A PERIODIC COINTEGRATION MODEL OF QUARTERLY CONSUMPTION

PHILIP HANS FRANSES AND TEUN KLOEK

Econometric Institute, Erasmus University, Rotterdam, P.O. Box 1738, NL-3000 DR, The Netherlands

SUMMARY

A periodic cointegration model is proposed to describe quarterly observed consumption. This model allows the cointegrating vectors and the adjustment parameters to vary with the seasons. Its links are discussed with an often considered standard economic theoretical model for macroeconomic variables like consumption. A simple empirical model specification strategy is given and applied to Austrian consumption and income data.

KEY WORDS   time series; seasonality; periodic models

1. INTRODUCTION

Error correction models (ECMs) are regularly used to describe sets of macroeconomic time series variables, which concern for example consumption, unemployment and money demand. An important motivation for using error correction models is that they have strong links with economic theoretical models. Furthermore, an error correction model is useful in practical occasions since it assumes long-run relationships between trending time series variables, i.e. it assumes cointegration—see Engle and Granger[1] for a discussion of the links between cointegration and error correction. Given that many macroeconomic time series like consumption and income are non-stationary in the sense that they contain a stochastic trend—see Nelson and Plosser[2] *inter alia*—ECMs provide a useful theoretical and practical modelling framework.

Many applications of ECMs concern annually or quarterly observed time series, where the quarterly observations are usually considered in seasonally adjusted form. Recently, however, there has been a growing interest in constructing ECMs for quarterly time series in seasonally *unadjusted* form. The main reason for this is that there has been a growing consensus that seasonal fluctuations in quarterly time series are worthwhile to study in their own right, and that such fluctuations may convey information on the behaviour of economic agents—see Hylleberg[3] and Miron.[4] One such ECM which allows for an explicit description of seasonal fluctuations is the so-called seasonal cointegration model proposed by Engle *et al.*,[5] which is applied to describe consumption and income in Japan. This seasonal error correction model (SECM) extends the usual ECM by investigating the presence of long-run relationships between stochastic seasonal trends. As noted by Osborn,[6] and as we indicate in Section 3 below, a drawback of the SECM is that it assumes that the cointegration relations vary with the *lags*, which may be hard to interpret

meaningfully in terms of economic theory. In this paper, therefore, the authors propose an alternative ECM for quarterly observed time series on consumption and income—the so-called periodic error correction model (PECM). This PECM extends the standard ECM by, for example, allowing the cointegration relations to vary with the *seasons*—see also Birchenhall *et al.*[7] for an empirical model for consumption in the United Kingdom.

The outline of the paper is as follows. In Section 2, the authors sketch a theoretical economic basis for ECMs that is useful for modelling consumption. In Section 3, the ECM is extended in order to be able to describe seasonal fluctuations explicitly. Then the authors discuss the seasonal and the periodic ECMs. In Section 4, the authors we present a step-wise empirical model specification strategy for a PECM, thereby extending the material of Birchenhall *et al.*[7] In Section 5, the authors apply this specification strategy to quarterly consumption and income data for Austria, 1954.1–1989.4. Finally, in Section 6, the paper is concluded with some remarks.

2. ERROR CORRECTION MODELS

An economic theoretical motivation for the empirically useful ECM is the so-called linear-quadratic adjustment cost model (LQAC)—see Nickell[8] and Gregory.[9] Applications of LQAC models can be found in describing labour and money demand—see Sargent,[10] Kennan[11] and Dolado *et al*[12]—and in the present paper we assume the usefulness of this theoretical framework for modelling consumption.

Consider an economic time series x_t, $t = 1, \ldots, n$, with its target value x_t^*, where the target is related to an $(m \times 1)$ vector of forcing explanatory variables z_t, or

$$x_t^* = \theta' z_t + u_t \tag{1}$$

where u_t is assumed to be a standard white noise process, i.e. a zero-mean uncorrelated process with constant variance. In this paper, x_t is taken to be consumption and z_t to be a univariate income series. Assume that at time t, economic agents choose the sequence x_{t+j}, with $j = 0, 1, 2, \ldots$ in order to minimize the expected intertemporal loss function

$$L_t = E\left[\sum_{j=0}^{\infty} \rho^j (\delta(x_{t+j}^* - x_{t+j})^2 + (x_{t+j} - x_{t+j-1})^2)\right] \tag{2}$$

where E denotes the expectation at time t on the basis of information set I_t, ρ is a certain discount factor and $\delta > 0$ is a weighting factor which reflects adjustment costs. The expression in (2) corresponds to a linear quadratic adjustment cost (LQAC) model—see Sargent[10] and Gregory,[9] *inter alia*. The first term in (2) concerns the adjustment to the target of x_t, and the second term reflects the adjustment costs.

