This chapter by Funke, Hall and Beeby (hereafter abbreviated as FHB) contains a wealth of material on the important relationship between money and prices. In this comment I focus on only one but very interesting part of their work. This concerns the analysis of long-run properties and short-run dynamics of money and price movements in and across Germany and the UK. The economic issue is whether the entrance of the UK into the Exchange Rate Mechanism (ERM) has established that monetary growth in Germany became important for the UK price level.

FHB put the economic question of a changing causality structure in an econometric framework, which in a simplified and stylized form looks like

\[ \Delta_1 x_t = \alpha_t (x_{t-1} - \beta y_{t-1}) + \varepsilon_{1,t} \]  
\[ \Delta_1 y_t = \gamma_t (x_{t-1} - \beta y_{t-1}) + \varepsilon_{2,t} \]

(1)  
(2)

where \( x_t \) and \( y_t \) are unit-root non-stationary time series variables that require the first differencing filter \( \Delta_1 \) to become stationary, where \( \varepsilon_{1,t} \) and \( \varepsilon_{2,t} \) are standard white-noise processes, \( \beta \) is the cointegration parameter that reflects a long-run relation between the two variables, \( \alpha_t \) and \( \gamma_t \) are the short-run adjustment parameters, where \( \alpha_t \) and \( \gamma_t \) can take different values over time. FHB assume that

\[ \alpha_t = 0 \quad \text{when } t < \tau \]  
\[ \alpha_t = \alpha \quad \text{when } t \geq \tau \]  

(3)

where in their case the break-date \( \tau \) would lie around the entrance of the UK into the ERM. FHB fit a model like (1), and use recursive \( t \)-tests to analyse if and when \( \alpha \) becomes significant, while it is assumed that the cointegration parameter \( \beta \) is non-zero and constant. In this comment I suggest two modifications of the procedure in FHB, which can be useful in practically relevant cases similar to the one studied by FHB.
Time-varying Adjustment

An important unknown parameter in the FHB approach is the break-date $\tau$. Hence, the distribution of the $t$-test statistic used by FHB is complicated and probably not normal (as is assumed in FHB). Additionally, one may want to extend the FHB framework by allowing the rank of the cointegration matrix to vary over time. Quintos (1995) proposes statistical tests for $\tau$ and for the change in the rank of $\Pi$ within the Johansen cointegration testing framework.

An alternative version of (1), which may be more easy to analyse since it does not require a search over possible values of $\tau$, assumes that (3) becomes a continuous function such as

$$
\alpha_t = \alpha \{1 + \exp(-\gamma(t - \tau^*))\}^{-1}, \quad \gamma > 0
$$

With (4), model (1) then becomes a so-called smooth transition error correction model, which extends in a sense the threshold error correction model proposed in Balke and Fomby (1996). The simulation results in Van Dijk and Franses (1996) show that, given short-run adjustment such as (4), the Johansen cointegration method yields unbiased estimates of the cointegration parameter $\beta$. In cases where $\alpha_t$ takes different values depending on, say, the business cycle states, one may consider Markov switching error correction models. Another so-called temporary cointegration model is given in Siklos and Granger (1996).

Time-varying Cointegration

A second more formal framework within which one may analyse changing causal structures concerns time-varying cointegration. A simple example is given by the model

$$
\Delta_1 x_t = \alpha(x_{t-1} - \beta_t y_{t-1}) + \varepsilon_{1,t}
$$

where

$$
\beta_t = 0 \quad \text{where } t < \tau
$$

$$
\beta_t = \beta \quad \text{where } t \geq \tau
$$

This model extends the periodic cointegration model proposed in Boswijk and Franses (1995), where $\beta_t$ can take different values in different seasons. Some diagnostic tests for versions of models such as (5) are proposed in Gregory and Hansen (1996) and Quintos and Phillips (1993). In the latter study, $\beta_t$ is described by a random walk time series model. Of course, in specific cases one may also want to consider, e.g., $\beta_t = \beta + \gamma w_{t-1}$, where $w_t$ is a certain variable, or $\beta_t = \beta + \gamma I[w_{t-1} > d]$, where $d$ is some threshold. At present, the statistical analysis of the latter type of models has not yet been developed.
Concluding Remarks

The FHB chapter on changing causal structures in money and price relations shows that economically meaningful hypotheses can be formulated in terms of non-linear cointegration models. It can be expected that a statistical analysis as well as economic applications of such models will become important in the near future.

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