Modeling Seasonality in New Product Diffusion

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ABSTRACT AND KEYWORDS	
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Availability	
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Modeling seasonality in new product diffusion

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Abstract

Although high frequency diffusion data is nowadays available, common practice is still to only use yearly figures in order to get rid of seasonality. This paper proposes a diffusion model that captures seasonality in a way that naturally matches the overall S-shaped pattern. The model is based on the assumption that additional sales at seasonal peaks are drawn from previous or future periods. This implies that the seasonal pattern does not influence the underlying diffusion pattern. The model is compared with alternative approaches through simulations and empirical examples. As alternatives we consider the standard Generalized Bass Model and ignoring seasonality by using the basic Bass model. One of our main findings is that modeling seasonality in a Generalized Bass Model does generate good predictions, but gives biased estimates. In particular, the market potential parameter will be underestimated. Ignoring seasonality gives the true parameter estimates if the data is available of the entire diffusion period. However, when only part of the diffusion period is available estimates and predictions become biased. Our model gives correct estimates and predictions even if the full diffusion process is not yet available.

Key words: New Product Diffusion, Seasonality

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1 Introduction

Sales of new products and services typically follow a diffusion process that has an S-shaped pattern for cumulative sales and the corresponding pattern for sales is hump-shaped. There is a variety of models that can capture such a diffusion pattern. By far, the Bass (1969) model is the most often used in marketing. This model has three key parameters that together fully capture any diffusion pattern. The main application of these models is forecasting sales. For new products one uses observed patterns of similar products. However, after having observed the diffusion for a product for a while, one can also forecast the remainder of the diffusion pattern.

The original Bass (1969) model is set in a continuous time context, and it assumes a smooth development of sales. This smooth development matches well with observed diffusion data at a yearly frequency. However, at a higher frequency the sales development tends to be less smooth. Within a year many products are likely to show a strong seasonal pattern. Seasonality creates periods with higher sales followed by periods of lower sales. This pattern of these periods of high and low sales tends to be systematic. For example, Christmas sales usually generate a sales spike in the month of December. In this paper we present a model that allows to take into account such seasonality while keeping intact the basic long-run S-shaped pattern.

The importance of having a diffusion model that incorporates seasonality is amplified by the increasing availability of high frequency data. Although high frequency data is nowadays often available, researchers usually opt to aggregate such data to the yearly level. For example Venkatesan et al. (2004) mention the practice of collecting annual data in order to get rid of seasonal fluctuations. Although this aggregation of data of course reduces or even removes seasonality, it comes at a loss of information. Other studies argue that the use of annual data in diffusion models results in too small a number of data points. Such a small number of data points leads to the so-called ill-conditioning problem, which in turn leads to biased estimates, see van den

Bulte and Lilien (1997) and Bemmaor and Lee (2002). Putsis (1996) and Non et al. (2003) find that the use of quarterly or monthly data significantly improves estimates of diffusion model parameters compared to only using annual data. The main reason for this improvement is the reduction in the data-interval bias that comes from the discrete time approximation of the underlying continuous time diffusion model. Both studies do not model seasonality for these monthly and quarterly data.

From a managerial point of view, we also believe that seasonal patterns hold valuable information for managers. This information can be used to answer questions concerning short-term demand as well as inventory management issues. Hence, filtering out seasonality effects, which is common practice in the literature of financial and macroeconomic time series, is not a preferable solution in case of diffusion models in marketing.

The conclusion is that seasonality must somehow be incorporated in the Bass type model. However, it is not immediately obvious how this should be done. In this paper we propose a natural way to incorporate seasonality in the Bass model. We treat seasonality in a peak period as a result of customers who delay or are speeding up their purchases. We also contrast our model with other approaches. The first alternative approach we consider is to include seasonal dummies in a way that matches with the Generalized Bass model [GBM] (Bass et al., 1994). We show that for this model estimates for the diffusion parameters are biased, where especially the market potential parameter is underestimated. Although our model is similar to the GBM specification, the results of our model are intuitively more appealing and do not lead to a bias. The second alternative approach is the traditional Bass model, which ignores seasonality even when it is present. Our results give reassuring results for the practice of 'guessing by analogy', because in case the full diffusion series is present we document that the traditional Bass parameters have no bias.[1] However,

[1]'Guessing by analogy' is a popular method among researchers as well as managers to find the diffusion parameters of a new product based on the diffusion parameters of earlier introduced similar products, see Ofek (2005) and Lilien et al. (2000). Thus, if published or obtained estimates are biased by ignoring

if seasonality is ignored when the diffusion process is before its saturation level, the estimates as well as the forecasts are biased. We use empirical examples and simulations to show that our new seasonal Bass model is most useful.

Next to our model we also propose another variation to the Generalized Bass Model with seasonality. This variation has the same nice behavioral statistical features of our model, that is the parameter estimates are unbiased, but it does not have the same nice interpretation that our model has. The main advantage, however, is that it is easy to implement in standard statistical software packages, and therefore we also pay attention to this model.

The outline of our paper is as follows. In Section 2 we propose our model and show that it fits an empirical sales series concerning flat-screen television sets (LCD and Plasma). In this section we also theorize why the alternative approaches are less useful when seasonality is present. In Section 3 we contrast our proposed model with the other approaches in a simulation study. We discuss parameter estimation and forecasting. In Section 4 we return to actual sales data and demonstrate that the new seasonal Bass model fits naturally to these data and that it gives plausible forecasts. In Section 5 we conclude with various suggestions for further research.

2 A Bass diffusion model with seasonality

Although seasonality is a major issue, literature on seasonality in marketing is scarce. Shugan and Radas (1999) give an overview of the types of seasonality issues in the context of service marketing. They consider how to overcome these issues and how managers should react to them. Fok et al. (2007) look at weekly seasonality in sales in a panel of fast moving consumer goods. There are to our knowledge only two papers that focus on modeling seasonality in diffusion, and these are Radas and Shugan (1998) and Einav (2007). Both these papers consider the movie industry.

seasonality this affects the prediction of the diffusion of the new product as well.

Einav (2007) uses a structural model to distinguish between the seasonal demand and supply effects. Sales could be higher because more people go to the movies in holiday seasons or because better movies are screened during these periods. The view on seasonality discussed in Radas and Shugan (1998) comes closest to our approach. These authors interpret seasonality as a time transformation process, that is, it is as if the service or product ages more quickly along its life cycle in peak seasons. In off-peak seasons it is as if the product ages slowly. The diffusion model they propose to explain this results is somewhat in line with the Generalized Bass Model (Bass et al., 1994). However the seasonal structure is imposed rather than estimated in that model. Furthermore, this paper does not consider the impact of the specification on the estimation and interpretation of the standard Bass-type parameters.

As an empirical example of the feature we study, we present in Figure 1 the monthly sales figures of flat-screen televisions in the United Kingdom. These figures clearly show a systematic seasonal pattern. Furthermore, the graph suggests that the seasonal pattern is proportional to the speed and position of the diffusion process, and therefore we should account for this.

[Figure 1 about here.]

In the following subsection we build up to our proposed methodology and we show that it is useful for this empirical case on consumer electronics. In the subsection thereafter we further discuss the alternative approaches, and indicate why these approaches are less satisfactory. Further, in this last subsection we deal with the consequences of ignoring seasonality.

2.1 The proposed model

Our aim is to create a model that is consistent with the standard Bass model, therefore our starting point is the discrete time specification of the Bass model. As the underlying Bass diffusion model is specified in continuous time it is up to the empirical modeler to shape it such that it can be fitted to actually observed data in discrete time. Bass (1969) simply puts the theory into the standard linear regression framework, whereas Srinivasan and Mason (1986) take aggregation bias into account and arrive at a non-linear structure. Recently, Boswijk and Franses (2005) took possible deviations from the underlying S-shape seriously. They introduced a specification that contains a heteroscedastic error process and a tendency for the diffusion to return towards an equilibrium growth path. In the present paper we start with the Srinivasan and Mason (1986) approach in combination with heteroskedasticity.

The heteroscedasticity implies that larger fluctuations are more likely to occur around the moment of peak sales. This phenomenon occurs in every diffusion process. For the estimation of the traditional Bass parameters it is known that ignoring heteroscedasticity only leads to a loss in efficiency, it does not lead to a bias. However, to disentangle seasonality from random shocks it is of importance to properly model the heteroscedasticity.

The dependent variable in all the models in this paper is the sales[2] of a new product at month t, S_t. The basis for our model, as well as the alternatives presented later, is

$$S_t = m(F(t) - F(t-1)) + f(t)\varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma^2), \tag{1}$$

where $F(t)$ is the fraction of cumulative adopters at time t and $f(t)$ the fraction of current adopters. $F(t)$ and $f(t)$ are the solutions of the differential equation that

[2]In this paper we take months as the basic frequency of the observed data. The empirical data we have available is also monthly data. Further, in the diffusion literature monthly data is the most often used data interval that contains seasonality. Our model, however, can be used for any other data interval.

underlies the continuous time Bass model, that is,

$$
\begin{aligned}
F(t) &= \frac{1-\exp\{-(p+q)t\}}{1+\frac{q}{p}\exp\{-(p+q)t\}}, &(2)\\
f(t) &= \frac{\frac{(p+q)^2}{p}[1-\exp\{-(p+q)t\}]}{(1+\frac{q}{p}\exp\{-(p+q)t\})^2}. &(3)
\end{aligned}
$$

Further, p, q and m are the traditional Bass parameters capturing the innovation, imitation and market potential, respectively. The error term is weighted by the fraction of current adopters, this implies that the error term at time t has variance $f(t)^2\sigma^2$. This specification is slightly different from the original proposal by Boswijk and Franses (2005). They weight the error term with the sales of the previous period. The advantage of our suggestion is that it leads to a smooth pattern for the variance. The lagged sales show a far more erratic pattern, especially in the case of seasonality. The technical disadvantage is that in this case the variance is dependent on the diffusion parameters.

