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A DIFFERENCING TEST

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ABSTRACT

This paper proposes and applies a test procedure for misspecification in a dynamic regression model with moving average errors. The test statistics are based on testing for unit roots in the moving average process when the model is deliberately overdifferenced.

1. INTRODUCTION

The class of misspecification tests for dynamic regression models can roughly be divided in tests which consider the inadequacy of the maintained model versus a well-specified alternative and tests which check the overall adequacy of a model without specifying an alternative hypothesis. An example of the first type of tests is an LM test for residual autocorrelation, and an example of the second type is the misspecification test proposed in Bierens (1987). An additional example of the latter type is the differencing test, advocated in Plosser, Schwert and White (1982) [PSW]. Basically, this test amounts to comparing the parameter estimates for the maintained model with those for the model in its first differenced version. Davidson, Godfrey and MacKinnon (1985) show that the PSW test is asymptotically equivalent to a test of parameter restrictions in an augmented regression.

Principally, one can apply the PSW test to any ARMAX type of model, i.e. the maintained model may include lagged dependent variables as well as moving average (MA) errors. In that case, an application of the PSW test necessitates the use of instrumental variables estimation. Further, when the model has MA errors, the PSW test should be used in two steps, i.e. first one estimates the MA parameters, then one transforms the time series variables, and finally one uses the PSW test for a maintained model that includes the latter variables. These two aspects of the PSW test may limit its empirical performance. In the present paper a differencing test is proposed which seeks to cope with these limitations. This test is based on testing for a unit root in the MA process in the first order differenced model. The test can be constructed for MA processes of any order, but for expository purposes only the MA(1) process will be considered.

The outline of the paper is as follows. In section 2, the differencing test is given. In the next section, this test is evaluated using a Monte Carlo study. Its empirical performance is also compared to that of the PSW test. In order to save space, the reader is referred to the original paper for details of the PSW test. The new differencing test will be applied in two applications in section 4. Both applications consider empirical ARMA(X) type models, which are implied by economic theories of consumption. The first is the theory in Mankiw (1982), which predicts that durable consumption follows an ARMA(1,1) process, and the second is the theory in Winder and Palm (1989), which states that total consumption can be described by an ARMAX model, where the X part of the model is given by dummy variables for structural changes. Since the Monte Carlo simulations in section 3 indicate that the empirical size of the PSW test is usually far from the nominal size in case a MAX model is the data generating process (DGP), only the new differencing test will be considered in section 4. This paper is concluded with some remarks in section 5.

2. A DIFFERENCING TEST

Consider the model

$$y_t = \sum_{i=1}^{m} \beta_i x_{it} + u_t \qquad \text{with } u_t = (1-\theta_1 B-..-\theta_q B^q)\varepsilon_t = \theta_q(B)\varepsilon_t \tag{1}$$

which is the maintained model to be tested for misspecification, where for convenience all variables are assumed to be mean-corrected. The x_{it} are m

stationary regressors, the u_t is an MA(q) process where B denotes the usual backward shift operator defined by $B^k y_t = y_{t-k}$, $k = 0,1,2,..$ The $\{\varepsilon_t\}$ is assumed to be a white noise process. The first order differenced version of (1) is

$$\Delta y_t = \sum_{i=1}^{m} \beta_i \Delta x_{it} + \nu_t \qquad \text{with } \nu_t = (1-B)\theta_q(B)\varepsilon_t \tag{2}$$

where Δ is defined by $\Delta y_t = y_t - y_{t-1}$, and where the MA($q$+1) error process ν_t contains a unit root.

One way to test for the adequacy of model (1) is to estimate model (2) and to test for a unit root in the moving average process. Unfortunately, the parameter estimates for this process are downward biased, see, e.g., the results in Plosser and Schwert (1977), and hence this hypothesis cannot be tested with conventional t tests, see also Sargan and Bhargava (1983). To circumvent this problem it seems more appropriate to consider testing for a moving average unit root, e.g., via testing for certain values of the serial correlations of the error process. The main argument for this is that Pierce (1971) has shown that the distribution of the residual autocorrelations of ν_t are independent of the regression part of the model.

