The Midweight Method to Measure Attitudes towards Risk and Ambiguity

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This paper introduces a parameter-free method for measuring the weighting functions of prospect theory and rank-dependent utility. These weighting functions capture risk attitudes, subjective beliefs, and ambiguity attitudes. Our method, called the midweight method, is based on a convenient way to obtain midpoints in the weighting function scale. It can be used both for risk (known probabilities) and for uncertainty (unknown probabilities). The resulting integrated treatment of risk and uncertainty is particularly useful for measuring the differences between them: ambiguity. Compared to existing methods to measure ambiguity attitudes, our method is more efficient and it can accommodate violations of expected utility under risk. An experiment demonstrates the feasibility and tractability of our method, yielding plausible results such as ambiguity aversion for moderate and high likelihoods but ambiguity seeking for low likelihoods, as predicted by Ellsberg.

Keywords: Prospect Theory; Probability Weighting; Pessimism

1. Introduction

Because of the many violations of expected utility (Starmer 2000, Gilboa 2004), nonexpected utility theories have been developed so as to better explain empirical findings. Most nonexpected utility theories use weighting functions that generalize (subjective) probabilities by relaxing additivity. Obviously, the increased flexibility for accommodating data comes at a price: eliciting nonadditive weighting functions takes extra work. This paper aims to simplify this work, both for risk (known probabilities) and for ambiguity (no probabilities are known or conceivable).

Since Keynes (1921) and Knight (1921), it has been understood that ambiguity is more important than risk. Probabilities are rarely known in practice. Nevertheless, until the late 1980s, virtually all papers in decision theory exclusively dealt with risk. Some, building on Savage (1954), did consider uncertainty (which includes both risk and ambiguity) but then only under the assumption that there exist subjective probabilities, to be used within the Bayesian expected utility model. This Bayesian approach stays close to risk and cannot capture ambiguity that, as demonstrated by Ellsberg (1961), entails a more fundamental breakaway from risk. For a long time, no one was able to develop behaviorally sound models for ambiguity. Only 68 years after Keynes (1921) and Knight (1921), Schmeidler (1989) and Gilboa & Schmeidler (1989) succeeded in doing so. Tversky & Kahneman (1992) incorporated these models into the psychologically founded prospect theory. Thus, only in the 1990s could a serious study of ambiguity begin. Up to today, however, there have only been few empirical measurements of weighting functions for ambiguity. They were all laborious and most measurements, such as those based on the popular multiple priors and α maxmin models (Gilboa & Schmeidler 1989), assumed the descriptively problematic expected utility model for risk.

For the special case of risk, many studies have measured probability weighting functions through parametric fitting techniques (Andersen et al. 2007). Advantages are that these techniques can be applied to virtually any data set, and that they smooth errors in the data. A drawback is that the techniques require prior commitment to particular parametric families. These impose particular shapes of the weighting function that may not hold in reality, and give no insights into the prevalence of alternative shapes. Some examples are Hey & Orme (1994) and Harless & Camerer (1994) who used power functions, excluding inverse-S shapes, and Donkers, Meelenberg, & van Soest (2001) who committed to inverse-S shapes, excluding all other shapes. Another drawback is that these methods are often subject to colinearity effects, where utility and the weighting functions have similar effects and cannot be reliably separated from the data, with errors in one generating errors in the other (Stott 2006 pp. 112, 121).

An obvious advantage of nonparametric measurements is that they need no pior commitment to any shape, and that they will uncover true patterns and phenomena irrespective of what those are. They also make clear to what extent utility and weighting functions overlap or can be separated. Further, they show how probability weighting and utility are related to decisions in a transparent manner. Hence they can be used in interactive measurement sessions.

This paper introduces a nonparametric method for eliciting weighting functions that can be used both for risk and for uncertainty. Our method is called the midweight method and is based on an easy way to obtain midpoints in the weighting function scale. The midweigth method is more efficient than existing methods both for risk (Abdellaoui 2000; Bleichrodt & Pinto 2000) and for uncertainty (Abdellaoui et al. 2009; Abdellaoui, Vossmann, & Weber 2005), because it minimizes the need to measure utility. The only restriction for utility is that for at least one pair of outcomes a utility midpoint has to be available. The method yields the correct weighting functions completely independently of what utility is, avoiding any colinearity. We implement our method in experiments both for risk and for uncertainty. Our findings agree with the common findings, although we find more pessimism for risk than mostly found.

Most studies of ambiguity up to today only measured a single number that should reflect a universal aversion towards ambiguity of a person. Abdellaoui et al. (2009) introduced source functions, and showed how these can capture the full richness of ambiguity and uncertainty attitudes in a tractable manner. We show how source functions can be measured more efficiently using the midweight method. Our experiments confirm Abdellaoui et al.’s (2009) finding that people are ambiguity averse for events of moderate and high likelihood, but are, on the contrary, ambiguity seeking for unlikely events. This pattern of ambiguity attitudes was already suggested by Ellsberg (2001, p. 203, p. 206). It underscores that ambiguity attitudes cannot be modeled through one single number to reflect a universal degree of ambiguity aversion.

The remainder of this paper is organized as follows. Section 2 briefly presents prospect theory. Section 3 introduces the midweight method, first for risk, then for uncertainty. An empirical measurement of the weighting function for risk is presented in Section 4. Section 5 applies the midweight method to measure general uncertainty attitudes, and Section 6 applies the method to measure source functions and ambiguity. Discussions and conclusions are in Sections 7, 8, and 9. Throughout this paper, we first present results for risk, and then extend them to uncertainty. In this way, we make this paper accessible to readers unfamiliar with the relatively new models of ambiguity. This presentation also illustrates that risk is a subcase of uncertainty rather than a separate case.

2. Prospect Theory for Risk and for Uncertainty

Outcomes are monetary, with $\mathbb{R}^+$ the outcome set. For simplicity, we do not consider losses (negative outcomes). Because the midweight method will require no more than three distinct outcomes, we focus on this case in this theoretical exposition. For discussions and motivations of the following theories, see Wakker (2009).

We first consider decision under risk. We use Tversky & Kahneman's (1992) prospect theory, which coincides with Quiggin's (1981) rank-dependent utility because we only consider gains. It is an improved version of Kahneman & Tversky's (1979) original prospect theory because it corrects a theoretical problem of probability weighting, and allows more than two nonzero outcomes (Wu, Zhang, & Abdellaoui 2005). A *prospect* $(p_1{:}x_1, p_2{:}x_2, p_3{:}x_3)$ yields x_j with probability p_j, j = 1,2,3. The p_js are nonnegative and sum to 1. The prospect is evaluated by:

$$\text{(for } x_1 \geq x_2 \geq x_3\text{):}\quad w(p_1)U(x_1) + w((p_1+p_2) - w(p_1))U(x_2) + (1 - w(p_1+p_2))U(x_3). \qquad (2.1)$$

Here U denotes *utility*, which is continuous and strictly increasing. The (*probability) weighting function* w maps [0,1] to [0,1] and is strictly increasing and continuous, with $w(0) = 0$ and $w(1) = 1$. In what follows, $x_p y$ denotes the two-outcome prospect yielding x with probability p and y with probability 1–p.

We now turn to decision under uncertainty. The major improvement of Tversky & Kahneman's (1992) prospect theory relative to the 1979 version was that the new theory could handle not only risk, but also the more important context of uncertainty (which includes ambiguity). We will use this extension in our study, where it coincides with Gilboa's (1987) and Schmeidler's (1989) rank-dependent utility because no losses are involved. Under uncertainty, prospects assign outcomes to uncertain events of which the probabilities need not be known. In our experiment, the uncertain events concern the average temperature in the Dutch city Eindhoven 11 days ahead. $(E_1{:}x_1, E_2{:}x_2, E_3{:}x_3)$ denotes the *prospect* yielding x_j if E_j obtains, where the E_js denote three temperature intervals, or unions of temperature intervals. It is always understood that the E_js are exhaustive and mutually exclusive. Our subjects had no statistics available so that they did not know the probabilities of these events. Statistics of the past, even if available, would not have eliminated all ambiguity because of changed circumstances today, such as because of global warming. $x_E y$ denotes the prospect yielding x under event E and y otherwise.

