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Impulse response functions for periodic integration

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Abstract

A quarterly observed time series is said to be periodically integrated [PI] if the stochastic trend needs to be removed by a seasonally varying differencing filter. In this paper we consider the impulse response functions [IRF] for such a PI time series. © 1997 Elsevier Science S.A.

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1. Introduction

Recently there have appeared several empirical studies which suggest that quarterly observed time series may be usefully described using periodic time series models, see, e.g., Osborn, Smith (1989) and Franses, Paap (1994). The typical feature of the class of periodic autoregressive [PAR] models is that the autoregressive parameters can take different values in different seasons. This periodic variation introduces the possibility that the stochastic trend can be removed using a seasonally varying differencing filter. The time series is then called periodically integrated [PI]. A consequence of the usefulness of a periodic differencing filter is that the stochastic trend is related to the seasonal fluctuations, see Franses (1996). In the latter study it is also found that quarterly observed macroeconomic time series tend to have only a single stochastic trend, which can be removed either by regular first order differences or by periodic differences.

In the present letter we focus on the impulse response functions [IRF] for periodically integrated time series. In Section 2 we briefly discuss periodic integration. In Section 3 we present a simple method to estimate the empirical IRF: In Section 4 we apply our method to quarterly observed industrial production in the USA.

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2. Periodic integration

Consider a quarterly observed time series y_t, where $t = 1, 2, \dots, n$ and consider its corresponding skip-sampled vector series Y_T, which is a (4×1) series with $Y_T = (Y_{1,T}, Y_{2,T}, Y_{3,T}, Y_{4,T})'$, where $Y_{s,T}$ is the observation in season s in year T, where $s = 1, 2, 3, 4$ and $T = 1, 2, \dots, N$. We assume that the $Y_{1,T}$ observations are the y_t data in case t equals $1, 5, \dots, n-3$, the $Y_{2,T}$ observations concern t is $2, 6, \dots, n-2$, and so on. The notion of skip-sampling a (periodic) time series was introduced in Gladyshev (1961), and it is useful for the analysis of stochastic trends in y_t, see Osborn (1991) and Franses (1996).

In case a quarterly series y_t is described by a periodic autoregression of order p [PAR(p)], it can be represented by

$$y_t = \mu_s + \phi_{1s} y_{t-1} + \cdots + \phi_{ps} y_{t-p} + \varepsilon_t, \tag{1}$$

where ε_t is a standard white noise process with variance σ^2. The μ_s is a seasonally varying intercept term. The ϕ_{is} are seasonally varying parameters, where $i = 1, 2, \dots, p$. The parameters in (1) can be estimated using least squares techniques. The regression model then concerns all terms on the right-hand side of (1) after multiplying them by seasonal dummies $D_{s,t}$, for $s = 1, 2, 3, 4$, see, e.g., Pagano (1978).

Before one can analyze the presence of stochastic trends in y_t using extensions of the familiar Dickey–Fuller type test statistics, one should decide on the model order p. The simulation results in Franses, Paap (1994) indicate that an F-type test for the significance of the $\phi_{p+1,s}$ parameters is most useful in selecting between PAR($p+1$) and PAR(p) processes. The empirical applications in that paper and also in Franses (1996) show that in practice it usually holds that $p \leq 4$. Hence, we confine further analysis to such model orders.

To investigate the presence of stochastic trends, it is useful to write (1) in vector notation, i.e. to write the PAR(p) model for y_t (with $p \leq 4$) as an AR(1) model for Y_T:

$$\Xi_0 Y_T = \mu + \Xi_1 Y_{T-1} + \varepsilon_T, \tag{2}$$

where $\mu = (\mu_1, \mu_2, \mu_3, \mu_4)'$ and $\varepsilon_T = (\varepsilon_{1,T}, \varepsilon_{2,T}, \varepsilon_{3,T}, \varepsilon_{4,T})$, with $\varepsilon_{s,T}$ is the observation on the error process in season s in year T. The Ξ_0 and Ξ_1 are (4×4) parameter matrices containing the parameters ϕ_{is} in (1). Since model (2) is a model for annually observed time series, the parameter matrices do not contain seasonally varying parameters. Model (2) can be used to investigate the presence of the common stochastic trend in Y_T by checking the solutions to the characteristic equation $|\Xi_0 - \Xi_1 z| = 0$, see also Lütkepohl (1991).

