# RECENT ADVANCES IN MODELLING SEASONALITY

**Philip Hans Franses**

*Erasmus University Rotterdam*

**Abstract.** In this paper we review recent developments in econometric modelling of economic time series with seasonality. The prime focus is on econometric models which incorporate explicit descriptions of seasonal variation, instead of removing this variation using a seasonal adjustment method. This review centres around developments in seasonal unit root models and in periodic parameter models, both in the univariate and multivariate context. Several empirical examples are used for illustration. We also discuss several areas for further research.

**Keywords.** Seasonality; Unit roots; Periodic integration/cointegration

## 1. Introduction

In this paper we review recent developments in econometric modelling of economic time series with seasonality. We confine ourselves to developments in seasonal unit roots models and periodic models since these have attracted some attention recently. A common feature of these two model classes is that seasonal variation is explicitly modelled. For further reference, we adopt the definition of seasonality as it is given in Hylleberg (1992, p.4), which is that

> Seasonality is the systematic, although not necessarily regular, intra—year movement caused by changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by the agents of the economy. These decisions are influenced by the endowments, the expectations and the preferences of the agents, and the production techniques available in the economy.

An important part in this definition of seasonality is that, although seasonal fluctuations can be deterministic because of, e.g., calendar and weather effects, some seasonal fluctuations may be caused by the behaviour of economic agents and may therefore not be constant. An example of such behaviour is that producers of commodities that are harvested seasonally like coffee or rubber, smooth their output using inventories, in turn generating a seasonal pattern in inventories. Hence, economic agents may take seasonal fluctuations in some variables into account when making plans and forming expectations for other variables. The latter seasonal patterns may then change, not only because of, e.g., changing weather conditions, but also because of changing habits and utility functions of economic agents. For example, improved storage capacities and

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production in greenhouses allows one to buy some vegetables in all seasons now. Another example is that nondurable consumption patterns may change when preferences for certain holiday seasons change. A final example is that, at least in countries where it is legal, the start of the winter clearance sales can depend on the current status of the economy. In fact, in recession periods one typically starts with clearance sales earlier, i.e. some weeks before Christmas instead of the first weeks of January, to get rid of the remaining stock, e.g., in order to be able to finance new stock.

The explicit focus in this review on econometric models for economic time series with seasonality, which include explicit descriptions of seasonal variation, necessarily excludes two important areas of research on seasonality. The first concerns the issue of seasonal adjustment. Although we will briefly touch upon the issue in Section 2.2 to sketch some of the key aspects of seasonal adjustment, a detailed review of developments in this area is beyond the scope of the present paper. The interested reader may consult the survey paper by Bell and Hillmer (1984) and some of the papers collected in Hylleberg (1992).

The second area of research which will not be dealt with in great detail in this survey concerns the construction of theoretical economic models for economic processes with seasonality. By now, the economics literature contains many important papers on such theoretical models. For example, Miron and Zeldes (1988) and Krane (1993) consider seasonality in production-smoothing models of inventories. Faig (1989) considers seasonal fluctuations in money demand. Miron (1986) and Osborn (1988) study how the life cycle-permanent income hypothesis can incorporate the marked seasonal patterns in consumption. Todd (1990), Chatterjee and Ravikumar (1992), Hansen and Sargent (1993) and Braun and Evans (1995) propose modifications of (variants of) real business cycle models to cope with apparent seasonal fluctuations in various essential macroeconomic variables as consumption and production. In sum, there are already several economic models that consider seasonality explicitly and these models cover various economic phenomena. The set of econometric models we survey in our review are linked with the theoretical economic models along two ways. The first is that the econometric models can be fitted to data when these are generated from the theoretical equations. The second way is that some theories directly imply certain aspects of econometric models. For example, the theory in Osborn (1988) implies that consumption can be described by a first order periodic autoregression (see Section 4 for more details on periodic models). Hence, although the econometric models we consider can have strong links with economic theory, we regard an extensive survey of relevant economic theory beyond the scope of this paper.

The outline of our review paper is as follows. In Section 2, we document some empirical regularities for a set of quarterly observed macroeconomic time series for the USA. We use these data in more than one occasion in this paper to illustrate several modelling approaches. The main empirical questions we investigate in Section 2 are: (i) what amount of fluctuations in macroeconomic variables can be attributed to seasonality?, (ii) are seasonal fluctuations constant over time?, and (iii) can seasonal, trend and cyclical fluctuations in a single time series be disentangled in a meaningful way? The latter question is of importance since current seasonal adjustment methods assume that seasonal and business cycle fluctuations are somehow independent. For completeness we discuss in Section 2.2 some properties of seasonally adjusted time series, which are documented in recent studies. As will be argued in Section 2.5, for some macroeconomic variables it holds that it may be difficult (if not impossible) to separate seasonal from nonseasonal fluctuations. This leads to the suggestion that it may sometimes be useful to explicitly incorporate seasonality in econometric models, even though these models can be much less parsimonious than models for seasonally adjusted data. First, in Section 3, we discuss econometric models with seasonal unit roots for univariate and multivariate time series. Then, in Section 4, we discuss advances in periodic models, which are models that allow dynamic parameters to vary with the seasons. Both sections contain several suggestions for further research. In Section 5, we conclude this paper with some final remarks.

We conclude this introduction with a discussion of the notation we adopt throughout the paper. In general, we use $x_t$ for some quarterly or monthly observed time series, where $t$ runs from 1 through $n$. We also use $X_{s,T}$ to denote the observation in season $s$, where $s = 1, 2, .., S$ and $S$ can be 4 or 12, and where the annual index $T$ runs from 1 through $N$. For convenience we assume that $n = SN$ is the number of effective observations, although in some empirical occasions $n$ may not be exactly equal to $SN$. In this review, we limit our attention mostly to quarterly time series for notational convenience. Extensions to, e.g., monthly data are relatively straightforward.

## 2. Empirical regularities

It seems useful to start this review with a discussion of several empirical regularities one can observe for seasonal time series, see also Hylleberg (1994) and Miron (1994) for similar approaches. In Section 2.1 we present typical graphs of macroeconomic time series, and we compute some summary statistics which give an indication of the relative size of seasonal fluctuations. Section 2.2 discusses some relevant aspects of seasonal adjustment. In Section 2.3, we document results of testing for seasonality in consumer expectations, which show that sometimes consumers appear to have difficulties in disentangling seasonal and trend patterns, and which emphasize the findings in Ghysels and Nerlove (1988). In the next subsection, we discuss the question to what extent seasonal fluctuations in macroeconomic time series are constant. We limit the amount of technical detail in this exposition since we seek to stress our main viewpoints mostly using intuitive arguments. In Section 2.5. we review recent time series evidence on the separability of seasonal, trend and business cycle fluctuations can be related. It appears that for some macroeconomic variables it does not appear straightforward to disentangle seasonal from nonseasonal fluctuations. In the summary we argue that it can sometimes be more useful to explicitly describe seasonality instead of removing it using seasonal adjustment.

## 2.1. Graphs and summary statistics

In this paper we use several economic time series to illustrate the various issues discussed. Six of these series, which serve as running examples throughout this paper, concern USA data on consumption variables, industrial production and money. These six time series are all quarterly observed and are used in seasonally unadjusted form. For specific purposes, we consider quarterly unemployment in Canada, where we use seasonally unadjusted (NSA) and seasonally adjusted (SA) data, and the well known 'airline' data in Box and Jenkins (1970). Details of all variables are given in Appendix 1. Except when otherwise indicated we use all data after applying the natural log transformation.

In Figure 1 we display four consumption series in the USA. These series concern Consumption of Services, Nondurables, Durables and Total Consumption. For all these series we observe a clear trending pattern and, except perhaps for Consumption of Services, also the seasonal fluctuations appear relevant. An additional feature is the cyclical pattern in Consumption of Durables, which seems to display significant business cycle behaviour more than the other components of Total Consumption. In Figure 2 we depict the growth rate of the USA industrial production series since the growth rate highlights the seasonal pattern better than the untransformed series itself. Apart from a possibly outlying observation in 1975, the graph in Figure 2 suggests a seasonal pattern which may be different across the three decades. A typical example of a graph that shows changing seasonal fluctuations is given in Figure 3, where we

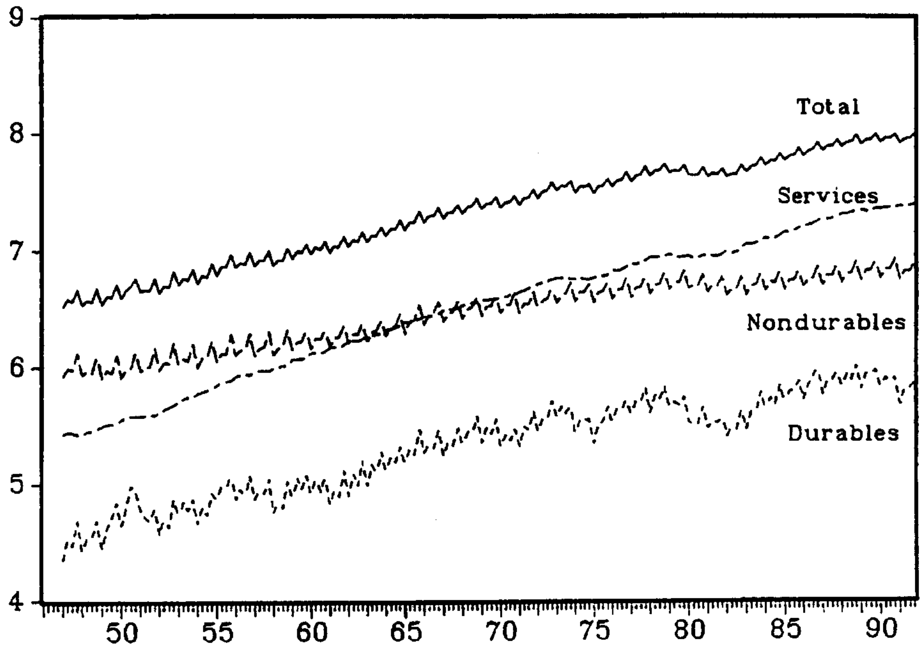


**Figure 1.** Consumption in the USA

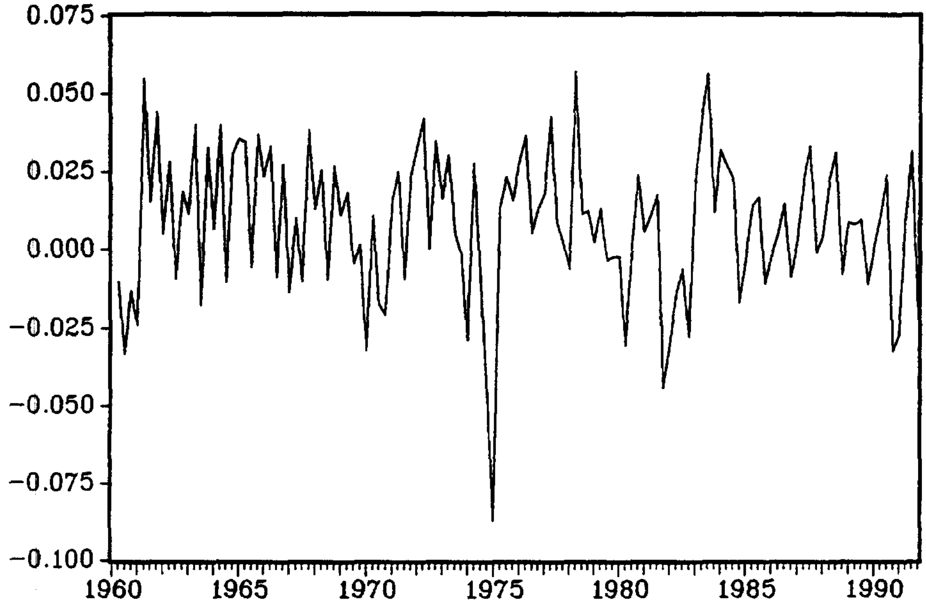


**Figure 2.** USA Industrial Production, growth rates

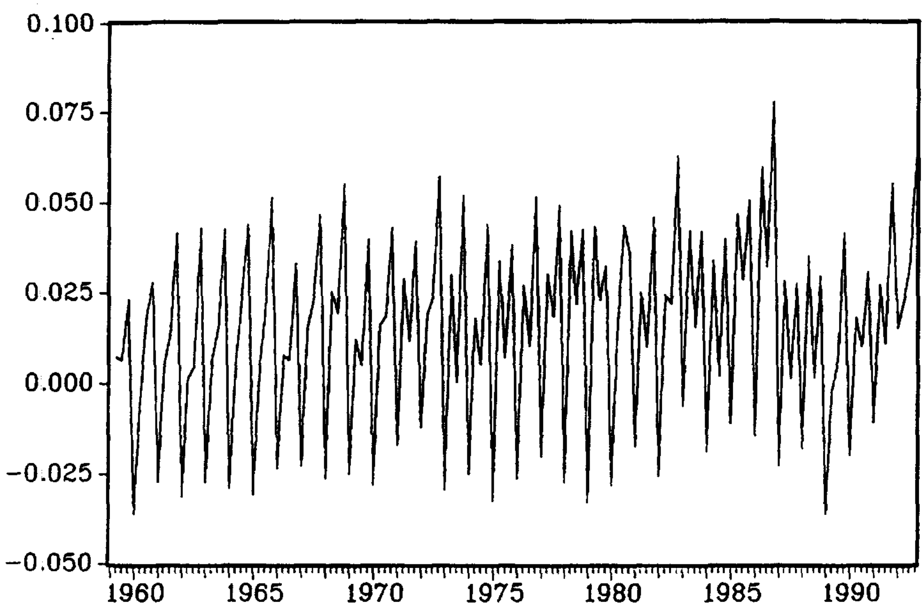


**Figure 3.** USA money, growth rates

present the growth rates of money in the USA. This graph indicates that seasonal variation appears a dominant component in the variation of the time series, and that seasonality changes after the beginning of the seventies. In sum, from these graphs one can observe that quarterly macroeconomic variables may display trending, seasonal and cyclical behaviour.

To obtain a tentative impression of the amount of seasonal variation, we apply the method advocated in Miron (1994), i.e. we transform our data into growth rates (we take first differences of the data in logs), and we regress these growth rates on four seasonal dummies,

$$\Delta_1 x_t = \delta_1 D_{1t} + \delta_2 D_{2t} + \delta_3 D_{3t} + \delta_4 D_{4t} + u_t \tag{2.1}$$

where $\Delta_k x_t \equiv (1 - B^k) x_t \equiv x_t - x_{t-k}$, where $D_{st}$ is a seasonal dummy variable which takes a value 1 in season $s$ and zero elsewhere, $s = 1, 2, 3, 4$ and where $u_t$ is some error process. In Miron (1994), *inter alia*, it is usually assumed that $u_t$ is a stationary and invertible autoregressive moving average [ARMA] process. In practice, this assumption may be debatable. see, e.g., Hylleberg, Jørgensen and Sørensen (1993) and we will return to this assumption in Sections 3 and 4 below.