We use utility U as before, but instead of the weighting function w for probabilities we use a function W defined on events. For reasons explained later, W is called a(n *event*) *weighting function*. W assigns weight 0 to the vacuous event and weight 1 to the universal event, and $A \supset B$ implies $W(A) \geq W(B)$. W shares these properties with probability measures. However, $W(A \cup B) \neq W(A) + W(B)$ may hold for disjoint events A,B, violating additivity, and this is where W generalizes probability measures. A prospect $(E_1{:}x_1, E_2{:}x_2, E_3{:}x_3)$ is evaluated by:

$$\text{(for } x_1 \geq x_2 \geq x_3\text{):} \;\; W(E_1)U(x_1) + (W(E_1 \cup E_2) - W(E_1))U(x_2) + (1 - W(E_1 \cup E_2))U(x_3). \quad (2.2)$$

Risk can be considered the special case of uncertainty where probabilities p_j are given for the events E_j, and $W(E_j) = w(p_j)$. So as to maximally clarify that risk is a special case of uncertainty rather than a separate case, we use the same terms for risk and uncertainty whenever no confusion arises.

Convexity of w can be defined as

$$w(a+b) - w(b) \leq w(a+b+i) - w(b+i) \text{ for all nonnegative a,b,i.} \quad (2.3)$$

It is naturally extended to uncertainty, with W *convex* if

$$W(A \cup B) - W(B) \leq W(A \cup B \cup I) - W(B \cup I) \text{ for all disjoint sets A,B,I.} \quad (2.4)$$

Concavity is defined by reversing the inequality signs. If W is a transform w(P) of a probability measure P, then under some richness assumptions convexity (concavity) of W is equivalent to convexity (concavity) of w (Wakker 2009). Hence, our terminology is consistent. In the domain investigated in our study, we equate the often found *inverse-S shape* with concavity for unlikely events and convexity for events of moderate and high likelihood (“cavexity”).

Assuming zero decision weight (and probability) for single temperature values, it is immaterial how we take openness and closedness of intervals. For convenience, we usually take intervals left-closed and right-open (except occasionally for bound 1).

3. The Midweight Method Defined

The midweight method, which will measure midpoints in the weighting scale, starts with measuring a midpoint of utility. To this end we measure:

$$x_{2_p}y \sim x_{1_p}Y \text{ and } x_{1_p}y \sim x_{0_p}Y \quad \text{for risk, and}$$

$$x_{2_E}y \sim x_{1_E}Y \text{ and } x_{1_E}y \sim x_{0_E}Y \quad \text{for uncertainty,} \quad (3.1)$$

with $x_2 > x_1 > x_0 > Y > y$ (as in the tradeoff method of Wakker & Deneffe 1996). Then, with $0 < \pi = w(p)$ or $0 < \pi = W(E)$,

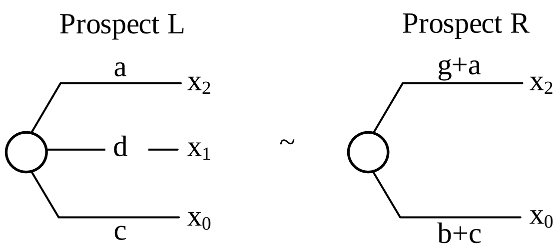
$$\pi(U(x_2) - U(x_1)) = (1-\pi)(U(Y) - U(y)) = \pi(U(x_1) - U(x_0)), \text{ which implies}$$

$$U(x_2) - U(x_1) = U(x_1) - U(x_0). \tag{3.2}$$

That is, x_1 is the utility midpoint of x_2 and x_0. These x-values will be used throughout what follows, and from here on the preference domain will be restricted to prospects that use only these three outcomes (called the probability triangle of x_0,. x_1,.x_2 for risk).

We first present the midweight method for risk. For any probability a and larger probability $d + a$ we will find their w-midpoint probability $g + a$, with $0 < g < d$. We start from the left prospect $L = (a{:}\,x_2, d{:}\,x_1, c{:}\,x_0)$ in Figure 3.1, with x_0, x_1, x_2 as in Eqs. 3.1 and 3.2 for risk. Here d, the probability mass of x_1 in the left prospect, will be divided (this is what d refers to) over the other outcomes to yield the equivalent right prospect R. g is moved to the high outcome x_2, and the remainder $b = d–g$ is moved to the low outcome x_0.

FIGURE 3.1. Distributing d's weight evenly over the upper and lower branch



Because the proof of the following theorem may be instructive, it is given in the main text.

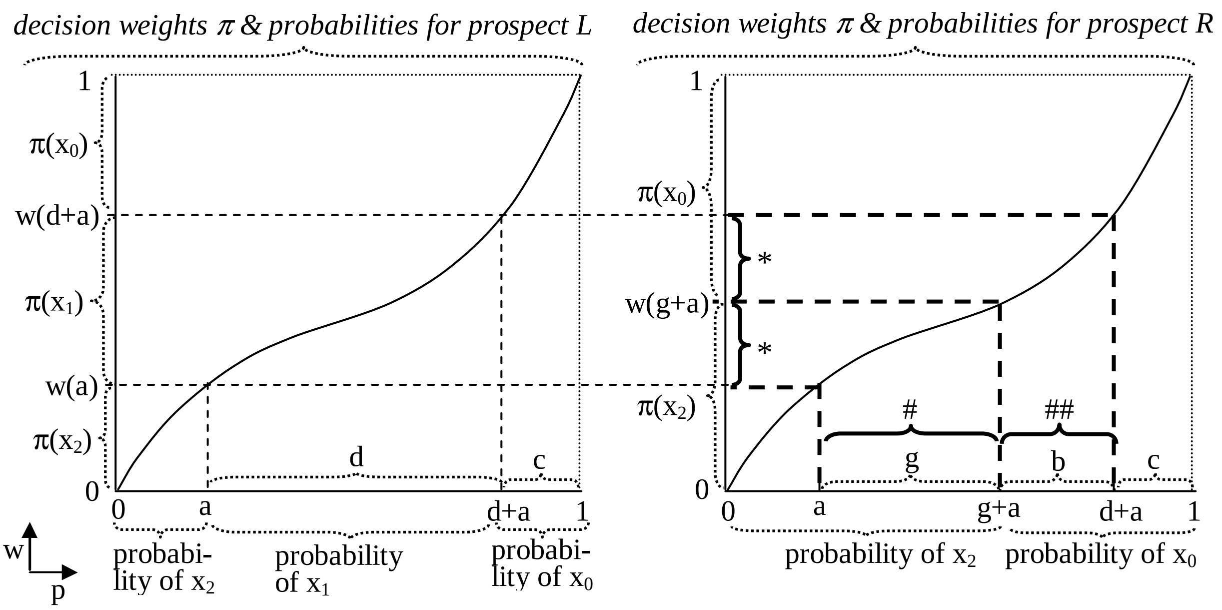
THEOREM 3.1. The indifference in Figure 3.1 implies that

$$w(g + a) = \frac{w(a) + w(d + a)}{2}$$

whenever $U(x_2) - U(x_1) = U(x_1) - U(x_0) > 0$.

PROOF. Figure 3.2 depicts the decision weights to be derived. The move of g probability mass from outcome x_1 up to outcome x_2 increases the prospect theory value by $\delta_{12} \times \big(U(x_2) - U(x_1)\big)$ where δ_{12} is the extra decision weight for the upper branch, $\big(w(g+a) - w(a)\big)$ (the lower * in Figure 3.2). The move of b probability mass from outcome x_1 down to outcome x_0 decreases the prospect theory value by $\delta_{10} \times \big(U(x_1) - U(x_0)\big)$ where δ_{10} is the extra decision weight for the lower branch, i.e. $(1-w(g+a)) - (1-w(d+a)) = w(d+a) - w(g+a)$ (the upper * in Figure 3.2). Dropping the equal utility differences, $(w(g+a) - w(a)) = w(d+a) - w(g+a)$ must hold so as to preserve indifference. The theorem follows.

FIGURE 3.2. Decision weights for prospects of Figure 3.1



*: **decision weight moved from $U(x_1)$ to $U(x_2)$**
= **decision weight moved from $U(x_1)$ to $U(x_0)$**

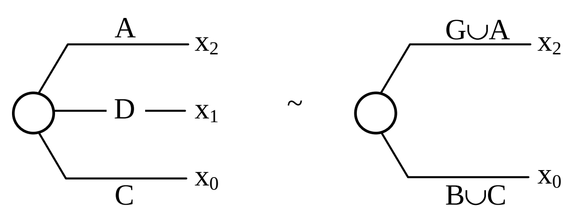
#: **probability moved from x_1 to x_2**
##: **probability moved from x_1 to x_0**

□

Our approach is general in the sense that the weight-midpoint between any two probabilities can be measured directly. The only richness of outcomes needed is that for at least one pair of outcomes a utility-midpoint exists. With a method available to measure midpoints of the weighting function, we can measure the weighting function to any desired degree of precision. For example, we can start with $p=0$ and $q=1$ to find $w^{-1}(½)$, i.e., the probability corresponding to weight ½. Then we use $p=0$ and $q=w^{-1}(½)$ to find $w^{-1}(¼)$, and so on.

