-2}.

Once price p_{N-1} is observed the continuation costs of search are defined by

$$\begin{aligned} C_{N-1}(p^s_{N-1}) &= c + E(p_N | p_N < p^s_{N-1} + b)F(p^s_{N-1} + b) + \\ &+ (1 - F(p^s_{N-1} + b))(p^s_{N-1} + b) = \\ &= c + \int_{\underline{p}}^{p^s_{N-1}+b} p dF(p) + (1 - F(p^s_{N-1} + b))(p^s_{N-1} + b) = \\ &= c + p^s_{N-1} + b - \int_{\underline{p}}^{p^s_{N-1}+b} F(p)dp. \end{aligned}$$

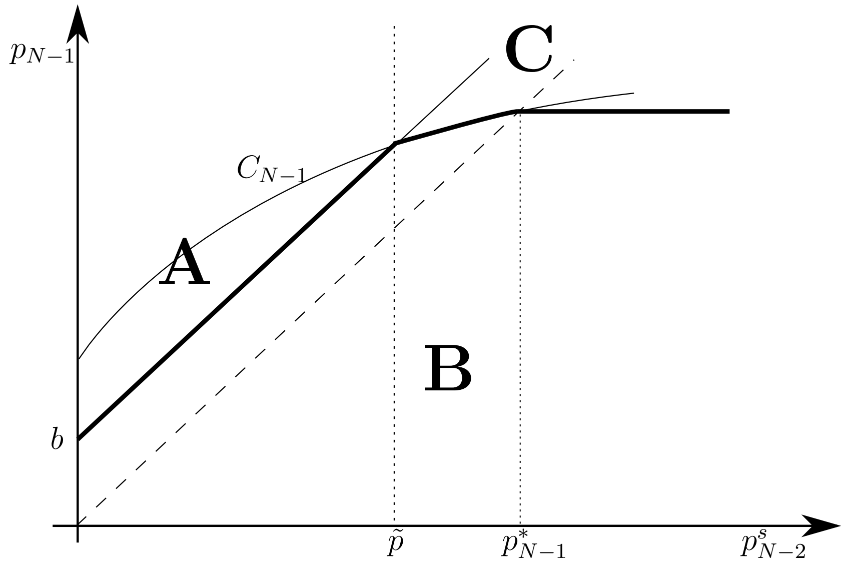
□

The following picture illustrates the lemma.

The reservation price as a function of p^s_{N-2} is presented by the bold curves. It is easy to see that this line consists of three parts:

- for $p^s_{N-2} < \tilde{p}$ the best alternative to buying at p_{N-1} is to go back to the lowest-priced firm in the sample so far. Thus, the reservation price is determined by $\rho_{N-1} = p^s_{N-2} + b$.

Figure 2.1: Reservation Price ρ_{N-1} as a function of p^s_{N-2}



- for $\tilde{p} \leq p^s_{N-2} < p^*_{N-1}$ the option to continue searching is always preferred to the option of going back to the lowest-priced firm in the sample so far. Thus, the consumer's optimal choice is based on a comparison between the current price and the expected continuation costs of continuing to search;

- for the region $p^s_{N-2} \geq p^*_{N-1}$ the situation is similar to the previous case, except that the current price is always the lowest price in the sample so far, implying that the continuation cost does not depend on p^s_{N-2}. Therefore, the reservation price is independent of p^s_{N-2} in this case.

Along the bold curve the consumer is indifferent between buying now at the shop he is currently visiting or either continuing to search or to go back to the lowest-priced firm in the sample so far.

Since optimal search behaviour is completely determined by the pair (p_{N-1}, p^s_{N-2}) we can characterize it in the same figure. Indeed,

- in region A which is bounded from below by ρ_{N-1} and from the right by $\tilde{p}$, the consumer always goes back and buys at the lowest-priced firm in the sample so far;

- in region B which is bounded from above by the reservation price, the consumer always buys at the current shop;
- finally in region C, which is bounded from below by reservation price and for which $p^s_{N-2} > \tilde{p}$, the consumer always continues to search.

Now we show that on any step $1 < k < N-1$ the reservation price as a function of the lowest price in the sample is uniquely defined and has essentially the same shape as in Figure 2.1. The proof is by induction. Before we give the formal statement of the result and the proof, we have to provide a technical result that turns out to be useful in making the induction step. To this end, assume that y is a random variable with a continuous distribution function $F(y)$. Let for a given search and return cost c and b, the following function be defined

$$\begin{aligned} C^*(x) = {} & \mathbb{P}(y < \min(x+b, C(\min(x,y))))\cdot \\ & \cdot \mathbb{E}(y | y < \min(x+b, C(\min(x,y))))+ \\ & + \mathbb{P}(y \geq \min(x+b, C(\min(x,y))))\cdot \\ & \cdot \mathbb{E}(\min(x+b, C(\min(x,y))) | y > \min(x+b, C(\min(x,y)))) + c. \end{aligned} \quad (2.3.3)$$

The function $C^*(x)$ can be interpreted as a generalized continuation cost of additional search given continuation cost on the next step.

For any function $f(x)$ let us define

$$f^-(x) = \liminf_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f^+(x) = \limsup_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Then the following lemma holds.

Lemma 2.3.2. *If $C(z)$ is a continuous function and for any z in he support of $F(\cdot)$ $0 \leq C^-(z) \leq C^+(z) < 1$ and $C(\underline{y}) > b$, where $\underline{y}$ is the lower bound of the support of $F(y)$, then $C^*(x)$ is a continuous function and for any x in the support of $F(\cdot)$ except the lower bound, $0 \leq C^{*-}(x) \leq C^{*+}(x) < 1$.*

Proof. Continuity follows immediately from the definition of C^*.

Consider the following inequality: $y < \min(x+b, C(\min(x,y)))$. Since $C^+(z) < 1$, this inequality can be rewritten in the form $y < g(x) = \min(x+b, C(x), a)$, where a satisfies equation $a = C(a)$. It is clear that $g^+(x) \leq 1$.

Thus, we can rewrite C^* in the following form:

$$C^*(x) = \mathbb{P}(y < g(x))\mathbb{E}[y|y < g(x)]+ \\ + \mathbb{P}(y \geq g(x))\mathbb{E}[\min(x+b, C(\min(x,y)))|y \geq g(x)] + c$$

Now note, that if $x \leq a$ then given that $y \geq g(x)$ we get $\min(x+b, C(\min(x,y)) = \min(x+b, C(x))$ which is just $g(x)$ for $x < a$. Then we get

$$C^*(x) = \mathbb{P}(y < g(x))\mathbb{E}[y|y < g(x)] + \mathbb{P}(y \geq g(x))\mathbb{E}[g(x)|y \geq g(x)] + c$$

and therefore

$$C^{*+}(x) = \left(\frac{F(g(x))}{F(g(x))} \int_{\underline{y}}^{g(x)} y f(y) dy + (1 - F(g(x)))g(x) \right)^+ =$$
$$= [g(x)f(g(x)) + (1 - F(g(x))) - g(x)f(g(x))]\, g^+(x) = [1 - F(g(x))]g^+(x) < 1.$$

with the second equality coming from continuity of $g(x)$. It is also clear that $C^{*-} \geq 0$.

Another case is if $x > a$. Here, given $y \geq g(x)$we get $\min(x+b, C(\min(x,y)) = C(\min(x,y))$. Then we get

$$C^*(x) = \mathbb{P}(y < g(x))\mathbb{E}[y|y < g(x)] + \mathbb{P}(y \geq g(x))\mathbb{E}[C(\min(x,y))|y \geq g(x)] + c$$

Or

$$C^*(x) = \int_{\underline{y}}^{g(x)} y f(y) dy + \int_{g(x)}^{x} C(y) f(y) d(y) + \int_{x}^{\infty} C(x) f(y) d(y) + c$$

Now, because of continuity of $g(x)$ and $C(x)$ again we get

$$C^{*+}(x) = [g(x)f(g(x)) - C(g(x))f(g(x))]g^+(x) + C^+(x)(1 - F(x))$$

Now note, that for $x > a$ we have $g(x) = a$ and therefore $C(g(x)) = a$. Thus,

$$C^{*+} = C^{+}(x)(1 - F(x)) < 1$$

In the same way

$$C^{*-} = C^{-}(x)(1 - F(x)) < 1$$

which completes the proof since $C^{-}(x) \geq 0, 1 - F(x) \geq 0$.

□

Given these two lemmas, we are now ready to state and prove the main result of the chapter. The result says that the reservation price as a function of p^s_{k-1} is well-defined and unique and a monotone function of p^s_{k-1}. In later results, we prove that the time- and history-dependency of these reservation prices cannot be neglected, unlike the case of costless recall.

Theorem 2.3.3. *The reservation price $\rho_k(p^s_{k-1})$ is uniquely defined for any k and any p^s_{k-1} from the support of $F(p)$. Moreover, the time- and history-dependent reservation prices $\rho_k(p^s_{k-1})$ are nondecreasing in p^s_{k-1}.*

Proof. Let $C_k(p^s_k)$ be a continuation cost of additional search on the k-th step given realizations of (p^s_{k-1}, p_k) (recall that $p^s_k = \min(p^s_{k-1}, p_k)$). Then, given the optimal search behaviour of the consumer, $C_k(p^s_k)$ is the expected payoff of two events: either the consumer buys at the next firm to be searched (first event) or he continues to search onwards or goes back (second event). Thus, we get that

$$\begin{aligned} C_k(p^s_k) = c + {} & \mathbb{P}(p_{k+1} < \min(p^s_k + b, C_{k+1}(p^s_{k+1})))\cdot \\ & \cdot \mathbb{E}(p_{k+1} | p_{k+1} < \min(p^s_k + b, C_{k+1}(p^s_{k+1})))+ \\ & + \mathbb{P}(p_{k+1} \geq \min(p^s_k + b, C_{k+1}(p^s_{k+1})))\cdot \\ & \cdot \mathbb{E}(\min(p^s_k + b, C_{k+1}(p^s_{k+1})) | p_{k+1} \geq \min(p^s_k + b, C_{k+1}(p^s_{k+1}))) \end{aligned} .$$

We prove that $0 \leq C^{-}_k(p^s_k) \leq C^{+}_k(p^s_k) < 1$. The proof is by backward induction. From lemma 2.3.1 it is easy to see that $0 \leq C^{-}_{N-1}(p^s_{N-1}) \leq C^{+}_{N-1}(p^s_{N-1}) < 1$, thus the base of induction is proven. We will now argue that this property also holds for any other period. For proving the induction step we can apply lemma 2.3.2 by substituting in the equation (2.3.3) $x = p_k^s$, $y = p_{k+1}$, $C^*(x) = C_k(p_k^s)$, $C(\min(x, y)) = C_{k+1}(p_{k+1}^s)$. Therefore, from $0 \leq C_{k+1}^-(p_{k+1}^s) \leq C_{k+1}^+(p_{k+1}^s) < 1$ it follows that $0 \leq C_k^-(p_k^s) \leq C_k^+(p_k^s) < 1$ and thus, by induction it follows that for any k $0 \leq C_k^-(p_k^s) \leq C_k^+(p_k^s) < 1$.

The rest of the proof is straightforward. If $p_k \geq p_{k-1}^s$, then $\rho_k(p_{k-1}^s) = \min(p_{k-1}^s + b, C_k(p_{k-1}^s))$, which is well-defined and unique. Moreover, it is non-decreasing in p_{k-1}^s since both $p_{k-1}^s + b$ and $C_k(p_k^s)$ are non-decreasing in p_{k-1}^s. If, on the other hand, $p_k < p_{k-1}^s$, then the reservation price is a solution to the equation $p_k = C_k(p_k)$, which is unique since $C_k(p_k)$ has a slope strictly smaller than 1. In this case, the reservation price does not depend on p_{k-1}^s and is thus nondecreasing in p_{k-1}^s.

□

The proof of the theorem basically shows that the function $\rho_{k+1}(p_k^s)$ is defined over three separate intervals and essentially looks like the reservation price for the last step (see Figure 2.1). When p_{k-1}^s is relatively small $\rho_k(p_{k-1}^s) = p_{k-1}^s + b$. Then for intermediate values of p_{k-1}^s, $\rho_k(p_{k-1}^s) = C_k(p_{k-1}^s)$ and for higher values $\rho_k(p_{k-1}^s)$ is independent of p_{k-1}^s. One can thus, define the price $\tilde{p}_k$ as the price such that the consumer is indifferent between going back to the shop charging this price and continuing to search, i.e., $\tilde{p}_k + b = C_k(\tilde{p}_k)$.

We are now in the position to prove some special properties of the reservation price function. To this end, define ρ^{pr} as the reservation price under perfect recall, i.e., as noted, e.g., by Stahl (1989),

$$c = \int_{\underline{p}}^{\rho^{pr}} F(p)dp.$$

By considering the limiting case where the cost of recall is zero we provide more insight into the reason why the cases of perfect recall and costly second visits are so different from one another. Moreover, the reservation price under perfect recall turns out to play an important role in further characterizing the optimal search behaviour under costly second visits.

Proposition 2.3.4. [1] *Let $b = 0$. Then for any k the reservation price is defined by:*

$$\rho_k = \min(p^s_{k-1}, \rho^{pr}).$$

Under perfect recall, the search rule is stationary, but (interestingly) slightly different from what is commonly thought as in any period the reservation price is still dependent on the lowest of previously sampled prices. When the current price is smaller than any of the previously sampled prices, then the consumer simply compares the current price with ρ^{pr} and decides whether or not to buy. If the current price is larger, the consumer simply forgets about the current price. Because of stationarity, previously sampled prices are in a full model including price setting behaviour of the firms, irrelevant. Either these previously sampled prices are below ρ^{pr}, but then the consumer simply does not continue to search, or they are above ρ^{pr}, but then the consumer never considers buying there unless he has visited all the stores and knows for sure that there are no lower prices in the sample.[2]

To further characterize the optimal search rule, under costly second visits we show that the price $\tilde{p}_k$ is intimately related to the price ρ^{pr} under perfect recall.

Proposition 2.3.5. *For all k, $\tilde{p}_k = \tilde{p} = \rho^{pr} - b$.*

Proof. Note that the price $\tilde{p}_k$ is defined such that after visiting k stores, the consumer is indifferent between continuing searching and going back to the lowest-priced store in the sample so far. Therefore, at $\tilde{p}_k$ the reservation price $\rho_k(\tilde{p}_k) = \tilde{p}_k + b$. The expected costs of continuing to search are:

$$c + F(\tilde{p}_k + b)\mathbb{E}(p_{k+1}|p_{k+1} < \tilde{p}_k + b) + (1 - F(\tilde{p}_k + b))(\tilde{p}_k + b)$$

By equating it to the best current option $(\tilde{p}_k + b)$ and some simplifications we have also used in previous proofs, we get

$$c = \int_{\underline{p}}^{\tilde{p}_k + b} F(p)dp.$$

[1] As this fact is intuitively obvious the proof is available upon request.

[2] However, in equilibrium even this could not be the case with $b = 0$ as then the traditional argument kicks in that no firm wants to charge the highest price above ρ^{pr} as no consumer will ever buy at this price, implying that no firm will want to choose a price above ρ^{pr}.

It follows therefore that $\tilde{p}_k$ does not depend on k and that (by comparing this equation to the definition of ρ^{pr}) it is actually just equal to $\rho^{pr} - b$.

□

Next, we show that rational consumers never use the option of going back to previously sampled stores, unless they have visited every store available.

Corollary 2.3.6. *Assume the consumer behaved optimally on all steps* $1 \leq k \leq K$*. Then if* $K < N$*, it is never optimal for this consumer to go back.*

Proof. Note, that the option of going back is preferred to continue searching or stopping only if $p_K^s < \tilde{p}$. On the first step any price $p_1 \leq \rho^{pr}$ would be accepted immediately. So, if the consumer continued his search it must be the case that $p_1 > \rho^{pr}$. Given $p_1^s > \rho^{pr}$ on the second step any price $p_2 \leq \rho^{pr}$ also would be accepted immediately. Thus, if consumer continued his search it must be the case that $p_2 > \rho^{pr}$. Then by induction if customer reached step K it must be the case that for any $1 \leq k \leq K$ it was the case that $p_k > \rho^{pr}$. Therefore $p_K^s > \rho^{pr} > \tilde{p}$ and it is never optimal to go back, except possibly at the last step.

□

Next, we show that reservation prices are non-decreasing over time. In particular, if a price smaller than $\tilde{p} = \rho^{pr} - b$ is sampled before, then the reservation price is simply $\rho_k(p_{k-1}^s) = p_{k-1}^s + b$ and therefore if $p_k^s = p_{k-1}^s$, then $\rho_{k+1}(p_k^s) = \rho_k(p_{k-1}^s)$. However, if a price strictly larger than $\tilde{p} = \rho^{pr} - b$ is the lowest price in the sample so far, then $\rho_{k+1}(p_k^s) > \rho_k(p_{k-1}^s)$. Thus, under costly second visits reservation prices are essentially nonstationary.

Proposition 2.3.7. *If* $p_k^s = p_{k-1}^s$*, then* $\rho_{k+1}(p_k^s) \geq \rho_k(p_{k-1}^s)$*, i.e., reservation prices are non-decreasing over time. Moreover,* $\rho_{k+1}(p_k^s) > \rho_k(p_{k-1}^s)$ *for all* p_k^s *and* p_{k-1}^s *such that* $p_k^s = p_{k-1}^s > \tilde{p} = \rho^{pr} - b$ *.*

Proof. Note, that the reservation price essentially represents the cost of the next-best available alternative to buying now at the shop the consumer is currently visiting. If the next-best available alternative is to go back to the lowest-priced firm in the sample before visiting this shop, i.e., $p_{k-1}^s < \tilde{p}$ the reservation price is simply independent of the periods, i.e., $\rho_{k+1}(p_{k-1}^s) = \rho_k(p_{k-1}^s) = p_{k-1}^s + b$.

Now consider the case where the next-best available alternative is to continue searching. Let $\{\rho_k(p^s_{k-1})\}_{k=1}^N$ be the sequence of the reservation price functions. Consider the following suboptimal strategy. If on step k the consumer makes a decision to visit one more firm he either buys at the firm he visits at step $k+1$ or continues his search but *forgets* about this firm later on (thus, he never comes back to that firm). Let us denote a reservation price under this suboptimal strategy by $\rho'_k(p^s_{k-1})$. Then $\rho_k(p^s_{k-1}) \le \rho'_k(p^s_{k-1})$. On the other hand for any $p^s_{k-1} > \tilde{p}$ we get

$$\begin{aligned}\rho'_k(p^s_{k-1}) &= F(\rho_{k+1}(p^s_{k-1}))\mathbb{E}(p_{k+1}|p_{k+1} < \rho_{k+1}(p^s_{k-1}) + \\ &+ (1 - F(\rho_{k+1}(p^s_{k-1})))\rho_{k+1}(p^s_{k-1}) < \rho_{k+1}(p^s_{k-1})\end{aligned}$$

which completes the proof.

□

We finally consider the limiting case (of perfect competition) where there are potentially infinitely many prices to sample. As the time dependency of the reservation prices disappears due to the fact that now the cost of continuing to search is independent of time, i.e., $\rho_k(p^s_{k-1}) = \rho_{k+1}(p^s_k)$. For prices below $\tilde{p}$, we knew already that this equality holds. Interestingly, with infinitely many firms and previously sampled prices above $\tilde{p}$, the reservation prices becomes independent of previously sampled prices and equal to the reservation price under perfect recall. Thus, the cost of going back to previously sampled firms does not play an important role under perfect competition.

Proposition 2.3.8. *Let $K \in \mathbb{N}$. Then for any $p \ge \tilde{p}$ $\lim_{N\to\infty} \rho_K(p) = \rho^{pr}$.*

Proof. Note, that for any $p \ge \tilde{p}$, $C_{N-1}(p)$ is fixed and does not depend on N. On the other hand for any $p \ge \tilde{p}$ we have

$$\begin{aligned}C_k(p) &= F(\rho_{k+1}(p))\mathbb{E}(p_{k+1}|p_{k+1} < \rho_{k+1}(p)) + \\ &+ (1 - F(\rho_{k+1}(p))\mathbb{E}(C_{k+1}(p_{k+1})|p_{k+1} \ge \rho_{k+1}(p)) \le \\ &\le C'_k(p) = F(\rho_{k+1}(p))\mathbb{E}(p_{k+1}|p_{k+1} < \rho_{k+1}(p)) + (1 - F(\rho_{k+1}(p))\rho_{k+1}(p)\end{aligned}$$

Note, that $C'_k(p)$ can be rewritten in the form:

$$C'_k(p) = \rho_{k+1}(p) + c - \int_{\underline{p}}^{\rho_{k+1}(p)} F(p)dp$$

Therefore, following our notation

$$C'^{+}_k(p) = \rho^{+}_{k+1}(p)(1 - F(\rho_{k+1}(p))) \leq C'^{+}_{k+1}(p)(1 - F(\rho_{k+1}(p)))$$

Then

$$C'^{+}_K(p) \leq \prod_{i=K}^{N-1} C'^{+}_{i+1}(p)(1 - F(\rho_{i+1}(p)))$$

As $1-F(\rho_{i+1}(p)) < 1$ for any $p > \tilde{p}$ and $i > K$ (note, that $\rho_{i+1}(p) < \rho_{i+2}(p) \Rightarrow 1 - F(\rho_{i+1}(p)) > 1 - F(\rho_{i+2}(p))$) we get

$$\lim_{N\to\infty} C'^{+}_K(p) = 0.$$

.

Now note that from proposition 3.5 it follows that $\rho_K(\tilde{p}) = \rho^{pr}$ and therefore $C_K(\tilde{p}) = \rho^{pr}$. Therefore, since $C'_K(p)$ is a continuous function we get that for any $p \geq \tilde{p}$,

$$\lim_{N\to\infty} C'_K(p) = \rho^{pr}.$$

Therefore

$$\lim_{N\to\infty} C_K(p) = \rho^{pr}.$$

□

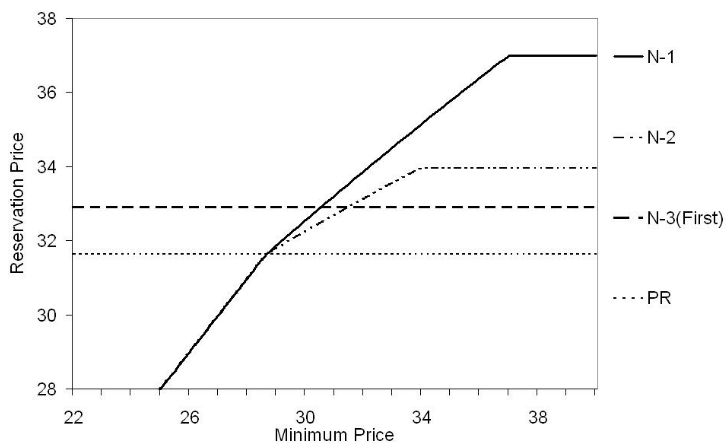
Thus, under perfect competition the reservation price under costly second visits is exactly identical to the case where consumers have perfect recall.

2.4 Example

In the previous section, we have provided a general characterization of the time- and history-dependency of the reservation price. In this Section we provide an example to illustrate the features of these reservation prices. The example clearly shows that it can be rational to accept a price in a future period even if a lower price has been observed in the past.

