Economic Dances for Two (and Three)

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Economic Dances for Two (and Three)

Economische dansen voor twee (en drie)

Thesis

to obtain the degree of Doctor from the
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by command of the
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by
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born in Moscow,
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ERASMUS UNIVERSITEIT ROTTERDAM

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Acknowledgements

It was a lovely Saturday afternoon. It could have been an even lovelier Saturday morning, but then – which PhD students wake up early on Saturdays? The sun was mild, occasionally obstructed by the passage of magnificent white clouds, a frequent wave would have its crest broken, only to reappear a moment later anew, or, when it felt like it, to become a spray of sweet water into our faces, the air around sounded like a forgiveable moderate breeze. On the near horizon there was a white noise of shapes and colours, the outline of IJburg. A small sailing boat went quietly on a beam reach, with only a squeak or two from old wooden rigging.

The whole atmosphere was inviting for an easy talk. “So, let me go back to my question. The problem is – when I add a third period to include effort, I get these six degree ...”. One man can spoil it all.

Only a credible threat of getting wet beyond repair persuaded the author to put his mind aside from the everlasting research questions and do something more immediately useful, like fetching cheese and apples and trimming the jib.

I would like to thank all my friends, who were around during these years and helped me relax from my research, thus, undoubtedly, indirectly contributing to its quality. Every break was welcome, whether it was as complex as a game of go and a cup of tea, as straightforward as skiing down a black slope in near zero visibility, or as challenging as balancing on one’s toe metres above the ground in a reasonable attempt to reach the next handhold.

My friends did a really good job in distracting me from my PhD, but fortunately they did not succeed completely, for which I would like to thank my supervisor, Maarten Janssen. More importantly, I would like to thank Maarten for shaping my understanding of how modern economic research is done. A skill no less useful than my PhD.

Contents

Introduction

The industrial revolution accompanied by economies of scale, the growth of consumer wealth along with modern communication technologies, the relative scarcity of entrepreneurial talent – these are some of the major factors that made oligopolies what they are today – a common, if not the most common, market form.[1] Simply think of the last durable good you have purchased, whether it was a household appliance, a computer gadget, or maybe a digital piano. It is likely that it came from a company that shares its market not with many but with a few competitors only.

For an economist the primary task is standard: to understand how oligopolistic markets operate and, in gaining this knowledge, to be able to judge how society shall organize oligopolistic markets so as to improve social welfare. This task gives challenges from theoretical and applied perspectives. The theoretical challenge is to discover appropriate mathematical concepts and methods, which would help to formalize oligopolistic interactions. The applied challenge is to enumerate all the important aspects of oligopolistic competition and to study those aspects in detail.

What is it that we shall look at in oligopolistic markets? Trivially, prices and quantities (capacities) are the first candidates. Cournot (1838) showed that quantity competition between duopolists attains an equilibrium – a situation where no duopolist wants to change his choice unilaterally. The work of Cournot was later reviewed by Bertrand (1883), who also brought up the discussion of price competition – if the marginal costs of competing firms are the same and are constant, then there is in an equilibrium in prices, where each competitor sets his price equal to his marginal cost. Cournot and Bertrand were pointing out that oligopolistic markets attain a state where quantities and prices become stable. From a methodological perspective, this stability allows a straightforward discussion about the welfare implications of different policies.

[1] See the opening paragraphs in Sylos-Labini (2008).

Edgeworth (1922, 1925) considered price competition with increasing marginal costs. In contrast to Cournot and Bertrand, he demonstrated that in his setting no equilibria exist, which left open the question whether general oligopolistic interactions can be described in any systematic way.[2] It was not until later, when von Neumann and Morgenstern (1944) laid the foundations for the theory of games and Nash (1950) introduced his concept of an equilibrium, that this conceptual question was, in a certain sense, answered. Today our toolbox is rich with equilibrium concepts: from games of incomplete information (Bayesian Nash equilibria, Harsanyi, 7 68) to games of imperfect information (sequential equilibria, Kreps and Wilson, 1982), to dynamic games (Markov perfect equilibria, see, for example, Dockner et al., 2000). Thus, from a mathematical perspective, analyzing a model of oligopolistic competition has become a standard task. However, the challenge of understanding and describing all the important facets of oligopolistic competition remains open.

The field in question, industrial organization, already covers topics like vertical and horizontal differentiation, mergers and acquisitions, dynamic competition and tacit collusion, advertisement, research and development and multiple others, and is still growing.[3] There are always new research questions, because companies always seek and employ new ways to compete, within the law and occasionally outside it. After all, investments into Pareto-improving technologies have diminishing returns, like any other economic activity, so companies also invest in new competition mechanisms – to gain at the expense of others. Contrast R&D investments with salaries for those top managers who posses acknowledged marketing skills. It is this natural process of seeking new ways to compete that makes the field of IO so rich and expanding.

In my thesis I also take the aforementioned applied challenge and I use game-theoretical tools to contribute to the following four topics in IO: competition in quality, dynamic oligopolistic competition, organizational choice and research and development. In all cases I consider competition between two or three firms, so that the relevant strategic interactions are most pronounced.

Chapter 1 is devoted to competition in quality. Together with my supervisor and coauthor, Maarten Janssen, we argue that firms are often able

[2]Today we know that Bertrand-Edgeworth competition posses only a mixed strategy equilibrium.

[3]See, for example, the Handbook of Industrial Organization, volumes 1-3.

to vary the quality of their products along with the price. In this framework we study whether prices can signal quality to those consumers who are aware of the prices of competing offers, but are unaware of the respective quality. In brief, we conclude that there can be an equilibrium, where the prices signal the quality precisely, so that even those consumers who do not know the quality can infer it perfectly. However, while such consumers are not "cheated" in the equilibrium, their mere presence creates an equilibrium characterized by price/quality dispersion and Pareto-inefficiency.

Chapter 2 is devoted to dynamic oligopolistic competition. Together with Alexei Parakhonyak we point at an ability of firms to target particular rivals, which can be achieved through product differentiation, comparative advertisement, lawsuits or a variety of other mechanisms. Despite such an ability being obvious, it has received little attention from the literature. We show, however, that it contains important economic insights. Our main result is that targeted competition coupled with sufficiently forward-looking firms stabilizes competition, i.e. all the firms remain in the market. If, on the other hand, the firms are myopic, then the market boils down to a monopoly.

Chapter 3 is devoted to organizational choice. I study the choice of duopolists between functional and divisional organizational structures as a means of committing to certain pricing strategies. I show that product differentiation – both between and within the firms – matters for their organizational choices. Another result is that in certain markets, those organizational forms are chosen that would have been sub-optimal for a monopolist.

Chapter 4 innovates in the field of research and development. Most literature today studies R&D in a framework where innovations are protected by patents. Contrary to the existing literature, I consider a framework where innovations are protected as trade secrets. Think, for example, of specific technologies behind websites or of IT infrastructures in private companies. In both cases there is only partial patent protection: while particular software implementations can be patented, algorithms and ideas can not be, so those technologies are often kept as trade secrets.[4] Within this trade secrets framework I explore the incentives of competing firms to form a research joint venture. The main result is that under Cournot competition, if there is a small chance of a major innovation, the firms prefer to keep their innovations as trade secrets, otherwise the firms prefer to form a research joint venture.

[4] To give a specific example, try finding out what software Google uses to operate its servers and how much it costs to run that software.

Chapter 1

Oligopolistic Competition in Price and Quality

1.1 Introduction

The very nature of competition implies that firms compete in as many ways as possible, and not just in price. Firms will choose that combination of strategic variables that serves their interests best. However, if consumers are homogeneous in their preferences among these variables and if they are fully informed about all relevant product characteristics, then this multi-dimensional form of competition can be expressed in terms of a one dimensional competition model, essentially identical in nature to that of price competition.

When consumer preferences differ, or when some consumers are better informed than others, the competitive process involving many dimensions does not have a single dimension analogue and should be analysed in its own right. In this chapter we restrict our attention to markets where firms compete in two dimensions: price and quality. There are different approaches known in the literature dealing with endogenous price/quality competition. Chan and Leland (1982), Cooper and Ross (1984), and Schwartz and Wilde (1985) emphasize the heterogeneity of information among consumers. Wolinsky (1983), Rogerson (1988), and Besancenot and Vranceanu (2004) additionally emphasize the heterogeneity of preferences. Another dimension along which these models differ is the type of market interaction considered. The above models either consider perfect competition or monopolistic competition.

Contrary to the aforementioned literature, we address the issue of price/quality competition in a *strategic* oligopoly model where price and quality are endogenously chosen and concentrate on the role of consumers having heterogeneous information (and therefore take consumer preferences to be identical). In a recent paper Armstrong and Chen (2009) consider a similar framework where they focus on boundedly rational consumers that observe the prices but do not infer the corresponding quality, even if such inference is possible. We, on the other hand, focus on rational consumers, and the signalling role of prices for quality inference plays a central role in our study.

We ask the following questions. First, do firms differentiate themselves with different prices and/or quality choices or do they make the same choices? Everyday experience suggests that price and/or quality dispersion is quite common in many markets. How to explain price dispersion was first addressed by Stigler (1961). The role of imperfect information in explaining quality dispersion has been less emphasized (but receives some attention in, e.g., Chan and Leland 1982). Second, can price act as a signal of quality to consumers who somehow cannot evaluate it? From previous literature with exogenous quality we know that the adverse selection problem can be mitigated if firms can signal quality choices to the consumers on the basis of the prices they charge (see, e.g., Bagwell and Riordan 1991). Third, how should we characterize the outcomes in terms of Pareto-efficiency?

Stigler (1961) has pointed out that acquiring information about market prices is costly. As consumers can have different search costs, different groups of consumers can be present in a market: those who know all prices and those who do not. This idea is central in Varian (1980). The idea also readily extends to quality. In Cooper and Ross (1984), for example, all consumers know prices, but some of them are informed about quality while the rest is uninformed about quality. We combine these approaches in the following way. Quality is a more complex notion than price and so it is more costly for a consumer to learn the quality than to learn the price a firm charges. We therefore assume there are three groups of consumers in our model: fully informed consumers know prices and quality of the products in a market, partially informed consumers know the prices but not the quality and fully uninformed consumers know neither prices nor quality. We emphasize the role of partially informed consumers. When they are present in a market, price is not just an instrument of competition between firms, but potentially also a signalling instrument. When they are absent, our model essentially reduces to a variation on Varian (1980) where price is replaced by a price/quality combination.

We analyse the consequences of this informational scenario in a model where two firms choose price and quality and consumers buy one good at most. Either firm is unaware of the quality choice of its competitor before it has to make its own choice over the price. The formal model is therefore one where firms choose prices and quality simultaneously. The case where firms have to choose prices while being unaware of the quality chosen by their competitors is also at the heart of Daughety and Reinganum (2005, 2007), Janssen and Roy (2010), and Janssen and van Reeven (1998). They provide examples of markets (such as markets with illegal practices and/or where safety standards are involved) where firms are indeed unaware of the quality rival firms produce. This type of markets is one motivation for our setup. The cited papers treat quality as exogenous, however, where we consider endogenous quality choice.

Another motivation for the simultaneous price/quality move stems from the observation that many firms, in consumer electronics, clothing, automobiles and other industries, compete in price/quality *bundles*: prices and quality tend to change together. For example, a new digital camera comes out with a certain price tag and usually keeps this tag during the period, in which the company itself and its competitors are quick enough to release other new cameras (hence new quality choices) along with new price tags. It is this kind of competition where quality can change as fast as price that suggests simultaneous decisions in prices and quality.

In this model with endogenous quality choice, we arrive at the following results. First, there is an equilibrium characterized by a dispersion of prices and quality where price signals quality precisely. This kind of equilibrium correspondence between price and quality is formally described as a curve in a price-quality strategy space resulting in a one-dimensional distribution of price/quality offers over that curve. In such an equilibrium partially informed consumers learn the true quality from the prices. Second, consumers' preferences over the resulting price/quality offers are monotone in price: a consumer always prefers either the cheapest offer or the most expensive one. Which of these two particular equilibria is to occur depends on how marginal utility of quality changes with respect to price. Third, though the preferences over equilibrium price/quality offers are monotone in price, equilibrium quality need not be so, e.g. the quality may be worse for average prices and better for low and high prices. Fourth, we show that price/quality combinations offered in equilibrium are Pareto-inefficient due to the signalling behaviour of firms.

This chapter is organized in the following way. Section 1.2 formally introduces the model. Section 1.3 provides the equilibrium analysis. Section 1.4 gives an example that illustrates the more complicated expressions of section 1.3. Section 1.6 concludes. The more technical proofs of propositions are given in the appendix.

1.2 Setup

We consider a market with two firms selling similar products.[1] The firms choose both price and quality of the product they offer. There is a unit mass of consumers who choose where to buy. The timing is as follows. First, firms simultaneously decide on the price and quality of their products. Second, each consumer decides from which firm to buy and whether to buy the product at all.

The production technology is such that producing higher quality comes at a higher cost. For simplicity we assume a linear dependency[2] so that the per-unit profits are given by

$$\Pi(p, q) = p - aq, \tag{1.1}$$

where p and q represent price and quality and the coefficient $a > 0$ characterizes the quality production technology. We take a to be the same across firms. We assume that the firms make their production costs at the moment of sale, so there are no excess goods that are produced but not sold.

Since the firms move simultaneously a strategy of either firm is simply a distribution over all possible (p, q) bundles. Let P_i and Q_i be the random variables that stand for the price and quality offered by firm i and let $\mathbb{P}_i$ be the probability measure that corresponds to the strategy of firm i.

Consumers are homogeneous in their preferences and are represented by a utility function $U(p, q)$. All consumers have the same reservation utility U_R and demand one unit of good. Total demand is normalized to 1. Consumers search for the best price/quality combination – the one that maximizes $U(p, q)$.

As explained in the introduction, we consider three groups of consumers: (i) fully informed consumers know the prices and quality offered by both

[1]The model is readily extendible to multiple firms. Besides the possibility to study the limiting case, we expect that our qualitative results will not change with that extension. However, as our model have parallels to Varian (1980), there can be additional asymmetric equilibria when there are many firms – see Baye et al. (1992).

[2]A more general concave function gives the same qualitative results.

firms, (ii) partially informed consumers know the prices offered but not the quality, and (iii) fully uninformed consumers know neither price nor quality. These groups are referred to as H, M and L consumers, respectively. Their relative sizes are given by λ_H, λ_M and λ_L with $1 = \lambda_H + \lambda_M + \lambda_L$.

The H group consists of expert consumers who know the firms and the products and can check costlessly for prices and quality. A new consumer or a consumer who lacks certain expertise to assess the quality, but who can take his or her time to check for different price offers belongs to the M group. A person with substantially high alternative costs of searching for a better price and quality is a typical member of the L group.

Consumers search for the best offer given the information they have. H consumers know exactly what the offers are and know how the utility from the best offer compares against their reservation utility U_R. M and L consumers search for the best offer based on their expectations and so they do not know how the offers they choose from actually compare against their reservation utility. We assume, however, that the firms have a return policy (as is required by law in many countries), and that any consumer can learn the quality of the product he purchased within the return period.[3] Therefore, if a consumer purchases a (p, q) bundle with $U(p, q) < U_R$, the product is returned, the firm makes no profit, and the consumer gets U_R.

Let us now put the behaviour of the consumers into a more formal context. Let (p_i, q_i) be the offer of firm i. A consumer of type H knows both offers in full detail and he compares $U(p_1, q_1)$, $U(p_2, q_2)$ and U_R and chooses the option that gives the highest payoff. A consumer of type M knows only p_1 and p_2 but not q_1 or q_2. Given p_1 and p_2, he has beliefs about the distribution of Q_1 and Q_2. We use the notation $\hat{U}_j(p_i; p_{-i})$ to denote consumer j's expected utility – where the expectation is taken over his beliefs – from the offer of firm i given both the price of firm i and the price of its competitor, firm $-i$. An M consumer can buy from either firm 1 or 2, the corresponding expected payoffs are $\hat{U}_j(p_1; p_2)$ and $\hat{U}_j(p_2; p_1)$. When he buys from firm i, he learns quality q_i. Now he can decide whether to keep or to return the product. If he keeps it, he gets $U(p_i, q_i)$, otherwise he gets U_R.

Consumers of type L go to only one of the firms and buy there, and we assume that *half* of them go to either firm. Once the product is purchased, an L consumer knows the price and learns the quality, and he can either keep

[3] It is feasible therefore that a consumer purchases from both firms, compares the quality at home, keeps the best offer and returns the other. Such a consumer, however, values his time less and so by definition belongs to the H group.

or return the product. He keeps it if $U(p_i, q_i) \geq U_R$.

The subsequent analysis is based on the assumption that the utility function $U(p, q)$ is well-behaved.

Assumption 1.1. *The utility function $U(p, q)$ is strictly decreasing in p, strictly increasing in q, strictly quasi-concave in (p, q) and twice differentiable in (p, q). Moreover, $U(p, q)$ is such that the optimization problem*

$$\max_{p,q} \Pi(p, q) \quad s.t. \quad U(p, q) \geq x \tag{1.2}$$

has a solution for any $x \geq U_R$ (in a sense, the utility function should be sufficiently quasi-concave).

1.2.1 Equilibria

This is a game with complete but imperfect information. So, we use the notion of sequential equilibrium (Kreps and Wilson 1982). In short, players' strategies and beliefs form a sequential equilibrium if the strategies are sequentially rational given the beliefs and the beliefs are consistent with the strategies.

In general, a certain price may signal a specific distribution of quality.[4] We restrict attention, however, to symmetric equilibria where a certain price p signals a certain quality $\hat{q}(p)$, we name function $\hat{q}(p)$ an equilibrium curve. This restriction considerably simplifies the analysis. We show that in this restricted class of equilibria interesting price and quality choices can be made. Formally, we restrict our attention to exact signalling equilibria defined as follows:

Definition 1.1. *A sequential equilibrium is called an exact signalling equilibrium if (i) the strategies of the firms are symmetric, i.e $\mathbb{P}_1 \equiv \mathbb{P}_2$, (ii) $supp(\mathbb{P}) = \{(p, q) : p \in [p_l, p_h], q = \hat{q}(p)\}$ and (iii) $\hat{q}(p)$ is continuously differentiable in p over $[p_l, p_h]$, where p_l and p_h are some arbitrary bounds and $\hat{q}(p)$ is some arbitrary function of p.*

It is important to note that conditions (i)-(iii) restrict the set of equilibrium strategies. We impose no restrictions on out-of-equilibrium strategies, i.e. a firm can deviate to playing any possible (p, q) bundles if it finds doing so profitable.

[4]This happens if firms play some mixed strategies over a region in (p, q) space.

As the sequential rationality of the consumers' strategies is obvious from the description above, it remains to discuss the consistency of beliefs and the sequential rationality of firms' strategies.

The consumers of the H group do not hold any beliefs, because they observe prices and quality directly. The consumers of the L group, as assumed, posses trivial beliefs: half of them beliefs firm 1 has a better offer, the other half beliefs firm 2 has a better offer (these beliefs do not and can not depend upon the prices or quality of the offers, because L consumers do not observe any of that information). As for the consumers of the M group, their expected utility is consistent with the strategies of the firms if it is computed over the probability measure that defines those strategies. So,

$$\hat{U}_j(p_i; p_{-i}) = \mathbb{E}(\max(U(P,Q), U_R) \mid P = p_i) \quad \forall p_i \in [p_l, p_h], \tag{1.3}$$

where $\mathbb{E}$ is the expectation operator. We take the maximum of $U(P,Q)$ and U_R, because a consumer can choose to return the product if the realization of $U(P,Q)$ is smaller than his reservation utility U_R.

Equation (1.3) tells us that consistent beliefs, when considered for $p_i \in [p_l, p_h]$, result in an expected utility, which is the same for all M consumers and does not depend upon the price of the rival firm. Therefore, we use the following notation: $\hat{U}_j(p_i; p_{-i}) = \hat{U}(p_i)$.

In an exact signalling equilibrium for any realization of P there is a unique corresponding realization of $Q = \hat{q}(P)$. So, $U(P,Q) = U(P, \hat{q}(P))$ and

$$\begin{aligned}\hat{U}(p) = \mathbb{E}(\max(U(P,Q), U_R) \mid P = p) = \\ \mathbb{E}(\max(U(P, \hat{q}(P)), U_R) \mid P = p) = \max(U(p, \hat{q}(p)), U_R)\end{aligned} \tag{1.4}$$

for all $p \in [p_l, p_h]$. Equation (1.4) gives the consistency condition for beliefs. Turning to the sequential rationality of the strategies of the firms, we begin by writing down the expected profits a firm gets if it selects a particular (p,q) bundle and if its rival is playing an equilibrium strategy.

Let $\mu_H(p,q)$, $\mu_M(p)$ and μ_L denote the expected number of H type, M type and L type consumers that a firm gets if it charges (p,q) such that $U(p,q) \geq U_R$. Given the sequentially rational strategies of consumers

$$\mu_H(p,q) = \mathbb{P}(U(P,Q) < U(p,q)) \cdot \lambda_H + \mathbb{P}(U(P,Q) = U(p,q)) \cdot \frac{\lambda_H}{2}. \tag{1.5}$$

Indeed, all H consumers go to firm i if its (p,q) bundle gives higher utility than that of the rival. In general, a rival plays a mixed strategy and hence the chance to get all H consumers is given by $\mathbb{P}(U(P,Q) < U(p,q))$. If both offers give the same utility consumers split evenly.

In a similar way we have

$$\mu_M(p,q) = \mathbb{P}(\hat{U}(P) < \hat{U}(p)) \cdot \lambda_M + \mathbb{P}(\hat{U}(P) = \hat{U}(p)) \cdot \frac{\lambda_M}{2}. \tag{1.6}$$

The only difference is that M consumers do not compare actual utilities, but rather the expected utilities given the prices. For $p \in [p_l, p_h]$ the expected utility is given by equation (1.4), for $p \notin [p_l, p_h]$ the expected utility comes from out-of-equilibrium beliefs and, in principal, can be be arbitrary. Theorem 1.3, which concerns the existence of exact signalling equilibria, touches on out-of-equilibrium beliefs in more detail (see the proof of the theorem).

Finally, for the L consumers we have $\mu_L = \frac{\lambda_L}{2}$.

Define, for convenience, $\mu(p,q) = \mu_H(p,q) + \mu_M(p) + \mu_L$. Then expected profits are given by

$$\pi(p,q) = \begin{cases} \mu(p,q) \cdot \Pi(p,q) & \text{if } U(p,q) \geq U_R, \\ 0 & \text{otherwise.} \end{cases} \tag{1.7}$$

As the firms choose simultaneously, a firm's sequentially rational strategy is simply a best response strategy. Choosing a (p,q) bundle over an equilibrium curve $\hat{q}(p)$ is a best response strategy if and only if the profit function $\pi(p,q)$ attains its maximum along that equilibrium curve:

$$\text{supp}(\mathbb{P}) \in \arg\max_{p,q} \pi(p,q). \tag{1.8}$$

Equations (1.4) and (1.8) give necessary and sufficient conditions for there to be an exact signalling equilibrium. To have non-trivial results we make the following additional assumption:

Assumption 1.2. *The model is non-degenerate, i.e. there exists* (p,q) *such that* $\Pi(p,q) > 0$ *and* $U(p,q) \geq U_R$.

Since $\pi(p,q) \geq \mu_L \cdot \Pi(p,q)$ if $U(p,q) \geq U_R$, a firm can always guarantee itself some positive profits given the above assumption. So, in an equilibrium no firm will offer (p,q) such that $U(p,q) < U_R$. Consequently,

$$\hat{U}(p) = \max(U(p,\hat{q}(p)), U_R) = U(p,\hat{q}(p)) \quad \forall p \in [p_l, p_h]. \tag{1.9}$$

Next we rewrite μ_H and μ_M in terms of a common distribution function. Let $F(u) = \mathbb{P}(U(P,Q) < u) = \mathbb{P}(\hat{U}(P) < u)$ and $dF(u) = \mathbb{P}(U(P,Q) = u) =$

$\mathbb{P}(\hat{U}(P) = u)$, then

$$\mu_H(p,q) = F(U(p,q)) \cdot \lambda_H + dF(U(p,q)) \cdot \frac{\lambda_H}{2}, \tag{1.10}$$

$$\mu_M(p) = F(\hat{U}(p)) \cdot \lambda_M + dF(\hat{U}(p)) \cdot \frac{\lambda_M}{2}. \tag{1.11}$$

In the next section we show that in any exact signalling equilibrium $U(p, \hat{q}(p))$ is strictly monotone in p. Therefore an exact signalling equilibrium is fully characterized by its equilibrium curve $\hat{q}(p)$, by the boundary points p_l and p_h, by the distribution of utilities along the equilibrium curve, namely $F(u)$, and by its out-of-equilibrium beliefs, namely $\hat{U}(p)$ for $p \notin [p_l, p_h]$.