The midweight method can be applied to uncertainty in a way very analogous to risk, as is explained next. For any event A and a larger event $D \cup A$ a W-midpoint $G \cup A$ ($G \subset D$) can be determined by eliciting indifference between the prospects $(A{:}\,x_2, D{:}\,x_1, C{:}\,x_0)$ and $x_{2_{G\cup A}}x_0$ as in Figure 3.3.

FIGURE 3.3. Distributing D's weight evenly over the upper and lower branch



THEOREM 3.2. The indifference in Figure 3.3 implies that

$$W(G \cup A) = \frac{W(A) + W(D \cup A)}{2}$$

whenever $U(x_2) - U(x_1) = U(x_1) - U(x_0) > 0$.

PROOF. The proof is similar to that for risk, with the value increase $(W(G \cup A) - W(A))(U(x_2) - U(x_1))$ of the right prospect equal to its value decrease $(W(D \cup A) - W(G \cup A))(U(x_1) - U(x_0))$, implying the theorem. □

A midpoint event $G \cup A$ as just constructed exists for all events A and $D \cup A$ if the event space is sufficiently rich (such as a continuum), as for instance in Gilboa's (1987) preference foundation.

4. Direct Measurement of the Weighting Function for Risk

This section describes an experiment measuring the weighting function for risk.

Participants. $N = 78$ undergraduate students participated from a wide range of disciplines recruited at the University of Amsterdam. They were self-selected from a mailing list of about 400 people. 14 participants were excluded from the analysis because they gave erratic or heuristic answers such as always choosing the left prospect or always choosing the right prospect. The practice choices of this experiment also served to detect such erratic and heuristic answers. These participants apparently did not understand the choices or did not seriously think about them. The following analysis is based on the remaining 64 participants (26 female; median age 21). Including the excluded participants would not alter the results presented hereafter.

Procedure. Participants were seated in front of personal computers in 7 different sessions with approximately 11 participants per session. Participants first received experimental instructions (see Appendix B), after which the experimental questions followed.

Stimuli; general. Participants were asked two practice choice questions to familiarize them with the experimental procedures. In each question they chose between a prospect L (left) and a prospect R (right). Both prospects yielded prizes depending on the outcome of a roll with two 10-sided dice, each determining one digit of a random number below 100. Prospects were framed as in Figure 4.1. Participants indicated their choice by clicking on the appropriate button. They were encouraged to answer at their own pace. The position of each prospect was counterbalanced between participants.

FIGURE 4.1. The framing of the prospect pairs

PROSPECT L		
roll	probability	prize
1 to p	p %	x_{i-1} euro
p+1 to 100	(100 − p)%	Y euro

PROSPECT R		
roll	probability	prize
1 to p	p %	x_i euro
p+1 to 100	(100 − p)%	y euro

Measuring utility. We set $x_0 = 60$ and obtained values x_1 and x_2 to generate indifferences

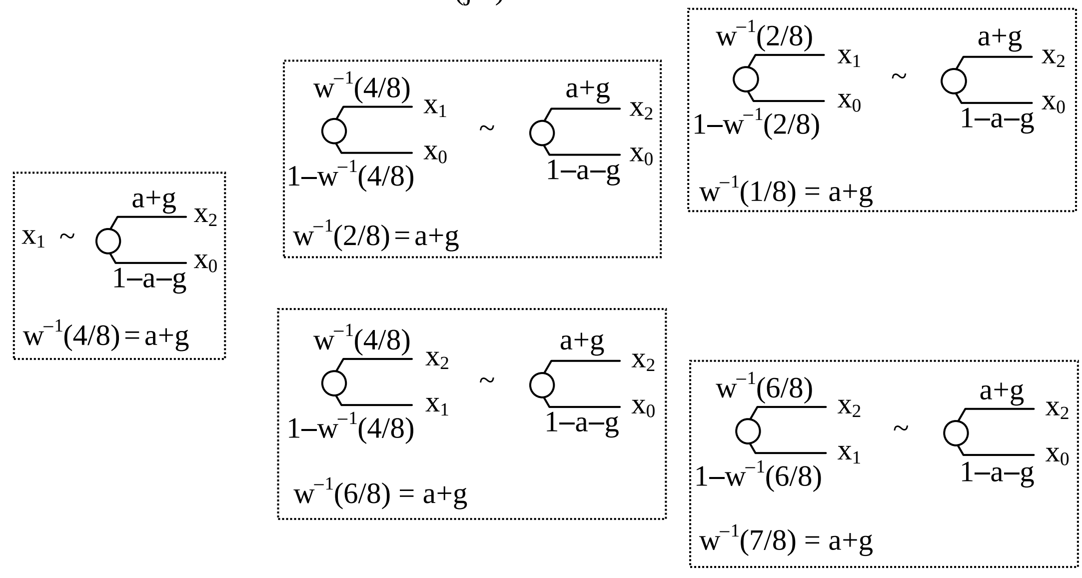
$$\mathbf{x_1}_{0.25}30 \sim 60_{0.25}40 \text{ and } \mathbf{x_2}_{0.25}30 \sim {x_1}_{0.25}40. \quad (4.1)$$

(The values that were elicited are printed in bold.) Then under prospect theory x_1 is the utility midpoint of x_0 and x_2 (Eq. 3.2). Because all further measurements in the experiment depended on the values x_1 and x_2, these values were elicited twice and the average of the two values obtained was used as input in the rest of the experiment, so as to reduce noise. Throughout this paper, indifferences are obtained using a bisection choice method. Such methods, while time-consuming, give more consistent results than direct matching (Bardsley & Moffat 2009; Bostic, Herrnstein, & Luce 1990; Noussair, Robbin, & Ruffieux 2004).

The particular bisection method that we used is similar to the method used by Abdellaoui (2000), and is explained in the rest of this paragraph. To obtain x_1 in ${x_1}_{0.25}30 \sim {x_0}_{0.25}40$, we iteratively narrowed down what we call indifference intervals containing the indifference value of x_1 as follows. Based on extensive pilots, we assumed that x_1 would not exceed $x_0 + 96$ and took $[x_0, x_0 + 96)$ as the first indifference interval, denoted $[\ell^1, u^1)$. To construct the j+1th indifference interval from the jth indifference interval $[\ell^j, u^j)$, we observed the choice between $(\ell^j + u^j)/2_{0.25}30$ and ${x_0}_{0.25}40$. A left choice meant that the midpoint $(\ell^j + u^j)/2$ exceeded x_1, so that x_1 was contained in $[\ell^j, \frac{\ell^j + u^j}{2})$, which was then defined as the j+1th indifference interval $[\ell^{j+1}, u^{j+1})$. After a right choice we similarly took $[\frac{\ell^j + u^j}{2}, u^j)$ as the j+1th indifference interval $[\ell^{j+1}, u^{j+1})$. We did five iteration steps, ending up with $[\ell^6, u^6)$ (of length $96 \times 2^{-5} = 3$), and took its midpoint as the elicited indifference value x_1. We similarly elicited x_2 (substitute x_2 for x_1 and x_1 for x_0 above).

Measuring probability weighting for risk. Using the midweight method we elicited five probabilities $w^{-1}(1/8)$, $w^{-1}(2/8)$, $w^{-1}(4/8)$, $w^{-1}(6/8)$ and $w^{-1}(7/8)$. We framed the prospects as in Figure 4.1. All left prospects used in the experiment are special cases of Prospect L in Figure 3.1 with at least one probability 0, so that at most two branches remain.

FIGURE 4.2. Indifferences to elicit $w^{-1}(j/8)$



The midweight method concerns indifference between prospect L = $(a\!:x_2,\ d\!:x_1,\ c\!:x_0)$ and prospect $R = x_{2_{g+a}}x_0$ which, as shown in §3, implies that probability $g + a$ is the weight midpoint between probability a and probability $d + a$. For example, to obtain $w^{-1}(1/2)$, the weight midpoint between 0 and 1, we take, as in the left panel of Figure 4.2, $a = 0$ and $d = 1$, so that prospect L is the degenerate prospect yielding x_1 with certainty. Figure 4.2 lists the indifferences elicited to obtain the probabilities $w^{-1}(1/8)$, $w^{-1}(2/8)$, $w^{-1}(4/8)$, $w^{-1}(6/8)$, and $w^{-1}(7/8)$. In general, to find the g's to generate the required indifferences, we used a bisection method as in the outcome part of the experiment, explained in Appendix A.