Consider the uniform distribution of prices on $[0, 100]$. Assume there are 4 firms in the market, search costs c are equal to 5 and the costs of going back to a previously sampled firm b equals 3. The reservation prices after visiting no, one and two firms as well as the reservation price under perfect recall are presented in Figure 2.2. In this case, the reservation price under perfect recall equals approximately 31.62, while the reservation price before visiting any shop under costly second visits equals 32.90. Thus, if a consumer faces, say, a price of 33 in the first period he decides to continue searching. From Figure 2 it is clear, however, that if the third price the consumer encounters is say 34 it is optimal for him to stop.

Figure 2.2: Simulation Results for Uniform Distribution.



Parameters of simulation: $N = 4, a = 100, b = 3, c = 5$.

The figure also illustrates most of the results we proved in the previous section. In particular, it is easy to observe that:

- all reservation price functions are non-decreasing in p_k^s (Theorem 2.3.3);

- all the reservation price functions have a kink at the same $\tilde{p} = \rho^{pr} - b \approx 28.62$ (Proposition 2.3.5);
- the sequence of reservation prices is non-decreasing in the number of firms left, and strictly increasing for all prices above $\tilde{p}$ (Proposition 2.3.7);

2.5 Conclusions

In this chapter, we built a consumer search model where we explicitly model the cost of going back to stores already searched. We show that in general the optimal search rule under costly second visits is very different from the optimal search rule under perfect recall. Under costly second visits, the optimal search behaviour is nonstationary and, moreover, the reservation price is not independent of previously sampled prices. Consequently, it may happen that the optimal search strategy tells consumers to reject relatively low prices early on in the search process and accept higher prices later on. Stationarity is obtained only in the special case of perfect competition where there are infinitely many firms.

Future work should incorporate the optimal search rule under costly second visits characterized here, in a full consumer search model where the pricing behaviour of firms is explicitly modelled. Due to the assumption of costless recall, consumers in most search models actually do not search very much in any symmetric equilibrium. This is because in a symmetric equilibrium where firms set prices above the reservation price under free recall, the firm charging the highest price in the market does not make any sales as there is always a firm present with a lower price and consumers will continue searching until they find this price. So, no firm would want to charge the highest price above the reservation price and, therefore, in equilibrium no prices above the reservation price will be charged. Future research should inquire whether this result continues to hold under costly second visits. Under costly second visits, it may well be possible that for example in a duopoly market all firms charging with some positive probability prices above the reservation price (for the first price observation) is part of an equilibrium as by doing so these firms still have two potential sources of revenue: (i) from consumers who first searched firm A, next search firm B and want to stop there even if B charges higher price than A, due to the costs of going back to firm A and (ii) from consumers who first search firm A, after that B and go back to firm A if it had sufficiently lower prices.

We have not analyzed a full model including price setting behaviour of firms in this chapter as this would require a choice of a specific market set-up. Here, we have characterized the optimal search rule under costly second visits in a general form that could be applied to any specific market environment.

Chapter 3

Costly Second Visits: Oligopolistic Competition

3.1 Introduction

In the previous chapter we studied the implications of introducing the assumption of costly second visits for consumer search behaviour. We argued that this assumption is better aligned with the general philosophy of consumer search literature than the assumption of perfect recall as when consumers have to make a cost to visit a store in the first place, they typically also have to incur some cost, possibly somewhat smaller, to execute the purchase at that shop when they have left the store to first consider other prices elsewhere; often the consumer simply has to travel back to the shop or to go back to a previously visited website. We showed that if the assumption of costless second visits (perfect recall) is relaxed to allow for costly second visits, the consumer's behaviour changes drastically. The optimal search rule is no longer characterized by a time-independent stationary reservation price. the reservation price depends both on the search history and the number of firms left.

The natural question is how this change in consumer's search strategies caused by introduction of costly second visits affects the market equilibrium? Is the assumption of perfect recall crucial for the analysis of search markets? The answer to these questions depend on the industry setup. In this chapter we use a conventional model of oligopolistic competition with homogeneous goods and sequential consumer search, which was pioneered by Stahl (1989), to study the implications of costly second visits. The distinguishing feature of the Stahl model is that there are two types of consumers, the informed and the uninformed consumers. Informed consumers have zero search cost and pay at the lowest price in the market. Uninformed consumers have positive search cost and engage in optimal sequential search. In the Stahl model N firms set prices simultaneously to maximize profits, where the demand potentially comes from both types of consumers.

In this chapter we show, that even though the consumer's search strategy is much more complicated, the market analysis essentially remains the same. We have two types of results that underline this general conclusion. First, the equilibrium that is found by Stahl (1989) remains an equilibrium In this equilibrium firms choose a price from a price distribution that is such that consumers with a positive search cost buy immediately in the first store they visit even the definition of the reservation price does not need to be adjusted. The second result is that there are no other types of symmetric equilibria. Thus, the Stahl equilibrium remains the unique symmetric equilibrium if we allow for costly second visits. With costly second visits in principle firms may benefit from setting prices above the reservation price of the first search round. The standard argument why firms will not set prices above the first search round reservation price is that a firm that charges a price equal to the upper bound will not sell to any consumer as even the uninformed consumers will then continue to search after observing such a price and have then at least two prices to compare and the other price(s) are strictly smaller with probability one. This argument does not hold with costly second visits as a competitors that are visited first may have prices that are lower, but not so much lower that it pays for consumers to pay the cost of going back to these previously visited stores. We show, however, that the structure of the profit function is such, that if firms charge prices above first round reservation prices, they can never compensate the loss of demand with higher revenue per consumer.

Armstrong and Zhou (2010) give a particular interpretation of costly second visits. They show that costly second visits can be re-interpreted as buy-now discounts, i.e. as discounts consumers only get when they visit a firm for the first time: as soon as they walk out of the store without buying the possibility to receive a discount disappears. there are two main differences between their models and ours. We study an oligopolistic setting, where Armstrong and Zhou (2010) employ a monopoly model. On the other hand, the buy-now discount in Armstrong and Zhou (2010) is endogenously determined by firms, whereas our costly second visit is exogenous. Our analysis may have various other applications, in particular they can be applicable to the analysis of shopping-malls (see Non (2010)).

It is important to mention that our result holds true for the Stahl model and that we cannot guarantee that it also holds for other search models. We find, however, that from a theoretical perspective studying costly recall in the context of a model where firms choose prices according to some mixed strategy equilibrium is more interesting than in a setup where the symmetric equilibrium is in pure strategies (as in Wolinsky (1986) and Anderson and Renault (1999)). In the search models with homogeneous goods where the equilibrium is in mixed strategies, the uncertainty for consumers what the next search will bring is endogenously determined. In the search models with heterogeneous goods where the equilibrium is in pure strategies, the uncertainty for consumers what the next search will bring (and the reason for continued search) is exogeneously imposed through the match function. This makes the introduction of costly second visits in models a la Stahl theoretically challenging.

The structure of this chapter is as follows. In section two we briefly review the industry setup introduced by Stahl (1989). In the next section we will use this setup together with the results of Chapter 2 to show that there is no symmetric equilibrium where consumers continue to search after having visited the first store they visited. The last section concludes.

3.2 Model

We analyse the oligopolistic industry model that was first introduced in Stahl (1989). Consider a market where N firms produce a homogenous good and have identical production costs, which we normalize to zero. Each firm decides upon the price at which it is going to sell the good in the market. There are two types of consumers in the market. A fraction λ of all consumers are "shoppers", i.e. these consumers like shopping or have zero search costs for other reasons. We assume that these consumers know all prices in the market. The remaining fraction $1-\lambda$ of consumers is uninformed. These consumers engage in sequential search and get their first price quotation for free, but any subsequent price quotation comes at a search cost c. All consumers have a unit demand and an identical valuation for the good which we denote by v and $v > c$. If the consumer decides to go back to the store she already visited before she incurs costs b where $0 \leq b \leq c$.

The timing in the model is as follows. First, firms simultaneously decide on their prices. A strategy of firm i is described by a probability distribution $F_i(p)$. We concentrate on symmetric equilibria with $F_i(p) = F(p)$ for all i. Let $\underline{p}$ be the lower bound of the support of the distribution and $\overline{p}$ be the upper bound of the support. After firms made their decisions, consumers decide. Shoppers simply buy at the firm with the lowest price. Uninformed consumers engage in optimal search as decribed above and analysed in greater detail in the previous chapter.[1]

3.3 Analysis

In this section we analyse the optimal way for firms to behave. We start with the question whether "Stahl-type" of pricing strategy, i.e. when all firms play mixed strategies with the support up to first reservation price is indeed an equilibrium in the model with costly second visits. Then we proceed with the investigation whether other types of equilibria are possible. First, we show that there is no equilibrium with "low" or "intermediate" level of the upper bound of the support, i.e. when the upper bound of the support is higher than first reservation price (Propositions 3.3.5, 3.3.6). Then we show that there is no equilibrium where the upper bound of the support is larger than the largest reservation price plus the costs of going back (Proposition 3.3.7).

We start the analysis with showing that the "Stahl-type" of equilibrium is also an equilibrium in the model with costly second visits.

Proposition 3.3.1. *There is a mixed strategy equilibrium where all firms charge prices below the first-round reservation price.*

[1]This assumption is often made in the search literature but can be relaxed; see Janssen, Moraga and Wildenbeest (2005) for details.

Proof. If the upper bound of the support $\overline{p} = \rho_1$, then $\max_p \rho_1(p) = \ldots = \max_p \rho_{N-1}(p) = \rho^{pr}$. Therefore, the equilibrium defined in Stahl (1989) is an equilibrium if none of the firms has a profitable deviation. The only (potentially profitable) way for firms to deviate is to charge prices above ρ_1. However, then this firm has a zero demand both from informed and uninformed consumers. Therefore, a profitable deviation does not exist, and the Stahl type of equilibrium is indeed an equilibrium. □

Our analysis showing the Stahl type of equilibrium is the unique equilibrium starts with a technical lemma establishing the relation between the highest and the lowest reservation prices.

Lemma 3.3.2. *For any p in the support of $F(p)$* $\frac{\rho_{N-1}(p)+b}{\rho^{pr}} < 2$.

Proof. Lemma 2.3.1 states that

$$\rho_{N-1}(p) = \min\left(p + b,\ c + p + b - \int_{\underline{p}}^{p+b} F(p)dp,\ p^*_{N-1}\right)$$

,

where p^*_{N-1} satisfies the equation

$$p^*_{N-1} = c + E(p_N | p_N < p^*_{N-1} + b)F(p^*_{N-1} + b) + (1 - F(p^*_{N-1} + b))(p^*_{N-1} + b).$$

Note, that first, $\rho_{N-1}(p) \leq p^*_{N-1}$, and, second, p^*_{N-1} satisfies the equation

$$c + b = \int_{\underline{p}}^{p^*_{N-1}+b} F(p)dp. \tag{3.3.1}$$

The reservation price under perfect recall is defined by:

$$c = \int_{\underline{p}}^{\rho^{pr}} F(p)dp. \tag{3.3.2}$$

From (3.3.1) and (3.3.2) it follows that $p^*_{N-1} + b < 2\rho^{pr}$. Indeed, if this were not true, by subtracting one equation from the other we get:

$$b = \int_{\rho^{pr}}^{p^*_{N-1}+b} F(p)dp > \int_{\rho^{pr}}^{2\rho^{pr}} F(p)dp \geq \int_{0}^{\rho^{pr}} F(p)dp > c, \tag{3.3.3}$$

which contradicts the assumption $b < c$. The second inequality stems from the fact that $F(p)$ in a non-decreasing function, the last form the definition of ρ^{pr}. Therefore $p^*_{N-1} + b < 2\rho^{pr}$ and since $\rho_{N-1}(p) \leq p^*_{N-1}$ the lemma is proved.

□

Corollary 3.3.3. $\forall k \in (2, N) : \frac{\rho_k(p)+b}{\rho_1} < 2$

Proof. Since $\rho_k(p) \leq \rho_{N-1}(p)$ (Proposition 2.3.7), $\rho_1 \geq \rho^{pr}$ we get

$$\frac{\rho_k(p) + b}{\rho_1} < \frac{\rho_{N-1}(p) + b}{\rho^{pr}} < 2$$

□

Using these results we can now formally prove the idea that there are no other symmetric equilibria than the generalized Stahl equilibrium. For such an equilibrium to exist it must be the case that the upper bound of the price distribution is strictly larger ρ^{pr}. To simplify notation we introduce the following definition.

Definition 3.3.4. *Let's denote r_k to be the maximum possible reservation price in the k-th search round, i.e., $r_k = \max_p \rho_k(p)$.*

The claim that there are no other symmetric equilibria is now proved in three consecutive steps. Proposition 3.3.5 shows that there are no equilibria where the upper bound of the support is smaller than to r_{N-1}). Proposition 3.3.6 shows that there are no equilibria where the upper bound of the support is in between r_{N-1} and $r_{N-1} + b$). Finally, Proposition 3.3.7 shows that there are no equilibria where the upper bound of the support is above $r_{N-1} + b$.

Proposition 3.3.5. *There is no equilibrium price distribution with $r_1 < \overline{p} < r_{N-1}$.*

Proof. It is easy to see that given the optimal search behavior all reservation prices are below or equal to the upper bound of the support of the distribution. Indeed, suppose $\overline{p} < r_{N-1}$. Recall, that

$$c + b = \int_{\underline{p}}^{r_{N-1}+b} F(p)dp$$

then

$$c = \int_{\underline{p}}^{r_{N-1}} F(p)dp$$

and therefore $r_{N-1} = \rho^{pr}$, which is not possible.

□

Now we analyze the "intermediate" case where the upper bound would be $\overline{p} \in [r_{N-1}, r_{N-1} + b]$. The proof of this proposition is based on the fact that in order to compensate firms for the loss in demand resulting from charging above r_{N-1}, the upper bound of the distribution has to be above $2r_1 - b$, which contradicts the relationship between reservation prices that is consistent with the search perspective as established Lemma 3.3.2.

Proposition 3.3.6. *There is no equilibrium price distribution with* $\overline{p} \in [r_{N-1}, r_{N-1}+b]$.

Proof. First, consider profits at r_1 and at $\overline{p}$:

$$\pi(r_1) = \lambda(1 - F(r_1))^{N-1} r_1 + \frac{1-\lambda}{N} S r_1$$

and

$$\pi(\overline{p}) = Y\overline{p}$$

It is clear, that $S \geq 2 - F(r_1)$. If firm charges $\overline{p} > r_{N-1}$ it only sells something, if all other firms set prices at least above r_1 (otherwise all consumers stop on the first step). Therefore, $Y < (1 - F(r_1))^{N-1} < (1 - F(r_1))$. Then it should be that

$$\frac{1-\lambda}{N}(2 - F(r_1))r_1 < \frac{1-\lambda}{N}(1 - F(r_1))\overline{p} \leq \frac{1-\lambda}{N}(1 - F(r_1))(r_{N-1} + b)$$

and therefore $\frac{r_{N-1}+b}{r_1} > 2$ which contradicts Corollary 3.3.3.
Thus, the theorem is proved.

□

Finally, we analyze the case where the upper bound is quite well above the highest reservation price. This part of the overall proof is the most complicated part. The idea of the proof is that if the upper bound of the support is larger than the highest reservation price, it is anyway bounded from above due to the structure of the upper part of the support of an equilibrium price distribution. This gives an upper bound on the profits firms receive from choosing a price equal to the upper bound. On the other hand, we argue that a price equal to the first-round reservation price should also be charged in equilibrium. Moreover, we show that this first-round reservation price should be larger than some lower bound, creating some lower bound on equilibrium profits. The last part of the proof shows that the upper bound we construct is smaller than the constructed lower bound yielding an inconsistency.

Proposition 3.3.7. *There is no equilibrium price distribution with* $\overline{p} > r_{N-1} + b$.

Proof. Let π_0 be the equilibrium profits. First, consider the profits of a firm that charges $\overline{p}$. As, by construction, $\overline{p}$ is in the support of the equilibrium price distribution, equilibrium profits are given by:

$$\pi_0 = \frac{1-\lambda}{N}(1 - F(\overline{p} - b))^{N-1}\overline{p} \tag{3.3.4}$$

As $\overline{p} > r_{N-1} + b$, a firm charging $\overline{p}$ does not get any informed consumers and only those uninformed consumers who have first visited all other firms, have observed these firms charge prices above r_{N-1} and then do not want to go back to these stores because of the cost of a second visit b. If a firm would charge $\overline{p} - b$ instead, its profits would be at least equal to

$$\left(\lambda(1 - F(\overline{p} - b))^{N-1} + \frac{1-\lambda}{N}(1 - F(\overline{p} - 2b))^{N-1}\right)(\overline{p} - b)$$

which is larger than or equal to

$$\left(\lambda(1 - F(\overline{p} - b))^{N-1} + \frac{1-\lambda}{N}(1 - F(\overline{p} - b))^{N-1}\right)(\overline{p} - b).$$

Whether or not $\overline{p} - b$ is in the support of the equilibrium price distribution, it should be the case that π_0 is larger than or equal to this expression, yielding

$$\overline{p} \leq \frac{1 - \lambda + \lambda N}{\lambda N} b \tag{3.3.5}$$

Therefore,

$$\pi_0 < \phi(\lambda, N) \equiv (1-\lambda)\frac{1-\lambda+\lambda N}{\lambda N^2}b \tag{3.3.6}$$

this is the upper bound on the equilibrium profit. Next, we will construct a lower bound on the equilibrium profit. To this end, consider profits at r_1. It is easy to see that r_1 should be in the support of the equilibrium price distribution as (i) by definition of r_1 it cannot be the case that the whole price distribution lies above r_1 and (ii) if the largest price in the support of the equilibrium price distribution below r_1 is strictly smaller than r_1, then a firm could increase to profits by deviating and charge r_1. To simplify notation, let $F(r_1) = m$. We then have that

$$\pi_0 = \lambda(1-m)^{N-1} r_1 + \frac{1-\lambda}{N} S r_1,$$

where $S \geq 1$ is the total probability that a consumer buys from the firm, arising form all possible search paths of consumers. The firm charging r_1 gets at least $1/N$ consumers who randomly arrive at its store in the first search round and $\frac{N-1}{N}\frac{1}{N-1}(1-m)$ of consumers who first visit another store, observe a price strictly larger than r_1 and then randomly visits the store under consideration. thus, it follows that $S \geq 2 - m$. Therefore, for any $p \leq r_0$ in the equilibrium support:

$$\pi_0 = \lambda(1-F(p))^{N-1} p + \frac{1-\lambda}{N} S p,$$

which gives,

$$F(p) = 1 - \left(\frac{\pi_0}{p\lambda} - \frac{1-\lambda}{N\lambda} S\right)^{\frac{1}{N-1}}$$

and

$$\underline{p}(r_1) = \frac{N\lambda(1-m)^{N-1} + (1-\lambda)S}{N\lambda + (1-\lambda)S} r_1.$$

Now consider a family of probability distributions:

$$F(p; K) = 1 - \left(\frac{\pi_0}{p\lambda} - \frac{1-\lambda}{N\lambda} S\right)^{\frac{1}{K-1}}$$

Then for $M \geq K$ $F(p, M) \leq F(p, K)$. Moreover, if we define $r_1(K)$ as

$$\int_{\underline{p}(r_1(K))}^{r_1(K)+b} F(p; K)dp = c + b,$$

then we get that the solution of this equation $r_1(K)$ is an increasing function of K, because $\underline{p}(r_1(K)$ is linearly increasing in $r_1(K)$ with slope less than 1 and $F(p, K)$ is decreasing in K. Therefore, $r_1(2) \leq r_1(K)$ for any K. It is also clear that $r_1(K)$ is increasing in c, therefore, $r_1(2)|_{c=b} \leq r_1(2)|_{c>b}$. Let's denote $r^* = r_1(2)|_{c=b}$. It follows that r^* is implicitly defined by

$$\int_{\underline{p}(r^*)}^{r^*+b} F(p, 2)dp = 2b$$

and therefore

$$\int_{\underline{p}(r^*)}^{r^*} F(p, 2)dp \geq b$$

or

$$\int_{\underline{p}(r^*)}^{r^*} \left(1 - \frac{\pi_0}{p\lambda} + \frac{1-\lambda}{N\lambda}S\right) dp = \left(1 + \frac{1-\lambda}{N\lambda}S\right)(r^* - \underline{p}(r^*)) - \frac{\pi_0}{\lambda} \ln \frac{r^*}{\underline{p}(r^*)} \geq b.$$

As $r^* \leq r_1$ for any N, b, c 8and fixed S, m it follows that

$$\pi_0 \geq \lambda(1-m)^{N-1} r^* + \frac{1-\lambda}{N} S r^*. \tag{3.3.7}$$

By plugging in the expressions for $\underline{p}(r^*)$ and this lower bound on π_0 we get

$$\left(1 + \frac{1-\lambda}{N\lambda}S\right)(r^* - \underline{p}(r^*)) = \frac{\lambda N + (1-\lambda)S}{\lambda N} \frac{N\lambda(1-(1-m)^{N-1})}{\lambda N + (1-\lambda)S} r^* = (1-(1-m)^{N-1})r^*$$

$$\frac{\pi_0}{\lambda} \ln \frac{r^*}{\underline{p}(r^*)} \geq \frac{r^*}{\lambda}\left(\lambda(1-m)^{N-1} + \frac{1-\lambda}{N}S\right) \ln \frac{N\lambda + (1-\lambda)S}{(1-m)^{N-1}N\lambda + (1-\lambda)S}$$

which gives a lower bound for r^*:

$$r^* \geq \frac{\lambda b}{\lambda(1-(1-m)^{N-1}) - \left(\lambda(1-m)^{N-1} + \frac{1-\lambda}{N} S\right) \ln \frac{N\lambda + (1-\lambda)S}{(1-m)^{N-1} N\lambda + (1-\lambda)S}}.$$

Therefore $\pi_0 \geq \psi_0(\lambda, m, N, S)$ where

$$\psi_0(\lambda, m, N, S) \equiv \frac{\lambda\left(\lambda(1-m)^{N-1} + \frac{1-\lambda}{N}S\right) b}{\lambda(1-(1-m)^{N-1}) - \left(\lambda(1-m)^{N-1} + \frac{1-\lambda}{N} S\right) \ln \frac{N\lambda + (1-\lambda)S}{(1-m)^{N-1} N\lambda + (1-\lambda)S}}.$$

This is the lower bound on equilibrium profits. It is straightforward to verify that $\frac{\partial}{\partial S}\psi_0(\lambda, m, N, S) > 0$ and because $S \geq 2 - m$ we get that

$$\pi_0 \geq \psi_0(\lambda, m, N, S) > \psi(\lambda, m, N) \equiv \psi_0(\lambda, m, N, 2-m).$$

Now, since $\pi_0 < \phi(\lambda, N)$ and $\pi_0 > \psi(\lambda, m, N)$ the equilibrium can only exist if the lower bound on profits is smaller than the upper bound, or $\xi(\lambda, m, N) \equiv \phi(\lambda, N) - \psi(\lambda, m, N) > 0$.

