TI 2010-109/2
Tinbergen Institute Discussion Paper

Pensions, Debt and Inflation Risk in a Monetary Union

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Pensions, debt and inflation risk in a monetary union

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October 2010

Abstract

This paper investigates the international spillovers of government debt and the associated risk of inflation within a monetary union when countries have different pension systems. I use a stochastic two-country two-period overlapping-generations model, where one country has PAYG pensions and the other country has funded pensions. The paper shows that the PAYG country can shift part of its long-term debt burden to the funded country. Moreover, the PAYG country gains from unexpected inflation at the cost of the funded country. In response to these conflicting interests about inflation, inflation *risk* may rise with the level of debt in the PAYG country. Higher inflation risk harms both countries. Actually, in contrast to the debt burden, the PAYG country cannot share the negative effects of a rise in inflation risk with the funded country. The scenarios analysed might be especially relevant for the years to come.

JEL codes: E31, F41, G11, G12, H55, H63

Keywords: spillovers, pensions, debt, inflation

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I am grateful to Lans Bovenberg, Manthos Delis, Giovanni Facchini, Bas van Groezen, Bas Jacobs, Frank de Jong, Lex Meijdam, Lorenzo Pozzi, Ward Romp, Harrie Verbon, Casper de Vries, Ed Westerhout, seminar participants in Rotterdam and Utrecht and conference audiences at the Netspar workshop 2008 in the Hague, the 2008 IIPF conference in Maastricht, the 2009 Conference on Macroeconomic Analysis and International Finance in Rethymno, and the 2009 EEA congress in Barcelona, for useful comments and discussions.

1 Introduction

On May 2, 2010, the countries of the euro zone and the IMF announced to support Greece with a rescue package of loans worth 110 billion euro. In the weeks before, financial markets had lost confidence in the possibilities of the Greek government to finance its debt. Interest rates on Greek sovereign bonds rose to double digits. However, the rescue package for Greece was not able to restore confidence in the stability of the euro zone as financial markets turned their attention to other vulnerable euro-zone countries like Spain and Portugal and there was fear of even more serious contagion effects in government bond markets. Therefore on May 9, 2010, only one week after the rescue package for Greece was announced, the Ecofin Council of the European Union announced an even more impressive rescue package to safeguard the stability of the euro zone. In addition the European Central Bank (ECB) decided to buy governments bonds to restore the functioning of markets. This intervention by the ECB was perceived as highly controversial and led to a lot of discussion about its independence and credibility in the popular press. The events in May 2010 show that high levels of public debt in one country can cast doubt on the overall stability of the euro zone.

The worries concerning government debt will stay prevalent in the future as population ageing will put even more pressure on the public finances of European countries. The European Commission estimates that the average debt level in the euro area will exceed 140 percent of GDP in 2030 if countries do not reduce their primary deficits and do not reform their pension systems (European Commission, 2009). Even when rescue facilities will remain unused, high debt levels will have their spillover effects in a monetary union, especially when high debt is associated with higher inflation risk. Higher levels of nominal government debt may lead to a higher risk of inflation through various channels. First of all there is the temptation to reduce the fiscal burden of debt service through higher inflation. Second, doubts about the stability of the euro zone will cause a depreciation of the euro (this is exactly what happened in Spring 2010), which can lead to imported inflation. Third, if investors think that the ECB will purchase government bonds again in case many euro countries have trouble financing their debt, this will be accompanied with higher inflation expectations. The actions of the ECB in response to the sovereign debt crisis and its lack of transparency about its decision making process (see for example Geraats et al., 2008) might lead to a situation in which markets perceive its monetary policy as less credible. The lack of transparency and credibility of the ECB's monetary policy creates uncertainty and implies that high levels of public debt can be associated with a higher risk of inflation. This paper analyses the international spillover effects of government debt within a monetary union, where higher debt levels might be associated with higher inflation risk. To the best of my knowledge this paper is one of the first attempts that provides a formal analysis of this issue.

These international spillover effects are analysed in a framework where countries differ in the pension schemes they use. Within the European Economic and Monetary Union (EMU) we can, broadly speaking, distinguish two groups of countries that have different pension systems which might lead to different interests concerning government debt and inflation. On the one hand there are countries like Greece, France, Spain and Italy that have extensive PAYG schemes, but almost no funded pensions. In these countries PAYG pensions finance more than 90 percent of the income of retirees (Boeri et al., 2006). On the other hand, there are countries like the Netherlands and Finland that have sizeable funded pension systems. The investments of pension funds in the latter group of countries are substantial (see Table 1), and part of these funds are invested in government bonds of countries that have installed large PAYG schemes. This paper shows that the fact that funded countries finance part of the government debt in PAYG countries creates conflicting interests concerning the creation of inflation between these two groups of countries. The main idea is then that the conflicts about inflation policy will increase in the coming decades as especially countries that have large public PAYG schemes will have problems financing their pensions when demographic pressures rise. In these schemes the working population pays taxes to finance the pension benefits of the elderly and the temptation for governments to use debt instead of raising taxes or lowering pension benefits will be large.

Table 1: Total investments pension funds (% GDP, 2009)

Greece*	0.01	Spain	8.1
France*	0.8	Portugal	13.4
Belgium*	3.3	Ireland	44.2
Italy	4.1	US	67.8
Austria	4.9	Finland	76.8
Germany	5.2	Netherlands	129.8

*Data for 2008, as data for 2009 were not available.
Source: OECD Global Pension Statistics.

To address both the issue of government debt and inflation risk I develop a stochastic two-period overlapping generations (OLG) model with two countries that form a monetary union. One country has fully funded pensions and the other country relies on PAYG-financed defined benefit pensions. Consumers allocate their investment portfolio to stocks and government bonds. Both assets are risky; there is productivity risk on stocks and inflation risk on government bonds. The major advantage of the stochastic general equilibrium model in this paper is that it is quite tractable, which allows us to focus on the main mechanisms that drive the results. Most papers that develop stochastic general equilibrium OLG models use computable models (e.g., Storesletten et al. (1999), Sánchez-Marcos and Sánchez-Martin (2006), and Krueger and Kubler (2006)). There are a few papers that develop an analytical stochastic general equilibrium OLG model, like Bohn (2001, 2009) and Beetsma and Bovenberg (2009). These papers do not, however, derive the optimal conditions for savings- and portfolio decisions of consumers as I do in my model. Moreover, this literature analyses the intergenerational risk-sharing properties of pension schemes, while the focus of my paper is on the international spillover effects of government debt and inflation risk. For that purpose I develop a two-country model instead of the one-country closed economy models considered in the aforementioned papers.

To obtain an expression for the optimal portfolio share I use the approach of Campbell and Viceira (2002). This approach is also taken by Matsen and Thøgersen (2004) who develop a partial equilibrium model where the PAYG pension system is treated as a 'quasi-asset' and derive the optimal share invested in this PAYG asset. They assume that people only consume in the second period of life, so that the complete net labour income received in the first period of life is saved. In contrast to Matsen and Thøgersen (2004) I model the savings decisions of individuals and more importantly, I develop a general equilibrium model where the effects on the rates of return are taken into account. This is one of the advantages of the model in this paper; because it is a general equilibrium model both the optimal portfolio choice and the equity premium can be derived at the same time. This contrasts with the 'finance' literature where typically only one of the two is derived. In case one would like to study asset pricing, the stochastic discount rate (i.e, the marginal value of financial wealth in the next period) is kept exogenous, see Campbell (2003) for an overview. For the explanation of portfolio choice, on the other hand, asset returns are taken as exogenous, see for example Campbell and Viceira (1999).

Persson (1985) also studies the effects of public debt in a two-country setting[1]. In contrast to Persson (1985), I consider two countries that differ in the pension scheme they use. Moreover, while Persson (1985) only distinguishes between the effects in the short run and the long run, I derive the whole transition path for the two economies. But most importantly, in contrast to Persson (1985), this paper considers the use of government debt in a *stochastic* two-country model, which allows me to also examine the case where rising debt levels lead to more inflation risk.

This paper is also related to the literature that studies the interaction between monetary and fiscal policy, see for example Beetsma and Uhlig (1999) and Chari and Kehoe (2007). They show that monetary unification leads to excessive debt accumulation and an inflation rate that is too high and therefore these papers provide a rationale for having fiscal restrictions in case countries form a monetary union. Although my

[1]Fried and Howitt (1988) also analyse public debt in an international setting. This paper uses a model based on assumptions that essentially differ from the ones made in this paper; they abstract from capital accumulation and examine the effect of government debt on capital gains and losses.

analysis also reaches the policy conclusion that fiscal restrictions are necessary in a monetary union because high debt levels can lead to higher inflation (risk), the set-up of the model and the focus of my paper is completely different than in Beetsma and Uhlig (1999) and Chari and Kehoe (2007). In these papers the countries in the monetary union are symmetric and the interaction between monetary and fiscal policy is modelled as a game. In my model countries are asymmetric and I show that this asymmetry creates conflicting interests between countries concerning the creation of inflation. Moreover, in contrast to Beetsma and Uhlig (1999) and Chari and Kehoe (2007), I develop a complete general equilibrium model where consumer- and firm behaviour are modelled as well, to study the international spillovers of debt and inflation risk.

The main results are as follows. First, for a country using a PAYG pension scheme it is easier to temporarily finance its pensions with debt when it shares one capital market with a country that uses a fully funded pension system, i.e., the PAYG country experiences positive spillover effects in the long run. The country with funded pensions, on the other hand, is in the long run adversely affected by the use of government debt by the PAYG country. In the short run, however, the spillover effects may be opposite: The initial generations in the funded country may gain from the rise in public debt in the PAYG country. The reason for this is that these initial generations mainly receive the gains from the higher rates of return that result from the rise in debt, they do not, or do not fully, have the negative effects from the lower wages that later generations experience.

Second, in an asymmetric monetary union where one country relies on PAYG pensions and the other country has a fully funded pension scheme, the former country gains from unexpected inflation at the cost of the latter country. This means that there is a conflict of interest on monetary policy when countries with different pension schemes form a monetary union. If market participants do not exactly know how the central bank will react to these conflicting interests (for example because the decision-making process on monetary policy is not completely transparent) they might perceive that the *risk* of inflation rises when the PAYG country increases its debt.

Higher inflation risk harms both countries. It makes government bonds more risky and less attractive to hold. The rate of return on bonds will rise relative to the return on equity to induce people to buy the existing stock of government debt. The increase of the rate of return on government bonds implies that the costs of government debt rise for both countries. Actually, the negative utility effects from a rise in inflation risk are larger for the PAYG country if it forms a monetary union with a country that uses a fully funded pension scheme instead of a PAYG system. This result arises because the funded country holds a relatively large part of the government bonds and individuals in the funded country do not receive a safe PAYG pension benefit.

Therefore these agents need to be compensated more in order to hold the more risky government bonds, that is, the rate of return on government bonds has to rise to a larger extent when a funded pension scheme is in place. The analysis in this paper thus shows that although the PAYG country can export part of the debt burden to the funded country, it cannot share the negative effects of higher inflation risk.

This paper is organised as follows. The next section presents the stochastic OLG model. In Section 3 I discuss the effects of government debt. Section 4 analyses the effects of an unexpected inflation shock where both countries try to compensate the people that lose from this shock; the elderly. Section 5 explores the spillover effects in case high debt levels raises perceived inflation risk. Section 6 concludes the paper.

2 Model

Following Buiter (1981) and Persson (1985), I will use a two-period overlapping-generations model of an open economy. The world consists of two countries, country P and country F, which differ in the way the pensions are financed. Country P relies on a PAYG pension system and country F has a fully funded pension scheme. I assume a constant population size[2] and dynamic efficiency in both countries. The countries may, however, differ in population size. In this way, I allow for scale differences between the two countries. The active population is L^i, where $i = P, F$. Define $\frac{L^F}{L^P} = \nu$ and normalize L^P to one, then ν tells us the relative size of L^F. The countries are identical in all other respects. All variables in the model are expressed as the amount per young individual and lowercase letters refer to the logarithm of the respective variable. Throughout the paper the notation 'log' refers to the natural logarithm.

2.1 Risk factors

There are two risk factors, there is productivity risk (σ_a^2) on stocks and inflation risk (σ_π^2) on government bonds. Investing in stocks is more risky than the investment in government bonds, i.e., $\sigma_a^2 > \sigma_\pi^2$.

[2]This assumption may come as a surprise as the main motivation of this paper, as sketched in the Introduction, is that population ageing will exert great pressure on public finances in the coming decades. To keep the results tractable, however, population ageing is left out of the analysis. The interested reader is referred to Adema (2008*a*) where the rise in government debt evolves endogenously after a rise in life expectancy. The international spillover effects of pensions under population ageing without the use of government debt are analysed in Adema et al. (2008).

