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Garch Effects on a Test of Cointegration

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Abstract. This article discusses the effects of GARCH type error processes on the use of the Engle and Granger cointegration test for two variables. Simulation results indicate that (nearly) integrated GARCH processes, as well as GARCH processes that are not covariance stationary, change the critical values. An application to testing for cointegration between spot and futures prices illustrates the practical relevance of using the appropriate critical values.

Key words: cointegration relations, GARCH errors, co-volatile

1. Introduction

A widely recognized stylized fact of empirical financial time series is that they are frequently nonstationary. This implies that one often relies on cointegration techniques to model dynamic econometric relations between these series. Currently there are several approaches to investigate cointegration relationships, the well-known of which are the Johansen (1988) maximum likelihood method, the Phillips (1991) approach and the Engle and Granger (1987) method. Descriptions and applications of these methods can be found in the financial empirical literature. Despite its shortcomings in certain cases, the Engle and Granger (1987) approach is often applied, especially in the case where it is relatively easy to classify the variables with respect to their causality structure. One example is when one wants to test whether spot and futures prices on the same financial asset are cointegrated.

A second statistical time series aspect of many financial series is that they often have conditionally varying variances. A class of models, which is also successful in practice, is the generalized autoregressive conditional heteroskedasticity (GARCH) class, see Bollerslev (1986). An aspect of these models is that time series are assumed to have time-varying variances. This specification incorporates the possibility that volatile periods are succeeded by volatile periods.

The question addressed in this article is whether GARCH patterns in two nonstationary time series can affect the behavior of cointegration tests. In particular we consider the Engle and Granger (1987) test in the bivariate case when the first differences of the series can be described by a GARCH(1,1) process. The Engle and Granger (1987) test is similar to the Dickey and Fuller test for a univariate series. For an investigation of the performance of this test for univariate GARCH processes, see Kim and Schmidt (1992). Although the statistical properties of cointegration methods hold under broad regularity conditions, it may be that GARCH patterns can have an effect on the empirical performance of cointegration tests. When two processes show more volatility in the same periods, and when this volatility can be described by the same model, one may expect that the probability that the levels of the series show similar patterns becomes higher. Hence, the null hypothesis of no cointegration is more easily rejected, even when it is true. In other words, the empirical size of the test is likely to be higher than the nominal size. In Section 2, we verify this intuitive result using Monte Carlo simulations. The results in Kim and Schmidt (1992), Pantula (1988) and Haldrup (1992) for a univariate series indicate that the Dickey-Fuller test is robust to stationary GARCH errors. Analogously, we find that for (nearly) integrated GARCH processes and for GARCH processes that are not covariance stationary, see Nelson (1990), the critical values for the cointegration test shift to the left. Thus, the distribution of the Engle and Granger test changes. Section 3 considers its implications for testing for cointegration between spot and futures prices. We argue that cointegration relations may become spurious. Section 4 concludes with several remarks.

2. A Monte Carlo Study

Consider two nonstationary time series x_t and y_t whose first differences, Δx_t and Δy_t, are stationary. When the univariate time series behavior of the series z_t, where $z_t = x_t, y_t$, is described by a GARCH(1,1) process, one writes

$$z_t = z_{t-1} + \epsilon_t \tag{1}$$

$$\epsilon_t \sim N(0, h_t) \tag{2}$$

$$h_t = \varphi + \alpha\epsilon_{t-1}^2 + \beta h_{t-1}, \tag{3}$$

where φ, α and β are positive constants, for which $\alpha + \beta < 1$, and where ϵ_t is an uncorrelated zero mean variable. Although the normality assumption in (2) is sometimes violated, the set of Equations (1) through (3) is often found to be appropriate in practical occasions, see for example Bollerslev, Chou and Kroner (1992). When $\alpha + \beta = 1$, one obtains a so-called integrated GARCH(1,1) model (IGARCH). This is easily seen by rewriting (3) as

$$\epsilon_t^2 = \pi + (\alpha + \beta)\epsilon_{t-1}^2 - \beta\tau_{t-1} + \tau_t \tag{4}$$

where $\tau_t = \epsilon_t^2 - h_t$. Since the τ_t are uncorrelated, the equation in (4) is similar to the usual ARMA(1,1) time series model, which becomes an IMA(1,1) model when $\alpha + \beta = 1$.

Nelson (1990) gives a concise account of the properties of the GARCH(1,1) process where the φ, α and β can also take values different from those listed below (3). When φ equals zero, the h_t converges to zero almost surely. The unconditional variance of an IGARCH process with $\varphi > 0$ is strictly stationary and ergodic. In that case, the conditional variance converges to the unconditional variance. There are parameter regions where $\alpha + \beta \geq 1$, for which it can be shown that the unconditional process $\epsilon_t h_t^{1/2}$ is strictly stationary but not covariance stationary.