It is possible to verify that $\psi(\lambda, m, N)$ is decreasing function of m and that

$$\lim_{m\to 1} \frac{1}{(1-\lambda)b} \cdot \xi(\lambda, m, N) = \frac{1-\lambda+\lambda N}{\lambda N^2} - \frac{\lambda}{N\lambda - (1-\lambda)\ln\frac{1-\lambda+N\lambda}{1-\lambda}}. \quad (3.3.8)$$

Therefore $\xi(\lambda, m, N) > 0$ only if the denominator of the second fraction in (3.3.8) is negative, which is equivalent to

$$\ln\frac{N\lambda + 1 - \lambda}{1-\lambda} > \frac{\lambda N}{1-\lambda}, \quad (3.3.9)$$

or, the denominator is positive, but the expression still holds, which is equivalent to

$$\ln\frac{N\lambda + 1 - \lambda}{1-\lambda} < \frac{\lambda N}{1-\lambda+N\lambda}. \quad (3.3.10)$$

Let us start with (3.3.9). It is clear that at $\lambda = 0$ both the right hand side and the left hand side of (3.3.9) are equal to 0. However,

$$\frac{\partial}{\partial\lambda}\left(\ln\frac{N\lambda+1-\lambda}{1-\lambda} - \frac{\lambda N}{1-\lambda}\right) = -\frac{\lambda N^2}{(1-\lambda)^2(1-\lambda+N\lambda)} < 0$$

Thus, the left hand side of (3.3.9) increases slower than the right hand side, and thus (3.3.9) can never hold.

Now we proceed with (3.3.10). Again, at $\lambda = 0$ both the right hand side and the left hand side of (3.3.10) equal to 0. If we take the derivative of the difference again we get

$$\frac{\partial}{\partial \lambda}\left(\ln \frac{N\lambda+1-\lambda}{1-\lambda} - \frac{\lambda N}{1-\lambda+N\lambda}\right) = \frac{\lambda N^2}{(1-\lambda)(1-\lambda+N\lambda)^2} > 0.$$

Therefore, the left hand side of (3.3.10) increases faster than the right hand side, and so (3.3.10) cannot hold either. Therefore, there is no equilibrium with $\overline{p} > r_{N-1}$. □

Thus, the "Stahl-type" of equilibrium is the only symmetric equilibrium in the model.

3.4 Conclusions

Most consumer search models assume that consumers have costless access to prices in stores they already visited, but have to pay a search cost to visit the store in the first place. In the previous chapter we have shown that without the assumption of costless second visits, the optimal sequential search rule is no longer characterized a unique, stationary reservation price. Thus, the costless recall assumption is, however, at odds with the general consumer search literature where the cost consumers have to make to buy at a particular store are central: if there is a cot to go to the store in the first place, there is often also a (smaller) cost of going back to that store.

In this chapter we have shown that search cost literature does not need to make this assumption of costless recall by analyzing whether the results presented in the celebrated paper by Stahl (1989) are robust to the assumption of costly second visits: our analysis shows that the equilibrium analyzed by Stahl remains an equilibrium under the alternative assumption of costly second visits and that, in addition, there do not exist other possible symmetric equilibrium outcomes in the oligopolistic competition setup. even though the optimal search behaviour of the consumers can be very complicated, firms behave in such a way that they do not change prices above the first search round reservation price. The main reason for this finding is that if a firm charges a price above this first search round reservation price, it loses relatively so many consumers that can never be sufficiently compensated by the increase in price. Given this striking result, it is interesting to see in future research whether a similar conclusion holds true for other search models.

Chapter 4

Minimum Price Guarantees In a Consumer Search Model

4.1 Introduction

It is well known that minimum price guarantees (MPGs) of one sort or the other are found in many sectors and industries. In retail markets, minimum price guarantees (MPGs) often take the form that sellers offer consumers who buy their products to match any other price a competitor charges for identical products provided that they have proof that an identical product is sold by a competitor at a nearby shop within a well-defined time period. It is this type of pricing matching policy that is our main interest in this paper. Major department stores, electronic goods stores and many other retail companies offer MPG in order to insure their potential clients against the possibility that they later regret buying the good if a lower price has been found in a competitor's store. Alternative forms of MPGs offer to give back (100+x)% of the price difference (so called price beating strategies) or offer a 'free lunch' in addition to matching prices (see, e.g., IKEA stores).[1] Most firms give the price difference only to consumers who provide evidence of lower prices elsewhere and do not commit to change list

[1]The biggest supermarket in the Netherlands, Albert Heijn, introduced in spring 2009 a policy that gave customers a free apple pie in addition to "all your money back policy" in case customers could show that other shops had lower prices for identical products.

prices.[1]

The effect of MPGs on the (pricing) behavior of competitors has been discussed in the economics as well as in the business and the law literature. The main conclusion from these literatures is that despite the appearance of creating additional competitive pressure on the pricing behaviour of firms, MPGs are in fact highly anticompetitive. One argument that has been made (cf., Salop (1986)) is that MPGs facilitate collusion as they remove the incentives to undercut. MPGs, so it is argued, do not just contain information for consumers, but in fact convey the information to competitors that any attempt to undercut will be automatically followed, i.e., MPGs work as a trigger strategy that helps firms to collude. Moreover, MPGs are an extremely cheap way of doing so as firms do not have to spend any resources on monitoring competitor's behaviour. Although some MPGs take the form that firms ex ante commit to change their list prices if they are informed that a competitor has a lower price (see above), most MPGs restrict the MPG to the client that has informed the firm of a lower price elsewhere, i.e., list prices are unaffected. This means that most MPGs actually are dissimilar to trigger strategies and it is therefore unclear whether they really support collusive practices.

A second argument that has been made (cf., Png and Hirschleifer (1987)) is that MPGs are an effective way to price discriminate between shoppers and non-shoppers. In the absence of MPGs, the activity of shoppers forces firms to reduce prices market- wide. Shoppers provide a positive externality to non-shoppers and force firms to set more competitive prices. With MPGs, however, the effect of the disciplining power of shoppers is limited to these shoppers themselves according to Png and Hirschleifer and act as a price discrimination mechanism for firms that can set high list prices and provide shoppers with discounts (see, also, Edlin (1997)). One shortcoming of the model proposed by Png and Hirschleifer is that the behavior of shoppers and non-shoppers is exogeneously imposed: shoppers always compare all prices, and more importantly, non-shoppers always go to one shop and buy if the price charged is below their willingness to pay. The effect MPGs may have on the search behavior of consumers is not analyzed.

[1] There are, however, some firms that commit to lowering list prices if competitors offer lower prices (see, e.g., Comet Services at comet.co.uk).

According to both arguments discussed so far, the imposition of MPGs reduces economic welfare and this had led Edlin (1997) to investigate the legal possibilities to prohibit MPGs under the Sherman Act. Recently, Moorthy and Winter (2006) have argued that MPGs may also have a pro-competitive effect in case products are horizontally differentiated and firms have different production costs. In such a context MPGs may signal to consumers that the firm under consideration really has a lower price. The lower price that is charged generates sufficient additional demand to compensate the firm for the lower profit per unit. High cost firms may find it too expensive to imitate the low pricing behavior of low cost firms, thereby allowing MPGs to work as a signalling device. Moorthy and Winter's model nicely illustrates how MPGs may work in markets with product heterogeneity. Most MPGs clauses, however, stipulate that the guarantee only comes into effect if prices of identical products at nearby shops are compared. This means that Moorthy and Winter's analysis is restricted to markets where geographical differentiation is important and transportation costs are high. Chen et al. (2001) also show that price matching policies may have pro-competitive effects in case they are pre-announced and there are consumers who prefer to shop at a particular store but are mindful of saving opportunities. In this paper the "search" behaviour of customers is also exogenously given as in Png and Hirschleifer (1987) and Varian (1980).

In this paper, we argue that MPGs have an important effect on the search behaviour of consumers which so far has not neglected in the literature on minimum price guarantees. To study this effect, we cast our model in a conventional sequential search setup a la Stahl (1989). Moreover, we assume that there is a certain probability that after the purchase a customer is informed about another price quotation. This probability represents the level of information communication among the customers (as in Galeotti (2009)). In such a setting, the main decision consumers have to make concerns their reservation price, i.e., the maximal price at which they will buy. We show that an MPG increases this reservation price as in the presence of MPGs consumers do not only buy the commodity under consideration, but in addition buy *an option* that if they are later informed of lower prices, they get the price difference back. Consumers value this option and this increases their reservation prices. Higher reservation prices, in turn, give firms the opportunity to raise their list prices, thereby increasing their profits. Thus, the key point of the paper is that the option value MPGs present impacts on the distribution of prices set by firms.

Another notable difference between our model and the existing literature is that we consider situations where consumers do not know in advance whether a firm that is visited has an MPG or not, i.e., MPGs are not pre-announced or advertised and consumers just encounter them when they arrive in a store. This setting where information about MPGs is revealed simultaneously with price information fits major consumer markets, such as electronics shops. In many of these shops, firms often put a label "minimum price guarantee" on their price labels, but not on their whole assortment. Moreover, at different points in time they have different products to which the MPG applies.[1]

We arrive at the following results. First, in our environment only two types of equilibria exist: one where firms do not set MPGs at all, and one where firms set MPGs with a certain positive probability, which is strictly smaller than one. The latter equilibrium only exists when the level of communication among consumers is relatively high. Thus, importantly, an equilibrium where firms set MPGs for sure does not exist. This explains that in markets where MPGs are not announced, but only discovered when a costumer arrives in the shop, firms randomize the products for which the MPG applies. This often happens in supermarkets and electronics stores. Second, the support of the equilibrium price distribution of a firm that provides MPG is always above the support of the distribution of a firm without MPG. This fact that firms setting MPGs have prices that are not below the prices of rivals firms without MPG is supported by empirical research (see Arbatskaya et al. (2004)). To understand the proper effect of MPGs empirically, our paper suggest, however, that one should not just compare prices in stores with and without MPGs, but instead one should also inquire whether the prices in stores without MPGs are shifted upwards when MPG *can be set* with some probability. Despite the fact that consumers can execute their

[1]We do not want to argue that this setting where MPGs are not pre-announced applies to all markets and it is certainly an interesting question to investigate what market characteristics are more prone to pre-announced MPGs and where pre-announcements are not observed. We leave this as a question for future research.

MPG with some positive probability if they are informed of lower prices, consumers are strictly worse off. Moreover, the better consumers communicate, the higher the equilibrium prices and the higher the prices consumers expect to pay even taking the probability into account that consumers can execute the MPG. Finally, we consider the possibility of firms offering price-beating strategies and show that they are always dominated by price-matching policies. The reason is that in markets where MPGs are not pre-announced, *in equilibrium* MPGs only affect the reservation prices.

The structure of the paper is as follows. In the next section we present the setup of the model. Section three contains the equilibrium analysis and main comparative statics results. In section four we show that price-beating is never optimal. Section five briefly concludes.

4.2 Setup

Consider a market where two firms produce a homogenous good and have identical production costs, which we normalize to zero. Firms set prices and decide whether or not to provide minimum price guarantees (MPGs). By providing an MPG, a firm commits to compensate the difference between its price and the price of the competitor, if the customer who has bought the product from the firm provides evidence that the lower price exists.

Like in the model of Stahl (1989) there are two types of consumers. A fraction $\lambda \in (0, 1)$ of all consumers are “shoppers”, i.e. these consumers like shopping or have zero search costs for other reasons. We assume that these consumers know all prices in the market as well as whether some of the firms set minimum price guarantees. The remaining fraction $1 - \lambda$ of consumers is uninformed. These consumers engage in sequential search and get their first price quotation for free, but any subsequent price quotation comes at a search cost c. All consumers have identical valuation for the good denoted by v and $v > c$. We assume that v is non-binding in the model, i.e. it is sufficiently large not to influence reservation prices. Whether a firm provides MPGs or not is revealed simultaneously with observing the price quotation of that firm. After the consumer has bought the good there is an exogenous probability $\mu \in (0, 1)$ that she observes (costlessly) the price of another firm. This information can come either form friends (as in Galeotti (2009)) or just accidentally because she noticed the price in the other store.

The timing in the model is as follows. First, firms simultaneously decide on their prices and minimum price guarantees. Firm i decides to set up minimum price guarantee with probability α^i, and then set prices with a probability distribution $F_0^i(p)$ if no minimum price guarantees set, and with $F_1^i(p)$ if it provides a MPG. Thus, the strategy of firm i is a tuple $\{\alpha^i, F_0^i(p), F_1^i(p)\}$. After firms made their decisions, consumers decide. Shoppers buy at the firm with the lowest price.[1] After observing a first price, uninformed consumers have to decide whether to buy at that firm or to continue search. After all purchasing decisions have been made, customers have some probability of getting a price quotation of the firm, which they did not search. If this price is less than the purchase price, and the purchase was made in a firm providing an MPG, the customer costlessly claims the price difference, which is paid back by the firm.

We look for symmetric perfect Bayesian equilibria. In such an equilibrium firms choose the same probability of setting MPGs and choose prices with the same probability density function in case they do and do not set MPGs, i.e., we look for equilibria where $\{\alpha^1, F_0^1(p), F_1^1(p)\} = \{\alpha^2, F_0^2(p), F_1^2(p)\}$.

4.3 Analysis

We start our analysis by investigating the search behaviour of uninformed consumers. To this end, let us denote by $\{\underline{p}_j, \overline{p}_j\}$ the lower and upper bounds of $F_j(p)$, $j = 0, 1$ and $\underline{p} = \min\{\underline{p}_0, \underline{p}_1\}$. Let $F(p) = (1-\alpha)F_0(p) + \alpha F_1(p)$ be the weighted average of the two equilibrium price distributions. Then the optimal search behaviour is defined by two reservation prices: one for firms with and another for firms without an MPG.

[1]In principle, if on of the firms charges the price lower than its competitor, while the competitor sets up the minimum price guarantees, shopper should be indifferent between the firms. We take one of possible models of their behaviour. One can think that there are infinitely small costs ϵ of claiming MPG, so shoppers prefer just to buy at the lowest price.

Lemma 4.3.1. *Uninformed consumers accept all prices at or below* r_0 *at a firm that does not provide MPG, and continue search otherwise; they accept all the prices at or below* r_1 *at a firm with MPG, and continue to search otherwise, where* $\{r_0, r_1\}$ *are defined by*

$$\begin{aligned} \int_{\underline{p}}^{r_0} F(p)dp = c \\ \int_{\underline{p}}^{r_1} F(p)dp = \frac{c}{1-\mu} \end{aligned} \tag{4.3.1}$$

Proof. After observing price r_0 at a firm without minimum price guarantees, a consumer has to be indifferent between buying now and continuing to search. If the consumer continues to search, she proceeds to the next firm. The next firm does not set MPG with probability $1-\alpha$, and in this case the consumer can choose the smallest price of r_0 and a random price p that is distributed according to F_0. Similarly, for when she continues to search and happens to visit a store with MPG, which occurs with probability α. Therefore

$$\begin{aligned} r_0 &= c + (1-\alpha)\left(F_0(r_0)\mathbb{E}_0(p|p<r_0) + (1-F_0(r_0))r_0\right) \\ &+ \alpha\left(F_1(r_0)\mathbb{E}_1(p|p<r_0) + (1-F_1(r_0))r_0\right) \end{aligned}$$

using integration by parts, this expression can be simplified to the usual rule determining the reservation prices.

$$\int_{\underline{p}}^{r_0} F(p)dp = c$$

Now consider the case when the customer found herself at a shop that provides MPG. In this case if the customer accepts the price there is a probability μ that later she observes another price, which is either from a no-MPG store (with probability $1-\alpha$) or from a MPG store (with probability α). If she decides to continues searching, the situation is similar to the case described above. Therefore, the reservation price is defined by

$$\begin{aligned} (1-\mu)r_1 &+ \mu[(1-\alpha)\left(F_0(r_1)\mathbb{E}_0(p|p<r_1) + (1-F_0(r_1))r_1\right) \\ &+ \alpha\left(F_1(r_1)\mathbb{E}_1(p|p<r_1) + (1-F_1(r_1))r_1\right)] = \\ &= c + (1-\alpha)\left(F_0(r_1)\mathbb{E}_0(p|p<r_1) + (1-F_0(r_1))r_1\right) \\ &+ \alpha\left(F_1(r_1)\mathbb{E}_1(p|p<r_1) + (1-F_1(r_1))r_1\right) \end{aligned}$$

which simplifies to

$$\int_{\underline{p}}^{r_1} F(p)dp = \frac{c}{1-\mu}$$

□

It immediately follows from the lemma that $r_1 > r_0$, i.e., a consumer is willing to buy at a higher price if the firm happens to provide an MPG. This is quite natural as the consumer has a probability to receive a pay-back in case a MPG is provided. One can clearly see this happening for values of μ close to one. Indeed, if μ is close to one, then the customer visiting a firm with a MPG clause almost surely pays the minimum of the two prices in the market. If she decides to proceed to search then she pays c and again buys at the minimum of the two prices that are set. Thus, for high values of μ a consumer prefers to stop searching in the MPG store, almost independently of the price it observes there.

Now we turn to the equilibrium pricing behaviour of firms. It is a standard result in the consumer search literature that both $F_0(p), F_1(p)$ are atomless and that $\overline{p}_0 = r_0$, $\overline{p}_1 = r_1$. To provide a full characterization of equilibrium, we first show that certain types of equilibria cannot exist.

Proposition 4.3.2. *There is no symmetric equilibria with* $0 < \alpha < 1$ *and* $r_0 > \underline{p}_1$.

Proof. First, consider the profits of a firm which sets no minimum price guarantees. these profits are given by

$$\pi_0 = \lambda(1 - F(p))p + \frac{1-\lambda}{2}p. \tag{4.3.2}$$

On the other hand, profits of a firm that provides MPG are equal to

$$\pi_1 = \lambda(1 - F(p))p + \frac{1-\lambda}{2}\left((1-\mu)p + \mu F(p)\mathbb{E}(p'|p' < p) + \mu(1 - F(p))p\right), \tag{4.3.3}$$

where the expectation is taken with respect to $F(p)$ and $p \leq r_1$. It is quite clear that equations (4.3.2) and (4.3.3) determine two different functional forms for $F(p)$. However, for the firm to be indifferent between setting and not setting a MPG it has to be the case that $\pi_0(p) = \pi_1(p)$ for all prices where the support of two distributions overlap. But this cannot be the case in more than one point, which together with that fact that $r_1 > r_0$ completes the proof.

□

Thus, if there is a positive probability that in equilibrium one firm provides MPGs, while the other does not, then it has to be the case that the price distributions are not overlapping. Given this result, we have three possible candidate equilibria: *(i)* $\alpha = 0$, *(ii)* $\alpha = 1$ and *(iii)* $0 < \alpha < 1$ but then $r_0 \leq \underline{p}_1$, i.e., a firm that does not provide an MPG will charge lower prices for sure than a firm with MPG.

We next argue that an equilibrium where both firms provide MPGs cannot exist either. In this, and the next propostions, it is important to realize that reservation prices are defined with respect to corresponding *equilibrium* price distributions. For example, if in an equilibrium both firms set MPG the consumer strategy is still represented by two reservation prices (r_0, r_1), both of them are defined by (4.3.1) using $F(p) = F_1(p)$. A similar point holds true for an equilibrium candidate where none of the firms sets MPG.

Proposition 4.3.3. *There is no equilibrium where both firms play* $\alpha = 1$.

Proof. If one firm chooses $\alpha = 1$ then the competitor has a profitable deviation by choosing $\alpha = 0$ and price r_0 which (as $F(p) = F_1(p)$ in this case) is defined by $\int_{\underline{p}_1}^{r_0} F_1(p)dp = c$. Indeed, since it has to be the case that $\underline{p}_1 < r_0 < r_1$, r_0 lies in the support of $F_1(p)$ and we get

$$\pi(r_0) = \lambda(1 - F(r_0))r_0 + \frac{1-\lambda}{2}r_0 >$$
$$> \lambda(1 - F(r_0))r_0 + \frac{1-\lambda}{2}\left((1-\mu)r_0 + \mu\mathbb{E}(\min(p, r_0))\right) = \pi_1.$$

Therefore, there is a profitable deviation. □

The idea behind this proposition is basically as follows. If a firm deviates from the proposed equilibrium where both firms provide MPG and simply sets the same price (in the lower end of the equilibrium distribution), but abandons the MPG, then the firm gets the same expected number of customers as with MPG, but the expected price paid by non-shoppers is higher since these consumers cannot exercise MPG anymore.

We now examine and characterize the remaining two candidate equilibria sequentially.

Proposition 4.3.4. *For all values of parameters there is an equilibrium where both firms choose $\alpha = 0$. The equilibrium price distribution in this case is*

$$F_0(p) = 1 - \frac{1-\lambda}{2\lambda}\frac{r_0 - p}{p} \tag{4.3.4}$$

Proof. Since $F(p) = F_0(p)$ we have

$$\int_{\underline{p}}^{r_0} F_0(p)dp = c$$
$$\int_{\underline{p}}^{r_1} F_0(p)dp = \frac{c}{1-\mu}.$$

In equilibrium each firms gets a profit of $\pi_0 = \frac{1-\lambda}{2} r_0$. Assume, one firm deviates and provides MPGs. Then the highest possible profit that can be obtained is by charging $p = r_1$. Indeed, it is clear that a firm only benefits from the deviation if $p > r_0$ (otherwise it gets the same number of customers, but might experience a loss from searchers who can exercise MPG), but then the shoppers would not buy from this firm anyway, so the firm has to extract maximum profits from shoppers, which is attained by charging $p = r_1$. Then

$$\pi_1 = \frac{1-\lambda}{2}\left((1-\mu)r_1 + \mu\mathbb{E}(p|p < r_1)\right) = \frac{1-\lambda}{2}\left((1-\mu)r_1 + \mu(r_0 - c)\right)$$

so that

$$\pi_1 > \pi_0 \Leftrightarrow r_1 - r_0 > \frac{\mu c}{1-\mu}$$

But we have

$$r_1 - r_0 = \int_{r_0}^{r_1} 1dp = \int_{r_0}^{r_1} F_0(p)dp = \int_{\underline{p}}^{r_1} F_0(p)dp - \int_{\underline{p}}^{r_0} F_0(p)dp =$$
$$\frac{c}{1-\mu} - c = \frac{\mu c}{1-\mu}$$

Thus, the best possible deviation gives the same payoff and a firm cannot strictly benefit from deviating. □

Finally, we focus on the intermediate case where firms do provide MPGs with some probability. The following proposition establishes existence of an equilibrium in mixed strategies.

Proposition 4.3.5. *An equilibrium with* $\alpha \in (0,1)$ *exists if and only if*

$$1 > \mu > \frac{4\lambda^2}{(1-\lambda)^2 \ln \frac{1-\lambda}{1+\lambda} + 2\lambda(1+\lambda)} > \frac{2}{3} \tag{4.3.5}$$

Proof. See Appendix. □

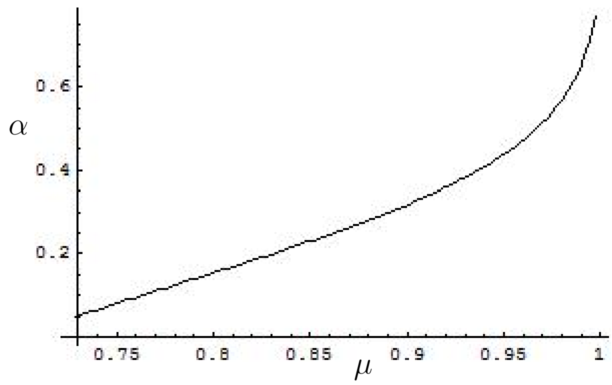
This result might seem to be a bit counterintuitive: firms set up (with some probability) MPGs only if there is sufficient probability that customers would exercise it. The explanation of course is that if μ is sufficiently high, consumers would accept higher prices at the store with MPG which more than offsets the adverse effect of exercising MPG on firms profits.