Motivating participants. We used a variation of the random incentive system, the almost exclusively used real-incentive system for individual choice experiments today (Holt & Laury 2002; Starmer & Sugden 1991), as follows. For each session there were as many envelopes as participants, with one envelop containing a blue card and all other envelopes containing a white card. Each participant was asked to choose an envelope, after which the participant who had selected the envelop containing the blue card could play for real. For this participant, one choice question was again selected randomly and the chosen prospect in that choice question was played out for real, with the participant paid according to the prospect chosen and the outcome that resulted from playing out this prospect. All other participants in a particular session, who had chosen a white card, received a fixed payment of €5. The possible monetary outcomes of the prospects used during the experiment ranged from €30 to approximately €250. All payments were done privately, immediately at the end of the experiment. The average payment under real play was €77.57, so that the total reward per participant was approximately €11.60, while it took participants about 20 minutes to complete the experiment. This version of the random incentive system where only some participants are paid for real was compared to the more popular rewarding scheme where all participants are paid for real, with no difference found for static choice, by Harrison et al. (2007, footnote 16) and Armantier (2006). These papers considered static choice, as does our paper.

Further Stimuli. Our questions were chained. It is well-known that chaining can give incentives for not truthfully answering questions (Harrison 1986). To check whether participants had been aware of this possibility, we asked two *strategy-check questions*: "Was there any special reason for you to specially choose left more often, or specially choose right more often?" and "Can you state briefly which method you used to determine your choice?" These questions were asked in a questionnaire at the end of the experiment, with further questions about age, study, and gender.

Results; utility. The first measurement of outcome x_1 (x_2) did not differ significantly from its second measurement (Wilcoxon signed-rank tests, $z = 1.23$, $p = 0.2$ and $z = -1.48$, $p = 0.14$). We, therefore, take averages of the two measurements in the following analyses. We had also used those averages for the stimuli in the experiment.

The median values of x_1 and x_2 are 92.25 and 123, respectively, which, together with $x_0 = 60$, suggests linear utility. The deviation from linearity is not significant (Wilcoxon signed-rank test, $z = 0.887$, $p = 0.3751$), in agreement with the common hypothesis that utility is approximately linear for moderate amounts of money (Rabin 2000). At the individual level, 22 (38) out of 64 participants exhibited a concave (convex) utility function. This result is robust for gender and field of study.

Results; probability weighting. There was no order effect for decision weights and we, hence, pooled the data. Figure 4.3 displays the median weighting function. Means were similar to medians, and standard deviations were approximately 0.2. Overall we find a convex (pessimistic) pattern.

FIGURE 4.3. Median probability weighting w

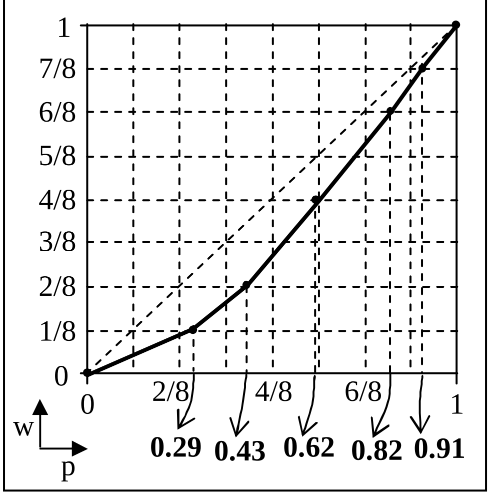


Table 4.1 confirms that participants did not process probabilities linearly, but mostly underweighted them. The probabilities $w^{-1}(\pi)$ all differ significantly from their corresponding weights π except for $w^{-1}(7/8)$.

TABLE 4.1. Counts of $w^{-1}(p) - p > 0$ and $w^{-1}(p) - p < 0$

$w^{-1}(p) - p$	> 0	< 0
$p = 1/8$	49**	15
$p = 2/8$	48**	16
$p = 4/8$	44**	20
$p = 6/8$	44**	18
$p = 7/8$	41	23

**denotes significance at the 1% level using a two-tailed Wilcoxon signed-rank test.

We used a classification system of individual weighting functions of participants of Bleichrodt & Pinto (2000), where details can be found. In short, we considered *slope differences*, i.e. changes in the average slope of the probability weighting function between two adjacent probability intervals. If, for the five adjacent probability interval pairs available in our data, at least three confirmed a particular shape (convex, concave, or linear) then the weighting function was classified as having this shape. Otherwise the weighting function was "unclassified." We found that 25% of the weighting functions were classified as concave, 62.5% as convex, 0% as linear, and 12.5% remained unclassified. Although this classification does not consider the inverse-S shape, it does confirm the prevalence of convex weighting. All the above analyses were nonparametric. For every participant we also estimated Prelec's (1998) two-parameter weighting function by minimizing the sum of squared residuals. This weighting function is given by

$$w(p) = e^{-\beta(-\ln p)^{\alpha}} \tag{4.2}$$

where α captures likelihood insensitivity (i.e. the degree to which behavior is sensitive towards changes in likelihood), and β captures the degree of optimism or pessimism. The median values of α and β were 1.1454 and 1.5781, while the values of α and β based on median data, as in representative agent analyses, were $\alpha = 1.054$, and $\beta = 1.763$. The former weighting function is depicted in Figure 6.4, and, obviously, accommodates the prevailing convexity. Further results, including individual, results are in the web appendix.

Results; strategy check questions. In the strategy-check questions, no participant revealed awareness of the chained nature of the questions, or an attempt to strategically exploit this chaining. 25 participants indicated a combination of (expected or maximal) value and safety, 5 went merely by expected value, and 4 went merely by highest value. Various other reasons were given.

5. Direct Measurement of the Weighting Function for Uncertainty

This section describes an experiment measuring the weighting function for uncertainty.

Participants. N = 44 undergraduate economics students from a wide range of disciplines were recruited from the student population at Tilburg University using an online recruitment system. The experiment was held on September 11, 2008. Participants were seated in front of personal computers in 4 different sessions with about 11 participants per session. 3 participants were excluded form the dataset because they gave erratic answers, such as always preferring left or right. The following analysis is based on the remaining 41 participants (21 female; median age 20). No conclusion would be altered if the 3 participants had been included.

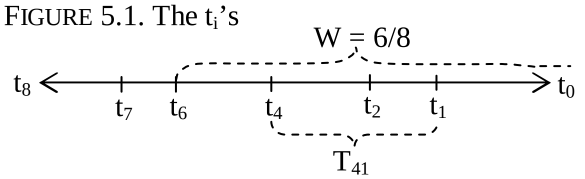
Procedure. Two practice choices served to familiarize the participants with the experimental procedure. In each question, the participants chose between a prospect L (left) and R (right) by clicking on the corresponding button. They were encouraged to answer the questions at their own pace.

Stimuli. Prospects yielded prizes depending on the mean temperature (described in °C) in Eindhoven 11 days after the experiment as measured by the Royal Dutch Meteorological Institute (KNMI). Prospects were framed in a way similar to the risk experiment. As for risk (Eq. 4.1), we set $x_0 = 60$ and then elicited indifferences:

$$\mathbf{x_1}_E 30 \sim 60_E 40 \text{ and } \mathbf{x_2}_E 30 \sim x_{1_E} 40, \qquad (5.1)$$

but now we used event E of mean temperature exceeding 15.7°C rather than a probability of 0.25. Again, x_1 and x_2 were elicited twice, their average was taken, and x_1 is the U midpoint of x_0 and x_2.

