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Modeling new product sales; an application of cointegration analysis

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Abstract

This paper considers an econometric model for new product sales, which extends the deterministic Gompertz process by allowing the saturation level to depend on forcing variables. It turns out that this model can be written in error correction form. Given that many marketing variables show nonstationary behavior, one may have to rely on cointegration techniques for proper estimation of its parameters. Empirical specification strategies are proposed and applied to modeling Dutch new car sales.

1. Introduction

The typical pattern of new product sales according to the product life-cycle hypothesis is characterized by periods in which distinct growth rates can be distinguished. Notably for durable products like cars or washing machines, it is commonly observed that sales do not return to its initial value. Further, in many practical cases an asymmetric growth pattern can be observed, i.e. the period of rapidly increasing sales is shorter than the period in which the sales converge to a certain saturation level. This implies that a Gompertz type of process can be useful to forecast new product sales; see e.g., Meade (1984) and Mahajan and Muller (1979) for surveys of market development applications of trend curves.

Properties of the standard Gompertz curve are that a fixed saturation level is assumed and that a deterministic trend is the only explanatory variable. For many marketing applications these two assumptions can be too restrictive. First, consumers may adapt their saturation level in case, e.g., the technical quality of the product changes or when the price of a product is structurally decreased. Second, the assumption that a deterministic trend dominates the sales of a product and that marketing efforts as, e.g., promotion and distribution, are assumed not to be effective, can also be regarded as a limitation of the determinis-

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tic Gompertz process. This calls for an extension of this process, which allows the saturation level to depend on marketing efforts or on other explanatory variables. A model that accounts for these extensions has first been proposed in Chow (1967). In the present paper this model will be analyzed again, although the primary focus will be on estimating the parameters in this model. This renewed interest in estimating the parameters is caused by the fact that the model involves variables which are possibly nonstationary, which implies that standard estimation techniques do not apply. Therefore, the main contribution of the present paper is to suggest appropriate estimation methods for an extended Gompertz model. This model can be relevant in marketing research involving sales response functions. Such functions are not necessarily limited to the level of product sales, but can also cover investigations into the effects of marketing instruments at, e.g., a brand level. The proposed estimation methods may prevent that applied marketing researchers incorrectly assume that their empirical model is an adequate description of a certain product life-cycle phenomenon and use such a model in, e.g., forecasting exercises.

The outline of the paper is as follows. In Section 2 below, I survey the main aspects of the Chow (1967) model. One of these aspects is that it can be written as a partial adjustment model in which the variables are measured in logarithms. Given that marketing variables as sales, advertising, distribution and price can be nonstationary (cf. Leong and Ouliaris, 1991) in the sense that their time series show time-varying means and variances, one cannot rely on standard estimation techniques for proper inference. In that case one may have to consider cointegration analysis; see e.g., Engle and Granger (1987) for a detailed exposition of this concept. In an appendix, I will briefly survey the concepts of nonstationarity, unit roots and cointegration. Several cointegration based estimation methods for the extended Gompertz model will be discussed in Section 3. In Section 4 they will be illustrated with an application to modeling new car sales in The Netherlands. The final section concludes with some remarks.

2. The model

The standard deterministic Gompertz curve for a sales variable X_t is

$$X_t = a \cdot \exp(-b \cdot \exp(-ct)), \tag{1}$$

where a, b and c are positive parameters, with a denoting the saturation level, with b and c characterizing the rate of growth and the point of inflexion, and where t is a deterministic trend variable (see also Meade, 1984 and Franses, 1994). It is not difficult to recognize that this Gompertz curve assumes (i) that the quantity of the existing sales has a positive influence on future X_t and (ii) that the difference between the equilibrium or saturation level a and X_t also has a positive effect. In fact, (1) is the solution of the differential equation

$$\mathrm{d}X_t/\mathrm{d}t = gX_t(\log a - \log X_t),$$

where g is some constant (see Chow, 1967, Eq. 1). These two assumptions make the Gompertz curve useful for marketing applications. The first assumption can be rationalized by the notion that a growing acceptation of the product ensures that more prospective buyers have learned about it. This effects the second assumption since the closer X_t comes to a, the smaller the influence of their difference.

A convenient alternative representation of the Gompertz process, which is already suggested by the differential equation above, is given by taking natural logarithms and by rewriting, i.e.

$$\Delta_1 x_t = (1 - \exp(-c)) \cdot (\log a - x_{t-1}), \tag{2}$$

where lower case letters indicate that the natural logarithm transformation has been applied, and where Δ_1 is the first order differencing filter defined by $\Delta_1 z_t = z_t - z_{t-1}$. Note that (2) follows exactly from (1).

In Chow (1967) it is proposed to model the log of the saturation level, $\log a$, as a time series variable x_t^*, which may depend on explanatory variables as the log of the price of the product, p_t, and the log of income, y_t, i.e.

$$x_t^* = \beta_0 + \beta_1 p_t + \beta_2 y_t, \tag{3}$$

where $\beta_1 < 0$ and $\beta_2 > 0$. Of course, specific applications of this model may require the inclusion of additional factors as, for example, measures for advertising and distribution efforts. Note that the standard interpretation of a constant saturation level does not apply anymore, but that now a certain equilibrium relation is assumed between a varying saturation level x_t^* and the explanatory variables.

For the moment, I assume that the variables on the right hand side of (3) are weakly exogenous explanatory variables (see Engle et al., 1983, for a survey of several exogeneity concepts). Of course, in some marketing applications one may encounter situations in which there is feedback between, e.g., sales and advertising. This would violate the weak exogeneity assumption. Therefore, I will deal with this assumption in more detail below.