From here we move to the inclusion of seasonal factors. To model seasonal peaks and dips we need to increase or decrease the sales in some months relative to the specification in (1). Above we already motivated that the seasonal effect should be proportional to the speed and position of the diffusion process (see also Figure 1). This proportionally additional effect of seasonality can be represented as

$$
S_t = m(F(t)-F(t-1))(1+\sum_{k\epsilon K}\delta_k D^{01}_{kt}) + f(t)\varepsilon_t \qquad \varepsilon \sim N(0,\sigma^2), \qquad (4)
$$

where D^{01} represents a zero-one dummy for each month k in the subset K, where K can consist of one or more months depending on the seasonal pattern. To put it more formally,

$$
D^{01}_{kt} = \begin{cases} 1 & \text{if observation } t \text{ is in month } k\text{, that is, } \kappa(t)=k \\ 0 & \text{otherwise,} \end{cases} \qquad (5)
$$

where $\kappa(t)$ gives the month number corresponding to observation t. In this formulation there is of course a maximum of eleven months that can be included in the set K, because otherwise the model parameters would not be identified.

Below we shall argue that the model in (4) changes the interpretation of the Bass parameters, especially m. In other words this model introduces a bias in the parameters.

To avoid this bias we need to introduce the seasonal pattern in such a way that it does not interfere with the long-run S-shape. To obtain this, the added seasonality effect should have mean zero. This means that the additional sales at a seasonal peak must be compensated in other months. For monthly data we therefore define the dummies such that the effect in the focal month is 11/12 whereas in the other months it is minus 1/12-th. Note that over an entire year this dummy has mean zero. We can use this zero-mean dummy (D^{ZM}) to replace the zero-one dummy (D^{01}) in (4). This zero-mean dummy is formally defined as

$$D_{kt}^{ZM} = \begin{cases} \frac{11}{12} & \text{if } \kappa(t) = k \\ -\frac{1}{12} & \text{otherwise.} \end{cases} \tag{6}$$

Later we show that the Generalized Bass Model with this zero-mean seasonal dummy has some preferable features. However there is a counter-intuitive feature as well. Again consider the case of a seasonal peak at some period t corresponding to month k. The size of the additional sales equals $\frac{11}{12}\delta_k(F(t) - F(t-1))$. In other words it is a fraction of the sales predicted by the underlying Bass model. The "compensating" decrease in sales in the next month is $\frac{1}{12}\delta_k(F(t+1) - F(t))$. The counter-intuitive thing here is that the compensation is relative to the "predicted" sales for the *next* period. Intuitively it would be more appealing if the compensation decrease equals $\frac{1}{12}\delta_k(F(t) - F(t-1))$, that is, a fraction of the increase itself. Stated differently, although the dummies have mean zero, the seasonal effect itself

$((F(t)-F(t-1))\delta_k D^{ZM}_{kt})$ does not have mean zero.

In the final model we therefore propose we correct for the above-mentioned counter-intuitive feature. As a result we take the idea that the seasonal peaks originate from customers delaying or speeding up their purchases. In this case the additional sales during a seasonal peak is the summation of the delayed and accelerated purchases of the other months. Hence we propose to make the additional seasonal effect dependent on the sum of a fraction of the underlying adoption curve of all the months influencing the focal month. First we define the set of months that influence a focal month k. We denote this set as H_k. For example, if $H_k = \{-3, -2, -1, 1, 2\}$ sales from up to three months before the focal month are delayed to the focal month and sales from unto two months after are accelerated towards the focal month. As we consider monthly data, we choose to take the subset to be all the other months $H_k = \{-6, \ldots, -1, 1, \ldots, 5\}$. This results in

$$S_t = m[(F(t) - F(t-1)) + \sum_{k \epsilon K} \delta_k (D^{OM}_{1kt} f(t) + D^{OM}_{2kt} \sum_{h \epsilon H_k} f(t+h))] + f(t)\varepsilon_t, \quad (7)$$

where still $\varepsilon_t \sim N(0, \sigma^2)$. The first dummy is defined as

$$D^{OM}_{1kt} = \begin{cases} -\frac{1}{12} & \text{if period } t \text{ influences month } k\text{, that is, } \kappa(t) - k \in H_k \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

This part of the specification concerns the decrease in sales at time t due to individuals delaying or speeding up their purchase. The second dummy in (7) makes sure that the delayed and accelerated sales are added to the sales of the focal month. The summation $\sum_{h \epsilon H_k} f(t+h)$ sums the sales from the months influenced by the focal period. The second dummy is defined as

$$D_{2kt}^{OM} = \begin{cases} \frac{1}{12} & \text{if } \kappa(t) = k \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

An additional advantage of this formulation is that the parameters for all months are identified. In case the monthly effects are not strong, a model with twelve monthly dummies is not advisable, but the intuition of our formulation is in this case still preferable.

This final model seems to capture the seasonal patterns found in practice very well. To show this we used this model to generate artificial data. Figure 2 shows the resulting diffusion pattern.

[Figure 2 about here.]

Panel (a) shows the results of a series generated by our proposed model, which is generated with values 0.00002, 0.075, and 42 for p, q and m. For this illustration we further add five seasonal dummies representing January, June, October, November and December, which we gave the parameter values (δ_k) 0.5, −0.1, 0.1, 0.3 and 0.9. All this seems very arbitrary, but if we zoom in on the months on the left side of the hump shaped curve (panel (b)), we see resemblance with the pattern of the sales of flat-screen televisions in the United Kingdom from February 2004 to June 2008 (panel (d)). In Section 4 we again analyze these data for the United Kingdom as well as for five other European countries[3]. In panel (c) of Figure 2 we also zoomed in on exactly the same part as panel (b), but now we generated the data without seasonality. From the latter it becomes clear that seasonality is an issue and that ignoring seasonality is likely to have consequences.

[3] The reason to zoom in and leave the left hand truncated is because we only have annual data before 2004, whereas flat-screen televisions were introduced in 2000 in the United Kingdom. This gives no problem for the estimation, because we use the method of Srinivasan and Mason (1986) to estimate the diffusion parameters. For this method the knowledge of the moment of introduction is sufficient for correct estimates (Jiang et al., 2006).

2.2 Alternative approaches

In this subsection we briefly discuss some alternative approaches. We consider the following models/approaches: (i) Bass model with additive dummies; (ii) GBM with 0/1 dummies; (iii) GBM with mean zero dummies; (iv) standard Bass model without seasonality; (v) Bass model applied to seasonally adjusted data. In this section we give intuitive arguments for the qualities and problems of the different approaches. If possible we support this using examples. In the next section some of the approaches are more formally compared using a large scale simulation study.

The Bass model with additive dummies does not capture the proportional effect of seasonality. Such a specification results in a seasonal effect that is the same throughout the diffusion process. In Figure 3 we show a simulated series for such a model. The data generating process for this simplified figure is, $p = 0.001$, $q = 0.1$, $m = 100$ and $\sigma^2 = 0$. Also note that the cumulative sales in the end exceed $m = 100$, that is, the seasonal effects result in a higher actual market potential than the parameter setting seems to imply. Summarizing, it is clear that the approach with additive dummies does not fit seasonality in diffusion models, and therefore we will not consider this approach in the simulation study.

[Figure 3 about here.]

The Generalized Bass Model with a zero-one dummy, see (4), makes sure that the seasonal effects are largest around the moment of peak sales. However, this model does not solve the bias for the market potential (true market potential is larger than m). This is shown in Figure 4 where we show simulated data based on this specification. However, the overall shape of the diffusion process for this model closely resembles that generated by our model and thus the real data. In the following sections we compare our model to this model, which we will refer to as the Seasonal Generalized Bass Model (SGBM).

[Figure 4 about here.]

Next we consider the SGBM after replacing the zero-one dummy by the zero-mean dummy as specified in (6). The difference between this specification and our full model can be small. We further discuss this in the next sections. In Figure 5 there is a simplified comparison between our model and this variation of the SGBM based on the data generating process mentioned before (p, q and m are 0.001, 0.1 and 100). At first sight both models give artificial data that seem to be equal. However, if we zoom in on the seasonal peaks, the (small) differences between the two models become clear.

[Figure 5 about here.]

As said, one can also decide to ignore seasonality altogether or to seasonally adjust the diffusion data. We argue that if seasonality is not dealt properly the estimation results are affected. However, if the full diffusion process is used for parameter estimation, that is, when the market potential has been reached, this problem disappears. In Figure 6 this is illustrated. The seasonal pattern goes above and below the dotted line of the underlying sales curve, but in the end they arrive at the same level of the market potential.

[Figure 6 about here.]

This reassuring result will be proven more thoroughly in the following section. In contrast, this will not hold if the diffusion is before its saturation level. If seasonality is not explicitly modeled, the fluctuations will be treated as random fluctuations. The latter makes that the parameter estimates can depend on the incorporation of one additional data point. If this data point is for example a month with a seasonal peak the market potential and/or the speed of diffusion will be overestimated.

In the literature some authors consider monthly and quarterly data in diffusion models. Some seem to ignore seasonality and in some cases it is smoothed out with the same statistical tool typically used in macroeconomics, like Census X12.

One example is Putsis (1996). To check if seasonal adjustment is in fact similar to ignoring seasonality and estimating the standard Bass Model we take a look at seasonally adjusted series generated by Census X12 (with all the default options). For data generated by our model as well as the zero-mean SGBM the seasonally adjusted series are almost the same as the underlying standard Bass Model, see Figure 7. Hence, it seems that adjusting for seasonality gives correct estimates if the full diffusion process is available, but has trouble coping with only a part of the diffusion process. This is the same as in case the seasonality is ignored altogether. In the rest of this paper we will not consider seasonal adjustment. We expect the results to be very similar to the case where seasonality is ignored. Hence, proper analysis of seasonally adjusted series can only be done if the saturation level is attained.

[Figure 7 about here.]

3 Simulation results

In this section we use simulations to compare our model to the two versions of the Generalized Bass Model. The first version uses zero-one dummies for the seasonal months, and this model will be abbreviated as SGBM_{01}. The second seasonal GBM uses a zero-mean dummy. This model we call SGBM_{ZM}. We further compare our model to the Bass model without seasonal dummies. Here we also make two distinctions. We consider one version where we leave out the seasonality, but do allow for heteroscedasticity. The other version excludes seasonality and heteroscedasticity. This final model resembles the current practice in the literature. We call the two versions BM_{het} and BM, respectively.