Since the x_t and y_t series are integrated in (1), it may be worthwhile to investigate whether a linear combination of x_t and y_t is stationary. Or, one can check whether x_t and y_t are cointegrated. An often applied method to test for the presence of cointegration is developed in Engle and Granger (1987). This tests the null hypothesis of no cointegration by checking the stationarity of the estimated residuals of the regression

$$x_t = m + ay_t + u_t \tag{5}$$

using the (augmented) Dickey-Fuller test statistic. The latter statistic is the t ratio of ρ in

$$\Delta u_t = \rho u_{t-1} + \eta_t \tag{6}$$

Critical values of this t statistic, t_ρ, which are asymptotically equivalent to those of the t test when lags of Δu_t are included, are given in Phillips and Ouliaris (1990).

A natural question to ask is whether a variance process like (2) and (3) has an effect on testing for cointegration along the lines of (5) and (6). This question is obviously related to the question of whether such processes have an effect on testing for unit roots in a univariate series. In Haldrup (1992) this question is addressed, and one of his conclusions is that processes like (2) and (3) do not very much affect the empirical distribution of univariate statistics when $\varphi > 0$, see also Kim and Schmidt (1992). However, the multivariate case may differ. One may expect that two processes with similar (I)GARCH parameters can be co-volatile, i.e., they are volatile at the same time and can move together in such periods, and hence that the distribution of the t_ρ test in (6) may change.

To verify this intuitive conjecture, we simulated the empirical distribution of the t statistic for many parameter values of α and β in (3). Most of the fractiles of these distributions were very similar to the fractiles reported in Engle and Granger (1987). Without reporting these results in detail, we can say that the effects of stationary GARCH error processes do not affect the empirical distribution of the t test when two random walk processes have equal or different GARCH parameters, or when these parameters do not approach the borderline case of $\alpha + \beta = 1$. This also applies to the case where the φ in (3) equals zero. However, differences between the fractiles emerge when the GARCH processes are of the (near-) integrated type, i.e., when $\alpha + \beta$ is close or equal to 1. Some of the relevant fractiles obtained from 10000 replications of sample size 250 are displayed in Table 1.

The standard error of the critical values is about 0.03. For other sample sizes similar results were obtained. The sample size of 250 roughly corresponds to one year of daily observations as will be used in the following section. Table 1 reports fractiles for various combinations of α and β. We also generated fractiles for intermediate α and β values with steps of 0.01. The first row gives the fractiles without GARCH persistence. The fractiles converge from −3.38 and −3.06 to −4.03 and −3.52, for the 5 and 10% significance level, respectively. Hence the distribution of the t ratio shifts to the left.

To verify whether this is caused by shifts in the numerator or in the denominator of the t_ρ statistic, we conducted some experiments for a limited range of α, β combinations. It showed that the impact of the numerator shift exceeds that of the denominator. Since both components of the t_ρ test vary with α and β, a linear transformation of the t_ρ statistic to correct for IGARCH effects does not seem appropriate. Furthermore, we observed that

Table 1. Garch effects on a test for cointegration, sample size 250, 10000 replications. Fractiles of the standard Dickey-Fuller test, calculated for the residuals of the regression $y_t = c + ax_t + u_t$, where $y_t = y_{t-1} + \epsilon_t$, and $x_t = x_{t-1} + \psi_t$ with $\epsilon_t \sim N(0, h_t)$, $h_t = 1 + \alpha\epsilon_{t-1}^2 + \beta h_{t-1}$ and $\psi_t \sim N(0, k_t)$, $k_t = 1 + \delta\psi_{t-1}^2 + \gamma k_{t-1}$.

