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Modeling seasonality in bimonthly time series

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Abstract: A recurring issue in modeling seasonal time series variables is the choice of the most adequate model for the seasonal movements. One selection method for quarterly data is proposed in Hylleberg et al. (1990). Market response models are often constructed for bimonthly variables, and hence the topic of the present paper is an extension of their method to such time series.

Keywords: Time series; seasonality; model selection.

1. Introduction

A recurring issue in modeling seasonal time series variables is the choice of the most adequate model for the seasonal movements. Given that a large number of models is available, see e.g., Ghysels (1990), it is important to have an appropriate model selection method. One such method is proposed in Hylleberg et al. (1990), where it is applied to quarterly time series. The extension of their method to monthly time series is treated in Beaulieu and Miron (1991) and Franses (1990). The influential paper on data interval bias by Clarke (1976) has established that several market response models make use of bimonthly time series variables. An extension of the Hylleberg et al. (1990) method to such time series may therefore be of interest to marketing researchers, and it is therefore the topic of this paper.

In Section 2, I discuss the general model selection issue of interest for seasonal time series. In Section 3, the Hylleberg et al. (1990) method is surveyed. In Section 4, I extend their approach to bimonthly variables, and apply it in several examples. A discussion concludes this paper.

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2. Modeling seasonality

In the literature on seasonality one can find several different views on modeling seasonal time series, see, e.g., Abraham and Box (1978) and Hylleberg (1986). One view is introduced in Box and Jenkins (1970, Chapter 9), and it results in a general multiplicative seasonal time series model. For a zero mean time series y_t, which is measured s times a year, this model is given by

$$\phi_{sP}(B^s)(1-B^s)^D y_t = \theta_{sQ}(B^s)\nu_t, \qquad (1)$$

with

$$\phi_p(B)(1-B)^d \nu_t = \theta_q(B)\varepsilon_t. \qquad (2)$$

where B is the backward-shift operator defined by $B^k y_t = y_{t-k}$, and where s is usually 2, 4, 6 or 12. The $\phi_{sP}(B^s)$ and $\theta_{sQ}(B^s)$ are polynomials in

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the operator B^s of orders P and Q, and the $\phi_p(B)$ and $\theta_q(B)$ are polynomials in B of order p and q. It is assumed that the usual conditions for stationary and invertibility of the processes in (1) and (2) hold. The values of d and D are often 0, 1 or 2, see Dickey and Pantula (1987). The ε_t in (2) is assumed to be a white noise process, i.e. an uncorrelated zero mean process with constant variance.

When dealing with empirical economic time series one may often observe that the observations in season i, $i = 1,\ldots,s$, are highly dependent on the value of those in the same season in previous years. Furthermore, they may show patterns which seem to be independent from the values in season j, where $j \neq i$. This may indicate that an appropriate value of D in (1) can be 1, i.e. that one may apply the annual differencing filter $1 - B^s$. With respect to the nonseasonal part, if can often be recognized that the observations at time t are highly dependent on $t-1$ such that a nonseasonal differencing filter $1-B$, i.e. $d = 1$ in (2), may be suitable. Therefore, a regularly applied transformation of economic time series is

$$x_t = (1 - B)(1 - B^s)y_t, \tag{3}$$

When this x_t is modeled with an MA model such that $q = Q = 1$, one obtains the well-known airline model

$$(1-B)(1-B^s)y_t = (1 + \theta_1 B)(1 + \theta_s B^s)\varepsilon_t, \tag{4}$$

which has been popularized by Box and Jenkins (1970). This airline model has already been applied and evaluated in a host of empirical studies and it seems to be useful in many of these, see Abraham and Ledolter (1983, Chapter 6), Granger and Newbold (1986, Chapter 3) and Hanssens, Parsons and Schultz (1990, Chapter 4).