We divide the simulation study in two parts, first we consider using data of full diffusion processes. Here we only look at how well the parameters are recovered. Second, we consider what happens when only part of the process is used for parameter estimation, that is, we look at the effect of right-hand truncation. In this case we look at the estimation results but also at forecasts. For both parts we use a cross design for the five models. We simulate the diffusion process based on each model and each of these simulated series is analyzed using all five models.

We vary the levels of the parameters for innovation (p) and imitation (q), as these two parameters define the curvature of the diffusion process. We use four values of p and four values for q, resulting in sixteen combinations for curvature. We use the moment of peak sales ($T^* = \frac{log(q/p)}{(p+q)}$) to represent this curvature, see Table 1 for the different settings. The standard deviation of the error is set at 0.01, 5 or 10.[4]

[Table 1 about here.]

We use three different seasonal structures. The first has a single monthly peak, say December, which we give a parameter value (δ_{12}) of 0.6. For the second structure we add a month, say January, with a parameter value (δ_1) of 0.3. And for the third structure we add two more monthly dummies, say June with a parameter value (δ_6) of -0.2 and November with a parameter value (δ_{11}) of 0.1.

We use two levels of truncation, that is one after the moment of peak sales and one prior to peak sales. The exact number of observations used depends on the level of curvature. The moment of peak sales (Table 1) lies a little before 50% of the total diffusion process, and therefore we make the number of data points of the full diffusion process dependent on the number of data points until the moment of peak sales[5]. The two levels of truncation are 20% before the moment of peak sales and 20% after. So, if the sample until peak sales has 50 data points, the first truncated series contains 40 data points and the second 60 data points.

There is no need to vary the level of the market potential, as it only shifts the effects of the entire model. The estimated market potential will be a key parameter

[4]The level of error-variance is proportional to the market potential. In our simulation we set the market potential to 100.

[5]The number of data points of the full diffusion process is not equal to two times the data points until peak sales, as the fraction of adopters at peak sales is actually less than 50%. How much less than 50% depends on the curvature.

when comparing the models. We set the level of market potential at 100 such that deviations can be interpreted as a percentage difference relative to the real market potential.

In sum, our design has 10800 cells. There are 5 simulated models, 5 estimated models, 16 curvature combinations, 3 noise levels, 3 truncation levels and 3 types of seasonality. In each cell we consider 100 replications.

3.1 Full Diffusion Process

In the first part of the simulation study we focus on the percentage difference of the estimated moment of peak sales (T^*) compared to the true value, $\frac{100(\widehat{T^*}-T^*)}{T^*}$, and the percentage difference for the market potential, $\frac{100(\widehat{m}-m)}{m}$. The means and standard deviations of these two criteria are given in Tables 2 and 3. These tables show the case with four seasonal months and the results are aggregated over all levels of curvature [6]. The results for fewer seasonal months are very similar.

[Table 2 about here.]

[Table 3 about here.]

First we note the large difference between the true and estimated moment of peak sales and between true and estimated market potential, in case the data is simulated without heteroscedasticity and a model with heteroscedasticity is used. Models with heteroscedasticity have problems finding correct estimates in case the true model has no heteroscedasticity, because relatively large fluctuations at the beginning and end of the diffusion curve cannot be explained fully by the heteroscedastic error term. Therefore these random fluctuations will change the diffusion parameters. This shows that if there is no heteroscedasticity it should not be included in the model. However, for most practical situations, we believe that the assumption of heteroscedasticity is

[6] Other results are available upon request from the authors.

a valid one. If there is uncertainty about the presence of heteroscedasticity in the diffusion series an additional parameter could be included in the model, see Boswijk and Franses (2005).

Table 2 shows that almost all models on average find the correct moment of peak sales. Apart from the case where the simulated model has no heteroscedasticity there is no cell above 1%. This is the average effect over all combinations of p and q for the case of four seasonal months. The small difference in the moment of peak sales means that these models find the correct innovation and imitation parameters. This is especially remarkable for the case where the estimated model does not account for seasonality, but the generated data does show seasonal fluctuations.

For the market potential, we see a very different picture. Here there is a large difference between the SGBM_{01} and the other models. All models overestimate the market potential by a little under 7% if the true model is the SGBM_{01}. If the SGBM_{01} is estimated when one of the other models is the true model the reverse effect of -7% is present. To see which of the models is correct we note that all models are simulated with a theoretical market potential of $m = 100$. In Table 4 the average value of the last data point of the series for each simulated model is given. Because the full diffusion process is considered this represents the true market potential. Thus, if the SGBM_{01} is simulated with $m = 100$ the true market potential is more than six percent higher, this is precisely the difference between the SGBM_{01} and the other models. In short, the SGBM_{01} substantially underestimates the market potential, as we predicted earlier.

[Table 4 about here.]

In case the full diffusion process is available, the main goal of estimation is to find the correct estimates. The SGBM_{01} fails in this objective. The standard Bass models do find the correct diffusion parameters, so if the goal is to find the underlying diffusion process and not the seasonal structure, the standard Bass models (with or without heteroscedasticity) can be used. Note that the use of the standard Bass model will lead to a loss of precision in the estimates relative to a model with seasonality.

3.2 Truncated Diffusion Process

In this subsection we look at the case when there is incomplete information. We consider the situation when the truncation is just before the moment of peak sales, and when it is just after. To evaluate the performance of the different models we use the same criteria as before, that is, the difference between true versus estimated moment of peak sales and market potential. As estimation is done on only a part of the diffusion curve, we encounter the usual problems of the Bass diffusion model that the model in some cases does not converge. This occurs mainly if only data before the moment of peak sales is used and/or the noise level is large. If the estimation procedure did not converge we do not take the estimates into account in the averages given in the tables below.

Tables 5 and 6 show the results based on the truncated series. In these tables we distinguish different seasonality structures and we aggregate over the levels of curvature. This distinction clearly shows that the complexity of the seasonal structure is an important factor. The level of noise, used to simulate the series, is set at the lowest level (0.01). Even with such a low noise level the consequences of truncation can clearly be seen. The main reason to only look at such a small noise level is that with a larger noise level the problem of non-convergence becomes larger. To illustrate this, consider the moment of peak sales given in Table 1. The number of months until peak sales is sometimes as few as 14 months. We subtract 20% from the data and try to estimate 4 to 8 parameters (diffusion parameters plus the seasonal parameters). It is not surprising that for larger noise levels the models have some problems. Especially both versions of the standard Bass model already have convergence problems for the minimal noise level, and this effect increases with the level of noise.

[Table 5 about here.]

[Table 6 about here.]

The results in Tables 5 and 6 show that the differences across the seasonal models are similar to the case of full diffusion series. The versions of the standard Bass model, on the other hand, do not produce the correct estimates. Not only are the results biased, but the large variation is also an indication of misspecification. Further, the bias and variation increase substantially with the complexity of the seasonality in the simulated process.

In Table 7 we compare the models based on the out of sample forecasting performance. The comparison is done based on the Root Mean Squared Prediction Error (RMSPE) for 12 months after the (truncated) estimation period. These results show that, despite the wrong estimates, the forecasts are comparable for all seasonal models. In particular, it shows that the SGBM_{01} generates similar forecasting results as the other two seasonal models. Hence, for short term forecasting the three seasonal models perform equally well. In the case of no seasonality in the true model, the difference between the seasonal models and the standard Bass version is small. If the diffusion shows seasonality, the RMSPE of the standard Bass model is more than two times as large compared to any seasonal model. Despite this, the magnitude of the difference indicate that, although the estimates of the standard Bass models are incorrect, the direction of the short term forecast is still correct. Of course the seasonal peaks are under- or overestimated for the standard Bass model.

[Table 7 about here.]

Summarizing, the simulations show that apart from the difficulties all models encounter with estimation on truncated diffusion series, the standard Bass model is not suited for estimating the underlying diffusion pattern when seasonality is present. For short-term forecasting all models are able to predict the overall direction of sales. However, if the objective is to forecast the precise sales in each month the seasonal structure should be taken into account as well. For forecasting it does not matter which seasonal model is used. Even the SGBM_{01}, which has a bias in the market potential, provides good forecasts.

4 Empirical illustration

Finally, we look at the performance of the different models on actual diffusion data. The data concern six countries in Europe from February 2004 until June 2009 and contains the sales figures (in millions) of flat-screen televisions. The first months of the diffusion process are not available. This has no major consequences for the estimation of our models as they are all based on the Srinivasan and Mason (1986) approach. With this approach it is only necessary to know the number of months since introduction (Jiang et al., 2006). In Tables 8 and 9 the estimation results for the different countries are shown. We show the estimated parameters together with their standard error, the root mean squared error (RMSE), and two information criteria (AIC and BIC).

We tried different sets of seasonal dummies and selected the combination with the best fit. Tables 8 and 9 show that the seasonal pattern, represented by the included dummies, differs across countries. Also the levels of the seasonal parameters show that seasonality is not similar across countries. For example, seasonal peaks in the Netherlands are higher in January compared to December, where the reverse is true for the United Kingdom.

[Table 8 about here.]

[Table 9 about here.]

For all countries the diffusion process seems to be around its inflection point. This explains why the results of the models ignoring seasonality show very different results for both versions of the standard Bass model. Additional evidence against the standard Bass model is given by the statistics for the fit of the model. The models incorporating seasonality outperform the standard Bass model. The model statistics do not clearly rule out one of the three models with seasonality. However, our proposed model performs best for 5 of the 6 countries and performs almost equally good for the other one.

If we compare the models based on the parameter estimates, the major difference between the three seasonal models is the market potential, where our proposed model and our variation to the SGBM (with zero-mean dummies) give a higher estimate. This matches our findings in the simulations. Note that the simulations showed that for the seasonal model with zero-one dummies the estimate of the market potential is biased. Another difference is that the seasonal parameters are estimated higher for the SGBM_{01}. This is also due to the fact that this model uses a different underlying diffusion curve. Finally, although our proposed model and the SGBM_{ZM} have approximately the same estimates, our model has smaller standard errors and therefore smaller confidence intervals.