To ensure the empirical usefulness of (2), we need to make additional assumptions on the time series process z_t. When we assume that these forcing variables can each be described by random walks, i.e. $\Delta_1 z_{it} = \varepsilon_{it}$ for $i = 1, \ldots, m$ and ε_{it} are each white noise error processes, with Δ_1 defined by $\Delta_k x_t = (1 - B^k)x_t = x_t - x_{t-k}$, it can be shown that the solution of the problem of minimizing L_t in (2) can be represented by the ECM

$$\Delta_1 x_t = \alpha(x_{t-1} - \theta' z_{t-1}) + \beta' \Delta_1 z_t + \varphi_t \tag{3}$$

where φ_t is again a white noise error process—see, for example, Gregory.[9] In case the Δz_t process can be described by some vector autoregressive process (VAR), (3) should be modified by including additional lags of $\Delta_1 x_t$ and $\Delta_1 z_t$.

The α parameter in (3) is related to the δ parameter in (2) in the sense that if $\delta > 0$ it holds that $\alpha < 0$. In case the time series x_t and z_t are both non-stationary, i.e. both time series contain a stochastic trend, the model in (3) assumes the presence of cointegration between x_t and z_t—see Engle and Granger.[1] The cointegration vector is $(1, -\theta')$. The adjustment parameters α 'correct' the errors from being out of equilibrium.

3. SEASONAL AND PERIODIC ERROR CORRECTION

In this section, two possible extensions are discussed of the ECM in (3), which can be considered when describing time series with pronounced and possibly non-constant seasonal fluctuations. Without loss of generality, we assume that z_t is a univariate process for notational convenience. The first model we discuss is the seasonal error correction model (SECM), which has been proposed by Engle et al.[5] The second is the periodic ECM.

3.1. Seasonal error correction

The SECM is motivated by the fact that quarterly observed time series may have stochastic trends at the non-seasonal and at the seasonal frequencies. Such a trend at the non-seasonal frequency, i.e. the familiar unit root 1, can be removed using the first-order differencing filter. However, for quarterly time series, one may use the $\Delta_4 = (1 - B^4)$ filter—see Box and Jenkins.[13] This fourth-order differencing filter can be decomposed as

$$
\begin{aligned}
(1 - B^4) &= (1 - B)(1 + B)(1 + iB)(1 - iB) \\
&= (1 - B)(1 + B)(1 + B^2) \\
&= (1 - B)(1 + B + B^2 + B^3)
\end{aligned}
\tag{4}
$$

where i is defined by $i^2 = -1$, and where $(1 + B)$ corresponds to the seasonal unit root -1 and $(1 + B^2)$ to the seasonal unit roots $\pm i$—see Hylleberg et al.[14] The seasonal unit roots correspond to the stochastic trends at seasonal frequencies. Hence, when a time series has seasonal unit roots, its seasonal fluctuations change over time. Hylleberg et al.[14] have proposed a test strategy for the presence of seasonal unit roots.

In the case of multivariate time series, one may, similar to the non-seasonal cointegration case, investigate whether sets of time series have seasonal unit roots in common. In the case of two time series x_t and z_t that both need to be differenced by Δ_4 and which are cointegrated at all non-seasonal and seasonal frequencies, the SECM can be written as