Boswijk, Franses (1996) study the case where y_t has one stochastic trend. There is a single stochastic trend in y_t when the differencing filter $(1 - \phi_s B)$ with $\phi_1 \phi_2 \phi_3 \phi_4 = 1$ is needed to render (periodic) stationarity, where B is the backward shift operator. Note that this (possibly periodic) filter nests the $(1 - B)$ filter. Boswijk, Franses (1996) show that a useful test is based on a rewritten version of (1), where the $(1 - \phi_s B)$ filter is imposed. For example, the PAR(2) model can be written as

$$y_t - \phi_s y_{t-1} = \mu_s + \psi_s (y_{t-1} - \phi_{s-1} y_{t-2}) + \varepsilon_t, \tag{3}$$

where $\phi_0 = \phi_4$. Obviously, the ϕ_s and ψ_s ($s = 1, 2, 3, 4$) parameters are nonlinear functions of the ϕ_{is}. The 12 parameters in (3), i.e. μ_s, ϕ_s and ψ_s for $s = 1$ to 4, can be estimated using non-linear least squares. A test for a single unit root in the y_t process is given by comparing the residual sums of squares [RSS] of (3) with or without the restriction $\phi_1\phi_2\phi_3\phi_4=1$. Boswijk, Franses (1996) show that a Likelihood Ratio test for this hypothesis follows the square of the familiar Dickey–Fuller distribution. If the hypothesis $\phi_1\phi_2\phi_3\phi_4=1$ cannot be rejected, a next step is to apply a Likelihood Ratio test for the hypothesis that $\phi_s=\phi$ for all s, where ϕ typically is 1. As expected, this test has a standard χ^2-distribution under the relevant null hypothesis. Franses, Paap (1994) apply this test procedure to a large set of UK macroeconomic time series and find that many series are periodically integrated.

3. Computing impulse responses

One of the most important features of PI time series, i.e. time series that need the $(1-\phi_s B)$ filter to remove the stochastic trend, is that the seasonal fluctuations and the stochastic trend are not independent, see Franses (1996), (Chapter 8) for details. In other words, the response to impulse shocks varies over the seasons. A useful measure to document how shocks affect future patterns of periodic time series is the impulse response function [IRF]. In this section we discuss the construction and estimation of such an IRF, where we explicitly take account of the fact that there are three cointegrating relations between the elements of the (4×1) vector Y_T.

In order to derive impulse response functions we need to specify the interesting type of shocks. Since we assume three cointegrating relations, there is a single type of permanent shock and there are three types of transitory shocks. Following Johansen (1991), (1994), the cointegrated system has the following stochastic trend representation

$$\Delta Y_T=\beta_\perp(\alpha_\perp{}'G\beta_\perp)^{-1}\alpha'_\perp\eta_T+C(B)(1-B)\eta_T, \tag{4}$$

where $\alpha_\perp$ and $\beta_\perp$ are (4×1) matrices obeying $\alpha'_\perp\alpha=0$ and $\beta'_\perp\beta=0$, and Δ is the first differencing filter. $C(B)$ is a (4×4) lag polynomial having all roots outside the unit circle. The matrix G is a nonsingular (4×4) matrix, which is of no specific interest here. In fact, the precise form of G can be found in Johansen (1991). For (2), we have $G=I_4$. From (4) we obtain the stochastic trends as

$$\tau_T=\tau_0+\sum_{i=1}^{T}\alpha_\perp{}'\eta_i. \tag{5}$$

Accordingly, the permanent shocks are defined by $\Delta\tau_T=\alpha_\perp{}'\eta_T$.

To estimate $\alpha_\perp$, several methods have been proposed in the literature. First, one can obtain an estimate of $\alpha_\perp$ by solving an eigenvalue problem similar to the one encountered when estimating β, see Johansen (1994). Second, Proietti (1994) presents a formula which allows one to estimate $\alpha_\perp$ using the parameters of the error correction representation of (2).

