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The central issue in the application of econometric and time series analysis (ETS) to market response models is the model-building process. The author proposes a specification strategy for ETS modeling and applies it to the primary demand for beer in The Netherlands.

Primary Demand for Beer in The Netherlands: An Application of ARMAX Model Specification

Systematic knowledge in marketing often is based on marketing generalizations, which can be viewed as "approximate summaries" of the available data on a certain marketing phenomenon (Hanssens, Parsons, and Schultz 1990). Such generalizations, which can provide guidelines for modeling new phenomena, can be given by empirical market response models. Hence, the construction of such models is of importance, especially because of their relevance to marketing theory development.

One useful approach to empirical model building in marketing consists of econometric and time series analysis (ETS). This approach combines the merits of econometrics, which focuses on relations between variables, with those of time series analysis, which specifies the dynamics in the model. In several marketing research studies, a dynamic relationship between time series variables has been investigated by means of ETS methodology (see, e.g., Bass and Pilon 1980; Hanssens 1980; Helmer and Johansson 1977; Leone 1983). The central issue in these applications of ETS is the model specification process. The purpose of this research note is to propose an empirical model-building strategy within the ETS approach.

Suppose most aspects of a market response model have been established, and only the lag structure has yet to be determined. Furthermore, consider the case in which there is more than one candidate explanatory variable. Possible lag specification strategies are transfer function analysis and the double prewhitening method (Hanssens, Parsons, and Schultz 1990). A crucial feature of these methods is the prewhitening of variables—that is, to fit univariate time series models to some or all individual series, and to base further analysis on the estimated white noise residuals.

The application of transfer function analysis is straightforward if one has a single input variable (see, e.g., Helmer and Johansson 1977). First the input series is prewhitened, and then the same filter is applied to the output data. The estimated residuals of the two models are cross-correlated to yield suggestions for the dynamic specification. The situation is more complicated if one has several input variables, because the prewhitening filters can vary across the variables. The method developed by Liu and Hanssens (1982) might offer a remedy, though their initial regression equation may have nonwhite residuals in the case of autocorrelation in the output variable.

With the double prewhitening method, one can draw inference on the relationship between variables by cross-correlating two prewhitened series (Haugh and Box 1977; Pierce 1977). Hanssens (1980) applies this method to the identification of a sales response equation's structural form by relating the sales variable with one candidate variable at a time. However, the double prewhitening approach has been subject to the criticism that spurious (in)dependence of time series can be due to omitted variables (Lütkepohl 1982; Sims 1977). In fact, one must recognize that in this method all test statistics are based on the assumption that the entire information set consists

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The author thanks Herman Bierens for making available the statistical package TSMOD, and Lourens Broersma, Marius Ooms, the Editor, and two anonymous *JMR* reviewers for making several helpful suggestions.

Journal of Marketing Research
Vol. XXVIII (May 1991), 240–5

of only two time series. So, if one can distinguish two or more possibly relevant variables in describing a certain phenomenon, it might be inconvenient to decide whether a variable enters a structural model conditionally on all other variables having zero influence. In that case one might end up with too small a model.

An alternative specification strategy in ETS modeling may be useful. It should ensure that all variables are considered simultaneously for eventual inclusion and that the models are prevented from being spurious regressions because of inappropriate lag structures. An ARMAX model specification can be such a strategy. An ARMAX model is an autoregressive moving average (ARMA) model for an endogenous dependent variable with additional explanatory exogenous variables (X). Bierens (1987) developed an estimation and testing methodology for such a model. In the next section, these results are summarized and a convenient model-building strategy is discussed. Then an ARMAX model for the product class demand for beer in The Netherlands is constructed, estimated, and evaluated to illustrate the specification strategy.