$$
\begin{aligned}
\Delta_4 x_t =\ & \gamma_{11}[(1 + B + B^2 + B^3)x_{t-1} - \alpha_{12}(1 + B + B^2 + B^3)z_{t-1}] \\
& + \gamma_{12}[(-1 + B - B^2 + B^3)x_{t-1} - \alpha_{22}(-1 + B - B^2 + B^3)z_{t-1}] \\
& - (\gamma_{13} + \gamma_{14}B)[(-1 + B^2)x_{t-2} - \alpha_{32}(-1 + B^2)z_{t-2} \\
& - a_{41}(-1 + B^2 x_{t-3} - a_{42}(-1 + B^2)z_{t-3}] + e_t
\end{aligned}
\tag{5}
$$

where the notation is similar to that of Engle et al.[6] The first cointegration term on the right-hand side corresponds to cointegration at the non-seasonal frequency, the second and third terms to cointegration at seasonal frequencies. The adjustment parameters are j_{1j}, with $j = 1, 2, 3, 4$.

With respect to the target relation (1), the target relations in (5) are written in terms of transformed x_t and z_t series. For example, the target relation at the non-seasonal frequency equals $(1 + B + B^2 + B^3)x_t^* = a_{12}(1 + B + B^2 + B^3)z_t$. In a comment on the work of Engle et al.,[5]

Osborn[6] demonstrates that (5) corresponds to

$$\Delta_4 x_t = \sum_{i=1}^{4} a_i(x_{t-i} - \theta_i z_{t-i}) + \varepsilon_t \tag{6}$$

indicating that the SECM implies four different target relationships associated with different lags. Indeed, such parameter variation may be hard to interpret in terms of economic behaviour.

3.2. Periodic error correction

An alternative model for seasonal observed time series that explicitly takes into account the seasonal variation in such variables is the periodic ECM, a simple version of which is given by

$$\Delta_4 x_t = \sum_{i=1}^{4} a_s(x_{t-4} - \theta_s z_{t-4}) + \beta \Delta_4 z_t + \varepsilon_t \tag{7}$$

where a_s and θ_s are periodically varying adjustment and cointegration parameters, respectively. There are four different target relations similar to (1) of the form $x_t^* = \theta_s z_t + u_t$, which in this case for the PECM vary with the season. These time-varying target relationships may originate from the time-varying availability of certain goods and services or from seasonally varying utility. Osborn[15] has derived a univariate consumption model in which it is assumed that consumers have a periodically varying utility function. Hence, the model in (7) in a sense amounts to a multivariate extension of the Osborn model.[15] Furthermore, seasonally varying adjustment parameters may reflect that adjustment is more easy in some seasons than in others. Additionally, economic agents may want to correct disequilibrium errors in some seasons more than in other seasons.

The model in (7) differs from most models for consumption which can be found in the literature, although it bears several similarities with the model due to Birchenhall et al.[7] For example, the model of Davidson et al.[16] assumes that $\theta_s = 1$ and $a_s = a$ for all s.

4. AN EMPIRICAL MODEL SPECIFICATION STRATEGY

In this section, an empirical model specification strategy is proposed for the PECM as in (7), possibly with the inclusion of additional lags of $\Delta_4 x_t$ and $\Delta_4 z_t$. This strategy follows the well-known two-step strategy given by Engle and Granger,[1] with one additional step for testing whether the various cointegration and adjustment parameter values are indeed periodically varying.

Consider the two quarterly observed time series x_t and z_t and the two (4×1) vector time series $X_T = (X_{1T}, X_{2T}, X_{3T}, X_{4T})$ and $Z_T = (Z_{1T}, Z_{2T}, Z_{3T}, Z_{4T})$, where X_{sT} and Z_{sT} are the observations on x_t and z_t in season s and year T, where $T = 1, ..., N$. Hence, the X_T and Z_T vector time series are observed annually. Periodic error correction as in (7) amounts to four cointegration relationships between x_t and z_t, i.e. one cointegration relation per quarter. It is clear that these cointegration relations can be written as $X_{sT} - \theta_s Z_{sT}$ for $s = 1, 2, 3, 4$. A first step in specifying models like (7) is then to regress X_{sT} on a constant and Z_{sT} for each of the seasons. In case of cointegration between X_{sT} and Z_{sT}, the residuals of one or more of these four regressions should be stationary time series. Of course, it can occur that only for a few seasons one obtains stationary residuals. In this case, the PECM can be called a partial PECM.

When all four residual series are stationary, one has a full PECM.