In case of a single common stochastic trend, however, it is appealing to apply the simpler procedure proposed in Breitung (1994). Since the permanent shock is only identified up to a scalar transformation, one may normalize one element of the vector $\alpha_\perp$ to unity. Letting $\alpha_\perp=(\alpha_{\perp,1}{}',1)'$ and $\alpha=(\alpha'_1,\alpha'_2)'$, where α_1 and α_2 are of order (3×3) and (1×3), respectively, we have

$$\alpha'\alpha_\perp=\alpha'_1\alpha_{\perp,1}+\alpha'_2=0. \tag{6}$$

From this system of equations the vector $\alpha_{\perp,1}$ can be estimated from $\alpha_{\perp,1} = -(\alpha_1')^{-1}\alpha_2'$, by substituting estimates of $\alpha = (\alpha_1', \alpha_2')'$.

The permanent shocks are the driving forces pushing the series along the "attractor set", while the "disequilibrium error" $\beta' Y_T$ reflects the distance to the attractor set, see Johansen (1994). Thus, it is natural to define the transitory shocks ν_T as shocks to the disequilibrium error so that $\nu_T = \beta' \eta_T$. This identification gives the "structural" errors

$$v_T = \begin{bmatrix} \Delta\tau_T \\ \nu_T \end{bmatrix} = \begin{bmatrix} \alpha_\perp{}' \\ \beta' \end{bmatrix} \eta_T + R\eta_T. \quad (7)$$

Using the techniques in Johansen (1995), (page 40), we can derive the impulse response functions for a PAR model of order less than four as

$$\text{IRF of } \Delta\tau_t \rightarrow Y_{T+j}: \quad \beta_\perp(\alpha_\perp{}'\beta_\perp)^{-1} \quad (8)$$

$$\text{IRF of } \nu_T \rightarrow Y_{T+j}: \quad \alpha(\beta'\alpha)^{-1}(I_3 + \beta'\alpha)^j. \quad (9)$$

We observe that the permanent shocks do not affect the equilibrium errors $\beta' Y_T$. Thus, the permanent shocks $\Delta\tau_T$ push the variables along the attractor line so that such shocks will not be "error corrected" by the system. Furthermore, the permanent shock does not induce any short-run dynamics and, thus, the expected time path of the variables is shifted permanently by a certain amount.

4. An application

In this section we illustrate the estimation of the IRF for PI processes for (logs of) the US industrial production index for the sample 1960.1–1991.4. Details of the specification strategy, of estimation results, and of the diagnostic results can be obtained from Breitung, Franses (1995).

Franses (1996) finds that this variable can be adequately described by a periodically integrated AR(2) model. For $\Pi = \Xi_0^{-1}\Xi_1 - I_4$, we obtain

$$\hat{\Pi} = \begin{bmatrix} -1 & 0 & -0.755 & 1.784 \\ 0 & -1 & -0.934 & 1.949 \\ 0 & 0 & -2.047 & 2.113 \\ 0 & 0 & -1.047 & 1.081 \end{bmatrix}$$

and for the cointegration vector for the Y_T process we get

$$\hat{\beta}' = \begin{bmatrix} -0.981 & 1 & 0 & 0 \\ 0 & -1.047 & 1 & 0 \\ 0 & 0 & -0.969 & 1 \end{bmatrix}.$$

Combining these, we obtain via the equality $\alpha = \Pi\beta(\beta'\beta)^{-1}$:

$$\hat{\alpha} = \begin{bmatrix} 1.013 & 0.973 & 1.784 \\ 0 & 0.955 & 1.949 \\ 0 & 0 & 2.113 \\ 0 & 0 & 1.081 \end{bmatrix}.$$

This matrix is used to give $\hat{\alpha}_{\perp} = (0, 0, -0.512, 1)$.

We can now calculate the IRFs with respect to permanent and transitory components and we display these in Fig. 1. From these graphs it can be observed that the IRFs with respect to permanent shocks seem about equal, as well as the IRFs with respect to the first and second equilibrium shock.