The following two Figures represent the relationship between the equilibrium probability of observing minimum price guarantees and the parameters of the model. Equation (4.3.5) shows that the probability of observing other prices should be relatively large. Figure 4.1 depicts the relation between the equilibrium probability of firms offering MPGs and the probability with which consumers observe another price quotation. The figure shows this relationship is positive: α is increasing with μ. Though high values of μ imply that *ex post* most of the consumers are informed ones, uninformed consumers are willing to buy at higher prices for higher μ. If μ is close to one, customer's are willing to accept virtually any price lower than v. Therefore firms are more likely to set MPGs and sell at higher prices when μ is large. Not surprisingly, Figure 4.2 shows that the greater the fraction of shoppers λ the lower the probability with which firms set MPG.

Now we proceed with the comparative statics analysis. The following proposition compares expected prices paid by consumers in the equilibrium with MPG and in the equilibrium without it.

Proposition 4.3.6. *Expected profits for firms in the equilibrium where MPGs are offered with positive probability are higher than the expected profits in the equilibrium without MPGs. As a consequence, in the equilibrium where MPGs*

Figure 4.1: Equilibrium probability of MPG as a function of μ at $\lambda = 0.2$.



are offered consumers pay higher expected prices (after a possible execution of MPG) than in the equilibrium without MPGs.

Proof. In fact, the equilibrium without MPG described by the same formulas as the equilibrium with MPG when α is set to be zero. The level of equilibrium profits for the equilibrium with MPG is

$$\pi(\alpha) = \frac{\lambda(1-\lambda+2\alpha\lambda)}{2(1-\alpha)\lambda + (1-\lambda+2\alpha\lambda)\ln\left(\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}c$$

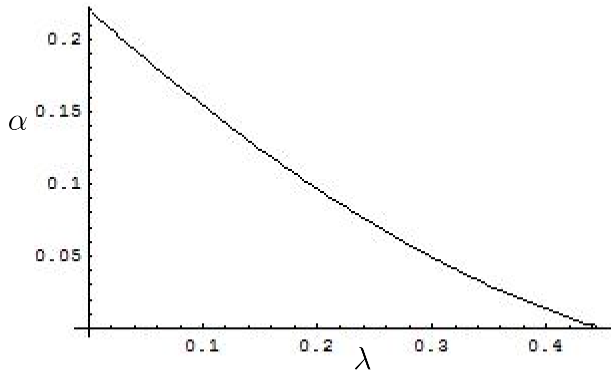
Then

$$\frac{\partial\pi}{\partial\alpha} = \frac{4(1-\alpha)\lambda^3}{\left(2(1-\alpha)\lambda + (1-\lambda+2\alpha\lambda)\ln\left(\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)\right)^2}c > 0$$

□

Proposition 3.7 shows the "anticompetitive" effect of a MPG in a search environment in the sense that in the equilibrium with MPGs the expected price is higher than in the equilibrium where MPGs are not offered for sure. The source of the anticompetitive effect is, however, different from that so far studied in the literature. It is not the case here that there is some type of collusive behaviour between the firms where MPGs play the role of a monitoring device. In our case the result is fully driven by consumer behaviour, namely by the willingness of

Figure 4.2: Equilibrium probability of MPG as a function of λ at $\mu = 0.8$.

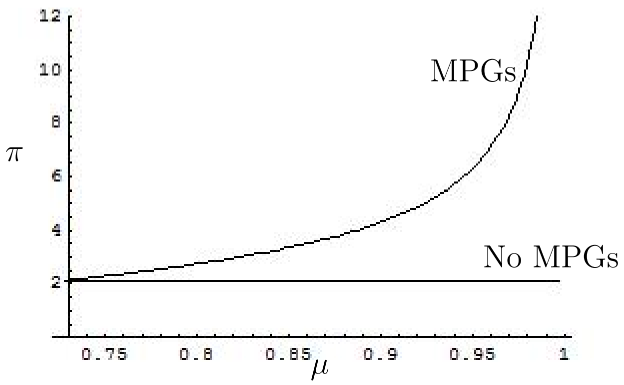


consumers to accept higher prices when firms do offer MPG. Another interesting observation is that the higher expected price paid in the equilibrium with MPG comes from two sources: (i) A firm charging an MPG can set a higher prices on average because of the higher reservation price of consumers at firms with an MPG and (ii) other firms without MPG react to these higher prices by setting higher prices themselves. Thus, also a firm that effectively does not charge MPGs has a higher price paid in the equilibrium where MPGs are charged with some positive probability compared to the equilibrium without MPGs. Figure 4.3 shows how a firm's expected profits depend on μ in the equilibrium where MPGs are offered with some probability and the equilibrium profits without MPGs. In the latter case, profits are, of course, a constant, whereas they are exponentially increasing in μ whenever this equilibrium exists.

4.4 Price-beating and free-lunch strategies

We next turn to the question whether firms will ever choose to provide price beating guarantees (PBG), if they do not announce (advertise) this policy in advance before consumers search for prices. To study this question, assume firms not only guarantee the purchase at the minimum price in the market if a lower price has been observed, but to compensate the customer even further by informing the

Figure 4.3: Expected profits as a function of μ at $\lambda = 0.2$.



customer that if she provides evidence of a lower price, say p', the effective purchase price will be $\beta p'$, where $\beta \leq 1$. The probability with which a price-beating strategy is chosen is (again) denoted by the probability α.

Even without a characterisation of the optimal search rule, we can argue that it is never optimal for a firm to offer a price beating strategies if it does not announce this policy in advance.[1] In the following two propositions we assume that prices charged are less or equal to corresponding reservation prices.

Proposition 4.4.1. *It is never optimal for a firm to offer a price beating policy with $\beta < 1$.*

Proof. Consider the profit function of a firm in an equilibrium where it provides a price-beating policy β:

$$\begin{aligned} \pi &= \lambda\left((1-\alpha)(1-F(p))p + F(p)\mathbb{E}(\beta p' | p' < p)\right) \\ &+ \tfrac{1-\lambda}{2}\left((1-\mu)p + \mu(1-F(p))p + \mu F(p)\mathbb{E}(\beta p' | p' < p)\right) \end{aligned} \tag{4.4.1}$$

The first term of this formula represent the profit the firm gets from informed customers. These consumers buy when this firm either has the highest price as

[1] http://www.besparingsmeter.nl:80/2009/06/17/winkelen/sla-je-slag-bij-ah/ reports cases where students bring cases with beer out of the supermarket Albert Heijn. After that Albert Heijn stopped this policy.

in that case they buy in order to exercise the price beating guarantee, or when the other firm has a higher price and does not offer a price beating policy. The last term represents the profit from uninformed customers. These consumers effectively buy at price p either because they are not informed about another price, or because the other firm charges a higher price. If the other firm charges a lower price, these consumers effectively pay a fraction of this lower price if they are informed about it. If the firm deviates and charges a higher β, it receives a higher profit as $\frac{\partial}{\partial \beta}\mathbb{E}(\beta p'|p' < p) > 0$ and the equilibrium price distribution is unaffected by the deviation. Thus, we have $\frac{\partial \pi}{\partial \beta} > 0$ and therefore it is optimal for a firm to set $\beta = 1$. □

This result shows that in markets where PBGs are not announced to consumers in advance so that consumers only are aware of these guarantees once they are in the shop and see the prices offered, price-matching is always preferred to price-beating. This comes from the fact that price-beating in these markets only affects the price the firm receives from those consumers who know the other firm has a lower price, but it does not affect the number of consumers. Therefore, for each individual firm it is better to choose $\beta = 1$, though if both firms were to stick to some $\beta < 1$ it would result in higher profits per firm.

Instead of offering price-beating strategies companies could compensate the price difference (match the price) and offer a "free lunch" on top of that. Thus, instead of offering $\beta p'$ the firm offers $p' - x$, where x is the value of the "lunch". The analysis and the results are very similar, as the following proposition shows.

Proposition 4.4.2. *It is never optimal for a firm to offer a price beating policy with $x > 0$.*

Proof. Consider the profit function of a firm in an equilibrium where it offers:

$$\begin{aligned} \pi &= \lambda\left((1-\alpha)(1-F(p))p + F(p)\mathbb{E}(p'-x|p'<p)\right) \\ &+ \tfrac{1-\lambda}{2}\left((1-\mu)p + \mu(1-F(p))p + \mu F(p)\mathbb{E}(p'-x|p'<p)\right) \end{aligned} \tag{4.4.2}$$

$\frac{\partial \pi}{\partial x} < 0$ because $\frac{\partial}{\partial \beta}\mathbb{E}(p' - x|p' < p) < 0$. Therefore it is optimal for the firm to choose x as low as possible, therefore $x = 0$. □

4.5 Conclusions

This paper analyzed the effect of firms offering price matching and price beating strategies in a consumer search model where reservation prices are endogenously determined. We restrict the analysis to markets where consumers are uninformed about whether or not firms offer minimum price guarantees before they come to the shop. We show that the effects of price matching and price beating are very different. Price matching can be observed in equilibrium, but only as an outcome of an equilibrium where firms randomize the decision to set minimum price guarantees. This may explain why multi-product firms (such as supermarkets and electronics shops) offer these policies over an ever changing group of products (if they offer them at all). We show that in the equilibrium where firms offer minimum price guarantees with strictly positive probability, the expected prices consumers pay are higher than in the equilibrium where no firm sets minimum price guarantees. The main reason for this result is that consumers' reservation prices increase considerably as they factor in the probability that they will be informed about lower prices later (and get their money back) and therefore they are more eager to buy now even if the price is relatively high. Importantly, even a shop that does not use minimum price guarantees sets higher prices on average. this basically follows as MPGs soften the competition.

Price beating strategies are different and it is never optimal to set them. The main reason is that firms do not gain additional consumers by setting such policies and only risk to get lower expected prices in case the competitor has a lower price.

Appendix: Proof of Proposition 3.6

To prove the proposition we explicitly construct and equilibrium and than show, that there is such a value of α that all the equilibrium conditions are satisfied.

Equilibrium price distribution support contains two parts: $[\underline{p}, r_0] \cup [\underline{p}_1, r_1]$.

The lower part of the support. The lower part of the support is defined by three equations:

$$\pi = \lambda(1 - F(p))p + \frac{1-\lambda}{2}p$$

$$F(\underline{p}) = 0$$

$$F(r_0) = 1 - \alpha$$

These three equations allow to write down everything as a function of (r_0, α). Indeed,

$$\pi(r_0, \alpha) = \frac{2\alpha\lambda + 1 - \lambda}{2} r_0 \tag{4.5.1}$$

$$\underline{p}(r_0, \alpha) = \frac{2\alpha\lambda + 1 - \lambda}{1 + \lambda} r_0$$

$$F(p; r_0, \alpha) = \frac{(1+\lambda)p - (1 - \lambda + 2\alpha\lambda)r_0}{2\lambda p}$$

Which using optimal search rule

$$\int_{\underline{p}}^{r_0} F(p)dp = c$$

gives an expression for r_0:

$$r_0 = \frac{2\lambda c}{2(1-\alpha)\lambda + (1 - \lambda + 2\alpha\lambda)\ln\left(\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)} \tag{4.5.2}$$

Thus, if the value of α is known the probability distribution on the lower part of the support is fully described.

The upper part of the support. Now let's consider the upper part.

The profit function is defined by:

$$\pi = \lambda(1 - F(p))p + \frac{1-\lambda}{2}\left(p - \mu \int_{\underline{p}}^{p} F(q)dq\right) \tag{4.5.3}$$

Since $\int_{\underline{p}}^{r_1} F(p)dp = \frac{c}{1-\mu}$ we get

$$\pi = \frac{1-\lambda}{2}\left(r_1 - \frac{\mu c}{1-\mu}\right)$$

Same way since $\int_{\underline{p}}^{p_1} F(p)dp = c + (1-\alpha)(\underline{p}_1 - r_0)$ we get

$$\pi = \alpha\lambda\underline{p}_1 + \frac{1-\lambda}{2}\Big(\underline{p} - \mu(c + (1-\alpha)(\underline{p}_1 - r_0))\Big)$$

Now, using equations (4.5.1) and (4.5.2) we can get expressions for r_1 and $\underline{p}_1$ as functions of α.

$$r_1 = \frac{2\lambda(1-\lambda(1-(2-\mu)\alpha)+\alpha\mu)+(1-\lambda)(1-\lambda+2\alpha\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}}{(1-\lambda)(1-\mu)\left(2(1-\alpha)\lambda+(1-\lambda+2\alpha\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}c$$

$$\underline{p}_1 = \frac{(1-\lambda+2\alpha\lambda)\left(2\lambda+(1-\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}{(1-\lambda(1-(2-\mu)\alpha-\mu)-\mu(1-\alpha))\left(2(1-\alpha)\lambda+(1-\lambda+2\alpha\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}c$$

Determination of α. To determine the value of α we use the following approach. We solve for the probability distribution on the upper part of the support using differential equation (4.5.3). The solution requires determination of the constant, say Q using boundary condition. We have two of them: $F(\underline{p}_1) = 1-\alpha$ and $F(r_1) = 1$, which gives us two values of the constant Q_1 and Q_2. Since the solution must satisfy both boundary conditions we get have to get $Q_1 = Q_2$ which gives us the equation on α. Note, that we do not calculate the optimal search integral here, since it is already incorporated in determination of r_1[1].

We start with the following differential equation:

$$Ay(x) + Bxy'(x) + Cx + D = 0 \tag{4.5.4}$$

The solution of this equation is

$$y(x) = Qx^{A/B} - \frac{Cx}{A+B} - \frac{D}{A}$$

[1] Another, may be more natural approach, is to use just one boundary condition and then explicitly calculate the search integral to get the equation for r_0 and α, as we did for the lower part of the support. However this approach results in more analytical complications, so we use the one presented in the text.

Now, if we compare (4.5.4) with (4.5.3) we can spot that it is the same equation with $x = p$, $y(x) = \int_{\underline{p}}^{p} F(p)dp$, $y'(x) = F(p)$, $A = -\frac{1-\lambda}{2}\mu$, $B = -\lambda$, $C = \frac{1+\lambda}{2}$, $D = \pi$.

Thus, the equilibrium price distribution is defined by

$$F(p) = \frac{1+\lambda}{2\lambda + (1-\lambda)\mu} - Q\frac{(1-\lambda)\mu}{2\lambda}p^{-\frac{2\lambda+(1-\lambda)\mu}{2\lambda}}$$

where Q is determined by initial conditions. $F(r_1) = 1$ and $F(\underline{p}_1) = 1 - \alpha$ give two values Q_1 and Q_2 which has to be equal.

$$Q_1 = -2\lambda(1-\lambda)\mu\left(1 - \frac{1+\lambda}{2\lambda+(1-\lambda)\mu}\right) \left(\frac{c\left(2\lambda(1-\lambda(1-\alpha(2-\mu)-\mu)-\mu(1-\alpha))-(1-\lambda)(1-(1-2\alpha)\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}{(1-\lambda)(1-\mu)\left(2(1-\alpha)\lambda+(1-(1-2\alpha)\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}\right)^{1+\frac{(1-\lambda)\mu}{2\lambda}}$$

$$Q_2 = -2\lambda(1-\lambda)\mu\left(1 - \alpha - \frac{1+\lambda}{2\lambda+(1-\lambda)\mu}\right) \left(\frac{c(1-(1-2\alpha)\lambda)\left(2\lambda+(1-\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}{(1-\lambda(1-\alpha(2-\mu)-\mu)-\mu(1-\alpha))\left(2(1-\alpha)\lambda+(1-(1-2\alpha)\lambda)\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}\right)^{1+\frac{(1-\lambda)\mu}{2\lambda}}$$

Equation $Q_1 = Q_2$ can be reduced to:

$$\left(\frac{(1-\lambda)(1-\mu)}{1-\lambda(1-\alpha(2-\mu)-\mu)-(1-\alpha)\mu}\right)^{\frac{(1-\lambda)\mu}{2\lambda}} = \left(\frac{2\lambda(1-\lambda+2\alpha\lambda-\alpha(1+\lambda)\mu)+(1-\lambda)(1-\lambda+2\alpha\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}}{(1-\lambda+2\alpha\lambda)\left(2\lambda+(1-\lambda)\mu\ln\frac{1-\lambda+2\alpha\lambda}{1+\lambda}\right)}\right)^{\frac{2\lambda+(1-\lambda)\mu}{2\lambda}} \tag{4.5.5}$$

First, we evaluate (4.5.5) at $\alpha = 0$. It is easy to spot that both LHS and RHS takes values of 1 for all (λ, μ). Second, we claim that the LHS of equation (4.5.5) evaluated at $\alpha = 1$ is greater than the RHS. Indeed, after canceling some terms the equation can be rewritten as $\left(\frac{1-\lambda}{1+\lambda}\right)^{\frac{(1-\lambda)\mu}{2\lambda}} = (1-\mu)$. Thus, the LHS is increasing in λ and as $\lambda \to 0$ it goes to e^{μ} which is greater than $(1-\mu)$. Finally, we examine the behaviour both of LHS and RHS around $\alpha = 0$. Obviously, if LHS decreases faster than the RHS, there must be an intersection point at

$\alpha \in (0,1)$. The derivative of the LHS with respect to α evaluated at $\alpha = 0$ equals to $-\frac{\mu(\lambda(2-\mu)+\mu)}{2\lambda(1-\mu)}$. The derivative of RHS evaluated at $\alpha = 0$ equals to $-\frac{(1+\lambda)(\lambda(2-\mu)+\mu)\mu}{(1-\lambda)\left(2\lambda+(1-\lambda)\mu \ln \frac{1-\lambda}{1+\lambda}\right)}$. Solving

$$-\frac{\mu(\lambda(2-\mu)+\mu)}{2\lambda(1-\mu)} < -\frac{(1+\lambda)(\lambda(2-\mu)+\mu)\mu}{(1-\lambda)\left(2\lambda+(1-\lambda)\mu \ln \frac{1-\lambda}{1+\lambda}\right)}$$

gives $\mu \in (-\frac{2\lambda}{1-\lambda}, 0) \cup (\frac{4\lambda^2}{(1-\lambda)^2 \ln \frac{1-\lambda}{1+\lambda}+2\lambda(1+\lambda)}, \infty)$. Given that μ is between 0 and 1 we get (4.3.5).

Now we show that $f(\lambda) \equiv \frac{4\lambda^2}{(1-\lambda)^2 \ln \frac{1-\lambda}{1+\lambda}+2\lambda(1+\lambda)} > 2/3$. First, this expression is increasing in λ with $f(1) = 1$. Second, we take a limit $\lim_{\lambda\to 0} f(\lambda)$. By applying l'Hopital's rule twice we get:

$$\lim_{\lambda\to 0} f(\lambda) = \frac{8}{4\frac{3+\lambda(3+\lambda)}{(1+\lambda)^2} + 2\ln\frac{1-\lambda}{1+\lambda}} = \frac{2}{3}$$

To prove that (4.3.5) is also a necessary condition we show that if the derivative of LHS of (4.5.5) is greater than the derivative of the RHS at $\alpha = 0$ than the LHS is higher than the RHS for any other α. That implies that there is no such a value of α which can equate both sides of the equation. Note, that both LHS and RHS of (4.5.5) are smooth functions in α, λ, μ. Therefore, we can directly verify the result on a mesh for $(\alpha, \lambda, \mu) \in (0,1)^3$. Numerical verification shows that (4.3.5) is indeed not only sufficient, but a necessary condition as well.

Chapter 5

On the relevance of irrelevant information

5.1 Introduction

The purpose of this chapter is to explore the role of information and interdependencies in the search process.

Consider a situation where you are planning to buy a book. You can read reviews on two different websites, say amazon.co.uk and amazon.com, and you are only interested in a book if it has positive reviews on both websites. Of course, reading reviews costs you some time. If you know that English readers are generally more skeptical than American readers, which website should you search first? Can it be optimal to stop and buy a book after searching just one site?

One can also think about an investment decision problem. Suppose the research department of a large company has to find out whether to buy a particular firm or to launch a new technology. The true value of the firm (the benefits of the technology) is unknown, and depends on two unknown parameters. For example, to calculate the value of the firm it is necessary to estimate future cash flows, and therefore it is necessary both to assess how the firm's costs will evolve and to predict demand fluctuations. These two factors are obviously somewhat correlated, at least due to mutual macroeconomic factors in prediction, such as the GDP growth rate or inflation. An accurate forecast can be quite costly and require both internal and external resources (such as marketing forecasts). Which of the parameters should the company research? What is the optimal order? Or perhaps some of the parameters do not require detailed investigation? For example, in an economy with high inflation both costs and revenues should follow the same pattern which is predetermined by the inflation rate.

What these examples have in common is that a researcher investigates some object with unknown attributes which she values. The researcher can investigate the attributes, and after learning one or more of their values she can either accept the object (buy, launch investment, etc.) or decline. Then the decision about the optimal investigation procedure is based on two factors: *(i)* utility concern, which is the value of a particular attribute for the researcher; *(ii)* informational concern, which is how knowing the true value of one attribute may provide information about the other.

If one attribute functionally determines the value of the other, it might be optimal to research the former first to save on further investigation costs, even if it is not particularly important from a utility point of view. More generally, if knowing the value of one attribute generates some information about another attribute, then an interesting search issue appears.

One of the obvious particular cases of this situation is when a customer wants to buy a bundle of goods and the price of each good is unknown (here prices play the role of attributes in the indirect utility function). This situation is quite well-studied in the literature. Burdett and Malueg (1981) and Carlson and McAfee (1984) studied optimal search rules for several commodities given various recall assumptions. However, in their setup the customer observes the entire price vector once she enters the store. Anglin (1990) pointed out that though the goods are consumed jointly, the customer can make a disjoint search for prices. Unfortunately all the results in the multicommodity search literature are based on the assumption of independent price distributions which makes it impossible to study the informational aspect of the problem. In reality searching for prices within a store is costly, and prices of different goods can be correlated. The customer's optimal search decision then can be based on informational concerns. This chapter attempts to cover this gap in the literature.

Moreover, most of consumer search literature (see, for example, Kohn and Shavell (1974)) assumes that the consumer who searches learns the exact value of the good. This is not a very realistic assumption: quite often the true value of a good is revealed to the customer after the purchase, because a thorough investigation of all the characteristics of the good is too time consuming and costly. Hey and McKenna (1981) study such a model with two characteristics: price and quality. However, in their model the search costs for the quality are infinite, it cannot be explored prior to purchase. In our setup decision whether to investigate the value of "quality" prior to purchase or not is endogenous.

To focus on the value of information, we develop a simple model. First, we study a research process with one object. The object possesses two random attributes with a known joint distribution[1]. The researcher can investigate these attributes sequentially at some cost. We study how the dependency between the attributes affects the optimal search (investigation) decision. To facilitate the analysis we consider a situation where each attribute can take either an acceptable or an unacceptable value and the researcher accepts the object only if both attributes are acceptable. Moreover, the utility function is symmetric between the attributes to eliminate the utility concern and so are the research costs. Symmetry of the utility function allows me to concentrate on the informational concern mentioned above. Finally, we consider a specific class of probability measures, such that the expectation and the variance of utility are constant for all measures in the class.