Production

Production per worker is described by the following production function:

$$F(A_t, K_t^i) = A_t (K_t^i)^\alpha \tag{1}$$

where A_t denotes productivity at time t, $\alpha \in [0, 1]$ the production elasticity, and K_t^i refers to the amount of capital per young individual at time t in country i, $i = P, F$. Profit maximization and perfect competition among producers results in the usual equilibrium conditions:

$$\begin{aligned} W_t^i &= (1-\alpha) A_t (K_t^i)^\alpha && (2) \\ R_{k,t}^i + \delta &= \alpha A_t (K_t^i)^{\alpha-1} && (3) \end{aligned}$$

where W_t^i is the real wage, $R_{k,t}^i$ the return to capital and δ the depreciation rate of capital. There is perfect capital mobility between the two countries, but labor is immobile. Since capital can freely move across countries, the rates of return will be equalized, i.e., $R_{k,t}^P = R_{k,t}^F = R_{k,t}, \forall t$. And because both countries are endowed with the same production technology, this implies that $K_t^P = K_t^F = K_t$, and consequently $W_t^P = W_t^F = W_t$. Following Campbell and Viceira (2002) I assume that the gross return on capital $(1 + R_{k,t})$ is lognormally distributed. To achieve this I assume that A_t is lognormally distributed and that there is 100% depreciation, i.e., $\delta = 1$. This implies that both W_t and $R_{k,t}$ are stochastic. People do not have to form expectations about W_t, however, as A_t is already known before W_t is paid (see Figure 1 in Section 2.2). People base their saving- and portfolio decisions on the future return of capital, so they *do* have to form expectations about this variable. The variance of the log of the gross return on capital is equal to the variance of the log of productivity (see Appendix A.1 for details), i.e.,

$$\begin{aligned} \mathrm{Var}_t[\log(1 + R_{k,t+1})] &= \mathrm{Var}_t[\log A_{t+1}] \\ \sigma_{k,t}^2 &= \sigma_{a,t}^2 && (4) \end{aligned}$$

Inflation

Government debt is denominated in nominal terms and therefore there is inflation risk on the return on government bonds. The two countries form a monetary union. In a monetary union, inflation is a common risk factor for all members and therefore inflation (risk) is the same in the two countries. As the main focus of this paper is on inflation risk I do not include country-specific risk, like default risk, on government bonds. Perfect capital mobility equalizes the rates of return on government bonds. The gross real return on government bonds is equal to:

$$1 + R_{b,t} = \frac{1 + R_{b,t}^N}{1 + \pi_t} \tag{5}$$

where $R^N_{b,t}$ is the nominal return on government bonds and π_t the inflation rate between $t-1$ and t. The nominal return $R^N_{b,t}$ is a predetermined variable and I assume that the inflation factor ($\frac{1}{1+\pi_t}$) is lognormally distributed. Appendix A.2 derives that:

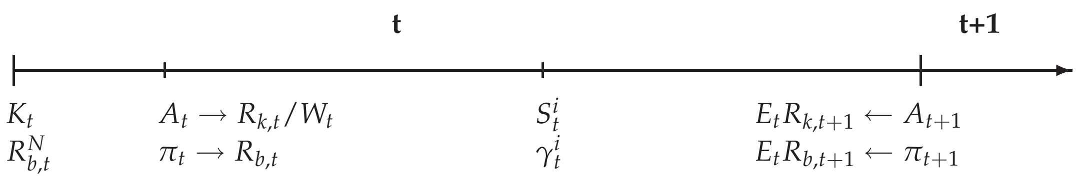
$$\begin{aligned} \mathrm{Var}_t\left[\log(1+R_{b,t+1})\right] &= \mathrm{Var}_t\left[\log\left(\frac{1}{1+\pi_{t+1}}\right)\right] \\ \sigma^2_{b,t} &= \sigma^2_{\pi,t} \end{aligned} \tag{6}$$

There are no risk-free inflation indexed bonds, which implies that there is a missing market. Safe income in the second period of life, like defined-benefit pensions, would (partly) fill this gap.

2.2 Timing

The sequence of events is shown in Figure 1. At the beginning of period t, the capital stock K_t and the nominal interest rate on government bonds $R^N_{b,t}$ are inherited from the previous period, as they are determined by the savings and portfolio decisions made in period $t-1$. Then, productivity and inflation are revealed. With this knowledge the return on capital, wages and the real return on government bonds are determined. Subsequently, households make their portfolio choice γ^i_t and saving decisions S^i_t (and thereby their consumption decisions), which are also based on the expected future asset returns. Consumers only face uncertainty about the return on their savings.

Figure 1: Timing of events



2.3 Pensions and government debt

Intially, the government in country P runs a balanced PAYG pension system, that is, pension benefits of the elderly (Z^P_t) are covered by lump-sum[3] taxes paid by the young (T^P_t)[4]:

$$Z^P = T^P \tag{7}$$

[3]Assuming proportional taxes, instead of lump-sum taxes, would reinforce the negative long-run welfare effects, as the fall in wages, after a rise in government debt, implies less tax revenues for the government and lower pension benefits for the elderly. Qualitatively, the results will not change however.

[4]By omitting time subscripts, I denote the initial steady state value of the respective variable.

PAYG pension benefits are guaranteed in real terms, that is, PAYG pension benefits are safe. The function of safe PAYG pensions in my stochastic model is twofold. First, it provides retirees with a certain minimum amount of income. Second, the PAYG pension scheme partly removes the market incompleteness that people cannot invest in any risk-free asset, as the system makes sure that part of the retirement income is non-stochastic. In this way I incorporate the fact that a PAYG pension scheme also serves as a risk-sharing and diversification device. PAYG pensions therefore have a comparable role as in the literature that focuses on the intergenerational risk-sharing properties of PAYG pension schemes, see for example Storesletten et al. (1999), Matsen and Thøgersen (2004), Krueger and Kubler (2006) and Miles and Černý (2006)[5].

Governments issue one-period debt, which yields the real interest rate $R_{b,t}$. The government budget constraint in the PAYG country is given by:

$$B_{t+1}^{P} = (1 + R_{b,t})B_t^P + Z_t^P - T_t^P - T_t^{B,P} \tag{8}$$

where public debt per young individual at time t is denoted by B_t^P. Instead of levying taxes on the young, the government in the PAYG country can also use public debt for a while to finance the pension benefits of the elderly. At a later stage, additional contributions ($T_t^{B,P}$) have to be raised to finance the interest obligations on the debt, so as to stabilize debt per worker. In case of a balanced PAYG system, i.e., $Z_t^P = T_t^P$, debt per worker is stabilized at B^P if:

$$T_t^{B,P} = R_{b,t}B^P \tag{9}$$

Note that $R_{b,t}$ is known at time t as both $R_{b,t}^N$ and π_t are already known at the beginning of period t (see Figure 1).

In country F, pension funds invest the contributions of the young (T_t^F) and return them with interest in the next period in the form of transfers to the then old agents (Z_{t+1}^F). The funded scheme is characterised by fixed contributions and the pension fund has the same investment strategy as individuals. This implies that contributions

[5] The idea of this literature is that financial markets are incomplete because there cannot be trade with unborn generations and human capital is not traded. As a result of these missing markets the young are too much exposed to wage risk and the old bear too much financial market risk. In case financial market returns are imperfectly correlated with wages, this results in suboptimal diversification. By linking PAYG pension benefits to wages, retired households obtain a claim to human capital which is not traded on financial markets. In this way PAYG pension schemes can contribute to better intergenerational risk sharing and diversification. In my model, however, wages are perfectly correlated with stock market returns and therefore wage-linked pension benefits will not improve diversification opportunities. To allow for imperfect correlation between labour and capital income one has to add an extra stochastic factor like depreciation risk. This would complicate the analysis to a large extent and therefore PAYG pension benefits are modelled as safe lump-sum benefits. In this way I capture the fact that PAYG pensions are imperfectly correlated with financial market returns and therefore contribute to better diversification.

to the pension scheme are exactly offset by an equal reduction in private savings. The funded pension system is neutral in the sense that the economy behaves in exactly the same way compared to the situation where there is no pension scheme. Therefore, I do not distinguish between contributions to the funded pension scheme and private savings, that is, pension contributions T_t^F are included in total savings S_t^F. Moreover, in contrast to the PAYG country, the pension benefits in the funded country are just as risky as savings.

As pensions in the funded country are organised by pension funds and not by the government, they do not enter the government budget constraint:

$$B_{t+1}^F = (1 + R_{b,t})B_t^F - T_t^{B,F} \tag{10}$$

where B_t^F denotes public debt per young individual. The government in the funded country keeps its debt constant at B^F by raising a debt tax $T_t^{B,F}$ that is equal to:

$$T_t^{B,F} = R_{b,t}B^F \tag{11}$$

It is assumed that the level of government debt is the same in both countries in the initial steady state, i.e., $B^P = B^F = B$.

2.4 Households

Expected lifetime utility of a representative individual born at t is given by the following utility function[6]:

$$E_t U(C_t^{Y,i}, C_{t+1}^{O,i}) = \log(C_t^{Y,i}) + \frac{1}{1+\rho} E_t \left[\log(C_{t+1}^{O,i})\right] \tag{12}$$

where $C_t^{Y,i}$ is consumption when young, $C_{t+1}^{O,i}$ is consumption in the second period of life, and ρ is the rate of time preference. Old-age consumption is uncertain at time t because the rates of return depend on the realizations of A_{t+1} and π_{t+1}.

People can either invest in firm stocks which yield the stochastic return $R_{k,t+1}$ or in government bonds with the stochastic return $R_{b,t+1}$. The share of savings that is invested in equities is denoted by γ_t^i, the return on the investment portfolio is therefore defined as:

$$R_{p,t+1}^i \equiv \gamma_t^i R_{k,t+1} + (1 - \gamma_t^i) R_{b,t+1} \tag{13}$$

[6]In the analytical part I assume a logarithmic utility function to keep the analysis tractable. To quantify the effects of the various shocks I also show some numerical simulation experiments. In these simulations I use Epstein-Zin preferences: $E_t U(C_t^{Y,i}, C_{t+1}^{O,i}) = \{(C_t^{Y,i})^{1-\frac{1}{\sigma}} + \frac{1}{1+\rho}[E_t\{(C_{t+1}^{O,i})^{1-\theta}\}]^{\frac{1-\frac{1}{\sigma}}{1-\theta}}\}^{\frac{1}{1-\frac{1}{\sigma}}}$. The main advantage of using Epstein-Zin preferences is that one can take separate values for the intertemporal substitution elasticity σ and the coefficient of relative risk aversion θ. In this way risk aversion can be increased without changing the intertemporal substitution elasticity, and one can make sure that the economy does not become dynamically inefficient.

Young agents inelastically supply one unit of labour. The consolidated lifetime budget constraint is:

$$C_t^{Y,i} + \frac{C_{t+1}^{O,i}}{1+R_{p,t+1}^i} = W_t - T_t^i - T_t^{B,i} + \frac{Z_{t+1}^i}{1+R_{p,t+1}^i} \tag{14}$$

where I assumed that the additional tax to stabilize government debt ($T_t^{B,i}$) is levied on the young. Maximizing lifetime utility with respect to the lifetime budget constraint gives the Euler condition:

$$1 = \frac{1}{1+\rho}\, C_t^{Y,i}\, E_t\left[\left(C_{t+1}^{O,i}\right)^{-1}(1+R_{j,t+1})\right] \tag{15}$$

where $j = p, k, b$. To derive a solution for the portfolio choice γ_t^i, I follow the approach of Hansen and Singleton (1983) and Campbell and Viceira (2002) and assume that the joint distribution of consumption and gross returns is lognormal. Optimal portfolio choice in the PAYG- and the funded country, γ_t^P and γ_t^F, are given by (see Appendix B for the details):

$$\gamma_t^P = \frac{\log E_t(1+R_{k,t+1}) - \log E_t(1+R_{b,t+1})}{(1-z_t)\sigma_{k-b,t}^2} - \frac{\sigma_{k-b,bt}}{\sigma_{k-b,t}^2} \tag{16}$$

$$\gamma_t^F = \frac{\log E_t(1+R_{k,t+1}) - \log E_t(1+R_{b,t+1})}{\sigma_{k-b,t}^2} - \frac{\sigma_{k-b,bt}}{\sigma_{k-b,t}^2} \tag{17}$$

where $\sigma_{k-b,t}^2$ is the variance of the excess log return of stocks over bonds, $\sigma_{k-b,bt}$ is the covariance between the excess log return and the log return on bonds, and z_t is equal to:

$$z_t = \frac{Z_{t+1}^P}{E_t(1+R_{p,t+1}^P)\exp(-\frac{1}{2}(\sigma_{pt}^2)^P)S_t^P + Z_{t+1}^P} \tag{18}$$

The denominator can be interpreted as the expected consumption in the second period of life. The $\exp(\cdot)$ term results from the assumption that the gross returns are lognormally distributed, see Appendix B for more details. The term z_t therefore represents the part of expected old-age consumption financed by PAYG pensions.

Comparing equations (16) and (17), it is easy to see that people in the country with the PAYG pension scheme invest relatively more in the risky asset, i.e., $\gamma_t^P > \gamma_t^F$. The reason is that the safe PAYG pension benefit reduces the variance of old-age consumption.

Optimal savings are given by (see Appendix C for details):

$$\begin{aligned}
S_t^P &= \frac{\exp\left[\frac{1}{2}\,z_t^2(\sigma_{pt}^2)^P\right]}{1+\rho+\exp\left[\frac{1}{2}\,z_t^2(\sigma_{pt}^2)^P\right]}\left(W_t - T_t^P - T_t^{B,P}\right) \\
&\quad - \frac{1+\rho}{1+\rho+\exp\left[\frac{1}{2}\,z_t^2(\sigma_{pt}^2)^P\right]}\;\frac{Z_{t+1}^P}{E_t(1+R_{p,t+1}^P)\exp[-\frac{1}{2}(\sigma_{pt}^2)^P]} \qquad (19) \\
S_t^F &= \frac{1}{2+\rho}\left(W_t - T_t^{B,F}\right) \qquad (20)
\end{aligned}$$

In general, a higher level of uncertainty affects savings in two ways (see for example Sandmo (1970) and Rothschild and Stiglitz (1971)). First, there is an income effect (precautionary savings) which induces people to save more if uncertainty increases. Secondly, there is a substitution effect: An increase in the degree of risk makes the consumer less inclined to expose his or her resources to the possibility of loss, so that savings fall. Without a PAYG pension scheme, the assumption of a logarithmic utility function implies that the income effect and the substitution effect exactly offset each other as the coefficient of relative risk aversion equals one. Savings in the funded country therefore do not react to changes in uncertainty (see equation (20)). The fact that people in country P receive a safe PAYG pension benefit during retirement makes that they act like a consumer with a coefficient of relative risk aversion between 0 and 1. In that case the substitution effect dominates the income effect, so that savings in the PAYG country fall when the risk on the portfolio rises[7].