The in-sample comparison of the models based on actual data shows that our model, closely followed by the SGBM_{ZM}, outperforms the other models. In Table 10 the models are compared based on their out-of-sample performance. In this table we compare the root mean squared prediction error (RMSPE) for the out-of-sample forecasts of the 1 and 12 months ahead forecasts[7]. For these RMSPE values we estimated the model again, but now we left out the last one or twelve months of the sales figures.

[Table 10 about here.]

[7]Here the one-step-ahead does not contain a seasonal peak for all the countries except the United Kingdom

As we found in the simulation study all seasonal models perform almost equally well regarding their predictions, including the SGBM_{01}. This shows that for the short term prediction all seasonal models can be useful. The Bass model with heteroscedasticity is outperformed by all seasonal models. However, the standard Bass model excluding both heteroscedasticity as well as seasonality has, in some cases, a RMSPE close to that of the seasonal models. In particular for Germany and Spain the one-period-ahead RMSPE for the Bass model is close to that of our model and even slightly better than the SGBM. For the twelve-month-ahead comparison the difference becomes larger, because the one-step-ahead forecast does not contain a seasonal month for Germany and Spain and the RMPSE for the twelve-month-ahead forecasts does contain several seasonal months. Nonetheless, this shows that in these cases ignoring the seasonality, despite wrongly estimating the parameters, still produces the right direction for short term forecasting.

To conclude the results, we observe that the SGBM_{01} estimates the market potential wrongly but can be used for short term prediction. The SGBM_{ZM} can also be used for short term forecasting and additionally produces correct estimates. The same holds for our model, with the additional advantage of smaller confidence intervals. The Bass model with heteroscedasticity is outperformed on all fronts. However, if the heteroscedasticity is excluded, the model can still be used to find the right direction of the short term forecasts. The standard Bass model still will not have the right estimates and is off on the seasonal peaks, but its forecasting performance can still be strong.

5 Conclusion

In this paper we looked at seasonality in diffusion models. The current availability of higher frequency data makes this a subject of increasing importance. The goal of study was to find a seasonal structure that can be used in combination with standard diffusion models. We based our models on the classic Bass diffusion model, but our seasonal structure works with any closed form diffusion model. Further, because estimated diffusion parameters are often used for the practice of 'guessing by analogy', the goal was also to find an extension which does not influence the estimates and interpretation of the underlying diffusion pattern.

Through simulations and an empirical case we showed that our proposed model lives up to all these goals. In contrast, the use of the Generalized Bass Model with seasonal dummies, which seems a straightforward way to take into account seasonality, finds biased estimates. In particular the market potential is biased. Next to our model we also put forward a variation of this Generalized Bass Model, which uses a zero-mean dummy. This variation gives similar results as our model, with the additional benefit that it can be used straightforwardly in standard statistical software packages.

That ignoring or adjusting seasonality is not suitable for estimating and predicting seasonal peaks is obvious. However, for estimation of the basic diffusion parameters, the current practice is often to ignore seasonality or to adjust the series. In this paper we found the reassuring result that, if the full diffusion series is available, estimation of the diffusion process indeed leads to the underlying diffusion pattern. However, if the series is truncated, this does not hold anymore.

Our model can estimate the seasonal pattern. The basic structure is that the seasonal peak in a focal month consists of sales drawn from the eleven months around the focal month. We showed that this works well for the empirical case in our paper, but in practice other underlying structures are possible as well. For, example it may be true that a focal month is influenced only by the quarter surrounding it. Such structures are all possible in our setup, the only restriction is that the seasonal structure should have the zero-mean feature.

The considerations for future empirical analysis of seasonal diffusion data are threefold. First, if the goal is to find a model that can be used for short term forecasting all three seasonal models described in this paper can be used. Second, if the only interest is to find the underlying diffusion, ignoring or adjusting seasonality seems tempting. However, this only works for full diffusion series and the seasonal GBM with zero-one dummy does not work. Finally, if the interest lies with both the correct estimation as well as short term forecasting our model is the way to go.

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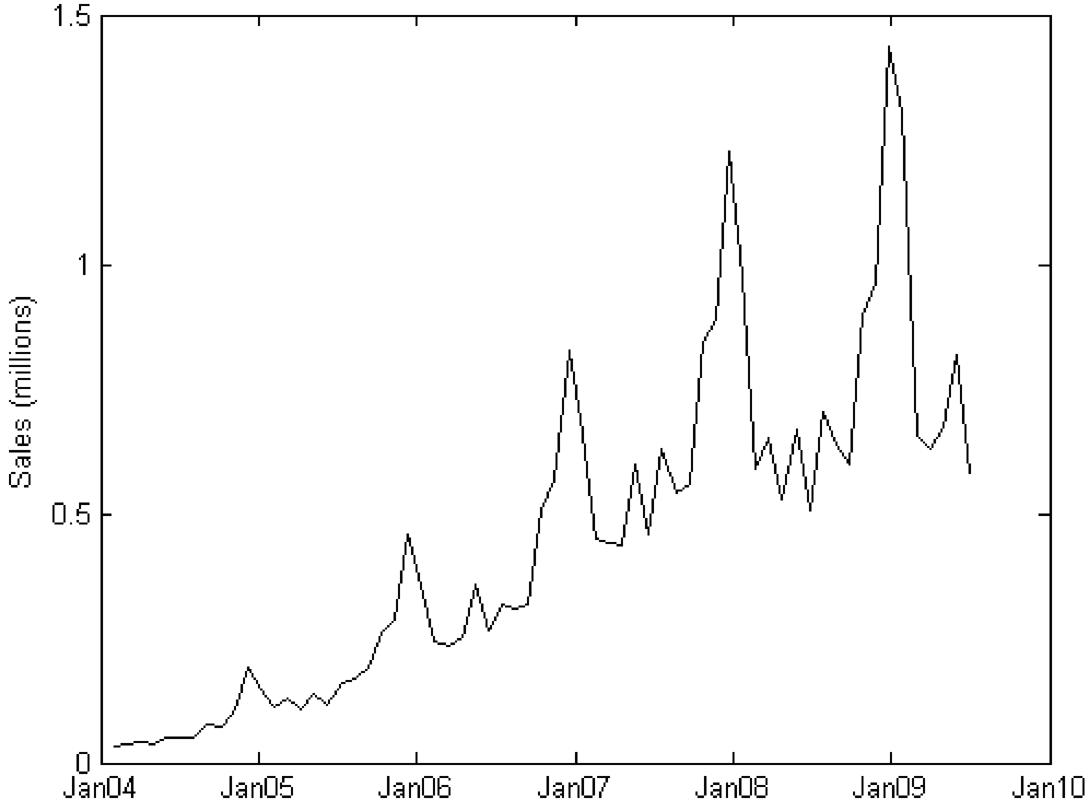

Figure 1: The actual flat-screen television sales data of the United Kingdom.

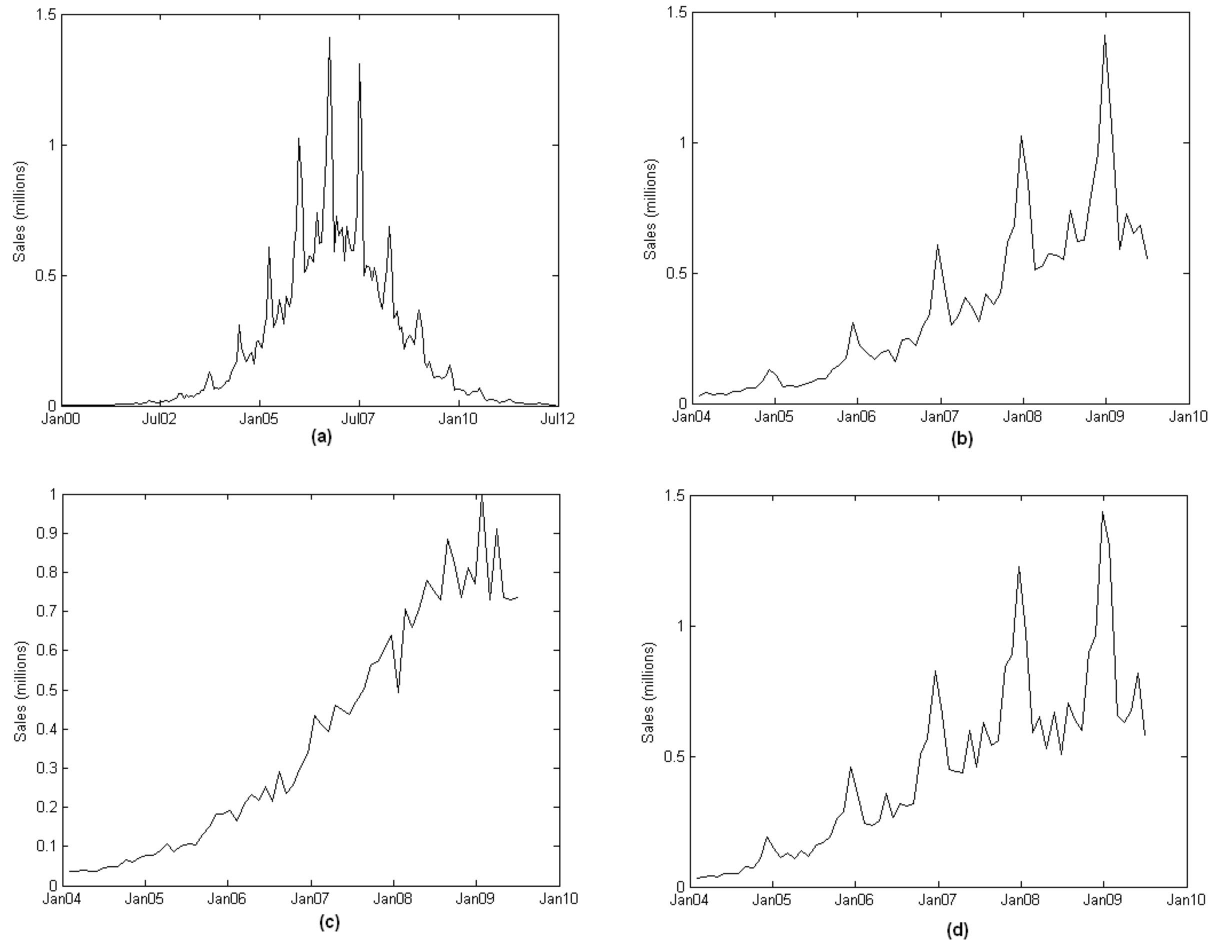

Figure 2: Panel (a) shows the artificial series generated by our model, with the diffusion parameters p, q and m being 0.00002, 0.075, and 42 respectively, and five seasonal dummies with parameter values (δ_k) 0.5, -0.1, 0.1, 0.3 and 0.9 representing the months January, June, October, November and December. Panel (b) is the same series as (a) but now zoomed in on the part that resembles the same period as the actual flat-screen television sales data of the United Kingdom, panel (d). Panel (c) is the same zoomed in period as panel (b), with the same diffusion parameters, but now the seasonal months are left out.