Despite its apparent success, there are also practical occasions in which the double differencing filter in (3) causes that model (4) is noninvertible, i.e., that one or both of its parameters are equal to minus 1. The choice of the $1-B$ or the $1-B^s$ filter may be guided by an inspection of the autocorrelation functions (ACFs) of the several distinctly differenced variables. A straightforward interpretation of ACFs can however be blurred by the presence of deterministic effects. A similar phenomenon may occur in seasonal time series where seasonal dummy variables may account for a part of the seasonal fluctuations. The observations in season i and in the same season the previous year are now highly correlated. Hence, the values of the ACF at seasonal lags may also die out only slowly. This establishes the need for a class of models in which seasonality is modeled deterministically, or

$$\phi_p(B)(1-B^d)y_t = \alpha_0 + \sum_{i=1}^{s-1} \alpha_i D_{it} + \phi_q(B)\varepsilon_t, \tag{5}$$

where $\phi_p(B)$ and $\theta_q(B)$ can include terms as B^{ks}, where $k = 1, 2,\ldots,$ and in which intermediate parameters can be set equal to zero. The seasonal dummy variables D_{it} take a value 1 in season i each year, and a value 0 in all other periods.

3. Model selection via testing for seasonal unit roots

Given the complicated expressions in (1) with (2) and (5), one can imagine that the selection of an appropriate model for seasonal time series may not be without difficulties. A recent proposal for a model selection method is given in Hylleberg et al. (1990). They consider for quarterly time series a process like (1) and (2) with d assumed to be 0 as their null model, a simple version of which is $(1-B^4)y_t = \varepsilon_t$. They recognize that its differencing filter can be written as

$$\begin{aligned}(1-B^4) &= (1-B)(1+B)(1-\mathrm{i}B)(1+\mathrm{i}B)\\ &= (1-B)(1+B)(1+B^2)\\ &= (1-B)(1+B+B^2+B^3).\end{aligned} \tag{6}$$

From this expression it can be seen that the $1-B^4$ filter assumes the presence of four unit roots, i.e. ± 1 and $\pm \mathrm{i}$, and that it can be written as the product of $1-B$ and a term which reflects an annual moving average. The roots -1 and $\pm \mathrm{i}$ are called seasonal unit roots, while the root 1 is called the nonseasonal unit root. There is now a large literature on the issue of unit roots, and hence to save space, the reader is referred to, e.g., the lucid discussion in Dickey, Bell and Miller (1986).

From (6) it can be seen that transforming a quarterly time series with a $1-B^4$ filter is appropriate only in the case of the simultaneous presence of 4 units roots. However, transforming the series with $1-B^4$ yields an overdifferenced series in case only one unit root is present such that, e.g., applying the $1-B$ filter is sufficient to make the series stationary and that seasonality can be handled by the inclusion of seasonal dummies. This overdifferencing may cause trouble for the construction of time series models because the (partial) autocorrelation patterns become hard to interpret. Furthermore, one may expect estimation problems because of the introduction of moving average polynomials with roots close to the unit circle. On the other hand, underdifferenced series may yield unit roots in their autoregressive parts, and so classical arguments as those in e.g., Granger and Newbold (1974), for time series containing neglected unit roots apply. So, it is important to test for the presence of (seasonal) unit roots.

The crucial proposition which makes the testing procedure relatively simple is given in Hylleberg et al. (1990, pp. 221–222). Since I need it in a subsequent section of this paper, this proposition is given almost literally in the technical appendix below. An application of this proposition to testing for (seasonal) unit roots in quarterly time series, i.e. applying (A.5) to (6), yields

$$\varphi(B)=\lambda_1B\varphi_1(B)+\lambda_2(-B)\varphi_2(B)+\lambda_3(-\mathrm{i}-B)B\varphi_3(B)+\lambda_4(\mathrm{i}-B)B\varphi_3(B)+\varphi^*(B)\varphi_4(B), \tag{7}$$