The logarithmic utility function also means that the intertemporal elasticity of substitutions is equal to one, implying that optimal savings in country F do not depend on the rates of return. For the same reason, optimal savings in country P only react to changes in the portfolio return because it changes the net present value of the pension benefit. A higher portfolio return decreases the net present value of Z_{t+1}^P and therefore affects savings in the PAYG country positively.

2.5 Equilibrium in the international capital market

Individuals invest their savings either in the home country or abroad. The international capital market is in equilibrium when total savings at time t finance the capital stock and the total amount of government debt in both countries in the next period:

$$S_t^P + \nu S_t^F = (1+\nu)K_{t+1} + (B_{t+1}^P + \nu B_{t+1}^F) \qquad (21)$$

[7] Actually, there is also a second-order effect via the safe PAYG benefit as this reduces the variance of old-age consumption. See Appendix D for details.

Moreover, the portfolio allocation has to be such that the right amount of savings goes to the capital stock and government debt:

$$\gamma_t^P S_t^P + \nu \gamma_t^F S_t^F = (1+\nu) K_{t+1} \quad (22)$$

$$(1-\gamma_t^P) S_t^P + \nu (1-\gamma_t^F) S_t^F = B_{t+1}^P + \nu B_{t+1}^F \quad (23)$$

where one of the equations is redundant. This implies that there are two equilibrium conditions and the predetermined variables K_{t+1} and $R_{b,t+1}^N$ adjust to make sure that these equilibrium conditions are satisfied[8]. As old-age consumption in country P is partly financed by a transfer from the young, while in country F old-age consumption has to be completely financed by savings, the latter country has higher savings, implying that country F exports capital abroad. The higher savings in the funded country combined with the fact that people in this country invest a larger part of their savings in government bonds also implies that country F finances part of the government debt of country P.

3 Government debt

As explained in the Introduction, the major threat to long-term fiscal solvency in Europe is the ageing of the population in the coming decades. This demographic pressure on public finances will be especially large in countries that have large public pension schemes financed on the basis of pay-as-you-go. Population ageing will increase the relative number of elderly compared to the number of young people, implying that PAYG contributions have to rise to a large extent if the government does not want to reduce pension benefits. These PAYG countries might therefore decide to finance their higher pension obligations by issuing more debt instead of raising PAYG contributions or lowering pension benefits. The purpose of this section is to analyse the (international) effects of the use of government debt by the PAYG country to temporarily finance its pension obligations. To keep the results tractable, I leave population ageing out of the analysis. The main results do not change if the rise in government debt evolves endogenously after an increase in the relative number of elderly people. The interested reader is referred to Adema (2008*a*) where this case is considered.

Suppose the government in the PAYG country temporarily uses debt to (partly) finance its pension obligations. This implies that contributions to the PAYG scheme fall short of the pension benefits ($T_t^P \neq Z_t^P$) for a while and this is modelled as follows:

$$T_t^P = \mu_t Z_t^P$$

[8]I checked for stability by verifying whether the two eigenvalues of the dynamic system given in equations (21) and (22) were within the unit circle. This was the case for realistic parameter values.

where $\mu_t \leq 1$. To calculate the effect of public debt over time analytically, I employ the method of comparative dynamics developed by Judd (1982). The process for μ_t is given by:

$$\mu_t = 1 + \zeta f_t$$

where $f_t \leq 0$ describes the time pattern of a perturbation of μ_t from its steady state value ($= 1$) and ζ reflects the magnitude of this perturbation. The effects of the use of government debt can be traced by linearizing the various equations with respect to ζ around the initial steady state. The change in PAYG contributions T_t^P is equal to:

$$\frac{\partial T_t^P}{\partial \zeta} = Z^P f_t \tag{24}$$

The government can only increase its debt for a certain number of periods, otherwise the government debt dynamics becomes unstable. Assume that the government uses debt until period t^* and from period $t^* + 1$ onwards PAYG contributions are back on their old level and the PAYG scheme is balanced again:

$$f_t = \begin{cases} \in [-1, 0) & \text{for } t = [0, t^*] \\ 0 & \text{for } t = [t^* + 1, \infty) \end{cases}$$

As soon as the PAYG scheme is back to balance, the debt tax $T_t^{B,P}$ is raised in such a way that the debt per capita is constant again, but at a higher level than in the initial steady state:

$$\frac{\partial T_t^{B,P}}{\partial \zeta} = \begin{cases} \dfrac{B^P}{1+\pi} \dfrac{\partial R_{b,t}^N}{\partial \zeta} & \text{for } t = [0, t^*] \\ \dfrac{B^P}{1+\pi} \dfrac{\partial R_{b,t}^N}{\partial \zeta} + R_b \dfrac{\partial B_t^P}{\partial \zeta} & \text{for } t = [t^* + 1, \infty) \end{cases} \tag{25}$$

The debt tax in the funded country will also change due to the change of the nominal interest rate on government debt:

$$\frac{\partial T_t^{B,F}}{\partial \zeta} = \frac{B^F}{1+\pi} \frac{\partial R_{b,t}^N}{\partial \zeta} \tag{26}$$

The debt dynamics for the PAYG country are derived by linearizing the government budget constraint (8):

$$\frac{\partial B_{t+1}^P}{\partial \zeta} = (1 + R_b) \frac{\partial B_t^P}{\partial \zeta} + \frac{B^P}{1+\pi} \frac{\partial R_{b,t}^N}{\partial \zeta} - \frac{\partial T_t^P}{\partial \zeta} - \frac{\partial T_t^{B,P}}{\partial \zeta} \tag{27}$$

We can now derive the dynamic equations for the two predetermined variables, the capital-labour ratio and the nominal return on public debt.

3.1 The change in the capital-labour ratio and the nominal return on debt

The first-order difference equations for the two predetermined variables K_{t+1} and $R^N_{b,t+1}$ are as follows (see Appendix D for details):

$$\frac{\partial K_{t+1}}{\partial \zeta} = \underbrace{\frac{\alpha W}{\Psi K}\frac{\partial K_t}{\partial \zeta}}_{5} \underbrace{- \frac{\Omega \nu}{\Psi^F}\frac{\partial T_t^{B,F}}{\partial \zeta} - \frac{1}{\Psi^P}\frac{\partial T_t^{B,P}}{\partial \zeta}}_{4} \underbrace{- \frac{1}{\Psi^P}\frac{\partial T_t^P}{\partial \zeta}}_{2}$$
$$- \underbrace{\frac{1}{1+\nu - S^P_{\sigma_p^2}\Delta_{\sigma_p^2} - S^P_{R_p}\Delta_{R_p}}}_{>0}\left[\underbrace{\frac{\partial B^P_{t+1}}{\partial \zeta}}_{1} - \Omega_{B^P}\frac{\partial B^P_{t+1}}{\partial \zeta}\right] \tag{28}$$

$$\frac{\partial S_t^F}{\partial \zeta} = \underbrace{\frac{\alpha W}{(2+\rho)K}\frac{\partial K_t}{\partial \zeta}}_{5} \underbrace{- \frac{1}{2+\rho}\frac{\partial T_t^{B,F}}{\partial \zeta}}_{4} \tag{29}$$

$$\frac{\partial R^N_{b,t+1}}{\partial \zeta} = -\Phi\frac{\partial K_{t+1}}{\partial \zeta} - \frac{\nu(\gamma^P - \gamma^F)\sigma^2_{k-b}(1+R^N_b)}{\nu S^F + \frac{S^P}{1-z}}\frac{\partial S_t^F}{\partial \zeta}$$
$$+ \underbrace{\frac{\gamma^P \sigma^2_{k-b}(1+R^N_b)}{\nu S^F + \frac{S^P}{1-z}}\frac{\partial B^P_{t+1}}{\partial \zeta}}_{3} \tag{30}$$

where Ψ, Ψ^F, Ψ^P, Ω, $S^P_{\sigma_p^2}$, $\Delta_{\sigma_p^2}$, $S^P_{R_p}$, Δ_{R_p}, Ω_{B^P}, and Φ are defined in Appendix D.

Equations (28) and (30) show the change in the capital-labour ratio and the nominal interest rate over time after an increase in goverment debt in the PAYG country when the two economies have a joint capital market. To analyze the international spillover effects, I derive the same kind of equations for the case where the two economies are closed. Obviously, in country F nothing happens when it is a closed economy, as pensions are arranged by private pension funds and no government debt is used to temporarily finance the pensions. By comparing the results in the closed-economy case with the effects when the two countries have integrated capital markets, I derive the pure spillover effects of the use of government debt in a common capital market[9]. I do not show the system of dynamic equations for the closed-economy case, however, as the system above can easily be used to explain the spillover effects.

Suppose the government in the PAYG country decides at $t = 0$ to use debt to (partly) finance its pension obligations during one period (so $t^* = 0$). This implies that the level of government debt increases at $t = 1$ and stays at this higher level afterwards. Public debt has a direct crowding-out effect on the capital stock (this effect is indicated by term 1 in equation (28)). The lower PAYG contributions at $t = 0$, on the other hand, induce higher savings and this positively affects the capital-labour ratio in period

[9] To exclude the effects of integration, it is assumed that the initial steady state is the same in all cases.

$t = 1$ (term 2 in equation (28)). The negative effect of public debt on the capital-labour ratio is larger than the positive effect that results from the rise in savings, however, so that the capital-labour ratio decreases at $t = 1$ (this is shown formally in Appendix E).[10]

To finance the higher level of government debt the nominal return on bonds will rise (term 3 in equation (30)), to induce people to invest a larger part of their savings in government bonds. This effect is reinforced by the fall in the capital stock as this implies that individuals have to reallocate their investment portfolio even more towards government bonds to have a clearing capital market (see the first term in equation (30); if the capital-labour ratio K_1 falls, $R^N_{b,1}$ will rise).

In case of an integrated capital market, the higher interest rate on government debt will increase the interest obligations on the debt at $t = 1$ in both countries, which increases the debt tax $T_t^{B,i}$. In the PAYG country the debt tax will rise to a larger extent as it also has to stabilize a higher level of debt (see equation (25)). A higher debt tax decreases the disposable income of people and reduces savings (term 4). This negative effect on savings is reinforced by the fact that a lower capital-labour ratio decreases wages (term 5). Both these effects imply that the capital-labour ratio continues to decline and the nominal return on government bonds continues to rise.

Figures 2 and 3 show some numerical simulation experiments to illustrate the mechanics of the model.[11] And we have the following result:

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[10]From equation (28) we can see that besides the direct negative crowding-out effect, government debt also has a positive effect on capital accumulation (the last term in equation (28)). This effect arises because people in the PAYG country also adjust their savings in response to changes in the portfolio return and the variance of the portfolio (see equation (19)). Simulations show, however, that Ω_{B^P} is fairly small. This indirect positive savings effect is therefore negligible and is dominated by the direct negative crowding-out effect of debt.

[11]The simulation graphs show the non-linear transition path where also the indirect effects of the portfolio return, the portfolio variance and savings on z_t are taken into account. Moreover, I use an Epstein-Zin utility function: $E_t U(C_t^{Y,i}; C_{t+1}^{O,i}) = \{(C_t^{Y,i})^{1-\frac{1}{\sigma}} + \frac{1}{1+\rho}[E_t\{(C_{t+1}^{O,i})^{1-\theta}\}]^{\frac{1-\frac{1}{\sigma}}{1-\theta}}\}^{\frac{1}{1-\frac{1}{\sigma}}}$ with values for the intertemporal substitution elasticity $\sigma = 1$ and the coefficient of relative risk aversion $\theta = 3$ to better account for the effects of risk. The countries are of equal size ($\nu = 1$) and the initial value of the PAYG contribution rate is 0.14: $T^P = 0.14W$. PAYG contributions are lowered by 20% at $t = 0$ during one period, i.e., $f_0 = -0.2$. I used the following production function $F(A_t, K_t) = A_t K_t^{0.3}$, with $E(A) = 1$. Capital fully depreciates after one period ($\delta = 1$) and $E(\pi) = 0$. Agents are relatively patient with a time preference rate of 1% per year, which gives $\rho = (1.01)^{30} - 1 = 0.3478$ when one period is assumed to equal 30 years. The initial level of government debt is chosen in such a way that there is still an equilibrium after the debt increase. I take $\sigma_a^2 = 0.2$ and $\sigma_\pi^2 = 0.01$, which roughly corresponds to an annual standard deviation of 8.2% for stock returns and 1.8% for bond returns, assuming that the returns are serially uncorrelated. Here I follow Campbell and Viceira (2005) who show that returns on stocks are significantly less volatile when the investment horizon is long and inflation risk on nominal bonds increases with the investment horizon. The qualitative results of these simulations are robust for changes in the adopted parameter values.

Figure 2: Change in K_t

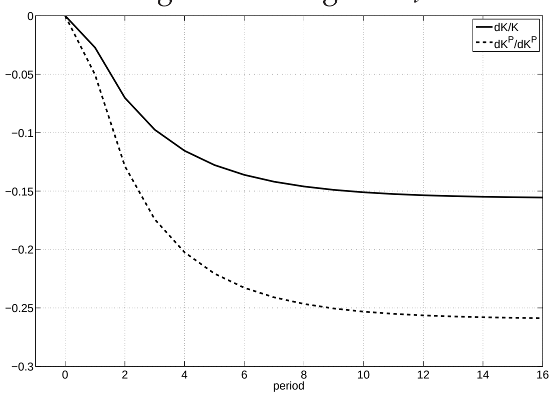
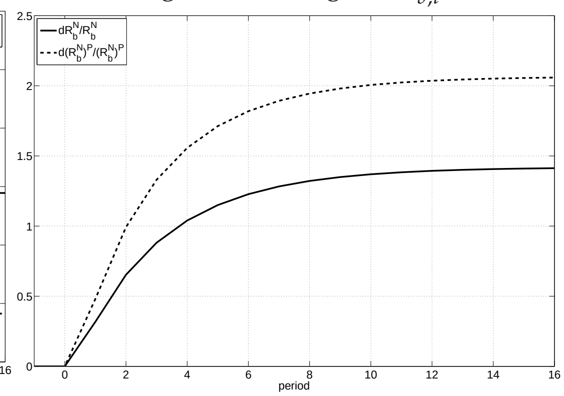


Figure 3: Change in $R_{b,t}^N$



Notes: These graphs show the relative changes in the capital-labour ratio and the nominal interest rate on government debt over time after an increase in debt in the PAYG country. The solid lines refer to the case where one country uses a funded pension scheme and the other country relies on a PAYG pension system. The dotted lines show the changes of the variables in case the PAYG country is closed.