ARMAX MODEL SPECIFICATION STRATEGY

The general form of an ARMAX model (Bierens 1987) can be written as

$$(1) \qquad \left(1 - \sum_{i=1}^{p} \alpha_i L^i\right) \cdot y_t = \mu + \sum_{i=1}^{r} \boldsymbol{\beta}_i' L^i \cdot \mathbf{x}_{t+1} + \left(1 + \sum_{i=1}^{q} \gamma_i L^i\right) \cdot \epsilon_t$$

where L is the lag operator, defined as $L^s z_t \equiv z_{t-s}$, y_t is the variable to be explained, $\mathbf{x}_t$ is a k-dimensional vector of explanatory variables, and the ϵ_t's are white noise disturbances. The expression on the left side in equation 1 is the AR part of the model, the second term on the right side represents the X part, and the third expression is the MA part in the ARMAX concept. If the lag polynomial in front of ϵ_t is invertible, equation 1 can be written as an ARX(∞) model. This can be seen by dividing both sides of equation 1 by the MA polynomial, which implies multiplying with an infinite polynomial in the operator L. Hence, the main advantage of an ARMAX model is that it can allow an infinite lag structure with a small number of parameters.

The first step in building an ARMAX model consists of identifying a suitable ARMA model for the endogenous variable that has been transformed appropriately to obtain stationarity—that is, deciding on the values of p and q. Furthermore, the ARMAX model concept requires that all exogenous variables also show stationary time series patterns. These eventually transformed variables are added to the ARMA model in the second step, in which also the lag length r is determined. Hence, data screening by time series methods is clearly an important feature.

This general ARMAX model can be estimated with nonlinear least squares, the method described by Bierens (1987, sect. 3.2). Given an invertibility assumption, equation 1 can be rewritten into an ARX model, a nonlinear regression model for which iterative estimation methods are available. In the present study, the presample data have been set equal to the mean values of the corresponding observable variables.

The estimated initial ARMAX model can be used to test hypotheses with economic content by imposing restrictions on the estimated parameters, and to estimate this simplified model again. Any simplification exercise, however, should start with testing the specification of the ARMAX model. Here, several tests and evaluation criteria are used. First, the standard error of the residuals (SE) and the adjusted coefficient of determination $\bar{R}^2$ are considered. Second, the normality of the residuals should be tested to check for outlying observations. The Wald test statistic for normality, T_1, follows a $\chi^2(2)$ distribution under the null hypothesis that simultaneously the skewness and the kurtosis of the residual distribution equal 0 and 3, respectively (Kiefer and Salmon 1983). Third, the possible presence of autocorrelation should be investigated. The Wald test statistics, $T_2(n)$, follow a $\chi^2(n)$ distribution under the null hypothesis of no serial correlation of order n. The construction of T_2 is explained by Bierens (1988). Fourth, the eventual occurrence of autoregressive conditional heteroscedasticity of order k, ARCH(k), in the residuals is tested (Engle 1982). This test also can indicate whether there are omitted variables or undetected structural breaks. Finally, a predictive test is performed on a holdout sample.

Simplification of the initial model is based on two criteria, the Wald test statistic for the null hypothesis that some parameters equal zero simultaneously, and the $\bar{R}^2$. The first criterion (like, e.g., the F-test) heavily penalizes the inclusion of too many variables and therefore often prefers the smallest model, whereas the $\bar{R}^2$ is a more conservative criterion for the larger model. It can be shown that when a model is selected with both criteria, the choice is robust to external influences (e.g., omitted variables; see Franses 1989 for some details). Finally, the simplified model is estimated, and obviously it also should pass the evaluation criteria.

PRIMARY DEMAND FOR BEER IN THE NETHERLANDS

Modeling the product class demand for beer has gained some attention in the marketing literature (see, e.g., Bourgeois and Barnes 1979; Franke and Wilcox 1987; Leeflang and Van Duijn 1982). From previous studies one can obtain several suggestions for candidate variables in an empirical model for the primary demand for beer in The Netherlands. The variables considered here are measures of advertising, temperature, price, and consumer expenditures. The ARMAX model specification strategy is used to specify the dynamics of this market response model.