A possible alternative interpretation of (3) is that x_t^* is the desired level of product sales, which depends on forcing variables as price and income. In that sense, one may also consider the inclusion of p_{t-1} and y_{t-1} in (3) instead of p_t and y_t. This decision can be based on the sampling interval of the available observations, i.e. model (3) may be an adequate description for annual time series, although for, e.g., monthly time series one may want to include p_{t-i} and y_{t-j} for some positive i and j. Additionally, x_t^* replaces a constant $\log a$, and there are no theoretical objections to replacing this constant by x_{t-1}^*.

Setting $\log a$ in (2) equal to x_t^*, denoting $(1 - \exp(-c))$ as α, substituting (3) in (2), and adding an error term ϵ_t, gives

$$\Delta_1 x_t = \alpha\beta_0 + \alpha\beta_1 p_t + \alpha\beta_2 y_t - \alpha x_{t-1} + \epsilon_t, \qquad (4)$$

which is an estimable partial adjustment model for time series variables in logarithms (see Chow, 1967). For estimation purposes one may assume that ϵ_t is a standard white noise variable, i.e. an uncorrelated zero mean process with constant variance σ^2. A rewritten version of (4) is

$$\Delta_1 x_t = \mu + \phi_1 \Delta_1 p_t + \phi_2 \Delta_1 y_t - \alpha(x_{t-1} - \gamma_1 p_{t-1} - \gamma_2 y_{t-1} - \gamma_0) + \epsilon_t, \qquad (5)$$

where the $x_{t-1} - \gamma_1 p_{t-1} - \gamma_2 y_{t-1} - \gamma_0$ is a zero-mean error correction (EC) variable, reflecting an equilibrium relation, and where the α is the corresponding short-term adjustment parameter. This term is called an error correction term since disequilibrium errors at $t-1$ are corrected at time t in case α is significant. Note that when there is no error correction term, there also will be no Gompertz-like pattern in X_t. In fact, (5) then reduces to a model containing only first differenced time series. In other words, a test for the presence of this EC variable is a test for the adequacy of a Gompertz model.

When (5) is exactly derived from (4), the ϕ_1, ϕ_2, γ_1 and γ_2 parameters in (5) are restricted by the parameters in (4). For example, $\phi_1 = \alpha\beta_1$ and $\gamma_1 = \beta_1$. In practice, these restrictions may be tested by comparing the residual sums of squares of (5) and (4), when (5) is estimated unrestrictedly. The model in (5) may however also emerge in case the current variables p_t and y_t in (3) are replaced by p_{t-1} and y_{t-1} and an empirical model like (4) needs the inclusion of current $\Delta_1 p_t$ en $\Delta_1 y_t$ variables. Of course, specific applications may require the inclusion of any additional lagged $\Delta_1 p_t$ and $\Delta_1 y_t$ variables to remove serial correlation in the error process. This inclusion does not violate the character of the underlying Gompertz type process. Usually, diagnostic tests for systematic patterns in the estimated residuals may guide the practitioner to construct an adequate empirical model. In the next section the construction of such a model will be illustrated with an example.

2.1. A further extension

The extension of the Gompertz curve proposed in Chow (1967), and which is analyzed in the present paper, only considers the saturation level. A further extension of the model is given by allowing the parameter c in (1) and (2) to vary over time. Similar to (3), one can argue that adjustment to equilibria may depend on explanatory variables as the level of the price or the availability of the product. For example, when income rises, and consumers are inclined to modify their desired purchases level, this modification may depend on, e.g., the availability of the product. Hence, one may replace the constant $\alpha = (1 - \exp(-c))$ in (2) and (4) by, e.g., $\alpha_t = \alpha_0 + \psi p_t$. In that case the expression in (4) changes to the nonlinear regression model

$$\Delta_1 x_t = (\alpha_0 + \psi p_t)(\beta_0 + \beta_1 p_t + \beta_2 y_t - \alpha x_{t-1}) + \epsilon_t,$$

the unrestricted version of which includes variables like $p_t y_t$ and $p_t x_{t-1}$. Given the time variation in the adjustment parameter, it may become difficult in practice to detect the equilibrium relationship on the right-hand side using the currently available cointegration based estimation techniques, which will be discussed in the next section. One possible route is of course to apply the cointegration techniques recursively. A detailed analysis of such an extended Gompertz model is however considered to be a research topic which is beyond the scope of the present paper.

2.2. Exogeneity

Until now the focus has been on model (4) with the assumption that p_t and y_t are weakly exogenous variables. Of course, when, e.g., the price-setting behavior of a firm depends on disequilibrium errors, this weak exogeneity assumption does not hold any more. For example, suppose that an equation similar to (4) such as

$$\Delta_1 p_t = \kappa(x_{t-1} - \gamma_1^* p_{t-1} - \gamma_2^* y_{t-1} - \gamma_0^*) + \nu_t, \quad (6)$$

may be added to (4) with ν_t denoting a standard white noise process. The relation $x_{t-1} - \gamma_1^* p_{t-1} - \gamma_2^* y_{t-1} - \gamma_0^*$ again refers to an equilibrium relationship, where κ is the parameter of adjustment to disequilibrium errors. For the sake of completeness, suppose further that

$$\Delta_1 y_t = \lambda + \pi_t, \quad (7)$$

where π_t is again a standard white noise process. The equation in (7) states that log income can be described by a random walk process with drift. The models (6) and (7) can be enlarged, but for the present illustrative purposes, the three equation system containing (4) or (5), (6) and (7) suffices. For example, (6) may include $\Delta_1 y_t$, but this would require complicated estimation procedures. A discussion of these does not contribute to the main points raised in the present paper.