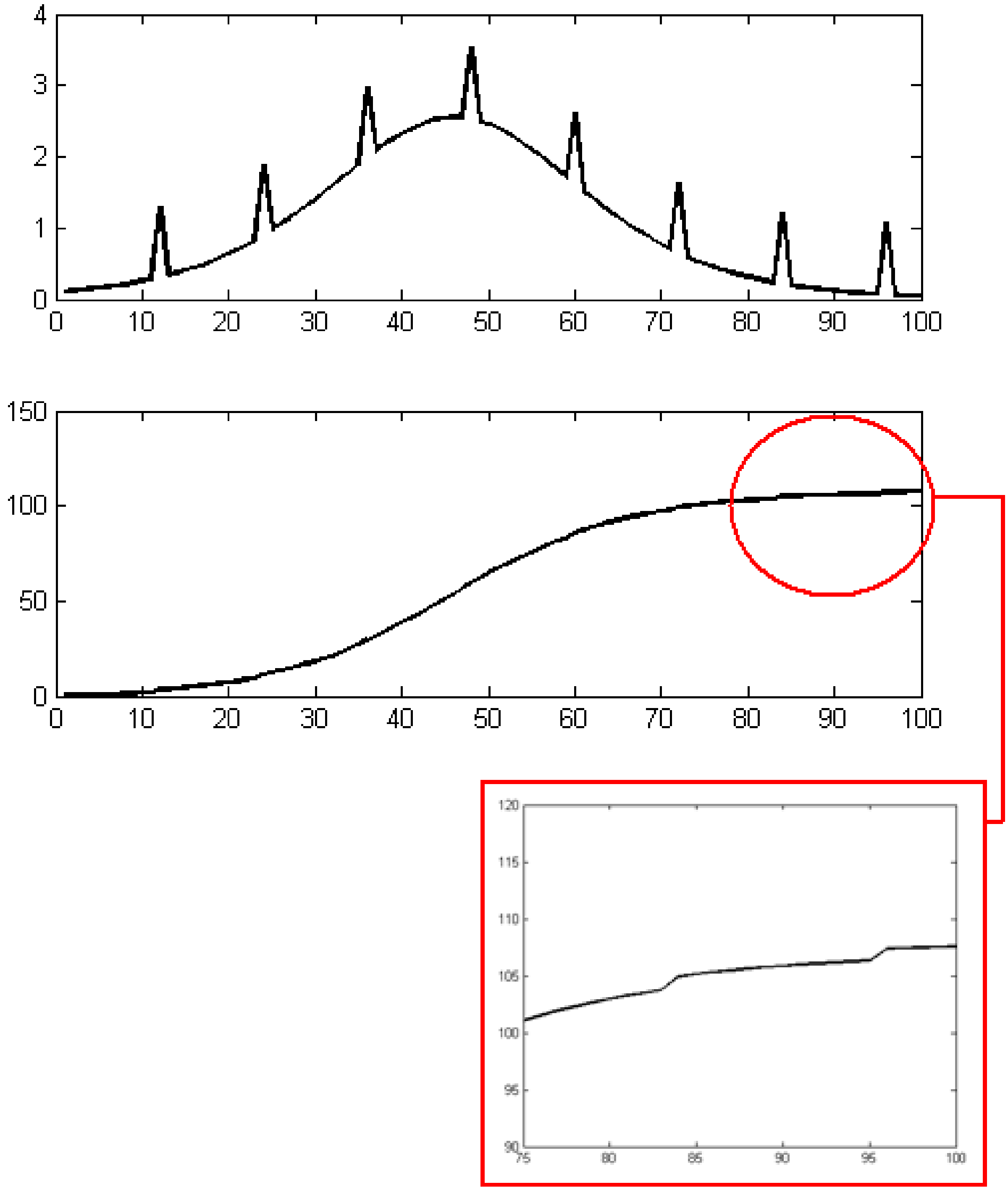

Figure 3: A simplified example of diffusion model with added constant seasonality. The diffusion parameters (p, q and m) in this simplified example are 0.001 0.1 and 100, there is no noise added. The actual sales (the upper panel) shows that the added model does not account for lower fluctuation at the beginning and end of the diffusion series. Further, the zoomed-in part of the cumulative sales shows that the true market potential is higher than $m = 100$.

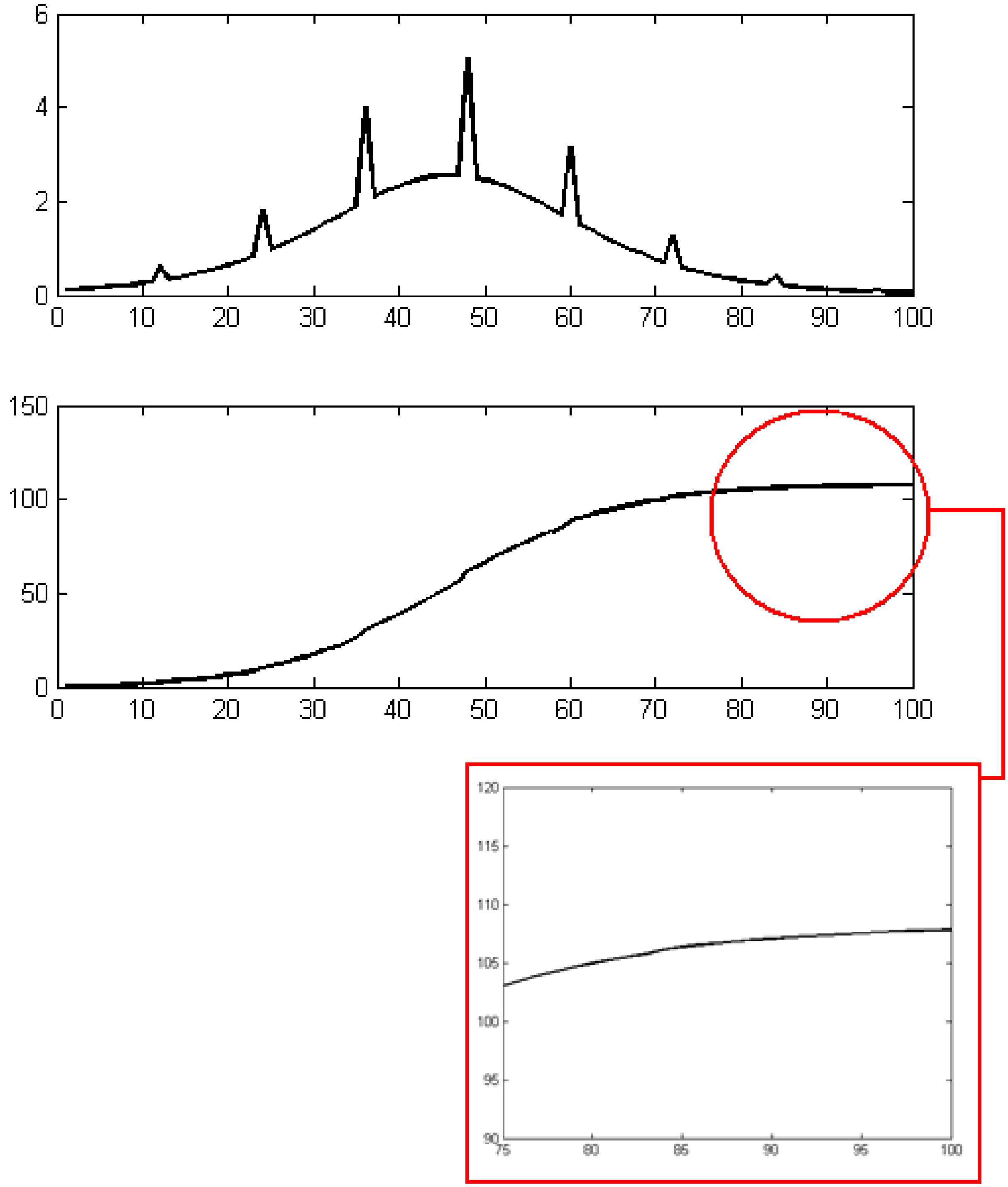

Figure 4: A simplified example of the SGBM diffusion model with a zero-one dummy. The diffusion parameters (p, q and m) in this simplified example are 0.001 0.1 and 100, and there is no noise added. The zoomed-in part of the cumulative sales shows that the true market potential is higher than the simulated market potential.