In Table 1 we report some estimation results of (2.1) for the six USA time series. The first statistic is the coefficient of determination $R^2$ for (2.1). For four of the six variables, this $R^2$ value is 0.8 or higher, while only for Industrial Production the $R^2$ value is as small as 0.123. These results correspond to those in, e.g., Miron (1994) and Beaulieu, MacKie-Mason and Miron (1992) (1). An interpretation is that once a trend is removed by taking first order differences, seasonal variation seems to dominate the overall variation in quarterly macroeconomic time series. In the last two columns of Table 1, we display the square roots of the components of the $R^2$ to illustrate the relative magnitudes of seasonal and nonseasonal variation.(2) In the same table, we further report the estimates of the $\delta_s$ parameters for the seasonal dummy variables in (2.1). The irregular patterns in the Figures 1 to 3, suggest that the $u_t$ process is unlikely to be

**Table 1.** Estimation results for the auxiliary regression

$$\Delta_1 x_t = \delta_1 D_{1t} + \delta_2 D_{2t} + \delta_3 D_{3t} + \delta_4 D_{4t} + u_t$$

where $D_{st}$ are seasonal dummy variables for USA variables

| Variable | $n$ | $R^2$ | $\hat{\delta}_1$ | $\hat{\delta}_2$ | $\hat{\delta}_3$ | $\hat{\delta}_4$ | $\hat{\sigma}_{sd}$ | $\hat{\sigma}_u$ |
|---|---|---|---|---|---|---|---|---|
| Consumption | 179 | 0.917 | −0.092 | 0.049 | −0.001 | 0.075 | 6.38 | 1.92 |
| Nondurables | 179 | 0.970 | −0.176 | 0.077 | <−0.001 | 0.116 | 11.22 | 1.99 |
| Durables | 179 | 0.869 | −0.203 | 0.133 | −0.036 | 0.135 | 13.99 | 5.44 |
| Services | 179 | 0.413 | 0.025 | <0.001 | 0.009 | 0.010 | 0.89 | 1.06 |
| Money | 135 | 0.805 | −0.023 | 0.022 | 0.015 | 0.044 | 2.40 | 1.18 |
| Production | 127 | 0.123 | 0.001 | 0.022 | 0.005 | 0.004 | 0.80 | 2.17 |

Note: The variables are defined in Appendix 1. These are considered after taking natural logs. The number of effective observations to estimate the auxiliary regression is $n$. The $\hat{\delta}_{sd}$ *and* $\hat{\delta}_u$ are the standard deviations of the fit of the regression and of the residuals, respectively (when multiplied by a white noise process, i.e. an uncorrelated process with constant variance, and therefore we do not report the estimated standard errors for the $\delta_s$ parameters from (2.1). The point estimates, however, indicate that $\hat{\delta}_1$ is typically negative, and that $\hat{\delta}_4$ typically takes the largest positive value.

## 2.2. On seasonal adjustment

Before we turn to several additional features of seasonally observed economic time series, we briefly discuss the issue of seasonal adjustment. The reason for this is that when seasonality in time series is as apparent as in Figures 1 to 3, a first approach may be to remove such seasonal fluctuations using a seasonal adjustment program.

A central assumption for seasonal adjustment is that the series $x_t$ can be decomposed into two unobserved components

$$x_t = x_t^{ns} + x_t^s, \tag{2.2}$$

where $x_t^{ns}$ denotes the nonseasonal component containing the trend, cycle and irregular components, and where $x_t^s$ denotes the seasonal component. Of course, in case seasonality appears multiplicative, i.e. seasonal variation increases with a trend, one may modify (2.2) to $x_t = x_t^{ns} x_t^s$. Since for many macroeconomic time series the trend and seasonal patterns are not perfectly constant, as may be observed from Figures 1 to 3, one usually considers certain moving average filters to characterize a changing trend and changing seasonality. In many practical occasions, such moving average filters are linear, symmetric and centred around the current observation. Denoting $F$ as the forward shift operator, i.e. $F = B^{-1}$ and hence $F^k x_t \equiv x_{t+k}$, such a linear moving average filter is given by

$$C_m(B, F) = c_0 + \sum_{i=1}^{m} c_i(B^i + F^i), \tag{2.3}$$

where $c_0, c_1, \ldots, c_m$ are the moving average weights. Details of the use of these filters are given in, e.g., Maravall (1995) and Crether and Nerlove (1970). In the latter study it is shown that filters as in (2.3) result in optimality properties of the estimates of seasonal and nonseasonal components. The well-known Census X-11 seasonal adjustment method makes extensive use of filters like (2.3), see Hylleberg (1986) for a documentation of this method. As an alternative to this Census X-11 method, one may consider the so-called model-based method, see Hillmer and Tiao (1982). Within this method, one seeks to explicitly describe the unobserved $x_t^{ns}$ and $x_t^s$ processes using ARIMA models. For example, for the seasonal component one typically assumes the model

$$(1 + B + B^2 + B^3)x_t^s = (1 + \theta_1 B + \theta_2 B^2 + \theta_3 B^3)v_t, \tag{2.4}$$

where $v_t$ is a white noise process with variance $\sigma^2$. In order to be able to identify the unobserved components $x_t^s$ and $x_t^{ns}$, it is required that $\sigma_v^2$ is very small. Obviously, this requirement implies that the seasonal fluctuations are set close to constant. Given the findings for e.g., money in Figure 3, it seems that this (technical) assumption which may not be beyond discussion.

Recently, there have appeared several studies that evaluate the impact on time series properties of (Census X-11) seasonal adjustment. Maravall (1995) shows that the resultant variable can be described by a non-invertible moving average process yielding possible difficulties in constructing univariate and multivariate time series models. Ghysels and Perron (1993) show that the power of unit root tests is lower for seasonally adjusted series. Furthermore, Ghysels, Granger and Siklos (1995) document that seasonal adjustment may lead to undesirable nonlinear properties in univariate time series. These studies all seem to suggest that seasonally adjusting a time series can distort some of its important properties and may complicate further analysis. On the other hand, Hansen and Sargent (1993) show that SA data may not dramatically bias inference from a class of real business cycle models. For earlier discussions on the use of SA data for univariate and multivariate time series modelling, see Sims (1974) and Wallis (1974).

Additional to the effect of filters as (2.3) on time series properties, it seems worthwhile to investigate the assumption in (2.2), i.e. the presumed possibility of identifying independent components $x_t^{ns}$ and $x_t^s$. A common aspect of several macroeconomic phenomena is that the seasonal variation can change because of changes in weather or calendar (Easter sometimes falls in March and sometimes in April), but also that changing seasonality may be caused by the behaviour of economic agents. In turn, this behaviour may be caused by perceived trends or cycles in the economy. In other words, it may occur that trend, seasonal and cyclical components are difficult to separate, and hence that identification of $x_t^{ns}$ and $x_t^s$ is difficult. A theoretical economic model that highlights this complication is presented in Ghysels (1988). If trend and seasons cannot be disentangled, it may occur that seasonal adjustment removes information from an economic time series which can be useful for describing the behaviour of economic agents.

### 2.3. Seasonality in expectations

Although the issue of seasonal adjustment is typically viewed as a statistical problem, a dominant motivation to seasonally adjust is that economic agents may prefer to make decisions on deseasonalized data. Indeed, the producer of, for example, ice-cream is aware of the seasonal demand for this product, and is therefore likely to be interested only in the direction of the underlying trend.

In case economic agents do not have seasonally adjusted data at their disposal, and when we suppose that they do want to separate nonseasonal from seasonal fluctuations, it is interesting to study to what extent agents are able to make this separation. A seminal contribution to such an investigation has been made in Ghysels and Nerlove (1988), where it is shown that seasonal effects appear in qualitative responses of monthly business surveys. In order to corroborate the findings in Ghysels and Nerlove (1988), we now briefly analyse some time series of consumer opinion on economic and financial conditions, collected by several statistical agencies in European Community countries, and which are summarized in issues of *European Economy, Supplement B*. We only consider data for the sample period 1975–1988, since after this period one has started seasonally adjusting these expectations series.(3) Our expectations series can be divided into seven categories, i.e. general consumer confidence index, the financial situation of households, the general economic situation, price trends, unemployment, major purchases, and savings. The relevant questions ask the respondent to compare the last 12 months with the next 12 months. We can collect observations on these indices three times per year for the period 1975 through 1988 for Belgium, Denmark, (West-)Germany, France, Ireland, Italy, The Netherlands and the United Kingdom. Our econometric procedure to test for seasonality is simply to run a regression of $index_t$ on a constant, two seasonal dummies and $index_{t-1}$. The results of the $F$ tests for the significance of the two seasonal dummies in the regression model are displayed in Table 2.

The results are that about one half of the indices display seasonal fluctuations, even though the consumers were asked to remove seasonality since the questions involved annual trends. Hence it seems that consumers have time-varying expectations. One reason may be that economic agents base their expectations on the current status of the economy and on expected trends which may again depend on the current economic situation. If this is true, economic agents appear to face difficulties disentangling seasonal from nonseasonal fluctuations.

**Table 2.** Testing for seasonality in consumer confidence indices. Based on 14 years of 3 observations (January, May, October) per year. The cells are the $F$ test values for the significance of the two seasonal dummy variables in $index_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \beta\ index_{t-1} + \varepsilon_t$

| Ind. (1) | Belgium | Denmark | Germany | France | Ireland | Italy | Neth. | U.K. |
|---|---|---|---|---|---|---|---|---|
| GCC | 7.463*** | 1.996 | 3.160* | 0.073 | 0.038 | 2.564* | 2.512* | 0.310 |
| GES | 3.102* | 1.468 | 3.864** | 0.690 | 0.263 | 1.388 | 4.365** | 2.436 |
| FP | 3.316** | 0.521 | 2.810* | 0.380 | 0.484 | 4.543** | 3.493** | 0.972 |
| PT | 6.176*** | 7.294*** | 1.354 | 2.493* | 1.484 | 5.279*** | 1.490 | 0.403 |
| U | 8.269*** | 6.382*** | 4.437** | 1.555 | 1.752 | 3.943** | 4.628** | 3.188* |
| MP (2) | 4.768** | 0.106 | 0.425 | 5.851*** | 4.519** | 8.660*** | 5.647*** | 1.877 |
| S | 0.019 | 2.666* | 0.777 | 12.679*** | 1.085 | 2.383 | 4.776** | 0.177 |

Note: From 1972 through 1983 there were only three consumer surveys per year, i.e. in January, May and October. In 1984 and 1985 measurements took place four times a year, i.e. in January/February, in March/June, in July/August, and in October/November. Starting from 1986, monthly data are available, which are seasonally connected from January 1989 onwards. To obtain time series that cover several years, we decide to construct series with three data points each year, i.e. in January, May, and October. For 1984 and 1985 the two first and the last observations were used. When there were no May observations available in 1986 through 1988, we took the observations of the months April or March. Since for some of the eight countries the surveys started in May 1974. the analysis of seasonal patterns considers all observations previous to 1975 as starting values.

*** Significant at a 1% level.

** Significant at a 5% level.

* Significant at a 10% level.

(1) The indices are General Consumer Confidence (GCC), General Economic Situation (GES), Financial Position (FP), Price Trends (PT), Unemployment (U), major Purchases (MP), and Savings (S).

(2) The sample sizes are 27 for Belgium, 35 for Ireland, and 30 for the Netherlands, respectively.

2.4. Changing seasonality

Additional to pronounced seasonal variation, a further aspect of macroeconomic time series is that the seasonal fluctuations may not be constant over time, see Figures 1 to 3 and, e.g., Hylleberg (1994). In Section 3 and 4 we review formal test procedures for such changing seasonal patterns. In this subsection we limit ourselves to the discussion of two simple graphs.

For the calculations in Table 1, we have assumed that the $\delta_s$ parameters are constant over time. Of course, this may be too restrictive an assumption for some time series. In fact, when we plot the growth rate $\Delta_1 x_t$ for each of the seasons, i.e. when we plot $X_{s,T} - X_{s-1,T}$ $(s = 1, \ldots, 4)$, where $X_{0,T} = X_{4,T-1}$, one may sometimes observe that the growth rates per season show irregular patterns.

In Figure 4, we depict such quarterly growth rates for Money in the USA. It is clear from this figure that the growth rates for the first, third and fourth quarter are reasonably constant until around 1990, but that the second quarter growth displays a trending pattern. The latter implies that a stochastic trend does not seem to have been effectively removed for the second quarter observations. Alternatively stated, the estimate for $\delta_2$ in (2.1) is unlikely to be constant over time.

To investigate the parameter constancy for seasonal dummy variables for M1, we estimate (2.1) recursively. To this end, we need a model for the error process $u_t$ in (2.1).[4] Some experimentation with these money data yields the following model for the effective sample 1960.2–1992.4,

$$\Delta_1\hat{x}_t = \underset{(0.005)}{-0.025}D_{1t} + \underset{(0.003)}{0.021}D_{2t} + \underset{(0.003)}{0.005}D_{3t} + \underset{(0.004)}{0.027}D_{4t} + \underset{(0.082)}{0.237}\Delta_1 x_{t-1} + \underset{(0.085)}{0.3241}\Delta_1 x_{t-4}$$

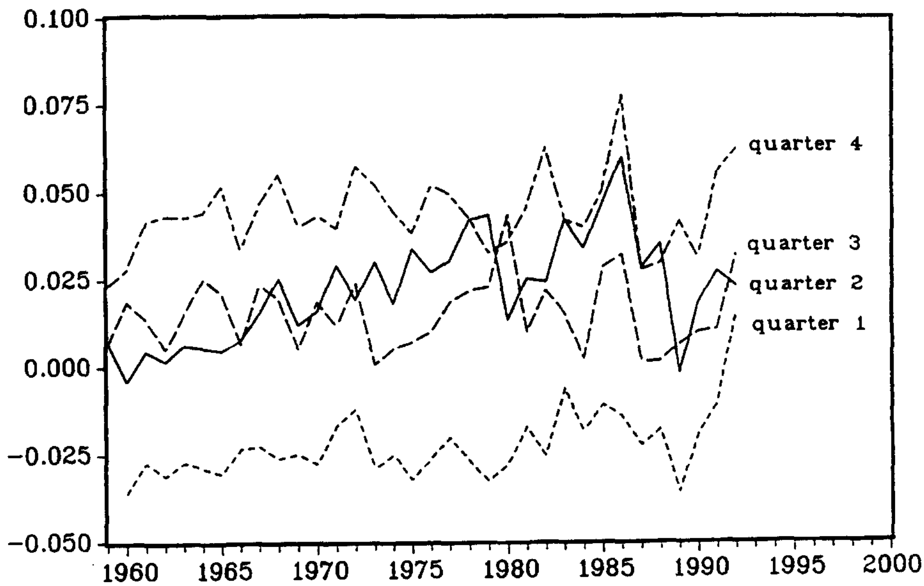


**Figure 4.** Growth rates in USA money: per quarter

where standard errors are given in parentheses. Various diagnostic tests do not reveal obvious misspecification of this estimated model. See Appendix 2 for an overview of these tests.[5] It should be mentioned here that all models in this review are checked for the absence of residual autocorrelation, the absence of autoregressive conditional heteroskedasticity (ARCH) and for the presence of normality in the estimated residuals. To save space, we do not report these diagnostic results everywhere.

In Figure 5, we depict the recursive estimates of the $\delta_s$ parameters using the above model specification. From these graphs it can be observed that $\hat{\delta}_4$ and $\hat{\delta}_3$ rapidly converge to constant values, that $\hat{\delta}_1$ shows some cyclical behaviour and that, as expected given Figure 4, $\hat{\delta}_2$ does not display a constant pattern. Even though the empirical model does not seem to be inadequate, the recursive estimates in Figure 5 reveal that its parameters are not constant over time. One implication of this result is that, apparently, the applied diagnostics are not informative enough. A further implication is that when one would use the above model for multi-step out-of-sample forecasting, it is likely that one finds large prediction errors in the second quarter. Hence, the simple autoregressive [AR] model above may be modified using seasonally varying variances or by modelling the second quarter observations differently. We will discuss such extensions in Section 4. In sum, the main conclusion to be drawn from this subsection is that changing seasonal patterns can easily be visualized.

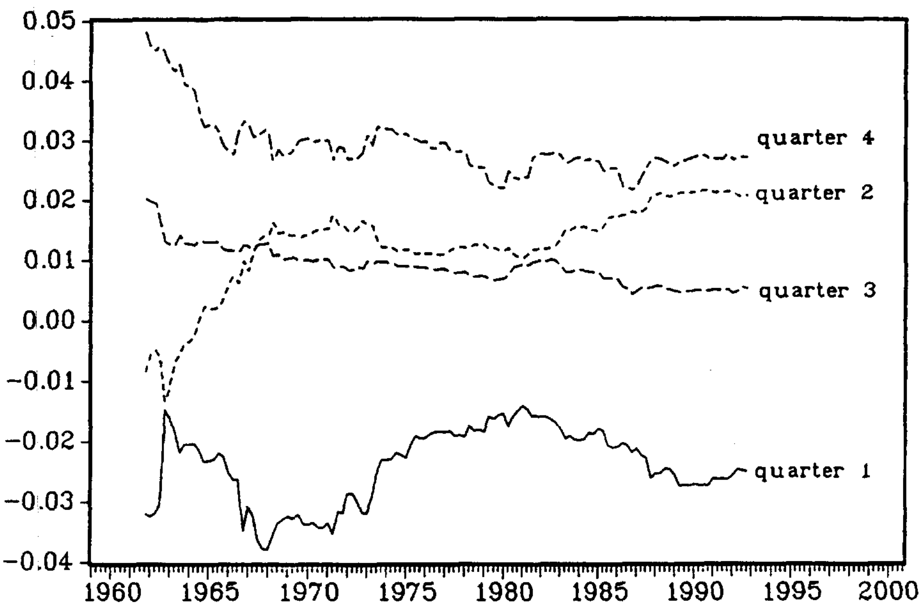


**Figure 5.** Recursive estimates of seasonal intercepts in AR model for growth rates in USA money

### 2.5. Seasons, Cycles and Trends

In this subsection, we review recent research that explicitly addresses the issue of independence of seasonal, trend and cyclical fluctuations in certain macroeconomic time series.