Based on similar arguments as above, it is clear that the model in (6) also reflects a Gompertz type of pattern for P_t. This pattern may or may not be similar to the Gompertz pattern for X_t. Hence, one may face a multivariate Gompertz process. Thus, the equilibrium relation in (6) is not necessarily equal to that in (5) since the values of γ_1^*, γ_2^* and γ_0^* do not necessarily have to be equal to the γ_2, γ_1 and γ_0 in (4). Hence, the three variable system $\{x_t, p_t, y_t\}$ can have two equilibrium relations and hence multiple error correction mechanisms. When (6) holds, the variable p_t is of course not weakly exogenous in the standard sense. This implies that one has to take account of (6) when estimating (5). In the next section, I will discuss several estimation methods for (4) or (5) in more detail, also with respect to the way these methods incorporate equations (6) and (7).

3. Estimation

When the time series for x_t, p_t and y_t are stationary, i.e. when they have means, variances and autocorrelations which do not depend on time, the models in (4) and (5) can be estimated using standard estimation techniques. In case (6) is part of the system, these estimation techniques should account for this equation. Several system estimation methods for stationary time series, like seemingly unrelated regression (SUR), are available in the standard econometric textbooks, and therefore I will not pursue this issue here.

However, as already noted above, a stylized fact for many marketing time series is that they are nonstationary in the sense that they contain a unit root (see e.g., Leong and Ouliaris, 1991). A time series z_t is said to have a single unit root if it must be transformed using the Δ_1 filter to make it stationary. An effect of this invalidation of standard assumptions is that the t ratio for α does no longer follow a standard t distribution, but a distribution that is shifted to the left. Otherwise formulated, what could seem to be a significant parameter, with, e.g., a t ratio of -2.1, turns out to be insignificant in case the variables are nonstationary. Given that (3) assumes that there is an equilibrium relation between the possibly nonstationary variables, one can rely on cointegration analysis to estimate the parameters in (4) and (5) and establish their significance. In Appendix 1, I display a brief survey of concepts as unit roots and cointegration.

In this paper, I illustrate the application of two estimation strategies. The first is the two-step approach proposed in Engle and Granger (1987), and the second is the method proposed in Boswijk (1992a,b). The first method is considered since it is often applied in practice. The second approach is used since it can easily be applied in case one can assume the weak exogeneity of the explanatory variables like p_t and y_t in the model. In case this assumption is valid, this Boswijk approach yields efficient estimates of the parameters. In case of the invalidity of this assumption, e.g., when a system contains an equation like (6), which is not unlikely in some marketing applications, one should consider another estimation method. I will come back to the latter issue at the end of this section.

3.1. Engle and Granger method

The first step in the Engle and Granger (1987) method is to test for cointegration, i.e. to test whether the estimated errors $\hat{u}_t$ from an OLS regression applied to

$$x_t = \gamma_0 + \gamma_1 p_t + \gamma_2 y_t + u_t, \tag{8}$$

are stationary. A relevant test statistic is the Dickey–Fuller test, which considers the t ratio of ρ in the auxiliary regression

$$\Delta_1 \hat{u}_t = \rho \hat{u}_{t-1} + \varphi_t. \tag{9}$$

In case the φ_t process is not a white noise process, one includes lagged $\Delta \hat{u}_t$ in (9). Decisions for this inclusion can be based on diagnostic checks for residual autocorrelation in (9) or on model selection criteria for the order of an autoregressive model for $\hat{u}_t$ (see e.g., Boswijk and Franses, 1992). Critical values of this t test for the hypothesis that the $\hat{u}_t$ process is nonstationary are tabulated in Engle and Yoo (1987) (see Appendix 1). Rejection of this hypothesis means that $\hat{u}_{t-1} = x_{t-1} - \hat{\gamma}_1 p_{t-1} - \hat{\gamma}_2 y_{t-1} - \hat{\gamma}_0$ is a stationary variable, and that the t ratio of $-\alpha$ in the second step, i.e.

$$\Delta_1 x_t = -\alpha \hat{u}_{t-1} + \mu + \phi_1 \Delta_1 p_t + \phi_2 \Delta_1 y_t + \epsilon_t, \tag{10}$$

follows a standard t distribution (see Engle and Granger, 1987).

3.2. Boswijk method

A possible drawback of this two-step method is that the long-run parameters γ_1 and γ_2, and the adjustment parameter α are not estimated simultaneously (see also Appendix 1). An alternative estimation strategy, which considers estimating (5) while abstracting from an equation like (6), and testing for cointegration at the same time is developed in Boswijk (1992a,b). Given that the parameter α is related to the error correction variable which includes x_{t-1}, p_{t-1} and y_{t-1}, a Wald test statistic is constructed to test for the joint significance of these variables. This Wald test statistic follows a nonstandard distribution for which some critical values are given in Appendix 1. Hence, this method starts with the estimation of (5), without imposing restrictions implied by (4), and it compares the residual sum of squares of

$$\Delta_1 x_t = \mu + \phi_1 \Delta_1 p_t + \phi_2 \Delta_1 y_t - \alpha x_{t-1} + \eta_1 p_{t-1} + \eta_2 y_{t-1} + \epsilon_t \tag{11}$$

with that of

$$\Delta_1 x_t = \nu + \phi_1 \Delta_1 p_t + \phi_2 \Delta_1 y_t + \epsilon_t \tag{12}$$

via a Wald test statistic. Note that the drift terms μ and ν in (11) and (12) do not necessarily reflect the same constant term. In fact, μ in (11) also contains the constant term in the equilibrium relation. Rejection of the null hypothesis that the simplification is valid implies that there is cointegration between x_t, p_t and y_t. The cointegration parameters are then estimated as $\hat{\gamma}_1 = \hat{\eta}_1 / \hat{\alpha}$ and $\hat{\gamma}_2 = \hat{\eta}_2 / \hat{\alpha}$ for p_t and y_t, respectively.