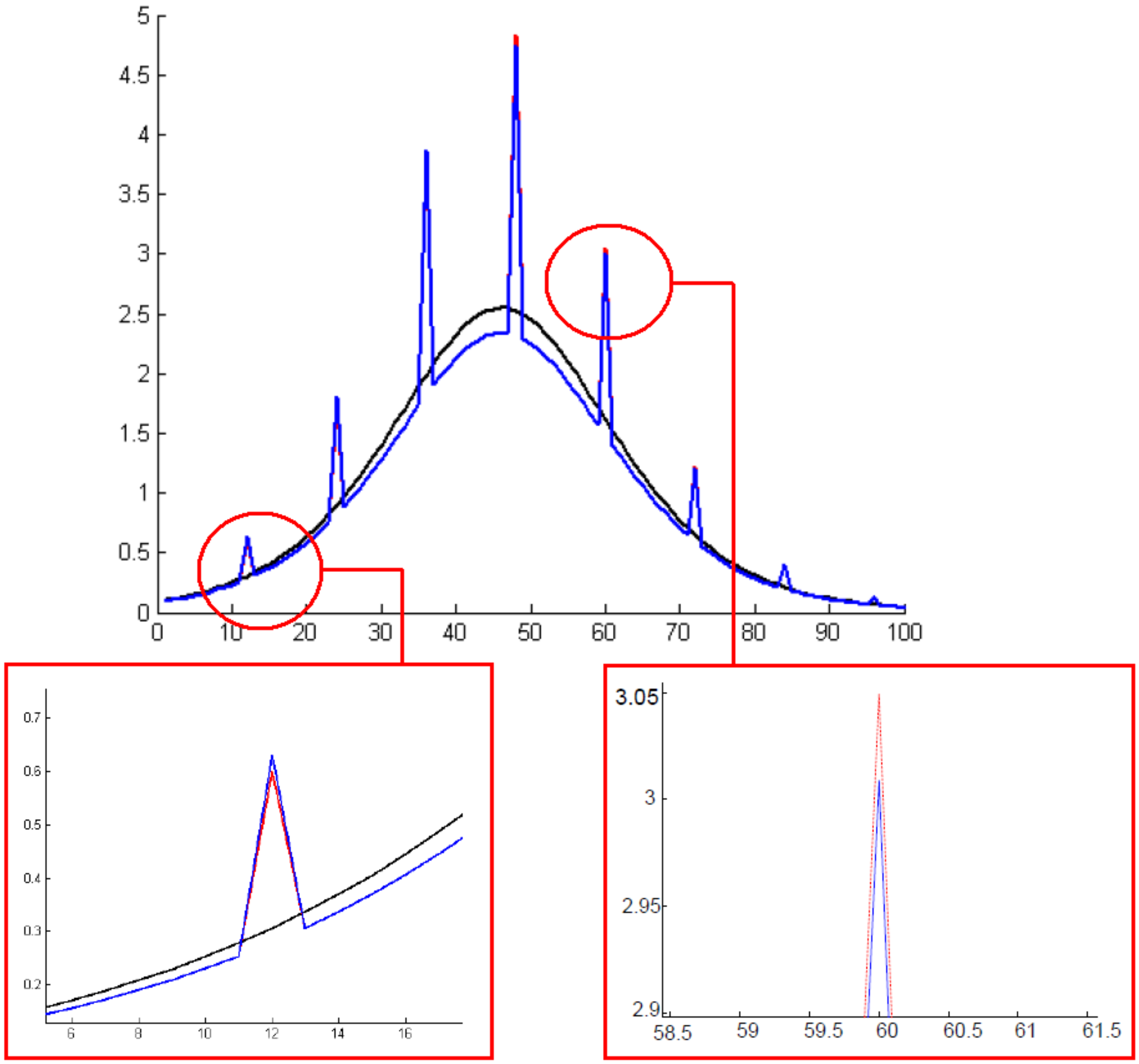

Figure 5: A simplified example of the SGBM diffusion model with zero-mean dummy compared to our model. The dotted line is the underlying diffusion pattern with diffusion parameters p, q and m being 0.001 0.1 and 100 respectively. The solid line is our model and the semi-dotted line is from the SGBM model, both models seem to overlap, but the two zoomed-in parts show the small differences in the seasonal peaks.

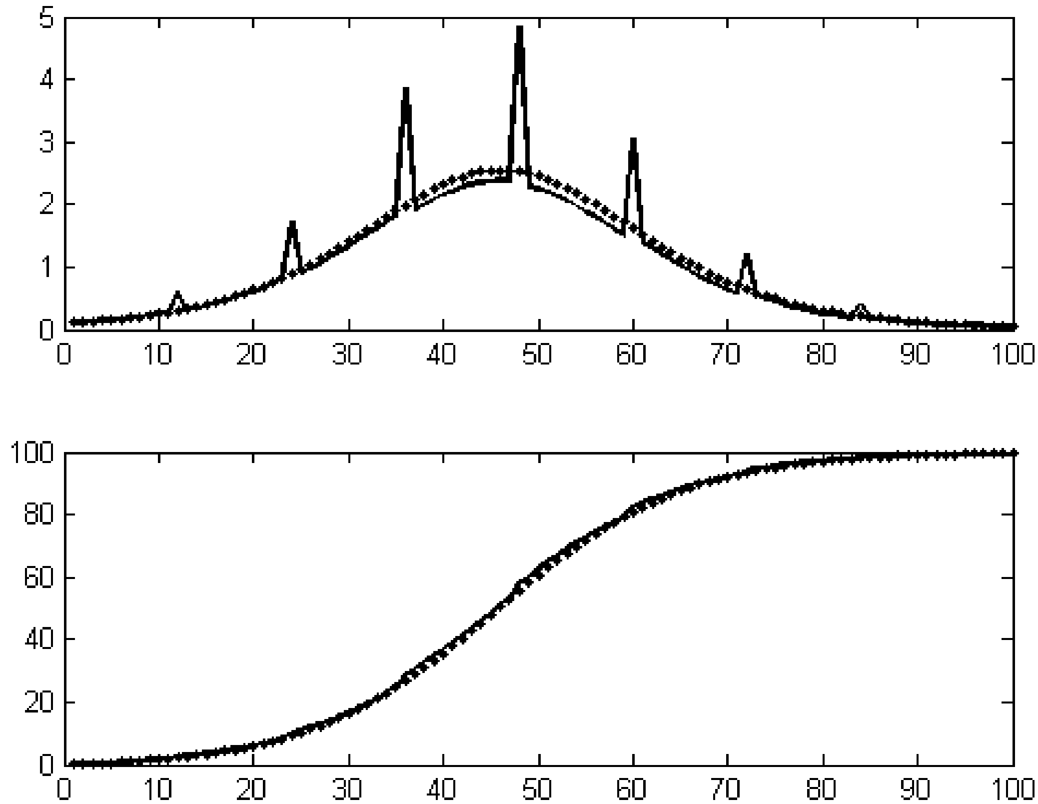

Figure 6: A simplified example of the standard Bass model (dotted line) when estimated on data generated by our model (solid line). The diffusion parameters (p, q and m) in this simplified example are 0.001 0.1 and 100, and there is no noise added.

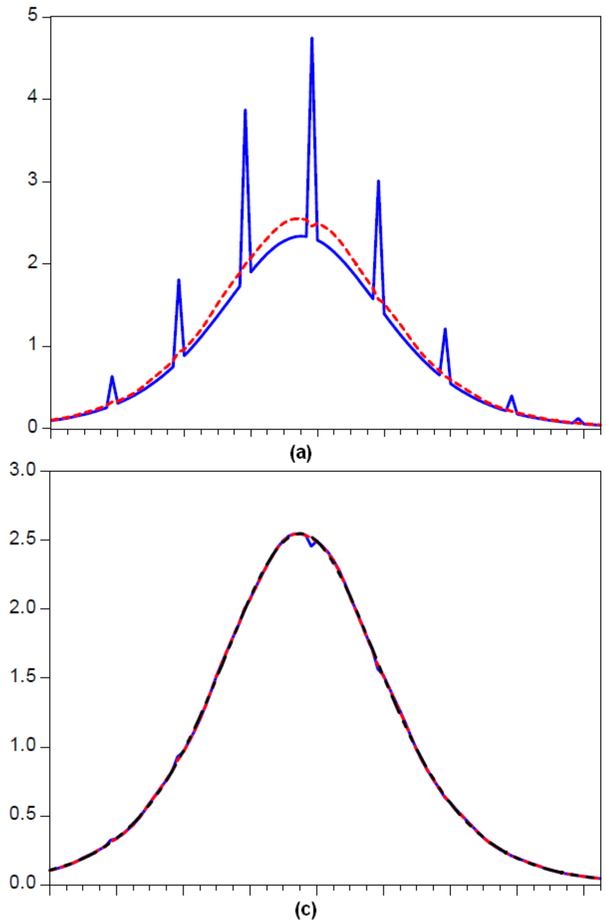

Figure 7: Panel (a) shows the simplified example of our model and the same series after seasonal adjustment with Census X12 moving average. Panel (b) is the same except here our proposed zero-mean version of the SGBM is used. Panel (c) shows the two adjusted series from panels (a) and (b) compared to the underlying Bass model, in case of full diffusion where they are an almost exact match. Panel (d) shows that when the adjustment is done over only a part of the series, there is a clear deviation between the adjusted series and the underlying standard Bass model.

Table 1: Combinations of parameters p and q used in the simulation design together with the corresponding curvature given by the moment of peak sales (T^*). For example, a value of 0.01 for p and 0.05 for q leads to a combination with a moment of peak sales T^* at 27 periods.

		q			
		0.05	0.10	0.15	0.25
p	0.01	27	21	17	14
	0.005	42	29	22	18
	0.001	77	46	33	26
	0.0005	91	53	38	30

Table 2: Results for the full diffusion series. The cells show the percentage difference of true moment of peak sales compared to the estimated moment of peak sales ($\frac{100(\widehat{T^*}-T^*)}{T^*}$), for the case with four monthly dummies and aggregated over all combinations of curvature. The columns are subdivided per noise level. For example the average difference across all combinations of curvature, for the case of four months, with noise level 5, with Our Model being the true model as well as the simulated model is 0.02 with a standard deviation of 0.35.