FIGURE 5.1. The t_i's

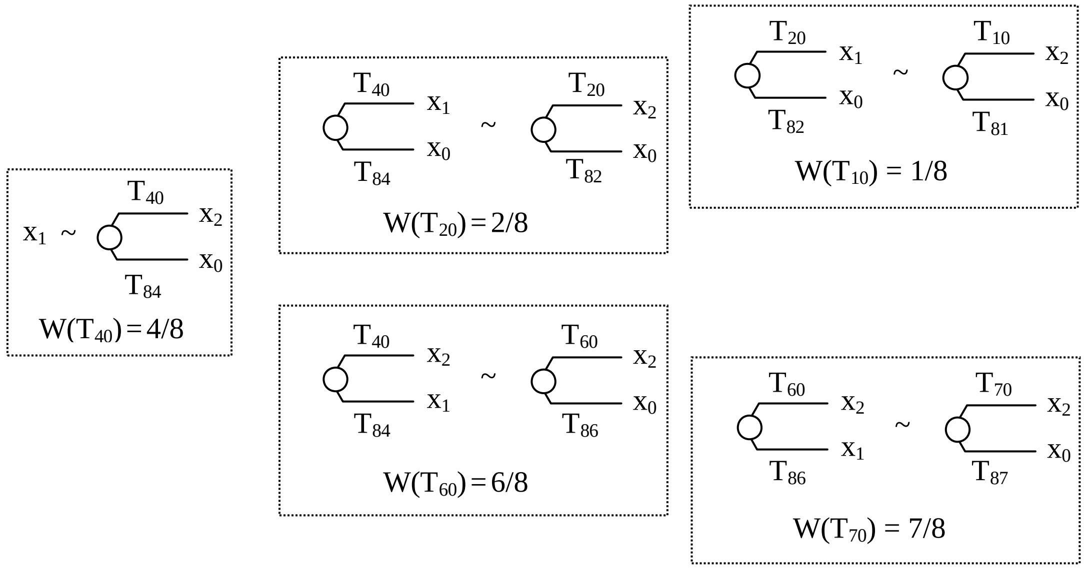


We then measured the W value of events $[t,\rightarrow)$ (temperature exceeding t). The temperatures measured were, in the order of elicitation, t_4, t_6, t_2, t_7, and t_1, satisfying:

$$W[t_i,\rightarrow) = i/8. \qquad (5.2)$$

Obviously, t_i decreases in i. T_{ij} denotes $[t_i,t_j)$ for $t_i < t_j$ $(i > j)$; see Figure 5.1. We write $t_0 = \infty$ and $t_8 = -\infty$. Indeed, $W[t_0,\rightarrow) = 0/8 = 0$ and $W[t_8,\rightarrow) = 8/8 = 1$, as in Eq. 5.2. $T_{i0} = [t_i,\rightarrow)$. A bisection choice method was again used to obtain indifferences between prospects. We used at most five iterations steps, stopping if the interval obtained was not broader than half a degree, and took its midpoint as the elicited indifference temperature t_i. Thus, a precision of a quarter degree results.

FIGURE 5.2. The elicited indifferences



Participants were informed that the average temperature in Eindhoven during the past 50 years had never been below 8.8°C or above 20.4°C. Therefore, the participants were told that the average temperature could be assumed to be in [7.2°C, 22°C), and this interval was the starting indifference interval containing t_4.

Motivating participants. This was done the same way as under risk, with a random incentive system, white and blue cards, and a show-up fee of €7.50. For each group, the participant who selected the blue card was invited to collect the possible prize at any day after the uncertainty about the temperature had been resolved.

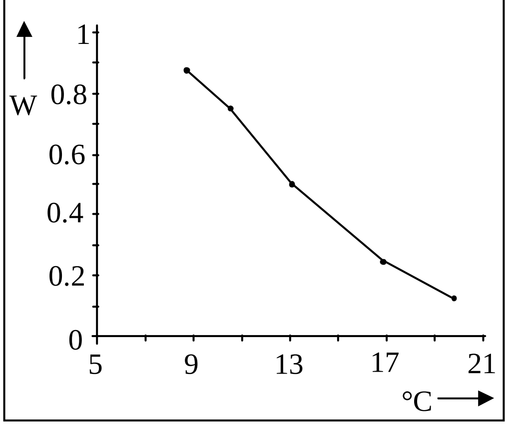
Results; utility. Again, the first measurement of outcome x_1 (x_2) did not differ significantly from the second measurement (Wilcoxon signed-rank tests, $z = 1.033$, $p = 0.3017$ and $z = -1.424$, $p = 0.1545$). The median values of x_1 and x_2 were 77.25 and 91.50, respectively, which, together with $x_0 = 60$, suggests linear utility on average (Wilcoxon signed-rank test, $z = 1.483$, $p = 0.1381$). Because the subjective likelihoods and subjective weightings may be different here than under risk, the values x_1 and x_2 can be expected to be different too; they were lower.[1] However, the absolute size of the x's is immaterial because only their equally spacedness in utility matters for our analysis. At the individual level, 22 (38) out of 64

[1] The historical probability of event E, based on data from the past 50 years, was 0.25, which is the same probability as used under risk. The participants were not informed about such historical data.

participants exhibited a concave (convex) utility function. This result is robust for gender and field of study.

Results; W. The median t_i values are $t_1 = 19.75$, $t_2 = 16.85$, $t_4 = 13.00$, $t_6 = 10.96$, and $t_7 = 9.70$, with means very similar, and standard deviations approximately 2.5. Figure 5.3 depicts the graph assigning the median W(t,→) to every temperature t.

FIGURE 5.3. The median W(t,→)



Direct Tests of Properties of W. If we obtain enough quantitative measurements of the weighting function then we can verify its properties such as additivity, convexity, and concavity. It is also possible to test such properties directly from qualitative preferences. Table 5.1 presents preferences that we observed through direct choices in the experiment (not allowing for indifferences but adding the top row for clarity), and the way in which they corroborate various properties of W. For example, with $U(0) = 0$, the value of $75_{T_{10}}0$ in the middle column is $W(T_{10})U(75)$, with W applied to the unlikely event T_{10}. The value of $0_{T_{87}}75$ in the right column is $W(T_{70})U(75)$, with W applied to the likely event T_{70}.

TABLE 5.1. Observed qualitative preferences.

W	W concerns unlikely events	W concerns likely events
additive	$75_{T_{10}}0 \sim 75_{T_{21}}0$	$0_{T_{87}}75 \sim 0_{T_{76}}75$
convex	$75_{T_{10}}0 \succcurlyeq 75_{T_{21}}0$ (34%)	$0_{T_{87}}75 \succcurlyeq 0_{T_{76}}75$ (44%)
concave	$75_{T_{10}}0 \preccurlyeq 75_{T_{21}}0$ (66%)*	$0_{T_{87}}75 \preccurlyeq 0_{T_{76}}75$ (56%)
inverse-S	$75_{T_{10}}0 \preccurlyeq 75_{T_{21}}0$ (66%)*	$0_{T_{87}}75 \succcurlyeq 0_{T_{76}}75$ (44%)

*: $p < 0.05$ (A two-sided Wilcoxon signed rank test with H_0: percentage is 50%.)

Proof for Table 5.1. We derive results for convexity of W. The other results are similar.

$75_{T_{21}}0 \preccurlyeq 75_{T_{10}}0 \Rightarrow W(T_{21}) \leq W(T_{10}) = 1/8 = W(T_{20}) - W(T_{10})$. Then T_{21} adds less weight to the vacuous event (which has weight zero) than to event T_{10}, to which it adds weight 1/8 because it augments the weight $W(T_{10}) = 1/8$ to $W(T_{20}) = 2/8$ there. This corroborates convexity of W.

$0_{T_{76}}75 \preccurlyeq 0_{T_{87}}75 \Rightarrow W(T_{87} \cup T_{60}) \leq W(T_{70})$ shows that T_{87} adds less than 1/8 weight to T_{60}, which is what it adds to its complement T_{70}. Again, the marginal W contribution of T_{87} to the larger T_{70} is larger than to the smaller T_{60}, corroborating convexity of W. □

For unlikely events, we find significantly more concavity than convexity, rejecting additivity and agreeing with inverse-S. For likely events the deviations from additivity were not significant.

Discussion. The values $W[t,\rightarrow)$ suffice to evaluate all prospects with outcomes increasing in temperature.[2] To evaluate other prospects, more measurements of W are needed. For example, for prospects with outcomes decreasing in temperature, we need to measure values $W(\leftarrow,t)$. In the absence of additivity, $W(\leftarrow,t)$ cannot be inferred from $W[t,\rightarrow)$ as just measured because these two values need not sum to 1. In general, to evaluate a prospect f, we have to measure W at all events $\{t: f(t) \geq \alpha\}$ for all outcomes α of the prospect. This added complexity is, as always, the price to pay for working with a more general model.