The cointegration relation to be estimated in the illustrative Gompertz model discussed in this paper is given in (3). It is clear that the previously constant saturation level is now assumed to be a nonstationary time series. This implies that shocks can have a permanent impact on the level of the 'saturation' (see Appendix 1). In other words, specific marketing efforts may cause permanent changes in the level of the sales, while such 'shocks' do not change the parameters in the system. This implies that the term 'saturation' may not be useful in this case, and it would be more appropriate to assume that there is a desired or equilibrium level of the sales of a new product, which can be nonstationary.

Fig. 1. New car sales, 1960–1988.

3.3. More than one cointegration relationship

Suppose that it is believed that there is only one cointegration relation in the system containing x_t, p_t and y_t, and that this relation is found using the Boswijk method. A test for weak exogeneity, which concerns the consistency and efficiency of the estimators in (11), can now be performed by testing the significance of the error correction variable $x_{t-1} - \hat{\gamma}_1 p_{t-1} - \hat{\gamma}_2 y_{t-1}$ when it is added to the econometric models for $\Delta_1 p_t$ and $\Delta_1 y_t$. Given that there is cointegration, this test can be evaluated using t tests following standard distributions.

However, when p_t is the log of the price of the product, it may well occur that the price-setting behavior of a firm depends on the current or lagged sales and that there may be another cointegration relation in the system containing the three variables x_t, p_t and y_t, which is for example reflected by (6). In case the practitioner suspects that there can be more than one cointegration relation, or that the same cointegration relation shows up in more than one equation, the Johansen and Juselius (1990) approach is more appropriate than the previous two approaches. With this method, which does not require the weak exogeneity of the variables, one can test for the number of cointegration relations and one can evaluate the significance of κ in (6). To save space, I do not give a detailed account of the Johansen and Juselius (1990) method, and refer the interested reader to the relevant literature.

4. Dutch new car sales

To illustrate the merits of the empirical specification strategies discussed in the previous section, consider the annual time series of the Dutch new car sales X_t for 1960–1988 as it is given in Fig. 1.

Because of the decline in the growth, it seems that there is a tendency towards a saturation level, and hence that the X_t series may be described by a Gompertz type of process. Furthermore, I have chosen to measure p_t by the log of the price of petrol after correction for the consumer price index, as a rough proxy for the costs of maintaining a car, and to measure y_t by the log of the Dutch real gross national product. Graphs of these two series are given in Fig. 2.

The observations of all three series are given in Appendix 2. One may now want to include additional variables in the model, as, e.g., the price of a new car or advertising expenditures, but this is not done here for the sake of brevity of exposition.

One may also want to modify the model in order to allow an increase in the technical quality of new cars, since such an increase might effect the speed of adjustment to a certain equilibrium level. In fact, this modification amounts to extending the model, as is discussed at the end of Section 2, by allowing the adjustment parameter to be time-dependent, i.e. α_t. For the same arguments as stated earlier, this involves a non-trivial extension of the cointegration model (5), and an investigation of this modification is beyond the scope of this paper. However, using Chow-type tests as well as recursive estimation methods, I will check whether the parameters in the estimated model have been reasonably constant over time.

The visual evidence from the graphs is that the three individual series are likely to be nonstationary. This suggestion seems verified by the formal unit root test results given in Table 1, where it is indicated that the null hypothesis of a unit root, or equivalently of nonstationarity, cannot be rejected for either series. Hence, we have to rely on the cointegration techniques described in the previous section to estimate a model like (5) for the Dutch new car sales.

The static regression in (8), i.e. the first step of the Engle and Granger (1987) approach, yields

$$x_t = \underset{(0.437)}{-1.048} - \underset{(0.221)}{1.204}p_t + \underset{(0.073)}{1.317}y_t, \tag{13}$$

where standard errors are denoted in parentheses. The value of the t ratio for the estimated ρ as in (9) is -4.296. Comparing this value with the critical values tabulated in Engle and Yoo (1987)

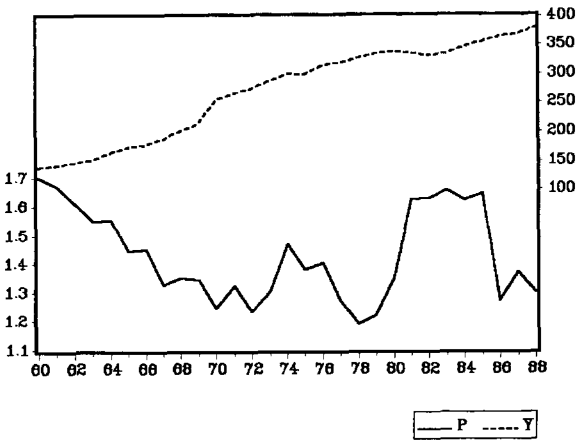

Fig. 2. The price of petrol and national income, 1960–1988.

Table 1
Unit root test results for the univariate series [a]

Model	Sales	Price	Income
Constant, trend	−1.608	−2.289	−0.808
Constant	−1.857	−2.372	−1.187

[a] It turns out that for all three variables a first order autoregressive model, AR(1), can not be rejected by the observations. The test statistic for the null hypothesis of a unit root is the t-value for the estimated parameter ρ in the model $\Delta y_t = \mu_t + \rho y_{t-1} + \xi_t$, where μ_t reflects deterministic elements. The 5% critical value of this t ratio is -3.45 when μ_t includes a constant and a trend, and -2.89 when the μ_t only includes a constant, see Fuller (1976). When $\mu_t = \mu_0 + \tau t$, the distribution of the t ratio for τ, i.e. the parameter for the deterministic trend variable, is not standard under the null hypothesis of a unit root. Hence, it seems sensible to evaluate the results in case μ_t is μ_0 only and in case it also contains a trend. Confidence in the test outcomes increases when similar conclusions can be drawn.

