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Moving average filters and periodic integration

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1. Introduction

Several quarterly observed macroeconomic time series may contain a stochastic trend which effects the seasonal fluctuations. An example is the unemployment rate which displays seasonality in business cycle expansion periods because of seasonal labor supply, and which shows much less seasonal fluctuations in the contraction periods because the dismissal of employees may be free of seasonal effects. Hence, the seasonal pattern of economic time series can change over time, and these changes may be caused by the stochastic trend. A class of univariate time series models that can describe such time series contains the periodic autoregressions with unit roots [PIAR], see, e.g., [1,2].

The basic assumption underlying seasonal adjustment methods is that, one way or another, seasonality, trend and cycles can be separated. However, when it is found that a PIAR can give an adequate description of a time series, the crucial requirement is violated. Hence, seasonal correction filters may either not remove all seasonal fluctuations or effect trend and cyclical patterns. In this paper, we focus on the effect of one particular filter, i.e. the linear moving average filter $(1+B+B^2+B^3)$ on testing for common stochastic trends across periodically integrated time series, where B is the familiar backward shift operator.

The outline of this paper is as follows. In Section 2, we discuss a few concepts related to PIAR processes. In Section 3, we discuss the linear moving average filter in relation to a PIAR. In Section 4, we present the results of some Monte Carlo exercises. The main conclusion is that the probability of finding true common trends across PIARs is dramatically reduced when moving average filters are used. In Section 5, we conclude with some remarks.

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2. Periodic integration

Consider the time series y_t, $t = 1, \ldots, n$, which is quarterly observed during N years, i.e., $n = 4N$. A periodically integrated autoregressive process of order 1 [PIAR(1)] is

$$y_t = \phi_s y_{t-1} + \varepsilon_t \tag{1}$$

with $\phi_1 \phi_2 \phi_3 \phi_4 = 1$, where ϕ_s is a parameter which value varies with the season, and where ε_t is a standard white noise process, see [1] and [2]. The restriction $\phi_1 \phi_2 \phi_3 \phi_4 = 1$ is given by the solution of the characteristic equation corresponding to a vector representation of (1),

$$A_0 Y_T = A_1 Y_{T-1} + \varepsilon_T \tag{2}$$

where A_0 and A_1 are

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\phi_2 & 1 & 0 & 0 \\ 0 & -\phi_3 & 1 & 0 \\ 0 & 0 & -\phi_4 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 & 0 & \phi_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and Y_T is the (4×1) vector $(Y_{1T}, Y_{2T}, Y_{3T}, Y_{4T})'$, where Y_{sT} is the observation in season s in year T, $T = 1, \ldots, N$. The characteristic equation for (2) is

$$| A_0 - A_1 z | = 0 \tag{3}$$

which is equal to $1 - \phi_1 \phi_2 \phi_3 \phi_4 z = 0$. Note that $\phi_1 \phi_2 \phi_3 \phi_4 = 1$ implies that there is one unit root in the system Y_T, or equivalently, that there are three cointegration relations between its elements Y_{sT}. In [2] a test is proposed for the hypothesis that $z = 1$ is a solution to (3).

For higher order periodic autoregressive processes one can consider similar characteristic equations as (3) to test for a unit root in Y_T. Sometimes it may however be more convenient to analyze such time series in a differenced form. For example, under the assumption that the solutions to (3) are not complex-valued, a PAR(2) process can be written as

$$y_t - \phi_s y_{t-1} = \beta_s (y_{t-1} - \phi_{s-1} y_{t-2}) + \varepsilon_t \tag{4}$$

where $\phi_0 = \phi_4$. Imposing the restriction $\phi_1 \phi_2 \phi_3 \phi_4 = 1$ in (4) implies that there is a unit root in the corresponding vector model. Further, model (4) can be easily used to check the hypothesis $\phi_s = \phi = 1$, see [3]. If so, one should apply the $(1 - B)$ filter to make a time series (periodically) stationary.

From (4) is can be observed that the differencing filter for a PIAR time series varies with the season. This indicates that seasons and trends are not independent. Hence, if one wants to investigate whether two or more periodically integrated time series have a common stochastic trend, one should take account of this dependency. In [4] a method is proposed to test for common trends which contains two steps. The first is the estimation of the most nonstationary linear combination between the Y_{sT} elements for each of the y_t series using the method in [5]. The second step is to use the regular cointegration methods to these combinations. Simulation and empirical evidence suggests that this method can yield useful results. A possible drawback of this method is that the second round of cointegration analysis uses only $n/4$ observations.

3. Moving average filters

In the practical occasions where PIAR models are found to be appropriate, the estimated values of ϕ_s are usually close to unity, as can be expected from the expressions in (1) and (4). The application of HEGY method in [6] to a process that is generated by a PIAR process is therefore likely to suggest that the differencing filter $(1-B)$ should be applied, although the presence of seasonal unit roots, i.e. that the $(1-B^4)$ filter should be used, can also often not be rejected, see [7]. The latter can be clarified by writing, e.g., (1) as

$$y_t = y_{t-4} + \varepsilon_t + \phi_s \varepsilon_{t-1} + \phi_s \phi_{s-1} \varepsilon_{t-2} + \phi_s \phi_{s-1} \phi_{s-2} \varepsilon_{t-3}, \tag{5}$$

see also [8].