In general, the family of nonadditive measures is large, and often special subfamilies are considered so as to increase tractability. In the next section we will consider a special subfamily, put forward by Abdellaoui et al. (2009). Based on ideas of Tversky & Fox (1995), Abdellaoui et al. (2009) distinguished different sources of uncertainty. A *source* (*of uncertainty*) is a group of events that are generated by the same random mechanism. In our study, the two tosses of the 10-sided die, used to generate risk, constitute one source of uncertainty. The temperature in Eindhoven is another source of uncertainty. Abdellaoui et al. (2009) assumed that within each source (generic notation So) there exist subjective probabilities P_{So}, and for each source, the weighting function W is a transform $w_{So}(P_{So})$ of those subjective probabilities. The transformation w_{So} depends on the source and is called a *source function*. Probabilistic sophistication within one source characterizes a uniform degree of ambiguity (Wakker 2008) for that source, and not absence of ambiguity as has sometimes been claimed (Epstein & Zhang 2001). In the next section we analyze the uncertain source concerning temperature in Eindhoven using Abdellaoui et al.'s (2009) method.

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[2] This can be inferred from Eq. 2.2. It holds for general prospects f, as can be inferred from the general prospect (= rank-dependent) theory formula $\int_{\mathbb{R}^+} W\big(f^{-1}(U^{-1}[\alpha,\rightarrow))\big)d\alpha$.

6. Using Subjective Probabilities to Measure Ambiguity

This section shows how the midweight method can simplify the analysis of uncertainty and ambiguity (the difference between uncertainty and risk) proposed by Abdellaoui et al. (2009). We assume that probabilistic sophistication holds with a subjective probability measure P (depending on the participant) for temperature in Eindhoven. For each temperature event E, $W(E) = w_t(P(E))$ with w_t the Eindhoven-temperature source function.

The measurement of W can now be simplified considerably. Thus this section, in combination with §5, provides a complete measurement of W. We, first, measure the subjective probability measure P, something which has to be done also under Bayesian expected utility. Next, W as measured in §5 is plotted as a function of P, yielding the source function w_t. Then, the whole weighting function $W = w_t(P)$ has been determined, and all prospects can be evaluated, including those whose outcomes do not increase in temperature. With W and w_t entirely determined we can, obviously, also investigate all their properties. For example, expected utility holds if and only if W equals P, i.e. if and only if the source function w_t is linear.

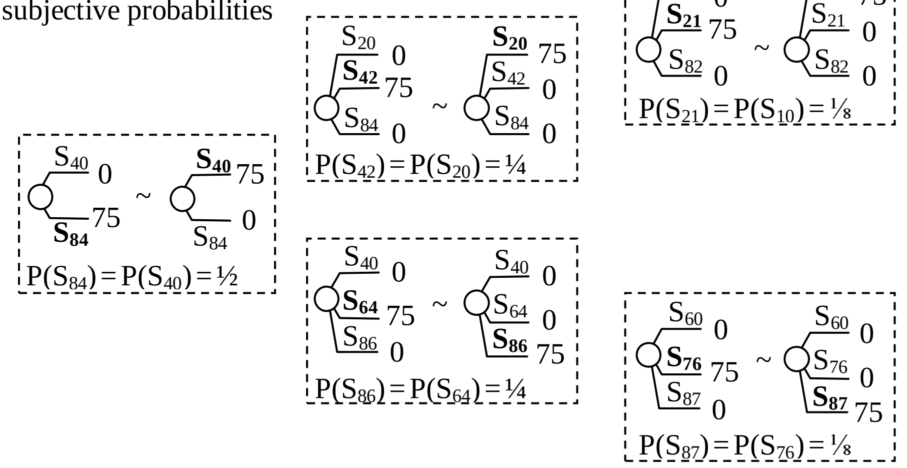
To measure P note that, with $x > 0$ and A and B temperature events, we have the following implication:

$$x_A0 \sim x_B0 \;\Rightarrow\; w_t(P(A))U(x) = w_t(P(B))U(x) \;\Rightarrow\; P(A) = P(B). \tag{6.1}$$

Events A and B as in Eq. 6.1 are called *equally likely*. Observations of equal likelihood can be used to measure P (Savage 1954). More specifically, we will use the method for eliciting subjective probabilities of Abdellaoui et al. (2009).

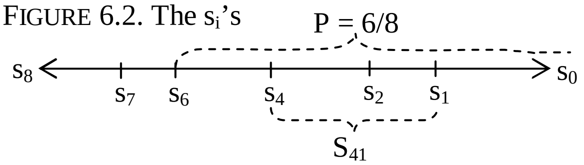
Stimuli.

FIGURE 6.1. Indifferences to elicit subjective probabilities



We measured, in the order of elicitation, temperatures s_4, s_6, s_2, s_7, and s_1, such that the indifferences in Figure 6.1 hold, with the notation $s_0 = \infty$, $s_8 = -\infty$, and $S_{ij} = [s_i,s_j)$. Then $P(s_i,\rightarrow) = i/8$ for all i, so that the notation is similar to that for the t_i's in preceding sections. The measurement procedure of indifference was the same as in Section 4. Under expected utility, $s_j = t_j$ for all j.

FIGURE 6.2. The s_i's



Results; subjective probabilities. Figure 6.3 displays the subjective probability distribution resulting from the median s_i's that we observed, together with the historical probability distribution from the past 50 years regarding September 22. Our participants generally considered high temperatures more likely than they were in the past, possibly because of global warming.

FIGURE 6.3. Median P[t,→) and historical (decumulative) distribution

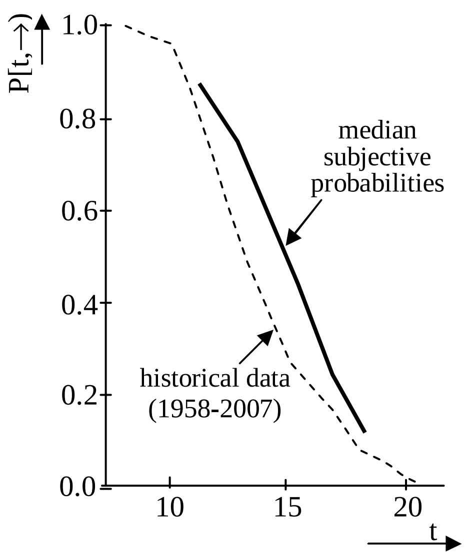
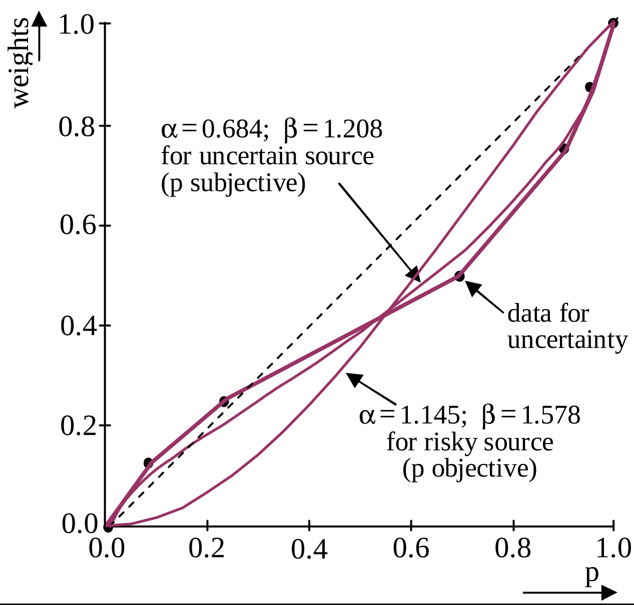


FIGURE 6.4. Source functions



Results; source function. Figure 6.4 displays the median source function. To fit domains, we used linear interpolation in the t_i scale. The source function displays an inverse-S shape with an intersection with the diagonal at about 0.3, which is confirmed by the values reported in Table 6.1. The differences between the W and P are always highly significant, both by t-tests and by Wilcoxon tests, except for t_2 (which determines T_{20}), which is no surprise because it is near the expected intersection point where overestimation changes into underestimation.

TABLE 6.1. Summary statistics for T-events

	W	Mean P	Median P	Standard deviation P
$P(T_{10})$	1/8*	0.133	0.081	0.144
$P(T_{20})$	2/8	0.310	0.229	0.218
$P(T_{40})$	4/8*	0.636	0.694	0.238
$P(T_{60})$	6/8*	0.836	0.903	0.155
$P(T_{70})$	7/8*	0.922	0.952	0.084

Again, we estimated Prelec's (1998) two-parameter weighting function (Eq. 4.2) for every individual by minimizing the sum of squared residuals. The median values of α and β were 0.684 and 1.208, respectively, while the values of α and β based on the median data were 0.622 and 1.166. The former weighting function is depicted in Figure 6.4, and, obviously, accommodates the prevailing inverse-S pattern. Individual results are in the web appendix.