We characterize the optimal investigation rule, which is quite counterintuitive: it is optimal to first investigate the attribute with the lowest probability of taking on an acceptable value. After that we show that any probability distribution which is asymmetric between the attributes is preferred by the searcher to the symmetric one. Moreover, changes in seemingly absolutely irrelevant information can be crucial for search behaviour, i.e. probabilities of unacceptable values of the attributes (given constant utility and risk), in other words probabilities of various outcomes when the searcher would not accept the good anyway. In particular,

[1] The study of objects with n attributes is also possible. However, it requires an optimization over all possible search sequences which is a problem of combinatorial complexity. Analysis of the $n = 2$ case shows the main ideas without obscuring them with algebraic complications.

the researcher may switch from non-researching to research, which in the case of consumer search may result in a purchase.

We also consider what would happen if there are multiple objects. Say Mr. Abramovich wants to buy a football club, he considers a few clubs in Premier League and each club varies in players and fan base. Weitzman (1979) derived the optimal search rule for objects with different distributions. The results of a single object search problem can be generalized using the same logic as Weitzman. The optimal search rule between the objects is characterized by reservation values, while within the object the researcher should follow the results developed for the single object. Moreover, once the researcher starts to investigate one object she never switches to another one unless she discover an unacceptable value of some attribute.

The structure of the paperchapter is as follows. Section 2 presents the model of the object with two attributes. Analysis the of single object case is presented in section 3, the investigation process with multiple objects is considered in section 4. Section 5 concludes. An interested reader can find the analysis of the continuous case of the model in the appendix.

5.2 Single Object: A Model

The object has two attributes: $a \in \{A, \overline{A}\}$ and $b \in \{B, \overline{B}\}$ which affect the researcher's utility. Thus, there are four possible types of the object. Each type can appear with a certain probability. The researcher is only interested in the object of type AB, i.e. then both the attributes take acceptable values. The utility function is symmetric, so the researcher values objects of type $\overline{A}B$ and $A\overline{B}$ equally, and has no utility grounds to prefer investigation of attribute a to attribute b.

The structure of the model is represented by the following two matrices.

Specification of the utility function is summarized by the following assumption.

Assumption 5.2.1. $u(AB) = u_2, u(\overline{A}B) = u(\overline{B}A) = u_1, u(\overline{A}, \overline{B}) = u_0$ *with* $u_2 > 0 > u_1 \geq u_0$.

Utilities

	B	$\overline{B}$
A	u_2	u_1
$\overline{A}$	u_1	u_0

Probabilities

	B	$\overline{B}$
A	α	γ
$\overline{A}$	δ	β

Utilities in the matrix are net of price (investments, etc.) After at least one attribute has been investigated the researcher can either accept or reject the object. If the object is rejected, the researcher gets reservation utility which we assume to be equal zero, if it is accepted she gets the utility corresponding to the type of the object.

Probabilities of the outcomes are presented in the above probability matrix.

At cost c the researcher can investigate whether $\omega \in A$ (if not then obviously $\omega \in \overline{A}$), and then we say that she investigates attribute a. For the same cost she can investigate whether $\omega \in B$, and then we say that she investigates attribute b.

To avoid utility-specific effects we consider a specific class of probability measures, which is characterized by constant probabilities of outcomes AB and $\overline{AB}$.

Definition 5.2.2. *Let $M(\alpha, \beta)$ be the class of probability measures such that $P(AB) = \alpha, P(\overline{AB}) = \beta$, $\alpha + \beta < 1$.*

Lemma 5.2.3. *For any probability measure $\mu \in M(\alpha, \beta)$*

$$\mathbb{E}u = \alpha u_2 + (1 - \alpha - \beta)u_1 + \beta u_0 \tag{5.2.1}$$

$$Var(u) = \alpha(u_2 - \mathbb{E}u)^2 + (1 - \alpha - \beta)(u_1 - \mathbb{E}u)^2 + \beta(u_0 - \mathbb{E}u)^2 \tag{5.2.2}$$

Therefore, all $\mu \in M(\alpha, \beta)$ are characterized by the same expected utility and risk. Thus, focusing on symmetric utility functions and $\mu \in M(\alpha, \beta)$ allows me to study the impact of information and dependencies on the investigator's behaviour.

5.3 Single Object: Analysis

The expectation and the variance of utility do not depend on (γ, δ). If (α, β) are fixed, the model is characterized by one degree of freedom γ (since $\delta = 1 - \alpha - \beta - \gamma$).

Intuitively, it seems that the value of γ should not play a relevant role in the model, since:

1. whatever the value of γ is, both expected utility and risk are constant;

2. changing the value of γ corresponds to changing the probabilities of two events with the same level of utility;

3. changing the value of γ corresponds to changing the probabilities of two events with negative utility, i.e. events when the searcher does not accept the object.

However, given that investigation of the attributes is possible, the intuition that the value of γ is irrelevant is incorrect. To illustrate this we derive an optimal investigation rule, and show that it heavily depends on γ, namely on how far the value of γ stays form the symmetric case $\gamma_0 = \delta_0 = \frac{1-\alpha-\beta}{2}$. The value of γ drives not only the optimal investigation order but sometimes can influence the decision whether to start research at all.

Consider a rational research process. Suppose the researcher decides first to investigate an attribute a. She pays c and observes whether the object is of type A or $\overline{A}$. In the latter case it is optimal to terminate the investigation and reject the object, because the best she can get is $u_1 < 0$. In the former case she can either accept the object immediately or investigate an attribute b. If she decides to proceed, she pays c and observes b. If $b = B$ she accepts, and if $b = \overline{B}$ she rejects. Thus, given the optimal behaviour, the value of the investigation process started with the attribute a denoted as V_a can be written as

$$V_a = -c + P(A) \max(P(B|A)u_2 + P(\overline{B}|A)u_1, P(B|A)(u_2 - c) + P(\overline{B}|A)(-c)) \tag{5.3.1}$$

Or equivalently

$$V_a = -c + \max(\alpha u_2 + \gamma u_1, \alpha(u_2 - c) - c\gamma) \tag{5.3.2}$$

If the investigation order is to first investigate b and then a, the value of the investigation process denoted as V_b is:

$$V_b = -c + \max(\alpha u_2 + (1 - \alpha - \beta - \gamma)u_1, \alpha(u_2 - c) - c(1 - \alpha - \beta - \gamma)) \tag{5.3.3}$$

This allows me to formulate the following result:

Theorem 5.3.1 (Optimal investigation rule). *The optimal investigation rule is:*

- *if* $\max(\alpha u_2 + \gamma u_1, \alpha(u_2 - c) - c\gamma, \alpha u_2 + (1 - \alpha - \beta - \gamma)u_1, \alpha(u_2 - c) - c(1 - \alpha - \beta - \gamma)) < c$ *then do not search;*
- *otherwise:*
  - *if* $\gamma < \frac{1-\alpha-\beta}{2}$ *it is optimal to investigate a first;*
  - *if* $\gamma > \frac{1-\alpha-\beta}{2}$ *it is optimal to investigate b first;*
  - *if* $\gamma = \frac{1-\alpha-\beta}{2}$ *both investigation orders yield the same expected utility;*
  - *if the researcher knows that the object is of type A then it is optimal to investigate the object further if* $u_1 < \frac{-c(\alpha+\gamma)}{\gamma}$*, and to terminate the investigation and accept the object otherwise;*
  - *if the researcher knows the object is of type B then it is optimal to invetigate the object further if* $u_1 < \frac{-c(1-\beta-\gamma)}{1-\alpha-\beta-\gamma}$ *and to terminate the investigation and accept the object otherwise;*
  - *if after the first investigation the researcher realizes that the object is either of type* $\overline{A}$ *or* $\overline{B}$ *then it is optimal to terminate the investigation and refrain from accepting.*

Proof. The proof comes naturally from comparing the benefits in different cases. If $\max(V_a, V_b) < 0$ then it is optimal not to search. By expanding this inequality we get $\max(\alpha u_2 + \gamma u_1, \alpha(u_2 - c) - c\gamma, \alpha u_2 + (1 - \alpha - \beta - \gamma)u_1, \alpha(u_2 - c) - c(1 - \alpha - \beta - \gamma)) < c$.

If $\max(V_a, V_b) > 0$ then it is optimal to investigate the attributes in the order which gives the highest expected value. Thus, if $V_a > V_b$ it is optimal to start with a. $V_a > V_b$ if and only if $\gamma < \frac{1-\alpha-\beta}{2}$. If an attribute a was investigated and $a = \overline{A}$ then it is optimal to terminate the search because $u_0 < u_1 < 0$. The same holds for b. If $a = A$, then it is optimal to continue the research if $\alpha u_2 + \gamma u_1 < \alpha(u_2 - c) - c\gamma$, or equivalently $u_1 < \frac{-c(\alpha+\gamma)}{\gamma}$. The analysis for $b = B$ is exactly the same. □

Note that the optimal investigation rule possesses quite a counterintuitive property: it is optimal to investigate an attribute with the lowest probability of taking on a positive value. Indeed, the probability of $a = A$ is $\alpha + \gamma$, and the probability $b = B$ is $1 - \beta - \gamma$. It is optimal to investigate a first when $\gamma < \frac{1-\alpha-\beta}{2}$ which implies that $\alpha + \gamma < 1 - \beta - \gamma$. The idea behind this fact is that the researcher sacrifices the high probability of a positive outcome in the first step for a more favorable probability distribution in the next step. In the case when it is optimal to explore both attributes before accepting the object the idea which drives the result is quite clear: the researcher tries to minimize the expected costs of investigation and therefore maximizes the probability to stop after the first investigation round. In the case when it is optimal to stop just after the first round (in some cases the second search can be simply prohibited due to lack of time, etc.) pure informational concerns play role: the researcher faces more favourable distribution in the second round (maximizes the probability of correct choice) at the cost of lower a probability in the first round.

It is important to emphasize that the value of γ affects the optimal investigation rule. Thus this information is not irrelevant given that investigation of the object is possible. Theorem 5.3.1 shows how the value of γ affects the optimal investigation order. The following proposition illustrates the importance of this information for the customer's preferences over distributions and welfare. Let's denote the value of the search process by $V(\gamma) = \max(V_a(\gamma), V_b(\gamma), 0)$, and $\gamma_0 = \frac{1-\alpha-\beta}{2}$

Proposition 5.3.2. *For any $\mu \in M(\alpha, \beta)$ $V(\gamma) \geq V(\gamma_0)$. If $\gamma_2 \leq \gamma_1 \leq \gamma_0$ or $\gamma_2 \geq \gamma_1 \geq \gamma_0$ then $V(\gamma_2) \geq V(\gamma_1)$.*

Proof. Note that $V_a(\gamma_0) = V_b(\gamma_0)$ and V_a is a decreasing function of γ, while V_b is an increasing. Therefore $\max(V_a(\gamma), V_b(\gamma)) \geq \max(V_a(\gamma_0), V_b(\gamma_0))$. In the same way, since either $V_a(\gamma_2) \geq V_a(\gamma_1)$ or $V_b(\gamma_2) \geq V_b(\gamma_1)$, the value of search $V(\gamma_2) \geq V(\gamma_1)$. □

Thus the researcher would prefer to have an asymmetric distribution to a symmetric one, and the more asymmetric the better. The reason is that given a positive outcome of investigation of the first attribute, the quality of information increases and the conditional probability distribution becomes more favorable. If the researcher prefers say extremely low values of γ to moderate ones it is reasonable to assume that for some γ close to γ_0 it is optimal not to search and to get reservation utility of zero, while for extreme values of γ it is optimal to start research and probably accept the object (buy the good, launch the investments). Thus, the seemingly irrelevant γ can dramatically affect the researcher's behaviour. Formally this result is shown in the following proposition.

Proposition 5.3.3. *There is a pair (c, γ) such that $V(\gamma) > 0 > V(\gamma_0)$.*

Proof. First, note that $V_a(\gamma_0) = V_b(\gamma_0)$. By expanding the expression of the value of the search we get that $V(\gamma_0) < 0$ if $c > \max\left(\alpha u_2 + \gamma_0 u_1, \frac{u_2}{1+\alpha+\gamma_0}\right)$. Note, that the right hand side of this expression is a decreasing continuous function of γ. Therefore it is possible to choose c and γ in such a way that

$$\max\left(\alpha u_2 + \gamma u_1, \frac{u_2}{1+\alpha+\gamma}\right) > c > \max\left(\alpha u_2 + \gamma_0 u_1, \frac{u_2}{1+\alpha+\gamma_0}\right)$$

with $V(\gamma) > 0$. □

Thus, the value of γ affects the optimal research order and sometimes can affect the decision to investigate itself. The researcher prefers probability distributions which are further from symmetric ones.

5.4 Multiple Objects

A search problem with multiple objects can be quite interesting. For example, in times of crisis a major financial company decides to take over one of the banks which has experienced some trouble. Each bank can be characterized by multiple attributes which affect the final decision. What is the optimal investigation rule? Weitzman (1979) showed that it is optimal to explore objects in order of reservation values. However, if each of the objects has a complex structure and there is a possibility to investigate the object itself, the reservation value has to be redefined. A situation when one object is partially explored (one attribute is known) deserves a particular interest. We show that it is never optimal to stop investigation and switch to another object if the attribute explored has a positive value. This section provides a study of the attribute search problem with multiple objects.

Assume there are multiple objects which satisfy the assumptions described above, i.e. they posses symmetric attributes (which in principle can be different in nature between objects). Then each object is characterized by a set $(u_{1i}, u_{2i}, u_{3i}, \alpha_i, \beta_i, \gamma_i, c_i)$, $i = \overline{1, n}$ is the object index. Attributes are independent between objects. Let $V_i = \max(V_{ai}, V_{bi})$ be an expected value given the optimal order of search of the attributes. For simplicity of notation assume that it is optimal to investigate a first for all the objects. and thus $V_i = V_{ia}$. Of course, attributes can be renamed in such a way that this holds for each object.

Assume that the researcher has some "sure thing" as a result of previous search, and denote it by z_i. Assume by now that only object i is left unexplored. Of course, if $z_i \neq 0$ the value of investigation changes in part which deals with reservation object. To take this into account let's denote

$$\tilde{V}_{ai}(z_i) = -c_i + \max(\alpha_i u_{2i} + \gamma_i u_{1i}, \alpha_i(u_{2i} - c_i) + (z_i - c_i)\gamma_i) \tag{5.4.1}$$

$$\tilde{V}_{bi}(z_i) = -c_i + \max(\alpha_i u_{2i} + \delta_i u_{1i}, \alpha_i(u_{2i} - c_i) + (z_i - c_i)\delta_i) \tag{5.4.2}$$

where $\delta_i = 1 - \alpha_i - \beta_i - \gamma_i$. It is clear that if $V_{ai} \geq V_{bi}$ then $\tilde{V}_{ai}(z_i) \geq \tilde{V}_{bi}(z_i)$ for all z_i and vice versa. If she searches object i in the optimal way the expected benefits equal

$$\max(\tilde{V}_{ai}(z_i + P(\overline{A})_i z_i, \tilde{V}_{bi} + P(\overline{B})_i z_i) \tag{5.4.3}$$

Note that from theorem 5.3.1 it follows that if $V_{ai} \geq V_{bi}$ then $P(\overline{A})_i \geq P(\overline{B})_i$. Therefore, since $z_i \geq 0$

$$\max(\tilde{V}_{ai}(z_i) + P(\overline{A})_i z_i, \tilde{V}_{bi}(z_i) + P(\overline{B})_i z_i) = \tilde{V}_{ai}(z_i) + P(\overline{A})_i z_i$$

The researcher is indifferent between researching and non-researching if $\tilde{V}_{ai}(z_i)+ P(\overline{A})_i z_i = z_i$. Therefore each object is characterized by the reservation value

$$z_i = \max\left(\frac{\alpha_i u_{2i} + \gamma_i u_{1i} - c_i}{\alpha_i + \gamma_i}, \frac{\alpha_i u_{2i} - (1 + \alpha_i + \gamma_i) c_i}{\alpha_i}\right) \tag{5.4.4}$$

Weitzman (1979) proved that it is optimal to search objects in the order of reservation values: from the highest z_i in descending order. Note that the search order in general is different from the order of expected values given the optimal search $\{V_i\}_{i=1}^n$, so it can be that $V_i > V_j$ but it is still optimal to search j before i.

It is clear that if object i is searched and an unacceptable value of an attribute is discovered, it is optimal to search another object or to terminate search if no object are left. But what if the first attribute takes the acceptable value? Let's denote a reservation value after a positive outcome in the first round ($a = A$) by $\tilde{z}_i$. If it is optimal to terminate the search and buy without looking at attribute b then $\tilde{z}_{i\text{stop}} = \frac{\alpha_i u_{2i} + \gamma_i u_{1i} - c_i}{\alpha_i + \gamma_i}$. If it is optimal to search for b before buying, object i is characterized by new reservation value $\tilde{z}_i$ is defined by:

$$\frac{\alpha_i u_{2i}}{\alpha_i + \gamma_i} + \frac{\gamma_i \tilde{z}_i}{\alpha_i + \gamma_i} - c_i = \tilde{z}_i \tag{5.4.5}$$

and then

$$\tilde{z}_{ib} = \frac{\alpha_i u_{2i} - c_i(\alpha_i + \gamma_i)}{\alpha_i} \tag{5.4.6}$$

Therefore

$$\tilde{z}_i = \max\left(\frac{\alpha_i u_{2i} + \gamma_i u_{1i}}{\alpha_i + \gamma_i}, \frac{\alpha_i u_{2i} - (\alpha_i + \gamma_i)c_i}{\alpha_i}\right) \tag{5.4.7}$$

Lemma 5.4.1. *If $a = A$, it is optimal either to terminate the search and accept the object, or search for another attribute of the same object.*

Proof. The lemma holds if $\tilde{z}_i > z_i$, which is true because

$$\frac{\alpha_i u_{2i} + \gamma_i u_{1i}}{\alpha_i + \gamma_i} > \frac{\alpha_i u_{2i} + \gamma_i u_{1i} - c_i}{\alpha_i + \gamma_i}$$

and

$$\frac{\alpha_i u_{2i} - (\alpha_i + \gamma_i)c_i}{\alpha_i} > \frac{\alpha_i u_{2i} - (1 + \alpha_i + \gamma_i)c_i}{\alpha_i}$$

□

The lemma shows that once the searcher has got a positive search result about characteristic a, the reservation value of the object is not decreasing and therefore it is optimal to search this object further. This allows me to formulate the optimal search rule for multicommodity research.

Theorem 5.4.2 (Optimal investigation rule with multiple objects). *The optimal investigation rule is:*

- *start with the object with the highest z_i;*
- *if $\max(\alpha_i u_{2i} + \gamma_i u_{1i}, \alpha_i(u_{2i} - c_i) - c_i\gamma_i, \alpha_i u_{2i} + (1 - \alpha_i - \beta_i - \gamma_i)u_{1i}, \alpha_i(u_{2i} - c_i) - c_i(1 - \alpha_i - \beta_i - \gamma_i)) < c_i$, then terminate the investigation;*
- *otherwise:*
  - *if $\gamma_i < \frac{1-\alpha_i-\beta_i}{2}$ it is optimal to investigate a first;*
  - *if $\gamma_i > \frac{1-\alpha_i-\beta_i}{2}$ it is optimal to investigate b first;*
  - *if $\gamma_i = \frac{1-\alpha_i-\beta_i}{2}$ both research orders yield the same expected utility;*
  - *if the researcher knows that the object is of type A, then it is optimal to investigate further if $u_{1i} < \frac{-c_i(\alpha_i+\gamma_i)}{\gamma_i}$ and to terminate the investigation and accept the object otherwise;*

– *if the searcher knows that object is of type B, then it is optimal to investigate further if* $u_{1i} < \frac{-c_i(1-\beta_i-\gamma_i)}{1-\alpha_i-\beta_i-\gamma_i}$ *and to terminate the investigation and accept the object otherwise;*

– *if after the first investigation the researcher realizes that the object is either of type* $\overline{A}$ *or* $\overline{B}$ *then it is optimal proceed with the object with the highest* z_i *among the objects left or to terminate the investigation if there are no more objects.*

5.5 Conclusions

This chapter illustrates the importance of information contained in one attribute of a good about another one. Even if two probability distributions provide the same expected utility and risk, the researcher might prefer one over another if she possesses the possibility of investigation of the attributes. Moreover, a change in the probability distribution which preserves the mean and variance of utility and only affects probabilities of outcomes when the researcher does not accept the object can dramatically affect her behaviour. She may switch from passive non-investigating (and hence rejection of the object) behaviour to active investigation, which might result in the acceptence of the object: purchase of a good, launching an investment project. The results can be illustrated for different distributions and utility functions, and the interested reader can find the continuous case with a uniform distribution in the appendix. The model allows one to consider the investigation problem with several heterogeneous objects, and the results preserve all essential properties of the single object solution. The results of the chapter can potentially be embedded in an equilibrium setup, when the firms make decisions about characteristics of the products (say, price and quality) or prices of two goods.

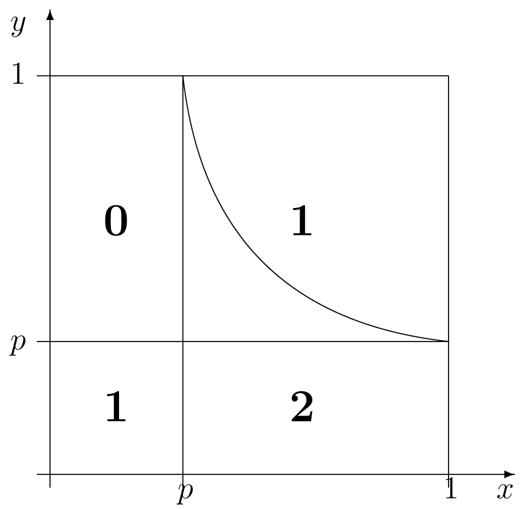
Appendix

For the sake of simplicity, in the main body of this chapter we restricted myself to the case where characteristics take binary values. However A and $\overline{A}$ can be considered as two regions where the utility function takes different signs. Here we show that though utility may not be constant over these regions, the analysis and results stay essentially the same. To do this we look at a continuous case: each attribute is a continuous variable bounded between zero and one.

Consider an example where attributes can take continuous values. Let $(x, y) \in [0, 1]^2$ be a characteristic space and $u = xy - p$ be a utility function, where $p < 1/2$ is the price of the good. Initially we assume that (x, y) are uniformly distributed.

It is obvious that if one of the characteristics is less then p, the researcher is never going to accept the object. Let us transform the probability distribution in this area in the following way: we set $\mathbb{P}(x \leq p, y \geq p) = 0$ and double density for $x \geq p, y \leq p$. See the figure for details.

Figure 5.1: Values of density function and reservation utility level



As before, assume that the researcher is allowed to investigate only one attribute, and then he has to make a decision.

1. x is investigated first, then y.

    (a) Given value of x density functions for y are:

    - $x \leq p$

$$f(y) = \begin{cases} 1/p & \text{if} \quad y \leq p \\ 0 & \text{if} \quad y > p \end{cases} \tag{5.5.1}$$

- $x > p$

$$f(y) = \begin{cases} \frac{2}{1+p} & \text{if} \quad y \le p \\ \frac{1}{1+p} & \text{if} \quad y > p \end{cases} \tag{5.5.2}$$

(b) The expected value of utility function conditional on x is:

- $x \le p$

$$Eu = \frac{1}{p}\int_0^p (xy - p)\,dy = \frac{p}{2}x - p < 0 \tag{5.5.3}$$

- $x > p$

$$\begin{aligned} Eu = & \frac{1}{1+p}\int_0^1 (xy-p)\,dy + \frac{1}{1-p}\int_0^p (xy-p)\,dy \\ = & \frac{1}{1+p}\left[\frac{1+p^2}{2}x - (p+p^2)\right] \end{aligned} \tag{5.5.4}$$

Note, that the object is accepted only if expected utility is greater than 0, therefore

$$x > \frac{2(p+p^2)}{1+p^2} \tag{5.5.5}$$

and the RHS is greater than p.