1.3 Analysis

In this section we solve for an exact signalling equilibrium, i.e. we solve for $F(u)$, $\hat{q}(p)$, p_l and p_h given $U(p,q)$ and given the other parameters of the model. At first we assume that there exists an exact signalling equilibrium and we derive its properties. Later, we also discuss existence conditions for an exact signalling equilibrium.

One of the functions that characterizes an exact signalling equilibrium is a CDF of utility over the equilibrium curve, namely $F(u)$. In this section we solve for $F(u)$.

$\hat{U}(p)$ is continuous in p because $U(p,q)$ is continuous in p and q and $\hat{q}(p)$ is differentiable. The continuity of $\hat{U}(p)$ allows us to define $[U_l, U_h] = \hat{U}([p_l, p_h])$. So, in equilibrium the corresponding utility is distributed over an interval. We next show that $F(u)$ does not have atoms.

Lemma 1.1. *$F(u)$ is continuous and $dF(u) \equiv 0$.*

In economic terms, the chance that the rivals provide the same utility level is zero, a result that is very similar in nature to the result that in the "model of sales" (Varian, 1980) the price distribution is atomless. The formal proof of this statement is therefore omitted. The next lemma argues that U_l must be equal to U_R. The main reason is that the firm offering the worst utility only gets uninformed consumers and if $U_l > U_R$, it could make more profit by providing them a worse deal.

Lemma 1.2. $U_l = U_R$.

Given lemmas 1.1, 1.2 and equation (1.9) (consistency of beliefs) it is straightforward to simplify the expression for $\pi(p,q)$.

Lemma 1.3. *For $p \in [p_l, p_h]$ the profits are given by*

$$\pi(p,q) = \left(F(U(p,q)) \cdot \lambda_H + F(U(p,\hat{q}(p))) \cdot \lambda_M + \frac{\lambda_L}{2} \right) \cdot \Pi(p,q) \qquad (1.12)$$

if $U(p,q) \geq U_R$ and they equal 0 otherwise.

To find the functional form of $F(u)$ we need to be able to define equilibrium per-unit profits as a function of utility (lemma 1.5 will clarify why this is necessary). The following lemma allows us to do so.

Lemma 1.4. *Given $u \in [U_l, U_h]$ per-unit profits $\Pi(p, \hat{q}(p))$ are the same for all $p \in \hat{U}^{-1}(u)$.*

To understand this lemma, take an arbitrary u from $[U_l, U_h]$. The iso-utility curve corresponding to u is implicitly given by $U(p,q) = u$. This iso-utility curve will intersect the equilibrium curve $\hat{q}(p)$ at least once. If $\{(p_i, \hat{q}(p_i))\}$ is the set of intersection points, then from the definition of $\hat{U}$, $\{p_i\}$ is precisely $\hat{U}^{-1}(u)$. At each intersection point $(p_i, \hat{q}(p_i))$ we can compute the per-unit profits $\Pi(p_i, \hat{q}(p_i))$. The lemma states that $\Pi(p_i, \hat{q}(p_i))$ does not depend upon a particular choice of the intersection point, it only depends upon u. So, we use the notation $\hat{\Pi}(u)$ to denote profits a firms obtains by offering a utility level $u(p,q)$.

Formally, take an arbitrary $\tilde{p}(u)$ such that $\tilde{p}(u) \in \hat{U}^{-1}(u)$ for all $u \in [U_l, U_h]$. Then

$$\hat{\Pi}(u) = \Pi(\tilde{p}(u), \hat{q}(\tilde{p}(u))). \qquad (1.13)$$

It is not possible to define $\hat{\Pi}(u)$ explicitly as it involves choosing a particular $\tilde{p}(u)$ and the functional form of $U(p,q)$ is not given. However, once a specific functional form of $U(p,q)$ is adopted, and once $\hat{q}(p)$ is known, it is possible to choose a particular $\tilde{p}(u)$ and hence solve for $\hat{\Pi}(u)$.

Now we can solve for the functional form of $F(u)$ using techniques that are known in the search literature.

Lemma 1.5.

$$F(u) = \frac{1}{2} \cdot \frac{\lambda_L}{\lambda_H + \lambda_M} \left(\frac{\hat{\Pi}(U_R)}{\hat{\Pi}(u)} - 1 \right) \quad \textit{for} \quad u \in [U_l, U_h]. \qquad (1.14)$$

Proof. It follows from lemma 1.3 that

$$\pi(p, \hat{q}(p)) = \left(F(U(p, \hat{q}(p))) \cdot (\lambda_H + \lambda_M) + \frac{\lambda_L}{2} \right) \cdot \Pi(p, \hat{q}(p)). \qquad (1.15)$$

Evaluating (1.15) at $\tilde{p}(u)$, noticing that $U(\tilde{p}(u), \hat{q}(\tilde{p}(u))) = u$ and using the definition of $\hat{\Pi}(u)$ gives

$$\pi(\tilde{p}(u), \hat{q}(\tilde{p}(u))) = \left(F(u) \cdot (\lambda_H + \lambda_M) + \frac{\lambda_L}{2}\right) \cdot \hat{\Pi}(u). \tag{1.16}$$

For there to be an equilibrium the strategies of the firms should be sequentially rational. Therefore $\pi(p, \hat{q}(p))$ is constant over this interval, and we denote its value by $\hat{\pi}$. Since $\tilde{p}(u) \in [p_l, p_h]$ we get

$$\hat{\pi} = \left(F(u) \cdot (\lambda_H + \lambda_M) + \frac{\lambda_L}{2}\right) \cdot \hat{\Pi}(u). \tag{1.17}$$

By definition, $F(U_l) = 0$. Also, $U_l = U_R$. So,

$$\hat{\pi} = \frac{\lambda_L}{2} \cdot \hat{\Pi}(U_R). \tag{1.18}$$

Plugging it back and solving for $F(u)$ gives the result. □

It then follows that U_h is implicitly given by $F(U_h) = 1$. Using lemma 1.5 it may alternatively be given by

$$\hat{\Pi}(U_h) = \frac{1/2 \cdot \lambda_L \cdot \hat{\Pi}(U_R)}{\lambda_H + \lambda_M + 1/2 \cdot \lambda_L}. \tag{1.19}$$

The previous lemma shows that the distribution of utility levels over the equilibrium curve is such that the fraction of uninformed consumers to the other consumers determines the spread of the utility. This is intuitive as the firms have market power over these uninformed consumers and if there are many of them, the price quality offers concentrate around the offers that are cheapest to provide. We next provide a description of the equilibrium curve $\hat{q}(p)$.

Lemma 1.6. *If there is an exact signalling equilibrium then $\hat{q}(p)$ has to satisfy*

$$\frac{d\hat{q}}{dp} = -\frac{\lambda_H + \lambda_M}{\lambda_M} \cdot \frac{U'_p(p, \hat{q}(p))}{U'_q(p, \hat{q}(p))} - \frac{\lambda_H}{a\,\lambda_M} \tag{1.20}$$

everywhere on (p_l, p_h).

Proof. It should be that

$$\frac{\partial \pi(p,q)}{\partial p}\Big|_{(\tilde{p}, \hat{q}(\tilde{p}))} = 0, \quad \frac{\partial \pi(p,q)}{\partial q}\Big|_{(\tilde{p}, \hat{q}(\tilde{p}))} = 0, \tag{1.21}$$

because otherwise a firm may get higher profits by deviating along the gradient vector. Using lemma 1.3 we get

$$\begin{aligned}\frac{\partial \pi(p,q)}{\partial p}\Big|_{(\tilde p,\hat q(\tilde p))} &= \Big(F'(U(p,q))\cdot U'_p(p,q)\cdot \lambda_H + \\ &F'(U(p,\hat q(p)))\cdot \big(U'_p(p,\hat q(p)) + U'_q(p,\hat q(p))\cdot \hat q'(p)\big)\cdot \lambda_M\Big)\cdot (p-aq)+ \\ &\Big(F(U(p,q))\cdot \lambda_H + F(U(p,\hat q(p)))\cdot \lambda_M + \frac{\lambda_L}{2}\Big)\cdot 1\Big|_{(\tilde p,\hat q(\tilde p))} = \\ &F'(U(\tilde p,\hat q(\tilde p)))\Big(U'_p(\tilde p,\hat q(\tilde p))(\lambda_H+\lambda_M) + U'_q(\tilde p,\hat q(\tilde p))\cdot \hat q'(\tilde p)\cdot \lambda_M\Big)\cdot \\ &(\tilde p - a\hat q(\tilde p)) + \Big(F(U(\tilde p,\hat q(\tilde p))\cdot(\lambda_H+\lambda_M) + \frac{\lambda_L}{2}\Big) = 0 \end{aligned} \quad (1.22)$$

and

$$\begin{aligned}\frac{\partial \pi(p,q)}{\partial q}\Big|_{(\tilde p,\hat q(\tilde p))} &= \Big(F'(U(p,q))\cdot U'_q(p,q)\cdot \lambda_H\Big)\cdot (p-aq)+ \\ &\Big(F(U(p,q))\cdot \lambda_H + F(U(p,\hat q(p)))\cdot \lambda_M + \frac{\lambda_L}{2}\Big)\cdot (-a)\Big|_{(\tilde p,\hat q(\tilde p))} = \\ &\Big(F'(U(\tilde p,\hat q(\tilde p)))\cdot U'_q(\tilde p,\hat q(\tilde p))\cdot \lambda_H\Big)\cdot (\tilde p - a\hat q(\tilde p))+ \\ &\Big(F(U(\tilde p,\hat q(\tilde p))\cdot(\lambda_H+\lambda_M) + \frac{\lambda_L}{2}\Big)\cdot(-a) = 0. \end{aligned} \quad (1.23)$$

From (1.22) and (1.23) it follows after some algebra that

$$\frac{d\hat q}{d\tilde p} = -\frac{\lambda_H+\lambda_M}{\lambda_M}\cdot \frac{U'_p(\tilde p,\hat q(\tilde p))}{U'_q(\tilde p,\hat q(\tilde p))} - \frac{\lambda_H}{a\,\lambda_M}. \quad (1.24)$$

□

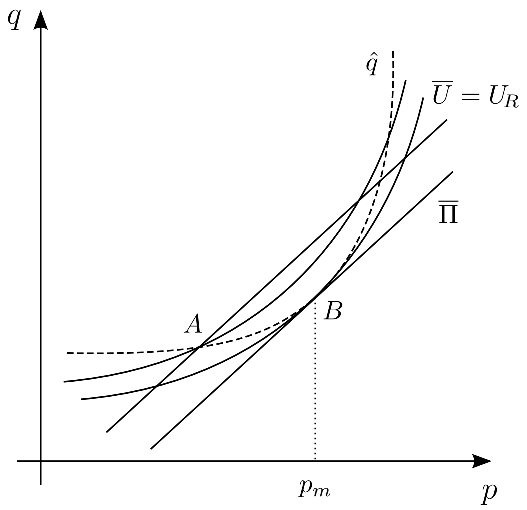
To illustrate the impact of the lemma, figure 1.1 depicts an equilibrium curve $\hat q(p)$ together with iso-utility curves and isolines of per-unit profits.

To see why an equilibrium curve has a shape as given in the figure, rewrite the differential equation for $\hat q(p)$ as follows:

$$\frac{d\hat q}{dp} = \frac{\lambda_H}{\lambda_M}\left(-\frac{U'_p(p,\hat q(p))}{U'_q(p,\hat q(p))} - \frac{1}{a}\right) - \frac{U'_p(p,\hat q(p))}{U'_q(p,\hat q(p))}, \quad (1.25)$$

and recall that the isoline of per-unit profits has a slope equal to $1/a$. The slope of the iso-utility curves is $-\frac{U'_p(p,q)}{U'_q(p,q)}$. Therefore it follows from (1.25) that if the slope of an iso-utility curve is less than $1/a$ (point A, for example), the slope of an equilibrium curve is even smaller at that point and vice versa. If the slope is exactly $1/a$ (point B), then an iso-utility curve and an equilibrium curve are tangent to each other and they are also tangent to

Figure 1.1: Equilibrium Curve



Notation: $\overline{X}$ stands for $X(p,q) = \text{const}$, where constant is arbitrary; $\overline{X} = X_0$ stands for $X(p,q) = X_0$, where X_0 is some specific value.

an isoline of per-unit profits at that point. Therefore an equilibrium curve relative to iso-utility curves should look as depicted in the figure.

According to lemma 1.2 the lowest attainable utility along an equilibrium curve equals U_R , thus the equilibrium curve in the figure "lies" on an iso-utility curve that corresponds to $U = U_R$.

To find out the boundary points of an equilibrium curve, let us refer to figure 1.1 once more. An equilibrium curve spans $[p_l, p_h]$, by definition. The figure shows that a choice of $[p_l, p_h]$ has important economic consequences: if $[p_l, p_h]$ is located to the left of the point of tangency p_m then $U(p, \hat{q}(p))$ is decreasing in p, i.e. *lower* prices signal higher utility. If, on the contrary, $[p_l, p_h]$ is to the right of p_m then $U(p, \hat{q}(p))$ is increasing in p and *higher* prices signal higher utility.

We find that both cases are possible depending upon the properties of $U(p,q)$, but that in both cases p_m is the boundary point of the price interval. To state the main result, we need two definitions. First, we formally define point (p_m, q_m):

Definition 1.2. *The point* (p_m, q_m) *is uniquely*[5] *defined by*

$$(p_m, q_m) = \arg\max_{(p,q):U(p,q) \geq U_R} \Pi(p,q). \tag{1.26}$$

[5]The solution to the optimization problem exists by assumption 1.1. Moreover, the solution is unique because $U(p,q)$ is strictly quasi-concave by the same assumption.

Lemma A1.1 (appendix) shows that the point (p_m, q_m) defined in this way belongs to the equilibrium curve $\hat{q}(p)$ and that the equilibrium utility $U(p, \hat{q}(p))$ attains its minimum at p_m – just like it is in the figure.

Second, we define a contract curve in the usual way as a curve that consists of all Pareto-efficient allocations in (p, q) plane:

Definition 1.3. *Let*

$$(p^*(x), q^*(x)) = \arg\max_{(p,q):U(p,q)\geq x} \Pi(p, q). \tag{1.27}$$

Then, if there exists a function g such that $q^*(x) = g(p^*(x))$, we shall refer to this function as a contract curve.

As, in principle, it is not necessary that $g(p)$ is defined for every p, we make the following technical assumption:

Assumption 1.3. *A contract curve $g(p)$ is defined in the neighbourhood of p_m and is differentiable at this point.*

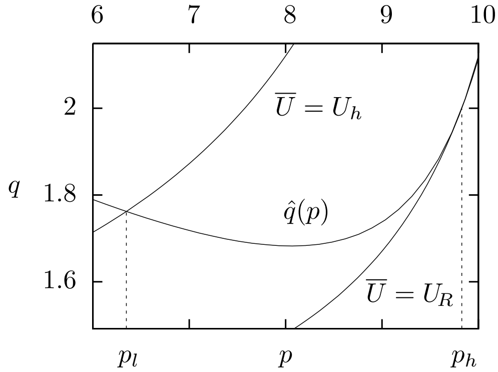
Now we can state the following theorem:

Theorem 1.1. *If $g'(p_m) < \frac{1}{a}$ and if there exists an exact signalling equilibrium then $[p_l, p_h] = [p_l, p_m]$ and $U(p, \hat{q}(p))$ is strictly decreasing in p over this interval. Hence in such an equilibrium higher prices signal lower utility.*

If $g'(p_m) > \frac{1}{a}$ and if there exists an exact signalling equilibrium then $[p_l, p_h] = [p_m, p_h]$ and $U(p, \hat{q}(p))$ is strictly increasing in p over this interval. Hence in such an equilibrium higher prices signal higher utility.

Theorem 1.1 explains that the signalling equilibrium relation between price and utility is monotone and can be of two different types. The main intuition for this result can be explained by reference to lemma 1.3 and figure 1.1. The profit of a firm along the equilibrium curve should be constant. Both the fully and partially informed consumers buy at the firm where the perceived utility is highest. Below the point p_m in figure 1.1, this implies that these consumers buy at the lowest possible price. To make that firms are indifferent between any point on the equilibrium curve, lemma 1.3 tells us that on that part of the equilibrium curve lower prices should be accompanied by lower per-unit profits and this is accomplished by a slope of the equilibrium curve strictly below $1/a$. The reverse argument holds true for points on the equilibrium curve above p_m. However, the theorem says nothing about the monotonicity of the equilibrium quality, which is $\hat{q}(p)$. It turns out that

Figure 1.2: Non-monotonic $\hat{q}(p)$



Notation: $\overline{U} = u$ stands for $U(p,q) = u$.

$\hat{q}(p)$ is not necessarily monotone. Figure 1.2 illustrates a particular exact signalling equilibrium with a non-monotone curve.[6]

If the marginal utility of quality declines as the price increases, i.e., if $U_{qp} < 0$, then $g(p)$ has a negative slope and $[p_l, p_h] = [p_l, p_m]$, so an equilibrium where *lower* prices signal higher utility results.

Given Theorem 1.1 we can solve for the boundary points p_l and p_h. If $g'(p_m) < \frac{1}{a}$ then $p_h = p_m$. As in this case $\hat{U}(p) = U(p, \hat{q}(p))$ is strictly decreasing in p, $p_l = \hat{U}^{-1}(U_h)$.Similarly, if $g'(p_m) > \frac{1}{a}$, $p_l = p_m$ and $p_h = \hat{U}^{-1}(U_h)$.

1.3.1 Existence and Uniqueness

In the previous sections we have uniquely determined all the parameters of an exact signalling equilibrium (except for out-of-equilibrium beliefs). Therefore we have the following theorem:

Theorem 1.2. *There is at most one exact signalling equilibrium(up to out-of-equilibrium beliefs).*

Proof. We recollect some of the results obtained so far. (p_m, q_m) was uniquely defined in def. 1.2. $\hat{q}(p)$ is given by a differential equation of lemma 1.6 and it is known to go through (p_m, q_m) (lemma A1.1), (p_m, q_m) is thus the boundary point to uniquely solve the differential equation. Depending upon the sign

[6] We used the following example to build the figure: $U(p,q) = (q-1)^{1/2}(10-p)^{1/2}$, $\lambda_H = 0.05$, $\lambda_M = 0.1$, $\lambda_L = 0.85$, $U_R = 1$ and $a = 1$. Section 1.4 gives another example and shows how to solve for an exact signalling equilibrium. If to apply that procedure to this example, one will get precisely fig. 1.2. However, this particular example we do not discuss in detail as the computations are harder comparing with the example of section 1.4.

of $g'(p_m)$ we know that an exact signalling equilibrium spans either $[p_l, p_m]$ or $[p_m, p_h]$. In either case $U(p, \hat{q}(p))$ is strictly monotone (theorem 1.1) and therefore $\tilde{p}(u)$ is uniquely determined by $U(\tilde{p}(u), \hat{q}(\tilde{p}(u))) = u$. In turn, $\tilde{p}(u)$ gives us $\hat{\Pi}(u)$ and $\hat{\Pi}(u)$ gives $F(u)$ (see eq. (1.13) and lemma 1.5 respectively). Equation $F(U_h) = 1$ uniquely determines U_h and from U_h we can determine the remaining p_l (or p_h). Hence we can uniquely determine the parameters of an exact signalling equilibrium, namely $\hat{q}(p)$, p_l, p_h and $F(u)$. □

Next, we address the existence issue. The main issue here is the following. Note that for any given utility function $U(p, q)$ and given the rest of the parameters (λ_H, λ_M, λ_L and a) we can always find an equilibrium curve $\hat{q}(p)$, its boundary points p_l and p_h and the distribution of utility over that curve, namely $F(u)$. We also know that profit function $\pi(p, q)$ will be constant along $\hat{q}(p)$ as required. But none of the results obtained so far guarantees that $\pi(p, q)$ will attain its maximum over $\hat{q}(p)$. Unfortunately, for a general utility specification it is impossible to provide sufficient existence conditions that do not involve a complete solution of the model. The difficulty that arises is that there is no explicit expression for the profit function at points that are off the equilibrium curve.[7] However, we can address the question of existence from a different angle: given an arbitrary equilibrium curve $\hat{q}(p)$, can we find such parameters of our model that there is a corresponding exact signalling equilibrium, i.e. one that has $\hat{q}(p)$ as its equilibrium curve? The following theorem 1.3 provides the answer, but first we need to formulate two additional necessary conditions.

Consider an arbitrary strictly increasing, strictly convex and twice differentiable equilibrium curve $\hat{q}(p)$ defined over some $[p_l, p_h]$. If we are looking for a corresponding equilibrium where lower prices signal higher utility, then $p_m = p_h$ and, consequently, $\hat{q}'(p_h) = \frac{1}{a}$. Hence, we know a and we can evaluate per-unit profits $\Pi(p, q)$. If there is an equilibrium it should be that

$$\Pi(p, \hat{q}(p)) = p - a\hat{q}(p) = p - \frac{\hat{q}(p)}{\hat{q}'(p_h)} > 0 \tag{1.28}$$

for all $p \in [p_l, p_h]$.

For a strictly increasing and strictly convex $\hat{q}(p)$ per-unit equilibrium profits $\Pi(p, \hat{q}(p))$ are strictly increasing in p over $[p_l, p_m]$ and therefore (1.28)

[7] In section 1.5 we provide an existence theorem for a specific linear-kinked utility function.

is equivalent to

$$\Pi(p_l, \hat{q}(p_l)) = p_l - \frac{\hat{q}(p_l)}{\hat{q}'(p_h)} > 0. \tag{1.29}$$

In a similar way, if we are looking for a corresponding equilibrium where higher prices signal higher utility, it should be that

$$\Pi(p_h, \hat{q}(p_h)) = p_h - \frac{\hat{q}(p_h)}{\hat{q}'(p_l)} > 0. \tag{1.30}$$

Given these two conditions we can state the theorem.

Theorem 1.3. *Consider an arbitrary strictly increasing, strictly convex and twice differentiable equilibrium curve $\hat{q}(p)$ defined over $[p_l, p_h]$ and satisfying* (1.29) *or* (1.30) *or both. Then there exist a utility function $U(p,q)$ satisfying assumption 1.1, parameters $(U_R, \lambda_H, \lambda_M, \lambda_L, a)$ and out-of-equilibrium beliefs such that there will be a corresponding exact signalling equilibrium, i.e one that has $\hat{q}(p)$ as its equilibrium curve.*

In other words, it may not be possible to have an exact signalling equilibrium for any $U(p,q)$, but at least there will be exact signalling equilibria for as many different forms of $U(p,q)$ as to generate every possible strictly increasing, strictly convex equilibrium curve $\hat{q}(p)$ that allows for positive per-unit profits.

1.3.2 Pareto-efficiency

An allocation is Pareto-efficient in this model if an iso-utility curve is tangent to an isoline of per-unit profits. Considering figure 1.1, one can see that for any $(p, \hat{q}(p))$ with $p \in [p_l, p_h)$ this is not the case. Therefore, equilibrium allocations are almost surely Pareto-inefficient.

This result may not be surprising as such, but it marks a sharp difference with Varian's model of sales, which is essentially this model (with prices being replaced by utilities) when there are no partially-informed consumers. In that model, all the equilibrium allocations will be Pareto-efficient. The presence of partially-informed consumers and the incentives they create for firms to signal quality with price is what brings Pareto-inefficiency. Fully uninformed consumers do not create Pareto-inefficiency on their own, they merely create a redistribution in welfare.

1.4 An Example

In this section, we illustrate an exact signalling equilibrium with an example. Take

$$U(p,q) = \frac{1}{2}\ln q - p,\ U_R = -2,\ \lambda_H = \lambda_M = \frac{1}{5},\ \lambda_L = \frac{3}{5},\ a = 1. \tag{1.31}$$

We begin by solving for (p_m, q_m). To do so we solve

$$\max_{p,q} \Pi(p,q) \quad \text{s.t.} \quad U(p,q) \geq U_R \tag{1.32}$$

and obtain

$$p_m = \frac{1}{2}\ln\frac{1}{2} - U_R = \frac{1}{2}\ln\frac{1}{2} + 2, \quad q_m = \frac{1}{2}. \tag{1.33}$$

Next we shall check whether it's an equilibrium where higher prices signal lower utility or the one where higher prices signal higher utility. To do it we need to know $g'(p_m)$. From (1.32) one can readily see that q_m does not depend upon U_R and hence contract curve $g(p) = q_m = \frac{1}{2}$. Therefore $g'(p_m) = 0 < \frac{1}{a} = 1$ and we have to search for an equilibrium to the left of p_m, i.e. $[p_l, p_h] = [p_l, p_m]$ (see theorem 1.1).