Data

The dataset used in this study covers the years 1978 through 1984 because of the limited availability of observations on the advertising variable, and consists of 42 bimonthly observations on the variables:

Q_t = primary demand for beer (liters/population above age 15),
$TEMP_t$ = average daytime temperature (degrees Celcius),
PB_t = price index of beer (1975 = 100),
AT_t = total advertising expenditures (cents/population above age 15), and
CE_t = consumer expenditures index.

The data on primary demand for beer are collected by the industry and include actual consumption and inventories. These data are obtained from the Central Bureau of Statistics (CBS). The advertising data are obtained from the Bureau of Budget Control. The advertising expenditures include those for radio, television, newspaper, and magazine advertising. The CBS is the source of the price data, which pertain to the final consumer expenditures of households. The price is a weighted average of actually paid prices and hence includes price promotion effects. This dataset is available from the author on request.

The variables Q_t and AT_t are measured per capita with age above 15, because it is assumed that beer consumption is restricted and hence that the advertising efforts are directed to that age category. The variables PB_t, AT_t, and CE_t are deflated with their corresponding price indices (1975 = 100). To smooth the dependent variable, the natural logarithm transformation is applied—that is, $\log Q_t$ is used in the sequel. The use of bimonthly observations has been propagated in the influential paper of Clarke (1976). The effects of the variables on the primary demand are supposed *a priori* to be all positive, with the exception of PB_t, but these effects may be insignificant.

Specification

Figure 1 is a plot of the dependent variable $\log Q_t$. In 1982 there was a major tax increase for alcoholic drinks. The effect can be spotted immediately by the nadir in that period. In the period before this tax change, hoarding is evident. A smaller tax increase in 1984 had a similar impact on $\log Q_t$. As tax changes are announced in advance, their influence on the price can be known exactly. Hence, hoarding effects on $\log Q_t$ might be explained by incorporating a price expectations variable, $PBEXP_t$, that is taken to be a perfect foresight one, PB_{t+1} in the present case.

A second aspect to be addressed is the obvious seasonality of $\log Q_t$. Two possible remedies can be considered, the incorporation of six seasonal dummy D_{1t} variables through D_{6t} in the model or a transformation of $\log Q_t$ into $\Delta_6 \log Q_t$, where $\Delta_i x_t = x_t - x_{t-i}$. The choice between these two options is made by using the autocorrelation function (ACF). The ACFs of $\log Q_t$, $\Delta_6 \log Q_t$, and $R\log Q_t$, the last being the residual vector after regressing $\log Q_t$ on the six dummy variables, are reported in Table 1.

One can see that the sixth- and twelfth-order autocorrelations of $\log Q_t$ are highly significant, that the first- and sixth-order autocorrelations of $\Delta_6 \log Q_t$ approach −.5 and hence indicate overdifferencing, and that $R\log Q_t$ has no significant autocorrelations, though the first is rather large. One therefore can conclude that $\log Q_t$ without its deterministic part is a stationary variable. For equation 1 this implies that $p = 6$, whereas α_1 through α_5 are restricted to zero, that μ is replaced by six seasonal dummy variables, and that q is set equal to 1, respectively.