Result 1 *In case country P uses debt to temporarily finance (part of) its pension benefits, the fall in the capital-labour ratio and the rise in the nominal interest rate on government debt are smaller in country P if it has a common capital market with a funded country compared to the case where it is closed.*

The reason for this result is that in case of an integrated capital market part of the public debt in country P is financed with savings of country F and the crowding-out of the capital stock will be smaller.

3.2 The change in utility

As soon as we have determined the changes in the two predetermined variables, the capital-labour ratio and the nominal return on bonds, we can derive the changes in all other variables. Appendix F presents the analytical expressions and the full interpretation for the change in consumption and utility in both countries. Here, I only show simulation graphs and I will focus on the main mechanisms behind the results. Figures 4 and 5 show the welfare effects over time if the PAYG country uses government debt to temporarily finance its pensions. Figure 4 shows the welfare effects for the PAYG country. The PAYG tax is lowered for one period at $t = 0$ and this creates a windfall gain for the generation born in this period and their welfare rises. Over time, the additional public debt crowds out part of the capital stock. Moreover, the return on government bonds has to increase in response to the rise in the debt level. As explained in Appendix F, a lower capital-labour ratio has unambiguously negative utility effects in the long run in a dynamically-efficient economy that is a net borrower on the international capital market. The rise in the nominal return on bonds

Figure 4: Welfare effects country *P*

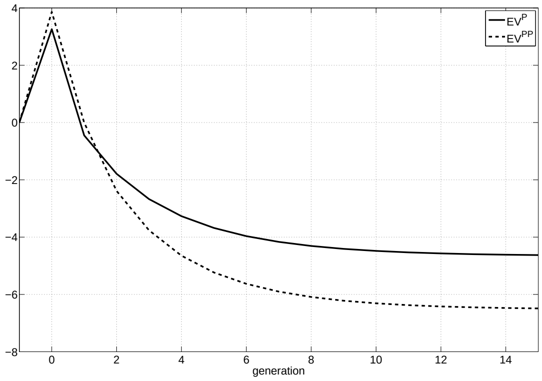
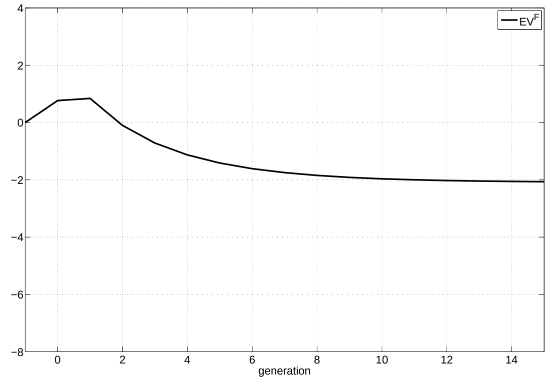


Figure 5: Welfare effects country *F*



Notes: These graphs show the welfare effects of an increase in government debt in the PAYG country, measured in terms of consumption equivalent variation. The y-axis shows the change in first-period consumption as a percentage of the wage that is needed to make an agent in the old equilibrium as well off as after the rise in debt. A distinction is made between the case where the PAYG country is closed (dotted line) and where it shares the capital market with the funded country (solid lines).

has adverse utility effects as well, because this increases the interest obligations on the debt and thus the debt tax that is levied on the working people. The long-run spillover effects for the PAYG country are given in the following result:

Result 2 *In case country P uses debt to temporarily finance (part of) its pensions, the long-run welfare losses in country P are smaller if it shares its capital market with country F, i.e., country P experiences positive spillovers in the long run.*

In the simulations the long-run welfare loss in the PAYG country is about 6.5 percent in the closed-economy case, while welfare declines by 4.7 percent in the case of integrated capital markets. The reason for this result is that the funded country absorbs part of the extra government debt so that the crowding out of capital is less and the interest rate on government debt rises to a lesser extent (see Figures 2 and 3 and Result 1). This implies that it is easier for the government in the PAYG country to temporarily finance its pensions with debt in case it forms a monetary union with a country that uses a funded system instead of a PAYG scheme.

The increase in public debt in the PAYG country will affect people living in the funded country through the change in factor prices and the interest rate on government bonds. The welfare effects for the various generations in the funded country are shown in Figure 5. The fact that people in the funded country partly finance the higher government debt in the PAYG country implies that they also experience a falling capital-labour ratio (see Figure 2). As explained in Appendix F, a lower capital-labour ratio will have negative long-run utility effects in the funded country for realistic parameter values. Moreover, the government in the funded country will also be confronted with a higher interest rate to be paid on its debt. The following result describes the spillovers for the funded country:

Result 3 *The rise in public debt in country P leads to: (i) Negative spillovers effects for country F in the long run. (ii) The initial generations in country F, however, experience positive spillovers.*

In the simulations the long-run welfare loss for the funded country is about 2 percent. The reason for the positive spillovers in the short run is that these initial generations only enjoy the gains from a higher portfolio return, while they do not (fully) incur the losses that result from the lower wages.

In the simulation graphs, the two economies have the same size ($\nu = 1$). However, as most EMU countries mainly use PAYG pension schemes, the group of countries that has sizeable funded pensions can be considered as relatively small. This means that $\nu < 1$ and when a large group of countries with extensive PAYG schemes uses public debt to temporarily finance their pensions, the spillover effects for the funded countries will be larger. For the PAYG countries, however, it holds that the larger they are relative to the funded countries, the more the effects resemble the effects of a closed economy.

As explained in the Introduction, Persson (1985) also considers the effects of government debt in a two-country model, but then in a deterministic setting. The following result summarizes the difference in welfare effects of debt between a stochastic and a deterministic set-up:

Result 4 *For a given rise in debt in country P, the negative long-run welfare effects in both countries are larger in the deterministic case compared to the stochastic case.*

The intuition for this result is as follows. In my stochastic model an increase in government debt also implies that the overall riskiness on the portfolio σ_{pt}^2 declines, as people hold relatively more bonds in their investment portfolio (γ decreases). The decline in the portfolio variance dampens the negative utility effects to some extent because individuals are risk averse, see also equation (72) in Appendix F. This effect is not present when there is no risk in the model and therefore the negative long-run utility effects of a given rise in public debt are larger in the deterministic case.

4 Unexpected inflation

High levels of nominal government debt give governments an incentive to lobby for surprise inflation at the central bank as this will reduce the fiscal burden of debt service. This section analyses the effects of such an unexpected inflation shock. I distinguish between the situation of a closed economy and a two-country setting where the two economies differ in the degree of funding of their pension schemes. It is shown that in the closed-economy set-up unexpected inflation is a zero-sum game, while in the latter case there is a conflict of interest on the creation of inflation between the two countries. To isolate the effects of unexpected inflation, the effects of government debt are not explicitly modelled in the analytical analysis below.

4.1 Closed economy

This section considers the effects of surprise inflation in a closed-economy setting. This case can be regarded as the situation in which both countries adopt the same type of pension scheme because this implies that there will be no capital flows between the two countries.

Define the inflation factor as:

$$\beta_t = \frac{1}{1+\pi_t} \tag{31}$$

The time pattern of β_t is as follows:

$$\beta_t = \beta + \zeta g_t \tag{32}$$

A rise in the inflation rate is reflected by a negative value of g_t. The effects of inflation can be analysed by taking the derivative with respect to ζ.

If inflation rises unexpectedly, the expected real return and the realised real return on government bonds will differ[12]:

$$\frac{\partial R_{b,t+1}}{\partial \zeta} = \frac{\partial E_t R_{b,t+1}}{\partial \zeta} + (1+R_b^N)\left[\frac{\partial \beta_{t+1}}{\partial \zeta} - \frac{\partial E_t \beta_{t+1}}{\partial \zeta}\right] \tag{33}$$

When people do not expect any inflation ($\frac{\partial E_t \beta_{t+1}}{\partial \zeta} = 0$) and inflation rises unexpectedly ($\frac{\partial \beta_{t+1}}{\partial \zeta} < 0$), the ex post real return on bonds will fall, while nothing has happened with the ex ante expected return.

Consumers have static expectations; their inflation expectations are based on the current inflation rate[13]:

$$\frac{\partial E_t \beta_{t+1}}{\partial \zeta} = \frac{\partial \beta_t}{\partial \zeta} \tag{34}$$

This implies that inflation can only be raised unexpectedly during one period, after that people adjust their inflation expectations.

[12] It is assumed that in the steady state $\beta = E(\beta)$.

[13] The main point is that investors could have the wrong inflation forecast: inflation is higher than what they expected when they bought their nominal government bonds. This assumption is not very unrealistic as one period is about 30 years. Over longer time periods the risk of inflation is substantially higher (see Campbell and Viceira, 2005).

Unexpected inflation affects the financial position of the government positively as it decreases the real value of government debt and the real interest rate to be paid on the debt. People that invested in government bonds lose from the unexpected fall in the real return on government bonds, however. There are various options for the government concerning how to use the interest gain. It can give the benefit to the young by lowering the debt tax in the same period or it can repay part of its debt so that future generations gain. In both these scenarios however, the elderly who own the government bonds will lose. The third possibility would therefore be to compensate these elderly. This last policy option will be analysed below.

Suppose that inflation rises unexpectedly at time τ. The real interest payments on the debt decrease by $-B(1+R_b^N)\frac{\partial \beta_\tau}{\partial \zeta}$. In case the elderly receive this entire gain, the compensation to the elderly Z_t^B is given by:

$$\frac{\partial Z_\tau^B}{\partial \zeta} = -B(1+R_b^N)\frac{\partial \beta_\tau}{\partial \zeta} > 0 \tag{35}$$

The change in old-age consumption is then equal to:

$$\begin{aligned}\frac{\partial C_\tau^O}{\partial \zeta} &= S\frac{\partial R_{p,\tau}}{\partial \zeta} + \frac{\partial Z_\tau^B}{\partial \zeta} \\ &= S(1-\gamma)(1+R_b^N)\frac{\partial \beta_\tau}{\partial \zeta} - B(1+R_b^N)\frac{\partial \beta_\tau}{\partial \zeta} \\ &= (1+R_b^N)\Big[S(1-\gamma) - B\Big]\frac{\partial \beta_\tau}{\partial \zeta} \end{aligned} \tag{36}$$

In a closed economy it holds that $S(1-\gamma) = B$, which implies that $\frac{\partial C_\tau^O}{\partial \zeta} = 0$ and the elderly are exactly compensated for the loss resulting from surprise inflation. Young people at time τ know that the inflation rate is higher and ask a higher nominal rate of return on government bonds, and the real rate of return will be back at its old value. An unexpected inflation shock where the government redistributes its gain to the debt holders that lose from surprise inflation, will therefore have no real effects in a closed-economy set-up. The government can set its policy in such a way that no generation is hurt after an unexpected rise in inflation. This is summarized in the following result:

Result 5 *In a closed-economy setting unexpected inflation is a zero-sum game, i.e., the gain for the government is exactly high enough to compensate the people who lose from the unexpected inflation shock.*

4.2 Open economy

In this subsection I analyse the effects of unexpected inflation in the open-economy case where one country uses a PAYG pension scheme, while the other country relies on funded pensions.

Equation (36) shows the change in old-age consumption in case the entire interest gain is transferred to the old at time τ. In the open-economy case, equation (23) has to hold. Residents of the funded country save more and invest a larger part of their savings in government bonds than people living in the PAYG country, therefore we know that $S^F(1-\gamma^F) > S^P(1-\gamma^P)$. Defining $S^F(1-\gamma^F) \equiv \varphi S^P(1-\gamma^P)$ where $\varphi > 1$, equation (23) can be used to derive[14]:

$$S^P(1-\gamma^P) = \underbrace{\frac{1+\nu}{1+\nu\varphi}}_{<1} B$$

so that $S^P(1-\gamma^P) < B$. This means that part of the debt of country P is financed by people in the funded country. For the PAYG country it therefore holds that the term between squared brackets in equation (36) is negative and combined with $\frac{\partial \beta_\tau}{\partial \zeta} < 0$ we know that:

Result 6 *In the open-economy case where country F finances part of the debt of country P, consumption of the elderly in country P at time τ will increase after an unexpected inflation shock if the government in this country transfers the entire interest gain to these people.*

In the closed-economy set-up this gain was just high enough to keep old-age consumption at time τ constant. In the two-country setting, however, unexpected inflation constitutes a Pareto improvement in the PAYG country; there is at least one generation that gains from the unexpected inflation shock, while no other generation loses.

For the funded country it holds that:

$$S^F(1-\gamma^F) = \underbrace{\frac{1+\nu}{\frac{1}{\varphi}+\nu}}_{>1} B$$

which implies that $S^F(1-\gamma^F) > B$. This shows that people in the funded country do not only finance their own debt, but also part of the debt of the PAYG country. The term between squared brackets in equation (36) is therefore positive for the funded country. So, we have:

Result 7 *In the two-country setting where country F finances part of the debt of country P, old-age consumption in country F at time τ still falls after an unexpected inflation shock even though the government in country F transfers the entire interest gain to the old at time τ.*

So in the funded country there is always at least one generation that experiences welfare losses from unexpected inflation.