(c) The density function of x is

$$f(x) = \begin{cases} p & \text{if} \quad x \le p \\ 1+p & \text{if} \quad x > p \end{cases} \tag{5.5.6}$$

(d) Expected utility of research equals to

$$\begin{aligned} Eu = & (1+p)\int_{\frac{2(p+p^2)}{1+p^2}}^1 \frac{1}{1+p}\left[\frac{1+p^2}{2}x - (p+p^2)\right] dx \\ = & \frac{(1-2p-p^2)^2}{4(1+p^2)} \end{aligned} \tag{5.5.7}$$

2. y is investigated first, then x.

(a) Given the value of y the density functions for x are:

- $y \leq p$

$$f(x) = \begin{cases} \frac{1}{2-p} & \text{if} \quad x \leq p \\ \frac{2}{2-p} & \text{if} \quad x > p \end{cases} \tag{5.5.8}$$

- $y > p$

$$f(x) = \begin{cases} 0 & \text{if} \quad x \leq p \\ \frac{1}{1-p} & \text{if} \quad x > p \end{cases} \tag{5.5.9}$$

(b) The expected value of utility function conditional on y is:

- $y \leq p$

$$\begin{aligned} Eu = & \frac{1}{2-p}\int_0^1 (xy-p)\,dx + \frac{1}{2-p}\int_p^1 (xy-p)\,dx \\ = & \frac{1}{2-p}\left[\frac{2-p^2}{2}y - (2p-p^2)\right] < 0 \end{aligned} \tag{5.5.10}$$

- $y > p$

$$Eu = \frac{1}{1-p}\int_p^1 (xy-p)\,dx = \frac{1+p}{2}y - p \tag{5.5.11}$$

Note, that the object is accepted only if expected utility is greater than 0, therefore

$$y > \frac{2p}{1+p} \tag{5.5.12}$$

and the RHS is greater than p.

(c) The density function of y is

$$f(y) = \begin{cases} 2-p & \text{if} \quad y \leq p \\ 1-p & \text{if} \quad y > p \end{cases} \tag{5.5.13}$$

(d) Expected utility of search equals to

$$\begin{aligned} Eu = & \frac{1}{1+2p}\int_{\frac{2p}{1-p}}^1 \left[\frac{1-p}{2}y - p\right] dy \\ = & \frac{(1-p)^3}{4(1+p)} \end{aligned} \tag{5.5.14}$$

3. It is easy to check that

$$\frac{(1-p)^3}{4(1+p)} > \frac{(1-2p-p^2)^2}{4(1+p^2)} \tag{5.5.15}$$

Thus, the key properties of the optimal investigation rule are preserved in the continuous setup. The investigator still prefers to look at the attribute with the lowest probability of an acceptable outcome first.

Part II

Dynamic Competition and Regualtion

Chapter 6

Targeted Competition: Choosing Your Enemies in Multiplayer Games

6.1 Introduction

Competition lies at the heart of economics and has been studied extensively. However, there is a class of competition mechanisms that abound in practice but that, to the best of our knowledge, have not yet been studied specifically in the literature – those are mechanisms providing a competitor with an ability to target his rivals on an individual basis. We group such mechanisms under the common label of *targeted competition*. The few examples that follow illustrate how pervasive targeted competition is. On product markets, firms may decide to develop a product that is closer along one characteristic to that of a particular competitor. A multinational corporation may decide to invest relatively more in a market shared with a particular rival (see, for example, surveys by Bailey and Friedlaender, 1982; Gabszewicz and Thisse, 1992; Lancaster, 1990). Another example of targeted competition is comparative advertisement (see, for example, Anderson et al., 2009; Barigozzi and Peitz, 2007), a practice of running ads that directly compare one's products to that of the rivals. Unethical practices, for example launching fabricated lawsuits against specific rivals, provide further ways to target competitors. Targeted competition is not restricted to economics only.

Think about the ways political parties and politicians compete through their support for specific programs, or how different governments try to protect local industries through trade barriers. Finally, a warfare stays as an ultimate example of selective competition.

Targeted competition includes a strategic consideration that does not arise in non-targeted competition: a player (a firm, a political party, an army) can influence the balance of powers among his rivals by choosing whom he competes against; in turn, that determines how much this player wins or loses competing with those rivals in the periods to come. In particular, one may intuitively expect the weaker players to direct more resources towards fighting the strongest player rather than fighting each other. Indeed, otherwise the strongest player stands a good chance of forcing the weaker ones out of the game (as time goes by).

Any model of targeted competition should have the following two characteristics: 1) there should be three players or more – otherwise the competition cannot be targeted; and 2) the analysis should be dynamic – the aforementioned strategic consideration can be only studied in a dynamic setting. The closest matching strand of the literature then is that of dynamic oligopoly models. Though many scenarios of dynamic competition are studied (inventories (Kirman and Sobel, 1974), evolution of sales (Dockner and Jrgensen, 1988), varying profit opportunities (Ericson and Pakes, 1995), collusion behaviour (Fershtman and Pakes, 2000), etc.), targeted competition is not part of the analysis. This chapter aims to be a first step towards filling this gap.

We develop a model of targeted competition that does not focus on case-specific aspects of competition but rather focuses on the general ability to target in competition. Each player in the model is characterized by his relative power – the amount of resources this player has. The power of a player can be distributed to fight each of the player's rivals. We first show that myopic players prefer to fight more with their weakest opponent. Consequently, the strongest player grows in power and eventually outcompetes the weaker players. Vice versa, we show that, if players are non-myopic and do not discount future payoffs too much, then the weaker players concentrate more on fighting their strongest opponent (provided no player is too strong to start with). Consequently, the strongest player becomes weaker over time and all the players converge in power to a common level and survive.

It is tempting to view the fact that weaker players focus on fighting with the stronger player together as a form of tacit collusion. It is, however, conceptually different. Whereas collusive behaviour in repeated games is sustained by the credible threat that other players will punish deviation away from collusion, the equilibrium concept of our game is Markov perfect equilibrium, hence the strategies do not depend upon past actions and so there are no strategies with retrospective punishment. In our case it is the dynamic structure of the game that pushes the weaker players to fight together against the stronger one: if they are to prefer fighting each other for the sake of immediate gains, then the power of the strongest player will grow up to the point at which, eventually, he can outcompete his rivals. If this threat of losing the game is large enough, then the weaker players will fight more against the strongest player and their behaviour will be alike to that of tacit collusion.

There are two related games that have been studied in the literature: colonel Blotto games (see, e.g., Roberson, 2006) and truel games (Kilgour, 1971).

A colonel Blotto game is a game between two players that share several battlefields. Each player divides his army between the battlefields, a battlefield is won by the larger force, a player who wins more battlefields wins the game. The game of targeted competition that we study can be viewed as a game of three players and three battlefields, where each pair of players share a battlefield and where there is no battlefield that is shared by all the players. Then the similarity of our game to colonel Blotto games is the ability of the players to choose how to split their powers against their opponents. The main differences are: 1) there are three players in our game, 2) our game is dynamic – the winner is not determined at once, rather the winner of this round becomes stronger and the game continues.

A truel game is an extension of a duel game. There are three players, each with a gun. Each round each player chooses whom to shoot and kills his opponent with a certain chance that depends upon his skill; if two or more players are still alive the game continues. Like in our game, there is a choice of the opponent, there are dynamics and there is a consideration that killing a certain player influences your chance of survival in the rounds to come. The main differences are: 1) in our game the payoff of the game is a discounted sum of the payoffs in each round, so each round is valuable, whereas in a truel game the payoff is 1 if the player survives and 0 otherwise; 2) in our game if the player is "shot", he does not die at once but rather becomes relatively weaker; 3) in a truel game a player chooses to fight either one opponent or the other, whereas in our game a player chooses *how much* to fight one opponent and *how much* to fight the other (a continuous choice).

So, our game has structural similarities to those of colonel Blotto and truel games, but we think the named differences make our model more appropriate for the aforementioned examples of targeted competition.

The rest of the chapter is organised as follows. The next section presents the model, which is inspired by the above examples about targeted competition between firms. Section three considers the simple case of myopic players and shows that only the strongest player survives as time goes by. In section four, we show that if players are not myopic, the discount factor is sufficiently small and if no player is too strong, then there is an equilibrium where all the players converge in power and remain in the game. The last section concludes.

6.2 Setup

There are three players, 1, 2, and 3 – firms, political parties, armies, etc. The players are involved in a dynamic competitive game. Each player i at time $t \in [0, \infty)$ is characterised by a state variable $x_i(t)$ being the amount of resources he can use in competition with his rivals at time t. We call this variable the "power" of player i. It can be the market share of a firm, the amount of personnel the firm has, how large and how good its credit resources are or how well the managers are connected; it can be the electoral base or the number of seats in parliament; it can be the number of military units.

For convenience, let $x = (x_1, x_2, x_3)$. The initial state is normalised so that $\sum_i x_i(0) = 1$ (later on we will see that $\sum_i x_i(t) = 1$ for any t) and also no player is too strong to start with. Formally, $x_0 \in X$, where

$$X = \left\{ x \in \mathbb{R}^3 \,\middle|\, \sum_i x_i = 1,\ x_i < \frac{2}{5}\ \forall i \right\} \tag{6.2.1}$$

Each player can fight by targeting his rivals. y_{ij} denotes the amount of power player i uses to fight against player j. We consider Markov strategies, i.e. the actions of the players are conditioned upon the state of the game, so y_{ij} are functions of x. Our choice is determined by the main idea of the chapter: we argue that collusive type of behaviour can arise even in Markov strategies, i.e. without threat of punishment from the other player.

For convenience, let $y_1 = (y_{12}, y_{13})$, $y_2 = (y_{21}, y_{23})$, $y_3 = (y_{31}, y_{32})$ and $y = (y_1, y_2, y_3)$.

Each player uses all his power to fight his opponents[1] and what amount he uses can not be negative, therefore

$$\begin{gathered} y_i \in Y_i(x) \\ Y_i(x) = \left\{ y_i \,\middle|\, y_{ij} \geq 0, \sum\nolimits_j y_{ij} = x_i \right\} \end{gathered} \tag{6.2.2}$$

Every "battle" between players i and j has two consequences: 1) the players receive instantaneous payoffs from the battle, 2) their powers change. The instantaneous payoffs can be, for example: profits in case of firms, or the salary and the bonus payments of a top manager; political contributions in case of political parties; access to natural resources in case of warfare for economic reasons.

The instantaneous payoffs for player i when he is fighting player j are given by $\varphi(y_{ij}, y_{ji})$, where 1) $\varphi(0, y_{ji}) = 0$, i.e. if a player doesn't fight, his instantaneous payoffs are always zero; 2) $\varphi(y_{ij}, y_{ji})$ is strictly increasing in y_{ij} and for $y_{ij} > 0$ it is strictly decreasing in y_{ji}; 3) $\varphi(y_{ij}, y_{ji})$ is strictly concave in y_{ij} (decreasing marginal returns).

Virtually the only class of differential games which allows analytical solutions are linear-quadratic games (see Lockwood (1996)). Therefore, to have an analytical solution to our model we take a quadratic specification for φ. A general

[1] In our model there are no alternative costs associated with fighting, therefore it is always optimal to use for fighting all the power.

quadratic specification that would also satisfy our assumptions on the relevant domain ($0 \leq y_{ij} \leq 1$, $0 \leq y_{ji} \leq 1$) is

$$\varphi(y_{ij}, y_{ji}) = (a - b_1 y_{ij} - b_2 y_{ji}) y_{ij} \tag{6.2.3}$$

where $b_1 > 0$, $b_2 > 0$ and $a \geq 2b_1 + b_2$. To simplify matters we take $b_1 = b_2 = b$, so

$$\varphi(y_{ij}, y_{ji}) = (a - b(y_{ij} + y_{ji})) y_{ij} \tag{6.2.4}$$

where $b > 0$ and $a \geq 3b$.

Let $\pi_i(y)$ denote the sum of all the instantaneous payoffs that player i receives from fighting his opponents with $\pi_i(y)$. We have

$$\pi_i(y) = \sum_{j \neq i} \varphi(y_{ij}, y_{ji}) \tag{6.2.5}$$

Per se, the power does not enter the instantaneous payoff function. However, becoming more powerful will yield higher payoffs as more power can be used competing with the rivals thus improving the outcomes of that competition.

If $x(t)$ reaches the boundary of X, the game ends. T denotes the ending time. Formally,

$$T = \inf\{t \geq 0 \,|\, x(t) \notin X\} \tag{6.2.6}$$

If the game never ends, then $T = \infty$.

If the game ends, each player i receives a terminal payoffs S_i, the strongest player wins, the weaker players loose:

$$S_i(x) = \begin{cases} M \text{ if } x_i > x_j \ \forall j \neq i \\ 0 \text{ otherwise} \end{cases} \tag{6.2.7}$$

where $M > 0$. If the game ends and two of the players are equally strong, they both loose (this assumption is not important for the results).

The rationale for ending the game if the boundary of X is approached is as follows. If one of the players becomes sufficiently strong, it is reasonable to expect him to eventually outcompete his rivals. To simplify the game we stop it at this time and assign a strictly positive payoff of M to the strongest player and a zero payoff to the weaker players.[1] As we will see later on, the results do not depend upon the size of M as long as M is positive, still it is helpful to think of it as of a payoff that is higher than what the strongest player could have got if he was to continue the competition. Loosing, on the other hand, means that a player quits the game (a firm looses its markets, etc) and the stream of the instantaneous payoffs ends – so loosing yields zero payoff.

The payoff for the whole game is the discounted stream of the instantaneous payoffs plus the discounted terminal payoff, so the payoff for player i is

$$U_i = \int_0^T e^{-\delta t} \pi_i(y(x(t)))dt + e^{-\delta T} S_i(x(T)) \tag{6.2.8}$$

where δ is a discount factor.

If player i fights player j more than player j fights player i ($y_{ij} > y_{ji}$), then player i becomes more powerful, while player j becomes less powerful. We call such dynamics a power shift. For example, if a company invests more in a market than its rival does, its customer base shall increase relatively to that of the rival; if a political party supports a certain program more than its rival does, its electoral base shall increase relatively to that of the rivalling party, etc. We assume these dynamics to be linear in y:

$$\begin{aligned} \dot{x}_i(t) &= f_i(y(x(t))) \\ f_i(y) &= \sum_{j \neq i} (y_{ij} - y_{ji})\, k \end{aligned} \tag{6.2.9}$$

where $k > 0$ stands for the power shift intensity.

We note here that from $\sum_i x_i(0) = 1$ and from (6.2.9) it follows that $\sum_i x_i(t) = 1$ for all t.

So, our setup is a differential game with simultaneous play (see Dockner et al., 2000) and we restrict our attention to Markov strategies. The strategies are functions $y(x)$ satisfying (6.2.2), the state variables x evolve according to (6.2.9) and the objective functions are given by (6.2.8).

[1] From $x \in X$ it follows that $x_i > \frac{1}{5}$, so a player i dies if $x_i(t)$ reaches $\frac{1}{5}$. An alternative specification is to say that a player i dies, e.g. a firm goes bankrupt, a political party dissolves, if $x_i(t)$ reaches 0 at some t. Such a specification seems to yield similar results, but requires a numerical solution (see the discussion at the end of section 6.4.2), so we have chosen against this latter specification.

6.3 Example: Cournot Competition

In the previous section we did not consider specific cases of targeted competition, rather we argued for a setup that can suit cases ranging from spatial competition among firms to warfare. In this section we show, with a particular example, that our setup can also stem from targeted Cournot competition with binding capacity constraints.

Suppose there are three universities and three areas (e.g. economics, management and sociology). Suppose that each university is active in two areas only – has two respective departments – and in each area there are only two active universities. Each university i is characterised by the number of professors, x_i, which the university can split between its departments, $\sum_i x_i = 1$. Let y_{ij} denote the number of professors of university i that are in the same area as professors of university j, $\sum_{j \neq i} y_{ij} = x_i$.

The amount of education a university department provides is proportional to the number of professors employed, we take the proportionality coefficient to be one.[1] For example, university 1 employs y_{12} professors in economics and y_{13} professors in sociology, so the supply of education by this university is y_{12} and y_{13} respectively. As for the demand, suppose it is the same in all the areas and is given by $Y = \frac{1}{b}(a - P)$, where P is the admission price and Y is the total amount of education demanded.

Suppose the universities compete a la Cournot and let us neglect the costs for simplicity. Then the profits of university i from an area shared with university j are given by

$$\varphi(y_{ij}, y_{ji}) = P(y_{ij} + y_{ji}) \cdot y_{ij} = (a - b(y_{ij} + y_{ji}))y_{ij} \qquad (6.3.1)$$

We additionally suppose that the demand for education is high compared to the number of professors to the extend that $a \geq 3b$ (in general terms, the capacity constraints are binding).

Finally suppose that as time goes by, the professors of different universities interact with each other within the same areas and tend to change their appointments toward the larger departments (for reasons of richer environment, better

[1] We are free to measure education in any units.

specialisation, etc). If we take these dynamics to be linear, then we get

$$\dot{x}_i = \sum_{j \neq i} (y_{ij} - y_{ji})\, k \tag{6.3.2}$$

So, we have presented an example of targeted Cournot competition that yields the same game structure, same instantaneous payoffs and same dynamics as in our model. If we further restrict the dynamics to X (a university has to close down if it becomes too small), then this example yields precisely our model.

Real life situations of targeted Cournot competition would be more complex, of course, but a simple example of three players is sufficient to study the implications of an ability to target particular competitors.

6.4 Analysis

We consider two cases: a case with myopic players and a general case. In both cases we solve our game for a Markov perfect equilibrium (MPE) and analyse the resulting equilibrium dynamics.

In what follows we denote the best response strategies with $\tilde{y}$ and the equilibrium strategies with $\hat{y}$.

6.4.1 Myopic Players

The players are myopic if they focus on the current gains only. For a myopic player i the payoff of the game at time t is

$$U_i(t) = \pi_i(y(x(t))) \tag{6.4.1}$$

The dynamics of the myopic case are summarised by the following proposition (we limit our attention to a general initial state, when one of the players is strictly stronger than the rest).

Proposition 6.4.1. *Suppose, without a loss of generality, that $x_1(0) > x_2(0)$, $x_1(0) > x_3(0)$. Then there exists a unique MPE. Moreover, the equilibrium dynamics are such that the game ends and the strongest player wins, i.e. $T < \infty$ and $x_1(T) > x_2(T)$, $x_1(T) > x_3(T)$*

Proof. Maximising $U_i(t)$ in (y_{ij}, y_{ik}) w.r.t. $y_{ij} + y_{ik} = x_i$ gives a unique best response

$$\tilde{y}_{ij}(x) = \frac{x_i}{2} + \frac{y_{ki}(x) - y_{ji}(x)}{4} \tag{6.4.2}$$

(a boundary solution is also possible but it is straightforward to check that it is never attained for $x \in X$).

Given the above best response functions we can solve for a unique equilibrium point. We get

$$\hat{y}_{ij}(x) = \frac{x_i}{2} + \frac{x_k - x_j}{10} \tag{6.4.3}$$

As we are considering Markov strategies, (6.4.3) constitutes a unique Markov perfect equilibrium.

Plugging (6.4.3) into (6.2.9) and using $x_1 + x_2 + x_3 = 1$ gives

$$\dot{x}_i(t) = \frac{9k}{5}\left(x_i(t) - \frac{1}{3}\right) \tag{6.4.4}$$

As $x \in X$, $x_1(0) > x_2(0)$ and $x_1(0) > x_3(0)$, we have that $x_1(0) > 1/3$ and $x_{2,3} < 1/3$. Consequently, $x_1(t)$ grows over time and

$$\dot{x}_1(t) \geq \frac{9k}{5}\left(x_i(0) - \frac{1}{3}\right) > 0 \tag{6.4.5}$$

while $x_2(t)$ and $x_3(t)$ decline. Since $\dot{x}_1(t)$ is bounded from below, $x(t)$ eventually reaches the boundary of X, the game ends and $x_1(T) > x_i(T)$ for $i \neq 1$. □

This case illustrates the intuition that if the players are myopic and pursue only their instantaneous payoffs then they may have no incentives to fight more against the stronger player. As a consequence, the weaker players loose.

6.4.2 Forward-looking Players

If the players are myopic, then the weaker players loose in the equilibrium. The question is, if the players are sufficiently non myopic, i.e. if δ is sufficiently small so that the players value their future profits high enough, will it be the case the dynamics are reversed? We give a positive answer to this question.

Proposition 6.4.2. *If $\delta < \frac{4k}{3}$, then there exists an MPE such that for all i $x_i(t) \to \frac{1}{3}$ as $t \to \infty$.*

Proof. We prove the proposition by construction: we state an equilibrium candidate possesing the property that $x_i(t) \to \frac{1}{3}$ and then check that it is an equilibrium indeed. Let

$$\hat{y}_{ij}(x) = \frac{x_i + c(x_k - x_j)}{2} \tag{6.4.6}$$

$$c = \frac{1}{18}\left(5\frac{\delta}{k} - 14 - \sqrt{\left(25\frac{\delta}{k} - 76\right)\left(\frac{\delta}{k} - 4\right)}\right) \tag{6.4.7}$$

From $\sum_i x_i(t) = 1$, from (6.2.9) and from (6.4.6) it follows that

$$\dot{x}_i(t) = \frac{3k(c+1)}{2}\left(x_i(t) - \frac{1}{3}\right) \tag{6.4.8}$$

If $\delta < \frac{4k}{3}$, then from (6.4.7) it follows that $c < -1$. Consequently, from (6.4.8) it follows that $x_i(t) \to \frac{1}{3}$ as $t \to \infty$.

Let us now prove that (6.4.6) constitute an MPE. To do so we need to show that $\hat{y}_i$ is a best response to $\hat{y}_j$ and $\hat{y}_k$. All the possible strategies of player i can be divided into two classes: those strategies that eventually end the game ($T < \infty$) – let it be class $\mathcal{B}$, and those that do not ($T = \infty$) – class $\mathcal{A}$. We proceed as follows. First, we restrict the strategies of player i to class $\mathcal{A}$ and show that in this class the strategy $\hat{y}_i$, as given by (6.4.6), is indeed a best response strategy. Second, we extend this result to $\mathcal{A} \cup \mathcal{B}$.