(see Appendix 1), indicates that the null hypothesis of no cointegration can be rejected at a 5% level. The estimated version of (10) is

$$\Delta_1 x_t = -\underset{(0.207)}{0.666}\hat{u}_{t-1} - \underset{(0.029)}{0.003} - \underset{(0.259)}{0.680}\Delta_1 p_t + \underset{(0.600)}{1.712}\Delta_1 y_t + \hat{\epsilon}_t, \tag{14}$$

and the t ratio for $-\alpha$ equals -3.225. The R^2 of this model is 0.431, and the sum of squared residuals is 0.247.

The model in (14) is evaluated using a range of diagnostic checks. A list of diagnostic test statistics used for this evaluation is given in Appendix 3. For (14) it emerges that F_{ar1} and F_{ar2} are 0.056 and 0.225, respectively, that F_{arch1} is 1.068, *JB* is 1.327, *White* is 0.856, and that the parameter constancy tests yield *Chow*(74) is 1.959, *Chow*(75) is 0.300, *Chow*(80) is 0.607, *Chow*(81) is 0.241, *PF*(74) is 1.788, *PF*(75) is 0.662, *PF*(80) is 1.207 and *PF*(81) is 0.871. The estimated parameters values in (13) and (14) have the expected signs since one would expect that the impact of price is negative, that the impact of income is positive, and that the adjustment parameter is negative, i.e. that α in (4) is positive. In summary, this model can not be rejected by the data.

The application of the Boswijk (1992a,b) procedure amounts to estimating (11) and comparing it with the model in (12). The estimation results for (11) are

$$\Delta_1 x_t = \underset{(0.572)}{-0.113} - \underset{(0.296)}{0.516}\Delta_1 p_t + \underset{(0.732)}{1.315}\Delta_1 y_t - \underset{(0.219)}{0.534} x_{t-1} - \underset{(0.361)}{0.508} p_{t-1} + \underset{(0.318)}{0.616} y_{t-1} + \hat{\epsilon}_t. \quad (15)$$

The Wald test for the presence of cointegration obtains a value of 13.412. Comparing this value with the critical values displayed in Appendix 1 indicates that the hypothesis of no cointegration can be rejected at a 10% level. It is usual practice to consider more than one significance level when testing for cointegration. This is suggested by simulation studies, like that in Boswijk and Franses (1992), which indicate that tests for cointegration can have low power. The R^2 is 0.494 and the residual sum of squares is 0.220. The diagnostic test results for this model are that F_{ar1} is 1.091 and that F_{ar2} is 0.502. The F_{arch1} is 0.625, *JB* is 2.195, and *White* is 0.960. The parameter constancy tests yield *Chow*(74) is 3.242, *Chow*(75) is 4.368, *Chow*(80) is 1.116, *Chow*(81) is 0.596, *PF*(74) is 4.980, *PF*(75) is 2.823, *PF*(80) is 0.931 and *PF*(81) is 0.653. The results for the parameter constancy tests for 1974 and 1975 suggest that the hypothesis of parameter constancy may be rejected, although the other diagnostic measures suggest that the model is adequate.

The t ratios in (15), as well as the parameter estimates for $\Delta_1 p_t$ and p_{t-1}, seem to suggest that the model may be simplified to a model like (4). When estimated, this simplified model is

$$\Delta_1 x_t = \underset{(0.514)}{0.113} - \underset{(0.180)}{0.551}\left(x_{t-1} + \underset{(0.334)}{1.120} p_t - \underset{(0.165)}{1.097} y_t\right) + \hat{\epsilon}_t \quad (16)$$

with an R^2 of 0.467 and a residual sum of squares of 0.232. Comparing the sums of squares of (15) with those of (16) indicates that this simplification cannot be rejected by the data. The diagnostic test results of (16) are very close to those for the model in (15). The exceptions are the tests for the constancy of the parameters. The *Chow*(74) and *Chow*(75) test values are now 1.694 and 1.227, while the *PF*(74) and *PF*(75) values are 1.004 and 0.704. The Wald test for the null hypothesis of no cointegration, which implies deleting x_{t-1}, p_t and y_t from (16), obtains the highly significant value of 21.021. Comparing the residual sums of squares of (16) and (14) suggests that the specification in (16) is to be preferred since it involves a smaller number of parameters to be estimated. Finally, regressing $\Delta_1 p_t$ and $\Delta_1 y_t$ on the EC term in (15) yields insignificant t ratios, indicating that the p_t and y_t variables are weakly exogenous, and hence that the equilibrium parameters in (16) have been consistently and efficiently estimated [1].

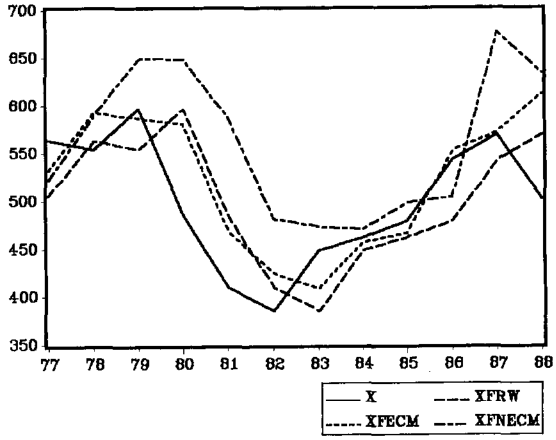

Fig. 3. One-step ahead forecasts for new car sales (x) based on error correction model (16), *xfecm*, on the random walk model, *xfrw* and on a model only using first differences, *xfnecm*.