$\alpha = \delta$	$\beta = \gamma$	0.01	0.025	0.05	0.10	0.50	0.90	0.95	0.975	0.99
0	0	−3.94	−3.63	−3.38	−3.06	−2.06	−1.00	−0.64	−0.33	0.06
.1	.89	−4.00	−3.65	−3.35	−3.05	−2.03	−0.94	−0.54	−0.11	0.43
.1	.9	−3.97	−3.61	−3.33	−3.02	−2.00	−0.86	−0.42	−0.06	0.63
.2	.79	−4.26	−3.85	−3.55	−3.19	−2.10	−0.98	−0.56	−0.16	0.36
.2	.8	−4.25	−3.86	−3.54	−3.20	−2.08	−0.95	−0.54	−0.08	0.46
.3	.69	−4.58	−4.11	−3.70	−3.31	−2.14	−0.99	−0.59	−0.22	0.30
.3	.7	−4.60	−4.12	−3.71	−3.32	−2.14	−0.98	−0.58	−0.18	0.34
.4	.59	−4.87	−4.26	−3.81	−3.39	−2.15	−0.99	−0.61	−0.27	0.25
.4	.6	−4.88	−4.29	−3.84	−3.40	−2.16	−0.99	−0.60	−0.23	0.31
.5	.49	−5.08	−4.38	−3.92	−3.44	−2.15	−1.01	−0.64	−0.26	0.30
.5	.5	−5.15	−4.43	−3.94	−3.46	−2.16	−1.00	−0.63	−0.27	0.33
.6	.39	−5.15	−4.50	−3.97	−3.46	−2.16	−1.02	−0.65	−0.28	0.26
.6	.4	−5.17	−4.55	−4.00	−3.49	−2.16	−1.02	−0.64	−0.27	0.32
.7	.29	−5.23	−4.53	−3.98	−3.49	−2.16	−1.02	−0.64	−0.29	0.28
.7	.3	−5.32	−4.57	−4.03	−3.51	−2.16	−1.02	−0.63	−0.27	0.30
.8	.19	−5.25	−4.56	−4.01	−3.50	−2.17	−1.02	−0.63	−0.26	0.25
.8	.2	−5.31	−4.58	−4.03	−3.52	−2.17	−1.02	−0.63	−0.27	0.26
.9	.09	−5.25	−4.55	−4.02	−3.51	−2.17	−1.02	−0.65	−0.25	0.21
.9	.1	−5.33	−4.58	−4.03	−3.52	−2.18	−1.02	−0.65	−0.25	0.20

Note: The unconditional processes are strictly stationary and ergodic, see Nelson (1990).

the 5% and 10% critical values do not change identically over the α, β range. This suggests that the distributional theory, which can generate the small-sample results in Table 1, is likely to be cumbersome.

To facilitate the practical use of the results in Table 1, we can however summarize the 5% critical values for the IGARCH process by regressing them on a constant, α, and on squares and cubics of α. For all cases except the case where α and β equal zero, some experimentation yields (for 101 observations)

$$crit = -3.20 - 2.02\alpha + 1.25\alpha^2 \tag{7}$$

where *crit* is the 5% critical values for t_ρ. The corresponding R^2 measure of (7) is 0.980. From Table 1 it can be seen that, though the sum of α and β remains constant, the value of the parameter β is more important in determining the critical values. This could have been expected from the expression in (4).

We also simulated some 5% critical values for the case where the GARCH process is not covariance stationary (region 2 in figure 1 in Nelson, 1990). The results are displayed in Table 2.

As expected, the critical values shift even more to the left. To investigate whether a similar equation as (7) can be obtained, we regressed the critical values (except for the $\alpha = \beta = 0$ case) in Table 2 on a constant, α, β, α^2, β^2. The result is

$$crit = -3.428 - 1.79\alpha \tag{8}$$

Table 2. Garch effects on a test for cointegration, sample size 250, 10000 replications. 5% fractile of the standard Dickey-Fuller test, calculated for the residuals of the regression $y_t = c + ax_t + u_t$, where $y_t = y_{t-1} + \epsilon_t$, and $x_t = x_{t-1} + \psi_t$ with $\epsilon_t \sim N(0, h_t)$, $h_t = 1 + \alpha\epsilon_{t-1}^2 + \beta h_{t-1}$ and $\psi_t \sim N(0, k_t)$, $k_t = 1 + \delta\psi_{t-1}^2 + \gamma k_{t-1}$.

$\alpha = \delta$	$\beta = \gamma$	Critical Value	$\alpha = \delta$	$\beta = \gamma$	Critical Value
0	0.0	−3.34	1.5	0.2	−6.17
1.5	0.1	−5.92	1.6	0.2	−6.41
1.6	0.1	−6.32	1.0	0.3	−5.16
1.7	0.1	−6.48	1.1	0.3	−5.48
1.8	0.1	−6.58	1.2	0.3	−5.62
1.9	0.1	−6.78	0.8	0.4	−4.83
2.0	0.1	−6.93	0.9	0.4	−5.11
2.1	0.1	−7.13	1.0	0.4	−5.45
1.2	0.2	−5.43	0.7	0.5	−4.65
1.3	0.2	−5.81	0.5	0.6	−4.19
1.4	0.2	−6.04	0.4	0.7	−4.19

Note: The unconditional processes are strictly stationary, but not weakly stationary, see Nelson (1990).

with an R^2 of 0.990. Note that the value of β does not seem to be relevant and that the relation between *crit* and α is linear. This suggests that it matters whether a process is IGARCH(1,1) or a GARCH process which is not covariance stationary. In practice, one usually finds examples of the first process.