Let us now find $\hat{q}(p)$. Plugging our utility and the parameters into the differential equation for $\hat{q}(p)$ (see lemma 1.6) gives

$$\frac{d\hat{q}(p)}{dp} = 4\hat{q}(p) - 1. \tag{1.34}$$

Solving it and using the boundary condition $\hat{q}(p_m) = q_m$ gives:

$$\hat{q}(p) = e^{4p-8} + \frac{1}{4}. \tag{1.35}$$

To find utility distribution $F(u)$ we need to know $\hat{\Pi}(u)$ and for that we need to find $\tilde{p}(u)$ such that $U(\tilde{p}(u), \hat{q}(\tilde{p}(u))) = u$. Writing down this latter expression gives

$$\frac{1}{2}\ln\left(e^{4\tilde{p}(u)-8} + \frac{1}{4}\right) - \tilde{p}(u) = u. \tag{1.36}$$

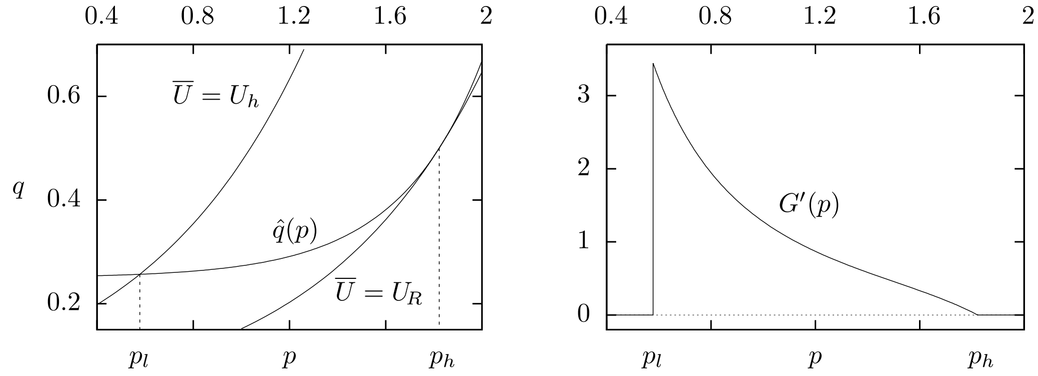
A little bit of algebra gives the solution:

$$\tilde{p}(u) = \frac{1}{2}\ln\left(\frac{1}{2}\left(e^{2u} - \sqrt{e^{4u} - e^{-8}}\right)\right) + 4. \tag{1.37}$$

Having (1.35) and (1.37) we therefore also have

$$\hat{\Pi}(u) = \tilde{p}(u) - a\hat{q}(\tilde{p}(u)) = \tilde{p}(u) - \hat{q}(\tilde{p}(u)) \tag{1.38}$$

Figure 1.3: Equilibrium Characteristic Functions



Notation: $\overline{U} = u$ stands for $U(p, q) = u$.

and

$$F(u) = \frac{1}{2} \cdot \frac{\lambda_L}{\lambda_H + \lambda_M} \left(\frac{\hat{\Pi}(U_R)}{\hat{\Pi}(u)} - 1 \right) = \frac{3}{4} \left(\frac{\hat{\Pi}(U_R)}{\hat{\Pi}(u)} - 1 \right). \tag{1.39}$$

Given $F(u)$ we can find U_h from $F(U_h) = 1$. Define

$$z = \frac{1}{2} \left(e^{2U_h+4} - \sqrt{e^{4U_h+8} - 1} \right). \tag{1.40}$$

This way $z \leq \frac{1}{2}$ and $F(U_h) = 1$ can be rewritten as

$$\frac{1}{2} \ln z - z^2 + \frac{7}{4} - \frac{3}{7} \hat{\Pi}(U_R) = 0. \tag{1.41}$$

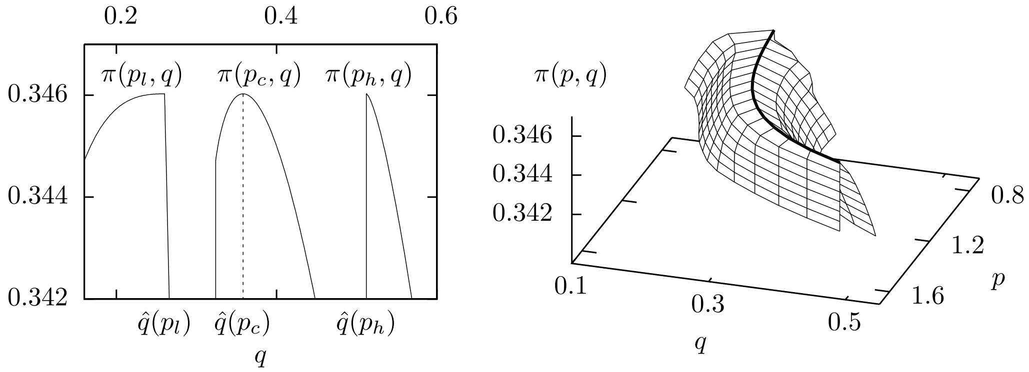
This equation can not be solved analytically but a numerical solution is easily obtainable: $z \approx 0.08226$. Then, from the definition of z,

$$U_h = \frac{1}{2} \ln \left(\frac{4z^2 + 1}{4z} \right) - 2 \approx -1.43089. \tag{1.42}$$

Finally, $p_l = \tilde{p}(U_h) \approx 0.75109$.

Figure 1.3 plots a few important functions of our equilibrium candidate. The left plot gives $\hat{q}(p)$ together with iso-utility curves that correspond to $U_l = U_R$ and U_h. It is easy to see in the plot that higher prices signal lower utility in an equilibrium. In other words, if a partially informed consumer faces two products with different prices he will go for the cheapest product and, though the expected quality will be lower, the expected utility will be higher. The right plot gives the density function of the price distribution. Earlier we exclusively worked with utility distribution $F(u)$, but there is an

Figure 1.4: Equilibrium Profits



Comments: $p_c = \frac{1}{4}p_l + \frac{3}{4}p_h$ (the left plot); the bold line depicts $\pi(p, \hat{q}(p))$, i.e. the profits along the equilibrium curve (the right plot); also, for convenience, only a summit of $\pi(p, q)$ is shown in the right plot.

easy transformation as

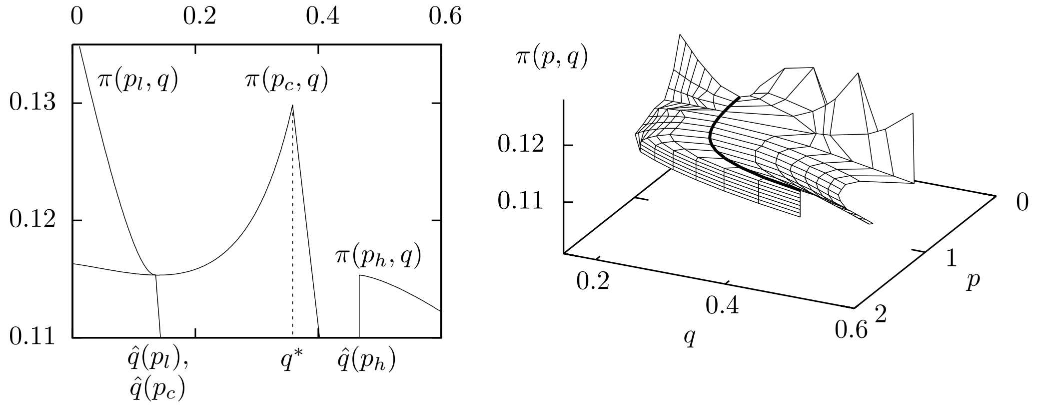
$$G(p) = \mathbb{P}(P < p) = \mathbb{P}(U(P, \hat{q}(P)) > U(p, \hat{q}(p))) = \\ 1 - \mathbb{P}(U(P, \hat{q}(P)) \leq U(p, \hat{q}(p))) = 1 - F(U(p, \hat{q}(p))). \quad (1.43)$$

The right plot gives the density of this distribution, namely $G'(p)$. From it we can see that the lower prices, lower quality occur more often than the higher prices, higher quality.

Recollect that $\pi(p, q)$ gives expected profits of one firm when the other firm is playing the equilibrium strategy. For there to be an equilibrium it should be that $\pi(p, q)$ attains its maximum along the equilibrium curve $\hat{q}(p)$. Figure 1.4 plots $\pi(p, q)$. The left plot gives 2D slices of $\pi(p, q)$ for various p, the right plot attempts a 3D presentation. One can readily see that the condition in question is satisfied indeed and so we have an exact signalling equilibrium.

Does an exact signalling equilibrium always exist? Not necessary. Consider the same example but with $\lambda_L = \frac{1}{5}$ and $\lambda_H = \lambda_M = \frac{2}{5}$. It can be solved in the same way as before. Figure 1.5 gives the same plots as before but for this new example. We know that if there was an equilibrium it should have had the same $\pi(p, q)$ as we have found, but we have found $\pi(p, q)$ that does not have its maximum along the equilibrium curve. Hence we can conclude that there is no exact signalling equilibrium in this latter case.

Figure 1.5: Disequilibrium Profits



Comments: $p_c = \frac{4}{5}p_l + \frac{1}{5}p_h$ and q^* is such that $U(p_c, q^*) = U_h$ (the left plot); the bold line depicts $\pi(p, \hat{q}(p))$, i.e. the profits along the equilibrium curve (the right plot); also, for convenience, only a summit of $\pi(p, q)$ is shown in the right plot; the sharp spokes to the back and right of the 3D picture are rendering artifacts – should a one steep "hill".

1.5 Linear Kinked Utility

One question remains partially unanswered so far: existence of an exact signalling equilibrium. For there to be an exact signalling equilibrium it is necessary and sufficient that profit function $\pi(p, q)$ attains its maximum along equilibrium curve $\hat{q}(p)$. We gave examples that sometimes this condition holds and an exact signalling equilibrium exists and sometimes it does not and so there is no exact signalling equilibria (see section 1.4). However, it seems impossible to provide sufficient conditions that do not involve a complete solution of the model. For a general utility specification the difficulty arises, because there is no explicit expression for profit function $\pi(p, q)$ at points that lie off the equilibrium curve.

So, there are two approaches to tackle the existence question. One approach was presented earlier in theorem 1.3. While the theorem does not answer whether there is an exact signalling equilibrium for a particular utility function, it says the equilibrium exists for at least as many utility functions as to generate every possible strictly increasing and strictly convex equilibrium curve. Another approach is to consider a particular class of utility functions. Then, potentially, the profit function can be explicitly solved for and sufficient conditions for the existence of an equilibrium can be derived. We pursue this approach next.

We have seen earlier that already for $U(p, q) = \frac{1}{2} \ln q - p$ the solution gets complicated and that an analytical expression for $\tilde{p}(u)$ is only possible for certain values of λ_H, λ_M and λ_L. A linear utility function is easier to analyze, e.g. $U(p,q) = bq - p$, but a linear utility function does not satisfy decreasing marginal returns to quality. Without decreasing marginal returns to quality the firms will either offer an infinite quality for an infinite price ($b > a$) or a negative infinite quality for a negative infinite price ($b < a$).[8]

A way to combine decreasing marginal returns to quality with linearity is to consider a linear but kinked utility function. Let

$$
\begin{aligned}
U(p,q) &= b(q) \cdot (q - q^*) - p, \\
b(q) &= \begin{cases} b_1 \text{ if } q \leq q^*, \\ b_2 \text{ if } q > q^*, \end{cases}
\end{aligned}
\tag{1.44}
$$

where $b(q) \cdot (q - q^*)$ is a maximum willingness to pay (up to a constant), b_1, b_2 and q^* are constants, and $b_2 < b_1$. A consumer values higher quality more, but once a certain threshold in quality is reached, namely q^*, his marginal willingness to pay for extra quality drops.

This case allows for a complete analytical solution. Theorem 1.4 presents the solution as well as necessary and sufficient conditions for the existence of an exact signalling equilibrium (the proof is in the appendix).

Theorem 1.4. *Consider a linear kinked utility function (eq. 1.44). Then an exact signalling equilibrium exists if and only if the following conditions are satisfied:*

$$(i) \quad -U_R - aq^* > 0, \tag{1.45}$$

$$(ii) \quad b_2 < a, \tag{1.46}$$

$$(iii) \quad \frac{b_1 - a}{a} > \frac{\lambda_M}{\lambda_H} \cdot \frac{2 - \lambda_L}{\lambda_L}. \tag{1.47}$$

If the conditions are satisfied, then the equilibrium is unique and is characterized by:

$$\hat{q}(p) = q^*, \quad p_l = \frac{\lambda_L(-U_R - aq^*)}{2 - \lambda_L} + aq^*, \quad p_h = -U_R, \tag{1.48}$$

$$F(u) = \frac{1}{2} \cdot \frac{\lambda_L}{1 - \lambda_L} \left(\frac{U_R + aq^*}{u + aq^*} - 1 \right), \quad U_l = -p_h, \quad U_h = -p_l. \tag{1.49}$$

Conditions (i) and (ii) guarantee that the model is non-degenerate, i.e. that the most profitable (p,q) bundle subject to $U(p,q) \geq U_R$ is finite and

[8] To avoid negative prices or qualities one can restrict the model only to non-negative values, but it is like considering a special case of a linear kinked utility.

gives positive profits to a firm. Condition (iii) guarantees that no firms will find it beneficial to undercut quality to exploit partially informed and fully uninformed consumers. This condition can be interpreted in two ways: an exact signalling equilibrium exists if the utility function is sufficiently quasi-concave (b_1 is sufficiently high), or, if the number of partially informed consumers is sufficiently small (λ_M is sufficiently small).

The intuition here is straightforward. If the firm undercuts on quality, it gains on partially informed and fully uninformed consumers and loses on fully informed. So, the more quasi-concave the utility function is, the more fully informed consumers the firm loses. The less is the number of partially informed consumers, the less the firm gains when undercutting on quality.

We expect that the same results – in terms of sufficient quasi-concavity or in terms of sufficiently small number of partially informed consumers – will also hold for general utility functions. However, so far we have not been able to analytically establish such results.

1.6 Conclusions

We have considered a market where oligopolistic firms compete for consumers by varying prices and quality of their products and where consumers are heterogeneous in their knowledge of the prices and quality of the products offered: some know both the quality and prices, some know only the prices and some know neither. We have derived a signalling equilibrium for this setting that is characterized by firms playing a mixed strategy over a curve in a price-quality space. We have shown that this signalling equilibrium can be of two types. Both types are characterized by a dispersion of prices and quality and by Pareto-inefficiency of the price/quality offers. But in one type of equilibrium *lower* prices signal better price/quality ratios, while in the other type *higher* prices signal better price/quality ratios. Which type results depends on consumers' preferences: the cheapest offer is the best deal from a consumer perspective if the marginal utility of quality is declining in prices.

Chapter 2

Targeted Competition: Choosing Your Enemies in Multiplayer Games

2.1 Introduction

Competition lies at the heart of economics and has been studied extensively. However, there is a class of competition mechanisms that is abound in practice but which, to the best of our knowledge, has not yet been studied specifically in the literature – those are mechanisms providing a competitor with an ability to target his rivals on an individual basis. We group such mechanisms under the common label of targeted competition. The few examples that follow illustrate how pervasive targeted competition is. On product markets, firms may decide to develop a product that is closer along one characteristic to that of a particular competitor. A multinational corporation may decide to invest relatively more in a market shared with a particular rival (see, for example, surveys by Lancaster, 1990; Gabszewicz and Thisse, 1992; Bailey and Friedlaender, 1982). Another example of targeted competition is comparative advertisement (see, for example, Barigozzi and Peitz, 2007; Anderson et al., 2009), a practice of running ads that directly compare one's products to that of the rivals. Unethical practices, for example launching fabricated lawsuits against specific rivals, provide further ways to target competitors. Targeted competition is not restricted to economics only. Think about the ways political parties and politicians compete through their support for specific programs, or how different governments try to protect local industries through trade barriers. Finally, a warfare stays as an ultimate example of targeted competition.

Targeted competition includes a strategic consideration that does not arise in non-targeted competition: a player (a firm, a political party, an army) can influence the balance of powers among his rivals by choosing whom to compete against; in turn, that determines how much this player wins or loses competing with those rivals in the periods to come. In particular, one may intuitively expect the weaker players to direct more resources towards fighting the strongest player rather than fighting each other. Indeed, otherwise the strongest player stands a good chance of forcing the weaker ones out of the game (as time goes by).

Any model of targeted competition should have the following two characteristics: 1) there should be three players or more – otherwise the competition cannot be targeted; and 2) the analysis should be dynamic – the aforementioned strategic consideration can be only studied in a dynamic setting. The closest matching strand of the literature then is that of dynamic oligopoly models. Though many scenarios of dynamic competition are studied (inventories (Kirman and Sobel, 1974), sticky prices (Fershtman and Kamien, 1987), evolution of sales (Dockner and Jørgensen, 1988), varying profit opportunities (Ericson and Pakes, 1995), collusive behaviour (Fershtman and Pakes, 2000), etc.), targeted competition is not part of the analysis. This study aims to be a first step towards filling this gap.

We develop a model of targeted competition that does not focus on case-specific aspects of competition but rather focuses on the general ability to target selected rivals. Each player in the model is characterised by his relative power – the amount of resources this player has. The power of a player can be distributed to fight each of the player's rivals. We first show that myopic players prefer to fight more with their weakest opponent. Consequently, the strongest player grows in power and eventually outcompetes the weaker players. Vice versa, we show that if players are non-myopic and do not discount future payoffs too much, then the weaker players concentrate more on fighting their strongest opponent (provided no player is too strong to start with). Consequently, the strongest player becomes weaker over time and all the players converge in power to a common level and survive.

So, if a competition on a certain market is targeted and the competitors are forward-looking, then this competition is sustainable. On the other hand, if there are no ways to target particular rivals, then the market becomes a monopoly. From a practical perspective, this result is relevant for policies that influence the effectiveness of targeted competition. Some recent examples of such policy questions are: whether or not to allow comparative advertisement (Barigozzi and Peitz, 2007); whether to legislate network neutrality – network neutrality prohibits internet providers to differentiate their traffic in any way, including price differentiation (Economides and Tåg, 2009; Kocsis and de Bijl, 2007).

It is tempting to view the fact that the weaker players fight together against their strongest rival as a form of tacit collusion. It is, however, conceptually different. Collusive behaviour in repeated games is sustained by the credible threat that other players will punish the one who deviates from the equilibrium. In our game the equilibrium concept is Markov perfect equilibrium, hence the strategies do not depend upon past actions and so there are no strategies with retrospective punishment. In our case it is the dynamic structure of the game that pushes the weaker players to fight together for the common cause: if they are to prefer fighting each other for the sake of immediate gains, then the power of the strongest player will grow up to the point at which, eventually, he can outcompete his rivals. If this threat of losing the game is large enough, then the weaker players will fight more against the strongest player and their behaviour will be alike to that of tacit collusion.

There are two related games that have been studied in the literature: colonel Blotto games (see, e.g., Roberson, 2006) and truel games (Kilgour, 1971).

A colonel Blotto game is a game between two players that share several battlefields. Each player divides his army between the battlefields, a battlefield is won by the larger force, a player who wins more battlefields wins the game. The game of targeted competition that we study can be viewed as a game of three players and three battlefields, where each pair of players share a battlefield and where there is no battlefield that is shared by all the players. Then the similarity of our game to colonel Blotto games is the ability of the players to choose how to split their powers against their opponents. The main differences are: 1) there are three players in our game, 2) our game is dynamic – the winner is not determined at once, rather the winner of this round becomes stronger and the game continues.

A truel game is an extension of a duel game. There are three players, each with a gun. Each round each player chooses whom to shoot and kills his opponent with a certain chance that depends upon his skill; if two or more players are still alive the game continues. Like in our game, there is a choice of the opponent, there are dynamics and there is a consideration that killing a certain player influences your chance of survival in the rounds to come. The main differences are: 1) in our game the payoff of the game is a discounted sum of the payoffs in each round, so each round is valuable, whereas in a truel game the payoff is 1 if the player survives and 0 otherwise; 2) in our game if the player is "shot", he does not die at once but rather becomes relatively weaker; 3) in a truel game a player chooses to fight either one opponent or the other, whereas in our game a player chooses *how much* to fight one opponent and *how much* to fight the other (a continuous choice).

So, our game has structural similarities to those of colonel Blotto and truel games, but we think the named differences make our model more appropriate for the aforementioned examples of targeted competition.

We use a linear-quadratic specification for our model. Among the types of differential games that tend to have analytical solutions (see Dockner et al., 2000), a linear-quadratic type is the only one that satisfies our assumptions on payoffs (diminishing marginal returns, etc). So, while restrictive, it is our only choice if we want to present a model that is analytically tractable. We discuss this point in greater detail in the following section.

The rest of the chapter is organised as follows. The next section presents the model, which is inspired by the above examples about targeted competition between firms. Section 2.3.1 considers the simple case of myopic players and shows that only the strongest player survives as time goes by. In section 2.3.2, we show that if players are not myopic, the discount rate is sufficiently small and if no player is too strong to start with, then there is an equilibrium where all the players converge in power and remain in the game. The last section concludes.

2.2 Setup

There are three players, 1, 2, and 3 – firms, political parties, armies, etc. The players are involved in a dynamic competitive game. Each player i at time $t \in [0, \infty)$ is characterised by a state variable $x_i(t)$ being the amount of resources he can use in competition with his rivals at time t. We call this variable the "power" of player i. It can be the market share of a firm, the amount of personnel the firm has, how large and how good its credit resources are or how well the managers are connected; it can be the electoral base or the number of seats in parliament; it can be the number of military units.

For convenience, let $x = (x_1, x_2, x_3)$. At any time t the powers of the players, $x(t)$, are common knowledge.

The initial state $x(0)$ is normalised so that $\sum_i x_i(0) = 1$ (later on we will see that $\sum_i x_i(t) = 1$ for any t) and also no player is too strong to start with. Formally, $x(0) \in X$, where

$$X = \left\{ x \in \mathbb{R}^3 \;\middle|\; \sum_i x_i = 1,\ x_i < \frac{2}{5}\ \forall i \right\}. \tag{2.1}$$

The reason for the restriction $x_i(0) < \frac{2}{5}$ is a technical one. Under this restriction the best responses (that we are to analyse later on) attain inner solutions and the whole problem is tractable analytically. If one considers a more natural restriction that $x_i(0) < \frac{1}{2}$, then a numerical solution for a system of differential equations shall be exercised. Not to overcomplicate the exposition of our ideas we circumvent the difficulty of solving the problem numerically by considering a smaller region for x.

In the following analysis we focus on Markov strategies: a strategy of any player depends only on the current state x and does not depend on the past actions of the players. Our choice for Markov strategies comes from the objective of the study – to see whether forward looking behaviour can produce collusive type outcomes without invoking the usual means of sustaining collusion (such as trigger strategies). Moreover, considering Markov strategies has appealing properties. First, an equilibrium in Markov strategies is also an equilibrium in a game with non-Markov strategies. Second, suppose a game with general strategies has multiple equilibria and one of them is a Markov equilibrium. One way to select an equilibrium is to explore whether there is a focal point (Schelling, 1960). If simplicity makes a focal point, then the Markov equilibrium is selected. There are also other reasons, both theoretical and practical, for opting for Markov strategies – see the introduction in Maskin and Tirole (2001).

A player can target his rivals, i.e. a player can decide how much he wants to fight each of his opponents. y_{ij} denotes the amount of power player i uses to fight against player j. Let $y_1 = (y_{12}, y_{13})$, $y_2 = (y_{21}, y_{23})$, $y_3 = (y_{31}, y_{32})$ and $y = (y_1, y_2, y_3)$.

As we consider Markov strategies, the actions of the players are conditioned upon the state of the game, and so y_{ij} are functions of x.

Each player uses all his power to fight his opponents[1] and what amount

[1]In our model there are no alternative costs associated with fighting, therefore it is

he uses can not be negative, therefore

$$y_i(x) \in Y_i(x),$$
$$Y_i(x) = \left\{ y_i \,\middle|\, y_{ij} \geq 0, \sum\nolimits_j y_{ij} = x_i \right\}. \tag{2.2}$$

Contemplating targeted competition brings forward two effects. First, players have immediate gains from "fighting", e.g. profits in case of firms, or utilities of top managers; political contributions in case of political parties; access to natural resources in case of warfare for economic reasons. We refer to these gains as instantaneous payoffs. Second, if for some time a player is opposed to another player with less power, then the former player becomes even stronger while his opponents becomes weaker. For example, if a company invests more in a market than its competitors do, or if a political party supports a certain program more than its rivals do, then the corresponding customer or electoral base increases relative to that of the rivals. We refer to such dynamics as power shift.

The instantaneous payoff for player i when he is fighting player j is given by $\varphi(y_{ij}, y_{ji})$, with 1) $\varphi(0, y_{ji}) = 0$, which happens if player i doesn't fight; 2) $\varphi(y_{ij}, y_{ji})$ strictly increasing in y_{ij} and, for $y_{ij} > 0$, strictly decreasing in y_{ji}; 3) $\varphi(y_{ij}, y_{ji})$ strictly concave in y_{ij} (decreasing marginal returns).