[14] It is assumed that the two countries have the same level of government debt in the initial steady state, i.e., $B^F = B^P = B$.

The above analysis shows that in the open-economy set-up the PAYG country gains from an unexpected inflation shock at the expense of the funded country. It takes advantage of the fact that the funded country owns a relatively large part of the total amount of government bonds. The PAYG country can therefore export part of the inflationary tax on debt holders to the funded country, while it still receives the full gain of a lower debt burden and a net gain results. This means that there is a conflict of interest on monetary policy when countries with different pension schemes form a monetary union.

In the coming decades, the ageing of the population will put the public finances more under pressure. The analysis is this section shows that if PAYG countries finance their increased pension obligations by issuing more debt, the incentive of governments in these countries to lobby for surprise inflation will rise. Unexpected inflation will, on the other hand, harm funded countries more if PAYG countries issue large amounts of nominal government debt. The conflict of interest on monetary policy between PAYG- and funded countries will therefore be reinforced if population ageing raises government debt in PAYG countries. If it is not clear to market participants how the central bank will react to these conflicting interests (for example because the decision-making process of the central bank is not completely transparent), there will be uncertainty about what the final outcome for inflation will be. In that case the market might perceive that there is a higher *risk* of inflation when public debt levels are high. This scenario will be analysed in the next section.

5 Inflation risk

This section analyses the effects of the use of government debt by the PAYG country if a higher debt level in this country is associated with more inflation risk. To keep the analysis tractable this relationship between changes in the debt level in the PAYG country and inflation risk is modelled by the following specification:

$$\frac{\partial \sigma^2_{\pi t}}{\partial \zeta} = \lambda \frac{\partial B^P_{t+1}}{\partial \zeta} \tag{37}$$

where λ shows the responsiveness of inflation risk to changes in government debt. Equation (37) reflects the fact that the incentive of PAYG countries to put political pressure on the central bank rises with their debt level, which increases the risk of inflation. Government debt in the funded country does not affect inflation risk, because the funded country does not have an incentive to lobby for surprise inflation as it always experiences a net loss from unexpected inflation (see Section 4). Inflation is a common risk factor for all members of a monetary union. This implies that if a higher debt level in the PAYG country raises perceived inflation risk this will not only affect the PAYG country but also the funded country that did not increase its debt[15].

If a rise in government debt is associated with more inflation risk, the first-order difference equations for K_{t+1} and $R^N_{b,t+1}$ change to:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \left.\frac{\partial K_{t+1}}{\partial \zeta}\right|_{\text{debt}} + \underbrace{\frac{\Omega_{\sigma_b^2}}{(1+\nu) - S^P_{\sigma_p^2}\Delta_{\sigma_p^2} - S^P_{R_p}\Delta_{R_p}}\frac{\partial \sigma^2_{bt}}{\partial \zeta}}_{7} \tag{38}$$

$$\frac{\partial R^N_{b,t+1}}{\partial \zeta} = \left.\frac{\partial R^N_{b,t+1}}{\partial \zeta}\right|_{\text{debt}} + \underbrace{\frac{[(1-\gamma^P)S^P + \nu S^F(1-\gamma^F)](1+R^N_b)}{\nu S^F + \frac{S^P}{1-z}}\frac{\partial \sigma^2_{bt}}{\partial \zeta}}_{6} \tag{39}$$

where:

$$\begin{aligned}\Omega_{\sigma_b^2} \equiv \frac{(1-\gamma^P)S^P + \nu S^F(1-\gamma^F)}{(\nu S^F(1-z) + S^P)\sigma^2_{k-b}} \Big\{ & S^P_{R_p}(1-\gamma^P)E(1+R_b)(1-z)\sigma^2_{k-b} \\ & - S^P_{R_p}[E(R_k) - E(R_b)] - S^P_{\sigma_p^2}(2\gamma^P\sigma^2_{k-b} + 2\sigma_{k-b,b})\Big\} \\ & + S^P_{R_p}\frac{[E(R_k) - E(R_b)](1-\gamma^P)}{\sigma^2_{k-b}} + S^P_{\sigma_p^2}(1-\gamma^P)\left(1+\gamma^P + \frac{2\sigma_{k-b,b}}{\sigma^2_{k-b}}\right)\end{aligned} \tag{40}$$

and:

$$\frac{\partial \sigma^2_{bt}}{\partial \zeta} = \frac{\partial \sigma^2_{\pi t}}{\partial \zeta} \tag{41}$$

The main effect of an increase in inflation risk works through the change in the nominal return on government bonds. Higher inflation risk increases the riskiness of government bonds, which makes them less attractive to hold. In response, the rate of return on bonds will rise relative to the return on equity (i.e., the equity premium falls) to induce people to buy the existing stock of government bonds. In equation (39) this effect is indicated by term 6. This rise in R^N_b increases the interest obligations on government debt in both countries and both countries have to increase their debt tax to stabilize their debt levels. These higher debt taxes in turn imply lower savings, which affects the capital-labour ratio negatively. This leads to the following result:

Result 8 *If a rise in public debt in country P is associated with higher inflation risk both the decline of the capital-labour ratio and the increase of the nominal return on government bonds will be larger compared to the benchmark case analysed in Section 3.*

A rise in inflation risk will also increase the overall riskiness of the total investment portfolio. Savings in the PAYG country react to changes in σ^2_{pt} and therefore the extra

[15]This was actually one the central issues behind the implementation of the Stability and Growth Pact for the Netherlands.

term 7 arises in the first-order difference equation for the capital-labour ratio. Simulations show, however, that the term $\Omega_{\sigma_b^2}$ is relatively small. Inflation risk therefore mainly affects the capital-labour ratio through the rise in R_b^N and the higher debt tax. The fact that individuals are risk averse implies that a higher variance of the portfolio has direct negative utility effects, however (see equation (72) in Appendix F).

Figures 6 and 7 compare the welfare effects in the different cases and we have the following result:

Result 9 *For both countries it holds that the negative long-run welfare effects of a rise in public debt in country P are larger when this also increases the perceived risk of inflation by investors.*

There are three factors that contribute to this result. First, because individuals are risk averse they are negatively affected by the increase of the overall risk on the portfolio. Second, the fact that investors ask a higher return on government bonds implies that debt taxes have to be higher which also has adverse welfare effects. Higher debt taxes also lead to less savings and a lower capital-labour ratio, which is the third factor that contributes to the lower welfare, as this leads to lower wages. In case the PAYG country forms a monetary union with a funded country (solid lines) welfare declines by about one percentage point more in the long run (from -4.7 to -5.7 percent). In the funded country long-run welfare worsens by about 0.3 percentage points (from -2.1 to -2.4 percent). This implies that the negative long-run spillover effects from the use of debt by the PAYG country are larger for the funded country.

If we compare the change in welfare effects for the PAYG country in case it forms a monetary union with the funded country (solid lines) with the closed-economy case (dotted lines), we can infer that the fall in welfare after a rise in inflation risk is larger in the open-economy case. This means that:

Result 10 *Residents in country P are affected more negatively by higher inflation risk when they form a monetary union with country F.*

This results from the fact that the nominal return on government bonds has to rise to a larger extent if the funded country finances part of the debt in the PAYG country compared to the case where the PAYG country completely finances its own debt. The intuition for this result is as follows. People in the funded country do not receive a safe PAYG benefit during retirement, which implies that these people need to be compensated more in order to hold the more risky government bonds. Combined with the fact that the funded country owns a relatively large part of the total amount of government bonds, the rate of return on government bonds has to rise to a larger extent to make sure that all government debt is financed. The rise in the debt tax and its negative consequences will therefore be larger for the PAYG country when it forms

Inflation risk

Figure 6: Welfare effects country P

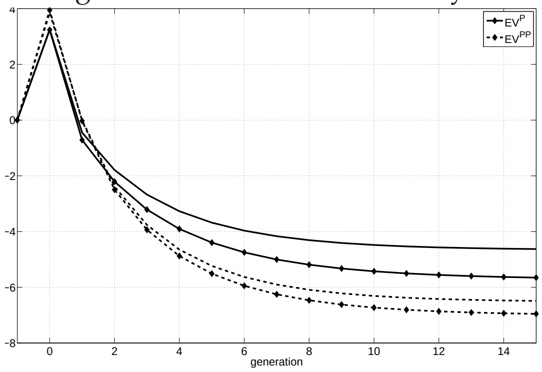
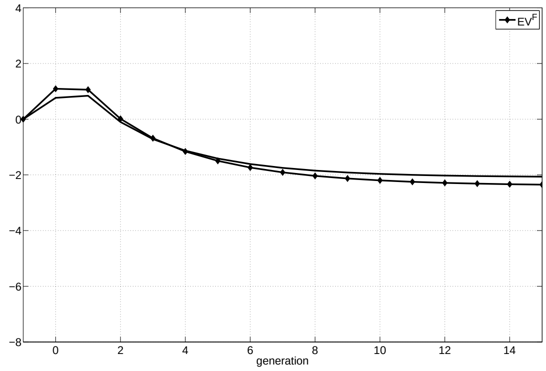


Figure 7: Welfare effects country F



Notes: These graphs show the welfare effects of an increase in government debt in the PAYG country which may raise inflation risk as well. The welfare effects are measured in terms of consumption equivalent variation, that is, the change in first-period consumption as a percentage of the wage that is needed to make an agent in the old equilibrium as well off as after the rise in debt (and inflation risk). A distinction is made between the case where the PAYG country is closed (dotted lines) and where it forms a monetary union with the funded country (solid lines). The lines with the diamonds indicate the welfare effects when not only government debt is used but inflation risk rises as well. Inflation risk (σ_π^2) rises from 0.01 to 0.04, which roughly corresponds to a rise of the annual standard deviation from 1.8% to 3.7%.

a monetary union with a country that uses a fully funded pension scheme instead of a PAYG pension system. This shows that although the PAYG country can shift part of the debt burden to the funded country, it cannot share the adverse effects of higher inflation risk in response to a higher level of public debt with the funded country. The positive long-run spillovers of forming a monetary union with a funded country are therefore smaller for people in the PAYG country if a rise in government debt also raises perceived inflation risk.

6 Conclusion

This paper has analysed the international spillover effects of government debt and the associated risk of inflation within a monetary union where countries rely to a different extent on PAYG pensions. It is shown that in a monetary union, the PAYG country can shift part of its long-run debt burden to the funded country. The funded country is therefore negatively affected in the long run by the use of government debt by the PAYG country. The initial generations in the funded country may, however, be positively affected by the rise in debt in the other country because these individuals only enjoy the gains from the higher rates of return.

High levels of nominal public debt give governments an incentive to lobby for surprise inflation at the central bank because this will reduce the fiscal burden of debt service. In a closed economy, unexpected inflation is only a matter of redistribution from the old to the young- or future generations and can be implemented in such a way that no generation gains or loses. In a monetary union where residents of the funded country finance part of the government debt of the PAYG country, the PAYG country experiences a net gain from unexpected inflation at the cost of the funded country. This implies that there are conflicting interests about the direction of monetary policy when countries with different pension schemes form a monetary union. In response to these conflicting interests investors might perceive that there is a higher *risk* of inflation when the PAYG country increases its debt. In contrast to unexpected inflation, a rise in inflation risk has adverse effects in both countries in the long run. Actually, the PAYG country experiences negative spillover effects from the fact that they form a monetary union with a funded country when inflation risk rises. The negative effects of a rise in inflation risk in response to a higher level of public debt can thus not be shared with the funded country. The fact that the PAYG country cannot share the negative effects of a rise in inflation risk with the funded country implies that it may be in the interest of both countries that government debt levels stay at low levels. This paper therefore shows that, in the light of the sovereign debt crisis in the EMU and the future ageing of the population, it is important that governments clarify which measures they will take to guarantee the sustainability of public finances and that the rules of the Stability and Growth Pact are re-established. Moreover, it is important for all countries that a central bank like the ECB is independent, credible *and* transparent to prevent an increase in inflation risk when debt levels are high. Another implication of the results is that investors in funded countries might require governments in countries that rely on extensive PAYG schemes to issue inflation-linked bonds to protect them against the negative effects of inflation. This is a development you already see in Europe.

Obviously, the model in this paper has oversimplified the real world in many ways. For example it was assumed that PAYG pension benefits are safe to account for the fact that PAYG pensions also serve as a risk-sharing and diversification device. The main results in the paper would still hold if one would change this assumption, however, as long as the funded country finances part of the government debt of the PAYG country. The analysis can also be extended and modified in several interesting directions. It would for example be interesting to study the role of country-specific or default risk. This is left for future research as the goal of this paper was to study the international spillover effects of government debt when this debt might be associated with a rise of a common risk factor, inflation risk. Moreover, in this paper the international capital flows arise endogenously because countries use different pension schemes. This situation was studied because within the EMU differences in financing methods of pension systems abound. There are of course other reasons why capital flows between countries. For example, countries may differ in their rate of time preference. If inhabitants of a certain country are relatively patient compared to the other country, savings will be relatively high in the former country and part of these savings will be invested in the latter country. The analysis in this paper would not change much if one would like to study this situation. The effects for the relatively patient country would resemble the effects for the funded country, while the effects for the more impatient country correspond to the effects for the PAYG country. The analysis in this paper can thus be applied to various settings where saving levels differ between countries, which lead to capital flows. The central result, i.e., that government debt and the associated risk of inflation will have adverse effects in the long run for capital-exporting countries will remain to stand out in a model where the source of capital flows differs from the one in this paper (differences in pension systems).

A Derivation variances

This appendix derives the variances of the log gross returns.