In a sequence of papers, Miron and associates have investigated the correlation between seasonal and cyclical variation in macroeconomic time series, see Barsky and Miron (1989), Beaulieu, MacKie-Mason and Miron (1992) and Miron (1994), *inter alia*. Their model is (2.1) and the correlation between the variance of the fit from (2.1), which is called the seasonal cycle, and the variance of the estimated error process, which is called the business cycle, is estimated for several macroeconomic variables for the US and other economies. In those studies it is typically found that these two variances are positively correlated, which results in the conclusion that countries and industries with large seasonal cycles also have large business cycles.[(6)]

Similar conclusions can be drawn from the empirical results reported in Canova and Ghysels (1994), where a modification of model (2.1) is used. This modification amounts to allowing the $\delta_s$ parameters to take different values across two business cycle stages, where the expansions and contractions are defined by the official NBER peaks and troughs. It is found that these $\delta_s$ estimates vary significantly across expansions and contractions, suggesting that cyclical and seasonal fluctuations may be related.

If seasonal and cyclical fluctuations are not independent, it is easily understood that the interpretation of seasonally adjusted time series becomes tedious. For example, it may be hard to assign a suddenly large observation to a change in seasonal pattern or to the business cycle. Typically, seasonal adjustment methods likely assign part of that observation to the seasonal component and part to the cyclical component. Hence, if one has to decide on peaks and troughs in a time series, which is usually done using seasonally adjusted time series, one may find peaks and troughs in certain seasons more than other seasons, especially if such seasons show more volatility like the first and last quarter of the year. This conjecture seems to be emphasized by the empirical findings in Ghysels (1994b) where it is documented that the NBER peaks and troughs are not equally distributed throughout the year. Of course, this may be caused by an underlying seasonal structure in the occurrence of business cycles, as is investigated in Ghysels (1993), but it may also be that seasonal adjustment techniques yield potentially unreliable data at and around business cycle turning points.

Related results on the relationship between seasonal and nonseasonal components are discussed in Franses en Ooms (1995). Consider again the time series $x_t$ and the decomposition (2.2). Obviously, one important aspect of a seasonally adjusted time series is that $x_t^{ns}$ does not contain seasonal fluctuations. Of course, in practical occasions one may expect some seasonality in the $x_t^{ns}$ series, but it should not be such that $x_t^s$ can be systematically predicted by $x_t^{ns}$. Formally, it is important that $\Delta_4 x_t^s$ is not systematically correlated with $\Delta_4 x_t^{ns}$. If there would be such a correlation, one can expect seasonality in the peaks and troughs as discussed above since a change in the trend is then related to a change in the seasonal component. An extreme case could then be that the economy is already in a recession while this is yet unknown because negative shocks have been assigned to the changing seasonal component.

As an illustration we depict in Figure 6 $\Delta_4 x_t^s$ versus $\Delta_4 x_t^{ns}$ for Unemployment in Canada observed for the period 1960.1–1987.4 (see Appendix 1 for more details on this variable). At first sight it seems that these two series are uncorrelated. However, if one draws the same picture for each of the seasons, as we have done in Figure 7, it is obvious that there exist large correlations between $\Delta_4 x_t^s$ and $\Delta_4 x_t^{ns}$ which vary across the seasons. This correlation is positive in the first two quarters, while it is negative in the last two quarters. Similar results emerge when we take logs of the unemployment series. Tests for the significance of these correlations in Section 4 below indicate that they are highly significant. Hence, it seems for the first two quarters that the seasonal component is overestimated in case unemployment increases. If so, the adjusted time series is underestimated since $x_t^{ns} = x_t - x_t^s$. An implication is that one can be too optimistic in the first quarter in case unemployment increases. In times of decreasing unemployment and also in other quarters, one may obtain similar or opposite results. The worst scenario is that too optimistic a view may prevent our recognition of the start of a recession.

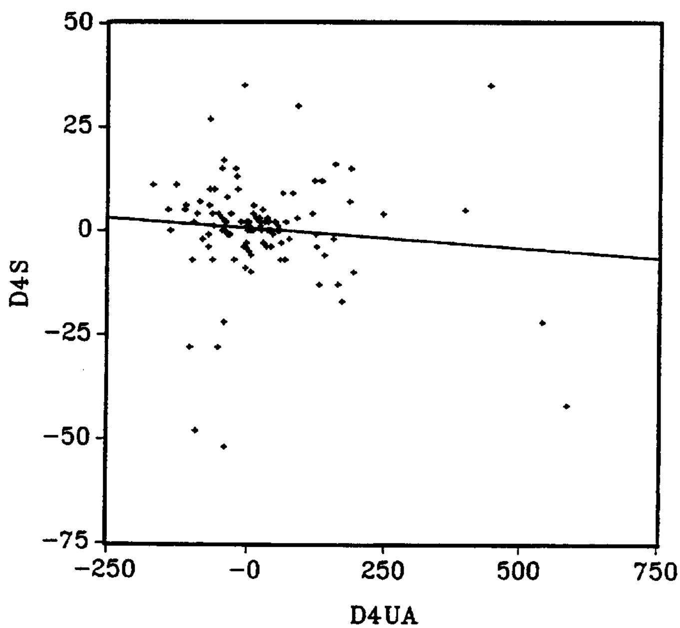


**Figure 6.** Annual changes in seasonal component versus annual changes in nonseasonal component for all quarters for unemployment in Canada

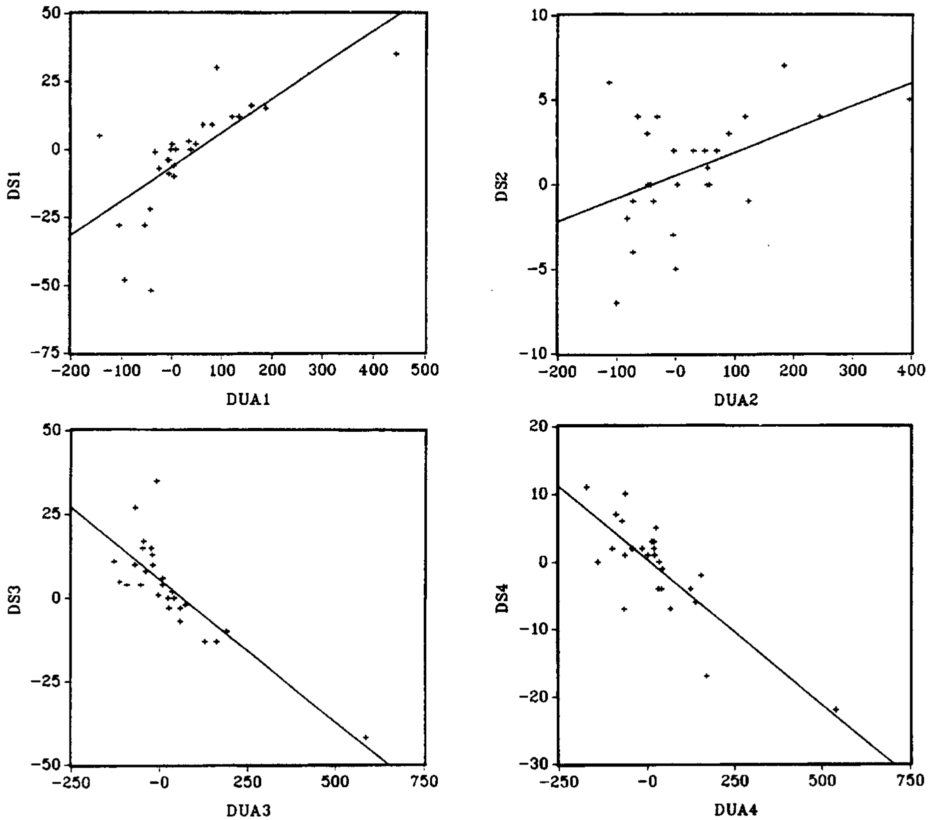


**Figure 7.** Annual changes in seasonal component versus annual changes in nonseasonal component per quarter for unemployment in Canada

## Summary

To summarize this section on empirical regularities in seasonal variables we can draw the following conclusions. The first is that seasonal variation can constitute a large part of total variation in macroeconomic time series. The second is that seasonality does not seem to be constant over time. The third is that seasonal and nonseasonal variation may not be independent for some economic variables. In fact, there is evidence of a relation between seasonality and business cycle stages and/or trends. This finding questions the usefulness of seasonal adjustment methods since the independence of seasonal and nonseasonal fluctuations is a key assumption. In the next two sections we review several approaches which explicitly incorporate seasonal variation within the context of an econometric model. It is our opinion that, even though these econometric models may not be as parsimonious as models for SA data, it is important at least to consider such models in practical occasions.

Finally, one important advantage of modelling seasonality instead of removing it is that anyone who analyses the same data may evaluate the merits of certain models. In fact, most of the approaches to be discussed below rely on formal statistical tests. Hence, nested and non-nested tests can be used to compare the various models. This is in sharp contrast to the current practice of seasonal adjustment where this practice is usually taken to be a black-box process. It is hard to obtain precise knowledge on how statistical agencies generate their adjusted time series, e.g., how they treat outliers or shifts in mean and trends. It is readily imaginable that two researchers considering the same variable in different seasonally adjusted forms, obtained from two distinct statistical agencies, may obtain different results which cannot be meaningfully compared.

## 3. Seasonal unit roots and differencing filter selection

In this section, we review recent developments in seasonal unit root models for univariate or multivariate time series. In the first subsection we discuss the prevailing practice in traditional time series analysis of seasonal variables which has been introduced by Box and Jenkins' pioneering work on time series analysis. One feature of this approach is that trend and seasons are assumed to be stochastic. In Section 3.2, we review recent research on (testing for) seasonal unit roots which questions the application of many differencing filters. In fact, using formal tests one usually finds that only a small number of unit roots is needed to removed stochastic trend behaviour. In Section 3.3, we discuss other methods to obtain insight into the most appropriate differencing filter, also in case the time series displays increasing seasonal variation. Obviously, the last issue is related to the question of taking logs or not. In Section 3.4, we discuss the so-called seasonal cointegration model, which is a multivariate extension of the seasonal unit root concept. When two time series have a seasonal stochastic trend, one can investigate whether these series have this trend in common. Hence, seasonal cointegration amounts to an extension of the well-known cointegration concept formulated in Engle and Granger (1987) to the seasonal frequencies. Finally, in Section 3.5, we discuss some extensions of the reviewed approaches and further research areas.

### 3.1. Typical autocorrelations

The initial step in time series analysis advocated by Box and Jenkins (1970) is the identification step. This usually involves the calculation of the empirical autocorrelation function [ACF] of (transformations) of the data. For example, in Table 3, we display the ACFs for the USA Consumption Nondurables, which are typical for many macroeconomic time series.

In the first column of Table 3 we give the first twelve autocorrelations for the untransformed time series. Obviously, these values do not die out quickly, suggesting that the time series has to be differenced in order to obtain a stationary time series. In the second column, we give the ACF for the first order differenced

**Table 3.** Typical autocorrelation functions of (transformations of) quarterly time series: USA Consumption Nondurables

| Lag | $x_t$ | $\Delta_1 x_t$ | Variable [1] $(\Delta_1 x_t)^c$ | $\Delta_4 x_t$ | $\Delta_1\Delta_4 x_t$ |
|---|---|---|---|---|---|
| 1 | 0.906 | −0.666 | −0.332 | 0.696 | −0.274 |
| 2 | 0.920 | 0.344 | −0.055 | 0.555 | 0.128 |
| 3 | 0.880 | −0.656 | −0.252 | 0.342 | −0.020 |
| 4 | 0.949 | 0.965 | 0.629 | 0.144 | −0.368 |
| 5 | 0.855 | −0.651 | −0.214 | 0.164 | 0.261 |
| 6 | 0.866 | 0.333 | −0.132 | 0.040 | −0.211 |
| 7 | 0.825 | −0.632 | −0.163 | 0.041 | 0.172 |
| 8 | 0.889 | 0.937 | 0.503 | −0.054 | −0.139 |
| 9 | 0.795 | −0.636 | −0.251 | −0.071 | −0.083 |
| 10 | 0.805 | 0.323 | −0.083 | −0.035 | 0.096 |
| 11 | 0.764 | −0.611 | −0.174 | −0.045 | −0.109 |
| 12 | 0.825 | 0.911 | 0.481 | −0.003 | 0.105 |
| SE | 0.075 | 0.075 | 0.075 | 0.075 | 0.075 |

[1] $(\Delta_1 x_t)^c$ refers to the residuals of the regression of $\Delta_1 x_t$ on four seasonal dummy variables.

time series $\Delta_1 x_t$. Again, it is clear that the values do not die out, although there now appears to be some alternating pattern, i.e. those values at lags 1, 3, 5, etc. take negative values, while at even lags the ACF has positive values. This latter pattern may be caused by some deterministic seasonal pattern, and in the third column we therefore display the ACF for $(\Delta_1 x_t)^c$, which are the residuals from the regression of $\Delta_1 x_t$ on four seasonal dummies.[7] Indeed, the large peaks in the ACF become somewhat smaller, though the ACF still does not die out rapidly at lags 4, 8 and 12. In the last two columns of Table 3 we report the ACFs for the $\Delta_4 x_t$ and $\Delta_1\Delta_4 x_t$ series. Note that $\Delta_4 x_t = x_t - x_{t-4}$ and $\Delta_1\Delta_4 x_t = x_t - x_{t-1} - x_{t-4} + x_{t-5}$. The ACF of $\Delta_4 x_t$ dies out rather quickly, and hence one may attempt to fit an ARMA model to this series. However, it is conventional practice to transform this $\Delta_4 x_t$ series with the $\Delta_1$ filter to obtain the doubly differenced series $\Delta_1\Delta_4 x_t$, for which the ACF usually is easily interpretable, i.e. only a few ACF values are significant. Examples of similar results can be found in standard time series textbooks as Nerlove, Grether and Carvalho (1979), Granger and Newbold (1986) and Mills (1990). In fact, the estimation of parameters in a simple ARIMA model for $\Delta_1\Delta_4 x_t$, suggested by the ACF values in last column of Table 3, results in

$$\underset{}{\Delta_1\Delta_4 x_t} = \underset{(0.001)}{-0.00004} - \underset{(0.075)}{0.229\hat{\varepsilon}_{t-1}} - \underset{(0.063)}{0.612\hat{\varepsilon}_{t-4}} + \underset{(0.074)}{0.163\hat{\varepsilon}_{t-5}}, \tag{3.1}$$

where the number of effective observations on USA Consumption Nondurables is 175, and where the model is estimated using non-linear least squares (based on the Gauss–Newton iteration scheme). This model is the unrestricted variant of the so-called 'Airline model', see Box and Jenkins (1970). The diagnostic test for residual autocorrelation $BP(12)$ obtains the insignificant value of 10.86, the $F$ versions of tests for ARCH effects are $F_{arch1} = 0.649$, $F_{arch1-4} = 0.113$ and the $\chi^2$ test for normality is $\chi^2_{norm} = 3.593$. Hence, the model does not display obvious misspecification.