Dockner et al. (2000) identify three types of differential games that admit analytical solutions: linear-quadratic, linear state and exponential games. Among those, only linear-quadratic games can exhibit payoffs satisfying the aforementioned assumptions[2]. So, we take a quadratic specification for φ:

$$\varphi(y_{ij}, y_{ji}) = (a - b_1 y_{ij} - b_2 y_{ji}) y_{ij}, \tag{2.3}$$

where $b_1 > 0$, $b_2 > 0$ and $a \geq 2b_1 + b_2$. This is the most general quadratic specification that would satisfy our assumptions on the relevant domain ($0 \leq y_{ij} \leq 1$, $0 \leq y_{ji} \leq 1$). Further, for simplicity, we assume that $b_1 = b_2 = b$, so

$$\varphi(y_{ij}, y_{ji}) = (a - b(y_{ij} + y_{ji})) y_{ij}, \tag{2.4}$$

where $b > 0$ and $a \geq 3b$. Let us note that if y_{ij} is interpreted as output, then $\varphi(y_{ij}, y_{ji})$ can be interpreted as the profit of a firm in a Cournot duopoly game with linear demand.

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always optimal to use all one's power for fighting.

[2]Solvability of linear-quadratic games makes them a popular tool for analysis of dynamic oligopolies – see, e.g. Fershtman and Kamien (1987), Cellini and Lambertini (1998).

Let $\pi_i(y)$ denote the sum of all the instantaneous payoffs that player i receives from fighting his opponents. We have

$$\pi_i(y) = \sum_{j \neq i} \varphi(y_{ij}, y_{ji}). \tag{2.5}$$

Power does not enter the instantaneous payoff function per se. However, becoming more powerful will yield higher payoffs as more power can be used to compete against rivals, thus improving the outcomes of future competition rounds.

If player i fights player j harder than player j fights player i ($y_{ij} > y_{ji}$), then player i becomes more powerful, while player j becomes less powerful. These power shift dynamics are assumed to be linear in y:

$$\begin{aligned} \dot{x}_i(t) &= f_i(y(x(t))), \\ f_i(y) &= \sum_{j \neq i} (y_{ij} - y_{ji})\, k, \end{aligned} \tag{2.6}$$

where $k > 0$ stands for the power shift intensity.

We note here that from $\sum_i x_i(0) = 1$ and from (2.6) it follows that $\sum_i x_i(t) = 1$ for all t.

If $x(t)$ reaches the boundary of X, the game ends. T denotes the ending time. Formally,

$$T = \inf\{t \geq 0 \,|\, x(t) \notin X\}. \tag{2.7}$$

If the game never ends we write $T = \infty$.

If the game ends, each player i receives a terminal payoffs S_i, the strongest player wins, the weaker players lose:

$$S_i(x) = \begin{cases} M \text{ if } x_i > x_j \ \forall j \neq i, \\ 0 \text{ otherwise}, \end{cases} \tag{2.8}$$

where $M > 0$.[3]

The rationale for ending the game if the boundary of X is approached is as follows. If one of the players becomes sufficiently strong, it is reasonable to expect him to eventually outcompete his rivals. To simplify the game we stop it at this time and assign a strictly positive payoff of M to the strongest player and a zero payoff to the weaker players. As we will see later on, the results do not depend upon the size of M. Yet it is helpful to think of it as

[3]If the game ends and two players are equally strong, they both lose. This assumption is made for simplicity and does not change the results.

of a payoff that is higher than what the strongest player could have got if he was to continue the competition. Losing, on the other hand, means that a player quits the game (a firm loses its markets, etc) and the stream of the instantaneous payoffs ends – so losing yields zero payoff.

The payoff for the whole game is the discounted stream of the instantaneous payoffs plus the discounted terminal payoff, so the payoff for player i is

$$U_i = \int_0^T e^{-rt} \pi_i(y(x(t)))dt + e^{-rT} S_i(x(T)), \tag{2.9}$$

where r is a discount rate.

So, our setup is a differential game with simultaneous play (see Dockner et al., 2000) and we restrict our attention to Markov strategies. The strategies are functions $y(x)$ satisfying (2.2), the state variables x evolve according to (2.6) and the objective functions are given by (2.9).

2.3 Analysis

We consider two cases: a case with myopic players and a general case. In both cases we look for Markov perfect equilibria (MPE) and analyse the resulting equilibrium dynamics.

In what follows we denote the best response strategies with $\tilde{y}$ and the equilibrium strategies with $\hat{y}$.

2.3.1 Myopic Players

The players are myopic if they only focus on the current gains. For a myopic player i the payoff of the game at time t is

$$U_i(t) = \pi_i(y(x(t))). \tag{2.10}$$

The dynamics of the myopic case are summarised by the following proposition (we limit our attention to a general initial state, when one of the players is strictly stronger than the rest).

Proposition 2.1. *Suppose, without a loss of generality, that* $x_1(0) > x_2(0)$, $x_1(0) > x_3(0)$. *Then there exists a unique MPE. Moreover, the equilibrium dynamics are such that the game ends and the strongest player wins, i.e.* $T < \infty$ *and* $x_1(T) > x_2(T)$, $x_1(T) > x_3(T)$.

Proof. Maximising $U_i(t)$ in (y_{ij}, y_{ik}) w.r.t. $y_{ij} + y_{ik} = x_i$ gives a unique best response

$$\tilde{y}_{ij}(x) = \frac{x_i}{2} + \frac{y_{ki}(x) - y_{ji}(x)}{4} \tag{2.11}$$

(a boundary solution is also possible but it is straightforward to check that it is never attained for $x \in X$).

Given the above best response functions we can solve for a unique equilibrium point. We get

$$\hat{y}_{ij}(x) = \frac{x_i}{2} + \frac{x_k - x_j}{10}. \tag{2.12}$$

As we are considering Markov strategies, (2.12) constitutes a unique Markov perfect equilibrium.

Plugging (2.12) into (2.6) and using $x_1 + x_2 + x_3 = 1$ gives

$$\dot{x}_i(t) = \frac{9k}{5}\left(x_i(t) - \frac{1}{3}\right). \tag{2.13}$$

Therefore

$$\dot{x}_1(t) - \dot{x}_i(t) = \frac{9k}{5}\left(x_1(t) - x_i(t)\right). \tag{2.14}$$

X is bounded and $x_1(0) > x_i(0)$ for $i \in \{2, 3\}$. It then follows from (2.14) that $x(t)$ reaches the boundary of X at some time T and that $x_1(T) > x_i(T)$ for $i \in \{2, 3\}$. □

This case illustrates the intuition that if the players are myopic and only pursue their instantaneous payoffs then they have no incentives to fight more against the stronger player. As a consequence, the weaker players lose.

2.3.2 Forward-looking Players

If the players are myopic, then the weaker players lose in the equilibrium. The question is, if the players are sufficiently non myopic, i.e. if r is sufficiently small so that the players value their future profits high enough, can it be the case the dynamics are reversed? We give a positive answer to this question.

Proposition 2.2. *If $r < \frac{4k}{3}$, then there exists an MPE such that for all i $x_i(t) \to \frac{1}{3}$ as $t \to \infty$.*

Proof. We prove the proposition by construction: we state an equilibrium candidate possesing the property that $x_i(t) \to \frac{1}{3}$ and then check that it is an equilibrium indeed. Let

$$\hat{y}_{ij}(x) = \frac{x_i + c(x_k - x_j)}{2}, \tag{2.15}$$

$$c = \frac{1}{18}\left(5\frac{r}{k} - 14 - \sqrt{\left(25\frac{r}{k} - 76\right)\left(\frac{r}{k} - 4\right)}\right). \tag{2.16}$$

From $\sum_i x_i(t) = 1$, from (2.6) and from (2.15) it follows that

$$\dot{x}_i(t) = \frac{3k(c+1)}{2}\left(x_i(t) - \frac{1}{3}\right). \tag{2.17}$$

If $r < \frac{4k}{3}$, then from (2.16) it follows that $c < -1$. Consequently, from (2.17) it follows that $x_i(t) \to \frac{1}{3}$ as $t \to \infty$.

Let us now prove that (2.15) constitute an MPE. To do so we need to show that $\hat{y}_i$ is a best response to $\hat{y}_j$ and $\hat{y}_k$. So, we fix the strategies of players j and k at $\hat{y}_j$ and $\hat{y}_k$ and we consider different strategies of player i. Given the strategies of players j and k, all the possible strategies of player i can be divided into two classes: those strategies that never end the game ($T = \infty$) – let it be class $\mathcal{A}$, and those that eventually do ($T < \infty$) – class $\mathcal{B}$. We proceed as follows. First, we restrict the strategies of player i to class $\mathcal{A}$ and show that in this class the strategy $\hat{y}_i$, as given by (2.15), is indeed a best response strategy. Second, we extend this result to $\mathcal{A} \cup \mathcal{B}$.

So, let the strategies of player i be restricted to class $\mathcal{A}$. Let us compute the value function V of player i if every player follows strategy $\hat{y}$ and if the game starts at $x(0) = x$. Solving (2.17) gives

$$x_i(t) = \left(x_i - \frac{1}{3}\right)e^{3k(c+1)/2\cdot t} + \frac{1}{3}. \tag{2.18}$$

Therefore (also using $x_1 + x_2 + x_3 = 1$) we have[4]

$$V_i(x) = \int_0^\infty e^{-rt}\pi_i(\hat{y}(x(t)))dt =$$
$$c_1\left(x_i - \frac{1}{3}\right)^2 + c_2\left(x_i - \frac{1}{3}\right) + c_3 + c_4(x_k - x_j)^2, \tag{2.19}$$

[4]See the appendix for the details of the derivation.

where

$$\begin{cases} c_1 = \dfrac{b(3c-1)}{4(r-3k(c+1))}, \\ c_2 = \dfrac{12a+b(3c-5)}{6(2r-3k(c+1))}, \\ c_3 = \dfrac{3a-b}{9r}, \\ c_4 = -\dfrac{bc(3c-1)}{4(r-3k(c+1))}. \end{cases} \tag{2.20}$$

Consider now the Hamilton-Jacobi-Bellman equations:

$$\hat{y}_i(x) \in \text{Arg} \max_{y_i \in Y_i(x)} \left(\pi_i(y_i, \hat{y}_{-i}(x)) + \sum_j \frac{\partial V_i(x)}{\partial x_j} f_j(y_i, \hat{y}_{-i}(x)) \right), \tag{2.21}$$

$$rV_i(x) = \pi_i(\hat{y}(x)) + \sum_j \frac{\partial V_i(x)}{\partial x_j} f_j(\hat{y}(x)). \tag{2.22}$$

If these equations are satisfied for all $x \in X$, then $\hat{y}_i$ is a best response to $\hat{y}_{-i}$ (when the strategies of player i are limited to class $\mathcal{A}$, so that $x(t)$ never leaves X) – see Dockner et al. (2000, chapters 3 and 4).

Equation (2.22) is automatically satisfied by the way V is constructed. We now check equation (2.21). Let

$$g(y_i, x) = \pi_i(y_i, \hat{y}_{-i}(x)) + \sum_j \frac{\partial V_i(x)}{\partial x_j} f_j(y_i, \hat{y}_{-i}(x)). \tag{2.23}$$

Using (2.15), (2.19) and the definitions for π_i, f_i to expand $g(y_i, x)$ and maximising the result w.r.t. $y_{ij} + y_{ik} = x_i$ gives

$$\tilde{y}_{ij}(x) = \frac{x_i + d(x_k - x_j)}{2}, \tag{2.24}$$

$$d = \frac{1-c}{4} - \frac{ck(3c-1)}{2(r-3k(c+1))}. \tag{2.25}$$

Strategy $\hat{y}_i$ is a best response strategy if (2.15) coincides with (2.24), i.e. if $c = d$. Using (2.25) to expand an equation $c = d$ and simplifying gives

$$18c^2 + \left(28 - 10\frac{r}{k}\right) c + \left(2\frac{r}{k} - 6\right) = 0. \tag{2.26}$$

It is straightforward to check that c as defined in (2.16) is a solution to the above equation. Hence $c = d$ and $\hat{y}_i$ is a best response.

In principle, it is possible that a corner solution is obtained when maximising $g(y_i, x)$, however it is never the case for $x \in X$.

Consider now an arbitrary strategy $\grave{y}_i(x) \in \mathcal{B}$. With a class $\mathcal{B}$ strategy the game ends at some T (that is determined by $\grave{y}_i(x)$). Let

$$y_i^n(x,t) = \begin{cases} \grave{y}_i(x) & \text{if} \quad t \leq T - \epsilon_n, \\ \hat{y}_i(x) & \text{if} \quad t > T - \epsilon_n, \end{cases} \tag{2.27}$$

where ϵ_n is a sequence, $\epsilon_n > 0$ and $\lim_{n\to\infty} \epsilon_n = 0$. This strategy $y_i^n(x,t)$ belongs to $\mathcal{A}$, therefore it gives the same or a lower payoff than the best response strategy $\hat{y}_i(x)$, i.e.

$$\int_0^\infty e^{-rt}\pi_i(\hat{y}(x(t)))dt \geq \int_0^\infty e^{-rt}\pi_i(y^n(x(t)))dt =$$

$$\int_0^{T-\epsilon_n} e^{-rt}\pi_i(\grave{y}(x(t)))dt + \int_{T-\epsilon_n}^\infty e^{-rt}\pi_i(\hat{y}(x(t)))dt. \tag{2.28}$$

Taking the limit as $n \to \infty$ gives

$$\int_0^\infty e^{-rt}\pi_i(\hat{y}(x(t)))dt \geq \int_0^T e^{-rt}\pi_i(\grave{y}(x(t)))dt + V_i(x(T)). \tag{2.29}$$

On the other hand, the payoff from employing strategy $\grave{y}_i(x)$ is

$$\int_0^T e^{-rt}\pi_i(\grave{y}(x(t)))dt + S_i(x(T)). \tag{2.30}$$

Therefore, if $S_i(x(T)) \leq V_i(x(T))$, then $\hat{y}_i$ is the optimal strategy in class $\mathcal{A} \cup \mathcal{B}$ as well.

As $x(0) \in X$, then from the definition of X it follows that $x_i(0) < \frac{2}{5}$. Whatever the strategy $\grave{y}(x)$ is, from (2.6), from (2.15) and from $x_1+x_2+x_3 = 1$ it follows that

$$\dot{x}_i(t) \leq \frac{3k(c+1)}{2}\left(x_i(t) - \frac{1}{3}\right). \tag{2.31}$$

Consequently, $x(T) < \frac{2}{5}$. At the same time, $x(T)$ belongs to the boundary of X. So, if it was true that $x_i(T) > x_j(T)$ for all $j \neq i$, then it should have been that $x_i(T) = \frac{2}{5}$. As it is not, we have that $x_i(T) \leq x_j(T)$ for at least some $j \neq i$. Therefore $S_i(x(T)) = 0$. But from $\varphi(\hat{y}_{ij}(x), \hat{y}_{ji}(x)) > 0$ it follows that $V_i(x(T)) > 0$.

So, $S_i(x(T)) \leq V_i(x(T))$ and $\hat{y}_i(x)$ is a best response strategy when all possible strategies are considered (class $\mathcal{A} \cup \mathcal{B}$).

In words, a weaker player can choose a strategy to reach the boundary of X, but doing so is not optimal. As for the strongest player, he may prefer to reach the boundary if he is still the strongest player when he does so, but he cannot achieve such dynamics if his rivals are playing the equilibrium strategies. □

So, for a sufficiently small r there is an equilibrium such that the strongest player declines in his power while the weaker players improve in their powers. Consequently, all the players converge. A notable property of this equilibrium is that each player fights his strongest opponent more.

2.4 Concluding Remarks

Stackelberg (1952) has argued that a duopoly will never achieve an equilibrium in price/quantity setting strategies. Moreover, the duopolists will engage into fighting for leadership and, consequently, one of them will become predominantly stronger in economic terms, or they will find it beneficial to collude.

"Duopoly is an unstable market form not only in the sense that price is apt to be indeterminate, but much more because it is unlikely to remain as a market form for any length of time. The inherent contradictions in the duopolistic situation press for a solution through the adoption of another market form – monopoly"

We do not say a market of three will attain an equilibrium in prices or quantities. Such strategic variables may as well stay indeterminate. Rather we consider the relative powers of the players. We show that if the three players are sufficiently forward looking and if there are ways for them to target their rivals, then everyone competes more against his stronger rival. Consequently the players converge in their power, and oligopolistic competition is sustainable – it does not boil down to a monopoly.

We have analysed but a basic setup of targeted competition and two possible extensions are worth mentioning – stochastic dynamics and multiple players. Arguably, both extensions would bring the model closer to judging real life situations as outcomes of competition are scarcely deterministic and many examples we talked about (e.g., multiproduct firms) often involve more than three players. The main question here will stay the same: given stochastic dynamics or given multiple (more than three) players in the game will it be more difficult or more easy for the weaker rivals to tacitly coordinate against the strongest one?

Chapter 3

Decentralized Pricing and Multiproduct Firms

3.1 Introduction

Back in 1962 Chandler coined the expression “strategy follows structure”. This link implies that a firm can commit to a certain strategy by choosing an appropriate organizational structure. In this spirit, the present chapter studies price competition between multiproduct firms together with their choices of organizational structures as means of committing to certain pricing strategies.

Two organizational choices are commonly available to a multiproduct firm. A multiproduct firm can assume a functional structure, which implies, from a marketing perspective, that all the prices are set centrally to maximize the total profits of the firm. Alternatively, a firm can assume a divisional structure, which implies that each product’s price is set individually by its respective division to maximize the divisional profits.

Ceteris paribus, decentralized pricing is suboptimal, but when the responses of the competitors are taken into account, decentralized pricing changes the equilibrium and can potentially result in higher total profits for the firm. The focus of the present chapter is on this strategic effect and on its implications for organizational choices and total welfare.

The idea that suboptimal response strategies, when they can be committed to, can provide higher profits for a competitive firm is well known. The classical example is Stackelberg competition (Stackelberg, 1952). A Stackelberg leader has higher profits than a Cournot duopolist. However, the quantity the leader chooses in the first period is not an optimal response in the second period (a Cournot response is). To be able to achieve higher profits a Stackelberg leader must be able to commit to that suboptimal response.

More recently, Fershtman et al. (1991) show that if some principles are engaged in a game, and if they can contract agents to play this game for them (so, a general commitment device is available), if these contracts can not be broken and are common knowledge, then any Pareto-efficient outcome can be delivered as a subgame perfect Nash equilibrium, where in the first period the contracts are signed and in the second period the game is played. In general these contracts do not prescribe an optimal response in the second period.

Which commitment devices are available to a firm in different situations, and which can be made common knowledge is a vast question and goes beyond the scope of this chapter. Here the commitment device is the organizational structure of a firm: a firm can either choose a functional structure and commit to centralized pricing or choose a divisional one and commit to decentralized pricing. The motivation is twofold. Intuitively, changing organizational structures is costly, is known to be costly, and hence an organizational structure is and is known to be a strong commitment. Additionally to that, much of the industrial organization literature takes the same path and discusses strategic effects of committing to different organizational structures.[1] The literature overview section discusses the relevant papers, here I would like to mention but a few well known ones.

McGuire and Staelin (1983) consider having retailing business in-house versus contracting with outside retailers. While leaving retailing in-house delivers higher profits ceteris paribus, the authors show that outsourcing retailing can be profitable if the final good market is competitive enough. Baye et al. (1996b) consider competition in quantity over a uniform good, where each competing firm can further split into multiple divisions. While divisionalization results in competition between divisions, it also increases the market share of the firm. The authors show that in equilibrium the firms find it profitable to divisionalize.

The present chapter focuses on a different setting – it focuses on price competition between multiproduct firms, where each firm's product range is composed of either substitutes or complements, and where the product ranges of different firms are always substitutes to one another. The following

[1]Though commitment via organizational structure is certainly not the only commitment mechanism studied. For example, Fershtman and Judd (1987) study managerial compensation that is linear in profits and sales. They show that contracting on sales can be profit improving.

examples confirm such a setting abounds in practice.

If the goods within a firm's product range are substitutes, then computer manufacturers all offering choices from small notebooks to server stations is one example, and car manufacturers offering choices from family hatchbacks to business class saloons is another one. The examples when the goods within a range are complements include: cosmetic manufacturers making all kind of products from a foot cream to a hair shampoo, or sports equipment manufacturers offering tennis shoes, light clothing, rackets, et cetera. Other examples of complementary products are cases when competing firms split their products into several offers: software and software support, vacuum cleaners and vacuum bags, etc.

The main result of this chapter is as follows. If the products within the product ranges are gross substitutes, then the firms prefer centralized pricing. If the product ranges consist of complementary products, then either case is possible. In particular, when competition between the firms is stronger, they are more likely to opt for decentralized pricing. So, decentralized pricing, which is suboptimal ceteris paribus, has a strategic advantage over centralized pricing when it comes to more competitive markets with competitors producing ranges of complementary goods.

Broadly speaking, this chapter shows that the product ranges of multiproduct firms – whether they consist of complements or substitutes – influence the organizational choices of those firms through market interactions. To the best of my knowledge this is a novel result.

From a welfare perspective, the case of complementary goods gives ambiguous results. However, the case of substitutes is unambiguous: the firms always choose functional structures, while divisional structures always deliver higher total welfare. So, on markets with competing multiproduct firms, each producing goods that are gross substitutes, a sufficiently high subsidy for divisionalization will be welfare improving.

There is also a novel but peculiar result that for certain parameters two equilibria coexist: in one equilibrium both competitors prefer centralized pricing and in the other equilibrium both competitors prefer decentralized pricing. As discussed in more detail at the end of section 3.4, this multiplicity of equilibria suggests there can be an industry-wise lag in the adjustment of organizational structures to the new market conditions.

While the focus of this chapter is on the commitment role of organizational structures, whether a company prefers a more centralized or a more decentralized structure does depend upon a variety of other factors. An interested reader is referred to Williamson (1981) for a discussion of transaction costs; to Jennergren (1981) for a discussion of scale economies, specialization, adaptiveness, etc.; to Mookherjee (2006) for a discussion of incentives, communication and information processing costs within a firm; and to Chandler (1962) for a seminal case study.

The next section presents further overview of the literature. Section 3.3 sets up the model. For simplicity of exposition a basic model with two goods, two firms and a linear demand system is taken. Section 3.4 solves for subgame perfect Nash equilibria of the model and discusses the results. Welfare analysis are done separately in section 3.5. Section 3.6 concludes.

3.2 Literature Overview

Baye et al. (1996b) study divisionalization: in the first period firms choose how many divisions to form, in the second period all the division of all the firms produce a uniform good and engage in Cournot competition. The authors show that the firms divisionalize in equilibrium. In the model the costs are assumed to be linear, so the focus is solely on the strategic role of divisionalization. The problem of divisionalization is then similar to a setting, where the number of divisions is fixed and the firms decide whether to set quantities centrally or to delegate quantity setting to the divisions. In this respect Baye et al. (1996b) can be compared to my setting of centralized versus decentralized pricing. The main differences are: I consider price competition instead of quantity competition and I consider differentiated goods. This different perspective allows to judge how the trade-off between centralized and decentralized pricing depends upon the degree of differentiation between the goods. A priori this perspective can not be justified by anything except curiosity, however, a posteriori it is justified as I achieve novel predictions concerning the dependence of organizational structures on the degree of differentiation between the goods.

Baye et al. (1996a) complement their earlier analysis with integral restrictions (the number of divisions is naturally an integer). Ziss (1998) extends Baye et al. (1996b) by considering a case, where the products of different firms are partial substitutes. He shows that differentiation of goods between firms alters the resulting equilibrium, most importantly a competitive solution is not approached as costs of divisionalization go to zero (which is the case in Baye et al. (1996b)).

Zhou (2005) compares a functional organizational structure to a divisional one in a market setting, where the goods within a firm are partial substitutes (or complements) but all the firms produce the same two goods and compete in quantities. If a firm chooses a functional structure, then it sets its quantities centrally and the cross-price effects are taken into account, but there is double marginalization: a production department charges transfer prices to a marketing department. If the firm chooses a divisional structure, the activities are split by product and not by function, so there is no double marginalization but each division sets its own quantity to maximize its own profits. So, the cross-price effects are not accounted for. The author shows that a divisional form is preferred for substitute goods and a functional form is preferred when the goods are sufficiently complementary.

In my setting the results are the opposite: a firm prefers a functional structure for gross substitutes and it might prefer a divisional structure for complementary goods if the inter-firm competition is strong enough. Besides me considering price competition, the other main reason for a such a difference in results is that I associate no additional costs with a functional structure (like double marginalization). I focus exclusively on the strategic role of different pricing mechanisms and do not look into internal costs that might be associated with them.