To test for cointegration between X_{sT} and Z_{sT}, we choose to check the value of the cointegrating regression Durbin–Watson (CRDW$_s$) statistic for each season s and the Dickey–Fuller (DF$_s$) test. The asymptotic distributions of these test statistics can be derived from the asymptotic distributions in cases where the parameters do not vary over the seasons. This is caused by the orthogonality of the regressors. For example, the DF$_s$ test should have the same distribution as the well-known cointegration Dickey–Fuller test in the case of two non-periodic variables—see Engle and Granger.[1] To verify these asymptotics, especially for small samples, we display some simulated critical values for these test statistics in Table I for the cases where z_t should be transformed using a Δ_4 filter or a Δ_1 filter. One can infer that the critical values for these two cases are similar and that the conjecture on the distributional results seem valid.

In case one obtains evidence that there is indeed cointegration in each season, a next step is to check whether the estimated parameters in the cointegration vectors, as well as the adjustment parameters, do indeed vary over the seasons. In other words, one may want to test the hypotheses $\theta_s = \theta$ and $\alpha_s = \alpha$. The test for $\theta_s = \theta$ can be performed by comparing the residual sums of squares (RSS) of the four regressions of X_{sT} on a constant and Z_{sT} with the RSS of the regression of x_t on four seasonal dummies and z_t. Assuming cointegration, one can construct an F test for the hypothesis $\theta_s = \theta$, which follows a standard F distribution under the null hypothesis. This result follows from Theorem 3.1 due to Johansen.[17] This test will be denoted as $F(\theta_s = \theta)$. Furthermore, in the case of cointegration, the F test for the hypothesis that $\alpha_s = \alpha$ also follows a standard F distribution, since $x_{t-4} - \hat{\theta}_s z_{t-4}$ are stationary variables—see Engle and Granger.[1] This test will be denoted as $F(\alpha_s = \alpha)$. Similar test statistics can be constructed in case of partial instead of full PECMs, although one should then make sure that the F tests for restrictions on the θ_s and α_s are only calculated for those seasons where one obtains cointegrating relationships. Otherwise these F tests do not follow standard F distributions.

Finally, a test for the weak exogeneity of z_t for the cointegration relations can be performed via testing the significance of the periodic error correction variables in a model for $\Delta_4 z_t$. Again, since these error correction variables are stationary variables, the relevant F test follows a standard F distribution under the null hypothesis—see Boswijk.[18]

Table I. Critical values of several test statistics for a sample of 25 years of quarterly observations, based on 20 000 Monte Carlo replications†

Test statistic		Significance level			
		0·20	0·10	0·05	0·01
$\Delta_4 z_t = v_t$	CRDW$_s$	0·86	1·08	1·26	1·63
	DF$_s$	−2·70	−3·13	−3·50	−4·27
$\Delta_1 z_t = v_t$	CRDW$_s$	0·86	1·09	1·27	1·63
	DF$_s$	−2·71	−3·12	−3·48	−4·31

†The CRDW$_s$ for $s = 1,2,3,4$, as well as the Dickey–Fuller t test statistic DF$_s$, for the null hypothesis of no cointegration are found by estimating the model: $X_{sT} = \mu_s + \theta_s Z_{sT} + \zeta_t$, for $s = 1,2,3,4$, where $\Delta_4 x_t = \varepsilon_t$, $\Delta_4 z_t = v_t$ or $\Delta_1 z_t = \psi_t$ and where ε_t, $v_t \sim N(0,1)$.

5. CONSUMPTION AND INCOME IN AUSTRIA 1954.1 TO 1989.4

To illustrate the steps of empirically specifying a periodic cointegration model for consumption and income, we consider two time series for Austria. The series are real private consumption and real disposable income for the period 1954.1 to 1989.4, which we use after taking natural logarithms. In 1964, the Austrian measurement system has been changed. Therefore, we ignore the observations for the period 1964.1 to 1965.1 by including five dummy variables in all forthcoming regressions. The estimation results for the periodic cointegration parameters θ_s and the related test outcomes are displayed in Table II. The values of the $CRDW_s$ and the DF_s statistics indicate that the hypothesis of no cointegration can be rejected for three of the four quarters, and that the evidence for cointegration in quarter 1 is weak, though not extremely weak. Since the power of cointegration tests is usually low, especially for small samples, we assume the presence of cointegration in all four seasons.

