Econometric Reviews
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713597248

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Online Publication Date: 01 January 1995

To cite this Article Hans, Franses Philip(1995)'A vector of quarters representation for bivariate time series',Econometric Reviews,14:1,55 — 63

To link to this Article: DOI: 10.1080/07474939508800303

URL: http://dx.doi.org/10.1080/07474939508800303

ECONOMETRIC REVIEWS, 14(1), 55-63 (1995)

A VECTOR OF QUARTERS REPRESENTATION FOR BIVARIATE TIME SERIES

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Key words and phrases: seasonality, cointegration, periodic models

JEL code: C22, C52

ABSTRACT

In this paper it is shown that several models for a bivariate nonstationary quarterly time series are nested in a vector autoregression with cointegration restrictions for the eight annual series of quarterly observations. Or, the Granger Representation Theorem is extended to incorporate, e.g., seasonal and periodic cointegration.

1. INTRODUCTION

The use of periodic models for univariate nonstationary seasonal time series has been advocated in recent studies as Osborn (1988) and Franses (1991). The basic idea is to treat the annual series containing the seasonal observations as separate series, and to consider multiple time series models for the vector of the annual variables. The present paper considers an extension of this idea to the bivariate case. Within this framework, it is argued that several non-nested bivariate models for nonstationary seasonal time series are implied by a vector autoregression with cointegration restrictions and parameter restrictions. Hence, the Granger Representation Theorem given in Engle and Granger (1987), is extended to incorporate, e.g., periodic cointegration as well as seasonal cointegration.

Section 2 deals with the representation issue. For notational convenience I only consider bivariate quarterly time series, though extensions are easily constructed. Section 3 covers some of the cointegration restrictions that imply often applied bivariate models. The final section discusses the possible practical relevance of considering vector autoregressions for empirical data, and focuses on the issue of model selection.

2. REPRESENTATION

Consider two time series x_t and y_t that are observed quarterly, i.e. 4 times per year, with $t = 1,..,n$. Further, consider the annual series X_T and Y_T that contain the 4 observations per year when stacked in a vector, with $T = 1,..,N$, i.e., $X_T = (X_{1T},X_{2T},X_{3T},X_{4T})$. It is assumed that $n = 4N$ and that $Z_T = (X_T,Y_T)'$ can be described by the multivariate model

$$A_0Z_T = \mu^* + A_1Z_{T-1} + \dots + A_pZ_{T-p} + \varepsilon_T \tag{1}$$

where the A_i, $i = 0,..,p$, are (8x8) parameter matrices, where μ^* is an (8x1) vector of constants, and where ε_T is an (8x1) vector white noise process with covariance matrix $\sigma^2 I_8$. The model in (1) is a general periodic model in which the means, variances and autoregressive dynamics are allowed to vary over the seasons.

For our purposes of checking cointegration restrictions between the elements of Z_T, it is most convenient to rewrite (1) by pre-multiplying with A_0^{-1}, and by writing the emerging VAR as

$$\Delta Z_T = \mu + \sum_{i=1}^{p-1} \Gamma_i \Delta Z_{T-i} + \Pi Z_{T-p} + \omega_T, \tag{2}$$

(cf. Johansen, 1991), where the Γ_i and Π are (8×8) parameter matrices, which are functions of the A_i, $i = 0,..,p$, and where the μ and ω_T are (8×1) vectors of constants and errors processes, respectively. The Δ is the usual first order differencing operator, defined by $\Delta Z_T = (1-L)Z_T = Z_T - Z_{T-1}$. Note that Δ for the annual series corresponds to the Δ_4 filter for quarterly series, with $\Delta_4 x_t = x_t - x_{t-4}$. The ω_T are independent variables with mean zero and variance matrix Λ, where this Λ is not a diagonal matrix.

With respect the general vector autoregressive model in (2), the Granger Representation Theorem roughly amounts to the following. When the matrix Π is of full rank, the system is stationary. When Π has rank zero, all variables in the vector Z_T are integrated of order one. And, when Π has a rank r such that $0 < r < 8$, one can write $\Pi = \alpha\beta'$, where β is an $(8\times r)$ matrix of cointegration vectors and α is an $(8\times r)$ matrix of adjustment parameters. For details of this theorem, see Engle and Granger (1987) and Johansen (1991). Linear restrictions on the cointegration space can be formulated as

$$\beta = H\varphi \tag{3}$$

where H and φ are $(8\times s)$ and $(s\times r)$ matrices, respectively, where $r \leq s \leq 8$. In Johansen (1991) a maximum likelihood method is developed to test for the value of r and for the validity of the restrictions H.