Figure 1
PRIMARY DEMAND FOR BEER IN THE NETHERLANDS, 1978–1984

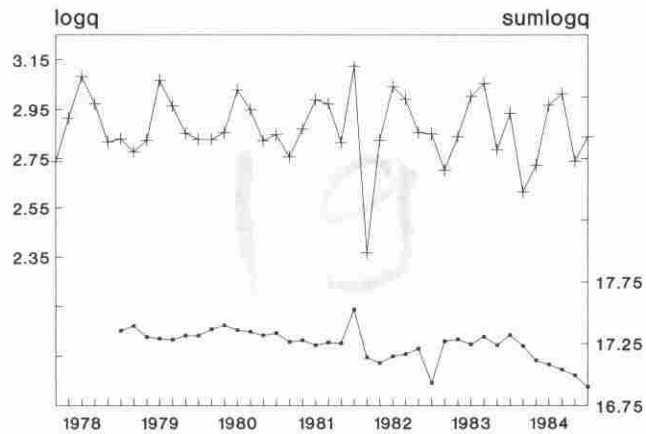


Table 1
AUTOCORRELATION FUNCTIONS OF SOME VARIABLES

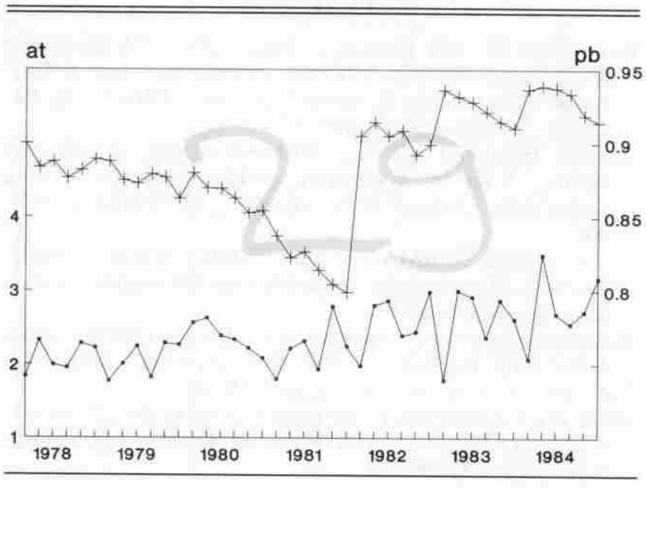
k	$\log Q_t$	$\Delta_6 \log Q_t$	$R\log Q_t$	PB_t	AT_t	CE_t
1	−.046	−.406	−.296	.790	.153	.930
2	−.277	−.021	.026	.589	.204	.872
3	−.278	.014	−.071	.406	.529	.816
4	−.179	.016	.101	.255	.138	.745
5	.022	.262	.077	.176	.046	.669
6	.556	−.494	−.047	.060	.400	.603
7	.106	.156	.029	−.043	.048	.539
8	−.208	.025	−.019	.178	.015	.465
9	−.238	−.037	−.080	.253	.294	.394
10	−.155	.048	.124	−.279	.041	.311
11	−.021	−.089	.038	−.287	−.040	.236
12	.483	.045	−.032	−.256	.300	.183
SD[a]	.154	.167	.154	.154	.154	.154

[a]Standard deviation of the autocorrelations. An autocorrelation coefficient ρ_k is significant at a 5% level if the interval $\rho_k \pm 2.SD$ does not include zero.

The detection of possible structural breaks in $\log Q_t$ is another aspect that must be discussed. In Figure 1, the mean of the dependent variable seems to have decreased after the tax shocks. When the plot of the six-period sum of $\log Q_t$ is considered (see also Figure 1), and especially the years 1979 and 1980 in comparison with 1983 and 1984, there seems to be a major drop in the mean for 1984. Therefore a dummy variable $DMEAN_t$, with ones for 1984 and zero for other years, is added to the nonstochastic part of the model.

The variable $TEMP_t$ shows an obvious seasonal pattern. The ACF of the residuals after regression of this variable on D_{1t} through D_{6t} has no significant autocorrelation, and $TEMP_t$ is a stationary variable. The real price of beer PB_t seems to be nonstationary, however (see Figure 2). Its ACF in Table 1 confirms this observation and shows that PB_t might have been generated by an AR(1) process (see Table 1). To investigate the possible presence of a unit root or stochastic trend, the method described by Hylleberg and Mizon (1989) is followed. To remove the effects of an eventual deterministic trend, $\Delta_1 PB_t$ first is regressed on a constant, which yields an estimate for the ratio of the mean $\hat{\mu}$ and the standard deviation of the regression. This ratio is of special importance, because the distribution under the null hypothesis of the test statistic depends on its value. Second, $\Delta_1 PB_t$ is regressed on 1 and PB_{t-1}, giving a t-value for the estimated parameter of the latter variable, to be denoted by τ. The $\hat{\mu}$ for $\Delta_1 PB_t$ is insignificant, and hence the first block of the table with critical values in Hylleberg and Mizon's article applies. The estimated τ equals -2.176, so there is no indication that the null hypothesis of a unit root should be rejected, which implies that the price variable is not stationary. The variables PB_t and $PBEXP_t$ therefore are included in the model in their first difference representations.