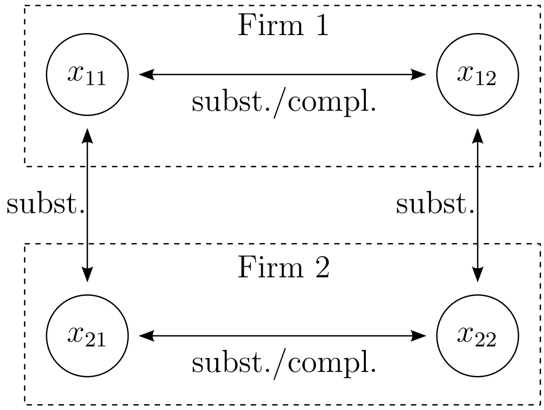
Ju (2003) considers multiproduct firms that produce differentiated goods and compete in prices, which is also what I do. However, his paper has a different focus. He assumes the pricing mechanism to be given (centralized pricing in his case) and studies instead how many differentiated products a firm finds optimal to produce and compares that to the social optimal. In principal, both choices – for optimal pricing mechanisms and for optimal product variety – deserve equal attention. This is so, because an ultimate goal is combining those choices into a unified theory of the behavior of oligopolistic multiproduct firms, a common arrangement on today's markets, and thus improving our understanding of how this arrangement influences our welfare.

3.3 Setup

3.3.1 Market Layout

There are two multiproduct firms and each firm $i \in \{1, 2\}$ produces two goods, x_{i1} and x_{i2}, which are either gross substitutes or gross complements

Figure 3.1: Market Setup



to one another. The corresponding goods produced by different firms are gross substitutes, namely x_{1k} is a substitute to x_{2k}, $k \in \{1, 2\}$. Such a setup is a simple schematic representation for a typical situation, when firms cover similar product ranges. Figure 3.1 illustrates the setup.

The following notation is used throughout the chapter: $i \in \{1, 2\}$ denotes one firm, while $j = 3 - i$ denotes the other firm; $k \in \{1, 2\}$ denotes one good, while $l = 3 - k$ denotes the other good.

For simplicity, a symmetric linear demand system is assumed:

$$x_{ik}(p) = 1 - p_{ik} + ap_{jk} + bp_{il}, \tag{3.1}$$

where $a > 0$ and $a + |b| < 1$. If $b > 0$ then the goods of the same firm are gross substitutes, if $b < 0$ then they are gross complements. The condition that $a+|b| < 1$ guarantees that the demand system is rationalizable, i.e. that there exist preferences that generate it (the technical details are provided in the appendix).

3.3.2 Pricing

The profits of division k of firm i are

$$\pi_{ik} = p_{ik} \cdot x_{ik}(\boldsymbol{p}), \tag{3.2}$$

where $\boldsymbol{p} = (p_{11}, p_{12}, p_{21}, p_{22})$. As the focus of the present study is not on the costs of production, they are omitted for simplicity.

The total profits of firm i are

$$\pi_i = \pi_{i1} + \pi_{i2}. \tag{3.3}$$

In general, each firm might be able to commit to a variety of pricing mechanisms. The managerial compensation can be contracted upon the total profits, the divisional profits or the combination of both, yielding different response functions. It can even be contracted upon prices and quantities directly, however it is natural to assume the owners do not posses complete information about the demand, in which case such direct contracting is not optimal (see, for example, Fershtman and Judd, 1987).

At the same time, any commitment shall be credible and common knowledge, otherwise it can never change the market equilibrium in favour of the firm. Arguably, the strongest public commitment is when a firm chooses a certain organizational structure that is known to result in a certain pricing mechanism. In this chapter I assume that every firm can adopt either a functional or a divisional organizational structure.

A functional structure slices the activities of a firm by function, so all the pricing decisions of the final goods fall within the same department (marketing, or sales). It is then natural to associate a functional structure with centralized pricing, under which the firm sets its prices so as to maximize the total profits:

$$(p_{i1}, p_{i2}) = \arg \max_{(p_{i1}, p_{i2})} \pi_i(\boldsymbol{p}). \tag{3.4}$$

A divisional structure, on the other hand, slices the activities of a firm by product, and it is customary to organize divisions as profit centers (see Jennergren, 1981, page 43). A divisional structure is then associated with decentralized pricing, under which each division sets its own price so as to maximize its own profits:

$$p_{ik} = \arg \max_{p_{ik}} \pi_{ik}(\boldsymbol{p}). \tag{3.5}$$

Under decentralized pricing the divisions within a firm compete with one another as well as with the rivalling firm.

3.3.3 Game Structure

Changing an organizational structure is more resource consuming than changing prices. Therefore, the prices have ample time to reach an equilibrium between the adjustments of the organizational structures (this is precisely the reason why an organizational structure is a credible commitment). Hence, when choosing an organizational structure, a firm is ought to compare the profits from the consequent price equilibria. The appropriate way to model this situation is to assume a sequential move game with perfect information: first the firms choose their organizational structures and second, after observing the organizational choices, they choose their prices.

It is additionally assumed no firm is a clear market leader, which means that no firm has a guaranteed first move when choosing an organizational structure or prices. So, organizational choices as well as price choices are modelled as simultaneous move games.

Overall, the game then is as follows. First the firms simultaneously choose their organizational structures, then they observe the organizational choices and simultaneously decide upon the prices. This is a standard way to model organizational choices of the firms that compete in prices or quantities (see, e.g., McGuire and Staelin, 1983; Baye et al., 1996b; Ziss, 1998; Zhou, 2005; González-Maestre, 2001).

3.4 Analysis

Consider the second stage of the game. If firm i has a functional structure, it chooses (p_{i1}, p_{i2}) so as to maximize

$$\pi_i(\boldsymbol{p}) = p_{i1}(1 - p_{i1} + ap_{j1} + bp_{i2}) + p_{i2}(1 - p_{i2} + ap_{j2} + bp_{i1}). \tag{3.6}$$

Solving the maximization problem gives the following best response function:

$$p_{ik}^{f}(p_{jk}, p_{jl}) = \frac{(1+b) + a(p_{jk} + bp_{jl})}{2(1-b^2)}. \tag{3.7}$$

If firm i opts for a divisional structure instead, then each division k chooses its price p_{ik} so as to maximize

$$\pi_{ik}(\boldsymbol{p}) = p_{ik}(1 - p_{ik} + ap_{jk} + bp_{il}). \tag{3.8}$$

Solving the maximization problem gives the following best response function for division k:

$$p_{ik}^{d}(p_{il}, p_{jk}) = \frac{1 + ap_{jk} + bp_{il}}{2}. \tag{3.9}$$

Given the best response functions we can solve for the equilibrium prices. Let p_{ik}^{fd} denote the equilibrium price for good k of firm i, when firm i has a functional structure, while firm j has a divisional structure. Similarly for p_{ik}^{df}, p_{ik}^{ff}, p_{ik}^{dd}.

Suppose firm i chooses a functional structure and its opponent j chooses a divisional one. Then from equations (3.7) and (3.9) it follows that

$$p_{ik}^{fd} = \frac{(2 + a - b)}{2b^2 + 4 - 6b - a^2}. \tag{3.10}$$

Similarly,

$$
\begin{aligned}
p_{ik}^{df} &= \frac{(2+a-2b)}{2b^2+4-6b-a^2},\\
p_{ik}^{ff} &= \frac{1}{2-a-2b},\\
p_{ik}^{dd} &= \frac{1}{2-a-b}.
\end{aligned}
\tag{3.11}
$$

These are the equilibrium prices in the respective subgames, i.e. the subgames defined by the organizational choices of the firms.

Now, let π_i^{fd} denote equilibrium total profits of firm i if it chooses a functional structure, and its opponent chooses a divisional structure. Similarly for π_i^{df}, π_i^{ff}, π_i^{dd}.

Then

$$
\pi_i^{fd} = p_{i1}^{fd}(1-p_{i1}^{fd}+ap_{j1}^{df}+bp_{i2}^{fd}) + p_{i2}^{fd}(1-p_{i2}^{fd}+ap_{j2}^{df}+bp_{i1}^{fd}) = \frac{2(1-b)(2+a-b)^2}{(4-a^2+2b^2-6b)^2}. \tag{3.12}
$$

Similarly,

$$
\begin{aligned}
\pi_i^{df} &= \frac{2(2+a-2b)^2}{(4-a^2+2b^2-6b)^2},\\
\pi_i^{ff} &= \frac{2(1-b)}{(2-a-2b)^2},\\
\pi_i^{dd} &= \frac{2}{(2-a-b)^2}.
\end{aligned}
\tag{3.13}
$$

As the symmetry of the model has resulted in symmetric equilibrium prices and symmetric equilibrium profits, in the following discussion the subscripts for p^{fd}, etc and π^{fd}, etc are dropped.

Consider now the first stage of the game. We know that $a > 0$ and $a + |b| < 1$. From equations (3.12) and (3.13) it then follows after some algebra that

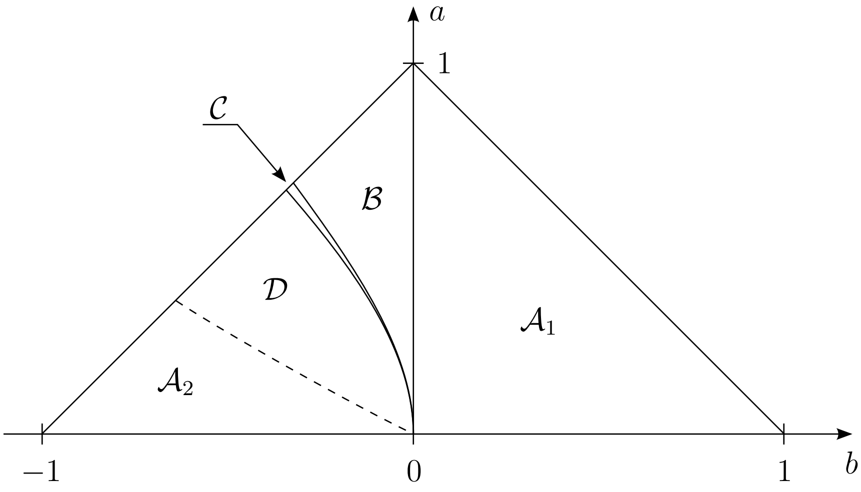
$$
\pi^{ff} \gtrless \pi^{df} \;\Leftrightarrow\; -b\left(a^4 - 4(b-1)^2a^2 + 4b(b-1)^2\right) \gtrless 0, \tag{3.14}
$$

$$
\pi^{fd} \gtrless \pi^{dd} \;\Leftrightarrow\; -b\left(a^4 - 2(b-1)(b-2)a^2 + b(b-1)(b-2)^2\right) \gtrless 0, \tag{3.15}
$$

$$
\pi^{ff} \gtrless \pi^{dd} \;\Leftrightarrow\; -b\left(a^2 + 2(b-1)a + b(b-1)\right) \gtrless 0. \tag{3.16}
$$

The expressions in (3.14) and (3.15) are quadratic equations in terms of a^2, the one in (3.16) is a quadratic equation in terms of a. Therefore for each $b \in (-1,1)$ it is straightforward to determine the values of a for which those

Figure 3.2: Parameters' Space



expressions become positive or negative. The results are plotted in figure 3.2, where

$$\begin{aligned}\mathcal{A}_1 \cup \mathcal{A}_2 &= \{(a,b)\,|\,\pi^{ff} > \pi^{df},\ \pi^{fd} > \pi^{dd},\ \pi^{ff} > \pi^{dd}\},\\ \mathcal{B} &= \{(a,b)\,|\,\pi^{ff} < \pi^{df},\ \pi^{fd} < \pi^{dd},\ \pi^{ff} < \pi^{dd}\},\\ \mathcal{C} &= \{(a,b)\,|\,\pi^{ff} > \pi^{df},\ \pi^{fd} < \pi^{dd},\ \pi^{ff} < \pi^{dd}\},\\ \mathcal{D} &= \{(a,b)\,|\,\pi^{ff} > \pi^{df},\ \pi^{fd} > \pi^{dd},\ \pi^{ff} < \pi^{dd}\}.\end{aligned} \tag{3.17}$$

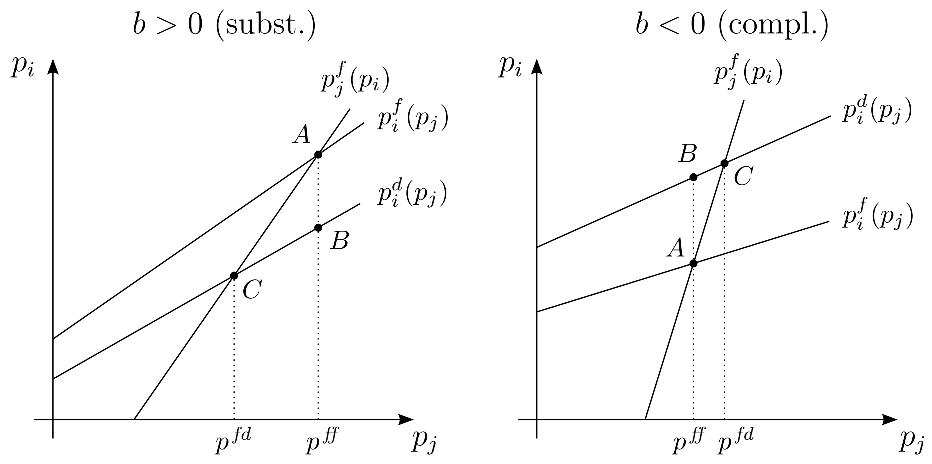
If $(a,b) \in \mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{D}$, then it is a dominant strategy to choose a functional structure and centralized pricing. If $(a,b) \in \mathcal{B}$, then it is a dominant strategy to choose a divisional structure and decentralized pricing. Finally, if $(a,b) \in \mathcal{C}$, then there are two Nash equilibria: either both firms choose a functional structure, or they both choose a divisional structure.

Moreover, if $(a,b) \in \mathcal{A}_1 \cup \mathcal{A}_2 \cup \mathcal{B}$, then the resulting unique equilibrium is also Pareto-efficient: in $\mathcal{A}_1 \cup \mathcal{A}_2$ the firms choose functional structures and at the same time we have that $\pi^{ff} > \pi^{dd}$, and in $\mathcal{B}$ the firms choose divisional structures and $\pi^{dd} > \pi^{ff}$. In contrast, if $(a,b) \in \mathcal{D}$, then the resulting unique equilibrium is not Pareto-efficient. In $\mathcal{D}$ the firms choose functional structures, whereas if they were to agree on divisional structures, then they would have been better off, because $\pi^{dd} > \pi^{ff}$ in $\mathcal{D}$. In $\mathcal{C}$ the divisional equilibrium is efficient, while the functional equilibrium is not.

Notably, there are no asymmetric equilibria, i.e. equilibria, where one firm chooses a functional structure while the other firm chooses a divisional structure.

To understand why it can be profitable for a firm to opt for suboptimal

Figure 3.3: Best Response Functions



decentralized pricing, it is instructive to look at the best response functions in prices. To make a graphical exposition possible, let us restrict our attention to symmetric prices, i.e. let

$$p_i = p_{i1} = p_{i2}. \tag{3.18}$$

Then the best response functions of a functionally or a divisionally organized firm (equations 3.7 and 3.9 respectively) can be rewritten as follows:

$$\begin{aligned} p_i^f(p_j) &= \frac{1 + ap_j}{2(1-b)}, \\ p_i^d(p_j) &= \frac{1 + ap_j}{2-b}. \end{aligned} \tag{3.19}$$

Suppose firm j has a functional structure and firm i considers whether to choose a divisional structure over a functional structure. Figure 3.3 depicts this situation for a case of substitutes ($b < 0$) and for a case of complements ($b > 0$).

Consider first the case when $b > 0$. In this case $\frac{1}{2(1-b)} > \frac{1}{2-b}$ and the functional best response line lies above the divisional one. If firm i chooses a functional structure, the resulting equilibrium prices are in point A, if it opts for a divisional one, the equilibrium prices are in point C. Let us analyse a move from A to C as a move from A to B and then to C. Between A and B the prices of firm j are constant, a divisional structure provides a suboptimal price response by definition, therefore the profits of firm i are lower at B than at A. As the goods between the firms are gross substitutes, the profits of firm i are increasing with p_j along any of its best response functions. Consequently, the profits of firm i are lower at C than at B. Summing up: the profits of firm i are lower at C than at A, i.e. there is no incentive to choose a suboptimal divisional structure when one's product range consists of gross substitutes.

The above argument is a general one and while I do not spell it formally for a general demand system, it shall be intuitively clear the result extends beyond the linear demand case.

Consider next the case when $b < 0$. In this case $\frac{1}{2(1-b)} < \frac{1}{2-b}$ and the divisional best response line lies above the functional one. For the same reason as before, the profits of firm i are lower at B than at A. However, switching to a divisional structure and decentralized pricing yields higher equilibrium prices in this case and, consequently, the profits of firm i are higher at C than at B. This a positive strategic effect of decentralized pricing. So, there is an ambiguity between A and C. Which effect prevails depends upon the specification of the demand and a general statement is difficult to make. However, it was shown earlier that for a case of linear demand a decentralized pricing is profitable if the competition between the firms is strong enough and the product range consists of complementary goods, though the complementarity effect shall not be to strong – namely, it shall be that $(a, b) \in \mathcal{B}$.

Finally, consider region $\mathcal{C}$.[2] There are two possible Nash equilibria there: either both firms choose functional structures and centralized pricing or they both choose divisional structures and decentralized pricing. Both equilibria coexist for the same demand parameters. Additionally, the latter equilibrium Pareto dominates the former equilibrium.

In comparison with the literature this result that there are multiple equilibria is uncommon – in the literature there is usually a unique Nash equilibrium (see McGuire and Staelin, 1983; Baye et al., 1996b; Ziss, 1998; Zhou, 2005; González-Maestre, 2001). This literature shows in various settings how an organizational choice depends upon the market parameters. Given the parameters the choice is unique. From the existence of region $\mathcal{C}$ it can be concluded that an organizational choice of a firm can depend not only on the market parameters, but also on the organizational choices of its competitors. Given the parameters, multiple outcomes can be possible.

[2]While it is small for a linear demand, it can be more pronounced for other demand specifications.

What equilibrium the firms will coordinate upon will likely depend upon the history of their interaction – the history can provide a focal point here, see Schelling (1960) for a rich discussion on the topic. For example, if the firms are originally in region $\mathcal{B}$ and the competition starts to loosen to such an extent that the firms move to region $\mathcal{C}$, then they are likely to end up with a divisional equilibrium, inheriting it from $\mathcal{B}$. On the other hand, if the firms were to start in region $\mathcal{D}$ and then move to $\mathcal{C}$, then they are likely to end up with a functional equilibrium. In this latter case, a Pareto-inefficient equilibrium is carried into a setting, where a Pareto-efficient equilibrium is already an option. Broadly speaking, this story illustrates how an industry can be *slow* in adjusting its organizational choices to the new market conditions, not because of transaction costs, but because of strategic considerations.

3.5 Welfare Analysis

One of the results of the previous section is that only two types of equilibria occur: either both firms choose functional structures or they both choose divisional structures.[3] Consider the corresponding prices, p^{ff} and p^{dd}. From (3.11) it follows that $p^{ff} > p^{dd}$ for $b > 0$ and $p^{ff} < p^{dd}$ for $b < 0$. So, in case of substitutes functional prices are higher than divisional prices. In turn, divisional prices would be higher than perfectly competitive prices, if we were to compare the considered duopoly against a perfectly competitive situation. Intuitively then, the divisional equilibrium shall deliver higher total welfare than the functional equilibrium, because the divisional prices are closer to the perfectly competitive case. Conversely, in case of complements the functional equilibrium shall be the better one. The following formal analysis confirms this intuition.

As before, $\boldsymbol{p} = (p_{11}, p_{12}, p_{21}, p_{22})$. Additionally, let $\boldsymbol{p}^{ff} = (p_{11}^{ff}, p_{12}^{ff}, p_{21}^{ff}, p_{22}^{ff})$, etc., and let $\boldsymbol{x} = (x_{11}, x_{12}, x_{21}, x_{22})$. Then

$$CS^{ff} - CS^{dd} = \int_{\boldsymbol{p}^{ff}}^{\boldsymbol{p}^{dd}} \boldsymbol{x}(\boldsymbol{p})d\boldsymbol{p} = \int_0^1 \boldsymbol{x}\big(\boldsymbol{p}^{ff} + t(\boldsymbol{p}^{dd} - \boldsymbol{p}^{ff})\big) \cdot (\boldsymbol{p}^{dd} - \boldsymbol{p}^{ff})dt = -\frac{2b\big((b-2)a + (b-1)(b-4)\big)}{(a+b-2)^2(a+2b-2)^2}, \quad (3.20)$$

where CS stands for consumer surplus.

[3]There are no asymmetric equilibria, i.e. equilibria, where one firm chooses a functional structure, while the other firm chooses a divisional one.

Producer surplus $PS = \pi_1 + \pi_2$, it can be computed from (3.13). Total welfare $W = CS + PS$. Straightforward computations give:

$$W^{ff} - W^{dd} = -\frac{2b(a+b-1)(2a+3b-4)}{(a+b-2)^2(a+2b-2)^2}. \tag{3.21}$$

For any (a, b) such that $a > 0$ and $a+|b| < 1$ it then holds that $W^{dd} > W^{ff}$ if $b > 0$ and $W^{ff} > W^{dd}$ if $b < 0$.

So, only when the product ranges of firms consist of complementary goods with the complementarity effects strongly pronounced, and only when the competition is not to strong, namely regions $\mathcal{A}_2$ and $\mathcal{D}$, the resulting unique Nash equilibrium is also welfare optimal.

In general, in the case of complements the welfare implications are ambiguous (region $\mathcal{A}_2 \cup \mathcal{D}$ versus region $\mathcal{B}$). However, if the product ranges consist of gross substitutes, then divisionalization creates more welfare than functional structures, while the firms unambiguously choose functional structures. Therefore, a sufficiently high subsidy for creating divisional units will be welfare improving in the case of gross substitutes.

3.6 Conclusions

This chapter studies commitment to different pricing strategies and the resulting price competition on a market with differentiated products and multiproduct firms. It extends a well-known intuition that public commitment to suboptimal responses can be beneficial: the main result of the chapter shows that choosing a divisional structure and decentralized pricing is more profitable than centralized pricing when firms' product ranges are composed of gross complements and when the market competition is strong.

In practice, different types of organizational structures have many different costs and benefits associated with them. These costs and benefits vary from a market to a market. Observing divisional structures for companies producing complementary goods and observing functional structures for companies producing substitutes does not alone support the proposed theory, because the comparison goes across different markets. Neither observing the opposite rejects the theory – for the same reason. However, there is a theoretical possibility to validate the theory. Obviously, a monopolist always chooses a functional structure. If this monopolist produces complementary goods and if a strong competitor enters the market, then the theory predicts that the monopolist will change his organizational structure to a divisional one. Correcting for other possible costs and benefits of different organizational forms, the prediction is as follows: a monopolist that produces complementary goods is likely to decentralize more once confronted with a strong competitor.

Broadly speaking, studying strategic effects of organizational choices enriches our understanding of internal structures of firms and our understanding of the resulting market equilibria. The field is still developing and larger questions, like making a link from team incentives to organizational structures to market equilibria to the resulting product variety, are still difficult to answer in a formal way. This chapter is a modest step in that general direction.

Chapter 4

Trade Secrets and Research Joint Ventures

4.1 Introduction

There are two major mechanisms to protect an innovation: patents and trade secrets. From a practical perspective, each mechanism is rich in detail and so its applicability and costs vary from industry to industry. From the perspective of economic theory, however, these mechanisms are the same but for one important difference – patent protection implies that the occurrence of an innovation, together with all its details, is public knowledge, whereas with trade secrets such information remains, at least partially, private.

Consider, specifically, cost reducing innovations. Under patent protection all the competitors are aware of each others' innovations and so are aware of each others' cost reductions. New computer hardware is one example: the innovations are protected by patents, which are publicly available from the moment of application, and the costs of producing a specific item are easily assessable, because most components come from third party supplies.[1] On the other hand, under trade secrets the information about each others' costs remains uncertain. For example, Google keeps its search algorithms secret and it is difficult to estimate how much it costs Google to run the corresponding software.[2] A broader example of trade secrets is internal IT and business infrastructures in private companies. What all these examples illus-

[1]For example, isuppli.com regularly publishes costs breakdowns for popular consumer electronics.

[2]To see the ambiguity on Google search algorithms, an interested reader is referred to `http://news.bbc.co.uk/2/hi/7823387.stm`. A US physicist Alex Wissner-Gross did an estimation that a Google search produces 7g of CO_2, while Google responded by saying it was a mere 0.2g.

trate is that in general competitors posses different information about each other depending on whether their innovations are protected by patents or as trade secrets. From a theoretical perspective, these informational differences imply that the strategy spaces are different and, consequently, the outcomes of the competition are different as well.

While theoretical analysis of various patent arrangements received much attention in the literature, to the best of my knowledge there is no theoretical work on trade secrets that highlights the aforementioned uncertainty that trade secrets inherently create. The present chapter contributes to this open topic.

The outlined difference between patent protection and trade secrets exists only when research and development has uncertain outcomes. The relevant literature on uncertain research and development includes: Reinganum (1990), Combs (1993), Choi (1993), Katsoulacos and Ulph (1998), Martin (2002), Miyagiwa and Ohno (2002), Hauenschild (2003), Erkal and Piccinin (2010). This literature focuses exclusively on innovations, that are public knowledge and can not be copied, or, equivalently, it focuses on patent protection. In contrast, in the present chapter I consider innovations that are protected as trade secrets. In particular, I consider quantity-competing duopolists that research into cost reducing innovations and ask the question whether these duopolists prefer to keep their innovations as trade secrets or whether they prefer to form a research joint venture (RJV).