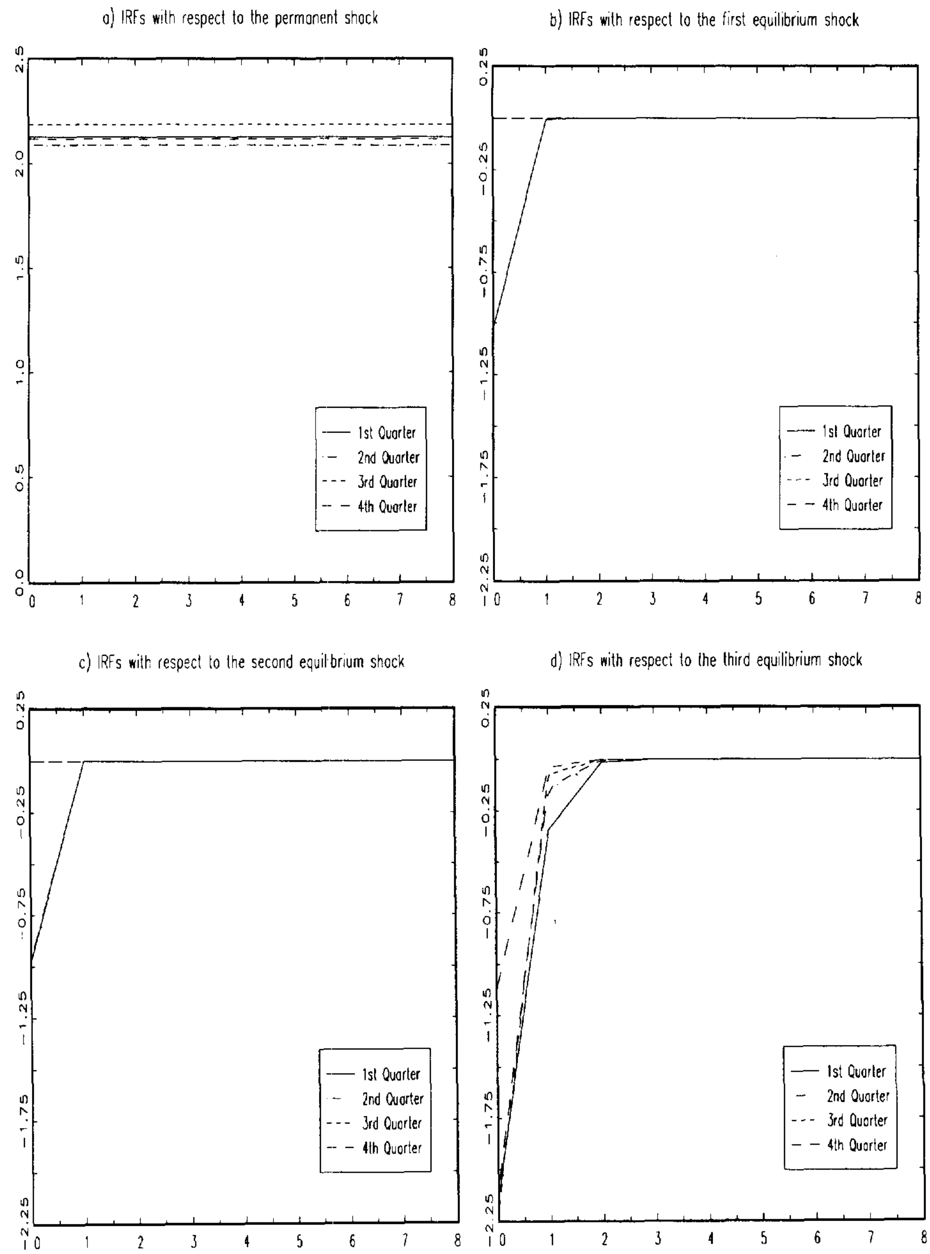

Fig. 1. Impulse response functions to permanent and transitory components in a periodically integrated autoregression of order 2: Industrial production in the USA.

Only for the IRFs with respect to the third equilibrium shock, we observe some differences between the seasons.

These empirical results seem to convey the following useful information. PAR models extend nonperiodic AR models by allowing the dynamic parameters to vary with the seasons. On the other hand, it is often found that the lag order in PAR models is reasonably small, see Franses, Paap (1994). Hence, there appears to be a trade-off between lag length and periodic parameter variation. With respect to the IRFs, it seems that periodically integrated AR models impose restrictions on the lag length, which in turn impose constraints on the permanent component and on the effect of transitory shocks. In summary, although periodic models serve well to approximate the stochastic process of seasonal time series, such models may imply less appealing dynamic structures.

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