		Estimated Model														
		Our Model			$SGBM_{ZM}$			$SGBM_{01}$			BM_{het}			BM		
		0.01	5	10	0.01	5	10	0.01	5	10	0.01	5	10	0.01	5	10
Simulated Model	Our Model	-0.00	0.02	0.00	0.01	0.03	0.02	0.13	0.15	0.14	0.32	0.33	0.31	-0.05	-0.03	-0.08
	(StDev)	0.00	0.35	0.69	0.13	0.37	0.70	0.16	0.38	0.70	0.37	0.51	0.78	0.44	0.66	1.08
	$SGBM_{ZM}$	-0.03	-0.02	-0.01	-0.00	0.00	0.01	0.12	0.12	0.13	0.30	0.30	0.30	-0.04	-0.06	-0.03
	(StDev)	0.13	0.37	0.65	0.00	0.34	0.64	0.05	0.34	0.64	0.24	0.40	0.67	0.50	0.69	1.10
	$SGBM_{01}$	-0.13	-0.13	-0.15	-0.11	-0.10	-0.12	0.00	0.00	-0.02	0.17	0.17	0.15	-0.15	-0.15	-0.19
	(StDev)	0.15	0.34	0.63	0.05	0.31	0.61	0.00	0.30	0.61	0.21	0.35	0.65	0.49	0.66	1.08
	BM_{het}	0.00	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.02	0.00	-0.01	0.03
	(StDev)	0.00	0.33	0.70	0.00	0.33	0.70	0.00	0.33	0.70	0.00	0.33	0.69	0.00	0.51	1.06
	BM	0.00	-14.49	-32.53	0.00	-14.56	-32.48	0.00	0.00	0.00	0.00	-14.82	-33.53	0.00	0.00	0.01
	(StDev)	0.04	33.27	101.90	0.04	33.35	100.42	0.00	0.00	0.00	0.04	33.91	104.79	0.00	0.28	0.60

Table 3: Results for the full diffusion series. The cells show the percentage difference of true market potential compared to the estimated market potential ($\frac{100(\hat{m}-m)}{m}$), for the case with four monthly dummies and aggregated over all combinations of curvature. The columns are subdivided per noise level. For example the average difference across all combinations of curvature, for the case of four months, with noise level 5, with Our Model being the true model as well as the simulated model is 0.02 with a standard deviation of 0.71.

		Estimated Model														
		Our Model			$SGBM_{ZM}$			$SGBM_{01}$			BM_{het}			BM		
		0.01	5	10	0.01	5	10	0.01	5	10	0.01	5	10	0.01	5	10
Simulated Model	Our Model	0.00	0.02	0.01	-0.14	-0.11	-0.13	-6.91	-6.88	-6.88	-0.47	-0.44	-0.45	-0.12	-0.09	-0.14
	(StDev)	0.00	0.71	1.37	0.15	0.73	1.38	0.25	0.80	1.54	0.74	1.03	1.56	0.54	0.94	1.61
	$SGBM_{ZM}$	0.13	0.12	0.14	-0.00	-0.00	0.01	-6.64	-6.64	-6.59	-0.24	-0.24	-0.22	-0.04	-0.04	-0.03
	(StDev)	0.15	0.72	1.43	0.00	0.70	1.43	0.02	0.77	1.59	0.56	0.92	1.55	0.53	0.93	1.67
	$SGBM_{01}$	6.77	6.77	6.77	6.65	6.64	6.65	0.00	-0.00	-0.03	6.40	6.39	6.41	6.61	6.61	6.61
	(StDev)	0.14	0.72	1.42	0.01	0.71	1.41	0.00	0.79	1.56	0.57	0.93	1.54	0.53	0.95	1.59
	BM_{het}	0.00	0.01	-0.00	0.00	0.01	-0.01	0.00	0.00	0.00	0.00	0.01	-0.01	-0.00	0.01	0.01
	(StDev)	0.00	0.70	1.37	0.00	0.70	1.37	0.00	0.75	1.53	0.00	0.70	1.37	0.00	0.74	1.46
	BM	-0.01	-25.28	-27.92	-0.01	-25.26	-27.90	0.00	-25.56	-28.42	-0.01	-25.43	-27.98	0.00	0.03	-0.01
	(StDev)	0.06	7.45	7.11	0.06	7.42	7.08	0.08	7.89	7.87	0.05	7.39	7.06	0.00	0.45	0.90

Table 4: The average value of the last datapoint of the cumulative sales across all simulations, this last point represents the true market potential.

			Cumulative Sales
Simulated	**Model**	Our Model	99.70
		SGBM_{ZM}	99.80
		SGBM_{01}	106.43
		BM_{het}	99.92
		BM	99.91

Table 5: Results for the truncated diffusion series. The cells show the percentage difference of real moment of peak sales compared to the estimated moment of peak sales ($\frac{100(\widehat{T^*}-T^*)}{T^*}$), for the case if minimal noise level (0.01) and aggregated over all combinations of curvature. The columns are subdivided per seasonality structure. The table shows both the results for the estimation of the series truncated before and after the moment of peak Sales. For example the average difference across all combinations of curvature for the case of minimal noise, for the case of one month, with Our Model being the true model as well as the simulated model and a series until 20% before the moment of peak sales is 0.00 with a standard deviation of 0.02.

		Estimated Model														
		Our Model			$SGBM_{ZM}$			$SGBM_{01}$			BM_{het}			BM		
		1	2	4	1	2	4	1	2	4	1	2	4	1	2	4
Simulated Model (Before peak sales)	Our Model	0.00	0.00	0.00	-0.52	-3.07	-3.06	-0.42	-2.91	-2.93	-11.83	-10.83	-11.84	-9.63	-1.30	-19.113
	(StDev)	0.02	0.02	0.03	1.16	3.45	3.67	1.17	3.47	3.69	13.91	14.83	15.77	16.36	35.45	20.34
	$SGBM_{ZM}$	0.56	4.80	5.01	0.00	0.00	0.00	0.09	0.15	0.14	-10.93	-9.56	-10.84	-9.47	-0.60	-18.45
	(StDev)	1.18	8.28	8.99	0.02	0.02	0.02	0.05	0.08	0.08	13.91	14.48	15.72	16.23	35.62	20.00
	$SGBM_{01}$	0.44	4.17	4.40	-0.09	-0.14	-0.13	0.00	0.00	0.00	-10.58	-7.49	-10.37	-9.36	-2.94	4.68
	(StDev)	1.14	7.26	7.99	0.05	0.08	0.07	0.02	0.02	0.02	13.47	13.58	15.04	15.50	28.95	54.68
	BM_{het}	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(StDev)	0.05	0.04	0.04	0.05	0.04	0.04	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	BM	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(StDev)	0.05	0.04	0.04	0.05	0.05	0.05	0.03	0.03	0.03	0.03	0.03	0.03	0.01	0.01	0.01
Simulated Model (After peak sales)	Our Model	0.00	0.00	0.00	-0.22	-0.60	-0.58	-0.13	-0.46	-0.46	-2.30	-1.22	-1.48	-0.59	0.87	1.51
	(StDev)	0.00	0.00	0.00	0.27	0.77	0.83	0.27	0.79	0.85	4.41	6.04	6.66	5.46	11.18	14.71
	$SGBM_{ZM}$	0.23	0.70	0.71	0.00	0.00	0.00	0.09	0.14	0.12	-2.01	-0.40	-0.66	-0.47	1.50	2.40
	(StDev)	0.28	0.98	1.05	0.00	0.00	0.00	0.04	0.07	0.07	4.70	6.88	7.55	5.81	12.88	17.37
	$SGBM_{01}$	0.13	0.52	0.54	-0.08	-0.13	-0.11	0.00	0.00	0.00	-1.97	-0.47	-0.71	-0.57	0.99	1.70
	(StDev)	0.27	0.90	0.97	0.04	0.07	0.06	0.00	0.00	0.00	4.50	6.38	7.10	5.50	11.21	15.01
	BM_{het}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(StDev)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	BM	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(StDev)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 6: Results for the truncated diffusion series. The cells show the percentage difference of real market potential compared to the estimated market potential ($\frac{100(\widehat{m}-m)}{m}$), for the case if minimal noise level (0.01) and aggregated over all combinations of curvature. The columns are subdivided per seasonality structure. The table shows both the results for the estimation of the series truncated before and after the moment of peak Sales. For example the average difference across all combinations of curvature, for the case of minimal noise, for the case of one month, with Our Model being the true model as well as the simulated model and a series until 20% before the moment of peak sales is 0.00 with a standard deviation of 0.03.

		Estimated Model														
		Our Model			$SGBM_{ZM}$			$SGBM_{01}$			BM_{het}			BM		
		1	2	4	1	2	4	1	2	4	1	2	4	1	2	4
Simulated Model (Before peak sales)	Our Model	0.00	0.00	0.00	-1.66	-6.11	-6.10	-6.89	-13.42	-12.51	-19.06	-15.51	-17.35	-11.28	93.34	-30.81
	(StDev)	0.03	0.04	0.04	4.51	5.52	6.00	4.48	5.22	5.71	24.85	27.04	29.81	40.41	297.32	38.28
	$SGBM_{ZM}$	2.16	9.26	9.65	0.00	0.00	0.00	-5.00	-7.49	-6.65	-16.88	-13.14	-15.62	-11.16	99.56	-29.70
	(StDev)	5.76	11.19	12.19	0.03	0.04	0.04	0.06	0.09	0.09	25.35	26.80	29.94	40.12	309.69	37.92
	$SGBM_{01}$	7.15	16.44	15.99	5.01	7.48	6.64	0.00	0.00	0.00	-12.14	-1.23	-9.38	-7.13	51.65	747.01
	(StDev)	5.77	10.63	11.65	0.08	0.10	0.10	0.03	0.04	0.04	24.84	28.94	28.94	38.81	174.16	1978.21
	BM_{het}	0.33	0.23	0.20	0.33	0.28	0.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(StDev)	1.26	0.90	0.79	1.26	1.07	0.99	0.03	0.04	0.04	0.03	0.03	0.03	0.05	0.05	0.05
	BM	0.33	0.23	0.20	0.33	0.27	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(StDev)	1.27	0.90	0.79	1.27	1.07	0.99	0.09	0.08	0.09	0.09	0.08	0.09	0.03	0.03	0.03
Simulated Model (After peak sales)	Our Model	0.00	0.00	0.00	-0.36	-0.99	-1.01	-5.45	-8.49	-7.63	-3.76	-2.03	-2.51	-1.06	1.29	2.49
	(StDev)	0.01	0.01	0.01	0.43	1.19	1.27	0.48	1.03	1.12	6.51	8.74	9.70	8.83	17.74	23.93
	$SGBM_{ZM}$	0.38	1.16	1.20	0.00	0.00	0.00	-5.00	-7.50	-6.67	-3.24	-0.57	-1.00	-0.90	2.39	4.25
	(StDev)	0.46	1.51	1.60	0.01	0.01	0.01	0.02	0.04	0.04	6.97	10.07	11.10	9.38	20.72	29.02
	$SGBM_{01}$	5.36	8.62	7.83	4.99	7.48	6.65	0.00	0.00	0.00	1.76	6.92	5.65	4.02	9.31	9.92
	(StDev)	0.47	1.48	1.58	0.02	0.04	0.04	0.01	0.01	0.01	6.98	10.00	11.11	9.31	19.16	26.14
	BM_{het}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(StDev)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	BM	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(StDev)	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00

Table 7: Results for the truncated diffusion series. The cells show the Root Mean Squared Prediction Error for 12 months after the truncated period, which is 20% before or after the moment of peak sales. The RMSPE is aggregated over all levels of curvature, levels of noise and the number of months. For example the average RMSPE where our model is the true model and the estimated model as well is 0.50 with a standard deviation of 0.8.