The model in (3.1) is found useful to forecast out-of-sample in many empirical occasions. One reason for this successful record is that (3.1) can describe quarterly time series with slowly changing seasonal patterns, see, e.g., Harvey (1984). In fact, Bell (1987) derives that for a model like

$$(1-B)(1-B^4)x_t = (1-\theta_1 B)(1-\theta_4 B^4)\varepsilon_t, \tag{3.2}$$

it holds true that when $\theta_4 \longrightarrow 1$

$$(1-B)x_t = \delta_1 D_{1t} + \delta_2 D_{2t} + \delta_3 D_{3t} + \delta_4 D_{4t} + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

emerges. Hence, the closer $\theta_4$ gets to 1, the more the seasonal fluctuations become deterministic. Note that for (3.1) the $\hat{\theta}_4$ would be about 0.6. Since the estimator of an MA parameter is downward biased in cases where the true parameter is equal to 1, see Plosser and Schwert (1977), one may expect to find values like 0.8 or 0.9 for $\hat{\theta}_4$ where the time series displays slowly changing seasonality.(8) One may also interpret a large value of $\hat{\theta}_4$ as an indication of overdifferencing. Indeed, the $\Delta_1\Delta_4$ filter assumes that the trend and the seasonal fluctuations are not only stochastic but also nonstationary. Strictly speaking, this implies, for example, that the seasonal pattern can change rapidly, in the sense that the usual peaks in the Winter (see Table 1) may appear in the Summer in one year, while it is the other way around in the next year. For many economic time series such highly volatile seasonality may not be realistic and the $\Delta_4$ differencing filter may be superfluous. The latter notion may lead to investigating processes which are somewhere in between $x_t$ and $\Delta_4 x_t$. In the next subsection we discuss these so-called seasonal unit root processes.

### 3.2. Seasonal unit roots

As is now well-known from the large literature on unit roots in time series, see, e.g., Dickey, Bell and Miller (1986) for a lucid exposition, that the assumption of a certain differencing filter amounts to an assumption of the number of seasonal and nonseasonal unit roots in a time series. This can most easily be understood by writing $\Delta_S = (1 - B^S)$, and by solving the equation

$$(1 - z^S) = 0 \text{ or } \exp(Si\phi) = 1, \tag{3.3}$$

for $z$ or $\phi$, where $i$ corresponds to the imaginary number. The general solution to (3.3) is $\{1, \cos(2\pi k/S) + i\sin(2\pi k/S)\}$ for $k = 1, 2, \ldots$, yielding $S$ different solutions which all lie on the unit circle. For example, in case $S = 4$, the solutions to $(1 - z^4) = 0$ are $\{1, i, -1, -i\}$. This means that we can write

$$\begin{aligned}\Delta_4 = (1-B^4) &= (1-B)(1+B)(1-iB)(1+iB)\\ &= (1-B)(1+B)(1+B^2)\\ &= (1-B)(1+B+B^2+B^3).\end{aligned} \tag{3.4}$$

The unit root 1 is called the nonseasonal unit root, while the unit roots $-1$ and $\pm i$ are called the seasonal unit roots, see Hylleberg *et al.* (1990). Any differencing filter $\Delta_S$ can be written as $\Delta_S = (1-B)(1+B+\cdots+B^{S-1})$, and any $\Delta_S$ can thus be decomposed in a part with one nonseasonal unit root and a part with $S-1$ seasonal unit roots. Note that in (2.4) the seasonal component is assumed to contain three seasonal unit roots. Furthermore, in the unobserved components model recently proposed in Harvey and Scott (1994) similar results apply.

Hylleberg *et al.* (1990) [HEGY] propose a method to test for the presence of seasonal and nonseasonal unit roots in quarterly time series. This method extends the Dickey, Hasza and Fuller (1984) approach, which considers testing whether $\rho = 1$ in $x_t = \rho x_{t-4} + \varepsilon_t$. The HEGY method is based on the auxiliary regression

$$\phi^*(B)z_{4t} = \mu_t + \pi_1 z_{1,t-1} + \pi_2 z_{2,t-1} + \pi_3 z_{3,t-2} + \pi_4 z_{3,t-1} + \varepsilon_t, \tag{3.5}$$

where $\phi^*(B)$ is some autoregressive polynomial in $B$, where $\mu_t$ can contain a constant, three seasonal dummies and/or a trend, and where

$$\begin{aligned} z_{4t} &= (1-B^4)x_t, \\ z_{1t} &= (1-B+B^2+B^3)x_t, \\ z_{2t} &= -(1-B)(1+B^2)x_t \text{ and} \\ z_{3t} &= -(1-B^2)x_t. \end{aligned}$$

For completeness, we summarize in Appendix 3 the key proposition and some of the derivations from HEGY which together lead to (3.5). An application of ordinary least squares (OLS) to (3.5), where the order of $\phi^*(B)$ is determined using diagnostic checks such that the estimated error process is approximately white noise, yields estimates of the $\pi_1$, $\pi_2$, $\pi_3$ and $\pi_4$. In HEGY it is shown that certain $\pi_j$ are zero in cases where the corresponding roots are on the unit circle. Hence testing for the significance of the $\pi_j$ implies testing for seasonal and nonseasonal unit roots. There are no seasonal unit roots when $\pi_2$, $\pi_3$ and $\pi_4$ are all different from zero. If $\pi_1 = 0$, the presence of the nonseasonal unit root 1 cannot be rejected. If $\pi_2 = 0$, the seasonal unit root $-1$ cannot be rejected. Critical values for the various test statistics for the $\pi_j$, $j = 1, 2, 3, 4$, as well as more details on, e.g., the construction of (3.5), are given in HEGY. The critical values in that paper concern $t$ tests for the individual $\pi_j$ parameters and an $F$ test for the joint significance of $\{\pi_3, \pi_4\}$. Ghysels, Lee and Noh (1994) extend HEGY by proposing $F$ tests for the joint significance of $\{\pi_1, \pi_2, \pi_3, \pi_4,\}$ and of $\{\pi_2, \pi_3, \pi_4,\}$[(9)]. Asymptotics for the test statistics can be found in HEGY and Ghysels, Lee and Noh (1994). In Appendix 4 (left panel) we display some critical values which are relevant for an empirical analysis of our six USA series.

In Table 4, we report the results of the application of the HEGY method to test for nonseasonal and seasonal unit roots in six quarterly macroeconomic variables for the USA. We find that Durables, Services and Money only have a nonseasonal unit root, that Consumption and Production additionally have the seasonal unit root $-1$, and that Nondurables should be transformed using the $\Delta_4$ filter. In sum, three of the six variables need to be first order differenced, and may thus be

**Table 4.** Testing for seasonal and nonseasonal unit roots in quarterly data. The test equation is (3.5). Critical values are given in Appendix 4. The data concern the USA

| | Test statistics | | | | | Model[(1)] | |
|---|---|---|---|---|---|---|---|
| Variable | $t(\pi_1)$ | $t(\pi_1)$ | $F(\pi_3, \pi_4)$ | $F(\pi_2, \ldots, \pi_4)$ | $F(\pi_1, \ldots, \pi_4)$ | $n$ | lags |
| Consumption | −1.058 | −2.109 | 12.965*** | 9.942*** | 7.753*** | 172 | 1,4 |
| Nondurables | −0.433 | −2.235 | 5.061 | 5.261 | 4.014 | 170 | 1–6 |
| Durables | −3.058 | −4.186 | 32.642*** | 27.362*** | 25.666*** | 175 | 1 |
| Services | −1.442 | −3.777*** | 8.697** | 9.534*** | 7.686** | 171 | 1,4,5 |
| Money | −2.086 | −4.035*** | 21.939*** | 21.136*** | 17.439*** | 131 | 1 |
| Production | −2.520 | −2.079 | 10.938*** | 8.702*** | 8.679*** | 122 | 1,2 |

*** Significant at the 1% level.
** Significant at the 5% level.
(1) The auxiliary regression is estimated using $n$ effective observations, and it can include lags at 1 ('1'), at 1 through 6 ('1–6') or specific lags ('1,4'). All regressions are tested for the absence of residual autocorrelation, ARCH and non-normality using the diagnostics given in Appendix 2. The models for Consumption, Durables, Services, and Production show non-normality in the residuals. ARCH patterns are not absent for Consumption and Durables. There are no obvious indications how to modify the model to obtain white noise residuals.

described using a model like (2.1)[(10)]. For one series, we find the $\Delta_4$ filter to be needed, and for two series $(1-B)(1+B) = (1-B^2)$ suffices to remove seasonal and nonseasonal stochastic trends.

These empirical results very much correspond to the results for different data sets in Osborn (1990), Otto and Wirjanto (1990), Hylleberg, Jørgenson and Sørensen (1993) and Mills and Mills (1992), where often only the nonseasonal unit root is found while in a few cases all four unit roots are found. The results suggest that the $\Delta_1\Delta_4$ and $\Delta_4$ differencing filters often may amount to overdifferencing. Hence, the often observed empirical adequacy of models like (3.2) may in some sense be spurious since then the MA part of the model, loosely speaking, 'repairs' the effect of the superfluous differencing filter on the AR part of the model.

An important question now is to what extent the encountered empirical results are caused by size distortions or low power properties of unit root tests. For example, the simulation experiments in Agiakloglou and Newbold (1992) show that nonseasonal unit root tests show severe size distortions in cases where the data generating process (DGP) is not an AR process (as is usually assumed in practice), but when it also contains a (neglected) MA component. In that case one finds the size of the test to be about 50% instead of 5%, and hence one too often incorrectly rejects the unit root. In terms of seasonal unit root testing, this implies that when the DGP is like (3.2), and one uses (3.5), one may too often find no seasonal unit roots. For example, the HEGY test indicates that a model like (2.1) is useful for M1 in the USA (see Table 4), while the graphs of the recursive estimates of the seasonal dummy parameters in Figure 5 indicate that seasonality does not seem to be constant over time. The ACF for $\Delta_1\Delta_4 x_t$ only has significant values at lags 1 and 4 and after some experimentation we find that an alternative empirical model for Money is (for 130 effective observations)

$$\underset{}{\Delta_1\Delta_4 x_t} = \underset{(0.0010)}{0.0004} + \underset{(0.073)}{0.204}\Delta_1\Delta_4 x_{t-1} + \varepsilon_t - \underset{(0.058)}{0.796}\varepsilon_{t-4}, \tag{3.6}$$

for which $BP(12) = 11.47$, $F_{arch1} = 2.195$, $F_{arch1-4} = 0.997$ and $\chi^2_{norm} = 3.064$, i.e. there seems no obvious misspecification. The roots of the MA polynomial are $\pm 0.945$ and $\pm 0.945i$, and these are very close to the unit disk. In cases where one generates data from (3.6), and applies the HEGY test to $x_t$, the results in Agiakloglou and Newbold (1992) indicate that one would reject the presence of seasonal unit roots, as we do indeed in Table 4. These issues on testing for seasonal unit roots in models like (3.1) suggest an interesting area for further research, i.e. to extend the HEGY type tests to models with MA terms, possibly along the lines of Said and Dickey (1985) or Hall (1989).

The HEGY test approach is in a sense a general-to-simple approach since it investigates the empirical adequacy of filters $(1 - B)$ and $(1 + B)$ against the 'more general' $(1 - B^4)$ filter. An alternative method which focuses on testing for seasonal stability is given in Canova and Hansen (1995). Hylleberg (1995) compares the two approaches in extensive simulation experiments and finds that the approaches appear to be complementary, i.e. one method is useful where the other is not, and *vice versa*. Hylleberg and Pagan (1994) provide a theoretical basis for both approaches within the so-called evolving seasonals model.

### 3.3. Differencing filter selection

The method to test for seasonal unit roots in HEGY assumes the maximum number of unit roots to be equal to 4, i.e. one nonseasonal and three seasonal unit roots. However, if one wants to allow for $\Delta_1\Delta_4$ type filters, one may follow two routes. The first is suggested by Dickey and Pantula (1987) (in the nonseasonal context) and it amounts to, first, applying the HEGY method to the $\Delta_1$ transformed time series, and, if there are no seasonal unit roots for this $\Delta_1 x_t$ series, to apply the HEGY method to the untransformed $x_t$ series[(11)]. Another approach is proposed in Osborn *et al.* (1988) [OCSB] which amounts to estimating the parameters in

$$\beta(\beta)\Delta_1\Delta_4 x_t = \delta_1\Delta_4 x_{t-1} + \delta_2\Delta_1 x_{t-4} + \alpha_0 + \sum_{i=1}^{3}\alpha_i D_{it} + \varepsilon_t, \tag{3.7}$$

where $\beta(B)$ is some autoregressive polynomial. When $\delta_1$ is equal to zero while $\delta_2$ is significantly smaller than 0, the $\Delta_1$ filter can be appropriate and the $\Delta_4$ filter is not. When $\delta_1$ is significantly smaller than 0 and $\delta_2 = 0$, the reverse holds. In case $\delta_1 = \delta_2 = 0$, the $\Delta_1\Delta_4$ filter is appropriate, and when both $\delta_1$ and $\delta_2$ are unequal to zero, the series may not need to be differenced. Again, it seems of interest to extend (3.7) to allow for MA terms.

In practice it seems useful to start with the OCSB method. When the outcome of this method is that at most the $\Delta_4$ filter may be needed, one can proceed with the HEGY approach. This sequence of steps may be particularly useful when one wants to describe time series with increasing seasonal variation. Usually one takes the natural logarithm to remove such increasing variation, although the straightforward use of this log transformation is not beyond discussion, see, e.g., Bowerman, Koehler and Pack (1990) and, earlier, Chatfield and Prothero (1973). A set of models for time series (which are not log transformed) with increasing seasonal variation is

$$\Delta_1 \Delta_4^2 x_t = \alpha_0 + u_t, \tag{3.8}$$

$$\Delta_4^2 x_t = \alpha_0 + \beta_0 t + u_t, \tag{3.9}$$

$$\Delta_1 \Delta_4 x_t = \alpha_0 + \sum_{i=1}^{3} \alpha_i D_{it} + u_t, \tag{3.10}$$

$$\Delta_4 x_t = \alpha_0 + \sum_{i=1}^{3} \alpha_i D_{it} + \beta_0 t + u_t. \tag{3.11}$$

$$\Delta_1 x_t = \alpha_0 + \sum_{i=1}^{3} \alpha_i D_{it} + \beta_0 t + \sum_{i=1}^{3} \beta_i D_{it} t + u_t, \tag{3.12}$$

$$x_t = \alpha_0 + \sum_{i=1}^{3} \alpha_i D_{it} + \beta_0 t + \sum_{i=1}^{3} \beta_i D_{it} t + u_t, \tag{3.13}$$

where $t$ is a deterministic trend variable, $t = 0, 1, 2, 3, \ldots$ One may now start model selection using HEGY type tests between (3.8), (3.9) and (3.10), since $\Delta_1 \Delta_4$ and $\Delta_4^2$ are both nested in $\Delta_1 \Delta_4^2$. In case the $\Delta_1 \Delta_4$ filter appears useful, one may proceed with the OCSB method to select between (3.10) to (3.13), where the auxiliary regression in (3.7) is extended with seasonally varying trends.

To illustrate the effect of taking logs on testing for the appropriate differencing filter, consider the results in Table 5 of the application of the OCSB test method, where we include seasonally varying trends in the auxiliary regression. In Table 5, we apply this extension of the OCSB test to the well-known airline data, as they are given in Box and Jenkins (1970), when we transform the original data using $y_t = (x_t)^\gamma$, for $\gamma \neq 0$, and $y_t = \log x_t$ for $\gamma = 0$, where $\gamma$ ranges from 0 to 1 with steps of 0.1. From the last column of this table, it can be observed that the decision ranges between the $\Delta_1$ and the $\Delta_1 \Delta_{12}$ filter, depending on the value of $\gamma$. The results in this table suggest that the number of unit roots one finds in practice may depend on the data transformation prior to analysis. Typically one applies the log transformation, but Table 5 indicates that different results may emerge for different transformations. It seems that there is much research to be done in this area, although some recent advances have already been made in Granger and Hallman (1991), *inter alia*.