where

$$\varphi_1(B)=(1+B+B^2+B^3), \tag{8a}$$

$$\varphi_2(B)=(1-B)(1+B^2)=(1-B+B^2-B^3), \tag{8b}$$

$$\varphi_3(B)=(1-B^2), \tag{8c}$$

$$\varphi_4(B)=(1-B^4). \tag{8d}$$

Defining

$$\lambda_1=-\pi_1, \tag{9a}$$

$$\lambda_2=-\pi_2, \tag{9b}$$

$$\lambda_3=\tfrac{1}{2}(-\pi_3+\mathrm{i}\pi_4), \tag{9c}$$

$$\lambda_4=\tfrac{1}{2}(-\pi_3-\mathrm{i}\pi_4), \tag{9d}$$

(7) can be written as

$$\varphi(B)=-\pi_1B\varphi_1(B)+\pi_2B\varphi_2(B)+(\pi_3B+\pi_4)B\varphi_3(B)+\varphi^*(B)\varphi_4(B). \tag{10}$$

Suppose that the quarterly observations are generated by an AR process

$$\varphi(B)y_t=\mu_t+\varepsilon_t, \tag{11}$$

where μ_t covers the deterministic elements, and might consist of a constant, seasonal dummies, or a trend. Now, the auxiliary test equation becomes

$$\varphi^*(B)y_{4t}=\mu_t+\pi_1y_{1,t-1}+\pi_2y_{2,t-1}+\pi_3y_{3,t-2}+\pi_4y_{3,t-1}+\varepsilon_t, \tag{12}$$

where

$$y_{1t}=\varphi_1(B)y_t,$$

$$y_{2t}=-\varphi_2(B)y_t,$$

$$y_{3t}=-\varphi_3(B)y_t,$$

$$y_{4t}=\varphi_4(B)y_t.$$

Applying OLS to (12), where the order of φ^*(B) is established in an experimental way using conventional autocorrelation checks, gives estimates of the π_i. Because the π_i are zero in case the corresponding unit roots are on the unit circle, see (9) and (A.2), testing the significance of the estimated π_i implies testing for unit roots. There will be no seasonal unit roots if π_2, and π_3 or π_4 are significantly different from zero. If $\pi_1=0$, then the presence of root 1 can not be rejected. This procedure for the nonseasonal unit root is similar to the Dickey and Fuller method when applied to the series y_{1t}. Note that, e.g., in case only $\pi_3 = \pi_4 = 0$ the transformation $1 + B^2$ can be applied to the raw data series. This kind of test outcomes ensures that the procedure considers also models which are intermediate between (1) and (5). For an exposition of such a case for a monthly time series, see Abraham and Box (1978). The alternative hypotheses for the first two unit roots are that the roots are smaller than one in an absolute sense, which implies that the t-test for π_1 and π_2 are one-sided. A test strategy for π_3 and π_4 may be to test π_4 in a two-sided procedure, and when the insignificance of π_4 is accepted, to check the significance of π_3 with a one-sided t-test. A sensible strategy may also be to jointly test $\pi_3 = \pi_4 = 0$ with an F-test. See Hylleberg et al. (1990) for further details.

To see why it is reasonable to have a two-sided test for π_4 and a one-sided test for π_3, consider for example

$$\varphi(B) = (1 + \alpha B^2)\left(\frac{1 - B^4}{1 + B^2}\right) = (1 + \alpha B^2)(1 - B^2), \tag{13}$$

where $|\alpha| < 1$ in the stationary case. Using (A.1) and (A.3) it can easily be established that for the $\varphi(B)$ in (13) its holds that

$$\varphi(-\mathrm{i}) = \varphi(\mathrm{i}) = 2(1 - \alpha),$$

$$\delta_1(-\mathrm{i}) \cdot \delta_2(-\mathrm{i}) \cdot \delta_4(-\mathrm{i}) = \delta_1(\mathrm{i}) \cdot \delta_2(\mathrm{i}) \cdot \delta_3(\mathrm{i}) = 4,$$

and hence that $\lambda_3 = \lambda_4 = \frac{1}{2}(1 - \alpha)$, such that λ_3, $\lambda_4 > 0$ under the stationary alternative. The transformations in (9) give that

$$\pi_3 = -\lambda_3 - \lambda_4,$$

$$\pi_4 = \mathrm{i}(\lambda_4 - \lambda_3),$$

and hence that tests for π_3 and π_4 may be one- and two-sided, respectively.