One of the important gains of considering cointegrated systems, at least theoretically, concerns out-of-sample forecasting; see e.g, Engle and Yoo (1987). To see whether the error correction variable in (16) is useful for one-step-ahead forecasting, consider the graphs in Fig. 3.This figure displays the new car sales throughout the

[1] A casual application of the Johansen and Juselius (1990) method to a vector autoregression of order 1 yields that there seems to be only one highly significant and also sensible cointegration relationship. This relation is broadly similar to that in (13) and (15), and hence the above obtained outcomes for weak exogeneity carry over. Details of these results can be obtained from the author.

years 1977–1988, and the forecasts from model (16), from a random walk with drift model and from a model for $\Delta_1 x_t$ when, it is regressed on $\Delta_1 p_{t-1}$ and $\Delta_1 y_{t-1}$, to be denoted as *ECM*, *RW* and *NECM*, respectively. The latter model is selected using t ratios and forecasting performance. In fact, a model containing a constant, $\Delta_1 p_t$ and $\Delta_1 y_t$ yields unreasonable forecasts like a sales of 1484 in 1988. The random walk model emerges from (16) when the EC variable is deleted. All models are estimated using the sample 1960–1976. The mean squared forecast errors (*MSPE*) over the forecast period 1977–1988 are $MSPE_{ecm} = 2562$, $MSPE_{rw} = 3158$ and $MSPE_{necm} = 8717$. The number of times, out of the twelve, that SPE_{ecm} is smaller than SPE_{rw} is 9 and that it is smaller than SPE_{necm} is 10. Using a nonparametric sign test (cf. Flores, 1989), one can conclude that the forecasts for x_t from (16) are significantly closer to the true observations than those from the two rival models.

5. Concluding remarks

The model for new product sales developed in Chow (1967), that accounts for Gompertz patterns and for saturation levels which are dependent on explanatory variables, is a partial adjustment model for the variables in logarithms. When these variables are nonstationary, one has to rely on cointegration methods to estimate the model parameters. Two empirical specification strategies, which focus on testing for cointegration and on parameter estimation, are discussed in detail and applied to the annual new car sales in the Netherlands.

It is not easy to recommend either one of the cointegration methods on the basis of a single application only. As can be observed, the methods can yield roughly similar models in particular applications. Theoretical arguments in favor of the Boswijk (1992a) procedure, which assumes weak exogeneity of some variables, are that the long-run cointegration parameters and the short-run adjustment parameters are estimated in one step. This facilitates, for example, the simplification of a model in case it might be appropriate, as it is in the application in the present paper. Of course, if weak exogeneity cannot be assumed, the Johansen and Juselius approach is more appropriate.

The main contribution of the present paper is the illustration of how cointegration techniques, which are only recently developed in econometrics, can be meaningfully applied in marketing research. In this paper the focus is only on modeling product sales. Of course, one may consider cointegration techniques when modeling sales at a brand level. A further example is modeling push–pull strategies (see e.g, Farris et al., 1989), where possibly nonstationary variables like market shares and distribution may be related via an equilibrium relationship.

The limitation of the paper is that it focuses on the Gompertz model. A topic for further research is therefore to extend the cointegration analysis in this paper to other growth processes like, e.g, the logistic process.

Appendix 1: Unit roots and cointegration

In this appendix a brief review of concepts as unit roots and cointegration is given. Detailed expositions can be found in the referenced literature. For our purposes it suffices to investigate the properties of univariate variables z_t and x_t, when they are generated by the first order vector autoregression,

$$\begin{bmatrix} z_t \\ x_t \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} z_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ u_t \end{bmatrix} \quad \text{(A.1)}$$

where ϵ_t and u_t are assumed to be two independent standard white noise variables, i.e. they are uncorrelated mean zero processes with variances σ_ϵ^2 and σ_u^2 and it is assumed that ϵ_t and u_{t-j} are uncorrelated for $j = \pm 1, \pm 2,..$

Using the backward-shift operator B, which is defined by $B^k v_t = v_{t-k}$, the model in (A.1) can be written as

$$\begin{bmatrix} 1-\alpha B & -\beta B \\ -\gamma B & 1-\delta B \end{bmatrix} \begin{bmatrix} z_t \\ x_t \end{bmatrix} = \begin{bmatrix} \epsilon_t \\ u_t \end{bmatrix} \quad \text{(A.2)}$$

This vector process is stationary if the roots of

$$1 - (\alpha + \delta)\xi + (\alpha\delta - \beta\gamma)\xi^2 = 0 \quad \text{(A.3)}$$

lie outside the unit circle, i.e. $|\xi_{1,2}| > 1$. When one solution of (A.3) is on the unit circle, there is one unit root in the multivariate system. A single unit root emerges when, e.g., $\alpha + \delta - \alpha\delta + \beta\gamma = 1$, i.e. when $\beta\gamma = (1-\alpha)(1-\delta)$. Since (A.2) can be written as

$$[1-(\alpha+\delta)B+(\alpha\delta-\beta\gamma)B^2]\begin{bmatrix} z_t \\ x_t \end{bmatrix} = \begin{bmatrix} 1-\delta B & \beta B \\ \gamma B & 1-\alpha B \end{bmatrix}\begin{bmatrix} \epsilon_t \\ u_t \end{bmatrix}, \quad \text{(A.4)}$$