Finally, when testing for seasonal unit roots the current focus is on quarterly

**Table 5.** Selection of the appropriate differencing filter for (transformations of) the Box and Jenkins' airline data $x_t$. The regression model is

$$\phi_3(B)\Delta_1\Delta_{12}y_t = \delta_1\Delta_{12}y_{t-1} + \delta_2\Delta_1 y_{t-12} + \alpha_0 + \sum_{i=1}^{11} \alpha_i D_{it} + \beta_0 t + \sum_{i=1}^{11} \beta_i D_{it} t + \varepsilon_t,$$

where $y_t = (x_t)^\gamma$

| $\gamma$ | | $t(\delta_1)$ | $t(\delta_2)$ | Decision (at 5%) |
|---|---|---|---|---|
| 0.0 (log $x_t$) | | −1.300 | −9.414*** | $\Delta_1$ |
| 0.1 | | −1.303 | −9.208*** | $\Delta_1$ |
| 0.2 | | −1.308 | −8.891*** | $\Delta_1$ |
| 0.3 | | −1.315 | −8.739*** | $\Delta_1$ |
| 0.4 | | −1.324 | −8.486** | $\Delta_1$ |
| 0.5 | | −1.337 | −8.227** | $\Delta_1$ |
| 0.6 | | −1.355 | −7.963* | $\Delta_1\Delta_{12}$ |
| 0.7 | | −1.378 | −7.694* | $\Delta_1\Delta_{12}$ |
| 0.8 | | −1.407 | −7.421 | $\Delta_1\Delta_{12}$ |
| 0.9 | | −1.443 | −7.143 | $\Delta_1\Delta_{12}$ |
| 1.0 ($x_t$) | | −1.486 | −6.859 | $\Delta_1\Delta_{12}$ |
| *Critical values*(1) | 1% | −2.85 | −8.74 | |
| | 5% | −2.11 | −8.04 | |
| | 10% | −1.71 | −7.67 | |

*** Significant at the 1% level.
** Significant at the 5% level.
* Significant at the 10% level.
(1) The critical values are simulated using 25000 Monte Carlo replications from the DGP: $\Delta_1\Delta_{12}x_t = \varepsilon_t$ with $\varepsilon_t \sim N(0,1)$.

and monthly time series, for which the components of the $(1 - B^4)$ or $(1 - B^{12})$ filter can also be used to difference the time series. Indeed, as can be seen from Table 4, one may transform the quarterly USA Consumption and Production data using the $(1 - B^2)$ filter. There are however time series for which the components that correspond to seasonal unit roots do not make much sense like, e.g., the cases where the number of seasons is 5 (daily data) or 13 (four-weekly data). In those cases one may be interested in testing whether the $(1 - B)$, $(1 + B + \cdots + B^{S-1})$, $(1 - B^S)$ or no filter is appropriate. This amounts to a modification of the OCSB or HEGY method, in the sense that one estimates the parameters in the auxiliary regression

$$\beta(B)\Delta_S x_t = \delta_1(1 + B + \cdots + B^{S-1})x_{t-1} + \delta_2\Delta_1 x_{t-(S-1)} + \alpha_0 + \sum_{i=1}^{S-1} \alpha_i D_{it} + \varepsilon_t, \quad (3.14)$$

and checks whether the $\delta_1$ and/or $\delta_2$ are significantly different from zero. Notice that for quarterly data, the $t$-test for $\delta_2$ in the regression model in (3.14) corresponds with the $F$-test for the joint significance of $\pi_2$, $\pi_3$ and $\pi_4$ in (3.5) proposed in Ghysels, Lee and Noh (1994).

### 3.4. Seasonal cointegration

In the previous three subsections, we confined ourselves to the review of developments in univariate time series analysis. Of course, most of this analysis is usually performed prior to the construction of multivariate time series models. Once one has decided on the most appropriate differencing filter, or number of seasonal and nonseasonal unit roots, the next step may be to relate the differenced time series using, e.g., transfer function analysis. Typically, in traditional time series analysis one constructs models for two or more $\Delta_1\Delta_4$ or $\Delta_4$ differenced time series, see, e.g., Mills (1990). However, we have seen above that probably only for a few time series these differencing filters are required, and that more often one may find the presence of only a few seasonal unit roots. Furthermore, it may be that univariate time series need the $\Delta_4$ filter, while a linear combination of $\Delta_1$ transformed series is already stationary. In other words, two time series with each a nonseasonal unit root and a few seasonal unit roots may have such roots in common, i.e. linear combinations of appropriately transformed time series do not possess such unit roots. This calls for an extension of the cointegration concept as defined in Engle and Granger (1987).

Engle *et al.* (1993) [EGHL] propose a test procedure for the presence of seasonal and nonseasonal cointegration relations. Suppose that two time series $x_t$ and $y_t$ have some or all unit roots at nonseasonal and/or seasonal frequencies. When there is cointegration at the zero frequency, i.e. when $x_t$ and $y_t$ have a common nonseasonal unit root, the process $u_t$ defined by

$$u_t = (1 + B + B^2 + B^3)x_t - \alpha_1(1 + B + B^2 + B^3)y_t \tag{3.15}$$

is a stationary process. Seasonal cointegration at the bi-annual frequency $\pi$, corresponding to unit root $-1$, amounts to the stationarity of the process $v_t$, which is defined by

$$v_t = (1 - B + B^2 - B^3)x_t - \alpha_2(1 - B + B^2 - B^3)y_t. \tag{3.16}$$

Finally, seasonal cointegration at the annual frequency $\pi/2$, corresponding to the unit roots $\pm i$, amounts to the stationarity of the process $\omega_t$, defined by

$$\omega_t = (1 - B^2)x_t - \alpha_3(1 - B^2)y_t - \alpha_4(1 - B^2)x_{t-1} - \alpha_5(1 - B^2)y_{t-1}. \tag{3.17}$$

In case all $u_t$, $v_t$ and $\omega_t$ series are stationary, the bivariate time series model for $(x_t, y_t)$ can be written in seasonal cointegration form as

$$\Delta_4 x_t = \mu_1 + \gamma_1 u_{t-1} + \gamma_2 v_{t-1} + \gamma_3 \omega_{t-2} + \gamma_4 \omega_{t-3} + \varepsilon_t, \tag{3.18}$$

$$\Delta_4 y_t = \mu_2 + \nu_1 u_{t-1} + \nu_2 v_{t-1} + \nu_3 \omega_{t-2} + \nu_4 \omega_{t-3} + \kappa_t, \tag{3.19}$$

where $\mu_1$ and $\mu_2$ are intercept terms, and where $\gamma_1$ to $\gamma_4$ and $\nu_1$ to $\nu_4$ are adjustment parameters. The right hand sides of these equations consider the long-run relations between $x_t$ and $y_t$ at the various frequencies. The errors of being out of equilibrium at the zero frequency are corrected by the $\gamma_1$ and $\nu_1$ parameters, at the bi-annual frequency by $\gamma_2$ and $\nu_2$, and at the annual frequency by $\gamma_3$, $\nu_3$, $\gamma_4$ and $\nu_4$.

The test method proposed in EGHL is a two-step method, similar to Engle and Granger's approach to nonseasonal time series. The first step involves the estimation of the $\alpha_1$ through $\alpha_5$ parameters in (3.15), (3.16) and (3.17) via simple regressions, where such regressions may include a constant, seasonal dummies and a trend if necessary, and a test whether the residual processes $\hat{u}_t$, $\hat{v}_t$ and $\hat{\omega}_t$ are stationary. The second step is to replace the $u_t$, $v_t$ and $\omega_t$ processes in (3.18) and (3.19) by their estimated counterparts, and to test the significance of the adjustment parameters. The latter step involves standard asymptotics for the $t$ values for the $\gamma_i$'s and $\nu_i$'s while the first step involves (extensions of the) Engle–Granger (1987) type asymptotics. For example, to test for nonseasonal cointegration, one checks whether $\rho = 0$ in the auxiliary regression

$$(1 - B)\hat{u}_t = \rho\hat{u}_{t-1} + \sum_{i=1}^{p-1} \lambda_i(1 - B)\hat{u}_{t-i} + \varepsilon_t. \tag{3.20}$$

The critical values of this so-called Augmented Dickey–Fuller (ADF) $t$ test for $\rho$ are those tabulated in Engle and Granger (1987), see also MacKinnon (1991) for extended tables. Similarly, to test for seasonal cointegration at frequency $\pi$, one tests whether $\rho = 0$ in the auxiliary regression

$$(1 + B)\hat{v}_t = -\rho\hat{v}_{t-1} + \sum_{i=1}^{p-1} \lambda_i(1 + B)\hat{v}_{t-i} + \varepsilon_t, \tag{3.21}$$

where again one can use the Engle and Granger (1987) critical values. The test for seasonal cointegration at the frequency $\pi/2$ is somewhat more complicated, and we refer the interested reader to EGHL for details on asymptotics and critical values.

To illustrate the EGHL seasonal cointegration method, we consider the USA Consumption and Production series, which both have a nonseasonal unit root and a seasonal unit root at the bi-annual frequency. We analyse these data for a sample from 1960.1–1991.4. (Unreported) HEGY test results reveal that also for this sample, the Consumption series does have the same number of unit roots as that obtained in Table 4. For the zero frequency we find that $\alpha_1$ is estimated as 0.870, with a standard deviation of 0.009, where the auxiliary regression includes a constant but no trend. The $R^2$ of this regression is 0.998, and the Durbin–Watson statistic is 0.081 (for 125 observations). The Augmented Dickey–Fuller test as in (3.19), where we need to include lags at 1, 4 and 5, obtains a value of $-3.467$, which is significant at the 5% level. When we include a trend in the first regression, we obtain an insignificant ADF value. For the bi-annual frequency we find that $\hat{\alpha}_2$ is 0.438 with a standard error of 0.044. The $R^2$ of this regression is 0.973, while the Durbin–Watson value is 2.373. The ADF test, with lags at 1, 4 and 5 obtains a value of $-2.384$, which is not significant at the 10% level, but only at about the 25% level. Hence, the evidence for seasonal cointegration in this case is not strong.

Kunst (1993), Lee (1992) and Lee and Siklos (1992) document empirical evidence in favour of seasonal cointegration. This evidence is obtained using a modification of the Johansen (1988) maximum likelihood method so that it is applicable in case of seasonal cointegration.

### 3.5. Miscellaneous issues

In the previous subsections, we reviewed aspects of traditional time series techniques and recent extensions in the areas of seasonal unit roots, of the selection of differencing filters and of the multivariate extension to seasonal cointegration. In this final subsection, we discuss some additional topics for further research.

The first research area has already been suggested in Section 3.2 and it amounts to extending tests for seasonal and nonseasonal unit roots and for the appropriate differencing filters to models that also include moving average terms. Since near-unity parameters at seasonal lags in the MA part of a model for seasonally differenced time series can describe slowly changing seasonal patterns, such extensions seem particularly useful for seasonal time series.

A second area of research is the derivation of the asymptotic distributions of tests for the appropriate differencing filters when such filters can be, e.g., $\Delta_1\Delta_4$. In fact, $\Delta_1\Delta_4$ assumes the presence of three seasonal unit roots and two nonseasonal unit roots. Hence, a time series which requires such a filter is an $I(2)$ type time series.

Seasonal unit root processes can describe time series with changing seasonal patterns. This is similar to nonseasonal unit root processes which describe time series with a stochastic trend. In the latter case it is well known that a one-time shift in the mean or trend can generate a time series which seems to display unit root behaviour, see Perron (1989). It seems then useful to extend the unit root testing framework to allow for the possibility of such deterministic shifts. This is particularly relevant for specifying multivariate models since one may be inclined to use inappropriate asymptotics in case all processes at hand are assumed to be $I(1)$ while these are in fact $I(0)$ with a break.

In the case of seasonal variables, one may expect tests for seasonal unit roots to be biased towards nonrejection in case one neglects shifts in one or more seasonal means. On the other hand, when one includes additional regressors for these deterministic shifts in the HEGY test equation in (3.5) one may expect the critical values of the test statistics to change, see also Perron (1990). For example, one may consider the auxiliary regression

$$\begin{aligned}\phi_k(B)z_{4t} = \mu_t + \pi_1 z_{1,t-1} + \pi_2 z_{2,t-1} + \pi_3 z_{3,t-2} + \pi_4 z_{3,t-1} \\ + \sum_{i=1}^{4} \delta_i [I_{t>\tau}] D_{st} + \sum_{i=1}^{4} \zeta_i D_{\tau+i} + \sum_{j=1}^{k} \gamma_j D_{\tau+i+j} + \varepsilon_t\end{aligned} \tag{3.22}$$

where $\mu_t$ contains a constant, seasonal dummies and a trend, where the $z_{it}$ are defined below (3.5), where the regression additionally contains four seasonal dummy variables which become effective after time $\tau$, $[I_{t>\tau}]D_{st}$, and where $D_{\tau+i+j}$ are single observation dummy variables which have a value of 1 when $t = \tau + i + j$

$(i = 1, 2, 3, 4,\ j = 1, \ldots, k)$ and a zero elsewhere. Note that the latter variables are included to allow the shift to occur gradually, i.e. (3.22) corresponds to the so-called innovative outlier model. The asymptotic results in Perron (1989, 1990) indicate that the distribution of the test statistics will shift to the left (for one-sided $t$ statistics) and to the right for the $F$ tests for the joint hypotheses on the $\pi_i$ values, $i = 1, 2, 3, 4$. In Appendix 4, we give for a sample of 20 years and a break halfway the sample the critical values for the various HEGY type test statistics, which are obtained via 25000 Monte Carlo replications. Franses and Vogelsang (1995) formally extend the HEGY method in order to allow for changing seasonal means, where the breakpoint is either known or unknown. An extension for the known break case of the Dickey, Hasza and Fuller (1984) test is suggested in Ghysels (1994a).

To illustrate the empirical relevance of allowing for structural shifts in seasonal means, we consider again the USA industrial production series. Similar to Perron (1989) we assume that the oil crisis in 1979.4 may have had a permanent effect on the USA economy. Using the HEGY test regression (3.5), we find that this series has a nonseasonal unit root and a seasonal unit root at the bi-annual frequency, see Table 4. However, when we set $\tau = 1979.4$, and estimate the auxiliary regression in (3.22) with $\Delta_4 x_{t-1}$ and $\Delta_4 x_{t-2}$ (hence $k = 2$), we obtain the test results $t(\pi_1) = -3.202$, $t(\pi_2) = -4.811^{***}$, $F(\pi_3, \pi_4) = 16.736^{***}$, $F(\pi_2, \ldots, \pi_4) = 18.804^{***}$ and $F(\pi_1, \ldots, \pi_4) = 18.139^{***}$, where $^{***}$ denotes significance at the 1% level. This auxiliary regression cannot be rejected using the diagnostic checks for autocorrelation, ARCH and normality. The HEGY test results now indicate that Industrial Production series seems most adequately differenced using the first order differencing filter $\Delta_1$ only while allowing for some seasonal mean shifts. Hence, the unit root at the bi-annual frequency disappears. Obviously, in practice one may not know the timing of the shifts nor the total number of such shifts, but the example in (3.22) indicates that seasonal unit roots may be found in case the possible presence of structural seasonal mean shifts is neglected.

Finally, an issue which is of further research interest for multivariate model building is to restrict the number of parameters to a manageable level. In fact, this is one of the obvious possible drawbacks of modelling seasonality, as opposed to analyzing seasonally adjusted data, since such models may include many parameters to estimate. Indeed, one intercept parameter for SA data compares to four seasonal dummy parameters for unadjusted series. It is therefore relevant to investigate the presence of simplifying structures for multivariate model building like common trends (or seasonal cointegration) and common deterministic seasonality. Reduced rank regression techniques may be useful for this purpose, see Hylleberg and Engle (1995).