		Estimated Model				
		Our Model	$SGBM_{ZM}$	$SGBM_{01}$	BM_{het}	BM
Simulated Model (Before peak sales)	Our Model	0.36	0.44	0.47	0.94	1.31
	(StDev)	0.39	0.48	0.50	0.84	1.16
	$SGBM_{ZM}$	0.45	0.37	0.41	0.92	1.32
	(StDev)	0.55	0.40	0.47	0.84	1.17
	$SGBM_{01}$	0.46	0.37	0.40	0.91	1.32
	(StDev)	0.55	0.40	0.45	0.82	1.17
	BM_{het}	0.37	0.37	0.74	0.36	0.56
	(StDev)	0.43	0.44	11.86[1]	0.38	0.78
	BM	0.35	0.34	0.55	0.32	0.23
	(StDev)	0.55	0.56	4.38[1]	0.48	0.42
Simulated Model (After peak sales)	Our Model	0.13	0.15	0.15	0.46	0.58
	(StDev)	0.09	0.10	0.10	0.25	0.37
	$SGBM_{ZM}$	0.15	0.13	0.13	0.48	0.61
	(StDev)	0.10	0.09	0.09	0.27	0.40
	$SGBM_{01}$	0.15	0.13	0.13	0.48	0.61
	(StDev)	0.10	0.09	0.09	0.26	0.39
	BM_{het}	0.13	0.13	0.13	0.12	0.13
	(StDev)	0.09	0.09	0.09	0.08	0.09
	BM	0.13	0.13	0.13	0.12	0.06
	(StDev)	0.11	0.11	0.11	0.11	0.08

[1]The high variation comes from a few extreme outliers

Table 8: Results of the diffusion model of the flat-screen television in Europe

	United Kingdom					Germany					The Netherlands				
	Our Model	$SGBM_{ZM}$	$SGBM_{01}$	BM_{het}	BM	Our Model	$SGBM_{ZM}$	$SGBM_{01}$	BM_{het}	BM	Our Model	$SGBM_{ZM}$	$SGBM_{01}$	BM_{het}	BM
p	1.8e-05	1.8e-05	1.8e-05	1.4e-05	3.0e-05	2.3e-05	2.4e-05	2.3e-05	1.9e-05	4.2e-05	7.0e-05	7.1e-05	7.1e-05	6.4e-05	9.4e-05
(Std.)	1.6e-06	1.9e-06	1.8e-06	3.8e-06	1.9e-05	2.6e-06	3.0e-06	2.9e-06	4.0e-06	1.8e-05	5.1e-06	5.5e-06	5.5e-06	8.5e-06	2.8e-05
q	0.078	0.078	0.078	0.082	0.071	0.072	0.071	0.071	0.075	0.062	0.072	0.071	0.071	0.074	0.067
(Std.)	0.001	0.002	0.002	0.004	0.008	0.002	0.002	0.002	0.003	0.006	0.001	0.002	0.002	0.003	0.005
m	43.49	43.43	36.19	41.34	46.28	34.32	34.38	29.63	32.08	41.17	8.91	8.90	7.86	8.63	9.34
(Std.)	1.21	1.31	1.22	2.97	5.53	1.70	1.87	1.67	2.40	5.67	0.26	0.27	0.26	0.41	0.66
Jan	0.52	0.54	0.65	-	-	0.40	0.41	0.48	-	-	0.43	0.44	0.49	-	-
(Std.)	0.06	0.05	0.07	-	-	0.08	0.07	0.09	-	-	0.05	0.05	0.06	-	-
May	-	-	-	-	-	0.06	0.07	0.08	-	-	0.20	0.21	0.24	-	-
(Std.)	-	-	-	-	-	0.06	0.06	0.08	-	-	0.05	0.05	0.05	-	-
Jun	-0.09	-0.09	-0.11	-	-	-	-	-	-	-	-	-	-	-	-
(Std.)	0.05	0.05	0.06	-	-	-	-	-	-	-	-	-	-	-	-
Oct	0.25	0.26	0.32	-	-	0.21	0.22	0.26	-	-	0.25	0.25	0.29	-	-
(Std.)	0.05	0.05	0.07	-	-	0.07	0.07	0.08	-	-	0.05	0.05	0.06	-	-
Nov	0.36	0.37	0.44	-	-	0.31	0.31	0.36	-	-	0.14	0.14	0.16	-	-
(Std.)	0.05	0.05	0.07	-	-	0.07	0.07	0.08	-	-	0.05	0.05	0.06	-	-
Dec	0.90	0.91	1.10	-	-	0.62	0.64	0.74	-	-	0.35	0.35	0.40	-	-
(Std.)	0.07	0.05	0.07	-	-	0.09	0.07	0.09	-	-	0.05	0.05	0.06	-	-
RMSE	0.057	0.056	0.056	0.168	0.165	0.046	0.046	0.046	0.087	0.082	0.010	0.010	0.010	0.019	0.019
AIC	-236.3	-236.0	-236.0	-108.3	-43.7	-250.8	-250.3	-250.2	-186.2	-137.1	-454.5	-454.6	-454.6	-385.1	-330.8
BIC	-225.4	-225.1	-225.1	-101.8	-37.1	-239.9	-239.4	-239.3	-179.6	-130.5	-443.6	-443.6	-443.6	-378.6	-324.2

Table 9: Results of the diffusion model of the flat-screen television in Europe, continued from Table 8

	Italy					Spain					Belgium				
	Our Model	$SGBM_{ZM}$	$SGBM_{01}$	BM_{het}	BM	Our Model	$SGBM_{ZM}$	$SGBM_{01}$	BM_{het}	BM	Our Model	$SGBM_{ZM}$	$SGBM_{01}$	BM_{het}	BM
p	3.7e-05	3.8e-05	3.8e-05	3.0e-05	8.0e-05	1.4e-05	1.4e-05	1.4e-05	1.2e-05	5.4e-05	7.9e-06	8.1e-06	8.0e-06	6.8e-06	3.5e-05
(Std.)	3.9e-06	4.4e-06	4.4e-06	6.8e-06	3.3e-05	1.9e-06	2.1e-06	2.1e-06	2.4e-06	2.1e-05	1.2e-06	1.4e-06	1.4e-06	1.6e-06	1.8e-05
q	0.067	0.067	0.067	0.071	0.055	0.079	0.078	0.078	0.080	0.059	0.087	0.087	0.087	0.089	0.068
(Std.)	0.002	0.002	0.002	0.004	0.007	0.002	0.002	0.002	0.003	0.006	0.002	0.003	0.003	0.003	0.007
m	24.87	24.91	21.37	23.10	31.47	20.42	20.44	18.10	19.85	27.75	3.94	3.94	3.58	3.86	4.72
(Std.)	1.15	1.27	1.13	1.81	5.45	0.95	1.01	0.95	1.21	3.87	0.16	0.17	0.16	0.21	0.52
Jan	0.19	0.20	0.23	-	-	0.44	0.45	0.51	-	-	0.59	0.61	0.67	-	-
(Std.)	0.06	0.06	0.08	-	-	0.08	0.08	0.10	-	-	0.09	0.09	0.10	-	-
May	0.12	0.13	0.15	-	-	0.15	0.15	0.17	-	-	-	-	-	-	-
(Std.)	0.06	0.06	0.07	-	-	0.07	0.07	0.08	-	-	-	-	-	-	-
Jun	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(Std.)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Oct	0.32	0.32	0.38	-	-	0.22	0.22	0.25	-	-	0.10	0.10	0.11	-	-
(Std.)	0.07	0.06	0.08	-	-	0.08	0.08	0.09	-	-	0.08	0.09	0.10	-	-
Nov	0.24	0.25	0.29	-	-	0.14	0.14	0.16	-	-	-	-	-	-	-
(Std.)	0.06	0.06	0.08	-	-	0.08	0.08	0.09	-	-	-	-	-	-	-
Dec	0.79	0.80	0.94	-	-	0.39	0.39	0.45	-	-	0.36	0.36	0.40	-	-
(Std.)	0.09	0.06	0.08	-	-	0.08	0.08	0.10	-	-	0.09	0.09	0.10	-	-
RMSE	0.032	0.032	0.032	0.067	0.063	0.040	0.040	0.040	0.056	0.049	0.010	0.010	0.010	0.014	0.013
AIC	-299.2	-298.5	-298.4	-216.3	-172.2	-296.3	-295.8	-295.8	-259.8	-204.8	-493.5	-493.1	-493.1	-450.8	-380.5
BIC	-288.2	-287.6	-287.5	-209.8	-165.7	-285.4	-284.9	-284.9	-253.2	-198.2	-486.9	-486.5	-486.5	-444.2	-373.9

Table 10: Forecasting results (RMSPE) of diffusion models for flat-screen televisions in Europe, for one and twelve month ahead forecasts

	Our Model		$SGBM_{ZM}$		$SGBM_{01}$		BM_{het}		BM	
	One	Twelve	One	Twelve	One	Twelve	One	Twelve	One	Twelve
United Kingdom	16.19	10.35	16.27	10.15	15.57	10.22	42.33	27.13	34.25	25.26
Germany	16.25	9.13	16.57	9.15	16.95	9.28	26.62	14.47	16.86	10.98
The Netherlands	2.44	2.64	2.50	2.62	2.44	2.67	4.69	4.30	4.12	3.10
Italy	9.09	4.87	9.15	4.87	9.24	4.91	17.77	10.11	12.31	9.59
Spain	9.10	5.18	9.95	5.11	9.99	5.22	13.93	7.55	9.37	6.03
Belgium	1.87	1.42	1.86	1.44	1.79	1.42	2.54	1.95	2.42	1.85

[a]The RMSPE are multiplied by 100 for convenience

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