This question is common in the literature on R&D, and the present chapter extends the earlier results by addressing the question in the framework of trade secrets. The angle of analysis is also different from the existing literature. The literature on R&D primarily focuses on the difference in effort between private and joint research, and the consequent welfare implications.[3] In this chapter the focus is on how the choice between private and joint research is influenced by the characteristics of the R&D process, like the probability of success and the potential impact of successful innovations.

Keeping innovations as trade secrets means that each duopolist does R&D on its own, and if he achieves a cost reducing innovation, his competitor is not aware of it. Forming a research joint venture, on the other hand, implies a twofold change: i) in an RJV the duopolists join their R&D effort, thus raising the chances for success, and they share any consequent innovations; ii) in an RJV, as the duopolists share their innovations, they are automatically

[3] Starting with the seminal paper by d'Aspremont and Jacquemin (1988).

aware of each others' cost reductions.

The second effect, in its own right, is well studied in the literature on information sharing: Fried (1984), Li (1985), Gal-or (1986) and Shapiro (1986), all consider Cournot competition with uncertain costs and show that an ex-ante commitment to share the information on those costs is a strictly dominant strategy (with Bertrand competition the results are the opposite – concealing the information is a strictly dominant strategy; I discuss Bertrand competition in more detail in the concluding section).[4]

Arguably, in the industries, where patent protection is weak and innovations are commonly guarded as trade secrets, it is impossible to credibly reveal one's reduction in costs without revealing the corresponding innovation. Therefore, these two effects, (i) and (ii), can not be separated for such industries and shall be studied together. As the present chapter demonstrates, the joint analysis of these two effects gives novel results.

The interaction in question is modelled as a two-period game. In period one the firms negotiate whether to form a research joint venture. In period two, if the RJV was formed, the firms observe their mutual cost reductions, otherwise – if the firms chose to conduct their research in private – they observe only their own cost reductions. In either case the firms simultaneously choose how much of the commodity to supply to the market, after that the price and profits are realized.

Studying subgame perfect Nash equilibria of this game gives the following results. If there is a small chance of a major innovation, then competing firms choose private R&D in period one and guard any consequent innovations as trade secrets. Otherwise – if the chances of an innovation are high, or if any possible innovation is minor at best, or both – then the firms join their R&D effort by forming a research joint venture.

Intuitively, these results can be explained as follows. A posteriori, a firm would prefer a trade secrets arrangement over a joint research one, if the firm itself acquires an innovation while its competitor does not. A priory then, the more profitable and more likely this event of an exclusive innovation is, the more attractive is the trade secrets arrangement. An exclusive innovation is more profitable, if it reduces the costs substantially, i.e. if it is a major innovation. Second, if there is an innovation, it is more likely to be exclusive when the chance of an individual innovation is small. Hence, a small chance of a major innovation favours trade secrets.

[4]See also Kühn and Vives, 1995 for a broader overview of the subject.

The results contrast those of the information sharing literature. As mentioned earlier, Cournot duopolists always prefer to share the information on their costs. In the present model sharing information additionally implies forgoing a possibility to enjoy an exclusive innovation, and, as the formal analysis shows, this consideration is important enough to support equilibria where no information is shared.

The joint research improves efficiency, so it is to be expected it is also welfare improving. The analysis confirms this intuition: RJVs always create higher consumer surplus and higher total welfare than do trade secrets. So, there is a normative implication: in industries where possible innovations are expected to be major but the chances of success are estimated to be small, a subsidy for research joint ventures can be welfare improving.

Finally, the chapter compares trade secrets and patents. When there is patent protection, a firm with a successful innovation applies for a patent and thus signals its success to other firm. This signalling reduces the output of the rival, what is not the case when innovations are protected as trade secrets. Consequently, in case of patents ex-ante expected profits of firms are larger than in case of trade secrets, and so in case of patents firms are less likely to form a research joint venture. Whether considering trade secrets or patents, RJVs always deliver the highest welfare. So, there are cases when trade secrets are preferable to patents, because they facilitate joint research.

The chapter is organized as follows. Section 4.2 formally sets up the outlined two-period game. First, the second period is analysed, for the case of trade secrets – section 4.3, and for the case of a research joint venture – section 4.4. Second, the first period is analysed to see whether the firms prefer to form an RJV or not, this is done in section 4.5. Section 4.6 discusses welfare implications, section 4.7 looks at how the ealier analysis changes when patent protection is considered, section 4.8 concludes. Possible extensions of the model – Bertrand competition, dependence of R&D on effort, multiple firms – are addressed in the concluding section.

4.2 The Model

Particular functional forms are assumed for the demand system, production costs and the distribution of R&D outcomes. Doing so gives a closed form solution to the model, and, consequently, comparative statics are straightforward. It is also a common practice in the literature on R&D as well as the literature on information sharing.

There are two firms, 1 and 2. Let $i \in \{1, 2\}$ denote either firm and let $j = 3 - i$ denote its competitor. Both firms produce a homogeneous good q and compete à la Cournot on the final good market. The inverse demand is given by

$$p(q) = a - bq, \tag{4.1}$$

where $q = q_1 + q_2$, $a > 0$, $b > 0$.

Each firm i has linear marginal costs. In case of private R&D the costs of firm i are

$$c_i(q_i) = (c - \varepsilon_i)q_i, \tag{4.2}$$

where ε_i is the random outcome of the firm's private R&D process, $0 \leq c \leq a$, $0 \leq \varepsilon_i \leq c$, and ε_1, ε_2 are independent.

In case of a research joint venture, the costs of either firm are

$$c_i(q_i) = (c - \eta)q_i, \tag{4.3}$$

where η is the random outcome of the joint R&D process.

Private research and development is modelled as follows. Each firm has a research team, and each team is given a certain amount of time to complete their research agenda. Over this time a team can make cost reducing innovations. The innovation process is a Poisson process with intensity λ. If one or more innovations are made, then one innovation gets implemented. For simplicity it is assumed that a successful innovation always results in a reduction d of the marginal costs, $d \leq c$. If no innovations are made, there is no reduction of the marginal costs. So, with probability $r = 1 - e^{-\lambda}$ there is a successful innovation and $\varepsilon_i = d$, and with probability $1 - r$ there is no innovation and $\varepsilon_i = 0$.

When the firms join their research and development, they join their research teams and the resulting intensity of the innovation process doubles. Then, with probability $s = 1 - e^{-2\lambda}$ there is a successful innovation and $\eta = d$, and with probability $1 - s$ there is no innovation and $\eta = 0$.

In principal, RJVs can exhibit synergy effects[5] as well as bear coordination costs. Either of those effects, if not too large, will change the quantitative results of the model, but not the qualitative results. Therefore those effects are omitted from the discussion.

[5]For example, within the current setup, if two or more innovations are required to achieve the reduction of marginal costs, then there will be additional synergy when forming RJVs.

While the research and development literature focuses on the amount of effort invested in R&D and whether that amount is socially optimal, the focus of this chapter is on the trade off between common knowledge and common innovations (RJVs) on one hand, and private knowledge and private innovations (trade secrets) on the other hand. So, effort considerations are omitted and R&D outcomes are simply modelled as unconditional random variables. The concluding section briefly discusses the question of adding effort to the present model.

There are two periods. In the first period the firms know the demand and have their expectations about the success of their own R&D programs as well as about the success of the joint research, if they are to conduct one. In this period the firms negotiate whether indeed to conduct the joint research. A simple two-stage negotiating procedure is assumed, which guarantees the selection of the most efficient outcome: firm 1 can offer firm 2 to form an RJV or it can make no offer at all; if firm 1 does make the offer, then firm 2 can either accept or reject it; if the offer is made and accepted, then the firms form an RJV, in any other case the firms resort to the trade secrets arrangement.

In the second period all the R&D programs are completed. In case of an RJV, both firms receive the same innovation and so are aware of each others' cost reductions. In case of private R&D each firm observes only the outcome of its own R&D process, not that of its competitor. At the end of the period the firms simultaneously choose their production levels, to the best of their knowledge.

The final payoff for each firm is its profit and the firms are assumed to be risk neutral.

4.3 Trade Secrets

This section analyzes the subgame, in which the firms do their own R&D. This subgame starts at the node, where nature moves to determine ε_1 and ε_2.

Once nature has moved, firm i sets its output q_i knowing ε_i, but not knowing ε_j, its expected profits conditional on ε_i are

$$\mathbb{E}(\pi_i^{ts} \,|\, \varepsilon_i) = \mathbb{E}\big((a - b(q_i + q_j) - (c - \varepsilon_i))q_i \,\big|\, \varepsilon_i\big) = \\ \big(a - c + \varepsilon_i - b(q_i + \mathbb{E}(q_j \,|\, \varepsilon_i))\big)q_i = \big(\alpha + \varepsilon_i - b(q_i + \mathbb{E}(q_j))\big)q_i, \quad (4.4)$$

where the notation $\alpha = a - c$ is assumed for convenience and $\mathbb{E}(q_j \,|\, \varepsilon_i) = \mathbb{E}(q_j)$, because q_j depends upon ε_j only and ε_j is independent from ε_i.

Maximizing (4.4) in q_i gives

$$\hat{q}_i = \max\left(\frac{\alpha + \varepsilon_i}{2b} - \frac{1}{2}\mathbb{E}(q_j),\, 0\right). \tag{4.5}$$

Potentially, if there is no innovation, i.e. $\varepsilon_i = 0$, and if the expected output of the competitor $\mathbb{E}(q_j)$ is large enough, then it can be that no level of output provides positive profits and it is best to produce nothing. We will see that this corner solution can indeed occur in an equilibrium.

In an equilibrium firm j plays its best response $\hat{q}_j$, therefore in an equilibrium

$$\hat{q}_i = \max\left(\frac{\alpha + \varepsilon_i}{2b} - \frac{1}{2}\mathbb{E}(\hat{q}_j),\, 0\right). \tag{4.6}$$

Taking an unconditional expectation of both parts gives

$$\mathbb{E}(\hat{q}_i) = \mathbb{E}\left(\frac{\alpha + \varepsilon_i}{2b} - \frac{1}{2}\mathbb{E}(\hat{q}_j) \,\middle|\, \varepsilon_i \geq b\mathbb{E}(\hat{q}_j) - \alpha\right) \cdot \mathbb{P}\big(\varepsilon_i \geq b\mathbb{E}(\hat{q}_j) - \alpha\big) =$$
$$\begin{cases} \dfrac{\alpha + rd}{2b} - \dfrac{1}{2}\mathbb{E}(\hat{q}_j) & \text{if} \quad \mathbb{E}(\hat{q}_j) \leq \dfrac{\alpha}{b}, \\ \dfrac{r(\alpha + d)}{2b} - \dfrac{r}{2}\mathbb{E}(\hat{q}_j) & \text{if} \quad \dfrac{\alpha}{b} < \mathbb{E}(\hat{q}_j) \leq \dfrac{\alpha + d}{b}, \\ 0 & \text{if} \quad \dfrac{\alpha + d}{b} < \mathbb{E}(\hat{q}_j). \end{cases} \tag{4.7}$$

As $i \in \{1, 2\}$, equation (4.7) defines expected best response functions for both firms. Their unique intersection is given by:

$$\mathbb{E}(\hat{q}_i) = \begin{cases} \dfrac{\alpha + rd}{3b} & \text{if} \quad 2\alpha \geq rd, \\ \dfrac{r(\alpha + d)}{(r + 2)b} & \text{if} \quad 2\alpha < rd. \end{cases} \tag{4.8}$$

Bringing together equations (4.6) and (4.8) gives the unique equilibrium strategy for either firm:

$$\hat{q}_i = \begin{cases} \dfrac{2\alpha - rd + 3\varepsilon_i}{6b} & \text{if} \quad 2\alpha \geq rd, \\ \max\left(\dfrac{2\alpha - rd + (r + 2)\varepsilon_i}{2b(r + 2)},\, 0\right) & \text{if} \quad 2\alpha < rd. \end{cases} \tag{4.9}$$

If $2\alpha \geq rd$, then both firms participate in the market no matter whether they achieved a successful innovation. If $2\alpha < rd$ on the other hand, i.e. if the maximum size of the market is relatively small in comparison with the expected benefits from the R&D, then only the innovating firms participate in the market (in this case, if $\varepsilon_i = 0$, then $q_i = 0$; if $\varepsilon_i = d$, then $q_i = \frac{\alpha+d}{b(r+2)} > 0$).

Given (4.9), it is straightforward to calculate expected equilibrium profits:

$$\mathbb{E}(\hat{\pi}_i^{ts}) = \begin{cases} \dfrac{4\alpha^2 + 8\alpha rd + 9rd^2 - 5r^2d^2}{36b} & \text{if} \quad 2\alpha \geq rd, \\ \dfrac{r(\alpha+d)^2}{(r+2)^2 b} & \text{if} \quad 2\alpha < rd. \end{cases} \tag{4.10}$$

So, we have the following result:

Proposition 4.1. *There is a unique Nash equilibrium in the trade secrets subgame. The strategies of the firms are given by* (4.9) *and the ex-ante expected equilibrium payoffs are given by* (4.10).

4.4 Research Joint Venture

This section analyzes the subgame, in which the firms form an RJV. This subgame starts at the node, where nature moves to determine η.

In this subgame the firms share the outcome of their research and development program and therefore no uncertainty is left at the stage of Cournot competition. It is then a straightforward textbook exercise. However, for the sake of completeness, the solution is provided below.

Conditional on η, the expected profits of firm i are

$$\mathbb{E}(\pi_i^{rjv} \,|\, \eta) = \big(\alpha + \eta - b(q_i + \mathbb{E}(q_j \,|\, \eta))\big)q_i. \tag{4.11}$$

Maximizing $\mathbb{E}(\pi_i^{rjv} \,|\, \eta)$ w.r.t. q_i gives

$$\hat{q}_i = \max\left(\frac{\alpha+\eta}{2b} - \frac{1}{2}\mathbb{E}(q_j \,|\, \eta),\, 0\right) \tag{4.12}$$

and in an equilibrium

$$\hat{q}_i = \max\left(\frac{\alpha+\eta}{2b} - \frac{1}{2}\mathbb{E}(\hat{q}_j \,|\, \eta),\, 0\right). \tag{4.13}$$

Taking a conditional expectation of both parts gives

$$\mathbb{E}(\hat{q}_i \,|\, \eta) = \max\left(\frac{\alpha+\eta}{2b} - \frac{1}{2}\mathbb{E}(\hat{q}_j \,|\, \eta),\, 0\right). \tag{4.14}$$

Equation (4.14) shall hold for $i \in \{1,2\}$, its unique solution is then

$$\mathbb{E}(\hat{q}_i \,|\, \eta) = \frac{\alpha+\eta}{3b}. \tag{4.15}$$

Hence, using (4.13),

$$\hat{q}_i = \frac{\alpha + \eta}{3b}. \tag{4.16}$$

Given that $\eta \sim B(1, s) \cdot d$ with $s = 1 - (1 - r)^2$ and given (4.16), it is straightforward to calculate expected equilibrium profits in case of a research joint venture:

$$\mathbb{E}(\hat{\pi}_i^{rjv}) = \frac{\alpha^2 + 4\alpha rd + 2rd^2 - 2\alpha r^2 d - r^2 d^2}{9b}. \tag{4.17}$$

So, we have

Proposition 4.2. *There is a unique Nash equilibrium in the RJV subgame. The strategies of the firms are given by* (4.16)*, and the ex-ante expected equilibrium payoffs are given by* (4.17)*.*

4.5 Trade Secrets vs. RJV

Consider now the whole game. If $\mathbb{E}\pi_i^{rjv} > \mathbb{E}\pi_i^{ts}$, then there is a unique SPNE, in which firm 1 makes the offer of a joint research and firm 2 accepts. If $\mathbb{E}\pi_i^{rjv} < \mathbb{E}\pi_i^{ts}$, then, formally, there are two SPNE: i) firm 1 makes no offer, ii) firm 1 makes a joint research offer and firm 2 rejects. In either case the firms continue in the trade secrets subgame.

We next turn to the comparison of the expected profits. If $r = 0$ or $r = 1$ then, obviously, $\mathbb{E}\pi_i^{ts} = \mathbb{E}\pi_i^{rjv}$. If $0 < r < 1$, then the following proposition holds.

Proposition 4.3.

$$\mathbb{E}\pi_i^{ts} \gtreqless \mathbb{E}\pi_i^{rjv} \Leftrightarrow \frac{\alpha}{d} \lesseqgtr \begin{cases} \frac{1}{8} & \textit{if} \quad 0 < r \leq \frac{1}{4}, \\ f(r) & \textit{if} \quad \frac{1}{4} < r \leq \frac{\sqrt{13}-3}{2}, \\ 0 & \textit{if} \quad \frac{\sqrt{13}-3}{2} < r < 1 \end{cases} \tag{4.18}$$

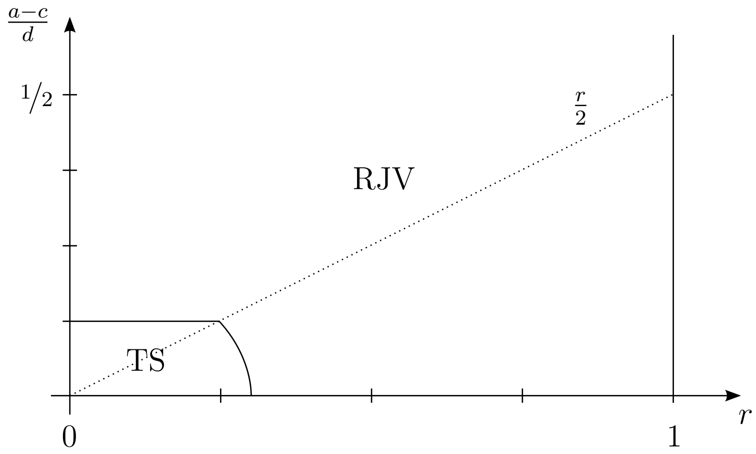
with

$$f(r) = \frac{r(1 - 3r - r^2) + (2 + r)\sqrt{r(1 - r)(1 - 3r - r^2)}}{4 - r}. \tag{4.19}$$

The proof is provided in the appendix.

Figure 4.1 plots the respective regions (recollect that $\alpha = a - c$). Within the trade secrets region (TS), the line $\frac{a-c}{d} = \frac{r}{2}$ separates two cases: above the

Figure 4.1: Comparative Statics



line both firms participate in the market irrespective of their R&D achievements, below the line only those firms participate in the market that have achieved a successful innovation (see equation 4.9 and the consecutive notes).

If $\frac{a-c}{d}$ is close to zero then the possible innovation is a *major* one: the costs are close to the maximum size of the market and a successful innovation reduces the costs substantially. On the other hand, if $\frac{a-c}{d}$ is large then the possible innovation is a *minor* one: either it does not reduce the costs much, or the costs are already small comparing to the maximum size of the market. Bringing together this terminology, figure 4.1 and the above discussion on SPNE delivers

Proposition 4.4. *When there is small chance of a major innovation the firms do their own R&D, otherwise they form a research joint venture.*

Intuitively, a small chance of a major innovations improves the likelihood of an exclusive innovation in the trade secrets subgame, given there is an innovation, as well as its profitability. Hence, trade secrets become more attractive.

4.6 Welfare Analysis

It remains to study the choices of the firms from a welfare perspective. As forming an RJV is more likely to be cost reducing, it is to be expected that RJVs always create higher total welfare than trade secret arrangements. The following formal analysis confirms this intuition.

Consider equilibrium consumer surplus

$$CS = \frac{(a - \hat{P})\hat{Q}}{2} = \frac{b\hat{Q}^2}{2} = \frac{b(\hat{q}_1 + \hat{q}_2)^2}{2}. \tag{4.20}$$

Substituting (4.6), (4.8) and (4.16) into (4.20) and taking expectations gives

$$\mathbb{E}(CS^{ts}) = \begin{cases} \dfrac{8\alpha^2 + 16\alpha rd + 9rd^2 - r^2d^2}{36b} & \text{if} \quad 2\alpha \geq rd, \\ \dfrac{r(r+1)(\alpha + d)^2}{(r+2)^2 b} & \text{if} \quad 2\alpha < rd, \end{cases} \tag{4.21}$$

and

$$\mathbb{E}(CS^{rjv}) = \frac{2(\alpha^2 + 4\alpha rd + 2rd^2 - 2\alpha r^2 d - r^2d^2)}{9b}. \tag{4.22}$$

Consider equilibrium total welfare: $TW = CS + \hat{\pi}_1 + \hat{\pi}_2$. Taking expectations and substituting the respective consumer surpluses and profits gives

$$\mathbb{E}(TW^{ts}) = \begin{cases} \dfrac{16\alpha^2 + 32\alpha rd + 27rd^2 - 11r^2d^2}{36b} & \text{if} \quad 2\alpha \geq rd, \\ \dfrac{r(r+3)(\alpha + d)^2}{(r+2)^2 b} & \text{if} \quad 2\alpha < rd, \end{cases} \tag{4.23}$$

and

$$\mathbb{E}(TW^{rjv}) = \frac{4(\alpha^2 + 4\alpha rd + 2rd^2 - 2\alpha r^2 d - r^2d^2)}{9b}. \tag{4.24}$$

The relationship between these expected values is given by

Proposition 4.5. $\mathbb{E}(CS^{rjv}) > \mathbb{E}(CS^{ts})$ *and* $\mathbb{E}(TW^{rjv}) > \mathbb{E}(TW^{ts})$ *for any* r *such that* $0 < r < 1$.

The proof is provided in the appendix.

So, forming a research joint venture always yields higher consumer surplus and higher total welfare in comparison with trade secrets. At the same time, the firms prefer the trade secrets arrangement when there is a small chance of a major innovation. Consequently, in this latter case a properly designed subsidy for research joint ventures can improve consumer surplus as well at total welfare. This perspective, for example, gives another basis to support governmental subsidies for fundamental research (in this case Cournot competition, when viewed as competition in capacities[6], is more relevant than Bertrand competition, because fundamental research is a long run issue).

[6] On capacities, see, for example, section 5.3 in Tirole (1988).

4.7 Patents

This section analyses the case of patent protection of innovations. The questions are analogous to those that we considered earlier in the case of trade secrets. Given patent protection of innovations, when do firms conduct private research and when do they form a research joint venture? What are the consequent consumer and total surpluses?

Additionally, given welfare analysis in case of trade secrets and in case of patents, we can compare these arrangements to one another and answer the following normative question: if innovations in an industry can be protected as trade secrets, shall the government also implement patent protection?

First let us consider the subgame where firms conduct private research. In this subgame the firms choose q knowing both ε_1 and ε_2. The firms know about innovations of their competitors, because any innovation is protected by patents and patents, by definition, are public knowledge. Maximizing the profit function

$$\mathbb{E}(\pi_i^p \,|\, \varepsilon_i, \varepsilon_j) = \mathbb{E}\big((a - b(q_i + q_j) - (c - \varepsilon_i))q_i \,\big|\, \varepsilon_i, \varepsilon_j\big) = \\ \big(\alpha + \varepsilon_i - b(q_i + \mathbb{E}(q_j \,|\, \varepsilon_i, \varepsilon_j))\big)q_i \quad (4.25)$$

in own output q_i gives:

$$\hat{q}_i = \max\left(\frac{\alpha + \varepsilon_i}{2b} - \frac{1}{2}\mathbb{E}(q_j \,|\, \varepsilon_1, \varepsilon_2),\, 0\right). \quad (4.26)$$

Compare (4.26) to (4.5).

In equilibrium,

$$\hat{q}_i = \max\left(\frac{\alpha + \varepsilon_i}{2b} - \frac{1}{2}\mathbb{E}(\hat{q}_j \,|\, \varepsilon_1, \varepsilon_2),\, 0\right). \quad (4.27)$$

Solving it for a fixed point gives

$$\hat{q}_i = \begin{cases} \dfrac{\alpha + 2\varepsilon_i - \varepsilon_j}{3b} & \text{if} \quad \alpha \geq d \text{ or } \varepsilon_i = \varepsilon_j, \\ \dfrac{\alpha + d}{2b} & \text{if} \quad \alpha < d,\ \varepsilon_i = d,\ \varepsilon_j = 0, \\ 0 & \text{if} \quad \alpha < d,\ \varepsilon_i = 0,\ \varepsilon_j = d. \end{cases} \quad (4.28)$$

If the maximum size of the market minus the costs (α) is smaller than the potential size of an innovation (d), then if one firm is successful in its research while the other firm is not, then only the successful firm participates in the market. This result is similar to what we obtained earlier for the case of trade secrets. Back then we derived that only the successful firms participate if $2\alpha < rd$. Comparing these results shows that monopoly outcomes occur for a larger range of parameters under patents than under trade secrets. Indeed, $2\alpha < rd \Rightarrow \alpha < d$. So we might expect that patents are, in general, less socially desirable that trade secrets. A later proposition 4.8 confirms this intuition. But to come to that proposition we first need to finish our analysis of the patents subgame.