## 4. Periodic models

In this section, we review recent developments in the area of univariate and multivariate time series models with periodically varying parameters. In Section 2, when discussing some empirical regularities for seasonal macroeconomic variables, we observed that sometimes it is found that a time series has different properties in different seasons. For example, when comparing the annual changes in the seasonal component with the annual changes in the (official) seasonally adjusted time series in Figure 7 for Canadian Unemployment, we observed that the correlation between these changes seems positive in the first two quarters and negative in the last two quarters. Hence, it seems that the trend and seasons are not only correlated, but also that this correlation varies with the quarter. Furthermore, the results in Table 2, where we report on the seasonal pattern in some consumer expectations, suggest that consumers may have different expectations in different seasons. Finally, Ghysels (1994b) documents that the NBER peaks and troughs do not seem to be equally distributed throughout the year, indicating, e.g, that shocks to the economy because of strikes or oil crises may have a time-varying impact. These observations may indicate that univariate and multivariate time series models with periodically varying parameters in the dynamic part of the models can be useful to describe macroeconomic time series. Periodic time series are often considered in environmetric studies concerning water resources and air pollution, see, e.g., McLeod (1993) and Vecchia and Ballerini (1991) for some useful references. It is only during the last few years that periodic models have been used to describe economic time series, in particular, Osborn (1988), Todd (1990) and Hansen and Sargent (1993) extend economic theories to allow for periodic parameters.

In Section 4.1, we discuss representation issues of a periodic autoregression [PAR]. Most of that material can be extended to models with periodic moving average terms [PARMA], but for the sake of convenience we confine ourselves to PAR models. See Vecchia and Ballerini (1991) for an empirical PARMA(1,1) model. In Section 4.2, we discuss unit roots in PAR models, and we analyse two model selection strategies. We illustrate the methods using the six USA macroeconomic time series we considered in Table 1. We also compare the obtained empirical results with those in Table 4. In Section 4.3 we focus on periodic integration as an alternative to the usual concept of integration. In Section 4.4, we discuss the periodic cointegration model. The main feature of this multivariate model is that equilibrium and adjustment parameters can vary across seasons. Effects of seasonal adjustment on periodic processes is discussed in Section 4.5, together with a few additional miscellaneous issues.

## 4.1. Periodic autoregression

A periodic autoregression of order $p$ [PAR($p$)] can be written as

$$x_t = \mu_3 + \sum_{i=1}^{p} \phi_{is} x_{t-i} + \varepsilon_t, \tag{4.1}$$

where $\mu_s$ and $\phi_{is}$ are parameters that may vary across the seasons. This variation is, at least in principle, unrestricted although there are some cases where particular (combinations of) values correspond to unit roots, as we will show below in

Section 4.2. The $\varepsilon_t$ process can allow for seasonal variation in the error variances. Most forthcoming results, however, do not depend on this extension, and hence we will confine ourselves to (4.1). Since some of the $\phi_{is}$ values can be zero, the $p$ denotes the maximum lag order across the four seasons. Hence, (4.1) not only allows for varying AR length, it also allows for a subset of AR models. The parameters in (4.1) can be estimated by OLS from

$$x_t = \sum_{s=1}^{4} \mu_s D_{st} + \sum_{i=1}^{p} \sum_{s=1}^{4} \phi_{is} D_{st} x_{t-i} + \varepsilon_t, \tag{4.2}$$

where the $D_{st}$ are seasonal dummy variables, see Pagano ( 1978 ) and Troutman (1979). Of course, given that (4.1) allows for a different model in different seasons, the number of observations to estimate each of the $\mu_s$ or $\phi_{is}$ parameters equals $N$, i.e. the number of years in the sample. Furthermore, note that one should always estimate (4.2), even when one suspects that the underlying constant term is equal across seasons. This is easily seen from the PAR(1) model $(x_t - \mu) = \phi_{1s}(x_{t-1} - \mu) + \varepsilon_t$, which can be rewritten as $x_t = \mu_s + \phi_{1s}x_{t-1} + \varepsilon_t$ with $\mu_s = \mu(1 - \phi_{1s})$.

Strictly speaking, the process $x_t$ generated by (4.1) is a nonstationary process since the autocovariance function is not constant over time. It can be useful in some occasions to rewrite (4.1) in a time-invariant representation, see, e.g., Osborn (1991), *inter alia.* Since (4.1) allows for an AR model for each of the seasons, it seems natural to consider the so-called vector of quarters [VQ] representation of (4.1), i.e.

$$\Phi_0 X_T = \mu + \Phi_1 X_{T-1} + \cdots + \Phi_P X_{T-P} + \varepsilon_T, \tag{4.3}$$

where $X_T$ is the $(4 \times 1)$ vector process $(X_{1,T}, X_{2,T}, X_{3,T}, X_{4,T})'$, where $X_{s,T}$ is the observation in quarter $s$ in year $T$. The $\Phi_0, \Phi_1, \ldots, \Phi_P$ are $(4 \times 4)$ parameter matrices, and the $\mu$ and $\varepsilon_T$ are $(4 \times 1)$ vectors containing the stacked $\mu_s$ and $\varepsilon_t$ from (4.1). The order $P$ is related to the order $p$ in (4.1) via $P \leqslant 1 + [p/4]$, where $[\cdot]$ denotes 'integer value of'. An example is the simple PAR(1) process, which can be written in the VQ notation as (4.3) with $\Phi_2 = 0, \ldots, \Phi_P = 0$, and

$$\Phi_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\phi_{2s} & 1 & 0 & 0 \\ 0 & -\phi_{3s} & 1 & 0 \\ 0 & 0 & -\phi_{4s} & 1 \end{bmatrix} \quad \text{and} \quad \Phi_1 = \begin{bmatrix} 0 & 0 & 0 & \phi_{1s} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{4.4}$$

From the many zero values in $\Phi_1$ it can be noticed that a PAR(4) process for $x_t$ can be written as a VQ process of order 1.

Since the number of parameters in a PAR($p$) process (4.1) is $4(p+1)$, which can become quite large for large $p$, and hence since the parameters are estimated using one-fourth of the sample only, it seems useful to test whether a time series $x_t$ has indeed periodically varying dynamics before one analyses a fully parameterized PAR model. Roughly speaking, there are two approaches, i.e. a general-to-simple approach and a simple-to-general approach. The first amounts to estimating the order $p$ of the PAR($p$) model using model selection criteria as $F$ tests for the $\phi_{p+1,s}$ parameters or the well-known Akaike and Schwarz information criteria, and next, to test the joint hypothesis that $\phi_{is} = \phi_i$ for all $i = 1, \ldots, p$ Boswijk and Franses (1996) show that the $F$ statistic for $\phi_{is} = \phi_i$ approximately follows a standard $F(3p, n-4(p+1))$distribution. Notice that this distributional result holds for any $\phi_{is}$ values and hence that it is independent of the number of unit roots in the process $x_t$. In the Monte Carlo experiments in Franses and Paap (1994) it is found that all model selection criteria in the first step lead to useful results, although the $F$ test for the significance of the $\phi_{p+1,s}$ parameters seems preferable. An alternative general-to-simple approach is proposed in Flores and Novales (1994). This approach compares an unrestricted VAR model for $X_T$ with a restricted VAR where all $\phi_{is}$ parameters are equal across $s$.

The simple-to-general approaches start with a nonperiodic AR($k$) model for $x_t$ and check whether the residuals display periodicity. Examples of such tests are given in Franses (1993) and Ghysels and Hall (1993). The LM test in the first study amounts to estimating an AR($k$) model, extracting the residuals $\hat{\varepsilon}_t$, and performing the auxiliary regression

$$\hat{\varepsilon}_t = \sum_{i=1}^{k} \omega_i x_{t-i} + \sum_{s=1}^{4} \psi_s D_{st} \hat{\varepsilon}_{t-1} + u_t, \tag{4.5}$$

and test the significance of the $D_{st}\hat{\varepsilon}_{t-1}$ variables if one wants to investigate periodic autocorrelation at lag 1. A test for seasonal heteroskedasticity can be constructed along similar lines.

To illustrate the use of the first model specification strategy, we estimate PAR($p$) models for the six USA variables. In Table 6 we report the estimate of $p$ for each of

**Table 6.** Testing for periodicity in the autoregressive dynamics of several quarterly observed time series

| Variable | Order PAR | $n$ | $F$ test for periodicity [1] | Diagnostics [2] |
|---|---|---|---|---|
| Consumption | 1 | 179 | $F(3,171) = 69.706^{***}$ | $\chi^2_{norm}$ |
| Nondurables | 4 | 176 | $F(12,156) = 9.797^{***}$ | $\chi^2_{norm}$ |
| Durables | 2 | 178 | $F(6,166) = 3.637^{***}$ | $\chi^2_{norm}, F_{arch1-4}$ |
| Services | 6 | 174 | $F(18,146) = 2.071^{**}$ | $\chi^2_{norm}$ |
| Money | 4 | 132 | $F(12,112) = 2.326^{**}$ | — |
| Production | 2 | 126 | $F(6,114) = 11.299^{***}$ | $\chi^2_{norm}$ |

*** Significant at the 1% level.
** Significant at the 5% level.
[1] The $F$ test for periodicity tests the hypothesis that $\phi_{is} = \phi_i$ for all $s$, $i = 1, .., p$, where $p$ is the order of the periodic autoregression. The $F$ test is asymptotically distributed as $F(3p, n-4-4p)$, where $n$ is the effective sample size.
[2] This column indicates which diagnostics for normality, autocorrelation and ARCH indicate the rejection of the corresponding null hypothesis at the 5% level. Augmentation of the univariate models using additional lags does not result in insignificant diagnostic measures. A '-' means that no diagnostic measure indicates misspecification.

the quarterly variables, the effective sample size and the $F$ test for the joint hypothesis $\phi_{is} = \phi_i$ for all $i$. The last column of Table 6 indicates that the conventional diagnostics do not suggest that the PAR($p$) models are severely misspecified, although the estimated residuals do not display normality for six series. When we compare the results on the lag length for Consumption, Durables and Industrial production with those in Table 4, where many variables can be described using at least an AR(5) model, it can be observed that intra-year variation of the parameters can lead to fewer autoregressive lags. Furthermore, the $F$ test results in the fourth column of the table indicate that for all six series we can reject the null hypothesis of no periodicity at least at the 5% level. This means that there appears to be considerable periodicity in the AR dynamics in these macroeconomic variables.

Before we turn to a discussion of unit roots in PAR($p$) models, we focus briefly on the effects of neglecting periodicity. In Tiao and Grupe (1980) and Osborn (1991) it is shown that any periodic time series has a time-invariant representation, i.e. one can always estimate the parameters in a nonperiodic AR($k$) approximation to a PAR($p$) time series. However, this time-invariant model is a *misspecified* model since it neglects periodicity. The AR($k$) model can be obtained from an inspection of the autocovariances of $x_t$ at lags $j$, say $\gamma_j$, for which it holds that

$$\gamma_j = (1/4) \sum_{s=1}^{4} \gamma_{js}, \tag{4.6}$$

where the $\gamma_{js}$ correspond to the autocovariance function for each of the seasons. Hence, based on $\gamma_j$, one can identify an AR($k$) process. However, if one would apply, e.g., the LM test in (4.5) to the estimated residuals of such an AR($k$) process, this LM test will usually reject the null hypothesis of no periodicity in the residuals. The intrinsic periodicity in $x_t$ cannot disappear, although of course the power of diagnostic tests determines our ability to detect this in practice. Furthermore, as discussed in Tiao and Grupe (1980) and Osborn (1991), the order $k$ in the nonperiodic AR model is likely to exceed the order $p$ in the periodic model. Hence there is a trade-off between the intra-year parameter variation and the number of lags in a nonperiodic AR model. As noted above, this seems to correspond to our results in Table 6 when we compare them with those in Table 4.

### 4.2. Unit roots in periodic autoregressions

The VQ representation in (4.3) is particularly useful for an investigation into the presence of unit roots in $X_T$ and hence $x_t$. In fact. the number of unit roots in $X_T$ equals the number of unity solutions to

$$|\Phi_0 - \Phi_1 z - \cdots - \Phi_P z^P| = 0, \tag{4.7}$$

which is the characteristic equation for (4.3), see, e.g., Lütkepohl (1991). A simple example of (4.7) is again the PAR(1) model in (4.4), for which

$$|\Phi_0 - \Phi_1 z| - 1 - \phi_{11}\phi_{12}\phi_{13}\phi_{14} z = 0. \tag{4.8}$$

Hence, there is a unit root in $x_t$ when $\phi = \phi_{11}\phi_{12}\phi_{13}\phi_{14} = 1$. Note that the maximum number of unit roots in a PAR(1) model equals 1. Furthermore, when all $\phi_{1s} = 1$, the PAR(1) model reduces to the random walk model $x_t = x_{t-1} + \varepsilon_t$, and when all $\phi_{1s} = -1$, it becomes $x_t = -x_{t-1} + \varepsilon_t$. The PAR(1) model nests the AR(1) model with either a nonseasonal unit root or a seasonal unit root at the bi-annual frequency. When $\phi = 1$ and not all $\phi_{1s}$ are equal, the $x_t$ process is called periodically integrated of order 1 [PI], see Osborn *et al.* (1988). In cases of PI, the time series $x_t$ needs to be differenced using the periodically varying differencing filter $(1 - \phi_{1s}B)$. In other words, the stochastic trend is removed using a filter that can be different for different seasons.

One approach to test for the number of unit roots in $X_T$, as well as to test whether such roots are, e.g., seasonal unit roots, is proposed in Franses (1994). It amounts to the application of the Johansen (1988, 1991) maximum likelihood cointegration method to a vector autoregressive model for $X_T$. In the case of the VQ model of order 1, a VAR model for $X_T$ is $X_T = \Phi_0^{-1}\Phi_1 X_{T-1} + \Phi_0^{-1}\varepsilon_T$. The Johansen method considers the estimation of the rank $r$ of the matrix $\Pi = \Phi_0^{-1}\Phi_1 - I$, and, in this case the rank is smaller than 4, i.e. in case $\Pi = \alpha\beta'$, with $\alpha$ and $\beta$ are $(4 \times r)$ matrices, whether certain restrictions on this $\beta$ matrix apply. For example, this $\Pi$ matrix for the PAR(1) process in (4.4) equals

$$\Pi = \Phi_0^{-1}\Phi_1 - I = \begin{bmatrix} -1 & 0 & 0 & \phi_{11} \\ 0 & -1 & 0 & \phi_{11}\phi_{12} \\ 0 & 0 & -1 & \phi_{11}\phi_{12}\phi_{13} \\ 0 & 0 & 0 & \phi_{11}\phi_{12}\phi_{13}\phi_{14} - 1 \end{bmatrix}. \tag{4.9}$$

As expected, given (4.8), the rank of this $\Pi$ matrix is 3 in case $\phi_{11}\phi_{12}\phi_{13}\phi_{14}$ equals 1.

As can be observed from (4.9) and (4.8), in cases where the rank of $\Pi$ is 3, the cointegration vectors can be $(1, -1)$ for pairs of annual time series $X_{i,T}$ and $X_{i-1,T}$ with $i = 2, 3, 4$ in case of a nonseasonal unit root, or they can be $(1, +1)$ in case of the seasonal unit root $-1$, or $(1, -\phi_{1s})$ in cases of periodic integration. It seems natural therefore to test restrictions on the $\beta$ matrix containing the $r$ cointegrating vectors to investigate which differencing filter is most useful for the univariate $x_t$ series. In cases where the rank of $\Pi$ equals 3, one can test the hypothesis $\beta = H\varphi$, where $H$ is a $(4 \times v)$ matrix with $r \leqslant v \leqslant 4$. An example of such a matrix is

$$H = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4.10}$$

When these restrictions cannot be rejected using a $\chi^2(3)$ test statistic, the $(1 - B)$ transformation can be applied to remove the stochastic trend in the $x_t$ series. Similar to (4.10) one can construct matrices which correspond to seasonal unit roots, see Franses (1994).