Discussion of results and ambiguity attitudes. The significant differences between the s_is and the t_is provide yet another falsification of expected utility. Relative to measurements under expected utility, Abdellaoui et al's (2009) method requires the measurement of one additional curve per source. We emphasize that w_t concerns the entire attitude towards uncertainty, rather than a risk attitude.

The difference between w_t and w (the probability weighting function for risk as measured in §4) reflects ambiguity. We can make such a comparison between subjects here. Within-subject comparisons can obviously be obtained by carrying out both measurements of §4 and §5 within individuals. For brevity, we have not carried out such a task here, and leave it to future studies. Under universal ambiguity aversion, w_t would be below w everywhere, but this clearly is not the case. Instead, w_t is more inverse-S shaped than w, in agreement with claims and findings by Curley & Yates (1989), Tversky & Fox (1995), Abdellaoui, Vossmann, & Weber (2005), Kahn & Sarin (1998, p. 270), Kahneman & Tversky (1979, p. 281), Kilka & Weber (2001), and Weber (1994). This phenomenon was predicted by Ellsberg (2001) himself, and shows that modeling ambiguity attitudes through one single number to reflect a universal degree of ambiguity aversion is crude.

7. Other Measurements in the Literature

Measuring weighting functions for risk. In parametric fittings, the weighting and utility functions are usually estimated simultaneously. Gonzalez & Wu (1999) did not commit to a parametric family but still used fitting techniques that minimize squared distances, based on a complex numerical system that requires much data per participant. In return, their results are very reliable. Abdellaoui (2000) and Bleichrodt & Pinto (2000) provided two more tractable methods for estimating probability weighting functions nonparametrically. As with all other measurements used before, but unlike our midweighting method, these methods need a detailed measurement of utility. From n observed indifferences we obtain n–2 data points of the weighting function (plus 1 data point of utility), whereas Abdellaoui (2000) and Bleichrodt & Pinto (2000), for instance, would obtain only (n–1)/2 data points of probability weighting (plus (n–1)/2 data points of utility).

Blavatskyy (2006) described the general procedure of starting with measurements in one dimension, then using this to obtain measurements in the other dimension, possibly using the latter again to obtain more refined measurements in the first dimension, and so on. He examined general efficiency principles regarding error propagation of such general procedures.

Measurements of weighting functions for uncertainty. We are only aware of measurements (of more than one or two values) by Diecidue, Wakker, & Zeelenberg (2007) and Kilka & Weber (2001) who assumed linear utility, Mangelsdorff & Weber (1994) who assumed expected utility for risk, Abdellaoui, Vossmann, & Weber (2005) who adapted the methods of Abdellaoui (2000) and Bleichrodt & Pinto (2000) to uncertainty, and Abdellaoui et al. (2009), Fox, Rogers, & Tversky (1996), Fox & Tversky (1998), Andersen et al. (2007), and Tversky & Fox (1995) who carried out complex measurements that included measurements of utility functions. Furthermore, some studies used direct judgments of subjective probabilities (Einhorn & Hogarth 1985; Hogarth & Einhorn 1990; Wu & Gonzalez 1999) which are based on introspection and not on revealed preference. This paper has focused on revealed-preference based methods.

Measuring endogenous midpoints. We used the tradeoff measurement technique of Wakker & Deneffe (1996) to obtain utility midpoints derived endogenously from preference, as suggested by Köbberling & Wakker (2003, p. 408). Abdellaoui, Bleichrodt, & Paraschiv

(2007) and Abdellaoui & Munier (1999, Eqs. 1 & 2) similarly used this method. They next obtained a probability q with $w(q) = 0.5$ through what amounts to a degenerate version of Figure 3.1 with $c = 1$ and $a = 0$. Finally, they used this probability to efficiently measure utility midpoints in general. Their approach can, like our approach, be interpreted as a special case of Blavatskyy's (2006) general procedure.

Vind (1991, p. 134; 2003, §IV.2, above Theorem IV.2.1) proposed an alternative method for obtaining endogenous utility midpoints under expected utility and, more generally, under state-dependent expected utility (from which he derived what he called a mean groupoid operation). He showed that y is the utility midpoint between x and z if the following indifferences hold:

$$x \sim x_{1_q}x_2,\ z \sim z_{1_q}z_2, \text{ and } x_{1_q}z_2 \sim z_{1_q}x_2 \sim y. \tag{8.1}$$

His method holds under prospect theory if we add the requirement that $x_1 > x_2$, $x_1 > z_2$, $z_1 > z_2$, and $z_1 > x_2$.

Ghirardato et al. (2003, Definition 4) proposed another method to derive utility midpoints endogenously from preferences. They showed that β is the utility midpoint between α and γ under prospect theory if the following indifferences hold:

$$\alpha_q\gamma \sim x_q y,\ x \sim \alpha_q\beta, \text{ and } y \sim \beta_q\gamma \tag{8.2}$$

with $\alpha > \beta > \gamma$.

With β a utility midpoint between α and γ, the tradeoff method has γ as dependent variable and α and β as independent variables, whereas the other two methods have β as dependent variable and α and γ as independent variables. In the former case, the experimenter has no control over the range (α,γ), which entails a drawback of the tradeoff method. We still preferred this method because it requires fewer indifferences to be measured and is easier to implement experimentally.

8. General Discussion

Empirical studies have found that individual weighting functions are mostly convex or inverse-S shaped, with the latter shape prevailing. Thus, the majority of studies found that a majority of participants exhibited the inverse-S shape. We are aware of some 50 such references (Web-Appendix F). Yet, the finding is not universal, and several studies did not only find convex weighting functions for some of their participants, but even for a majority, as we did for risk.[3] Many other studies found other evidence against inverse-S.[4]

Thus, although we believe that inverse-S is the prevailing phenomenon, it certainly is not universal. It is not clear at this stage why different studies have found different results. Much about weighting functions remains yet to be discovered. Our findings and literature search suggest once more that probability weighting is a volatile phenomenon, with results depending on framing and ways of measurement, and with no phenomena holding in great generality. As one admittedly after-the-fact explanation, our design may have suppressed inverse-S somewhat because we kept outcomes fixed and focused on uncertainty, enhancing sensitivity towards uncertainty. Inverse-S entails insensitivity towards uncertainty. For risk this effect may have been enough to suppress the inverse-S shape. Because inverse-S is more pronounced for unknown probabilities, it may still have shown up for those. Our restriction to prospects from the boundary of the probability triangle may also have contributed to the extra pessimism.

In the experiment we used the midweight method to measure the weighting function over its whole domain. The method can also be used to investigate the local curvature of the weighting function. For example, if we want to know whether the weighting function is convex on a particular domain [a,c), then we can use our method to find the w-midpoint q between a and c, and then the w-midpoint between a and q, and so on, and in this manner we obtain local tests of convexity on [a,c).

The values x_1, x_2, and $w^{-1}(p)$ that were elicited from participants returned as inputs in later questions (chaining), and bisection also involves chaining. It is well known that participants can exploit chaining by not answering truthfully at particular questions so as to improve stimuli in future questions (Harrison 1986). Such a distortion is unlikely to have arisen in our experiment. It is difficult for participants to understand that their anwer to one question will influence future stimuli. For example, we did not directly ask for the indifference values used in future questions, but derived indifference values indirectly as midpoints between values used in choices, so that participants had not seen the indifference

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[3] See Goeree, Holt, & Palfrey (2002), Jullien & Salanié (2000), Kühberger, Schulte-Mecklenbeck, & Perner (1999, p. 217), Li et al. (2009), Mosteller & Nogee (1951 in their student population), and Qiu & Steiger (2008).

[4] See Barron & Erev (2003), Bearden, Wallsten, & Fox (2007), Birnbaum (2008, in particular pp. 484-486, and the many references to his preceding studies), Bleichrodt (2001), Fatas, Neugebauer, & Tamborero (2007), Goeree, Holt, & Palfrey (2003), Hartinger (1999), Henrich & Mcelreat (2002), Humphrey & Verschoor (2004), Kunreuther & Pauly (2003), Loomes (1991), Loomes, Moffat, & Sugden (2002), Luce (1996), and Stott (2006).

values before and in this way could not recognize them. In addition, to exploit chaining, not only the presence of chaining must be understood, but also the way in which future questions will depend on current answers, which will be very hard for subjects. Finally, our strategy-check questions revealed no strategic exploitation of chaining. We carefully formulated our instructions (end of Appendix B) in order to avoid deception.