So, let the strategies of player i be restricted to class $\mathcal{A}$. Let us compute the value function V of player i if every player follows strategy $\hat{y}$ and if the game starts at $x(0) = x$. Solving (6.4.8) gives

$$x_i(t) = \left(x_i - \frac{1}{3}\right)e^{3k(c+1)/2 \cdot t} + \frac{1}{3} \tag{6.4.9}$$

Therefore (also using $x_1 + x_2 + x_3 = 1$) we have[1]

$$V_i(x) = \int_0^\infty e^{-\delta t}\pi_i(\hat{y}(x(t)))dt = c_1\left(x_i - \frac{1}{3}\right)^2 + c_2\left(x_i - \frac{1}{3}\right) + c_3 + c_4(x_k - x_j)^2 \tag{6.4.10}$$

[1]See the appendix for the details of the derivation.

where

$$\begin{cases} c_1 = \dfrac{b(3c-1)}{4(\delta - 3k(c+1))} \\ c_2 = \dfrac{12a + b(3c-5)}{6(2\delta - 3k(c+1))} \\ c_3 = \dfrac{3a-b}{9\delta} \\ c_4 = -\dfrac{bc(3c-1)}{4(\delta - 3k(c+1))} \end{cases} \tag{6.4.11}$$

Consider now the Hamilton-Jacobi-Bellman equations:

$$\hat{y}_i(x) \in \arg \max_{y_i \in Y_i(x)} \left(\pi_i(y_i, \hat{y}_{-i}(x)) + \sum_j \frac{\partial V_i(x)}{\partial x_j} f_j(y_i, \hat{y}_{-i}(x)) \right) \tag{6.4.12}$$

$$\delta V_i(x) = \pi_i(\hat{y}(x)) + \sum_j \frac{\partial V_i(x)}{\partial x_j} f_j(\hat{y}(x)) \tag{6.4.13}$$

If these equations are satisfied for all $x \in X$, then $\hat{y}_i$ is a best response to $\hat{y}_{-i}$ (when the strategies of player i are limited to class $\mathcal{A}$, so that $x(t)$ never leaves X) – see Dockner et al. (2000, chapters 3 and 4).

Equation (6.4.13) is automatically satisfied by the way V is constructed. We now check equation (6.4.12). Let

$$g(y_i, x) = \pi_i(y_i, \hat{y}_{-i}(x)) + \sum_j \frac{\partial V_i(x)}{\partial x_j} f_j(y_i, \hat{y}_{-i}(x)) \tag{6.4.14}$$

Using (6.4.6), (6.4.10) and the definitions for π_i, f_i to expand $g(y_i, x)$ and maximising the result w.r.t. $y_{ij} + y_{ik} = x_i$ gives

$$\tilde{y}_{ij}(x) = \frac{x_i + d(x_k - x_j)}{2} \tag{6.4.15}$$

$$d = \frac{1-c}{4} - \frac{ck(3c-1)}{2(\delta - 3k(c+1))} \tag{6.4.16}$$

Strategy $\hat{y}_i$ is a best response strategy if (6.4.6) coincides with (6.4.15), i.e. if $c = d$. We check it now. Using (6.4.16) to expand $c = d$ and simplifying gives

$$18c^2 + \left(28 - 10\frac{\delta}{k}\right) c + \left(2\frac{\delta}{k} - 6\right) = 0 \tag{6.4.17}$$

It is straightforward to check that c as defined in (6.4.7) is a solution to the above equation. Hence $c = d$ and $\hat{y}_i$ is a best response.

In principle, it is possible that a corner solution is obtained when maximising $g(y_i, x)$, however it is never a case for $x \in X$.

Consider now an arbitrary strategy $\grave{y}_i(x) \in \mathcal{B}$. With a class $\mathcal{B}$ strategy the game ends at some T (that is determined by $y_i(x)$). Let

$$y_i^n(x,t) = \begin{cases} \grave{y}_i(x) & \text{if} \quad t \leq T - \epsilon_n \\ \hat{y}_i(x) & \text{if} \quad t > T - \epsilon_n \end{cases} \tag{6.4.18}$$

where ϵ_n is a sequence, $\epsilon_n > 0$ and $\lim_{n\to\infty} \epsilon_n = 0$. This strategy $y_i^n(x,t)$ belongs to $\mathcal{A}$, therefore it gives the same or a lower payoff than the best response strategy $\hat{y}_i(x)$, i.e.

$$\int_0^\infty e^{-\delta t}\pi_i(\hat{y}(x(t)))dt \geq \int_0^\infty e^{-\delta t}\pi_i(y^n(x(t)))dt = \int_0^{T-\epsilon_n} e^{-\delta t}\pi_i(\grave{y}(x(t)))dt + \int_{T-\epsilon_n}^\infty e^{-\delta t}\pi_i(\hat{y}(x(t)))dt \tag{6.4.19}$$

Taking the limit as $n \to \infty$ gives

$$\int_0^\infty e^{-\delta t}\pi_i(\hat{y}(x(t)))dt \geq \int_0^T e^{-\delta t}\pi_i(\grave{y}(x(t)))dt + V_i(x(T)) \tag{6.4.20}$$

On the other hand, the payoff from employing strategy $\grave{y}_i(x)$ is

$$\int_0^T e^{-\delta t}\pi_i(\grave{y}(x(t)))dt + S_i(x(T)) \tag{6.4.21}$$

Therefore, if $S_i(x(T)) \leq V_i(x(T))$, then $\hat{y}_i$ is the optimal strategy in class $\mathcal{A} \cup \mathcal{B}$ as well.

As $x(0) \in X$, then from the definition of X it follows that $x_i(0) < \frac{2}{5}$. Whatever the strategy $\grave{y}(x)$ is, from (6.2.9), from (6.4.6) and from $x_1 + x_2 + x_3 = 1$ it follows that

$$\dot{x}_i(t) \leq \frac{3k(c+1)}{2}\left(x_i(t) - \frac{1}{3}\right) \tag{6.4.22}$$

Consequently, $x(T) < \frac{2}{5}$. At the same time, $x(T)$ belongs to the boundary of X. So, if it was true that $x_i(T) > x_j(T)$ for all $j \neq i$, then it should have been that $x_i(T) = \frac{2}{5}$. As it is not, we have that $x_i(T) \leq x_j(T)$ for at least some $j \neq i$. Therefore $S_i(x(T)) = 0$. But from $\varphi(\hat{y}_{ij}(x), \hat{y}_{ji}(x)) > 0$ it follows that $V_i(x(T)) > 0$.

So, $S_i(x(T)) \leq V_i(x(T))$ and $\hat{y}_i(x)$ is a best response strategy when all possible strategies are considered (class $\mathcal{A} \cup \mathcal{B}$).

In words, a weaker player can choose a strategy to reach the boundary of X, but doing so is not optimal. As for the strongest player, he may prefer to reach the boundary if he is still the strongest player when he does so, but he cannot achieve such dynamics if his rivals are playing the equilibrium strategies. □

So, for a sufficiently small δ there is an equilibrium such that the strongest player declines in his power while the weaker players improve in their powers. Consequently, all the players converge. A notable property of this equilibrium is that each player fights his strongest opponent more.

6.5 Concluding Remarks

Stackelberg (1952) has argued that a duopoly will never achieve an equilibrium in price/quantity setting strategies. Moreover, the duopolists will engage into fighting for leadership and, consequently, one of them will become predominantly stronger in economic terms, or they will find it beneficial to collude.

"Duopoly is an unstable market form not only in the sense that price is apt to be indeterminate, but much more because it is unlikely to remain as a market form for any length of time. The inherent contradictions in the duopolistic situation press for a solution through the adoption of another market form – monopoly"

We do not say a market of three will attain an equilibrium in prices or quantities. Such strategic variables may as well stay indeterminate. Rather we consider the relative powers of the players. We show that if the three players are sufficiently forward looking and if there are ways for them to target their rivals, then everyone competes more against his stronger rival. Consequently the players converge in their power, and oligopolistic competition is sustainable – it does not boil down to a monopoly.

We have analysed but a basic setup of targeted competition and two possible extensions are worth mentioning – stochastic dynamics and multiple players. Arguably, both extensions would bring the model closer to judging real life situations as outcomes of competition are scarcely deterministic and many examples we talked about (e.g., multiproduct firms) often involve more than three players. The main question here will stay the same: given stochastic dynamics or given multiple (more than three) players in the game will it be more difficult or more easy for the weaker rivals to tacitly coordinate against the strongest one?

Appendix

Here we give a detailed derivation of (6.4.10), (6.4.11).

Let $z_i = x_i - \frac{1}{3}$. As $x_1 + x_2 + x_3 = 1$, so $z_1 + z_2 + z_3 = 0$. Next we derive $\pi_i(\hat{y}(z))$. First,

$$\hat{y}_{ij}(x) = \frac{x_i + c(x_k - x_j)}{2} = \frac{z_i + c(z_k - z_j)}{2} + \frac{1}{6} \tag{6.5.1}$$

Then (using $\sum_i z_i = 0$ where appropriate)

$$\begin{aligned}
\pi_i(\hat{y}(z)) = (a - b(\hat{y}_{ij} + \hat{y}_{ji}))\hat{y}_{ij} + (a - b(\hat{y}_{ik} + \hat{y}_{ki}))\hat{y}_{ik} = \\
\left(a - b\left(\frac{z_i + c(z_k - z_j)}{2} + \frac{z_j + c(z_k - z_i)}{2} + \frac{1}{3}\right)\right) \cdot \\
\left(\frac{z_i + c(z_k - z_j)}{2} + \frac{1}{6}\right) + \\
\left(a - b\left(\frac{z_i + c(z_j - z_k)}{2} + \frac{z_k + c(z_j - z_i)}{2} + \frac{1}{3}\right)\right) \cdot \\
\left(\frac{z_i + c(z_j - z_k)}{2} + \frac{1}{6}\right) = \\
\left(a - \frac{b}{3}\right)\left(z_i + \frac{1}{3}\right) - \frac{b(3c-1)}{2}\left(z_k\left(\frac{z_i + c(z_k - z_j)}{2} + \frac{1}{6}\right) + \right. \\
\left. z_j\left(\frac{z_i + c(z_j - z_k)}{2} + \frac{1}{6}\right)\right) = \\
\frac{b(3c-1)}{4}z_i^2 + \frac{12a + b(3c-5)}{12}z_i + \frac{3a-b}{9} - \frac{bc(3c-1)}{4}(z_k - z_j)^2
\end{aligned} \tag{6.5.2}$$

Let $m = 3k(c+1)/2$, then $z_i(t) = z_i e^{mt}$. So,

$$V_i(z) = \int_0^\infty e^{-\delta t}\pi_i(\hat{y}(z(t)))dt = \int_0^\infty e^{-\delta t}\left(\frac{b(3c-1)}{4}\left(z_i e^{mt}\right)^2 + \frac{12a+b(3c-5)}{12}z_i e^{mt} + \frac{3a-b}{9} - \frac{bc(3c-1)}{4}\left(z_k e^{mt} - z_j e^{mt}\right)^2\right)dt = \frac{b(3c-1)}{4}\frac{1}{\delta-2m}z_i^2 + \frac{12a+b(3c-5)}{12}\frac{1}{\delta-m}z_i + \frac{3a-b}{9}\frac{1}{\delta} - \frac{bc(3c-1)}{4}\frac{1}{\delta-2m}(z_k-z_j)^2 \quad (6.5.3)$$

Plugging in $z_i = x_i - \frac{1}{3}$ and $m = 3k(c+1)/2$ gives precisely (6.4.10) and (6.4.11).

Chapter 7

Selection Effects in Regulated Markets

7.1 Introduction

Consider yourself arriving after a long trip at the railway station of your final destination. You know it is not too far to your hotel, but you want to take a taxi because of your luggage and because of the fatigue. Taxis are standing in line and have regulated non-negotiable fees. You walk up to the first taxi waiting in line and after hearing where you want to be taken, the driver tells you that you better walk because he refuses to take on passengers for such a short distance. As an economist, you may wonder: is this rational behaviour on the part of the taxi driver? If so, what is the role of the fare structure and does a fare structure exists where potential passengers are not refused? Can it be socially optimal that potential passengers are refused?

There are a certain number of elements that are crucial to the above example. First, consumers arrive sequentially to demand some service and come in different treatment times or "complexities". In the taxi example: different passengers have different travel destinations and thus require different travel time. Second, the fare structure (how price depends on treatment time or "complexity") is fixed by a regulating authority or central company management. In the taxi example: in different countries around the world, taxi drivers are not free to determine their own fare structure, but the fare structure is centrally programmed in the taxi meter. Third, agents who actually provide the service can either accept or reject customers based on comparison of benefits and costs. In the taxi example: taxi drivers are "free" to tell potential clients that they do not take them.[1] It is optimal for a taxi driver to refuse a passenger if the expected discounted revenue of waiting for the next passenger (the chance of getting " big fish") is larger than the revenue of taking the current passenger and waiting before a next passenger can be taken on. This, of course, assumes that the taxi driver receives at least a part of the revenue of the ride.

The taxi market is not the only market where these features are present. In many countries, many parts of the medical sector also satisfy the main features outlined above. First, patients demanding some treatment enter a hospital or private clinic sequentially. Second, medical doctors are not free to set their own fees, but instead the fees per treatment are set by government authorities. Finally, medical doctors can refuse to take on patients and send them to other hospitals sometimes giving the argument that other doctors are better equipped to provide the proper treatment. Instead of selection, one may also observe what is called demand inducement in health care markets where medical doctors provide either more or a different treatment than what would be socially optimal for a patient with a particular disease. The phenomenon of demand inducement is of the same nature as the selection that appears in taxi markets: it exists because the provider prefers to deal with a different type of customer than the one they actually face. Though taxi drivers cannot induce demand (there is no possibility of bringing the customer to some other location), for medical doctors it is possible to provider some unnecessary treatment. Since demand inducement by its nature is similar to selection, it can also be studied as part of our general framework. Other markets that have features that are described above include the market for social attorneys[2] and some repair markets (shoes, electronics) where prices for standard repairs are set at the central management level and franchise holders bear the revenues and costs.

In this chapter we analyze markets that are characterized by the three features mentioned above. The underlying idea of our analysis is the fact that both selection and demand inducement exist because the providers of the service *prefer one type of the customers to another* (longer or shorter travel times, more or less severe illnesses). We show that for a large class of price schedules, selection (or demand inducement) is a crucial aspect of the equilibrium in these markets: depending on the price schedule

[1]A more official way to say this is that it is very difficult to enforce a system where taxi drivers have to take all passengers.

[2]In quite a few countries, low income families can apply for legal aid (attorneys) at a regulated fee. The fee structure has been modified recently in The Netherlands and this has lead to selection effects as attorneys argued the fee structure is such that they cannot provide legal aid to certain clients.

either consumers with a low level of complexity or consumers with a high level of complexity are refused. We then characterize the (class of) price structures for which selection does not arise. These fare structures can be set *a priori*, i.e. the regulator does not have to observe the complexity of particular customer. As the number of customers increases or agents become more patient this class of selection-free price structures shrinks and in the limit it is unique. We also show that selection is always bad from a welfare point of view in the sense that for any price structure that gives rise to selection, there exists another price structure without selection that generates a higher total surplus.

To the best of our knowledge there are relatively few papers on selection effects in regulated markets. There is, however, some literature on demand inducement in health care markets, where the modeling and measurement of induced demand is one of the main topics. This literature started with Evans (1974) who studied "supply-induced demand". McGuire and Pauly (1991) studied the demand inducement problem under regulated fees in a static setup and they characterized possible demand inducement based on physician's utility function (where intrinsic motivation also plays a role). They studied physicians responses to changes in fees as well. The model is static, although time is taken into account through the physician's preference for leisure. Gruber and Owings (1996) extended the model by introducing a parameter capturing overall demand and supply conditions. Ellis and McGuire (1995) in their empirical paper study selection and moral hazard problems in hospitals based on the reimbursement scheme and created a panel date methodology for analysis of these effects. Ellis (1998) analyzed over-provision, under-provision of a service as well as an explicit avoidance of high severity patients. He analyzed these effects in static model with transportation costs and showed that cost-based payment schemes lead to over-provision of a service, while lump-sum lead to under-provision and selection. Wright (2007) described selection of the patients between public and private hospitals based on different fee structures. The key difference of our setup is that we consider a dynamic model, which has two main advantages. First, this allows us to avoid imposing too restrictive assumptions on demand or supply sides of the market, selection arises naturally from intertemporal considerations of the agents. Second, we show that selection (or demand inducement) can be avoided even if agents are completely selfish (so they do not care about customer benefits like in Ellis (1998) and some other papers), and they do not have any reputation concerns. We also show that in the general model not only lump-sum price structures (like in Ellis (1998)) lead to selection, but cost-base (linear) can lead to the same outcome, but on the other side of the market, i.e. for low complexities.

On taxi markets, there is some literature on the desirability of entry and price regulation. Tullock (1975) and Williams (1980), among others, have argued in favour of deregulation, basically using standard arguments on the welfare effects of perfect competition. Proponents of regulation (such as Beesley (1973), Beesley (1979) and Teal and Berglund (1987) have argued that due to the peculiarities of the taxi market, some form of regulation may be necessary for a proper functioning of the market. Cairns and Heyes (1996) also mention the rather mixed success with experimentation with deregulation in some US cities in the 1980s leading those cities to back away from the deregulation policy. Whatever one's views on the (theoretical) desirability of regulating the taxi market, fact is that most taxi markets around the globe are heavily regulated. Cities as diverse in nature as New York, London, Tokyo, Amsterdam, Shanghai and Singapore all have price structures which are regulated (see, e.g., Yang et al. (2004)). Although every city has a structure with an initial charge and a distance-based charge, the precise nature of the price structure differs from city to city.[1] What is striking is that certainly for smaller distances, almost all price structures are linear in distance. In some cities the price proportional to distance becomes lower, if distance is beyond a certain threshold. In this chapter we show that such price structures always lead to selection of some customers. Glazer and Hassin (1983) were first who analyzed selection and cheating in the taxi market and derived cheat proof prices, their results can be obtained as a particular case of our model given two types of customers and specific assumptions on relation between processing time and equilibrium level of selection.

The rest of the chapter is structured as follows. The next section describes in detail the general model of selection we are analyzing in some detail. Section 3 presents the main results for generic price structures. Section 4 extends the analysis to study demand inducement effects where suppliers have the possibility of giving consumers a treatment that is different from the socially most optimal treatment given their complexity. Section 5 briefly analyzes the case where the arrival rate of customers follows a Poisson process. Section 6 concludes.

7.2 General Model

Consider a market where a consumer arrives and demands some service in each certain time interval Δt. Assume each consumer is characterized by a level of complexity θ which is randomly distributed over the interval $[\theta_{\min}, \theta_{\max}]$, with distribution $F(\theta)$.

[1] In addition, the fare may depend on delay-based charges, and additional week-end or night charges.

A customer of complexity θ derives a utility $u(\theta)$ from a service and a customer's reservation utility is normalized to 0. The market is (centrally) regulated, which means that the price structure per unit $g(\theta)$ is fixed by a central authority or by central management. Note, that the central authority does not observe the complexity θ of a particular customer; it just sets up a complete price structure for any θ. There are N (sufficiently large to meet the demand) agents who provide the service and in the basic model they simply decide whether or not to accept a customer on the basis of expected costs and benefits. This decision takes place after the customer reveals the information about his complexity θ. Sometimes it is more beneficial for the agent not to take the first customer, but to wait for the next one. For simplicity, agents have infinite planning horizons and maximize the expected present value of future cash flows. Payment is made just at the moment when the customer is accepted. It is not possible for agents to influence the price structure. There are material costs per $c(\theta)$ and the time to treat a consumer of complexity θ is given by $t(\theta)$. The time cost implies that when a consumer is accepted no other consumer can be treated during the time period $t(\theta)$. We define $f(\theta) = g(\theta) - c(\theta)$ to be the net price. We assume that for any $\theta \in [\theta_{\min}, \theta_{\max}]$ $f(\theta) > 0$ to exclude trivial selection cases.

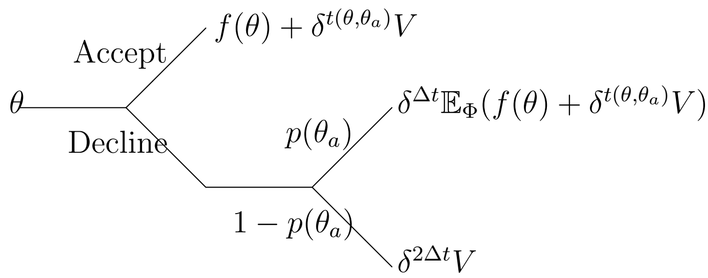
Obviously for some price structures it can be the case that it is more profitable to refuse a customer now, and to wait for the next one. In general it is possible that multiple groups of customers are excluded due to selection, but there is a subcase that is of particular interest, namely where selection is monotone. We say that selection is monotone if it is not the case that a customer with complexity θ_1 is accepted and both customers with $\theta_0 < \theta_1$ and $\theta_2 > \theta_1$ are not accepted. Though the analysis below is applicable to the more general case where selection is not monotone, monotone selection allows us to simplify notation as it can be characterized by a unique cut-off level θ_a for which either all customers with smaller or larger complexities are not accepted (so we can parameterize selection). In case of low-θ selection, we get that the support of $\Phi(\theta;\theta_a)$ is $[\theta_a;\theta_{\max}]$, where θ_a is the lowest accepted complexity and $\Phi(\theta;\theta_a) = \frac{F(\theta)-F(\theta_a)}{1-F(\theta_a)}$. For high-$\theta$ selection, we obtain that the support of $\Phi(\theta;\theta_a)$ is $[\theta_{\min};\theta_a]$, where θ_a is the highest accepted complexity and $\Phi(\theta;\theta_a) = \frac{F(\theta)}{F(\theta_a)}$.

If there is selection, the probability that an agent faces an acceptable customer each period is $p(\theta_a)$ which is equal $1 - F(\theta_a)$ for the low-selection case and $F(\theta_a)$ for the high-selection case.

We assume that the objective function of agents is their discounted *expected* value of future cash flows *conditional on* θ: $v(\theta)$.[1] The *unconditional* expected value of future

[1] We abstract from discussions whether providers are also lead by different considerations,

Figure 7.1: The decision tree of the service provider



cash flows we denote by V. It is important to emphasize that the value of V, which is the value prior to the moment when an agent knows the complexity of a particular customer, should be distinguished from the value of the process after this complexity is revealed. This latter value is $v(\theta)$ and can be either larger or smaller than V. The discount factor δ is assumed to be the same for all agents. As the model horizon is infinite and each time the agent has to make a decision whether or not to take a customer he faces the same situation, V has to be constant over time.

In case of monotone selection, the agent's decision-making problem can be presented as in the tree on the Figure 7.1.

Once the agent knows the particular θ of a customer, he can either accept or reject the customer. If he accepts, he receives the price $f(\theta)$ from the customer and some expected continuation value. If he rejects the customer, he faces an acceptable customer with probability $p(\theta_a)$ in the next period, or he has to wait until the next period after that which gives him discounted value V. In an equilibrium with selection, the agent compares the value of accepting a customer now $f(\theta) + \delta^{t(\theta,\theta_a)}V$ with the expected value of waiting, which equals

$$p(\theta_a)\left[\delta^{\Delta t}\mathbb{E}_{\Phi}(f(\theta) + \delta^{t(\theta,\theta_a)}V)\right] + (1 - p(\theta_a))\,\delta^{2\Delta t}V.$$

If the pricing structure is such that every consumer is provided the service, then the agent should at least weakly better off by taking the consumers. As we will see, this invokes some restrictions on the parameters of the model.

such as the health of their patients or the socially optimal level of service. We only say that if the provider is indifferent between providing any type of service, he will choose to provide the optimal service.

7.3 Analysis

We are now ready to proceed with the analysis of the model. We first consider situations where every consumer is provided the service and then consider the case when selection may occur.

7.3.1 Full participation case

We now first characterize the (class of) price structure(s) that is such that all customers participate in equilibrium. Note, that in case of full participation $p(\theta_a) = 1$ so that the continuation value V is defined by:

$$V = \mathbb{E}(f(\theta) + \delta^{t(\theta,\emptyset)}V), \tag{7.3.1}$$

where $t(\theta, \emptyset)$ is just a $t(\theta, \theta_a)$ in case there is no selection.