When the rank of $\Pi$ is 0, there are no cointegrating relations between the annually observed $X_{s,T}$ series and hence one should apply the $(1 - B)$ filter to the $X_T$ vector process, which is equivalent to the $(1 - B^4)$ filter for the $x_t$ series. Notice that no cointegration between the elements of $X_T$ implies that the integrated processes $X_{s,T}$ are not tied together in the long-run. Hence, peaks in series that typically appear in a certain season may show up in another season some years later. In Table 4 we found, using the HEGY test in a nonperiodic AR model, that the USA Consumption Nondurables series appears to require the $(1 - B^4)$ filter to obtain stationarity. In Table 6 we find that the same series can be described using a PAR(4) model, which exactly fits into a VAR(1) model for the $X_T$ series. This VAR(1) model is estimated using 44 annual observations. The rank of the $\Pi$ matrix is estimated to be equal to 2 using the 5% significance level, and equal to 3 using the 15% significance level. Given the small sample, we set $r$ equal to 3. Next, we test the validity of the restrictions in (4.10), and those corresponding to the seasonal unit root $-1$. The last hypothesis is rejected at the 0.1% level, while the first is rejected at the 5% level. In sum, it seems that not $(1 - B)$ but the $(1 - \alpha_s B)$ filter is most appropriate, where $\alpha_1 \alpha_2 \alpha_3 \alpha_4 = 1$ with some $\alpha_s$ values close to unity. Indeed, we find that $\alpha_2$ and $\alpha_3$ are close to unity, but that $\alpha_1 \simeq 1.05$ and $\alpha_4 \simeq 0.92$. Most importantly, however, is the outcome that the number of cointegrating relations between the $X_{s,T}$ series for USA Nondurables Consumption does not seem to be equal to zero as found using the HEGY method, i.e. there seem to be two or more long-run relationships that tie the $X_{s,T}$ series together.

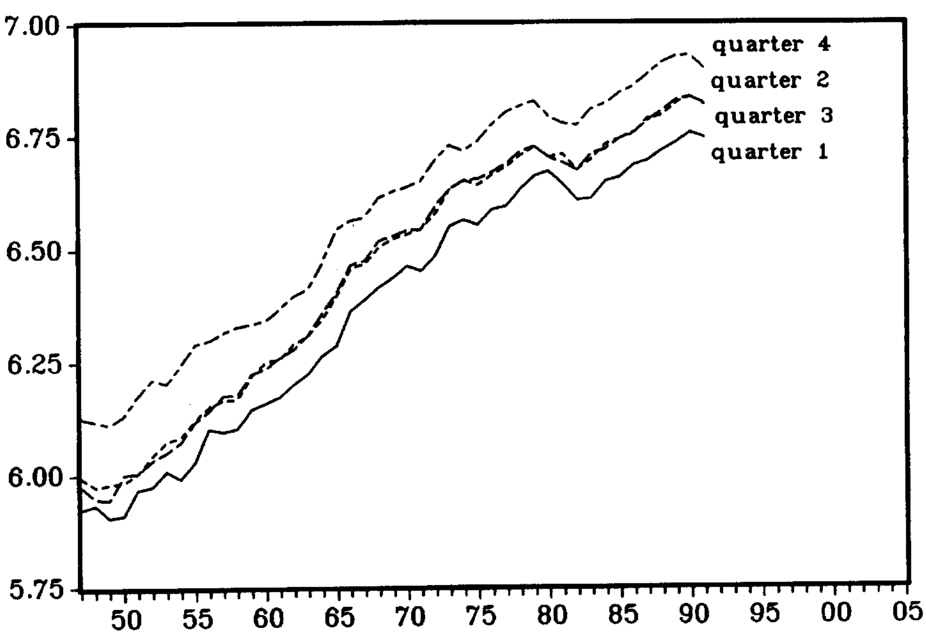


**Figure 8.** Consumption Nondurables in the USA, per quarter

An informal glance at the four graphs for the $X_{s,T}$ series in Figure 8 seems to confirm this empirical finding, i.e. the $X_{s,T}$ series seem to evolve over time in a common pattern, see also Wells (1994) for similar results.

In cases where the PAR process is of order less than 4, the $\Pi$ matrix may contain many zero-valued parameters, see for example (4.9). Hence, unrestricted estimation of this $\Pi$ matrix may decrease the power of the tests for the rank of $\Pi$ For low order PAR models, it seems useful therefore to consider an alternative approach to test for unit roots. Such an approach is given in Boswijk and Franses (1996). This approach amounts to testing the validity of the nonlinear parameter restriction that corresponds to a unity solution of (4.7). For a PAR(1) model this restriction is $\phi_{11}\phi_{12}\phi_{13}\phi_{14} = 1$. Boswijk and Franses (1996) propose the test statistic

$$LR_{\tau} = (\text{sign}(\hat{\phi} - 1))(n \log(RSS_r/RSS_u))^{1/2}, \tag{4.11}$$

where $RSS_r$ and $RSS_u$ are the residual sums of squares in the restricted and unrestricted models, respectively. In the PAR(1) case, $\hat{\phi}$ is the product of the four $\hat{\phi}_{1s}$ parameters in the unrestricted model. In higher order cases, like the PAR(2) model, which can be written as

$$(1 - \alpha_s B)x_t = \mu_s + \beta_{1s}(1 - \alpha_{s-1}B)x_{t-1} + \varepsilon_t, \tag{4.12}$$

where $\alpha_0 = \alpha_4$, the $\hat{\phi}$ in (4.11) corresponds to $\hat{\alpha}_1\hat{\alpha}_2\hat{\alpha}_3\hat{\alpha}_3$. The restricted models can be estimated using NLS. In Boswijk and Franses (1996) it is shown that the $LR_{\tau}$ test statistic follows a Dickey–Fuller distribution under the null hypothesis of a unit root. As usual, the distribution depends on the deterministic terms in the model. Furthermore, in cases where the unit root cannot be rejected, one can test restrictions like $\alpha_s = 1$ for all $s$ using standard $\chi^2$ type test statistics in the restricted model.

An example of the application of the test statistic in (4.11) is given by USA Total Consumption, which can be described using a PAR(1) process. Unrestricted estimation of this model yields $\hat{\phi} = 0.988$ and $RSS_u = 0.029128$, while restricted NLS estimation results in $\hat{\phi}_{11} = 1.048$, $\hat{\phi}_{12} = 0.984$, $\hat{\phi}_{13} = 1.014$ and $\hat{\phi}_{14} = 0.957$ with $RSS_r = 0.029424$. The $LR_{\tau}$ test obtains a value of $-1.345$, and the unit root hypothesis cannot be rejected. The $RSS$ of the PAR(1) model where all $\phi_{1s}$ are restricted to be equal to unity is 0.065323, which can be compared tot $RSS_r$ using an $F(3,172)$ test. The value of this test statistic is 69.95, which rejects the nonseasonal unit root hypothesis at the 0.5% level. In sum, the Total Consumption series appears to be periodically integrated.

To summarize this subsection, a sensible model selection strategy for PAR models may contain four steps. The first is to determine the order $p$ of the PAR, which can be done using standard model selection methods. The second is to test whether the parameters in the estimated PAR process are seasonally varying indeed. This can be done using a standard $F$ test. When the null hypothesis of no periodicity cannot be rejected, one can proceed with testing for the presence of nonseasonal or seasonal unit roots using the HEGY method discussed in Section 3.2. Otherwise, the third step is to apply, e.g, the $LR_{\tau}$ test in (4.11). This test only considers a single unit root versus no unit root. Empirical evidence in Franses and Paap (1994) indicates that this is usually the most interesting case in practice. Finally, the fourth step is to check for nonseasonal or seasonal unit roots in case the $LR_\tau$ test does not reject the null hypothesis.

### 4.3. Aspects of periodic integration

The above model selection strategy has been applied to a large set of macroeconomic time series from the United Kingdom in Franses and Paap (1994) with the main result that several variables are PI, i.e. quite often the $(1-\alpha_s B)$ filter is needed to remove the stochastic trend. Given this empirical finding, it is worthwhile to review some specific properties of PI in this subsection.

A first aspect of PI is that it allows the seasonal fluctuations and the stochastic trend to be interdependent. Given the empirical regularities noted in Section 2 of this paper, this aspect can be useful for modelling macroeconomic time series. This relation between seasons and trends can be noticed from the periodically integrated autoregression [PIAR] of order 1:

$$x_t = \mu_s + \phi_{1s}x_{t-1} + \varepsilon_t \text{ with } \phi_{11}\phi_{12}\phi_{13}\phi_{14} = 1 \tag{4.13}$$

which after some rewriting can be given as

$$X_T = X_0^* + \Lambda\mu T + \Phi_0^{-1}\varepsilon_T + \Lambda\sum_{i=1}^{T-1}\varepsilon_i \tag{4.14}$$

where $X_0^*$ is some function of $X_0$, $T = 1, 2, 3, \ldots$, and where

$$\Lambda = \begin{bmatrix} 1 & \phi_{11}\phi_{13}\phi_{14} & \phi_{11}\phi_{14} & \phi_{11} \\ \phi_{12} & 1 & \phi_{11}\phi_{12}\phi_{14} & \phi_{11}\phi_{12} \\ \phi_{12}\phi_{13} & \phi_{13} & 1 & \phi_{11}\phi_{12}\phi_{13} \\ \phi_{12}\phi_{13}\phi_{14} & \phi_{13}\phi_{14} & \phi_{14} & 1 \end{bmatrix}, \tag{4.15}$$

see Franses (1996) for more details. Note that the matrix $\Lambda$ has rank 1 under the restriction $\phi_{11}\phi_{12}\phi_{13}\phi_{14} = 1$. In fact, the matrix $\Lambda$ distributes the accumulation of shocks in the $(4 \times 1)$ vector process $\Sigma_{i=1}^{T-1}\ \varepsilon_i$. For PI processes the values of the $\phi_{1s}$ are not all equal to 1, and hence the accumulations of shocks have different impacts in different seasons.

An illustrative example may clarify this. Consider again the USA Total Consumption series, with the parameter estimates $\hat{\phi}_{1s}$ as in Section 4.2. The $\Lambda$ matrix for this variable is

$$\hat{\Lambda} = \begin{bmatrix} 1 & 1.017 & 1.003 & 1.048 \\ 0.984 & 1 & 0.987 & 1.031 \\ 0.998 & 1.014 & 1 & 1.046 \\ 0.955 & 0.970 & 0.957 & 1 \end{bmatrix} \tag{4.16}$$

Since the values in the first and third rows of $\hat{\Lambda}$ are highest (compared to the second and fourth rows), the first and third quarter observations are affected by accumulations of shocks more than the other two quarters. Furthermore, the values in the fourth column are highest across columns, and hence the shocks in the fourth quarter have the largest impact[12]. Since the accumulation of the $\varepsilon_t$ process has a time-varying impact on the $x_t$ series, one may observe the seasonal fluctuations to change slowly over time. Otherwise formulated, the persistence of shocks varies across the seasons, see Wells (1994) for related examples.

Indeed, a second aspect of PI processes is that they can describe time series with slowly changing seasonal patterns. Additional to the expression in (4.15), this can also be observed from rewriting, e.g., a PIAR(1) process as

$$x_t = x_{t-4} + \varepsilon_t + \phi_s \varepsilon_{t-1} + \phi_{s-1}\phi_s \varepsilon_{t-2} + \phi_{s-2}\phi_{s-1}\phi_s \varepsilon_{t-3} \tag{4.17}$$

where we dropped one index from $\phi_{is}$ for convenience, and where $\phi_{-1} = \phi_3$[13], which (in case the $\phi_s \simeq 1$) is approximately equal to

$$(1 - B)(1 - B^4)x_t = (1 - \theta B^4)u_t, \text{ with } \theta \simeq 1 \tag{4.18}$$

This model in (4.18) is again similar to the model in (3.2), where it was noted that such a model can describe slowly changing seasonality.

A third aspect of PI is that changes in the seasonal fluctuations are caused by (accumulations of) realizations of the stochastic process $\varepsilon_t$. Hence, in practice one may want to check whether a model where seasonal changes are caused by deterministic shifts can be rejected in favour of a PIAR model. Assuming the presence of a unit root, such an alternative model could be

$$(1 - B)x_t = \mu_s + [I_{t \geq \tau}]\mu_s^* + \sum_{s=1}^{4} [I_{t=\tau+s}]\kappa_s + \varepsilon_t, \tag{4.19}$$

where $\mu_s^*$ are the seasonal mean shifts which become effective at time $t = \tau$. As an example, consider the USA Consumption series and assume (similar as in Section 3.5) that a break occurred in 1979.4. The $RSS$ of the model

$$(1 - \alpha_s B)x_t = \mu_s + [I_{t \geq 1979.4}]\mu_s^* + \sum_{s=1}^{4} [I_{t=1979.4+s}]\kappa_s + \varepsilon_t, \tag{4.20}$$

with $\alpha_1\alpha_2\alpha_3\alpha_4 = 1$ is 0.024427. The $F(8,164)$ test for PI versus (4.20) obtains the 1% significant value of 4.194. The $RSS$ of (4.19) equals 0.046384, which results in an $F(3,164)$ test for (4.19) versus the general model in (4.20) with a value of 49.139. Hence, the general model seems to be preferred over (4.19) and the PI model.

### 4.4. Periodic cointegration

Obviously, for many economic applications, one is interested in modelling more than one time series. Birchenhall *et al.* (1989) introduce a concept of periodic cointegration, which can be viewed as a multivariate extension of periodic integration. In its simplest form for two time series $x_t$ and $y_t$ such a periodic cointegration model can be written as

$$\Delta_4 x_t = \gamma_s(x_{t-4} - \theta_s y_{t-4}) + \varepsilon_t, \tag{4.21}$$

$$\Delta_4 y_t = \kappa_s(x_{t-4} - \theta_s y_{t-4}) + \kappa_t, \tag{4.22}$$

where $\lambda_s$ and $\kappa_s$ are time-varying adjustment parameters and $\theta_s$ are periodically varying cointegration parameters. These models in (4.21) and (4.22) can be extended in a variety of directions. One may include lagged $\Delta_4$ transformed time series with periodic parameters. One may also consider $\Delta_1$ transformations, or even $(1 - \alpha_s B)$ transformations in this case, e.g., $x_t$ is periodically integrated. All possible extensions, at least for a bivariate time series $(x_t, y_t)'$ are nested within a VQ model for the $(8 \times 1)$ vector series $(X_T, Y_T)'$. In Franses (1995) it is shown that this VQ model also nests (variants of) the seasonal cointegration model, discussed in Section 3.4.

There are several possible model selection strategies for, e.g, (4.21). A first is a straightforward extension of the method of Engle and Granger (1987) which amounts to regressing $X_{s,T}$ on a constant and $Y_{s,T}$, giving an estimate of $\hat{\theta}_s$, and estimating the adjustment parameters $\lambda_s$ in the second step where $x_{t-4} - \hat{\theta}_s y_{t-4}$ are added to (4.21). An application to New Zealand consumption and income data is given in Smith (1993). An alternative estimation method is proposed in Boswijk and Franses (1995), which amounts to testing whether the $\lambda_s$ and/or $\kappa_s$ in (4.21) are significant. This method is applied to consumption and income data in Sweden.

A formal model selection strategy for multivariate periodic models for seasonal time series is not yet available. Given the results in Sections 4.1 and 4.2, it seems that a reasonable strategy amounts to a test for the number of unit roots in a periodic VAR, and a sequence of tests for parameter restrictions that can correspond to seasonal cointegration or periodic cointegration. Further research in this area is needed.

## 4.5. Miscellaneous issues

We conclude this section with a discussion of miscellaneous issues for periodic models. The first considers the impact of seasonal adjustment on periodic time series. The second deals with seasonal variation in business cycle turning-points in seasonally adjusted data. Finally, we review some current and future research issues.

Current seasonal adjustment methods usually treat all observations similarly, i.e. the moving average filters as in (2.3) are applied equally to all observations. If a periodic time series process is considered in a seasonal adjustment exercise using the Census X-11 method, it is then easily understood that the intrinsic periodicity in the time series cannot be removed. Of course, in practice where one uses possibly low power tests it may be difficult to detect periodic dynamics in seasonally adjusted time series. Hansen and Sargent (1993) show that neglecting periodicity using SA data may not always be harmful.