3. Basis Cointegration at Different Maturities

For practical purposes the implications of the results in the previous section are that the currently applied critical values for cointegration may not always be appropriate. In fact, using these critical values can give the impression that cointegration is present where it is not. This section focuses on an application for which the critical values in Table 1 prove to be relevant.

An empirical illustration of this phenomenon is given for a financial asset (Deutschemark). This application is based on the presumed contemporaneous relationship between spot and futures prices for a single asset. The strength of this so-called basis relationship, i.e., spot minus futures price, measures market integration between both markets. For financial assets this link is established through the covered interest parity theorem. This link acts upon arbitrage on the same asset in different markets, see for the theoretical foundations Garbade and Silber (1983) and Brenner and Kroner (1992). These authors claim that such a relation is quite strong for the near basis where the futures contract expires in the nearby future. For the far basis, however, there seems to be little evidence for such a relation. Bessler and Covey (1991) and Quan (1992) test this claim by means of a cointegration approach where the degree of cointegration is related to the strength of any existing link between two intertemporal prices. Schroeder and Goodwin (1991) extend this study by incorporating nonstorables for which intertemporal prices are assumed independent. These studies find some evidence of the presence of cointegration for the near basis but for the far basis this relation is rejected.

Table 3. Some estimation results for GARCH and cointegration in basis relations at different maturities.

		Spot[1]		Futures[1]		Augmented[2]
Asset		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	Dickey-Fuller
Deutschemarks	Near	0.463 (0.079)	0.483 (0.081)	0.225 (0.049)	0.590 (0.083)	−4.646*
	Far	0.463 (0.079)	0.483 (0.081)	0.211 (0.043)	0.608 (0.082)	−3.533

*Significant at a 5% level.
[1]Standard errors are given in parentheses.
[2]The augmentation is two lags. The appropriate 5% critical value is −3.75, which is found by substituting $\alpha = (0.463 + 0.225)/2$ in (7).

For illustrative purposes we discuss the Deutschemark futures contract (Chicago Mercantile Exchange, Deutschemarks per US dollar) for May 3, 1978 through May 1, 1979 and the corresponding spot (or cash) prices over the same period. Each series consists of 250 observations. A further splitting of the futures contract has been adopted into a nearby contract, i.e., 6-month ahead, and a far contract, i.e., 12-month ahead. Transition from one futures contract to the next is made upon the contract's termination date for the nearby, and it is made at the day when trading in a new (further out) contract is initiated for the far basis.

Like most financial price series they require first differencing to achieve stationarity. For all transformed series a GARCH(1,1) model appears to be adequate, i.e., the usual diagnostic checks do not indicate severe misspecification. The estimation results are given in Table 3.

This table also displays the estimates of the augmented Dickey-Fuller test statistic, calculated for the residuals of the cointegrating regression (6). When we use standard critical values, for example, −3.38 at the 5% level, a cointegration relation seems to exist in the nearby basis, and also in the far basis at a 5% significance level. However, the variance seems to be time-varying, and the $\hat{\alpha}$ and $\hat{\beta}$ and their estimated standard errors indicate a nearly integrated GARCH process to be appropriate. In that case one should preferably compare the DF test statistic with the critical values in Table 1. Then, cointegration in the far basis seems to disappear.

4. Conclusion

The results in this article suggest that there can be empirical occasions in which a detected cointegration relationship is spurious due to a high degree of co-volatility. Therefore, we would suggest the following practical procedure. First, examine the autocorrelation of the squares of the series for any evidence of potential integrated GARCH effects as in Bollerslev (1986). If there is no such evidence, one can rely on the standard tables with critical values for cointegration. If there is IGARCH, estimate α and β as a second step. We admit that estimation of these parameters requires a model for the mean and variance and that they are therefore sensitive to omitted variables in the specification for conditional variance. However, the article clearly demonstrates the sensitivity of a cointegration test to GARCH effects. If the researcher lacks a model for the mean and variance, he must accept the reduced power of the ADF test. The third step is to conduct a cointegration test using the critical values given in this article for the previously determined levels of α and β.

Our study then confirms previous empirical findings in a more robust fashion. The farther from delivery the more futures and cash prices tend to diverge from each other. Two reasons are available to economically explain this phenomenon. Either futures markets do not operate efficiently for longer maturities or the futures contract and the spot contract do not represent the same asset.

As noted by Quan (1992), the rejection of cointegration in the basis for longer maturities implies a limitation to the usefulness of futures markets' long contracts for hedgers. If hedgers require long-term cover, roll-over strategies in nearby contracts offer more basis predictability.

Topics for future research are the analysis of the empirical distribution of the Engle-Granger cointegration test in case of more than two variables. Of course, the evaluation of alternative methods for cointegration may then also be incorporated. Additionally, our study suggests that there is a need for the construction of cointegration methods robust to conditionally heteroskedastic error patterns.

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