The $\ell_p$-function on trees

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Abstract

A $p$-value of a sequence $\pi = (x_1, x_2, \ldots, x_k)$ of elements of a finite metric space $(X, d)$ is an element $x$ for which $\sum_{i=1}^k d^p(x, x_i)$ is minimum. The $\ell_p$-function on $X$ is defined to have domain the set of all finite sequences on $X$ and $\ell_p(\pi) = \{x : x$ is a $p$-value of $\pi\}$. The Median Function and the Mean Function are the $\ell_p$-functions with $p = 1$ and $p = 2$ respectively. In this note the $\ell_p$-function on finite trees is characterized axiomatically.

Keywords: location function; $\ell_p$-function; median function; mean function, tree.