The null hypothesis, i.e. $(1 - B^4)y_t = \varepsilon_t$, in the test procedure in (12) is a nonstationary model, and therefore the critical values for the several test statistics have to be tabulated. Tables with critical values are displayed in Hylleberg et al. (1990) for the cases where μ_t can contain several combinations of deterministic elements. The distribution of the t-test statistic for π_1 depends on the inclusion of trend and intercept. The distribution of the t-tests for the π_2, π_3 and π_4 are dependent on the inclusion of seasonal dummy variables. In general it applies that deterministic elements make that the empirical as well as asymptotic distributions shift to the left, or, equivalently, that the critical values become larger in an absolute sense. The tables are based on Monte Carlo replications, and therefore the tabulated critical values are not exact. This naturally implies that it may not be wise to establish the significance of a parameter on the basis of test values which differ from critical values only by their second decimal point.

Extensions of this procedure to testing for seasonal unit roots in monthly time series are Franses (1990) and Beaulieu and Miron (1991). The results of some empirical power investigations are also reported there. One conclusion is that it is difficult to distinguish between stationary and integrated seasonality in a model like $y_t = \rho y_{t-12} + \varepsilon_t$, with ρ equal to 0.9 and 0.5, although this difficulty mainly concerns the test for the presence of the nonseasonal unit root. The test procedure does seem to have reasonable power with respect to the detection of the seasonal unit roots, especially when ρ equals 0.5. Secondly, it can be seen that a clear recognition of the alternative hypothesis, and in particular the elements in μ_t, does have a significant impact on the power. Finally, the power of the test procedure can be low for a small number of observations.

4. Testing for seasonal unit roots in bimonthly time series

The influential study of Clarke (1976) on data interval bias in empirical marketing models has motivated the consideration of bimonthly observations on time series variables in, e.g., market response models which are built according to the econometric and time series analysis approach (ETS), see Hanssens, Parsons and Schultz (1990). Recent examples of studies which consider such bimonthly time series are Leeflang and Reuyl (1985), and Franses (1991b).

Consider the decomposition of the polynomial as (6), which is relevant to bimonthly time series, or

$$\begin{aligned}1-B^6 &= (1-B^2)(1-B+B^2)(1+B+B^2)\\ &= (1-B^2)(1+B^2+B^4)\\ &= (1-B)(1+B+B^2+B^3+B^4+B^5)\\ &= (1-B)(1+B)\left(1-\tfrac{1}{2}(\mathrm{i}\sqrt{3}+1)B\right)\\ &\quad\times\left(1+\tfrac{1}{2}(\mathrm{i}\sqrt{3}-1)B\right)\left(1+\tfrac{1}{2}(\mathrm{i}\sqrt{3}+1)B\right)\\ &\quad\times\left(1-\tfrac{1}{2}(\mathrm{i}\sqrt{3}-1)B\right). \qquad (14)\end{aligned}$$

Similar to the derivations in (6) through (9), which for the bimonthly case are displayed in the technical appendix, a test equation for testing for seasonal unit roots in this case can be found to be

$$\begin{aligned}\varphi^*(B)y_{5,t} &= \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2}\\ &\quad + \pi_4 y_{3,t-1} + \pi_5 y_{4,t-2} + \pi_6 y_{4,t-1}\\ &\quad + \mu_t + \varepsilon_t, \qquad (15)\end{aligned}$$

with

$$\begin{aligned}y_{1t} &= (1+B)(1+B^2+B^4)y_t,\\ y_{2t} &= -(1-B)(1+B^2+B^4)y_t,\\ y_{3t} &= -(1-B^2)(1+B+B^2)y_t,\\ y_{4t} &= -(1-B^2)(1-B+B^2)y_t,\\ y_{5t} &= (1-B^6)y_t.\end{aligned}$$