Figure 2
TOTAL ADVERTISING EXPENDITURES, *AT* (IN THOUSANDS), AND PRICE, *PB*



The same procedure applies to the variables AT_t and CE_t, of which the first also is displayed in Figure 2. The advertising variable shows some seasonality and CE_t does not (see also their ACFs in Table 1). To prevent deterministic seasonal elements from blurring inference on the presence of a stochastic trend, the residuals of the regression of AT_t on six seasonal dummy variables, RAT_t, are used for unit root testing. The estimate of the constant in the regression of $\Delta_1 RAT_t$ on 1 is insignificantly different from zero. The estimated parameter of RAT_{t-1} in the regression of $\Delta_1 RAT_t$ on 1 and RAT_{t-1} has a t-value of -3.733; hence the unit root hypothesis is rejected and stationarity of AT_t is accepted. For the CE_t variable, the estimated mean is also zero and the t-value equals -1.034, so the variable $\Delta_1 CE_t$ is incorporated in the model.

The variables $TEMP_t$, $\Delta_1 PB_t$, $\Delta_1 PBEXP_t$, and $\Delta_1 CE_t$ are assumed not to influence $\log Q_t$ beyond the present period (i.e., $r = 1$). However, to accommodate lagged advertising influences, AT_{t-1} is included. Note that this is equivalent to setting r at 2 and restricting the parameters for the other lagged variables to zero. The initial general ARMAX model to be estimated, evaluated, and possibly simplified is now completely specified.

$$(2) \quad \log Q_t = \alpha . \mathrm{lo6}Q_{t-6} + \delta_0 . DMEAN_t + \sum_{i=1}^{6} \delta_i \cdot D_{it}$$
$$+ \beta_1 \cdot TEMP_t + \beta_2 \cdot \Delta_1 PB_t$$
$$+ \beta_3 \cdot \Delta_1 PBEXP_t + \beta_4 \cdot AT_t + \beta_5 \cdot AT_{t-1}$$
$$+ \beta_6 \cdot \Delta_1 CE_t + \epsilon_t + \gamma \cdot \epsilon_{t-1}$$

The nonlinear least squares estimation results are given in column 1 of Table 2. All parameter estimates have the expected signs, with the exception of those for $\Delta_1 CE_t$ and AT_{t-1}, both of which in fact do not differ significantly from zero. As these estimates are not the only insignificant ones, model 2 can be simplified.

Evaluation and Simplification

The values of the evaluation criteria for model 2 are reported in Table 2. There is no evidence that the ARMAX specification should be rejected, and hence it is allowed to simplify the model. The Wald test statistic for the restriction that β_1, β_4, β_5, and β_6 equal zero simultaneously yields a value of 3.353, which does not exceed the 95% critical value of the $\chi^2(4)$ distribution. Note that this test encompasses the hypothesis of no advertising influence. To calculate the $\bar{R}^2$, the simplified model, denoted model 2^s, must be estimated. The results for this model on the evaluation criteria are reported in Table 2. From these outcomes it is clear that model 2 can be simplified to model 2^s and that there is no reason to simplify the latter model further.