In Franses (1991) a test for a moving average unit root in univariate time series is given, which is based on the autocorrelations of the error process. Consider n observations on a univariate MA(1) series

$$y_t = (1-\theta_1 B)\varepsilon_t \tag{3}$$

where ε_t is defined to be an uncorrelated zero mean process with constant variance. Applying a first difference filter Δ to both sides of (3) gives

$$\Delta y_t = (1-\theta B)(1-\theta_1 B)\varepsilon_t \qquad \theta = 1 \tag{4}$$

where the θ has been introduced to describe the alternative hypotheses.

A procedure to test whether the θ is equal to 1 indeed, can be based on the sample autocorrelations, r_k, of the variable Δy_t. For model (4) it is not difficult to show that for the first and second order theoretical autocorrelations ρ_1 and ρ_2 applies that $\rho_1+\rho_2$ equals -0.5 for any θ_1. Moreover, it can be shown that $\rho_1+..+\rho_q$ equals -0.5 for an MA(q) process with a $(1-B)$ component.

The distributional results for sample autocorrelations of moving average processes, given and proved in Anderson and Walker (1964), may now be useful, see also Hannan and Heyde (1972). Consider n observations on the zero mean linear process

$$w_t = \sum_{i=-\infty}^{+\infty} \eta_i \varepsilon_{t-i} \qquad t=0,\pm1,\pm2,.. \tag{5}$$

where $\sum_{i=-\infty}^{\infty}|\eta_i| < \infty$ and $\sum_{i=-\infty}^{\infty}|i|\eta_i^2 < \infty$. Then it can be shown that the s-dimensional vector $n^{1/2}(r_k-\rho_k)$ asymptotically follows an s-variate normal distribution with mean zero and with covariances given by

$$n\mathrm{cov}(r_k,r_l) = \sum_{j=-\infty}^{+\infty}(\rho_j\rho_{j+k-l} + \rho_j\rho_{j+k+l} + 2\rho_k\rho_l\rho_j^2 - 2\rho_k\rho_j\rho_{j+l} - 2\rho_l\rho_j\rho_{j+k}) \tag{6}$$

This result holds for all admissible values of the ρ's, and power calculations can be easily carried out. Moreover, note that the restrictions for η_i apply in the MA cases considered here.

The only nonzero autocorrelations of Δy_t, when modeled by (4), are those at lags 0, ±1 and ±2, where $\rho_{-i}=\rho_i$ for i=1,2. Application of (6) results, after some straightforward algebra, in the asymptotic result

$$n^{1/2}\begin{bmatrix} r_1-\rho_1 \\ r_2-\rho_2 \end{bmatrix} \sim N\left\{\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}A_{11} & A_{12}\\ A_{12} & A_{22}\end{bmatrix}\right\} \tag{7}$$

where

$$A_{11} = 1 + 2\rho_2 + 4\rho_1^4 - 3\rho_1^2 + 2\rho_2^2 + 4\rho_1^2\rho_2^2 - 8\rho_1^2\rho_2$$
$$A_{12} = A_{21} = 2\rho_1 - 2\rho_1\rho_2 + 4\rho_1\rho_2^3 + 4\rho_1^3\rho_2 - 2\rho_1^3 - 4\rho_1\rho_2^2$$
$$A_{22} = 1 + 2\rho_1^2 - 3\rho_2^2 + 4\rho_2^4 + 4\rho_1^2\rho_2^2 - 4\rho_1^2\rho_2$$

Under the joint null hypothesis that θ_1=0, or ρ_2=0 and θ=1, model (4) reduces to a first order differenced white noise process, and it is easy to see that ρ_1=-0.5 and that A_{11} reduces to 0.5. The test statistic for non-invertibility in case $y_t = \varepsilon_t$ is the correct model is now given by

$$T_0 = (2n)^{1/2}(r_1+0.5) \tag{8}$$

which asymptotically follows a standard normal distribution under its null hypothesis. Using simulations, the normality of the distribution of the T_0 statistic has been verified in Franses (1991). Hence, this T_0 test can be applied as a misspecification test for (1) in case $u_t = \varepsilon_t$. rejection of the null hypothesis of a moving average unit root indicates that the model is not adequate.