Tables for the critical t-values of the individual π_i are given in the appendix. The critical values are generated similar to those for quarterly data. This means that 5000 replications of the data generating process $y_t = y_{t-6} + \varepsilon_t$, with $\varepsilon_t \sim$ N(0, 1), are constructed, that model (15) is estimated by OLS, and that t-ratios and F-tests are calculated and ordered. For example, the 5% critical value is then obtained by taking that value of a test statistic below, or above, which 5% of the estimated values can be found. This is repeated for several combinations of deterministic elements in μ_t, i.e. constant, seasonal dummies and trend. Similar to the quarterly case, one can verify that the tests for π_1 and π_2 are one-sided, the tests for π_4 and π_6 are two-sided and that for π_3 and π_5 these are one-sided. The tables for F-tests of $\pi_3 = \pi_4 = 0$ and $\pi_5 = \pi_6 = 0$ are displayed in the second part of the appendix.

To illustrate the application of the procedure in (15), I have chosen to consider the log of the Dutch car sales series for the period 1975–1988, LNQC, and the Dutch series over 1978–1987 for the log of the primary demand for beer, LNQB, and over 1978–1984 for total advertising, AT. The graph of the first series, when observed as monthly data, is given in Franses (1991a). It appears that a sensible alternative hypothesis is that seasonality can be modeled by seasonal dummies, and that a trend does not seem to govern the series. The graphs of LNQB and AT are displayed in Franses (1991b), and for these series a similar alternative hypothesis can be assumed. The test results are summarized in Table 1. The results for LNQC are quite close to those already obtained for monthly data. In Franses (1991b) it has been argued that for LNQB as well as for AT seasonal dummies may be appropriate, and that both series should not be first order differenced. For the primary demand variable this seems to be valid, although the AT series seems to contain several unit roots.

Table 1
Testing for (seasonal) unit roots in some empirical bimonthly time series

		Variable		
		LNQC [a]	LNQB [b]	AT [b]
t-statistics	π_1	−2.439	−3.544**	−0.775
	π_2	−2.482*	−2.667*	−1.394
	π_3	−3.552**	−3.198*	−3.000*
	π_4	3.679**	4.135**	0.881
	π_5	−7.342**	−3.633**	−2.288
	π_6	−3.578**	−0.893	−0.817
F-statistics	π_3, π_4	9.787**	10.170**	5.152*
	π_5, π_6	26.812**	7.677**	2.753

* Significant at a 10% level.
** Significant at a 5% level.
[a] Auxiliary regression contains a constant and seasonal dummies. The polynomial $\varphi^*(B)$ is $1-\varphi_3 B^3$, the estimated parameter of which is highly significant. The number of observations is 75.
[b] The auxiliary regressions contain constant and seasonal dummies, while $\varphi^*(B)$ is 1, and the number of observations are 54 and 36.

5. Discussion

Many modeling strategies for seasonally unadjusted time series start with the formulation of a suitable model for seasonality. It is current practice to remove seasonality via automatically transforming the series with the double differencing filter as in (5). Some adjustment procedures also use filters as $1-B^s$, and so the presence of several (seasonal) unit roots is often assumed. The method to test for seasonal unit roots developed in Hylleberg et al. (1990), and extended to bimonthly time series in the present paper, may be useful to establish the adequacy of these filters.