The evidence in Table II suggests that the parameter values of θ_s can be different from 1 as well as from each other. The $F(\theta_s = \theta)$ test yields a value of 12·289, which is significant at the 1% level. However, the F test for the restriction $\theta_1 = \theta_2 = \theta_4$ obtains an insignificant value of 1·232. In sum, we restrict the periodic error correction term to $ecm_t = x_t - \theta z_t - \theta_3 z_t$.

An adequate periodic cointegration model as in (7) is found after a short search in which a general model is estimated, tested, and simplified. The model passes diagnostic checks for the presence of residual autocorrelation of order 1 and 4, ARCH of order 1 and 4, and normality. The simplified model for $\Delta_4 x_t$ includes $\Delta_4 x_{t-1}$, $\Delta_1 \Delta_4 x_{t-4}$, $\Delta_4 z_t$, four error correction variables ecm_{t-4} for each of the seasons and four seasonal intercept terms corresponding to the error correction terms. The values of the t statistics for the four α_s adjustment parameters are $-2·955$, $-4·382$, $-5·824$ and $-4·452$ for the four quarters, respectively. Compared with the standard normal distribution, these t ratios suggest significant error correction in all four seasons. The $F(\alpha_s = \alpha)$ test gives a value of 2·205, which is not significant at the 5% level.

Table II. Testing for cointegration per quarter between consumption x_t and income z_t in Austria for the sample 1954–1989 ($N = 36$)

Quarter	θ_s	$CRDW_s$	DF_s
1	0·944 (0·012)	0·623	$-2·387$
2	0·923 (0·010)	0·946	$-3·433$†
3	1·007 (0·008)	1·353‡	$-4·144$‡
4	0·933 (0·010)	1·396‡	$-4·185$‡

† Significant at a 10% level.
‡ Significant at a 5% level.
Note. The θ_s refers to the estimated parameter for Z_{sT} of the regression of X_{sT} on 1 and Z_{sT} per quarter s. Estimated standard errors are given in parentheses and should be treated with care since θ_s is a superconsistent estimator—see Engle and Granger.[1] The $CRDW_s$ is the cointegrating regression Durbin–Watson statistic, and the DF_s refers to the Dickey–Fuller t ratio. The critical values of these statistics are displayed in Table I.

Imposing the $\alpha_s = \alpha$ restriction, the final PECM for Austrian consumption is

$$\Delta_4 x_t = 0 \cdot 114 D_{1t} + 0 \cdot 107 D_{2t} - 0 \cdot 053 D_{3t} + 0 \cdot 156 D_{4t} + 0 \cdot 374 \Delta_4 z_t$$
$$(0 \cdot 014) \quad (0 \cdot 013) \quad (0 \cdot 009) \quad (0 \cdot 019) \quad (0 \cdot 071)$$
$$+ 0 \cdot 213 \Delta_4 x_{t-1} - 0 \cdot 164 \Delta_1 \Delta_4 x_{t-4} - 0.493 [x - 0.933 z - 0.074 D_3 z]_{t-4} \qquad (8)$$
$$(0 \cdot 067) \qquad (0 \cdot 053) \qquad (0 \cdot 062) \quad (0 \cdot 006) \quad (0 \cdot 013)$$

where D_{st} is a seasonal dummy variable for season s, $s = 1, 2, 3, 4$. Model (8) is estimated for 144 observations, and the estimated standard errors are given in parentheses. Diagnostic test statistics indicate that this model cannot be rejected. Chow forecast tests, one-step-ahead forecast tests, CUSUM tests as well as the standard Chow split estimation tests do not indicate any severe misspecification.