(see Granger and Newbold, 1986), it is easily seen that a single unit root in a first order bivariate time series model implies that the z_t and x_t series both follow autoregressive integrated moving average processes of order (1,1,1), i.e. ARIMA(1,1,1). The integrated (I) part of these univariate models originates from the fact that when $\beta\gamma = (1-\alpha)(1-\delta)$, the $[1-(\alpha+\delta)B + (\alpha\delta - \beta\gamma)B^2]$ in (A.4) becomes $(1-(\alpha+\delta-1)B)(1-B)$. The univariate time series z_t and x_t are now said to have a unit root. An often applied test statistic for unit roots is based on the t ratio of the parameter ρ in the auxiliary regression

$$\Delta_1 v_t = \mu + \tau T_t + \rho v_{t-1} + \psi_1 \Delta_1 v_{t-1} + .. + \psi_p \Delta_1 v_{t-p} + \lambda_t \quad \text{(A.5)}$$

where μ is the intercept, T_t is a linear deterministic trend variable, Δ_1 is the first order differencing transformation. This auxiliary model is called the Dickey–Fuller regression (see Dickey and Fuller, 1979). The t ratio for ρ does not follow a standard distribution under the null hypothesis of a unit root. Tables with critical values for this t ratio can be found in Fuller (1976).

The impact of the presence of a unit root in a univariate time series is that shocks have a permanent impact on the future pattern of the series. This is most easily seen by rewriting a simple random walk process

$$z_t = z_{t-1} + \epsilon_t$$

as

$$z_t = z_0 + \sum_{i=1}^{t} \epsilon_i$$

The latter expression clearly indicates that the effect of a shock at some time i does not become smaller, but is constant. Hence, a large ϵ_i value can change the pattern of z_t permanently.

A natural implication of finding unit roots in univariate series is to transform each of the series using the Δ_1 filter, and construct multivariate models for $\Delta_1 z_t$ and $\Delta_1 x_t$. However, in case the number of unit roots in the multivariate system like (A.1) is less than the number of variables, though the number exceeds zero, a multivariate model containing only Δ_1 transformed variables is misspecified. This is easily understood by rewriting (A.1) as

$$\begin{bmatrix} z_t - z_{t-1} \\ x_t - x_{t-1} \end{bmatrix} = \begin{bmatrix} \alpha-1 & \beta \\ \gamma & \delta-1 \end{bmatrix}\begin{bmatrix} z_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ u_t \end{bmatrix} \quad \text{(A.6)}$$

and recognizing that under the parameter restriction $\beta\gamma = (1-\alpha)(1-\delta)$ holds

$$\begin{bmatrix} \alpha-1 & \beta \\ \gamma & \delta-1 \end{bmatrix} = \begin{bmatrix} \beta\gamma/(\delta-1) & \beta \\ \gamma & \delta-1 \end{bmatrix} = \begin{bmatrix} \beta\gamma/(\delta-1) \\ \gamma \end{bmatrix}[1, -(1-\delta)/\gamma] \quad \text{(A.7)}$$

This decomposition implies that (A.6) can be rewritten as

$$\Delta_1 z_t = (\beta\gamma/(\delta-1))[z_{t-1} - [(1-\delta)/\gamma]x_{t-1}] + \epsilon_t \quad \text{(A.8)}$$

$$\Delta_1 x_t = \gamma[z_{t-1} - [(1-\delta)/\gamma]x_{t-1}] + u_t \quad \text{(A.9)}$$

Since the error processes ϵ_t and u_t and the Δ_1 transformed time series are stationary, the error correction variable $z_{t-1} - [(1-\delta/\gamma]x_{t-1}$ is stationary as well. It is now said that the z_t and x_t are cointegrated (see Engle and Granger, 1987). The cointegration vector is $(1, -(1-\delta)/\gamma)$, and the adjustment parameters are $\beta\gamma/(\delta-1)$ and γ.

There are several routes to specifying models like (A.8) and (A.9). The first step in the Engle and Granger two-step approach, proposed in Engle and Granger (1987), is to regress z_t on a constant and x_t, to calculate residuals e_t, and to check whether the residual process $\{e_t\}$ is stationary along similar lines as in (A.5). Critical values of the corresponding t ratios for ρ are displayed in Table 2 in Engle and Yoo (1987) for 1 through 5 variables in a VAR like (A.1). In case the model contains an intercept and not a linear trend, the 5% and 10% critical values of this test are -3.37 and -3.03 for one weakly exogenous variable, -3.93 and -3.59 for two, -4.22 and -3.89 for three, and -4.58 and -4.26 for four such variables (see Engle and Yoo, 1987). When $\{e_t\}$ is stationary, the adjustment parameters are found in the second step, i.e. from the regressions of $\Delta_1 z_t$ and $\Delta_1 x_t$ on e_{t-1}. Note that this two-step method estimates the long-run cointegration parameters and the short-run adjustment parameters separately.

Two alternative cointegration methods which estimate the long-run and short-run parameters at the same time are proposed in Johansen and Juselius (1990) and Boswijk (1992a,b). The first approach estimates the parameter matrix in models like (A.6) and makes the decomposition in (A.7) using the maximum likelihood method. The underlying assumption of this method is that all variables in the multivariate system may be endogenous. The Boswijk (1992a,b) method assumes that one or more variables in an m-vector system are weakly exogenous. In that case equations as (A.9) are estimated unrestrictedly, i.e.

$$\Delta_1 x_t = \theta_1 z_{t-1} + \theta_2 x_{t-1} + u_t \quad \text{(A.10)}$$

and, here, an estimate of the cointegration parameter is obtained from θ_2/θ_1. Usually, models like (A.10) are enlarged by at least including $\Delta_1 z_t$. Testing for weak exogeneity can be performed by testing whether the adjustment parameter is significant in the regression of $\Delta_1 z_t$ on h_{t-1}, with h_t is the error correction variable obtained from (A.10). This so-called weak exogeneity concept concerns the consistency and efficiency of the estimators in (A.10). The test for cointegration is performed using a Wald test for the joint significance of z_{t-1} and x_{t-1} in (A.10). In case the model contains an intercept and not a linear trend, the 5% and 10% critical values of this Wald test are 11.41 and 9.54 for one weakly exogenous variable, 14.38 and 12.22 for two, 17.18 and 14.93 for three, and 19.69 and 17.38 for four such variables (see Boswijk, 1992a).