Under the null hypothesis $\theta=1$ in (4) it can be shown that the statistic $n^{1/2}(r_1+r_2+0.5)$ asymptotically follows a univariate normal distribution with mean zero and variance $A_{11}+2A_{12}+A_{22}$, which when substituting $\rho_2=-\rho_1-0.5$, can be written as $(1+2\rho_1+2\rho_1^2)$. Note that this variance is smallest, i.e. 0.5, in case $\rho_1=-0.5$. Estimating ρ_1 by r_1 gives the test statistic for non-invertibility in case of an MA(2) model, or

$$T_1 = n^{1/2}(r_1+r_2+0.5)/(1+2r_1+2r_1^2)^{1/2} \tag{9}$$

which asymptotically follows a standard normal distribution under its null hypothesis. This T_1 test can be applied for the model in (1) in case the error process is MA(1), i.e. when q equals 1. Extensions to u_t being MA processes of order q yields standard normal test statistics T_q which are functions of the first q+1 sample autocorrelations of ν_t.

Similar to the PSW test, the differencing test in this paper can easily be extended to regression models where the error process u_t follows an ARMA process of order p and q, i.e.

$$y_t = \sum_{i=1}^{m} \beta_i x_{it} + u_t \qquad \text{with } \phi_p(B)u_t = \theta_q(B)\varepsilon_t \tag{10}$$

The first step is now to estimate (10), then to transform (10) to

$$\hat{\phi}_p(B)y_t = \sum_{i=1}^{m} \beta_i \hat{\phi}_p(B)x_{it} + u_t^* \qquad \text{with } u_t^* = \theta_q(B)\varepsilon_t \tag{11}$$

and to apply T_q type of tests to (11). Alternatively, one may want to consider differencing tests using the autocorrelations of Δu_t, when u_t is as in (10). Such tests would then check for a moving average unit root in an ARMA(p,q+1) process. Given that this process can only be approximated by a MA(m) process, where m can be very large, and also given the expression in (6), it seems however most convenient to proceed along the lines in (11).

TABLE I
Monte Carlo evaluation of empirical size of the differencing tests T_0, T_1 and the PSW test. The cells report the rejection rate in 5000 replications The nominal size is set equal to 5%

DGP(1)	Maintained model(2)	Rejection rate(3) n	T_i	PSW
$y_t = x_t + \varepsilon_t$	$y_t = x_t + u_t$	25	7.88	6.60
		100	5.52	5.34
$y_t = x_t + \varepsilon_t + \theta\varepsilon_{t-1}$	$y_t = x_t + u_t + \phi u_{t-1}$			
$\theta = -0.9$		25	13.82	17.56
		100	15.00	15.20
-0.5		25	14.12	16.18
		100	14.46	10.48
-0.2		25	13.78	19.32
		100	14.56	6.14
0.2		25	14.70	45.68
		100	14.84	51.78
0.5		25	10.84	47.08
		100	10.42	45.70
0.9		25	6.02	46.06
		100	5.04	40.88

(1) The ε_t and x_t are drawn from N(0,1) distributions.
(2) When $y_t = x_t+u_t+\theta u_{t-1}$ is the maintained model, the test statistic T_1 is used, and T_0 is considered when $y_t = x_t+u_t$ is the maintained model.
(3) The effective sample size is n. T_i denotes either the T_0 or the T_1 test.

3. SIMULATIONS

The PSW test is an asymptotic χ^2 test. This ensures that asymptotic local power of the test can be investigated using the noncentrality parameter. The differencing tests proposed in the previous section all follow standard normal distributions under the null hypothesis. Hence, it seems most appropriate to rely on Monte Carlo simulations to assess the empirical performance of the test. In table I, the empirical size of the differencing tests T_0 and T_1 in (8) and (9) and of the PSW test is displayed. The results in this table suggest that the empirical performance of the T_0 test is adequate, that the empirical size of the T_1 test is overestimated, but that the over-estimation bias is by far not as large as that of the PSW test. Clearly, when the maintained model is a MAX(1) model, the PSW test should not be used.