Plugging $\hat{q}_i$, $\hat{q}_j$ into the profit function and taking expectations gives:

$$\mathbb{E}(\hat{\pi}_i^p) = \begin{cases} \dfrac{\alpha^2 + 2\alpha rd + 5rd^2 - 4r^2d^2}{9b} & \text{if} \quad \alpha \geq d, \\ \dfrac{4\alpha^2 + \alpha^2 r + 18\alpha rd + 9rd^2}{36b} & \\ \quad + \dfrac{-\alpha^2 r^2 - 10\alpha r^2 d - 5r^2d^2}{36b} & \text{if} \quad \alpha < d. \end{cases} \tag{4.29}$$

So,

Proposition 4.6. *There is a unique Nash equilibrium in the patents subgame. The strategies of the firms are given by* (4.28) *and the ex-ante expected equilibrium payoffs are given by* (4.29).

If the firms agree to form an RJV when there is patent protection, then the analysis is identical to that in section 4.4 on RJVs.

Once again, to see what the firms agree upon we need to compare their expected profits. Comparing (4.29) to (4.17) gives

$$\mathbb{E}(\hat{\pi}_i^p) - \mathbb{E}(\hat{\pi}_i^{rjv}) = \begin{cases} \dfrac{rd(1-r)(3d-2\alpha)}{9b} & \text{if} \quad \alpha \geq d, \\ \dfrac{r(1-r)(d+\alpha)^2}{36b} & \text{if} \quad \alpha < d. \end{cases} \tag{4.30}$$

Hence $\mathbb{E}(\hat{\pi}_i^{rjv}) \gtrless \mathbb{E}(\hat{\pi}_i^p) \Leftrightarrow \frac{\alpha}{d} \gtrless \frac{3}{2}$. So, we have

Proposition 4.7. *If there is patent protection of innovations, then firms form a research joint venture if and only if the maximum size of the market minus costs (α) exceeds $\frac{2}{3}d$, where d is the potential size of an innovation.*

Under patent protection firms prefer to form an RJV in fewer cases than under trade secrets (compare propositions 4.7 and 4.3). This is so, because patent protection gives a higher chance of capturing the whole market when a firm is acting alone. To look at the welfare side of the story we now turn to total and consumer surpluses.

Doing the same calculations as in earlier section 4.6 gives:

$$\mathbb{E}(CS^p) = \begin{cases} \dfrac{2\alpha^2 + 4\alpha rd + rd^2 + r^2d^2}{9b} & \text{if} \quad \alpha \geq d, \\ \dfrac{8\alpha^2 - 7\alpha^2 r + 18\alpha rd + 9rd^2}{36b} \\ \quad + \dfrac{7\alpha^2 r^2 - 2\alpha r^2 d - r^2 d^2}{36b} & \text{if} \quad \alpha < d, \end{cases} \tag{4.31}$$

and

$$\mathbb{E}(TW^p) = \begin{cases} \dfrac{4\alpha^2 + 8\alpha rd + 11rd^2 - 7r^2d^2}{9b} & \text{if} \quad \alpha \geq d, \\ \dfrac{16\alpha^2 - 5\alpha^2 r + 54\alpha rd + 27rd^2}{36b} \\ \quad + \dfrac{5\alpha^2 r^2 - 22\alpha r^2 d - 11r^2 d^2}{36b} & \text{if} \quad \alpha < d. \end{cases} \tag{4.32}$$

Comparing these results against the earlier results on RJV is straightforward:

$$\mathbb{E}(CS^{rjv}) - \mathbb{E}(CS^p) = \begin{cases} \dfrac{rd(1-r)(3d+4\alpha)}{9b} & \text{if} \quad \alpha \geq d, \\ \dfrac{7r(1-r)(d+\alpha)^2}{36b} & \text{if} \quad \alpha < d, \end{cases} \tag{4.33}$$

and

$$\mathbb{E}(TW^{rjv}) - \mathbb{E}(TW^p) = \begin{cases} \dfrac{rd(1-r)(8\alpha-3d)}{9b} & \text{if} \quad \alpha \geq d, \\ \dfrac{5r(1-r)(d+\alpha)^2}{36b} & \text{if} \quad \alpha < d. \end{cases} \tag{4.34}$$

Hence we have

Proposition 4.8. $\mathbb{E}(CS^{rjv}) > \mathbb{E}(CS^p)$ *and* $\mathbb{E}(TW^{rjv}) > \mathbb{E}(TW^p)$ *for any* r *such that* $0 < r < 1$.

Now we are ready to compare trade secrets to patents. If $\alpha > d$, then under both regimes firms form a research joint venture and the welfare is maximized. If $\alpha < d$, but innovations are either likely to occur or are relatively minor (see figure 4.1, RJV region), then under patents firms conduct private research and under trade secrets they form an RJV. The latter option delivers a higher welfare. So in this case, given there is an established practice of keeping innovations as trade secrets, imposing additional laws to protect those innovation with patents will deteriorate welfare. Finally, if there is a small chance of a major innovation, then under both regimes firms conduct private research and the resulting welfare is suboptimal, properly designed subsidies for joint research will be welfare improving.

A word of caution ought to be said: the outlined analysis takes effort as exogenous, so it does not compare how trade secrets, patents or research joint ventures influence the amount of effort that gets allocated towards R&D. How much the results can change if effort is endogenized is briefly discussed in the conclusions.

4.8 Conclusions

When it comes to the study of joint research ventures, most literature focuses on patent protection of innovations. This chapter provided an alternative perspective on joint research ventures by studying innovations that are protected as trade secrets.

It is shown that Cournot duopolists prefer to form research joint ventures when potential innovations are minor, or when the chances of a success are high, or both. However, if there is a small chance of a major innovation, then they prefer to conduct R&D in private and keep any consequent innovations trade secrets. From a welfare perspective, research joint ventures are always better. This result implies that a subsidy for Cournot industries, where there are small chances of major innovations, can be welfare improving.

Additionally, in certain cases a practice of protecting innovations as trade secrets is better from a welfare perspective than patent protection, because trade secrets facilitate joint research more than patents do.

As the present chapter raises a relatively new issue, many questions automatically appear. In particular, there is a question how trade secrets would compare against research joint ventures in other settings? Next I discuss three possible extensions: differentiated Bertrand, effort, and multiple firms.

Considering differentiated Bertrand would require a new demand setting, the technical analysis would be similar, but more importantly the intuition is straightforward. If the goods are highly differentiated, then both firms are essentially local monopolists, they do not suffer from each other cost reducing innovations, hence they will prefer to form a research joint venture to maximize the chance of a successful innovation. On the other hand, the less differentiation there is between the goods, the more profitable trade secrets become. At the extreme, if the goods are identical, then the profits are zero unless one firm gets an exclusive innovation, hence in this case trade secrets are chosen over joint research.

Adding effort is technically involving. Even within this simple framework of linear demand and constant marginal costs, if to link the chance of an innovation or its size to effort and if to include quadratic effort costs, the resulting expressions are polynomials of sixth degree (because of the extra optimization step). However, one important, though trivial observation can still be made. The expected profits are continuous in r and d, therefore, if effort changes the initial r or d but only to a limited extent, then the qualitative results will not change. The only change will be in the shape of the indifference curve – those initial r and d, where trade secrets and a research joint venture yield the same expected profits. A preliminary analysis shows that this change is ambiguous – it is not the case that having effort unanimously facilitates either trade secrets or joint research.

Considering multiple firms is a complex problem. If there are n symmetric firms, then there are $n+1$ possible events: nobody innovates, one innovates, etc. The consequent analysis of best responses is technically involved. Additionally, multiple firms bring the issue of different coalition sizes. Strictly speaking, one needs to consider every division of n firms into k RJV coalitions with n_i members each, i.e. $n = n_1 + \ldots + n_k$ and study the incentives of any of those RJV coalitions to merge. Alternatively, networks of bilateral contracts can be used as a basis to study research collaborations among multiple firms, see, e.g. Goyal and Moraga-González (2001). Intuitively, I expect that larger number of firms will favour research joint ventures: with multiple firms there is a smaller chance of an exclusive innovation and hence the trade secrets arrangement will be less attractive.

Summary & Discussion

The four chapters of the thesis cover the following four topics in industrial organization: (i) competition in price and quality, (ii) targeted competition, (iii) the commitment role of organizational structures, and (iv) research and development with trade secrets. The first chapter was written together with Maarten Janssen, the second chapter – together with Alexei Parakhonyak. The following paragraphs briefly discuss the relevance of the aforementioned topics, what particular question the chapters ask and what answers emerge, as well as the possible venues for future research.

(i) Today many companies compete in price/quality bundles. Consider, for example, consumer electronics, automobiles or clothing – in all these cases the producers are able to vary the quality and prices of their products, and all these products are released along with their own price tags. Hand in hand with such competition there exists an informational asymmetry among consumers. Some consumers become experts and can easily compare the prices and qualities of existing offers, others care less and shop at random, yet many of us look at the prices and give the quality a guess judging by the price.

Chapter 1 models this price/quality competition and these informational asymmetries among consumers and, within this new framework, asks an old question: do prices signal quality to those observing the prices but not the quality?

In its most general form, the question is technically challenging, because the strategy of every firm is a distribution over a two-dimensional space (price and quality). Instead of tackling this technical challenge in full, chapter 1 focuses on the most relevant economic aspect of it by asking: is there an equilibrium, where prices signal quality precisely? We term such an equilibrium an *exact signalling equilibrium.*

The answer to this question is: yes, for certain parameters, i.e. for those certain parameters there is an equilibrium where prices signal quality precisely. The chapter fully characterizes this exact signalling equilibrium when it exists, and discusses the existence conditions. The equilibrium can be of two types: one where higher prices signal lower utility and one where higher prices signal higher utility. For common utility functions it the former type, and lower prices signal a better deal.

An exact signalling equilibrium is almost surely Pareto-inefficient. So, while partially informed consumers (those who know prices but not qualities) can learn qualities precisely in an exact signalling equilibrium, and in being able to do so seem to be losing nothing, their mere presence changes the incentives of the firms and makes the subsequent equilibrium inefficient.

An interesting venue for future research is to try to tackle the aforementioned technical challenge in full and see what other types of equilibria exist, if any, besides the exact signalling equilibrium. For example, I have a conjecture that if a dense price dispersion is observed, then the prices signal quality precisely; and if the prices do not signal quality precisely, than there is no dense price dispersion.

(ii) Firm A can locate its outlet closer to firm's B outlet than to firm's C outlet, effectively competing more against firm B. Additionally, firm A can run a comparative advertisement comparing its products to those of firm B. Finally, firm A can launch a legal suit against firm B for violating one of firm's A many patents. These are but a few examples how firms can target particular rivals in their competition. Neither are these examples exclusive to firms – countries target their rivals through trade barriers, political parties organize debates, etc. We call such competition targeted.

While targeted competition abounds in practice, there is little general research on this question. Chapter 2 aims to fill the gap. We consider dynamic (differential) model of competition, between three firms, and each firm is allowed to split its "power" to selectively target its rivals. The firms that receive little competition grow relatively stronger, and those who receive more competition grow relatively weaker. The question is: what do firms find optimal to do and where does it lead them? Formally, we study Markov perfect equilibria of this dynamic game.

The results are as follows. Myopic firms compete more against weaker rivals, hence the strongest rival grows stronger and becomes a monopolist in the end. Sufficiently forward looking firms compete more against their stronger rivals, accounting for the future consequences of not doing so, hence the strongest rival grows weaker and, eventually, all the firms become equal in their powers. In brief, an ability to target competitors stabilizes competition.

We might be accused of complicating the exposition, because we consider a differential game with a continuous state space instead of considering a finite discrete game with a discrete state space. Surprisingly, a differential game is easier from an analytical perspective and is more transparent: in a discrete game many subcases emerge, value functions change over time, and the analysis becomes complex. Unfortunately, there is only one functional form that we can use to have an analytical solution in this differential game (linear-quadratic). There is then a standard question: how robust are our results to changes to the functional form that we use? The only sensible way to answer this question is to develop a procedure of solving the model numerically, test it against the current analytical solution and then apply it to a wider class of specifications. This is a possible venue for future research.

On a more general note, I would like to mention that stable competition is not necessarily beneficial. Stable competition provides security and hence less incentive to invest into R&D. Augmenting targeted competition with R&D and effort and studying the resulting welfare might prove useful in understanding whether targeted competition is socially beneficial or not.[7]

(iii) With the advent of the global economy, multinational and multiproduct companies become the norm rather than the exception. How do such companies influence prices? How do they affect R&D and product variety? In answering these and similar questions one can no longer abstract away from the internal organizations of multiproduct companies by assuming that every company has a benevolent dictator (manager), who posses all the information and is capable of making first-best decisions. To put it simply, the size of multiproduct companies makes single person decision making exorbitantly costly, whether due to information processing costs, moral hazard costs or any other. Decision making is distributed in large companies, hence their market strategies are different in comparison with smaller companies, and so are the outcomes of their competition.

Chapter 3 makes a step in this direction of studying internal structures of multiproduct companies. Namely, it studies the commitment role of organizational structures. While different organizational structures imply different transaction costs for a company, they also imply different strategy

[7]Policy questions on targeted competition arise often: some years ago there was the discussion on comparative advertisement, today there is much discussion on net neutrality, which partially forbids differentiation of goods on the Internet and thus inhibits targeted competition.

spaces when it comes to market competition. Thus, a company can commit to a certain strategy space by choosing a certain organizational structure, and thus influence the outcome of that market competition. As the chapter demonstrates, this commitment role can make a decentralized organizational structure be more attractive, even if it is otherwise suboptimal.

In particular, chapter 3 posits the following questions. Abstracting from all the information processing costs, do multiproduct companies have incentives to decentralize their pricing decisions? And if so, do these decisions depend upon similar decisions of the competitors? The answer to both questions is yes. Multiproduct companies find it profitable to decentralize pricing decisions, when each company produces a range of complementary goods (say, vacuum cleaners and vacuum bags) and when the competition between the companies is strong (the brands are close substitutes). Moreover, for certain parameters two equilibria coexist: in one equilibrium all the companies choose to decentralize their pricing decisions, in the other equilibrium all the companies choose not to do so. Hence, organizational choices of companies can be interdependent through their market interactions.

Chapter 3 simplifies its analysis by considering only two multiproduct companies, each producing only two goods. Extending the analysis to more goods and more firms opens a possibility of linking organizational choices of multiproduct companies to the question of product variety. This, in my view, is an interesting venue for future research.

(iv) In many industries patent protection is weak and innovations are protected as trade secrets. Specific website technologies, internal IT and business processes in private companies, customer databases – these are some of the modern examples. The past has even more examples to offer: watch making, trading routes, etc.

While, in principle, trade secrets can protect innovations just as well as patents do, there is an important difference between these two mechanisms in the amount of information that is revealed. Patents, by definition, are public and thus reveal all the information. If a firm designs and patents some new hardware that is 10% more energy efficient then the current one, all its competitors are fully aware of this firm's cost reductions. On the other hand, if this firm discovers a better software algorithm that allows it to run the current hardware with 10% less load to achieve the same results, the firm keeps its discovery a trade secret and no competitor is aware of its cost reductions.

Chapter 4 formalizes this story and asks the following question: considering Cournot competition, and given that innovations are protected as trade secrets, what are the incentives of firms to form research joint ventures? I obtain the following results. If there is a small chance of a major innovation, then competitors prefer to keep their R&D private and guard any consequent innovations as trade secrets. Otherwise, if only minor innovations are expected, or if the chances of success are high, then competitors prefer to form a research joint venture. From a social perspective, research joint ventures are always better. This observation provides a reason to subsidize joint research in industries where there can be a major breakthrough but the chances of success are small. Fundamental research is one example.

Chapter 4 mainly focuses on the information asymmetries that trade secrets create, keeping other variables, like effort, exogenous. While it serves the purpose of the chapter, it prohibits direct comparison between patents and trade secrets in terms of how well these arrangements facilitate R&D effort. Endogenizing effort is a possible venue for future research.

Are we to expect that patents will prove better than trade secrets, when it comes to effort and R&D? If anything, the open source "revolution" suggests the opposite. Software patent protection is weak and much specific software (for example, Google search algorithms) is kept as trade secrets. On the other hand, just as chapter 4 predicts, when it comes to minor but likely innovations – and constant small improvements in open source software can be viewed as such – companies cooperate. If strong patent protection substitutes the current status quo, many argue the welfare effects will be negative – see, for example, `http://www.fsf.org`.

Appendices

Appendix to Chapter 1

Lemma 1.2. $U_l = U_R$.

Proof. Consider $p \in [p_l, p_h]$ such that $U(p, \hat{q}(p)) = U_l$. Such p should exist because U_l belongs to the support of $F(u)$ by definition. Also, by definition, $F(U_l) = 0$. Therefore

$$\pi(p, \hat{q}(p)) = \frac{\lambda_L}{2} \cdot \Pi(p, \hat{q}(p)). \tag{A1.1}$$

Clearly, $U_l \geq U_R$. Suppose that $U_l = U(p, \hat{q}(p)) > U_R$. Since $\Pi(p, q)$ is strictly decreasing in q and $U(p, q)$ is continuous in q, it is possible to choose such $\varepsilon > 0$ that

$$U(p, \hat{q}(p) - \varepsilon) > U_R \quad \text{and} \quad \Pi(p, \hat{q}(p) - \varepsilon) > \Pi(p, \hat{q}(p)). \tag{A1.2}$$

Also, $F(U(p, \hat{q}(p) - \varepsilon)) = 0$ and therefore

$$\pi(p, \hat{q}(p) - \varepsilon) = \frac{\lambda_L}{2} \cdot \Pi(p, \hat{q}(p) - \varepsilon) > \frac{\lambda_L}{2} \cdot \Pi(p, \hat{q}(p)) = \pi(p, \hat{q}(p)). \tag{A1.3}$$

This contradicts

$$(p, \hat{q}(p)) \in \arg\max_{(\tilde{p}, \tilde{q})} \pi(\tilde{p}, \tilde{q}). \tag{A1.4}$$

So, $U_l = U_R$. □

Lemma 1.4. *Given $u \in [U_l, U_h]$ per-unit profits $\Pi(p, \hat{q}(p))$ are the same for all $p \in \hat{U}^{-1}(u)$.*

Proof. Take $p \in [p_l, p_h]$. It follows from lemma 1.3 that

$$\pi(p, \hat{q}(p)) = \left(F(\hat{U}(p)) \cdot (\lambda_H + \lambda_M) + \frac{\lambda_L}{2} \right) \cdot \Pi(p, \hat{q}(p)). \tag{A1.5}$$

If there are different p_1, p_2 such that

$$\hat{U}(p_1) = \hat{U}(p_2) = u, \tag{A1.6}$$

then $\Pi(p_1, \hat{q}(p_1)) = \Pi(p_2, \hat{q}(p_2))$. Indeed, if this is not the case, then equilibrium profits, i.e. the profits along an equilibrium curve, will differ between p_1 and p_2 as readily seen from (A1.5). But profits have to attain their maximum along the equilibrium curve and hence they have to be constant along it as well. □

Theorem 1.1. *If $g'(p_m) < \frac{1}{a}$ and if there exists an exact signalling equilibrium then $[p_l, p_h] = [p_l, p_m]$ and $U(p, \hat{q}(p))$ is strictly decreasing in p over this interval. Hence in an equilibrium higher prices signal lower utility.*

And vice versa. If $g'(p_m) > \frac{1}{a}$ and if there exists an exact signalling equilibrium then $[p_l, p_h] = [p_m, p_h]$ and $U(p, \hat{q}(p))$ is strictly increasing in p over this interval. Hence in an equilibrium higher prices signal higher utility.

Proof. The formal proof is fully contained in the following lemmas A1.1-A1.5. Next we only give a bit of explanation. For there to be an equilibrium, the profit function $\pi(p, q)$ should attain its maximum along the equilibrium curve $\hat{q}(p)$ or otherwise the firms will deviate from playing (p, q) bundles over it. The idea of the proof is to apply second order necessary conditions, i.e. the conditions for local concavity, to check whether $\pi(p, q)$ can indeed attain its maximum over $\hat{q}(p)$ given different choices of p_l and p_h. In general we have to consider a Hessian to check that but for this proof it suffices and it is convenient to check concavity only in q, i.e. we look at the following second order necessary condition:

$$\frac{\partial^2 \pi(p, q)}{\partial^2 q}\Big|_{(p, \hat{q}(p))} \leq 0 \quad \text{for} \quad p \in [p_l, p_h]. \tag{A1.7}$$

Lemma A1.3 provides us with $\frac{\partial^2 \pi(p,q)}{\partial^2 q}\Big|_{(p,\hat{q}(p))}$. However, the expression is complicated and it is hard to evaluate its sign for an arbitrary p from $[p_l, p_h]$, but important conclusions can be made when considering a limiting case with $p \to p_m$. Lemma A1.4 considers the limiting case and concludes that either $[p_l, p_h] = [p_l, p_m]$ or $[p_l, p_h] = [p_m, p_h]$. Which of these cases occurs depends upon the sign of $U_{pq}(p_m, \hat{q}(p_m)) + \frac{1}{a} U_{qq}(p_m, \hat{q}(p_m))$. This latter expression does not have an immediate interpretation but it can be rewritten so as to allow for an economic one. Namely, this expression can be formulated in terms of a slope of a contract curve at p_m, which is $g'(p_m)$. Lemma A1.5 does so. Together with lemma A1.4 they give: if $g'(p_m) < \frac{1}{a}$ then $[p_l, p_h] = [p_l, p_m]$ and if $g'(p_m) > \frac{1}{a}$ then $[p_l, p_h] = [p_m, p_h]$. Finally, lemma A1.2 gives that $U(p, \hat{q}(p))$ is strictly decreasing in p for $p < p_m$ and is strictly increasing in p for $p > p_m$. □

Lemma A1.1. *$q_m = \hat{q}(p_m)$ and $p_m \in [p_l, p_h]$, i.e. point (p_m, q_m) belongs to an equilibrium curve. Moreover, $U(p, \hat{q}(p)) > U(p_m, \hat{q}(p_m))$ for all $p \neq p_m$, i.e. point (p_m, q_m) gives the minimum utility among all the points of an equilibrium curve.*

Proof. Let $A = \{p \in [p_l, p_h] \,|\, U(p, \hat{q}(p)) = U_R\}$. A denotes the prices that, together with their equilibrium qualities, provide the lowest possible utility. Set A is nonempty as follows from the definition of U_l and from the result that $U_l = U_R$ (see lemma 1.2). Pick an arbitrary $p_0 \in A$. Let $q_0 = \hat{q}(p_0)$. Since $U(p_0, q_0) = U_R$ we have that $F(U(p_0, q_0)) = 0$ and therefore

$$\pi(p_0, q_0) = \frac{\lambda_L}{2}\Pi(p_0, q_0). \tag{A1.8}$$

As (p_0, q_0) belongs to the equilibrium curve, it maximizes the profits. Hence $\pi(p_0, q_0) \geq \pi(p, q)$ for any (p, q). Trivially, $\pi(p, q) \geq \frac{\lambda_L}{2}\Pi(p, q)$ if $U(p, q) \geq U_R$, therefore

$$\frac{\lambda_L}{2}\Pi(p_0, q_0) \geq \frac{\lambda_L}{2}\Pi(p, q) \tag{A1.9}$$

for any (p, q) such that $U(p, q) \geq U_R$. Hence, (p_0, q_0) is a solution to the following optimization problem:

$$\max_{p,q} \Pi(p, q) \quad \text{s.t.} \quad U(p, q) \geq U_R. \tag{A1.10}$$

We already know that this optimization problem has a unique solution, which is denoted by (p_m, q_m). So, $(p_0, q_0) = (p_m, q_m)$. To prove the second proposition it suffices to notice that since (p_0, q_0) is uniquely defined, set A consist of a single point. □

Lemma A1.2. *$\frac{d}{dp}U(p, \hat{q}(p)) > 0$ for $p > p_m$ and $\frac{d}{dp}U(p, \hat{q}(p)) < 0$ for $p < p_m$.*

Proof. For convenience let $\hat{U}$ stand for $U(p, \hat{q}(p))$ and let the same be for the derivatives, e.g. $\hat{U}_p$ stands for $U_p(p, \hat{q}(p)) = \frac{\partial U(p,q)}{\partial p}\Big|_{(p,\hat{q}(p))}$. Suppose $\frac{d\hat{U}}{dp} < 0$ at some point $p_0 > p_m$. Utility function $U(p, q)$ and equilibrium curve $\hat{q}(p)$ are continuous by assumption, therefore $U(p, \hat{q}(p))$ is continuous. Also $U(p_0, \hat{q}(p_0)) > U(p_m, \hat{q}(p_m))$ by lemma A1.1 and $U(p, \hat{q}(p))$ is decreasing at point p_0 by the above supposition. Since $p_0 > p_m$ we can then find $p_1 \in (p_m, p_0)$ such that

$$U(p_1, \hat{q}(p_1)) = U(p_0, \hat{q}(p_0)) \quad \text{and} \quad \frac{d\hat{U}}{dp}\Big|_{p_1} > 0. \tag{A1.11}$$