A.1 Return on capital/stocks

Take the logarithm of optimality condition (3) and recall the assumption that $\delta = 1$:

$$\log(1 + R_{k,t}) = \log(\alpha) + \log(A_t) + (\alpha - 1)\log(K_t)$$

Now define $\log(1 + R_{k,t}) \equiv r_{k,t}$ and $\log X_t \equiv x_t$, where X_t can be any variable, with the exception of the returns. We can then write:

$$r_{k,t+1} = \log(\alpha) + a_{t+1} + (\alpha - 1)k_{t+1} \tag{42}$$

The expectation of $r_{k,t+1}$ at time t is:

$$E_t r_{k,t+1} = \log(\alpha) + E_t a_{t+1} + (\alpha - 1)k_{t+1} \tag{43}$$

Using equations (42) and (43) we can derive the variance of $r_{k,t+1}$ given in equation (4). Note that there is no expectations operator E_t in front of k_{t+1} in equation (43). The capital stock is determined by the savings and portfolio decisions made in the previous period. This implies that the capital stock at time $t + 1$ is already known at the end of period t.

A.2 Return on bonds

Taking logs of equation (5) gives:

$$\log(1 + R_{b,t}) = \log(1 + R^N_{b,t}) + \log\left(\frac{1}{1 + \pi_t}\right)$$

which can be used to write:

$$r_{b,t+1} = r^N_{b,t+1} + \log\left(\frac{1}{1+\pi_{t+1}}\right) \tag{44}$$

$$E_t r_{b,t+1} = r^N_{b,t+1} + E_t \log\left(\frac{1}{1+\pi_{t+1}}\right) \tag{45}$$

These two equations can be used to the derive the variance of $r_{b,t+1}$ in equation (6).

B Portfolio choice

This appendix derives the solution for portfolio choice γ^i_t, $i = P, F$. Following the approach of Hansen and Singleton (1983) and Campbell and Viceira (2002), I assume that the joint distribution of consumption and gross returns is lognormal. For a log-normal random variable X it holds that:

$$\log E_t X_{t+1} = E_t \log X_{t+1} + \frac{1}{2}\mathrm{Var}_t \log X_{t+1} \tag{46}$$

Using this condition, the log form of the portfolio-return Euler condition, i.e., $j = p$ in equation (15), can be written as:

$$\begin{aligned}\log 1 = \; & \log\left(\frac{1}{1+\rho}\right) + \log(C^{Y,i}_t) + E_t\left[-\log(C^{O,i}_{t+1}) + \log(1+R^i_{p,t+1})\right] \\ & + \frac{1}{2}\mathrm{Var}_t\left[-\log(C^{O,i}_{t+1}) + \log(1+R^i_{p,t+1})\right]\end{aligned}$$

which can be rewritten to:

$$E_t c^{o,i}_{t+1} - c^{y,i}_t = \log\left(\frac{1}{1+\rho}\right) + E_t r^i_{p,t+1} + \frac{1}{2}\,(\sigma^2_{c^o t})^i + \frac{1}{2}\,(\sigma^2_{pt})^i - \mathrm{Cov}_t\left(c^{o,i}_{t+1}; r^i_{p,t+1}\right) \tag{47}$$

where $(\sigma^2_{c^o t})^i \equiv \mathrm{Var}_t\left[\log(C^{O,i}_{t+1})\right]$ and $(\sigma^2_{pt})^i \equiv \mathrm{Var}_t\left[\log(1+R^i_{p,t+1})\right]$.

In the same way we can derive the log form of the Euler equation of the return on bonds, the so-called benchmark-return Euler condition:

$$E_t c^{o,i}_{t+1} - c^{y,i}_t = \log\left(\frac{1}{1+\rho}\right) + E_t r_{b,t+1} + \frac{1}{2}\,(\sigma^2_{c^o t})^i + \frac{1}{2}\,\sigma^2_{bt} - \mathrm{Cov}_t\left(c^{o,i}_{t+1}; r_{b,t+1}\right) \tag{48}$$

Subtracting the benchmark-return equation (48) from the portfolio-return equation (47) gives:

$$E_t r^i_{p,t+1} - E_t r_{b,t+1} + \frac{1}{2}((\sigma^2_{pt})^i - \sigma^2_{bt}) = \mathrm{Cov}_t(c^{o,i}_{t+1}; r^i_{p,t+1}) - \mathrm{Cov}_t(c^{o,i}_{t+1}; r_{b,t+1}) \tag{49}$$

First I derive the terms on the left-hand side of equation (49). To derive $E_t r^i_{p,t+1}$ we need to relate the log portfolio return to the log returns on stocks and bonds. As the simple return on the portfolio is a linear combination of the simple returns on stocks and bonds (see equation (13)) and the log of a linear combination is not the same as a linear combination of logs, I follow Campbell and Viceira (2002) and use a second-order Taylor approximation of the nonlinear function relating the log individual-asset returns to the log portfolio return[16]. First rewrite equation (13) to:

$$\frac{1+R^i_{p,t+1}}{1+R_{b,t+1}} = 1 + \gamma^i_t \left[\frac{1+R_{k,t+1}}{1+R_{b,t+1}} - 1\right]$$

Taking logs gives:

$$r^i_{p,t+1} - r_{b,t+1} = \underbrace{\log\left[1 + \gamma^i_t(\exp(r_{k,t+1} - r_{b,t+1}) - 1)\right]}_{=f(r_{k,t+1}-r_{b,t+1})} \tag{50}$$

The function $f(\cdot)$ is approximated using a second-order Taylor expansion around the point $r_{k,t+1} - r_{b,t+1} = 0$, so that equation (50) can be written as:

$$r^i_{p,t+1} \approx r_{b,t+1} + \gamma^i_t(r_{k,t+1} - r_{b,t+1}) + \frac{1}{2}\gamma^i_t(1-\gamma^i_t)\sigma^2_{k-b,t} \tag{51}$$

where $\sigma^2_{k-b,t} \equiv \text{Var}_t[\log(1+R_{k,t+1}) - \log(1+R_{b,t+1})] = \sigma^2_{kt} + \sigma^2_{bt} - 2\sigma_{kb,t}$. From equation (51) we know that:

$$E_t r^i_{p,t+1} \approx E_t r_{b,t+1} + \gamma^i_t(E_t r_{k,t+1} - E_t r_{b,t+1}) + \frac{1}{2}\gamma^i_t(1-\gamma^i_t)\sigma^2_{k-b,t} \tag{52}$$

Equations (51) and (52) can be used to derive the variance of the log gross portfolio return:

$$(\sigma^2_{pt})^i = \sigma^2_{bt} + (\gamma^i_t)^2\sigma^2_{k-b,t} + 2\gamma^i_t\sigma_{k-b,bt} \tag{53}$$

where $\sigma_{k-b,bt} \equiv \text{Cov}_t[\log(1+R_{k,t+1}) - \log(1+R_{b,t+1}); \log(1+R_{b,t+1})] = \sigma_{kb,t} - \sigma^2_{bt}$. The expressions for $E_t r^i_{p,t+1}$ and $(\sigma^2_{pt})^i$ in equations (52) and (53) will be substituted into the left-hand side of equation (49).

The lifetime budget constraint of an individual is used to derive the covariances between old-age consumption and the returns on the right-hand side of equation (49). Rewriting equation (14) and taking logs gives:

$$c^{o,i}_{t+1} = r^i_{p,t+1} + \log\left[\exp w_t - \exp \tau^i_t - \exp \tau^{B,i}_t + \exp(z^i_{t+1} - r^i_{p,t+1}) - \exp c^{y,i}_t\right]$$

For the funded country it holds that $Z^F_{t+1} = (1+R^F_{p,t+1})T^F_t$, implying that the pension terms in the above equations can be substituted out. Combined with the fact that a

[16]This approximation holds exactly in continuous time and is an accurate approximation over short discrete time periods. The question is, however, whether this approximation can still be used in a two-period OLG model, where one period is around 30 years. Viceira (2001), Campbell and Viceira (2002) and Barberis (2000) show that the magnitude of the horizon effects are negligible, which implies that the approximation is still satisfactory for longer holding periods. Own simulations also indicate that the error is 0.7% at most.

logarithmic utility function implies that agents consume a fixed proportion of their wealth, it is not necessary to take a first-order approximation of the consumer's budget constraint to obtain the covariance between the returns and old-age consumption. See Appendix 5.A in Adema (2008*a*) for more details. For the PAYG country I approximate the term between the brackets with a first-order Taylor expansion around $r^P_{p,t+1} = E_t r^P_{p,t+1}$[17]:

$$c^{o,P}_{t+1} \approx r^P_{p,t+1} + \log \underbrace{\left[\exp w_t - \exp \tau^P_t - \exp \tau^{B,P}_t + \exp(z^P_{t+1} - E_t r^P_{p,t+1}) - \exp c^{y,P}_t\right]}_{\equiv l_t}$$
$$- \underbrace{\frac{\exp(z^P_{t+1} - E_t r^P_{p,t+1})}{l_t}}_{\equiv z_t} (r^P_{p,t+1} - E_t r^P_{p,t+1}) \tag{54}$$

The term z_t can be rewritten to:

$$z_t = \frac{Z^P_{t+1}}{E_t(1 + R^P_{p,t+1}) \exp(-\frac{1}{2}(\sigma^2_{pt})^P)\, S^P_t + Z^P_{t+1}} \approx \frac{\text{PAYG pension benefit}}{\text{expected old-age consumption}} \tag{55}$$

where I used the fact that $E_t r^P_{p,t+1} = E_t \log(1 + R^P_{p,t+1}) = \log E_t(1 + R^P_{p,t+1}) - \frac{1}{2}(\sigma^2_{pt})^P$ and $S^P_t = W_t - T^P_t - T^{B,P}_t - C^{y,P}_t$. The term z_t is the part of expected old-age consumption financed by PAYG pensions.

Equation (54) can be used to derive the variance of the log of old-age consumption and the covariance between the log of old-age consumption and the log gross returns:

$$(\sigma^2_{c^o t})^P = (1 - z_t)^2 (\sigma^2_{pt})^P \tag{56}$$
$$\mathrm{Cov}_t(c^{o,P}_{t+1}; r^P_{p,t+1}) = (1 - z_t)(\sigma^2_{pt})^P \tag{57}$$
$$\mathrm{Cov}_t(c^{o,P}_{t+1}; r_{b,t+1}) = (1 - z_t)\sigma^2_{bt} + (1 - z_t)\gamma^P_t \sigma_{k-b,bt} \tag{58}$$

The expressions for the funded country are the same, except that $z_t = 0$. Equation (56) shows very clearly that safe PAYG pension benefits lower the variance of old-age consumption.

Substituting equations (52), (53), (57) and (58) into equation (49) gives:

$$\gamma^i_t(E_t r_{k,t+1} - E_t r_{b,t+1}) + \frac{1}{2}\gamma^i_t \sigma^2_{k-b,t} + \gamma^i_t \sigma_{k-b,bt} = (1 - z_t)(\gamma^i_t)^2 \sigma^2_{k-b,t} + (1 - z_t)\gamma^i_t \sigma_{k-b,bt}$$

Dividing by γ^i_t and rearranging gives a solution for portfolio choice:

$$\gamma^i_t = \frac{E_t r_{k,t+1} - E_t r_{b,t+1} + \frac{1}{2}\sigma^2_{k-b,t}}{(1 - z_t)\sigma^2_{k-b,t}} + \frac{z_t}{1 - z_t}\frac{\sigma_{k-b,bt}}{\sigma^2_{k-b,t}}$$

[17] This is comparable to the approximation of a linear intertemporal budget constraint in Campbell (1993).

Using the fact that the gross returns on stocks and bonds are lognormally distributed gives the expressions for γ_t^P and γ_t^F $(z_t = 0)$ in terms of simple returns (equations (16) and (17)).

C Savings

This appendix derives the solutions for savings given in equations (19) and (20). As the optimal condition for the funded country can be directly deduced from the optimal savings condition for the PAYG country, setting $z_t = T_t^P = Z_{t+1}^P = 0$, I only derive the optimal condition for individuals living in the PAYG country.

Using equation (54) I can write:

$$E_t c_{t+1}^{o,P} = E_t r_{p,t+1}^P + \log l_t$$

Substitute this equation into the log portfolio-return Euler condition (47):

$$\log l_t - c_t^{y,P} = \log\left(\frac{1}{1+\rho}\right) + \frac{1}{2}\left(\sigma^2_{c^o t}\right)^P + \frac{1}{2}\left(\sigma^2_{pt}\right)^P - \mathrm{Cov}_t\left(c_{t+1}^{o,P}; r_{p,t+1}^P\right)$$

Using equations (56) and (57) this equation can be rewritten to:

$$\left[W_t - T_t^P - T_t^{B,P} + \frac{Z_{t+1}^P}{E_t(1+R_{p,t+1}^P)\exp(-\frac{1}{2}(\sigma^2_{pt})^P)} - C_t^{Y,P}\right]\frac{1}{C_t^{Y,P}}\left(\frac{1}{1+\rho}\right)^{-1} = \exp\left[\frac{1}{2}z_t^2\left(\sigma^2_{pt}\right)^P\right]$$

Rearranging this equation gives the optimal condition for consumption when young:

$$C_t^{Y,P} = \frac{1+\rho}{1+\rho+\exp\left[\frac{1}{2}z_t^2(\sigma^2_{pt})^P\right]}\left[W_t - T_t^P - T_t^{B,P} + \frac{Z_{t+1}^P}{E_t(1+R_{p,t+1}^P)\exp(-\frac{1}{2}(\sigma^2_{pt})^P)}\right]$$

This equation can be used to derive the optimal condition for savings in the PAYG country, equation (19).

D Government debt

This appendix shows the derivation of the first-order difference equations for the capital-labour ratio and the nominal return on government debt in case the government in country P uses debt instead of taxes to (partly) finance the pension benefits of the elderly.