We used the term “prospect” not only in our theoretical analysis, but also in the instructions and in the experiment. We did so because the term is neutral and avoids potential confounding effects resulting from connotations with terms such as lottery or gamble.

Because existing empirical evidence suggests that the most interesting behavioral phenomena occur when uncertain events are very likely or very unlikely to occur, we partitioned the events T_{02}, T_{68}, S_{02}, and S_{08}, but not the events T_{24}, T_{26}, S_{24}, and S_{26}. Following Abdellaoui et al. (2009), we chose not to partition the latter events so as to reduce the burden on participants.

9. Conclusion

We have introduced a new method for measuring functions that weigh risk and uncertainty. It is almost double as efficient as methods that have been used before because it minimizes the required measurements of utility. Experiments have demonstrated the feasibility of our method for both risk and uncertainty. A desirable feature of our method is that it serves well to study ambiguity, because it can be used for risk and uncertainty in the same way.

Appendix A. Bisection to Measure Indifference

The bisection method to find g to generate an indifference $(a\!: x_2,\, d\!: x_1,\, c\!: x_0) \sim x_{2_{g+a}}x_0$ as in Figure 3.1 proceeded as follows. We iteratively narrowed down so-called indifference intervals containing $g + a$, as follows. The first indifference interval $[b^1, u^1)$ was $[a, d + a)$, i.e. the interval of which the weighting-midpoint was to be found.[5] By stochastic dominance, it contains $g + a$ indeed. Each participant was first asked to make two practice choices between a particular prospect L and prospect $R = x_{2_{g^+ + a}}x_0 = x_{2_{g^- + a}}x_0$, where probability $g^+ + a$ $(g^- + a)$ was

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[5] The first indifference interval is, thus, $[0,1]$ for $w^{-1}(4/8)$, $[0, w^{-1}(4/8))$ for $w^{-1}(2/8)$, $[w^{-1}(4/8), 1]$ for $w^{-1}(6/8)$, $[0, w^{-1}(2/8))$ for $w^{-1}(1/8)$, and $[w^{-1}(6/8), 1]$ for $w^{-1}(7/8)$.

set equal to the upper (lower) limit of the range of the first indifference interval of probability $g+a$ minus (plus) 1/100. Then the iterative process started.

To construct the $j+1^{th}$ indifference interval $[b^{j+1},u^{j+1})$ from the j^{th} indifference interval $[b^j,u^j)$, we elicited whether the midpoint of $[b^j,u^j)$ was larger or smaller than $a+g$. To do so, we observed the choice between $(a{:}\,x_2,\ d{:}\,x_1,\ c{:}\,x_0)$ and $x_{2_{(b^j+u^j)/2}}x_0$. A right choice meant that the midpoint was larger than $g+a$, so that $g+a$ was contained in $[b^j,\frac{b^j+u^j}{2})$, which was then defined as the $j+1^{th}$ indifference interval $[b^{j+1},u^{j+1})$. A left choice meant that the midpoint was smaller than $g+a$, so that $g+a$ was contained in $[\frac{b^j+u^j}{2},u^j)$, which was then defined as the $j+1^{th}$ indifference interval $[b^{j+1},u^{j+1})$. We did five iteration steps like this, ending up with $[b^6,u^6)$, and took its midpoint as the elicited indifference probability $a+g$.[6]

As an illustration, Figure A.1 replicates the bisection procedure followed to obtain the probability corresponding to the weight of 0.5. The particular pattern of answers depicted there, preferring the right prospect twice and the left prospect three times, was exhibited by 6 of our participants. After the fifth iteration step, the midpoint of the last indifference interval was taken as the final indifference probability. Thus, individual indifference between the certain prospect (x_1) and the prospect $x_{2_{0.615}}x_0$ was inferred from the choices made by the 6 participants whose choices are replicated in Figure A.1.

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[6] Because prospects yielded prizes depending on the result of a roll with two ten-sided dice, we only allowed values j/100 for probabilities. When a particular midpoint probability was not a value j/100, the computer took the closest value j/100 on the left of this value if the value was lower than half and on the right of this value if the value was higher than half. The order of elicitation was varied between participants to prevent potential order effects. For some participants the order of elicitation was $w^{-1}(.5)$, $w^{-1}(2/8)$, $w^{-1}(6/8)$, $w^{-1}(1/8)$, $w^{-1}(7/8)$, whereas for other participants the order of elicitation was $w^{-1}(.5)$, $w^{-1}(6/8)$, $w^{-1}(2/8)$, $w^{-1}(7/8)$, $w^{-1}(1/8)$.

FIGURE A.1. The bisection method for measuring $w^{-1}(0.5)$

Choice Question	Indifference interval	Prospect L	Prospect R	Prospect chosen	Inference
1	$[b^1,u^1] = [0, 1]$	x_1	0.50: x_2; 0.50: x_0	L	$w^{-1}(0.5) > 0.50$
2	$[b^2,u^2] = [0.50, 1]$	x_1	0.75: x_2; 0.25: x_0	R	$w^{-1}(0.5) < 0.75$
3	$[b^3,u^3) = [0.50, 0.75)$	x_1	0.63: x_2; 0.37: x_0	R	$w^{-1}(0.5) < 0.63$
4	$[b^4,u^4) = [0.50, 0.63)$	x_1	0.57: x_2; 0.43: x_0	L	$w^{-1}(0.5) > 0.57$
5	$[b^5,u^5) = [0.57, 0.63)$	x_1	0.60: x_2; 0.40: x_0	L	$w^{-1}(0.5) > 0.60$
Conclusion	$[b^6,u^6) = [0.60, 0.63)$;	x_1	0.615: x_2; 0.385: x_0	–	$w^{-1}(0.5) \approx 0.615$

Appendix B. Experimental Instructions

[Instructions have been translated from Dutch into English]

Welcome to this experiment. If you have any question while reading these instructions, please raise your hand. The experimenter will then come to your table to answer your question. This experiment will take about half an hour. We ask you to make a number of decisions during this experiment. Each time, you choose between two so-called “prospects.” Both prospects yield prizes depending on the roll of the two 10-sided dice similar to the ones that are on your table right now.

As you can see, one 10-sided die has the values 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 and the other 10-sided die has the values 00, 10, 20, 30, 40, 50, 60, 70, 80, and 90. If we code the sum of the roll “a 0 and a 00” as 100, then the sum of a roll with both 10-sided dice yields a random number from 1 up to 100.

The prospects from which you have to choose are called Prospect L (left) and Prospect R (right), and are presented in the following way:

PROSPECT L		
roll	probability	prize
1 to 40	40%	100 euro
41 to 100	60%	50 euro

PROSPECT R		
roll	probability	prize
1 to 20	20%	150 euro
21 to 100	80%	20 euro

In the case depicted here, Prospect L yields a prize of 100 Euro if the sum of the roll with both 10-sided dice is 1 up to 40 and if the sum of a roll is 41 up to 100, Prospect L yields a prize of 50 Euro, as you can see. Similarly, Prospect R yields a prize of 150 Euro if the sum of a roll with both 10-sided dice is 1 up to 20 and otherwise Prospect R yields a prize of 20 Euro.

Both the prizes as well as the probabilities of yielding certain prizes can vary across decisions. We ask you to choose between Prospect L and Prospect R each time, by clicking the corresponding button with the mouse.

For your participation in this experiment, you receive 5 Euro at any rate. In addition, one participant will be selected at random at the end of this experiment. Each participant will then randomly pick a sealed envelope containing either a white or a blue card. Participants selecting an envelope containing a white card receive 5 Euro for their participation. For the participant whose envelope contains a blue card, one of their decisions will be selected at random by rolling both 10-sided dice. Thereafter, the prize of the chosen prospect in the decision selected will be determined by rolling the two 10-sided dice again. The resulting prize, always larger than 5 Euro, will be paid out to the participant with the blue card.

There are no right or wrong answers in this experiment. The experiment exclusively concerns your own preferences. Those are what we are interested in. At every decision it is best for you to choose the prospect that you want most. If you select the envelope containing the blue card at the end of the experiment, that decision can be selected at the end of the experiment. Then, the chosen prospect will be played out. Of course you want that prospect to be your preferred prospect. If you have no further questions then you can now start with the experiment by clicking on the “Continue” button below.

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