TABLE II
Monte Carlo evaluation of empirical power of the differencing test T_0 and the PSW test in case of omitted variables
The cells report the rejection rate in 5000 replications
The nominal size is set equal to 5%
The maintained model is $y_t = \beta x_t + u_t$, sample size is 100

Data generating process[(1)]		Rejection frequency T_0	PSW
$y_t = x_t + \varepsilon_t + \theta\varepsilon_{t-1}$	$\theta = -0.9$	62.20	5.50
	-0.5	48.38	5.06
	-0.2	14.42	5.32
	0.2	43.30	5.64
	0.5	99.50	4.64
	0.9	100.0	5.84
$y_t = x_t + 0.5y_{t-1} + \varepsilon_t$		98.22	77.44
$y_t = x_t + 0.5y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$	$\theta = -0.9$	5.70	83.52
	-0.5	21.58	87.36
	-0.2	74.02	84.26
	0.2	100.0	70.20
	0.5	100.0	57.90
	0.9	100.0	44.72
$y_t = x_t + 0.5x_{t-1} + \varepsilon_t$		14.48	90.80
$y_t = x_t + 0.5x_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$	$\theta = -0.9$	40.34	69.68
	-0.5	20.60	83.34
	-0.2	4.30	88.84
	0.2	61.84	90.22
	0.5	99.80	83.60
	0.9	100.0	70.44

(1) The ε_t and x_t are drawn from N(0,1) distributions. The starting-value of y_t is set equal to zero.

An evaluation of the power of the T_0 test with respect to the PSW test is displayed in table II. For most DGPs, the power of the T_0 test exceeds that of the PSW test. Note that in case the DGP is $y_t = x_t + \varepsilon_t + \theta\varepsilon_{t-1}$, the PSW test has no power. An additional conclusion of these outcomes is that neither of the two differencing tests dominates the other. Hence, it seems appropriate to use both tests in practice.

Since the PSW test does not perform well in case a MAX model is the maintained model, only the empirical power of T_1 test is evaluated in table III. Given that the size of the test can be overestimated, as can be observed from table

TABLE III
Monte Carlo evaluation of empirical power of the differencing test T_1 in case of omitted variables
The cells report the rejection rate in 5000 replications
The nominal size is set equal to 1% and 5%
The maintained model is $y_t = \beta x_t + u_t + \phi u_{t-1}$, sample size is 100

Data generating process[1]		Rejection frequency 1%	5%
$y_t = x_t + 0.5y_{t-1} + \varepsilon_t$		37.88	56.94
$y_t = x_t + 0.5y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$	$\theta = -0.9$	2.92	15.18
	-0.5	9.58	21.04
	-0.2	25.78	41.44
	0.2	45.02	68.30
	0.5	42.12	73.10
	0.9	34.48	70.90
$y_t = x_t + 0.5x_{t-1} + \varepsilon_t$		3.38	15.26
$y_t = x_t + 0.5x_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$	$\theta = -0.9$	1.14	15.02
	-0.5	1.66	15.06
	-0.2	2.42	15.76
	0.2	3.30	14.92
	0.5	1.62	9.80
	0.9	0.28	6.58

(1) The ε_t and x_t are drawn from N(0,1) distributions.

I, the rejection frequency of the T_1 test is investigated using a nominal 1% and 5% level. The results in table III suggest that this frequency is not very high. In fact, when the DGP contains an x_{t-1} variable, which is omitted in the maintained model, the power of the T_1 test does not seem to exceed the size, which is given in table I.