For periodically integrated time series like in (4.12), one may expect that seasonal adjustment smooths the $\alpha_s$ values in the differencing filter $(1-\alpha_s B)$ towards 1, but that the periodic variation in the other parameters remains detectable, i.e. (4.12) may become

$$(1-B)x_t^{ns} = \mu_s^* + \beta_{1s}^*(1-B)x_{t-1}^{ns} + \varepsilon_t. \tag{4.23}$$

Consider an empirical example using the Unemployment in Canada series discussed in Section 2.4. A PAR(2) process appears to be adequate for this unadjusted variable, and the $F(6,98)$ test for periodicity in the autoregressive dynamics obtains the highly significant value of 9.791. A PAR(2) can also be fitted to the official $x_t^{ns}$ series, and the $F(6,98)$ test for periodicity still produces a 5% significant value of 2.969. The $LR_\tau$ in (4.11) results in $-1.830$ and $-2.038$, respectively, and hence both models appear to have a unit root. An $F$ test for the adequacy of the $(1-B)$ filter in the $x_t$ and $x_t^{ns}$ series yields the values 3.959 and 0.382, of which the first is highly significant. Hence, the stochastic trend in the $x_t^{ns}$ series can be removed using the $(1-B)$ filter, while a filter like $(1-\alpha_s B)$ is needed for the unadjusted time series.

Obviously, the question now is how harmful it is to seasonally adjust a periodic time series in certain occasions. For example, does it matter for forecasting or business cycle analysis? With respect to the latter question, consider the regression proposed in Franses and Ooms (1995), which is

$$\Delta_4 x_t^s = \delta_s + \pi_s \Delta_4 x_t^{ns} + \varepsilon_t + \theta_{1s}\varepsilon_{t-4} + \cdots + \theta_{qs}\varepsilon_{t-4q}, \tag{4.24}$$

where all parameters are allowed to vary across the seasons, and where the MA residual process is included to whiten the errors. This MA process can also be different across seasons, since the variation in $x_t^s$ may depend on the season. When the $\pi_s$ parameters are significant, one can conclude that the $x_t^s$ process is over- or underestimated, depending on the direction of the trend $\Delta_4 x_t^{ns}$. As an example, consider again the Canadian Unemployment series. When we set $q$ equal to 3, 4, 5, and 1 in the quarters 1 to 4, respectively, where this decision is based on the $BP$ and $\chi^2_{norm}$ tests, we obtain the following $t$ ratios: $t(\pi_1) = 5.111$, $t(\pi_2) = 3.795$, $t(\pi_3) = -7.623$, and $t(\pi_4) = -9.369$. Evidently, these are all highly significant $t$ values. The results for this unemployment series can then be interpreted as follows. When unemployment increases, one overestimates the seasonal component in quarters 1 and 2, and hence the $x_t^{ns}$ is too small. This implies that one is too optimistic in those seasons. For the last two quarters, the opposite result holds. These empirical results indicate that sometimes changes in the trend are attributed too much to the seasonal component, and hence that one may think the trend is moving upward, while in reality it is moving downward. One important implication of this is that, when relying on SA data, one may assign peaks and troughs to inappropriate months or quarters. Given the underlying periodicity of the Unemployment series, as discussed above, this inappropriate assignment of peaks and troughs may occur with seasonal variation, and hence one may erroneously observe periodicity in business cycle turning-points. In sum, seasonally adjusting periodic processes may yield inappropriate business cycle inference.

There are several issues for further research. As noted earlier, the test for a unit root in PAR models in (4.11) only concerns one versus no unit root. An extension to multiple unit roots is principally straightforward, although the asymptotic distributions may be somewhat more involved. Another research area considers the empirical analysis of periodic VAR models. Such models may involve quite a number of parameters, and hence one may want to search for simplifying structures. Examples of such structures are common trends and common periodic features. Extensions of the methods in Engle and Kozicki (1993) and Vahid and Engle (1993) may then be useful in this respect.

## 5. Concluding remarks

In this paper we reviewed several recent developments in econometric modelling seasonality in economic time series. We first described some empirical regularities in such time series, which seem to indicate the need for relatively sophisticated models to describe such variables. Examples of such regularities are that seasonal patterns do not appear to be constant over time, and that they sometimes change together with changes in the trend. Consumers also appear to face difficulties in disentangling seasonal from cyclical variation. The latter phenomenon, which invalidates the assumption of independence of seasons and trend, suggests that it may be useful to model seasonality explicitly instead of its removal using seasonal adjustment. In Section 3, we reviewed developments in seasonal unit roots models and in multivariate time series models with long-run relations that cancel univariate seasonal stochastic trends. In Section 4, we treated periodic models with an explicit focus on unit roots. In the sections we have suggested several specific research topics. Additional econometric topics include the out-of-sample forecasting performance of more complicated models, and how seasonality can be incorporated in nonlinear models.

The main focus in the present paper is on econometric models for seasonal time series variables. An important next step is to link these models to current economic theories. Apparently, economic agents may take seasonal fluctuations into account when forming expectations and making plans. Given that seasonal fluctuations often appear to vary over time, economic models may incorporate related changing behaviour of the agents. The question is, of course, how economic models can be modified such that their implied statistical counterparts contain, e.g., seasonal unit roots or that these are periodically integrated. One approach is given in Osborn (1988), where consumers are assumed to have a time-varying utility function. Under strong assumptions, this economic model generates a periodically integrated AR(1) model. It seems worthwhile to explore how the adequacy of certain econometric models for seasonal variables can be incorporated in theories on, e.g., labour demand, money supply, production and consumption functions, and real business cycles.

Another important question concerns what causes seasonal fluctuations to change over time? Changing seasonality is likely to be caused by economic behaviour, and it may be that economic agents view shocks that coincidentally occur in the same quarter for seasonal shocks. Further theoretical and empirical research is needed into this area.

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## Notes

1. The interpretation of the $R^2$ value from regression (2.1) is conditional on the empirical adequacy of model (2.1). Typically, empirical studies to be discussed in section 3 find that (2.1) may be improved by allowing some form of stochastic seasonality. Hence, one may not interpret a large $R^2$ value as evidence in favour of *deterministic* seasonality, see Franses, Hylleberg and Lee (1995) for details. Furthermore, the significance of the $R^2$ can only be established when $u_t$ is explicitly modelled as a certain stationary and invertible process.
2. In some empirical work, most of which is summarized in Miron (1994), one proceeds with an additional step in analyzing seasonal patterns. This step involves the regression of $\hat{\sigma}_u$ on a constant and $\hat{\sigma}_{sd}$. In several occasions one then finds that there is a significant positive correlation between $\hat{\sigma}_u$ and $\hat{\sigma}_{sd}$, which is interpreted as that business cycles and seasonal cycles display similar features. This interpretation may critically depend on the adequacy of the auxiliary regressions, see Ghysels (1994a) and Hylleberg, Jørgensen and Sørensen (1993).
3. It is worthwhile to note that nowadays almost all consumer confidence data are published only in *seasonally adjusted* form, also in the USA and Canada.
4. In fact, if one estimates (2.1) recursively while neglecting dynamics in the error process, the recursive estimates for $\delta_s$ are likely to display time-varying behaviour. Only when $u_t$ is a white noise process, such recursive estimates can be meaningfully interpreted.
5. $F_{ar1} = 0.679$, $F_{ar1-4} = 0.708$, $F_{arch1} = 0.770$, $F_{arch1-4} = 0.989$, $\chi^2_{norm} = 2.294$. None of these diagnostic statistics is significant at the 10% level. An explanation of these diagnostics is given in Appendix 2.
6. In fact, if we regress the $\hat{\sigma}_u$ on a constant and $\hat{\sigma}_{sd}$, where the estimated standard deviations are obtained from Table 1, we also find a significant positive relationship between $\hat{\sigma}_u$ and $\hat{\sigma}_{sd}$.
7. Note that such a regression on seasonal dummies was not proposed by Box and Jenkins (1970) since they focused on stochastic trends and seasonality. In Abeysinghe ( 1994 ) it is shown that one has to be careful interpreting the ACF of $(\Delta_1 x_t)^c$, since even time series with stochastic seasonality may seem to be stationary after the regression on seasonal dummies. To overcome such possible ambiguities, we recommend the use of formal tests for the appropriate differencing filter, see sections 3.2 and 3.3.
8. Notice that the MA polynomial in (3.1) contains the roots $-0.009 \pm 0.887i$, $-0.892$, 0.871 and 0.267, of which the first four are quite close to $\pm i$, $-1$ and 1. In section 3.2, we discuss such roots which are close to the unit circle.

9. Extensions of the HEGY procedure to monthly time series are given in Franses (1991) and Beaulieu and Miron (1993).
10. Suppose we compare $\hat{\sigma}_u$ and $\hat{\sigma}_{sd}$ from (2.1) for only those three variables which seem indeed adequately described by (2.1), we still observe a positive relationship. This result suggests that the empirical findings in Miron (1994) may not be caused by neglecting seasonal unit roots, as one may hypothesize.
11. In case the order of an unrestricted AR model for the $x_t$ series does not exceed 4, the application of the HEGY test may too often indicate the presence of a nonseasonal unit root in case there is no such root. This is because moving average filtering of, e.g., a stationary AR(1) time series yields higher valued first order autocorrelations, and hence more often the incorrect impression that there is a nonseasonal unit root.
12. When shocks in certain seasons have more impact than similar shocks in other seasons, it can occur that business cycle turning-points appear to occur more frequently in some seasons than in others. In fact, as we discuss in Section 4.5, seasonal adjustment procedures do not entirely remove periodicity, and hence that such seasonal variation in peaks and troughs may also be found in seasonally adjusted time series.
13. Notice that this expression indicates that a PIAR(1) process can be described using an ARMA(4,3) model in case one neglects the periodicity, see Osborn (1991). In case one considers only nonperiodic AR models for tests for seasonal and nonseasonal unit roots it is clear that, because of the MA component, auxiliary HEGY type regressions need additional lags to whiten the errors.

## Appendix 1: Data

The data we use in this paper are:

\+ The airline data

Source: Box and Jenkins (1970). The data are monthly observed time series for 1949.01–1960.12, and are measured in millions of passengers.

\+ Consumption in the USA

Source: USA Dept. of Commerce, Bureau of Economic Analysis, Washington DC. The data are measured in billions of US dollars, and are deflated using the CPI. The observation period is 1947.1–1991.4. The data concern Total Consumption, Consumption Nondurables, Consumption Durables and Consumption of Services.

\+ Money in the USA

Source: International Financial Statistics. The data are the demand for M1 in the USA, observed for 1959.1–1992.4.

\+ Industrial production in the USA

Source: OECD Main Economic Indicators. The data are Total Manufacturing Industrial Production and they are observed for 1960.1–1991.4.

\+ Unemployment in Canada

Source: OECD Main Economic Indicators. The data are the Total Number of Unemployed (measured in thousands of persons) for the period 1960.1–1987.4.

These data are used in this paper in NSA and SA form, both obtained from the same source.
All data can be obtained from the author upon request in ASCII or in MicroTSP (version 7.0) format. This statistical package is used for most calculations in this paper.

## Appendix 2: Diagnostic tests

In this paper we use the following diagnostic tests to investigate the adequacy of the estimated empirical models. Detailed accounts of these tests can be found in standard econometric time series books.

$F_{ar1}$: The $F$-version of the Lagrange Multiplier test for first order residual autocorrelation in the estimated residuals.

$F_{ar1-4}$: The $F$-version of the Lagrange Multiplier test for first-to-fourth order residual autocorrelation in the estimated residuals.

$F_{arch1}$: The $F$-version of the Lagrange Multiplier test for first order ARCH patterns in the estimated residuals.

$F_{arch1-4}$: The $F$-version of the Lagrange Multiplier test for first-to-fourth order ARCH patterns in the estimated residuals.

$\chi^2_{norm}$: The $\chi^2(2)$ LM test for residual normality.

$BP(12)$: The Box–Pierce test for the joint significance of the first twelve autocorrelations in the estimated residuals. This test is only used in case the model contains moving average components.

Together with the parameters in various models, these diagnostic measures are calculated using MicroTSP (version 7.0).

## Appendix 3

In this appendix the proposition in HEGY ( pp. 221–222) is reproduced which is necessary to construct the auxiliary regression (3.5)

Proposition:
Any (possibly infinite or rational) polynomial, $\phi(z)$, which is finite valued at the distinct, non-zero, possibly complex points, $\theta_1, \ldots, \theta_p$, can be expressed in terms of elementary polynomials and a remainder as follows:

$$\phi(z) = \sum_{k=1}^{p} \lambda_k \Delta(z)/\delta_k(z) + \Delta(z)\phi^{**}(z), \tag{A.3.1}$$

where the $\lambda_k$ are constants defined by

$$\lambda_k = \phi(\theta_k) \Big/ \prod_{j \neq k} \delta_j(\theta_k), \tag{A.3.2}$$

$\phi^{**}(z)$ is a (possibly infinite or rational) polynomial and

$$\delta_k(z) = 1 - (1/\theta_k)z, \tag{A.3.3}$$

$$\lambda_k = \phi(\theta_k) \Big/ \prod_{k=1}^{p} \delta_k(z), \tag{A.3.4}$$

An alternative form of (A.3.1) is

$$\phi(z) = \sum_{k=1}^{p} \lambda_k \Delta(z)(1 - \delta_k(z))/\delta_k(z) + \Delta(z)\phi^*(z), \tag{A.3.5}$$

where $\phi^*(z) = \phi^{**}(z) + \sum \lambda_k$. From the definition of $\lambda_k$ in (A.3.2) it can be seen that the polynomial $\phi(z)$ will have a root at $\theta_k$ if and only if the corresponding $\lambda_k$ equals zero.

An application of (A.3.5) to an AR polynomial $\phi(B)$ for a quarterly time series results in

$$\begin{aligned}\phi(B) = {} & \lambda_1 B\phi_1(B) + \lambda_2(-B)\phi_2(B) + \lambda_3(-i - B)B\phi_3(B) \\ & + \lambda_4(i - B)B\phi_3(B) + \phi^*(B)\phi_4(B),\end{aligned} \tag{A.3.6}$$

where the $\phi_i(B)$ polynomials are defined by

$$\begin{aligned}\phi_1(B) &= 1 + B + B^2 + B^2 \\ \phi_2(B) &= 1 - B + B^2 - B^3 \\ \phi_3(B) &= 1 - B^2 \\ \phi_4(B) &= 1 - B^4\end{aligned}$$

When the $\lambda_i$ $(i = 1, 2, 3, 4)$ are redefined as

$$\begin{aligned}\lambda_1 &= -\pi_1 \\ \lambda_2 &= -\pi_2 \\ \lambda_3 &= (-\pi_3 + i\pi_4)/2 \\ \lambda_4 &= (-\pi_3 - i\pi_4)/2,\end{aligned}$$

the expression in (A.3.6) can be written as

$$\phi(B) = -\pi_1 B\phi_1(B) + \pi_2 B\phi_2(B) + (\pi_3 B + \pi_4)B\phi_3(B) + \phi^*(B)\phi_4(B), \tag{A.3.7}$$

which yields (3.5).

## Appendix 4: Some critical values for unit root tests

Critical values of HEGY test statistics with and without structural shifts in seasonal constants based on 25000 Monte Carlo replications for quarterly observations. The auxiliary regressions contain a constant, seasonal dummies and a deterministic trend.

The DGP is $\Delta_4 x_t = \varepsilon_t$, with $\varepsilon_t \sim N(0,1)$.

| Statistic | Years | Without shifts: Regression (3.5) 1% | 5% | With shits at $n/2$: Regression (3.21) 1% | 5% |
|---|---|---|---|---|---|
| $t(\pi_1)$ | 20 | −3.97 | −3.37 | −4.28 | −3.64 |
| | 40 | −3.96 | −3.39 | | |
| $t(\pi_2)$ | 20 | −3.41 | −2.81 | −3.85 | −3.25 |
| | 40 | −3.41 | −2.82 | | |
| $F(\pi_3, \pi_4)$ | 20 | 8.86 | 6.57 | 11.76 | 9.01 |
| | 40 | 8.79 | 6.55 | | |
| $F(\pi_2, \pi_3, \pi_4)$ | 20 | 7.86 | 6.03 | 10.78 | 8.37 |
| | 40 | 7.62 | 5.93 | | |
| $F(\pi_1, \pi_2, \pi_3, \pi_4)$ | 20 | 8.26 | 6.47 | 10.96 | 8.66 |
| | 40 | 7.93 | 6.31 | | |

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