To gain more confidence in these results, a predictive test should be applied on a holdout sample. However, the dataset used to estimate the models contains only 36 observations, too few to be separated into an estimation

Table 2
ESTIMATION AND EVALUATION RESULTS

	Model 2		*Model 2^s^*	
Parameter	*Estimation results (t-values)*			
α	.1811	(3.201)	.1703	(3.377)
δ_0	−.0610	(−7.363)	−.0557	(−7.141)
δ_1	2.241	(12.87)	2.297	(17.80)
δ_2	2.254	(12.97)	2.341	(16.02)
δ_3	2.424	(12.32)	2.508	(16.11)
δ_4	2.415	(12.00)	2.498	(16.35)
δ_5	2.214	(12.10)	2.303	(16.19)
δ_6	2.298	(12.77)	2.373	(16.07)
β_1	.0014	(.4183)		
β_2	−4.133	(−14.07)	−3.976	(−12.31)
β_3	2.410	(10.21)	2.267	(9.956)
β_4	.00003	(1.423)		
β_5	−.00002	(−.6537)		
β_6	−.1755	(−.7419)		
γ	−.5777	(−3.491)	−.5406	(−3.902)
Criterion[a]	*Evaluation results*			
SE	.0293		.0305	
$\bar{R}^2$	.9293		.9353	
T_1	.0865		.0453	
$T_2(3)$	.4889		.5063	
$T_2(6)$	1.861		1.726	
$T_2(9)$	5.233		6.469	
ARCH(1)	.2959		.8908	
ARCH(6)	2.568		2.919	

[a]Under the null hypotheses of normality, no autocorrelation of order n, and no conditional autoregressive heteroscedasticity of order k, the test statistics T_1, $T_2(n)$ through ARCH(k) follow chi square distributions with 2, n, and k degrees of freedom, respectively.

set and a holdout sample. Furthermore, observations on the advertising variable are limited to those already used. Therefore observations were collected on the primary demand for beer and on the price variables for the period 1985 through 1987, and the root mean squared prediction error (*RMSPE*) was calculated for model 2^s. Also, a Chow test statistic was computed to investigate the stability of the parameters when model 2^s was estimated again on the extended dataset. For this the structural break in 1984 was assumed to be a permanent one, so *DMEAN* has the value 1 for 1985 through 1987. The computed *RMSPE* equals .0555, which compares favorably with the estimated standard error of model 2^s. Furthermore, the Chow test statistic yields the value of 2.006, which does not exceed the 95% critical value of the $F_{18,25}$ distribution, and hence parameter constancy cannot be rejected.

DISCUSSION

The central issue in the application of econometric and time series analysis to empirical market response models is the model-building process. An ARMAX model specification strategy is proposed that addresses the drawbacks of some other model-building strategies. The ease of use of the strategy is illustrated with an application to the product class demand for beer in The Netherlands. The initial general ARMAX model, which incorporates all variables and dynamics simultaneously, is evaluated with an extensive set of checks for model adequacy. The next step is a simplification step, in which a test for restrictions is performed, and the simplified model is evaluated thoroughly.

An obvious limitation of the ARMAX model specification approach is that estimation problems for the unrestricted general model may arise in case of too many candidate variables and/or dynamics. Furthermore, though the simplification step in the illustration seems to be straightforward, one might encounter inadequate simplified models. Considering several sequential simplification steps may then be an option.

The final model for the primary demand for beer contains a lagged endogenous variable, a moving average part, six seasonal dummy variables, a dummy variable for a structural break, and two price variables, one of which is the expected price. Though the quality of the data is limited because, for example, the primary demand does not include only actual consumption, some tentative conclusions with economic content can be drawn. One conclusion is that a tax change seems to be a good instrument to induce a permanent change in the consumption of beer. However, the structural break was modeled with a dummy variable, which may be hazardous (see Broersma and Franses 1990). Possibly the shift in mean is not caused by tax changes, but reflects omitted variables such as lifestyle or health concerns or the substitution of other types of beverage. Furthermore, advertising expenditures do not seem to influence the primary demand for beer in the model. An explanation might be the saturated beer market in which advertising might not be the buyer's major information source. Another possibility is that brand advertising is indeed effective in establishing increases in market shares, but that all such efforts cancel out in the total advertising effect on primary demand.

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Reprint No. JMR282111