3. COINTEGRATION RESTRICTIONS AND IMPLIED MODELS

This section considers particular restrictions on the cointegration space and the models that are implied by these restrictions. Of course, there are many examples one can think of, but in order to save space, I only review some of the models which are currently the most often used in practice.

The value of the rank r of Π, as well as the adequacy of the restrictions H, can imply the adequacy of models for the quarterly $z_t = (x_t,y_t)'$ series. To save space, I assume that a model for x_t is the equation of interest. For example, the annual series X_{sT} and Y_{sT}, $s = 1,..,4$, are integrated processes when $r = 0$, and one may want to consider

$$\phi_s(B)\Delta_4 x_t = \mu_s + \gamma_s(B)\Delta_4 y_t + \varepsilon_{st} \tag{4}$$

as the first row of a bivariate model. The $\phi_s(B)$ and $\gamma_s(B)$ are seasonally varying polynomials in the backward shift operator B, which operates on quarterly observations. The μ_s are the seasonal means, and the ε_{st} is an error process with a periodic variance. Of course, in practice one can check whether any assumptions on ε_{st} and on periodicity of the polynomials are valid.

When the matrix β contains seven columns, i.e., when r equals 7, and the restrictions

$$H = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ -\alpha & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -\alpha & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -\alpha & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -\alpha & 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

apply, there are (1,–1) relations between the X_{sT}, (1,–1) relations between the Y_{sT}, and the variables $X_{sT} - \alpha Y_{sT}$ are stationary. A corresponding bivariate model is now

$$\phi_s(B)\Delta_1 x_t = \mu_s + \gamma_s(B)\Delta_1 y_t + \psi_s(x-\alpha y)_{t-k} + \varepsilon_{st}. \tag{6}$$

Note that the parameters which reflect adjustment to disequilibrium errors are seasonally varying. The Δ_1 filter is the first order differencing filter for quarterly time series, and it is implied by the (1,–1) cointegration relations between the annual series. The value of k corresponds to the order of the VAR in (2), i.e., k equals $4p$. In case there are error correcting variables that represent equilibrium relations which are valid for all quarters, such as in fact the $(x_t-\alpha y_t)$ in (6), the k can take other values.

The model used in Davidson *et al.* (1978) is a restricted, i.e. nonperiodic version of

$$\phi_s(B)\Delta_4 x_t = \mu_s + \gamma_s(B)\Delta_4 y_t + \psi_s(x-y)_{t-k} + \varepsilon_{st}, \tag{7}$$

and it is implied by the restrictions

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{8}$$

When the -1 in (8) are replaced by $-\phi_s$, where ϕ_s is a seasonally varying parameter, (7) becomes

$$\phi_s(B)\Delta_4 x_t = \mu_s + \gamma_s(B)\Delta_4 y_t + \psi_s(x-\phi_s y)_{t-k} + \varepsilon_{st}. \tag{9}$$

This model is called a periodic cointegration model, which is found useful in some practical occasions, see, e.g., Franses and Kloek (1991). In Birchenhall *et al.* (1989) a similar model is proposed, in which it is assumed that y_t may be described by a periodically integrated model, such as $y_t = \lambda_s y_{t-1} + \varepsilon_t$ with $\lambda_1\lambda_2\lambda_3\lambda_4 = 1$, albeit that not all λ_s are equal to one, see also Osborn (1988), see also Boswijk and Franses (1992).

Under certain restrictions, one cointegration relationship between the elements of X_{sT} and Y_{sT} can imply the model

$$\phi_s(B)\Delta_4 x_t = \mu_s + \gamma_s(B)\Delta_4 y_t + \psi_s[(1+B+B^2+B^3)x-\alpha_{12}(1+B+B^2+B^3)y]_{t-k} + \varepsilon_{st}. \tag{10}$$

This model reflects that the x_t and y_t series are seasonally integrated, and that there is cointegration at frequency 0, see Engle *et al.* (1993). The full seasonal cointegration model, without lagged $\Delta_4 x_t$ and $\Delta_4 y_t$, is

$$\begin{aligned} \Delta_4 x_t = {} & \gamma_{11}[(1+B+B^2+B^3)x_{t-1} - \alpha_{12}(1+B+B^2+B^3)y_{t-1}] \\ & + \gamma_{12}[(-1+B-B^2+B^3)x_{t-1} - \alpha_{22}(-1+B-B^2+B^3)y_{t-1}] \\ & - (\gamma_{13}+\gamma_{14}B)[(-1+B^2)x_{t-2} - \alpha_{32}(-1+B^2)y_{t-2} - \alpha_{41}(-1+B^2)x_{t-3} \\ & - \alpha_{42}(-1+B^2)y_{t-3}], \end{aligned} \tag{11}$$