The empirical relevance of adequate model selection in seasonal time series may be emphasized by the results in e.g., Beaulieu and Miron (1991), Osborn (1990) and Franses (1991a). These studies indicate that the use of the annual differencing filter often implies overdifferencing. The third study shows, via simulations as well as empirical series, that an incorrect use of this differencing filter is hard to recognize on the basis of the conventional autocorrelation checks. Furthermore, it is illustrated there that overdifferencing can yield a deterioration of forecasting performance. Together with the fact that univariate time series analysis often preceeds the construction of empirical market response models, and that it is easy to imagine that inadequate models for seasonality can blur inference, it seems preferable to include testing for seasonal unit roots in empirical marketing research.

Technical appendix

This appendix contains the proposition given in Hylleberg et al. (1990), which is useful for a method to test for seasonal unit roots. The second part of this appendix contains an application of this proposition to bimonthly time series.

Proposition. *Any (possibly infinite or rational) polynomial,* $\varphi(z)$, *which is finite valued at the distinct, non-zero, possibly complex points,* $\theta_1,\ldots,\theta_p$, *can be expressed in terms of elementary polynomials and a remainder as follows:*

$$\varphi(z)=\sum_{k=1}^{p}\lambda_k\Delta(z)/\delta_k(z)+\Delta(z)\varphi^{**}(z), \tag{A.1}$$

where the λ_k *are constants defined by*

$$\lambda_k=\varphi(\theta_k)/\prod_{j\neq k}\delta_j(\theta_k), \tag{A.2}$$

$\varphi^{**}(z)$ *is a (possibly infinite or rational) polynomial and*

$$\delta_k(z)=1-(1/\theta_k)z, \tag{A.3}$$

$$\Delta(z)=\prod_{k=1}^{p}\delta_k(z). \tag{A.4}$$

An alternative form of (A.1) *which will be used in the sequel, is*

$$\varphi(z)=\sum_{k=1}^{p}\lambda_k\Delta(z)(1-\delta_k(z))/\delta_k(z)+\Delta(z)\varphi^{*}(z), \tag{A.5}$$

where $\varphi^*(z)=\varphi^{**}(z)+\Sigma\lambda_k$. *From the definition of* λ_k *it can be seen that the polynomial* $\varphi(z)$ *will have a root at* θ_k *if and only if the corresponding* λ_k *equals zero.* □

To test for the presence of seasonal unit roots in bimonthly series, apply (A.5) to the first row of (14). This gives

$$\begin{aligned}\varphi(B)=&\lambda_1B\varphi_1(B)+\lambda_2(-B)\varphi_2(B)\\&+\lambda_3\left(\tfrac{1}{2}(\mathrm{i}\sqrt{3}+1)-B\right)B\varphi_3(B)\\&+\lambda_4\left(-\tfrac{1}{2}(\mathrm{i}\sqrt{3}-1)-B\right)B\varphi_3(B)\\&+\lambda_5\left(-\tfrac{1}{2}(\mathrm{i}\sqrt{3}+1)-B\right)B\varphi_4(B)\\&+\lambda_6\left(\tfrac{1}{2}(\mathrm{i}\sqrt{3}-1)-B\right)B\varphi_4(B)\\&+\varphi^*(B)\varphi_5(B),\end{aligned} \tag{A.6}$$

where

$$\varphi_1(B)=(1+B)(1+B^2+B^4), \tag{A.7a}$$

$$\varphi_2(B)=(1-B)(1+B^2+B^4), \tag{A.7b}$$

$$\varphi_3(B)=(1-B^2)(1+B+B^2), \tag{A.7c}$$

$$\varphi_4(B) = (1-B^2)(1-B+B^2), \quad \text{(A.7d)}$$

$$\varphi_5(B) = (1-B^6). \quad \text{(A.7e)}$$

To get rid of the complex terms in (A.6), it is suitable to define

$$\lambda_1 = -\pi_1, \quad \text{(A.8a)}$$

$$\lambda_2 = -\pi_2, \quad \text{(A.8b)}$$

$$\lambda_3 = -\tfrac{1}{3}\mathrm{i}\sqrt{3}\,\pi_4 - \tfrac{1}{2}\left(1+\tfrac{1}{3}\mathrm{i}\sqrt{3}\right)\pi_3, \quad \text{(A.8c)}$$