Appendix 2: The data[a]

Year	Sales	Price	Income
1960	92.8	1.695800	136.9452
1961	109.5	1.666374	140.9174
1962	130.6	1.604603	146.9759
1963	155.5	1.546875	151.8562
1964	208.6	1.548610	164.8974
1965	223.6	1.442709	173.5110
1966	238.8	1.447489	178.4692
1967	231.0	1.324397	188.1538
1968	301.6	1.348224	200.7906
1969	345.8	1.341699	214.2343
1970	378.9	1.240470	255.2124
1971	443.9	1.320656	265.9427
1972	408.6	1.228985	274.7583
1973	443.6	1.302104	287.6065
1974	382.4	1.465992	298.8530
1975	434.1	1.376426	298.5785
1976	504.4	1.402455	313.7838
1977	562.1	1.268687	318.2484
1978	552.4	1.186157	326.2041
1979	595.8	1.215405	333.9297
1980	484.5	1.348000	336.7400
1981	408.7	1.619012	334.4664
1982	383.6	1.622262	329.5086
1983	447.2	1.653343	334.0173
1984	461.0	1.617361	344.7010
1985	477.6	1.641066	353.7428
1986	541.7	1.268248	362.3892
1987	568.0	1.369584	366.8273
1988	500.9	1.297891	377.8372

[a] The sales is measured in thousands of new car sales, the price is the price (in guilders) of petrol when corrected for the consumer price index (1980 = 100), and income is measured as the real gross national product (in billions of guilders).

Appendix 3: Diagnostic test statistics

All calculations in this paper are performed using the statistical package MicroTSP, version 7.0. This package facilitates thorough diagnostic checking of an estimated model, since it incorporates a wide range of test statistics. In this paper, I use F_{ar1} and F_{ar2}, which are F versions of the Lagrange Multiplier (LM) tests for residual autocorrelation of order 1 and 2; F_{arch1}, which is an F version of the LM test for ARCH type errors of order 1; *JB*, which is a $\chi^2(2)$ distributed test for normality of the residuals; *White*, which is a test for heteroskedasticity in general (see White, 1980); *CHOW*(..), which is the familiar Chow test for parameter stability, and finally; *PF*(..), which is a Chow type test for predictive failure. In the present application, I apply the latter two tests to check whether structural breaks occurred in 1974, 1975, 1980 and 1981. These years are chosen since, a priori, one may expect structural breaks given the two economic crises caused by oil price shocks. The Chow type parameter constancy tests are also F type tests. Expressions for all the test statistics used can be found in the standard econometric text books like Judge et al. (1985).

References

Boswijk, H.P., 1992a, Cointegration, identification and exogeneity: Inference in structural error correction methods, Thesis (Amsterdam).

Boswijk, H.P., 1992b, Testing for an unstable root in conditional and structural error correction models, Journal of Econometrics forthcoming.

Boswijk, H.P. and P.H. Franses, 1992, Dynamic specification and cointegration, Oxford Bulletin of Economics and Statistics 54, 369–381.

Chow, G.C., 1967, Technological change and the demand for computers, American Economic Review 57, 1117–1130.

Dickey, D.A. and W.A. Fuller, 1979, Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association 79, 427–431.

Engle, R.F. and C.W.J. Granger, 1987, Co-integration and error correction: Representation, estimation, and testing, Econometrica 55 (March), 251–276.

Engle, R.F., D.F. Hendry and J.-F. Richard, 1983, Exogeneity, Econometrica 51, 277–304.

Engle, R.F. and B.S. Yoo, 1987, Forecasting and testing in co-integrated systems, Journal of Econometrics 35, 143–159.

Farris, P., J. Olver and C. de Kluyver, 1989, The relationship between distribution and market share, Market Science 8 (Spring), 107–128.

Flores, B.E., 1989, The utilization of the Wilcoxon test to compare forecasting methods: A note, International Journal of Forecasting 5, 529–535.

Franses, P.H., 1994, Fitting a Gompertz curve, Journal of the Operational Research Society 45, 109–113.

Fuller, W.A., 1976, Introduction to statistical time series (Wiley, New York).

Granger, C.W.J. and P. Newbold, 1986, Forecasting economic time series (Academic Press, San Diego).

Johansen, S. and K. Juselius, 1990, Maximum likelihood estimation and inference on cointegration – with applications to the demand for money, Oxford Bulletin of Economics and Statistics 52, 169–210.

Judge, G. G., W. E. Griffiths, R. Carter Hill, H. Lükepohl and T.-C. Lee, 1985, The theory and practice of econometrics, 2nd ed. (Wiley, New York).

Leong, S.M. and S. Ouliaris, 1991, Applying the econometrics of cointegration to time series data in marketing research, Working paper series 91–52 (National University of Singapore).

Mahajan, V. and E. Muller, 1979, Innovation diffusion and new product growth models in marketing, Journal of Marketing 43, 55–68.

Meade, N., 1984, The use of growth curves in forecasting market development – a review and appraisal, Journal of Forecasting 3, 429–451.

White, H., 1980, A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity, Econometrica 48, 817–838.