see equation (5) in Engle *et al.* (1993). It can now easily be derived that cointegration at frequency ½ is implied by the restrictions

$$H' = [1,\ -1,\ 1,\ -1,\ -\alpha_{22},\ \alpha_{22},\ -\alpha_{22},\ \alpha_{22}], \tag{12}$$

and that cointegration at frequency ¼ is related to

$$H' = \begin{bmatrix} -\alpha_{41} & 1 & \alpha_{41} & -1 & -\alpha_{42} & -\alpha_{32} & \alpha_{42} & \alpha_{32} \\ 1 & \alpha_{41} & -1 & -\alpha_{41} & -\alpha_{32} & \alpha_{42} & \alpha_{32} & -\alpha_{42} \end{bmatrix}, \tag{13}$$

In summary, it can be seen from the expressions in (4) through (13) that restrictions on the parameters of (2) can imply several nonnested models for a bivariate quarterly time series.

4. DISCUSSION

In this paper it is shown that a vector autoregression with cointegration restrictions for the annual series of quarterly observations nests several models for bivariate nonstationary quarterly time series. An implication is that the Granger Representation Theorem is extended to cover the seasonal, as well as the periodic, cointegration model.

The main conclusion to be drawn from the exercise in the previous section is that when one assumes the adequacy of a certain bivariate time series model for quarterly observations in practice, one implicitly makes an assumption on the number of unit roots in the multivariate system and on the validity of particular restrictions on the cointegration vectors. Naturally, this calls for a model selection strategy, with which one can verify whether one's empirical model is compatible with the assumed number of unit roots and the parameter restrictions.

Given the vector autoregressive model framework, one may now be inclined to use the Johansen (1991) maximum likelihood method for model selection in empirical occasions. However, there are several practical problems involved. First, the number of annual observations may not be very large, while the number of parameters to be estimated can be. For example, in a VAR(1) for Z_T, the number of parameters in Π equals 64. Second, the empirical performance of the method deteriorates in case the system gets large. Finally, the small sample behavior of the tests for the linear restrictions on the cointegration vectors may be different from the asymptotic behavior.

There are several alternative methods. An informal approach is given in Franses (1992). This method amounts to checking whether the annual series X_{sT} and Y_{sT} are cointegrated using the standard Engle and Granger (1987) tests. When the null hypothesis of no cointegration is rejected for each of the four seasons, there are at least 4 cointegration relations between the elements of Z_T. Only when there is cointegration at all the frequencies, as in (11), there are four such relations. Hence, when there is cointegration at only some of the frequencies, this informal method may be useful.

A more formal approach consists of two steps. The first is to check the number of unit roots in the individual systems of X_T and Y_T. The sum of these numbers is then the maximum number of unit roots in the Z_T system. Conversely, the sum of the cointegration relations in models for X_T and Y_T is the mimimum value of the rank of Π in (2). The methods in Boswijk and Franses (1992) and in Franses (1992b) can be used for detecting the numbers of unit roots and cointegration relations in the individual series, respectively. The sum of these numbers may already indicate the exclusion of several bivariate models. The second step consists of estimating a bivariate time series model with periodic parameters for x_t and y_t. The emerging parameter estimates are then used for A_i in (1), with which one can calculate the elements of the Π matrix in (2). In many empirical occasions, this Π matrix will not contain only nonzero elements. Hence, it may be possible to relate the rank of Π to particular parameter restrictions in the bivariate model for (x_t,y_t). The latter restrictions can then be tested using nonlinear estimation techniques. When one has decided on the number of cointegration relations between the elements of Z_T, F type tests can be used for further model selection steps. For example, when the rank of Π equals 4, and one considers selection between (9) and (7), one can estimate (9) in the unrestricted form

$$\phi_s(B)\Delta_4 x_t = \mu_s + \gamma_s(B)\Delta_4 y_t + \psi_s x_{t-k} + \kappa_s y_{t-k} + \varepsilon_{st} \qquad (14)$$

and test whether κ_s equals $-\psi_s$ for all s. Given that the number of unit roots is equal under the null and alternative hypothesis, the F test asymptotically follows a standard F distribution.

An extensive comparison of the various methods in empirical examples and in Monte Carlo experiments is an interesting topic for further research.

NOTES

This paper was initiated while I visited the Institute of Economics in Aarhus (February 1991). It is an extension of part of chapter 4 of my Ph.D. thesis. Thank are due to Svend Hylleberg, Søren Johansen, Teun Kloek and Denise Osborn for helpful suggestions, and to two anonymous referees for their constructive suggestions.

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