Linearize equation (21) with respect to ζ around the initial steady state and rearranging gives:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \frac{1}{1+\nu}\left[\frac{\partial S_t^P}{\partial \zeta} + \nu\frac{\partial S_t^F}{\partial \zeta} - \frac{\partial B_{t+1}^P}{\partial \zeta}\right] \tag{59}$$

From equation (20) we can derive the change of savings in the funded country:

$$\frac{\partial S_t^F}{\partial \zeta} = \frac{1}{2+\rho}\left(\frac{\partial W_t}{\partial \zeta} - \frac{\partial T_t^{B,F}}{\partial \zeta}\right) \tag{60}$$

The total derivative of savings in the PAYG country is[18]:

$$\frac{\partial S_t^P}{\partial \zeta} = \frac{\partial S_t^P}{\partial W_t}\frac{\partial W_t}{\partial \zeta} + \frac{\partial S_t^P}{\partial T_t^P}\frac{\partial T_t^P}{\partial \zeta} + \frac{\partial S_t^P}{\partial T_t^{B,P}}\frac{\partial T_t^{B,P}}{\partial \zeta} + \frac{\partial S_t^P}{\partial E_t R_{p,t+1}^P}\frac{\partial E_t R_{p,t+1}^P}{\partial \zeta} + \frac{\partial S_t^P}{\partial (\sigma_{pt}^2)^P}\frac{\partial (\sigma_{pt}^2)^P}{\partial \zeta} \tag{61}$$

Using equation (19) we can derive the partial derivatives of S_t^P. The derivatives with respect to W_t, T_t^P and $T_t^{B,P}$ are very straightforward and will not be shown here. The partial derivatives with respect to $E_t R_{p,t+1}^P$ and $(\sigma_{pt}^2)^P$ are:

$$\frac{\partial S_t^P}{\partial E_t R_{p,t+1}^P} = \frac{(1+\rho)Z^P}{\left\{1+\rho+\exp[\frac{1}{2}z^2(\sigma_p^2)^P]\right\}\exp[-\frac{1}{2}(\sigma_p^2)^P]\left[E(1+R_p^P)\right]^2} > 0 \tag{62}$$

$$\begin{aligned}\frac{\partial S_t^P}{\partial (\sigma_{pt}^2)^P} = {} & \frac{\frac{1}{2}z^2(1+\rho)\exp[\frac{1}{2}z^2(\sigma_p^2)^P]}{\left\{1+\rho+\exp[\frac{1}{2}z^2(\sigma_p^2)^P]\right\}^2}\left\{W - T^P - T^{B,P} + \frac{Z^P}{E(1+R_p^P)\exp[-\frac{1}{2}(\sigma_p^2)^P]}\right\} \\ & - \frac{\frac{1}{2}(1+\rho)Z^P}{\left\{1+\rho+\exp[\frac{1}{2}z^2(\sigma_p^2)^P]\right\}E(1+R_p^P)\exp[-\frac{1}{2}(\sigma_p^2)^P]} \lesseqgtr 0\end{aligned} \tag{63}$$

A higher expected portfolio return increases savings in the PAYG country as this decreases the net present value of the PAYG pension benefit. As explained in Section 2.4, in principle individuals in country P act like a consumer with a coefficient of relative risk aversion between 0 and 1, because of the safe PAYG pension benefit. In that case the substitution effect of more uncertainty dominates the income effect, so that savings fall when the risk on the portfolio rises. The last term in equation (63) shows this direct negative effect of uncertainty on savings in the PAYG country. There is, however, an extra indirect effect of safe PAYG pensions that induces people to save more when the level of uncertainty rises, which is shown by the first term in equation (63). The intuition for this term is as follows. The non-stochastic PAYG pension benefit reduces the variance of old-age consumption and this affects savings positively as the substitution effect of uncertainty dominates. This positive effect on savings becomes more important when the level of uncertainty rises. The ultimate effect of an increase in the riskiness of the portfolio on individual savings is therefore ambiguous. Simulations show that for a wide range of parameter values the indirect positive effect shown in the first term in equation (63) is of a second-order nature. Actually I could not find cases where $\frac{\partial S_t^P}{\partial (\sigma_{pt}^2)^P} > 0$. Therefore I conclude that savings in the PAYG country fall when the risk on the portfolio rises.

[18] The second-order effects via $E_t R_{p,t+1}^P$, $(\sigma_{pt}^2)^P$ and S_t^P on z_t are not taken into account, as this complicates the analytical expressions to a large extent. These indirect effects are taken into account in the simulation graphs presented in the main text, however.

Using equation (2), the change of W_t is given by:

$$\frac{\partial W_t}{\partial \zeta} = \frac{\alpha W}{K}\frac{\partial K_t}{\partial \zeta} \tag{64}$$

In order to derive the change in $E_t R^P_{p,t+1}$ and $(\sigma^2_{pt})^P$ we first need to know how γ^P_t changes. From equation (16), using equations (3) and (5), we obtain:

$$\frac{\partial \gamma^P_t}{\partial \zeta} = \frac{1}{(1-z)\sigma^2_{k-b}}\left(\frac{\alpha-1}{K}\frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R^N_b}\frac{\partial R^N_{b,t+1}}{\partial \zeta}\right)$$

Using equations (13) and (53), the changes of $E_t R^P_{p,t+1}$ and $(\sigma^2_{pt})^P$ are given by:

$$\begin{aligned}
\frac{\partial E_t R^P_{p,t+1}}{\partial \zeta} &= \frac{E(R_k)-E(R_b)}{(1-z)\sigma^2_{k-b}}\left(\frac{\alpha-1}{K}\frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R^N_b}\frac{\partial R^N_{b,t+1}}{\partial \zeta}\right) \qquad (65)\\
&\quad + \frac{\gamma^P(\alpha-1)E(1+R_k)}{K}\frac{\partial K_{t+1}}{\partial \zeta} + \frac{(1-\gamma^P)E(1+R_b)}{1+R^N_b}\frac{\partial R^N_{b,t+1}}{\partial \zeta}\\
\frac{\partial (\sigma^2_{pt})^P}{\partial \zeta} &= \frac{2\gamma^P\sigma^2_{k-b}+2\sigma_{k-b,b}}{(1-z)\sigma^2_{k-b}}\left(\frac{\alpha-1}{K}\frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R^N_b}\frac{\partial R^N_{b,t+1}}{\partial \zeta}\right) \qquad (66)
\end{aligned}$$

Substituting the partial derivatives of S^P_t with respect to W_t, T^P_t and $T^{B,P}_t$, and equations (62)-(66) in equations (60) and (61) gives:

$$\begin{aligned}
\frac{\partial S^F_t}{\partial \zeta} &= \frac{\alpha W}{(2+\rho)K}\frac{\partial K_t}{\partial \zeta} - \frac{1}{2+\rho}\frac{\partial T^{B,F}_t}{\partial \zeta} \qquad (67)\\
\frac{\partial S^P_t}{\partial \zeta} &= \frac{\exp[\frac{1}{2}z^2(\sigma^2_p)^P]\alpha W}{\{1+\rho+\exp[\frac{1}{2}z^2(\sigma^2_p)^P]\}K}\frac{\partial K_t}{\partial \zeta} - \frac{\exp[\frac{1}{2}z^2(\sigma^2_p)^P]}{1+\rho+\exp[\frac{1}{2}z^2(\sigma^2_p)^P]}\left(\frac{\partial T^P_t}{\partial \zeta} + \frac{\partial T^{B,P}_t}{\partial \zeta}\right)\\
&+ S^P_{R_p}\left[\frac{E(R_k)-E(R_b)}{(1-z)\sigma^2_{k-b}}\left(\frac{\alpha-1}{K}\frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R^N_b}\frac{\partial R^N_{b,t+1}}{\partial \zeta}\right)\right.\\
&+ \left.\frac{\gamma^P(\alpha-1)E(1+R_k)}{K}\frac{\partial K_{t+1}}{\partial \zeta} + \frac{(1-\gamma^P)E(1+R_b)}{1+R^N_b}\frac{\partial R^N_{b,t+1}}{\partial \zeta}\right] \qquad (68)\\
&+ S^P_{\sigma^2_p}\frac{2\gamma^P\sigma^2_{k-b}+2\sigma_{k-b,b}}{(1-z)\sigma^2_{k-b}}\left(\frac{\alpha-1}{K}\frac{\partial K_{t+1}}{\partial \zeta} - \frac{1}{1+R^N_b}\frac{\partial R^N_{b,t+1}}{\partial \zeta}\right)
\end{aligned}$$

where $S^P_{R_p} \equiv \frac{\partial S^P_t}{\partial E_t R^P_{p,t+1}}$ and $S^P_{\sigma^2_p} \equiv \frac{\partial S^P_t}{\partial (\sigma^2_{pt})^P}$, see equations (62) and (63).

Now use the other dynamic equation (22) to derive:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \frac{1}{1+\nu}\left(\gamma^P\frac{\partial S^P_t}{\partial \zeta} + S^P\frac{\partial \gamma^P_t}{\partial \zeta} + \nu\gamma^F\frac{\partial S^F_t}{\partial \zeta} + \nu S^F\frac{\gamma^F_t}{\partial \zeta}\right)$$

Combining this equation with equation (59) gives:

$$\frac{\partial K_{t+1}}{\partial \zeta} = \frac{1}{(1-\gamma^P)(1+\nu)} \left[\nu(\gamma^F - \gamma^P)\frac{\partial S_t^F}{\partial \zeta} + \gamma^P \frac{\partial B_{t+1}^P}{\partial \zeta} + S^P \frac{\partial \gamma_t^P}{\partial \zeta} + \nu S^F \frac{\partial \gamma_t^F}{\partial \zeta} \right] \tag{69}$$

Substituting the changes of γ_t^P and γ_t^F into equation (69), simplifying and rearranging gives:

$$\frac{\partial R_{b,t+1}^N}{\partial \zeta} = -\Phi \frac{\partial K_{t+1}}{\partial \zeta} - \frac{\nu(\gamma^P - \gamma^F)\sigma_{k-b}^2(1+R_b^N)}{\nu S^F + \frac{S^P}{1-z}} \frac{\partial S_t^F}{\partial \zeta} + \frac{\gamma^P \sigma_{k-b}^2 (1+R_b^N)}{\nu S^F + \frac{S^P}{1-z}} \frac{\partial B_{t+1}^P}{\partial \zeta} \tag{70}$$

where $\Phi \equiv \frac{[(1+\nu)K(1-\gamma^P)\sigma_{k-b}^2 + (1-\alpha)(\nu S^F + \frac{S^P}{1-z})](1+R_b^N)}{(\nu S^F + \frac{S^P}{1-z})K} > 0$. Use this equation to substitute for $\frac{\partial R_{b,t+1}^N}{\partial \zeta}$ in equation (68). Then substitute this expression and equation (67) into equation (59) and simplifying gives:

$$\begin{aligned} \frac{\partial K_{t+1}}{\partial \zeta} = {} & \frac{\alpha W}{\Psi K}\frac{\partial K_t}{\partial \zeta} - \frac{\Omega \nu}{\Psi^F}\frac{\partial T_t^{B,F}}{\partial \zeta} - \frac{1}{\Psi^P}\frac{\partial T_t^{B,P}}{\partial \zeta} - \frac{1}{\Psi^P}\frac{\partial T_t^P}{\partial \zeta} \\ & - \frac{1}{1+\nu - S_{\sigma_p^2}^P \Delta_{\sigma_p^2} - S_{R_p}^P \Delta_{R_p}} \left[\frac{\partial B_{t+1}^P}{\partial \zeta} - \Omega_{B^P} \frac{\partial B_{t+1}^P}{\partial \zeta} \right] \end{aligned} \tag{71}$$

where:

$$\begin{aligned}
\Delta_{R_p} &\equiv \frac{[E(R_k) - E(R_b)](1+\nu)(1-\gamma^P)}{\nu S^F(1-z) + S^P} \\
&\quad - \frac{\gamma^P(1-\alpha)E(1+R_k)}{K} - \frac{(1-\gamma^P)E(1+R_b)\Phi}{1+R_b^N} \lesseqgtr 0 \\
\Delta_{\sigma_p^2} &\equiv \frac{(2\gamma^P\sigma_{k-b}^2 + 2\sigma_{k-b,b})(1+\nu)(1-\gamma^P)}{\nu S^F(1-z) + S^P} > 0 \\
\Psi &\equiv \frac{\left\{1+\rho+\exp[\frac{1}{2}z^2(\sigma_p^2)^P]\right\}(2+\rho)\left(1+\nu - S_{\sigma_p^2}^P\Delta_{\sigma_p^2} - S_{R_p}^P\Delta_{R_p}\right)}{\exp[\frac{1}{2}z^2(\sigma_p^2)^P](2+\rho) + \Omega\nu\left\{1+\rho+\exp[\frac{1}{2}z^2(\sigma_p^2)^P]\right\}} > 0 \\
\Psi^P &\equiv \frac{\left\{1+\rho+\exp[\frac{1}{2}z^2(\sigma_p^2)^P]\right\}\left(1+\nu - S_{\sigma_p^2}^P\Delta_{\sigma_p^2} - S_{R_p}^P\Delta_{R_p}\right)}{\exp\left[\frac{1}{2}z^2(\sigma_p^2)^P\right]} > 0 \\
\Psi^F &\equiv (2+\rho)\left(1+\nu - S_{\sigma_p^2}^P\Delta_{\sigma_p^2} - S_{R_p}^P\Delta_{R_p}\right) > 0 \\
\Omega &\equiv 1 + \frac{\gamma^P - \gamma^F}{\nu S^F(1-z) + S^P}\Big\{ S_{\sigma_p^2}^P(2\gamma^P\sigma_{k-b}^2 + 2\sigma_{k-b,b}) \\
&\quad - S_{R_p}^P(1-\gamma^P)E(1+R_b)(1-z)\sigma_{k-b}^2 + S_{R_p}^P[E(R_k) - E(R_b)]\Big\} \approx 1 \\
\Omega_{B^P} &\equiv \frac{\gamma^P}{\nu S^F(1-z) + S^P}\Big\{ S_{R_p}^P(1-\gamma^P)E(1+R_b)(1-z)\sigma_{k-b}^2 - S_{R_p}^P[E(R_k) - E(R_b)] \\
&\quad - S_{\sigma_p^2}^P(2\gamma^P\sigma_{k-b}^2 + 2\sigma_{k-b,b})\Big\} \approx 0
\end{aligned}$$

Simulations show that the last two (negative) terms in the equation for Δ_{R_p} are much larger than the first term. For a large range of parameter values Δ_{R_p} was always smaller than -0.6. In the following I therefore assume that $\Delta_{R_p} < 0$. Moreover, simulations show that the term between brackets in the expression for Ω, and thus Ω_{B^P} as well, is fairly small. Therefore it holds that Ω is approximately equal to 1 and Ω_{B^P} is close to 0.