Full participation means that agents have no incentives to reject customers. Therefore, for any θ from the support of $F(\theta)$ we must have:

$$f(\theta) + \delta^{t(\theta,\emptyset)}V \geq \delta^{\Delta t}V \tag{7.3.2}$$

where V is defined above.

Thus, we can formulate the following proposition:

Proposition 7.3.1. *A price structure $f(\theta)$ insures full participation of customers if and only if equations* (7.3.1),(7.3.2) *hold for $f(\theta)$.*

In general for a given price structure $f(\theta)$, agents may still prefer one type of customer to another even if all customers are taken. But since Δt is some finite number, the agents still have no incentives to reject any consumer as the customer with the least profitable θ now is better than a costumer with an average θ later.

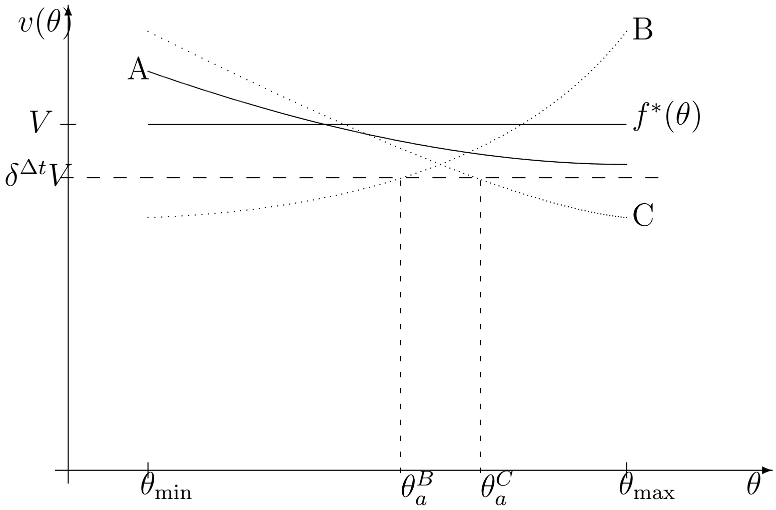
A particular case of an optimal price structure is when agents are purely indifferent between the customers, which means that even if Δt goes to 0, selection does not arise in a market equilibrium. We define this particular price structure in the following proposition.

Proposition 7.3.2. *The tariff structure that makes agents indifferent between customers and thereby eliminating selection is defined by:*

$$f^*(\theta) = V(1 - \delta^{t(\theta,\emptyset)}) \tag{7.3.3}$$

where V is continuation value.

Figure 7.2: Agent's pay-off under different price structures



Proof. Since agents are indifferent among the customers, the value must be constant in θ and equal to the expected value V. Thus, the equation

$$V = f^*(\theta) + \delta^{t(\theta,\emptyset)}V \tag{7.3.4}$$

must hold for any θ as an identity. Therefore,

$$f^*(\theta) = V(1 - \delta^{t(\theta,\emptyset)}). \tag{7.3.5}$$

Now we need to check conditions (7.3.1) and (7.3.2). For (7.3.1) we obtain:

$$V = \mathbb{E}\left(V(1 - \delta^{t(\theta,\emptyset)}) + \delta^{t(\theta,\emptyset)}V\right) = V \tag{7.3.6}$$

For (7.3.2) we obtain:

$$V(1 - \delta^{t(\theta,\emptyset)}) + \delta^{t(\theta,\emptyset)}V = V > \delta^{\Delta t}V \tag{7.3.7}$$

which completes the proof.

□

Note that the value of V is not determined here – it is a free parameter determining the level of prices in the model and we can assign any arbitrary value to it.

We summarize our results so far by means of the following graph (see Figure 7.2).

The expectation of $v(\theta)$ for all prices must be equal to V. For the price structure $f^*(\theta)$ we have that $v(\theta)$ is a constant, i.e., $v(\theta) = V$. Price structure A does not imply a constant value: agents prefer customers with lower complexity. But given the waiting time Δt it is still optimal to accept all the customers. If the arrival frequency of customers increases this price can induce high-complexity selection. Price structure B induces low-complexity selection: customers with θ lower than θ_a^B are not accepted. price structure C leads to selection from the top, all customers with complexity higher than θ_a^C are not accepted.

7.3.2 Dynamic Selection at busy places: general case

A natural question to ask is what will happen at busy places, where the time that elapses between consecutive customers arriving is very small. Analytically, we analyze situations where Δt is arbitrarily small. We will show that in this case, all selection-free price structures are sufficiently close to $f^*(\theta)$, i.e., price structures that are significantly different from the price structure where agents are indifferent between any of the customers lead to selection.

To this end, we first introduce the class of selection-free price structures. Since the value V determines the level of prices, we consider structures with the same V. Let $\mathcal{F}_V(\delta, \Delta t)$ be the class of all price structures such that the following three requirements are met:

- for any $f(\theta) \in \mathcal{F}_V(\delta, \Delta t)$ there is full participation in the market, i.e. condition (7.3.2) is satisfied for all θ in the support of the distribution;
- any $f(\theta) \in \mathcal{F}_V(\delta, \Delta t)$ gives to the agent the expected value V;
- for any $f(\theta) \in \mathcal{F}_V(\delta, \Delta t)$ and all θ $u(\theta) > f(\theta)$.

We assume that for all customers the utility derived from a service is large enough so that at least $f^*(\theta)$ is in $\mathcal{F}_V(\delta, \Delta t)$, and hence, so that $\mathcal{F}_V(\delta, \Delta t)$ is not empty. With this definition, we can state the following proposition.[1]

Proposition 7.3.3. *For any $\epsilon > 0$ there is a Δt^* such that for any $\Delta t^< \Delta t^*$ all selection-free price structures defined on $[\theta_{\min}, \theta_{\max}]$ are ϵ-close to the optimal one, i.e. satisfy the following two conditions:*

[1] A similar proposition can be proven for the case where the discount factor δ goes to 1. To economize on space, this proposition is not included in the text.

1. $f^*(\theta) - f(\theta) < \epsilon$;
2. $\int_{\theta_{\min}}^{\theta_{\max}} |f^*(\theta) - f(\theta)| dF(\theta) < 2\epsilon(\theta_{\max} - \theta_{\min})$.

Proof. 1. Recall, that by definition

$$f^*(\theta) + \delta^{t(\theta,\emptyset)} V = V$$

for all θ in the support of $F(\theta)$. Also note, that since $f(\theta) \in \mathcal{F}_V(\delta, \Delta t)$ we have that for all θ

$$f(\theta) + \delta^{t(\theta,\emptyset)} V \geq \delta^{\Delta t} V.$$

By taking difference we obtain

$$f^*(\theta) - f(\theta) \leq (1 - \delta^{\Delta t}) V$$

then by choosing $\Delta t^* = \frac{\ln(1-\frac{\epsilon}{V})}{\ln \delta}$ for any $\Delta t < \Delta t^*$ we get that

$$f^*(\theta) - f(\theta) < \epsilon$$

which proves the first part of the proposition.

2. To prove the second part recall, that since both price structures belong to $\mathcal{F}_V(\delta, \Delta t)$ we have

$$\mathbb{E} f^*(\theta) + V \mathbb{E} \delta^{t(\theta,\emptyset)} = V$$
$$\mathbb{E} f(\theta) + V \mathbb{E} \delta^{t(\theta,\emptyset)} = V$$

and therefore

$$\int_{\theta_{\min}}^{\theta_{\max}} [f^*(\theta) - f(\theta)] dF(\theta) = 0 \tag{7.3.8}$$

Let A^+ be a set of all θ in the support of distribution, such that $f^*(\theta) \geq f(\theta)$, and A^- be a set such that $f^*(\theta) < f(\theta)$, $A^+ \bigcup A^- = [\theta_{\min}, \theta_{\max}]$. Then

$$\int_{\theta_{\min}}^{\theta_{\max}} |f^*(\theta) - f(\theta)| dF(\theta) = \int_{A^+} [f^*(\theta) - f(\theta)] dF(\theta) - \int_{A^-} [f^*(\theta) - f(\theta)] dF(\theta)$$

From (7.3.8) we get

$$0 = \int_{A^+} [f^*(\theta) - f(\theta)] dF(\theta) + \int_{A^-} [f^*(\theta) - f(\theta)] dF(\theta)$$

Therefore, by taking the difference and using the first part of the proposition we obtain:

$$\int_{\theta_{\min}}^{\theta_{\max}} |f^*(\theta) - f(\theta)| dF(\theta) = 2 \int_{A^+} [f^*(\theta) - f(\theta)] dF(\theta) < 2\epsilon(\theta_{\max} - \theta_{\min})$$

□

Intuitively, when Δt becomes arbitrarily small, the cost of waiting for the next customer vanishes as well and waiting gives you the expected continuation pay-off. In this case, the only price structure that does not give rise to selection is the one where every complexity yields (approximately) the same revenue (which is equal to the expected value). This is exactly how the price structure $f^*(\theta)$ is characterized.

It is interesting to next investigate the welfare issues arising from selection. Our analysis shows that from the point of view of social welfare price structures with full participation are better than structures inducing selection.

We define social welfare as the sum of consumer and producer (agents) surplus. Producer surplus is simply equal to the discounted value of future pay-offs for the agents multiplied by the number of agents in the market. Each consumer has a utility of $u(\theta) - f(\theta)$ per service taken, which is his or her surplus. Integrating over all consumers (or over all services taken by consumers) that are accepted in equilibrium we arrive at the expected consumer surplus. So social welfare conditional on V is given by:

$$SW(V) = N \cdot V + M \int_{S_\Phi} (u(\theta) - f(\theta)) dF(\theta) \qquad (7.3.9)$$

where S_Φ is the support of the distribution under selection and M is some weighting parameter. Integration over S_Φ implies that we calculate surplus only for the customers who are accepted in equilibrium, since the others receive their reservation utility, which is zero.

Proposition 7.3.4. *Consider a tariff structure $g(\theta)$ which induces selection, and delivers value V to the agents. Then any $f(\theta) \in \mathcal{F}_V(\delta, \Delta t)$ yields social welfare that is larger than under $g(\theta)$.*

Proof. Consider some $f(\theta) \in \mathcal{F}_V(\delta, \Delta t)$. Since both fare structures $g(\theta)$ and $f(\theta)$ deliver value V to the agents, producer surplus equals NV in both cases. On the other hand, under fare structure $g(\theta)$ less customers participate in the market. Therefore to provide the same V they must pay on average more:

$$\int_{S_\Phi} f(\theta)dF(\theta) < \int_{S_\Phi} g(\theta)dF(\theta). \tag{7.3.10}$$

Let W be consumer surplus. Then, using (7.3.10) we obtain:

$$\begin{aligned} W(f(\theta)) = \int_{\theta_{\min}}^{\theta_{\max}} (u(\theta) - f(\theta))dF(\theta) > \int_{S_\Phi} (u(\theta) - f(\theta))dF(\theta) > \\ > \int_{S_\Phi} (u(\theta) - g(\theta))dF(\theta) = W(g(\theta)) \end{aligned}. \tag{7.3.11}$$

Since agents' surplus is constant, it follows that social welfare is larger in the full participation case for any positive value of M. □

Whether or not the socially optimal price is necessarily also a Pareto-improvement depends on the consumers processing time function. Indeed, for the optimal price structure $f^*(\theta)$ we have:

$$f^*(\theta) = (1 - \delta^{t(\theta,\emptyset)})V. \tag{7.3.12}$$

For a price structure with selection at θ_a we have:

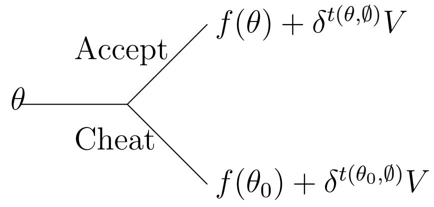
$$g(\theta_a) = (\delta^{\Delta t} - \delta^{t(\theta,\theta_a)})V. \tag{7.3.13}$$

Note, that if $t(\theta, \emptyset)$ is quite close to $t(\theta, \theta_a)$ (e.g. when consumer processing time is exogenous) customers with θ close to θ_a are better off under $g(\theta)$. Thus, if each customer has a fixed level of service complexity θ, then the optimal price structure may not be Pareto-optimal. However, if consumers' complexities vary in time so that each customer is interested in expected surplus rather than surplus generated for a particular value of θ, then the price that maximizes social welfare is also Pareto-improving.

7.4 Demand Inducement

The framework developed so far also allows us to analyze the demand inducement (or moral hazard) problem which can arise in health care markets. A medical doctor is, in principle, able to provide a customer of complexity θ with some other service θ_0 as no

Figure 7.3: Decision tree under demand inducement



one apart from him knows the exact value of θ. This problem of "demand inducement" can be analyzed in our framework as follows.

Assume the agent, say a medical doctor, can provide the customer with the level of service which is not required by his complexity. For example, he can prescribe some extra (unnecessary) treatment. If the customer (and the regulator who determines the price structure) does not know the true level of complexity, payment for this (unnecessary) treatment is made. We assume that pure fraud (reporting that a certain treatment is given while this is actually not the case) is not possible: if the true value is θ and the agent decides to provide service with level of complexity θ_0, then the required treatment time is $t(\theta_0, \theta_a)$ and the payment to the service provider is given by the net price $f(\theta_0)$. Thus, the agent can substitute the true θ with θ_0 but then it is necessary to perform the treatment that is required by θ_0 and he is payed for that.

It is clear that if there is a possibility for moral hazard the option "do not accept" is no longer relevant to the agent: a medical doctor can always treat the customer as the *best customer from his perspective*. We denote the level of complexity of this best customer as $\theta_0 \in \text{Argmax}_\theta(f(\theta) + \delta^{t(\theta,\emptyset)}V)$. Then, the decision tree of the agent looks as presented on Figure 7.3.

Thus, the agent can either accept the customer and treat him truthfully or cheat (we skip non-optimal ways of cheating since they are dominated by θ_0). Note, that the processing time is $t(\theta, \emptyset)$ or $t(\theta_0, \emptyset)$ which indicates that in the equilibrium of the model with the possibility of demand inducement the equilibrium level of selection is zero as the agents can always take on the costumer and provide a treatment that yields the highest possible pay-off.

The next Proposition argues that under the optimal price structure, the agent does not have an incentive to induce extra demand.

Proposition 7.4.1. *Under the optimal price structure 7.3.3 there is no pair* (θ, θ_0) *from the support of* $F(\theta)$ *such that* θ_0 *is better for the agents than* θ*.*

Proof. Recall, that the optimal fare structure is defined by

$$f^*(\theta) = V(1 - \delta^{t(\theta,\emptyset)})$$

Then the expected value of the service process for any θ after it has been observed is defined by:

$$f^*(\theta) + \delta^{t(\theta,\emptyset)}V = (1 - \delta^{t(\theta,\emptyset)})V + \delta^{t(\theta,\emptyset)}V = V = f^*(\theta_0) + \delta^{t(\theta_0,\emptyset)}V \tag{7.4.1}$$

□

Thus, the optimal price structure $f^*(\theta)$ we characterize allows to avoid both selection and moral hazard (demand inducement) problems in regulated markets. Given the optimal price structure, service providers are indifferent between providing (and getting paid for) any possible treatment and so they do not have an incentive to cheat. If they have a slight preference for given the optimal socially efficient treatment they will do so.

7.5 Stochastic arrival process

So far, we have studied the case where in any given time interval, only one potential costumer arrives. In this section we show that our results are robust to the case where customers arrive according to a Poisson process instead of having one customer arriving at a particular moment in time. To this end, assume Δt is now distributed according to an exponential distribution with parameter λ. Self-selection of customers leads to a decrease in the intensity of the Poisson process: if $p(\theta_a)$ is the fraction of customers that is taken in a selection equilibrium, then the intensity parameter of the arrival process equals $\lambda(\theta_a) = \lambda p(\tau_a)$. This means that the average waiting time until the next customer comes is $\frac{1}{\lambda p(\theta_a)}$.

Taxi drivers form expectations concerning the arrival time of the next customer. To derive the optimal decision-making rule for drivers we need the following result.

Lemma 7.5.1. *If* x *is exponentially distributed, then*

$$\mathbb{E}\delta^{ax} = \frac{\lambda}{\lambda - a \ln \delta} \tag{7.5.1}$$

Proof. Indeed,

$$\mathbb{E} = \delta^{ax} = \int_0^\infty \lambda \delta^{at} e^{-\lambda t} dt = \int_0^\infty \lambda e^{(a \ln \delta - \lambda)t} dt = \frac{\lambda}{\lambda - a \ln \delta} \tag{7.5.2}$$

□

Now we can apply this result to all previous sections. To do so we just need to replace a predetermined discount factor $\delta^{a\Delta t}$ by the expected discount factor $\frac{\lambda}{\lambda - a \ln \delta}$. Then we can formulate the following proposition.

Proposition 7.5.2. *The price structure $f^*(\theta)$ ensures full participation of customers if and only if:*

$$V = \mathbb{E}(f(\theta) + \delta^{t(\theta,\emptyset)} V) \tag{7.5.3}$$

and

$$f(\theta) + \delta^{t(\theta,\emptyset)} V \geq \frac{\lambda}{\lambda - \ln \delta} V. \tag{7.5.4}$$

The optimal price structure is not affected by changing the arrival process, therefore Proposition 7.3.2 still holds, and the price structure which ensures full participation for any value of the intensity λ is given by:

$$f^*(\theta) = V(1 - \delta^{t(\theta,\emptyset)}).$$

For an arbitrary net price structure $f(\theta)$, which leads to selection, the lowest (highest) accepted type is defined by the following equation:

$$\begin{aligned} f(\theta_a) + \delta^{t(\theta,\theta_a)} V = p(\theta_a) \frac{\lambda}{\lambda - \ln \delta} \mathbb{E}_\Phi (f(\theta) + \delta^{t(\theta,\theta_a)} V) + \\ + (1 - p(\theta_a)) \frac{\lambda}{\lambda - 2 \ln \delta} V \end{aligned}, \tag{7.5.5}$$

which corresponds to (7.3.2) (put as equality) in the non-stochastic case.

Finally, since the proof of proposition 7.3.4 is invariant with respect to the nature of Δt the price structure (7.3.3) is still socially optimal under stochastic customer arrival conditions.

7.6 Conclusions

In this chapter we have analyzed dynamic selection effects that arise in some regulated markets. Our framework applies when three core conditions are satisfied: (i) consumers arrive sequentially at some moments of time (a stochastic arrival process is also possible) and differ in their types (complexity of service required); (*ii*) price structures, with price depending on the level of complexity of a costumer, are fixed by a regulator or another authority or by central management and (*iii*) agents (providers of the service) can either accept or reject the customers based on a comparison of benefits and costs (or, in an extension, can decide to give a different treatment).

We have shown that for a large class of fare structures customers with a low level of complexity or customers with a high level of complexity are refused the service. Equilibria with selection are welfare inferior to equilibria without selection. We have characterized the class of price structures for which selection does not arise. For markets with very many customers, this price structure is unique up to a scaling factor. The optimal price structure also prevents moral hazard to arise if service providers can induce demand (as in the medical sector).

Chapter 8

Conclusions

This thesis consists of two parts. The first part analyses different issues of consumer search, and the part analyses competition and regulation in dynamic settings.

Chapter 2 studies the implication of the introduction of a costly second visits (costly recall) assumption on the optimal consumer search rule. We propose a model which explicitly takes the costs of going back into account. We show that the reservation price (i) depends on the number of firms left in the sample, and (ii) depends on the search history (the best price found so far). We show that consumers accept higher prices in the latter rounds of search, i.e. when the number of firms left decreases. We also show that the reservation price is a non-decreasing function of the best price searched. Thus, the optimal search rule implies that prices rejected in the first round of search might be acceptable in subsequent rounds. A notable property of the optimal search rule is that consumers never come back to the sampled store till they learn all the prices in the market. The results of this chapter can be applied to equilibrium settings, say in a model similar to Stahl (1989) or other settings.

Chapter 3 takes the first step towards applying the results of the first chapter. We show that though consumers' strategies can be very complicated, all the potential complexity in the outcome boils down to the usual results under perfect recall due to the equilibrium strategies employed by the firms. We show that the equilibrium defined in Stahl (1989) is still an equilibrium in a model with costly second visits. Moreover, this is a unique symmetric equilibrium of the model. Thus, uninformed consumers always stop at the first store. However, the result of this chapter must be interpreted with caution. We show that the perfect recall and costly second visits equilibria coincide in this particular industry setup. This might not be true for other setups, either monopolistic competition (e.g. Anderson and Renault (1999)) or even somewhat different oligopolistic setups (e.g. Chapter 4).

Chapter 4 looks at implications of minimum price guarantees (MPGs) for consumer markets. For our analysis we use the conventional sequential consumer search framework developed by Stahl (1989). We concentrate on the case when MPGs are not preannounced, i.e. the fact whether MPGs are set or not is revealed simultaneously with the price observation. The main part of the chapter focuses on price-matching strategies. We show, that due to the free-rider problem it is not possible that all firms set MPGs for sure. There are two equilibria in the model: when MPGs are never set, and when MPGs are played with a certain positive probability. The latter exists only if there is a sufficiently high level of consumer interaction which translates into a relatively high chance of exercising MPGs. In the equilibrium with MPGs prices are higher than in the equilibrium without it. We see further research in (i) empirical application of the results of the chapter, (ii) analysis of the case with preannounced MPGs in the similar setting.

Chapter 5 concentrates on investigating a particular object's properties of and their interdependence. We show that objects which are ex ante identical in terms of expected utility and risk might be evaluated very differently given the possibility of investigating the attributes. Asymmetry in the probability distribution over the characteristics, which is seemingly irrelevant, plays an important role in consumer behaviour. The optimal search rule is such that the searcher first observes an attribute which is least likely to take an acceptable value. We show that sometimes a sufficiently high level of asymmetry can induce search, which is not observed when the attributes are (almost) symmetric. This result provides interesting grounds for further research: can firms benefit from the fact that even though consumers have ex ante negative utility of buying the good, they might start to search?

In chapter 6, we consider a dynamic (differential) game with three players competing against each other. In each period, each player can allocate his resources so as to direct his competition towards particular rivals – we call such competition selective. The key feature of selective competition is that the power of the players changes with time based upon their actions. This brings an important strategic consideration into the game: players might want to target particular opponents and reduce the level of competition with others in order to influence the balance of powers. We show that if the players are myopic, the weaker players eventually lose the game to their strongest rival. Vice versa, if the players value their future payoffs high enough, each player concentrates more on fighting his strongest opponent. Consequently, the weaker players grow stronger, the strongest player grows weaker and eventually all the players converge and remain in the game. We leave two questions for further research: (i) what happens if there are more players in the game; (ii) how do the results of the model change if we in a stochastic rather than deterministic setting? The strategic considerations emphasized in this chapter create a field for future research in different applied settings.

In chapter 7, we analyse markets with regulated price structures, for example taxi markets, doctors' services, some repair markets. These markets quite often are characterized either by selection or by demand inducement. We show that for a large class of price structures some group of customers is refused the service. Equilibria with selection are welfare inferior to equilibria without selection. We characterize the class of price structures for which selection does not arise. As the number of customers increases or agents become more patient, the class of selection-free price structures shrinks and in the limit it is unique. Moreover, all other price structures induce selection. We show that this unique fare structure not only avoids the selection problem, but also eliminates demand inducement. The results of this chapter can be directly applied to policy-making decisions in regulated markets.

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