Finally, we test for weak exogeneity of income for the parameters in the error correction part of (8). It appears that $\Delta_4 z_t$ can be described by a first-order autoregressive model. Adding the error correction variables for each of the seasons yields an insignificant F statistic of $0 \cdot 655$. Hence, for this empirical example, we do not seem to need full system estimation methods like that in, for example, Johansen[17]

6. CONCLUDING REMARKS

In this paper, a periodic cointegration model has been proposed for quarterly consumption and income. This model allows that each quarter shows its own equilibrium relation between these variables, and that being out of equilibrium has distinct effects on future consumption. Hence this model may be rationalized using economic arguments such as that consumers have seasonally varying utility functions and adjustment costs within the theoretical framework of a linear quadratic adjustment cost model. An empirical specification strategy is discussed and applied to Austrian data on quarterly consumption and income. It appears that the proposed model gives a satisfactory description of the relations between these variables for Austria, and that the third quarter target relations for consumption relative to income is different from that in other quarters.

Extensions of the proposed model are the inclusion of deterministic trends, as well as more than two variables, into the periodically varying cointegration relations. Promising routes to test for cointegration seem to be given by extending the Boswijk[18] or Johansen methods[17]. The effects of seasonal adjustment on estimating and testing for periodic cointegration, as well as on forecasting in cases where a periodic model is the most appropriate, are also worthwhile investigating. Finally, a topic for future research is to develop simple test procedures to select between seasonal and periodic cointegration models.

REFERENCES

1. R. F. Engle and C. W. J. Granger, 'Cointegration and error correction: representation, estimation, and testing,' *Econometrica*, **55**, 251–76 (1987).
2. C. R. Nelson and C.I. Plosser, 'Trends and random walks in macroeconomic time series: some evidence and implications', *J. Monetary Economics*, **10**, 139–162 (1982).
3. S. Hylleberg, 'Modelling seasonal variation', in C. P. Hargreaves (ed.), *Nonstationary Time Series Analysis and Cointegration*, Oxford University Press, Oxford, U.K., 1994.
4. J. A. Miron, 'The economics of seasonal cycles', in C. A. Sims (ed.), *Advances in Econometrics, Sixth World Congress of the Econometric Society*, Cambridge University Press, Cambridge U.K., 1994.
5. R. F. Engle, C. W. J. Granger, S. Hylleberg and H. S. Lee, 'Seasonal cointegration: the Japanese consumption function', *J. Econometrics*, **55**, 275–298 (1993).
6. D. R. Osborn, 'Comment on Engle *et al.* (1993)', *J. Econometrics*, **55**, 299–303 (1993).
7. C. R. Birchenhall, R. C. Bladen-Hovell, A. P. L. Chui, D. R. Osborn and J. P. Smith 'A seasonal model of consumption', *Economic J.*, **99**, 837–43 (1989).
8. S. Nickell, 'Error correction, partial adjustment and all that: an expository note', *Oxford Bull. Economics & Statist.*, **47**, 119–129 (1985).
9. A. W. Gregory, 'Testing for cointegration in linear quadratic models', *J.Business & Economic Statist.*, **12**, 347–360 (1994).
10. T. J. Sargent, 'Estimation of dynamic labour demand schedules under rational expectations', *J. Political Economy*, **86**, 1009–1044 (1978).
11. J. Kennan, 'The estimation of partial adjustment models with rational expectations', *Econometrica*, **47**, 1441–1455 (1979).
12. J. J. Dolado, J. W. Galbraith and A. Banerjee, 'Estimating Euler equations with integrated series', *Int. Economic Rev.*, **32**, 919–936 (1991).
13. G. E. P. Box and G. M. Jenkins, *Time Series Analysis, Forecasting and Control*, Holden-Day, San Francisco, 1970.
14. S. Hylleberg, R. F. Engle, C.W.J. Granger and B.S. Yoo, 'Seasonal integration and cointegration', *J.Econometrics*, **44**, 215–38 (1990).
15. D. R. Osborn, 'Seasonality and habit persistence in a life cycle model of consumption', *J. Appl. Econometrics*, **3**, 255–66 (1988).
16. J. E. H. Davidson, D. F. Hendry, F. Srba and S. Yeo, 'Econometric modelling of the aggregate time-series relationship between consumers's expenditure and income in the UK', *Economic J.*, **88**, 661–692 (1978).
17. S. Johansen, 'Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models', *Econometrica*, **59**, 1551–80 (1991).
18. H. P. Boswijk, H.P. 'Testing for an unstable root in conditional and structural error correction models', *J. Econometrics*, **63**, 37–60 (1994).