$$\lambda_4 = \tfrac{1}{3}\mathrm{i}\sqrt{3}\,\pi_4 - \tfrac{1}{2}\left(1-\tfrac{1}{3}\mathrm{i}\sqrt{3}\right)\pi_3, \quad \text{(A.8d)}$$

$$\lambda_5 = \tfrac{1}{3}\mathrm{i}\sqrt{3}\,\pi_6 - \tfrac{1}{2}\left(1+\tfrac{1}{3}\mathrm{i}\sqrt{3}\right)\pi_5, \quad \text{(A.8e)}$$

$$\lambda_6 = -\tfrac{1}{3}\mathrm{i}\sqrt{3}\,\pi_6 - \tfrac{1}{2}\left(1-\tfrac{1}{3}\mathrm{i}\sqrt{3}\right)\pi_5. \quad \text{(A.8f)}$$

Substituting (A.8) into (A.6) gives

$$\begin{aligned}\varphi(B) = & -\pi_1 B\varphi_1(B) + \pi_2 B\varphi_2(B) \\ & + (\pi_4 + \pi_3 B) B\varphi_3(B) \\ & + (\pi_6 + \pi_5 B) B\varphi_4(B) \\ & + \varphi^*(B)\varphi_5(B),\end{aligned} \quad \text{(A.9)}$$

and the test equation becomes (16).

Appendix: Critical values

The following notation is used in Tables A.1 and A.2:

(n)c: (no) constant term in auxiliary regression.

(n)d: (no) seasonal dummies.

(n)t: (no) trend.

T: number of observations.

Table A.1
Critical t-values based on 5000 Monte Carlo simulations; DGP: $y_t = y_{t-6} + \varepsilon_t$, $\varepsilon_t \sim \mathrm{N}(0, 1)$

Parameters π_1 and π_2

Regression	T	π_1				π_2			
		0.01	0.025	0.05	0.10	0.01	0.025	0.05	0.10
nc, nd, nt	60	−2.45	−2.15	−1.84	−1.54	−2.52	−2.21	−1.88	−1.53
	120	−2.61	−2.22	−1.94	−1.60	−2.54	−2.21	−1.90	−1.58
c, nd, nt	60	−3.42	−3.05	−2.81	−2.48	−2.54	−2.18	−1.89	−1.53
	120	−3.40	−3.09	−2.85	−2.51	−2.54	−2.21	−1.90	−1.58
c, nd, t	60	−3.88	−3.61	−3.30	−3.03	−2.54	−2.15	−1.88	−1.54
	120	−3.90	−3.62	−3.34	−3.08	−2.54	−2.20	−1.91	−1.58
c, d, nt	60	−3.40	−2.99	−2.74	−2.42	−3.33	−3.01	−2.73	−2.40
	120	−3.38	−3.06	−2.80	−2.48	−3.35	−3.01	−2.77	−2.48
c, d, t	60	−3.84	−3.55	−3.26	−2.96	−3.33	−2.99	−2.73	−2.41
	120	−3.89	−3.58	−3.32	−3.05	−3.36	−3.01	−2.77	−2.47

Parameter π_4

Regression	T	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
nc, nd, nt	60	−2.16	−1.81	−1.48	−1.11	1.36	1.76	2.14	2.34
	120	−2.15	−1.76	−1.48	−1.12	1.39	1.77	2.06	2.50
c, nd, nt	60	−2.12	−1.79	−1.45	−1.09	1.34	1.72	1.98	2.32
	120	−2.13	−1.76	−1.46	−1.11	1.38	1.75	2.05	2.47
c, nd, t	60	−2.12	−1.74	−1.44	−1.10	1.31	1.65	1.91	2.23
	120	−2.10	−1.72	−1.45	−1.10	1.36	1.70	2.05	2.40
c, d, nt	60	−1.50	−1.15	−0.82	−0.40	2.20	2.53	2.87	3.26
	120	−1.64	−1.18	−0.80	−0.40	2.32	2.67	3.00	3.38
c, d, t	60	−1.41	−1.05	−0.72	−0.36	2.18	2.51	2.83	3.18
	120	−1.52	−1.12	−0.76	−0.36	2.32	2.65	2.99	3.33