When the HEGY method is applied to two quarterly observed time series, say y_t and x_t, and it is found that they both have a nonseasonal unit root, as well as three seasonal unit roots, a usual next step is to check whether the series have a nonseasonal unit root in common. For that purpose, y_t and x_t are transformed using the $(1+B+B^2+B^3)$ filter, see [9] [EGHL]. This is because the polynomial $1-B^4$ can be decomposed as $(1-B)(1+B+B^2+B^3)$. The new time series, say y_{1t} and x_{1t}, are then compared in a cointegration exercise. In EGHL it is shown that the standard critical values of [10] apply.

As already indicated in the previous section, the $(1+B+B^2+B^3)$ filter does not correspond to the appropriate differencing filter for a PIAR series. This is because it deals with each of the seasons in a similar way, while the main property of a PIAR process is that the seasons display distinct behavior. This can easily be recognized from (5), where it can be seen that a $(1+B+B^2+B^3)$ transformed PIAR(1) model results in an ARIMA(0, 1, 3) process with periodically varying MA structures. The results in, e.g., [11] suggest that tests for unit roots can be effected by the presence of neglected MA components. A further drawback of applying $(1+B+B^2+B^3)$ filter to a PIAR process is that it assumes too many unit roots for the y_t series. From a framework as (2), it can readily be derived that a $(1+B+B^2+B^3)$ filter assumes that there is one cointegrating relation between the Y_{sT} variables, see also [7]. Hence, in some sense the $(1+B+B^2+B^3)$ transformed PIAR series is not an invertible time series. In summary, all this suggests that the standard EGHL type of analysis applied to PIAR processes may yield inappropriate results.

4. Some simulation results

To verify the conjecture that the moving average filter has an impact on tests for common trends across PIAR time series, we conduct several Monte Carlo exercises based on 5000 replications using Gauss 386 VMI version 3.1.1. The data generating processes (DGP) are

(i) $y_t = \alpha_s y_{t-1} + \varepsilon_t, \quad x_t = \gamma_s x_{t-1} + \nu_t,$

(ii) $y_t = \alpha_s y_{t-1} + \varepsilon_t, \quad x_t = y_t + \eta_t,$

(iii) $y_t = \alpha_s y_{t-1} + \varepsilon_t, \quad x_t = y_t + \eta_t/(1-0.5B),$

(iv) $y_t = \alpha_s y_{t-1} + \varepsilon_t, \quad x_t = y_t + \eta_t/(1-0.8B),$

(v) $y_t = \alpha_s y_{t-1} + \varepsilon_t, \quad x_t = y_t + \eta_t/(1-0.9B)$

Table 1
Rejection frequencies of the hypothesis of no cointegration. Based on 5000 replications of sample size 100. Critical values are taken from [10].

Data generating process	Nominal size	EGHL CRDW	CRDF [(1)]	Franses 2-step CRDW	CRDF [(2)]
(i)	5%	0.0	4.2	5.7	5.2
	10%	0.0	8.7	11.8	11.3
	20%	0.0	15.7	20.4	21.4
(ii)	5%	95.4	35.7	97.4	95.2
	10%	99.4	48.2	99.3	98.5
	20%	100.0	60.8	99.8	99.7
(iii)	5%	4.7	32.6	95.6	91.5
	10%	18.8	44.3	98.7	96.8
	20%	52.7	56.8	99.7	99.3
(iv)	5%	0.0	23.6	71.2	59.5
	10%	0.1	35.5	84.6	74.9
	20%	1.2	47.8	93.8	88.3
(v)	5%	0.0	15.9	38.4	29.1
	10%	0.0	26.2	55.2	43.5
	20%	0.1	37.7	72.5	61.9

(1) The lag length in the augmented Dicky–Fuller regression is determined by LM tests for residual autocorrelation.
(2) The lag length here is set equal to zero.

where ε_t, ν_t and η_t are standard white noise processes, and where α_s, γ_s is set equal to 1.25, 0.8, 2 and 0.5 for the four seasons. For each of these DGPs we calculate the Durbin–Watson (CRDW) and Dickey–Fuller (CRDF) statistics for the residuals of the regression of $(1+B+B^2+B^3)y_t$ on $(1+B+B^2+B^3)x_t$, reflecting the EGHL method, and the same statistics for the residuals of the regression of the two most nonstationary linear combinations of the Y_T and X_T elements, reflecting the Franses two-step method. The results for DGP (i) can be interpreted as the empirical size of the tests, while the results for the other DGPs correspond to the empirical 'power' of the methods.

The results in this table seem very easy to interpret. The EGHL method yields an incorrect size of the CRDW statistic, while the size of the CRDF test is still reasonable. The size of both test statistics in Franses' 2-step method seems adequate. The (not size-corrected) 'power' of the CRDW test in the EGHL case is only high for DGP (ii) but converges to zero when autocorrelated residuals are allowed. The 'power' of the CRDF test in the EGHL case is low, though it may sometimes adequately suggest the presence of common trends. On the other hand, the values of the CRDW test are likely to be extremely low. The 'power' of the Franses 2-step method is quite reasonable, even in the case where the errors are strongly autocorrelated.

5. Concluding remarks

An application of the moving average filter to periodically integrated autoregressive time series in order to check for common trends across such series cannot be recommended. The model selection strategy proposed in [3] may be useful in deciding which filters are the most useful for such series. Whether the simulation results in this paper extend to officially seasonally adjusted time series using, e.g., the Census-X11 method, is a topic for future research.

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