The tables II and III only display the results of a Monte Carlo study of the empirical power of the T_q tests in case of omitted variables. Many simulations have also been performed for the case of errors in variables. The design of the experiment is that of PSW, table II. Roughly speaking, the outcome of the simulations is that, in contrast to the PSW test, the differencing tests T_0 and T_1 proposed in the present paper have almost zero power. Intuitively, this can be understood by considering the maintained model $y_t = x_t + u_t$, where the DGP is $y_t = z_t + e_t$ and $x_t = z_t + \varepsilon_t$. When ε_t, e_t and u_t are $N(0,1)$ variables, the first order autocorrelation of $\Delta\hat{u}_t$ will be close to -0.5. Detailed results of several simulation experiments can be obtained from the author.

4. APPLICATIONS

Mankiw (1982) shows that the life cycle–permanent income hypothesis implies that consumer expenditure goods on durable goods, say cd_t, can be described by an ARMA(1,1) model, i.e.

$$cd_t = \mu + \phi_1 cd_{t-1} + \varepsilon_t + (\delta-1)\varepsilon_{t-1} \tag{12}$$

where δ is the depreciation rate of the consumer's stock, and where ϕ_1 equals $(1+\gamma)/(1+r)$, with γ the rate of subjective time preference and r the real rate of interest. In many practical occasions this ϕ_1 can be set equal to unity.

The estimation results of (12) for the sample 1964.1 through 1988.4 for the Swedish data are

$$\underset{}{\Delta cd_t} = \underset{(0.018)}{0.322} - \underset{(0.026)}{0.596}D_{1t} - \underset{(0.026)}{0.177}D_{2t} - \underset{(0.026)}{0.480}D_{3t} + \underset{(0.066)}{0.449}D_t + \hat{\varepsilon}_t - \underset{(0.091)}{0.494}\hat{\varepsilon}_{t-1}$$

where D_{it} are seasonal dummies, $i = 1,2,3$, and where D_t is a dummy variable with value 1 in 1974.3, –1 in 1974.4, and 0 elsewhere. This model passes diagnostic checks for normality, for residual autocorrelation of order 1 and 4, and for ARCH effects of order 1 and 4. To apply the differencing test T_1, the first two residual autocorrelations r_1 and r_2 of the estimated error process of the regression of $\Delta\Delta cd_t$ on a constant, three seasonal dummies and ΔD_t are calculated. These correlations are –0.584 and 0.082. For 100 observations, the T_1 test statistic obtains a value of –0.028, and model (12) cannot be rejected for the Swedish durables consumption data.

A second application of the differencing test T_1 is given by the model in Winder and Palm (1989). Extending the life cycle theory to incorporate moving planning horizons caused by structural changes in income, Winder and Palm show that total consumption c_t can be described by

$$\Delta c_t = \sum_{j=1}^{7} D_{jt} + \phi\Delta c_{t-1} + \varepsilon_t + \eta\varepsilon_{t-1} \tag{13}$$

where the D_{jt} correspond to the structural changes, see Winder and Palm (1989, page 43). Diagnostic checks indicate that this model can not be rejected by the data, although there may be some question about the appropriateness of the dummies D_{jt}, see, e.g., Broersma and Franses (1990). Regressing $\Delta\Delta c_t$ on ΔD_{jt}, where $j = 1,..,7$, yields the residual autocorrelations $r_1 = -0.200$ and $r_2 = -0.322$. Hence, the T_1 statistic calculated for 69 observations is -0.222, and the model in (13) cannot be rejected by the data.

5. CONCLUSION

The simple differencing test procedure proposed in this paper seems useful for the detection of misspecification in dynamic regression models with or without moving average errors in case this misspecification is of the omitted variables type. Extensions to higher order moving average processes than the MA(0) and MA(1) process considered in the present paper are straightforward, and can be based on the results given in Anderson and Walker (1964).

The empirical size of the test in case the dynamic model has MA(1) errors exceeds the nominal size. This suggests that perhaps small-sample corrections may improve the empirical performance. On the other hand, alternative methods to test for moving average unit roots, see, e.g., Tanaka and Satchell (1990), may be worthwhile to consider. However, a drawback of such methods can be that the test statistics do not follow standard normal distributions in contrast to those proposed in the present paper.

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