Table A.1 (continued)

Parameter π_6									
Regression	T	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
nc, nd, nt	60	−2.62	−2.11	−1.75	−1.37	1.13	1.49	1.78	2.17
	120	−2.38	−2.02	−1.73	−1.38	1.11	1.44	1.72	2.15
c, nd, nt	60	−2.59	−2.07	−1.73	−1.36	1.10	1.47	1.79	2.11
	120	−2.37	−2.02	−1.73	−1.38	1.09	1.42	1.70	2.13
c, nd, t	60	−2.55	−2.03	−1.73	−1.35	1.06	1.42	1.72	2.08
	120	−2.36	−2.01	−1.73	−1.38	1.07	1.41	1.68	2.07
c, d, nt	60	−3.33	−2.94	−2.62	−2.29	0.41	0.78	1.16	1.48
	120	−3.39	−3.04	−2.69	−2.29	0.46	0.83	1.15	1.56
c, d, t	60	−3.31	−2.91	−2.58	−2.20	0.41	0.78	1.14	1.47
	120	−3.30	−3.01	−2.68	−2.27	0.46	0.82	1.13	1.54

Parameters π_3 and π_5									
Regression	T	π_3				π_5			
		0.01	0.025	0.05	0.10	0.01	0.025	0.05	0.10
nc, nd, nt	60	−2.49	−2.14	−1.87	−1.54	−2.68	−2.17	−1.84	−1.49
	120	−2.62	−2.18	−1.87	−1.57	−2.61	−2.20	−1.87	−1.55
c, nd, nt	60	−2.45	−2.15	−1.85	−1.54	−2.67	−2.19	−1.82	−1.48
	120	−2.61	−2.18	−1.87	−1.57	−2.60	−2.20	−1.87	−1.55
c, nd, t	60	−2.44	−2.14	−1.86	−1.56	−2.72	−2.19	−1.82	−1.49
	120	−2.58	−2.17	−1.84	−1.57	−2.61	−2.20	−1.87	−1.55
c, d, nt	60	−3.77	−3.47	−3.21	−2.90	−3.83	−3.48	−3.21	−2.87
	120	−3.94	−3.57	−3.31	−3.00	−3.93	−3.57	−3.28	−2.98
c, d, t	60	−3.76	−3.45	−3.20	−2.89	−3.82	−3.49	−3.19	−2.87
	120	−3.91	−3.58	−3.32	−3.00	−3.91	−3.58	−3.32	−3.00

Table A.2
Critical F-values based on 5000 Monte Carlo simulations; DGP: $y_t = y_{t-6} + \varepsilon_t$, $\varepsilon_t \sim N(0, 1)$

Regression	T	$\pi_3 = \pi_4 = 0$			$\pi_5 = \pi_6 = 0$		
		0.90	0.95	0.99	0.90	0.95	0.99
nc, nd, nt	60	2.28	2.95	4.66	2.32	3.15	5.10
	120	2.38	3.11	4.81	2.34	3.06	4.64
c, nd, nt	60	2.25	2.89	4.68	2.26	3.08	5.01
	120	2.35	3.07	4.78	2.31	3.05	4.56
c, nd, t	60	2.21	2.84	4.64	2.22	3.03	4.86
	120	2.32	3.03	4.73	2.29	3.01	4.45
c, d, nt	60	5.09	6.15	8.44	5.15	6.22	8.77
	120	5.44	6.46	8.71	5.46	6.45	8.76
c, d, t	60	5.08	6.05	8.19	5.04	6.14	8.79
	120	5.42	6.47	8.66	5.43	6.42	8.73

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