Equations (71), (70) and (67) together give the system of equations in Section 3.

E Change capital-labour ratio at $t = 1$

The change in the capital-labour ratio at $t = 1$ is:

$$\frac{\partial K_1}{\partial \zeta} = -\frac{1}{\Psi^P}\frac{\partial T_0^P}{\partial \zeta} - \frac{1}{1 + \nu - S^P_{\sigma_p^2}\Delta_{\sigma_p^2} - S^P_{R_p}\Delta_{R_p}}\frac{\partial B_1^P}{\partial \zeta}$$

Using equation (27) in the main text, we know that:

$$\frac{\partial B_1^P}{\partial \zeta} = -\frac{\partial T_0^P}{\partial \zeta}$$

So that we can write:

$$\frac{\partial K_1}{\partial \zeta} = \frac{1+\rho}{\{1+\rho+\exp[\frac{1}{2}z^2(\sigma_P^2)^P]\}(1+\nu - S^P_{\sigma_p^2}\Delta_{\sigma_p^2} - S^P_{R_p}\Delta_{R_p})}\frac{\partial T_0^P}{\partial \zeta}$$

As $\frac{\partial T_0^P}{\partial \zeta} < 0$ we know that $\frac{\partial K_1}{\partial \zeta} < 0$. The intuition for this result is obvious. The young generation at $t = 0$ receives a windfall gain as its PAYG tax is lowered, while their future pension benefit does not change. Only part of this gain is saved, the other part will be consumed. As the gain this generation receives equals the created debt, the increase in savings at $t = 0$ is smaller than the created debt, so that public debt crowds out part of the capital stock.

F Effects on utility

To analyse the utility effects, we first need to know what happens with consumption in both periods of life. The change in consumption when young in country i, $i = P, F$, is given by:

$$\frac{\partial C_t^{Y,i}}{\partial \zeta} = \frac{\partial W_t}{\partial \zeta} - \frac{\partial T_t^i}{\partial \zeta} - \frac{\partial T_t^{B,i}}{\partial \zeta} - \frac{\partial S_t^i}{\partial \zeta}$$

where $\partial T_t^i / \partial \zeta = 0$ for $i = F$. Country P lowers the PAYG tax T_t^P at $t = 0$ for one period. This creates a windfall gain for the generation born at $t = 0$ and part of this gain will be saved and part will be consumed, i.e., $\frac{\partial C_0^{Y,P}}{\partial \zeta} > 0$. Consumption when young in country F does not change at $t = 0$ as individuals in this country will only be affected through the change in factor prices and the interest rate on government bonds. The created public debt will crowd out part of the capital stock in the next period, which lowers wages and affects consumption when young (and savings) negatively in both countries. Consumption in the first period of life is further reduced by the rise in the debt tax $T_t^{B,i}$ that results from the increase in the nominal interest rate on the debt and for the PAYG country also because of the rise in public debt itself. So from period $t = 1$ onwards $\frac{\partial C_t^{Y,i}}{\partial \zeta} < 0$.

The change in expected old-age consumption is:

$$\frac{\partial E_t C_{t+1}^{O,i}}{\partial \zeta} = S^i \frac{\partial E_t R_{p,t+1}^i}{\partial \zeta} + E(1 + R_p^i) \frac{\partial S_t^i}{\partial \zeta}$$

The expected portfolio return will increase after a rise in government debt because debt crowds out capital (ER_k increases) and a higher debt level makes it more difficult to finance the total amount of debt (higher ER_b). The higher portfolio return has a positive effect on old-age consumption. As explained above, savings in country P will increase at $t = 0$ due to the windfall gain of the lower PAYG contributions. This implies that the old-age consumption of the generation born at $t = 0$, i.e., $C_1^{O,P}$ will unambiguously rise. From period $t = 1$ onwards, however, savings in both countries fall as a result of the lower wages and the higher debt tax[19], which affects old-age consumption negatively. The negative effect of lower savings dampens the positive effect of the higher portfolio return, but simulations show that for realistic parameter values old-age consumption in both countries still increases compared to the initial steady state.

The change in lifetime utility is equal to[20]:

$$\frac{\partial E_t U(C_t^{Y,i}; C_{t+1}^{O,i})}{\partial \zeta} = \frac{1}{C^{Y,i}} \frac{\partial C_t^{Y,i}}{\partial \zeta} + \frac{1}{(1+\rho)E(C^{O,i})} \frac{\partial E_t C_{t+1}^{O,i}}{\partial \zeta} - \frac{(1-z)^2}{2(1+\rho)} \frac{\partial (\sigma_{pt}^2)^i}{\partial \zeta} \tag{72}$$

where I used the assumption that $C_{t+1}^{O,i}$ is a lognormally distributed variable so that $E_t \log(C_{t+1}^{O,i})$ can be written as $\log E_t C_{t+1}^{O,i} - \frac{1}{2} \text{Var}_t \log C_{t+1}^{O,i}$, see equation (46). And for country F it holds that $z = 0$. I will separately discuss the utility effects for the PAYG country and the funded country.

[19] Savings in the PAYG country also react to changes in the expected portfolio return and the variance on the portfolio, see equation (61). Both the rise in the portfolio return and the fall in the variance on the portfolio (resulting from the fact that people switch from stocks to bonds) have a positive effect on savings. So these indirect saving effects via $E_t R_{p,t+1}^P$ and $(\sigma_{pt}^2)^P$ reduce the negative saving effects of lower wages and the higher debt tax to some extent.

[20] As before I do not show the indirect effect of changes in z_t in the analytical expression, but this is taken into account in the simulation graphs in the main text.

PAYG country

The generation born at $t = 0$ has higher consumption levels in both periods of life and will unambiguously experience positive utility effects. The next generations have less consumption possibilities when young (due to the lower wage and the higher debt tax) and the rise in old-age consumption will be less because savings fall over time. In a dynamically efficient economy a fall in the capital-labour ratio implies that the economy moves further away from the Golden Rule point where steady-state lifetime utility is maximized. This implies that a lower capital-labour ratio has negative utility effects in the long run. This negative Golden-Rule effect is reinforced by the rise in the debt tax and an extra negative effect via the current account. The PAYG country is a net borrower on the international capital market and will therefore be adversely affected by the increase in the returns. The fall in the variance on the portfolio, which results from the fact that people hold relatively more bonds in their investment portfolio, dampens the negative utility effects to some extent.

The negative long-run utility effects of government debt for the PAYG country will be smaller if it shares the capital market with a funded country. In that case part of the public debt can be financed with savings of the funded country, which implies that the crowding-out of the capital stock and the rise in the interest rate on debt will be less. The PAYG country therefore experiences positive spillover effects from the fact that it has an integrated capital market with the funded country.

Funded country

The fall in consumption when young affects lifetime utility negatively, while the increase in consumption possibilities during old-age has positive utility effects. The initial generations only experience the gains from a higher return on the portfolio; they do not (fully) incur the losses that result from the lower wages. This means that these initial generations experience positive spillovers from the rise in public debt in the PAYG country. In the long run, however, it holds that the fall in the capital-labour ratio has negative utility effects in a dynamically efficient economy as it moves away from the Golden Rule point where total lifetime consumption is maximized. On the other hand, the funded country is a net lender on the international capital market and will therefore be affected positively by the rise in the portfolio return. If the economy finds itself far enough from the Golden-Rule level of capital accumulation, the negative Golden-Rule effect will dominate the positive current-account effect. In Adema (2008*b*, Appendices 2.C.2 and 3.A.2) it is shown that this is the case for realistic parameter values. Therefore, I conclude that lifetime utility decreases if the capital-labour falls in a dynamically-efficient open economy, and the funded country will be negatively affected in the long run by the rise in debt in the PAYG country. These negative utility effects are reinforced by the fact that the debt tax in the funded country also rises as a result of the higher interest rate on government debt. The long-run spillovers for the funded country are therefore negative.

References

Adema, Y. (2008*a*), Ch. 5: Ageing, debt and inflation in a monetary union, *in* 'The international spillover effects of ageing and pensions', CentER Dissertation Series nr. 212, CentER, Tilburg.

Adema, Y. (2008*b*), *The international spillover effects of ageing and pensions*, Vol. 212 of *CentER Dissertation Series*, Tilburg University, Tilburg.

Adema, Y., Meijdam, L. and Verbon, H. A. A. (2008), 'Beggar thy thrifty neighbour: The international spillover effects of pensions under population ageing', *Journal of Population Economics* **21**, 933–959.

Barberis, N. (2000), 'Investing for the long run when returns are predictable', *Journal of Finance* **55**(1), 225–264.

Beetsma, R. and Bovenberg, A. (2009), 'Pensions and intergenerational risk-sharing in general equilibrium', *Economica* **76**(302), 364–386.

Beetsma, R. and Uhlig, H. (1999), 'An analysis of the stability and growth pact', *Economic Journal* **109**(October), 546–571.

Boeri, T., Bovenberg, L., Coeuré, B. and Roberts, A. (2006), 'Dealing with the new giants: Rethinking the role of pension funds.', *Geneva Reports on the World Economy* **8**.

Bohn, H. (2001), Social security and demographic uncertainty: The risk-sharing properties of alternative policies, *in* J. Y. Campbell and M. Feldstein, eds, 'Risk Aspects of Investment-Based Social Security Reform', University of Chicago Press, Chicago, pp. 203–246.

Bohn, H. (2009), 'Intergenerational risk sharing and fiscal policy', *Journal of Monetary Economics* **56**(6), 805–816.

Buiter, W. (1981), 'Time preference and international lending and borrowing in an overlapping-generations model', *Journal of Political Economy* **89**(August), 769–797.

Campbell, J. (1993), 'Intertemporal asset pricing without consumption data', *American Economic Review* **83**(3), 487–512.

Campbell, J. (2003), Consumption-based asset pricing, *in* G. M. Constantinides, M. Harris and R. M. Stulz, eds, 'Handbook of the Economics of Finance', 1, part 2 edn, Vol. 1, Elsevier, Chapter 13, pp. 803–887.

Campbell, J. and Viceira, L. (1999), 'Consumption and portfolio decisions when expected returns are time varying', *Quarterly Journal of Economics* **114**(2), 433–495.

Campbell, J. and Viceira, L. (2002), *Strategic asset allocation - Portfolio choice for long-term investors*, Oxford University Press, New York.

Campbell, J. and Viceira, L. (2005), 'The term structure of the risk-return trade-off', *Financial Analysts Journal* **61**(1), 34–44.

Chari, V. and Kehoe, P. J. (2007), 'On the need for fiscal constraints in a monetary union', *Journal of Monetary Economics* **54**(8), 2399–2408.

European Commission (2009), 'Sustainability report 2009', *European Economy* **9**.

Fried, J. and Howitt, P. (1988), 'Fiscal deficits, international trade and welfare', *Journal of International Economics* **24**.

Geraats, P., Giavazzi, F. and Wyplosz, C. (2008), *Transparancy and Governance*, Monitoring the European Central Bank 6, Centre for Economic Policy Research, London.

Hansen, L. and Singleton, K. (1983), 'Stochastic consumption, risk aversion, and the temporal behavior of asset returns', *Journal of Political Economy* **91**(2), 249–265.

Judd, K. (1982), 'An alternative to steady-state comparisons in perfect foresight models', *Economics Letters* **10**, 55–59.

Krueger, D. and Kubler, F. (2006), 'Pareto-improving social security reform when financial markets are incomplete!?', *American Economic Review* **96**(3), 737–755.

Matsen, E. and Thøgersen, O. (2004), 'Designing social security - a portfolio choice approach', *European Economic Review* **48**(4), 883–904.

Miles, D. and Černý, A. (2006), 'Risk, return and portfolio allocation under alternative pension systems with incomplete and imperfect financial markets', *Economic Journal* **116**(April), 529–557.

Persson, T. (1985), 'Deficits and intergenerational welfare in open economies', *Journal of International Economics* **19**(1-2), 67–84.

Rothschild, M. and Stiglitz, J. (1971), 'Increasing risk II: Its economic consequences', *Journal of Economic Theory* **3**, 66–84.

Sánchez-Marcos, V. and Sánchez-Martin, A. (2006), 'Can social security be welfare improving when there is demographic uncertainty?', *Journal of Economic Dynamics and Control* **30**, 1615–1646.

Sandmo, A. (1970), 'The effect of uncertainty on saving decisions', *Review of Economic Studies* **37**(3), 353–360.

Storesletten, K., Telmer, C. I. and Yaron, A. (1999), 'The risk-sharing implications of alternative social security arrangements', *Carnegie-Rochester Conference Series on Public Policy* **50**, 213–259.

Viceira, L. M. (2001), 'Optimal portfolio choice for long-horizon investors with non-tradable labor income', *Journal of Finance* **56**(2), 433–470.