Let us expand $\frac{d\hat{U}}{dp}$:

$$\frac{d}{dp}U(p,\hat{q}(p)) = \hat{U}_p + \hat{U}_q\frac{d\hat{q}}{dp} = \frac{\lambda_H \hat{U}_q}{\lambda_M}\left(-\frac{\hat{U}_p}{\hat{U}_q} - \frac{1}{a}\right), \tag{A1.12}$$

where the expression for $\hat{q}'(p)$ comes from lemma 1.6. Using the above to rewrite $\left.\frac{d\hat{U}}{dp}\right|_{p_0} < 0$ and $\left.\frac{d\hat{U}}{dp}\right|_{p_1} > 0$ gives

$$-\frac{U_p(p_0,\hat{q}(p_0))}{U_q(p_0,\hat{q}(p_0))} < \frac{1}{a} \quad \text{and} \quad -\frac{U_p(p_1,\hat{q}(p_1))}{U_q(p_1,\hat{q}(p_1))} > \frac{1}{a}. \tag{A1.13}$$

Let us now consider an iso-utility curve that goes through $(p_0, \hat{q}(p_0))$ and $(p_1, \hat{q}(p_1))$. It's the same iso-utility curve because $U(p_0, \hat{q}(p_0)) = U(p_1, \hat{q}(p_1))$. Denote this curve by $\tilde{q}(p)$, i.e. $\tilde{q}(p)$ is implicitly defined by

$$U(p,\tilde{q}(p)) = U(p_1,\hat{q}(p_1)) = U(p_0,\hat{q}(p_0)). \tag{A1.14}$$

This definition is valid since $U(p, q)$ is strictly increasing in q and so there is only one solution for $\tilde{q}$ in the above equation. For the same reason

$$\hat{q}(p_0) = \tilde{q}(p_0) \quad \text{and} \quad \hat{q}(p_1) = \tilde{q}(p_1). \tag{A1.15}$$

Differentiating (A1.14) gives

$$\frac{d\tilde{q}}{dp} = -\frac{U_p(p,\tilde{q}(p))}{U_q(p,\tilde{q}(p))}. \tag{A1.16}$$

Bringing together (A1.13), (A1.15) and (A1.16) gives

$$\tilde{q}'(p_0) < \frac{1}{a} < \tilde{q}'(p_1). \tag{A1.17}$$

At the same time $U(p, q)$ is strictly decreasing in p, strictly increasing in q and strictly quasi-concave, therefore $\tilde{q}(p)$ is convex, i.e. $\tilde{q}''(p) > 0$. It shall follow then that $\tilde{q}'(p_0) > \tilde{q}'(p_1)$ since $p_0 > p_1$, but that contradicts (A1.17). Therefore the earlier supposition that $\left.\frac{d\hat{U}}{dp}\right|_{p_0} < 0$ is wrong. Suppose now that $\left.\frac{d\hat{U}}{dp}\right|_{p_0} = 0$. From (A1.12) it then follows that

$$\hat{q}'(p_0) = -\frac{U_p(p_0,\hat{q}(p_0))}{U_q(p_0,\hat{q}(p_0))} = \frac{1}{a}. \tag{A1.18}$$

Also,

$$\tilde{q}'(p_0) = -\frac{U_p(p_0,\tilde{q}(p_0))}{U_q(p_0,\tilde{q}(p_0))} = -\frac{U_p(p_0,\hat{q}(p_0))}{U_q(p_0,\hat{q}(p_0))} = \frac{1}{a}. \tag{A1.19}$$

Taking $\frac{d^2\hat{U}}{dp^2}$, considering it at point p_0 and plugging in the above expression for $\hat{q}'(p_0)$ gives

$$\frac{d^2\hat{U}}{dp}\Big|_{p_0} = \frac{\lambda_H}{\lambda_M}\left(-U_{pp} - 2\frac{1}{a}U_{pq} - \frac{1}{a^2}U_{qq}\right)\Big|_{(p_0,\hat{q}(p_0))}. \tag{A1.20}$$

Using (A1.16) to get $\frac{d^2}{dp^2}\tilde{q}$, considering it at p_0 and plugging in the expression for $\tilde{q}'(p_0)$ gives

$$\frac{d^2}{dp^2}\tilde{q}\Big|_{p_0} = \frac{1}{U_q}\left(-U_{pp} - 2\frac{1}{a}U_{pq} - \frac{1}{a^2}U_{qq}\right)\Big|_{(p_0,\tilde{q}(p_0))}. \tag{A1.21}$$

But $\frac{d^2}{dp^2}\tilde{q} > 0$ because iso-utility curves are convex, $U_q > 0$ because $U(p,q)$ is strictly increasing in q, $(p_0, \tilde{q}(p_0)) = (p_0, \hat{q}(p_0))$. Therefore $\frac{d^2\hat{U}}{dp^2}\Big|_{p_0} > 0$. So, $\hat{U}$ is strictly convex at p_0 with $\frac{d\hat{U}}{dp}\Big|_{p_0} = 0$. Consequently $\exists p_2 \in (p_m, p_0) : \frac{d\hat{U}}{dp}\Big|_{p_2} < 0$. As was shown before this can not be the case and therefore the supposition that $\frac{d\hat{U}}{dp}\Big|_{p_0} = 0$ is also wrong. Summarizing both arguments gives that $\frac{d\hat{U}}{dp} > 0$ for $p > p_m$. Analogues arguments give that $\frac{d\hat{U}}{dp} < 0$ for $p < p_m$. □

Lemma A1.3. *For any $p \neq p_m$*

$$\frac{\partial^2\pi(p,q)}{\partial^2 q}\Big|_{(p,\hat{q}(p))} = \frac{a^2\lambda_L\lambda_M}{\lambda_H}\cdot\frac{\hat{\Pi}(U_R)}{\Pi(p,\hat{q}(p))}\left(\frac{1}{2}\frac{\hat{U}_{pq} - \frac{\hat{U}_p}{\hat{U}_q}\hat{U}_{qq}}{\hat{U}_q + a\hat{U}_p} + \frac{1}{\Pi(p,\hat{q}(p))}\right), \tag{A1.22}$$

where $\hat{U}_p = \frac{\partial U(p,q)}{\partial p}\Big|_{(p,\hat{q}(p))}$ and similarly for $\hat{U}_q$, $\hat{U}_{pq}$ and $\hat{U}_{qq}$.

Proof. We prove this lemma in a straightforward way. By lemma 1.3

$$\pi(p,q) = \left(F(U(p,q))\cdot\lambda_H + F(U(p,\hat{q}(p)))\cdot\lambda_M + \frac{\lambda_L}{2}\right)\cdot\Pi(p,q), \tag{A1.23}$$

where $\Pi(p,q) = p - aq$ and $F(u)$ is given by lemma 1.5. If we are to differentiate $\pi(p,q)$ we need to know $\hat{q}'(p)$ and $\hat{\Pi}'(u)$. The former derivative we take from lemma 1.6:

$$\frac{d\hat{q}}{dp} = -\frac{\lambda_H + \lambda_M}{\lambda_M}\cdot\frac{U_p'(p,\hat{q}(p))}{U_q'(p,\hat{q}(p))} - \frac{\lambda_H}{a\,\lambda_M}. \tag{A1.24}$$

As for the latter derivative, recollect that

$$\hat{\Pi}(u) = \Pi(\tilde{p}(u), \hat{q}(\tilde{p}(u))), \tag{A1.25}$$

where $\tilde{p}(u)$ could be any function such that $U(\tilde{p}(u), \hat{q}(\tilde{p}(u))) = u$. We'll be looking at the second order derivative of $\pi(p,q)$ at point $(p_0, \hat{q}(p_0))$ of an equilibrium curve with $p_0 \neq p_m$. For this point we can be more precise about $\tilde{p}(u)$. Indeed, from lemma A1.2 we know that

$$\frac{d}{dp}U(p, \hat{q}(p)) \neq 0 \quad \text{for} \quad p \neq p_m. \tag{A1.26}$$

Also $U(p, \hat{q}(p))$ is twice differentiable because for $U(p, q)$ it was assumed and $\hat{q}(p)$ is itself defined by a differential equation that involves only differentiable functions. So, by an inverse function theorem there is a unique continuously differentiable $\tilde{p}(u)$ defined in the neighbourhood of $u_0 = U(p_0, \hat{q}(p_0))$ by $U(\tilde{p}(u), \hat{q}(\tilde{p}(u))) = u$, with its derivative given by

$$\frac{d\tilde{p}(u)}{du} = \frac{1}{U_p(\tilde{p}(u), \hat{q}(\tilde{p}(u)) + U_q(\tilde{p}(u), \hat{q}(\tilde{p}(u)) \cdot \hat{q}'(\tilde{p}(u))} = \\ -\frac{a\lambda_M}{\lambda_H}\frac{1}{a \cdot U_p(\tilde{p}(u), \hat{q}(\tilde{p}(u)) + U_q(\tilde{p}(u), \hat{q}(\tilde{p}(u))}. \tag{A1.27}$$

Expressions (A1.24), (A1.25) and (A1.27) allow one to calculate the second order derivative of $\pi(p, q)$ in q in a straightforward way. Evaluating the resulting derivative at $(p_0, \hat{q}(p_0))$, noticing that

$$\tilde{p}(U(p_0, \hat{q}(p_0))) = p_0 \tag{A1.28}$$

and noticing that p_0 was chosen arbitrary just not to equal p_m immediately gives the result of the lemma. □

Lemma A1.4.

$$U_{pq}(p_m, \hat{q}(p_m)) + \frac{1}{a}U_{qq}(p_m, \hat{q}(p_m)) < 0 \quad \Rightarrow \quad [p_l, p_h] = [p_l, p_m], \tag{A1.29}$$

$$U_{pq}(p_m, \hat{q}(p_m)) + \frac{1}{a}U_{qq}(p_m, \hat{q}(p_m)) > 0 \quad \Rightarrow \quad [p_l, p_h] = [p_m, p_h]. \tag{A1.30}$$

In other words, either the equilibrium lies in the segment to the left of p_m where higher prices signal lower utility or the equilibrium lies in the segment to the right of p_m where higher prices signal higher utility.

Proof. Consider the case when $U_{pq}(p_m, \hat{q}(p_m)) + \frac{1}{a}U_{qq}(p_m, \hat{q}(p_m)) < 0$. Suppose $p_h > p_m$. Then we can consider the limit of $\left.\frac{\partial^2\pi(p,q)}{\partial^2 q}\right|_{(p,\hat{q}(p))}$ as p approaches p_m from the right. To do so let us start with the limit of $-\frac{\hat{U}_p}{\hat{U}_q}$. By the definition of p_m and by lemma A1.1

$$-\frac{U_p(p_m, \hat{q}(p_m))}{U_q(p_m, \hat{q}(p_m))} = \frac{1}{a}. \tag{A1.31}$$

By lemma A1.2

$$\frac{d}{dp}U(p,\hat{q}(p)) > 0 \quad \text{for} \quad p > p_m. \tag{A1.32}$$

Taking the derivative shows that this condition is equivalent to

$$-\frac{U_p(p,\hat{q}(p))}{U_q(p,\hat{q}(p))} > \frac{1}{a} \quad \text{for} \quad p > p_m. \tag{A1.33}$$

Moreover, $-\frac{\hat{U}_p}{\hat{U}_q}$ is continuous in p. Therefore we have that

$$-\frac{\hat{U}_p}{\hat{U}_q} \downarrow \frac{1}{a} \quad \text{as} \quad p \downarrow p_m. \tag{A1.34}$$

Hence

$$\lim_{p\downarrow p_m} \frac{\partial^2 \pi(p,q)}{\partial^2 q}\bigg|_{(p,\hat{q}(p))} = \lim_{p\downarrow p_m} \frac{a^2\lambda_L\lambda_M}{\lambda_H} \cdot \frac{\hat{\Pi}(U_R)}{\Pi(p,\hat{q}(p))} \left(\frac{1}{2a\hat{U}_q} \frac{\hat{U}_{pq} - \frac{\hat{U}_p}{\hat{U}_q}\hat{U}_{qq}}{\frac{1}{a} - \left(-\frac{\hat{U}_p}{\hat{U}_q}\right)} + \frac{1}{\Pi(p,\hat{q}(p))} \right) = +\infty. \tag{A1.35}$$

The sign comes from the preceding discussion and from the observation that $\hat{\Pi}(U_R)$, $\Pi(p,\hat{q}(p))$ and U_q are all strictly positive. But (A1.35) contradicts the necessary condition that $\frac{\partial^2\pi(p,q)}{\partial^2 q}\Big|_{(p,\hat{q}(p))} \leq 0$ for all $p \in [p_l, p_h]$. Therefore if there is an exact signalling equilibrium it should be that $p_h \leq p_m$.[8] But $p_m \in [p_l, p_h]$ (lemma A1.1), hence $p_m = p_h$. Analogous arguments hold for $U_{pq}(p_m,\hat{q}(p_m)) + \frac{1}{a}U_{qq}(p_m,\hat{q}(p_m)) > 0$. □

Lemma A1.5.

$$U_{pq}(p_m,\hat{q}(p_m)) + \frac{1}{a}U_{qq}(p_m,\hat{q}(p_m)) \gtrless 0 \quad \Leftrightarrow \quad g'(p_m) \gtrless \frac{1}{a}. \tag{A1.36}$$

Proof. Writing down the necessary conditions for the optimization problem that defines $g(p)$ gives

$$-\frac{U_p(p,g(p))}{U_q(p,g(p))} = \frac{1}{a}. \tag{A1.37}$$

Or, equivalently,

$$aU_p(p,g(p)) + U_q(p,g(p)) = 0. \tag{A1.38}$$

[8] In this case we can not consider a limit from the right and the contradiction does not hold.

Differentiating (A1.38) in p and rearranging the terms gives

$$g'(p) = -\frac{U_{pp} + \frac{1}{a}U_{qp}}{U_{pq} + \frac{1}{a}U_{qq}} = \frac{-U_{pp} - 2\frac{1}{a}U_{pq} - \frac{1}{a^2}U_{qq}}{U_{pq} + \frac{1}{a}U_{qq}} + \frac{1}{a}, \tag{A1.39}$$

where $U_{pp} = \frac{\partial^2 U(p,q)}{\partial p^2}\Big|_{(p,g(p))}$, and so on. Consider now an iso-utility curve going through (p_m, q_m). Namely, consider $\tilde{q}(p)$ defined by

$$U(p, \tilde{q}(p)) = U(p_m, q_m). \tag{A1.40}$$

Twice differentiating this expression, evaluating it at (p_m, q_m), noticing that

$$\tilde{q}'(p_m) = -\frac{U_p(p_m, \tilde{q}(p_m))}{U_q(p_m, \tilde{q}(p_m))} = \frac{1}{a} \tag{A1.41}$$

due to the definition of (p_m, q_m), and rearranging the terms gives

$$\tilde{q}''(p_m) = \frac{1}{U_q}\left(-U_{pp} - 2\frac{1}{a}U_{pq} - \frac{1}{a^2}U_{qq}\right)\Big|_{(p_m, \tilde{q}(p_m))}. \tag{A1.42}$$

Iso-utility curves are strictly convex (assumption 1.1), so $\tilde{q}''(p_m) > 0$. Also, (p_m, q_m) belongs to the contract curve $g(p)$, to the equilibrium curve $\hat{q}(p)$ and to the iso-utility curve $\tilde{q}(p)$, so $q_m = g(p_m) = \hat{q}(p_m) = \tilde{q}(p_m)$. So, we can use $U_{pp}(p_m, g(p_m))$, $U_{pp}(p_m, \hat{q}(p_m))$ and $U_{pp}(p_m, \tilde{q}(p_m))$ and the others interchangeably. But then the statement of the lemma readily follows from (A1.39), (A1.42), from $\tilde{q}''(p_m) > 0$ and from $U_q(p_m) > 0$. □

Theorem 1.3. *Consider an arbitrary strictly increasing, strictly convex and twice differentiable equilibrium curve $\hat{q}(p)$ defined over $[p_l, p_h]$ and satisfying* (1.29) *or* (1.30) *or both. Then there exist such a utility function $U(p,q)$ satisfying assumption 1.1, such parameters $(U_R, \lambda_H, \lambda_M, \lambda_L, a)$ and such out-of-equilibrium beliefs that there will be a corresponding exact signalling equilibrium, i.e one that has $\hat{q}(p)$ as its equilibrium curve.*

Proof. We only discuss how to find such parameters of our model as to get an equilibrium where *lower* prices signal higher utility. Construction of an equilibrium where *higher* prices signal higher utility is analogous. To prove the theorem we have to find $(U_R, \lambda_H, \lambda_M, \lambda_L, a)$, $U(p,q)$ and out-of-equilibrium beliefs such that a) $U(p,q)$ satisfies assumption 1.1, b) the resulting equilibrium curve is precisely $\hat{q}(p)$ and the resulting boundary points are precisely p_l and p_h, c) the expected profits $\pi(p,q)$ attain their maximum over the equilibrium curve $\hat{q}(p)$. We proceed as follows. First, we choose some specific parameters $(U_R, \lambda_H, \lambda_M, \lambda_L, a)$ and we choose a specific utility function

$U(p, q)$. Second, we show that a) and b) hold for those parameters and utility function. Third, we choose some specific but reasonable out-of-equilibrium beliefs and we show that c) holds as well. Take

$$a = \frac{1}{\hat{q}'(p_h)} \tag{A1.43}$$

and consider $\frac{d}{dp}\Pi(p, \hat{q}(p))$:

$$\frac{d}{dp}\Pi(p, \hat{q}(p)) = 1 - a\hat{q}'(p) = 1 - \frac{\hat{q}'(p)}{\hat{q}'(p_h)}. \tag{A1.44}$$

As assumed, $\hat{q}(p)$ is strictly increasing and strictly convex, i.e. $\hat{q}'(p) > 0$ and $\hat{q}''(p) > 0$. Therefore $\hat{q}'(p) < \hat{q}'(p_h)$ for $p < p_h$ and, consequently, $\frac{d}{dp}\Pi(p, \hat{q}(p)) > 0$ for $p < p_h$. In other words, equilibrium per-unit profits are strictly increasing in p over $[p_l, p_h]$. Define

$$\Pi_l = \Pi(p_l, \hat{q}(p_l)), \quad \Pi_h = \Pi(p_h, \hat{q}(p_h)). \tag{A1.45}$$

Take

$$\lambda_H + \lambda_M = \frac{\Pi_h - \Pi_l}{\Pi_h + \Pi_l}, \quad \lambda_L = 1 - (\lambda_H + \lambda_M) = \frac{2\Pi_l}{\Pi_h + \Pi_l}. \tag{A1.46}$$

We choose precise values for $\lambda_H + \lambda_M$ and λ_L. As for λ_H and λ_M, they can be chosen arbitrary but with λ_M sufficiently small, more precisely, we take λ_M such that

$$\lambda_M < \left(\frac{\Pi_l}{\Pi_h}\right)^2 \frac{\Pi_h - \Pi_l}{\Pi_h + \Pi_l}. \tag{A1.47}$$

Let

$$D = \{(p, q) : p \in [p_l, p_h], \Pi_l \leq \Pi(p, q) \leq \Pi_h\}. \tag{A1.48}$$

To define $U(p, q)$ and to show that it satisfies assumption 1.1 we proceed as follows. First, we define $U(p, q)$ for $(p, q) \in D$ and we show that $U(p, q)$ satisfies assumption 1.1 on D. Second, we argue that $U(p, q)$ can be extended beyond D in such a way that the assumption is still satisfied. As a result we will have a utility function $U(p, q)$ that satisfies assumption 1.1 in general and has an analytical expression for $(p, q) \in D$. Take

$$U(p, q) = \frac{\Pi_h}{p - aq} - \frac{\lambda_M}{\lambda_H + \lambda_M} \frac{\Pi_h}{p - a\hat{q}(p)} - \frac{\lambda_H}{\lambda_H + \lambda_M} \quad \text{for} \quad (p, q) \in D \tag{A1.49}$$

and take $U_R = 0$. As the equilibrium curve $\hat{q}(p)$ was taken to be twice differentiable, $U(p, q)$ is also twice differentiable on D. Consider U_q:

$$U_q(p, q) = \frac{\Pi_h}{(p - aq)^2} \cdot a > 0. \tag{A1.50}$$

Hence, $U(p,q)$ is strictly increasing in q. Next, consider U_p:

$$U_p(p,q) = -\frac{\Pi_h}{(p-aq)^2} + \frac{\lambda_M}{\lambda_M+\lambda_H}\frac{\Pi_h}{(p-a\hat{q}(p))^2}(1-a\hat{q}'(p)). \qquad \text{(A1.51)}$$

For $(p,q) \in D$ it holds that

$$p - aq \le \Pi_h, \quad p - a\hat{q}(p) \ge \Pi_l, \quad 0 \le \hat{q}'(p) \le \frac{1}{a}. \qquad \text{(A1.52)}$$

Therefore

$$U_p(p,q) \le -\frac{\Pi_h}{\Pi_h^2} + \frac{\lambda_M}{\lambda_M+\lambda_H}\frac{\Pi_h}{\Pi_l^2} < 0, \qquad \text{(A1.53)}$$

where the last inequality follows directly from (A1.46) and (A1.47). Hence, $U(p,q)$ is strictly decreasing in p on D. If $U(p,q)$ is strictly decreasing in p and strictly increasing in q then it is strictly quasi-concave if and only if its iso-utility curves $\tilde{q}(p)$ are strictly convex, i.e. it should be that $\tilde{q}''(p) > 0$. To check that $\tilde{q}''(p) > 0$ we start with $\tilde{q}'(p)$:

$$\tilde{q}'(p) = -\frac{U_p(p,\tilde{q}(p))}{U_q(p,\tilde{q}(p))} = \frac{1}{a} - \frac{1}{a}\frac{\lambda_M}{\lambda_M+\lambda_H}\frac{(p-a\tilde{q}(p))^2}{(p-a\hat{q}(p))^2}(1-a\hat{q}'(p)). \qquad \text{(A1.54)}$$

Next,

$$\tilde{q}''(p) = \frac{2}{a}\frac{\lambda_M}{\lambda_M+\lambda_H}\frac{(p-a\tilde{q}(p))^2(1-a\hat{q}'(p))^2}{(p-a\hat{q}(p))^3}\left(1 - \frac{\lambda_M}{\lambda_M+\lambda_H}\frac{p-a\tilde{q}(p)}{p-a\hat{q}(p)}\right) + \frac{\lambda_M}{\lambda_M+\lambda_H}\frac{(p-a\tilde{q}(p))^2}{(p-a\hat{q}(p))^2}\hat{q}''(p). \qquad \text{(A1.55)}$$

The equilibrium curve $\hat{q}(p)$ was taken to be strictly convex, so $\hat{q}''(p) > 0$. Also, on D

$$p - a\tilde{q}(p) \le \Pi_h, \quad p - a\hat{q}(p) \ge \Pi_l \qquad \text{(A1.56)}$$

and then

$$1 - \frac{\lambda_M}{\lambda_M+\lambda_H}\frac{p-a\tilde{q}(p)}{p-a\hat{q}(p)} \ge 1 - \frac{\lambda_M}{\lambda_M+\lambda_H}\frac{\Pi_h}{\Pi_l} > 1 - \frac{\Pi_l}{\Pi_h} > 0, \qquad \text{(A1.57)}$$

where the second inequality follows directly from (A1.46) and (A1.47). Consequently, $\tilde{q}''(p) > 0$ and $U(p,q)$ is strictly quasi-concave.

Consider $U(p,q)$ as a map of iso-utility curves on D. These iso-utility curves, when viewed as functions of p, are strictly increasing, strictly convex and are as sufficiently smooth as to make $U(p,q)$ twice differentiable. Also, D is a convex set. Clearly then, these iso-utility curves can be extended beyond D as to still be strictly increasing, strictly convex and sufficiently smooth. Moreover, if necessary, these iso-utility curves can be made convex enough outside D so as to have each of them attain a slope of $\frac{1}{a}$ at some point. This latter condition guarantees that $\max_{(p,q)} \Pi(p,q)$ s.t. $U(p,q) \geq x$ has an inner solution.

Now we proceed with verifying b). Given $(U_R, \lambda_H, \lambda_M, \lambda_L, a)$ and given $U(p,q)$ we can solve for the equilibrium curve and for the boundary points. We denote the equilibrium curve and the boundary points that we get as a solution to the model by $\hat{q}_s(p)$ and by p_l^s, p_h^s respectively, This way we can distinguish them from the given $\hat{q}(p)$ and p_l, p_h. Then to verify b) means to verify that $\hat{q}_s(p) \equiv \hat{q}(p)$, $p_l^s = p_l$ and $p_h^s = p_h$. In general, $[p_l^s, p_h^s] = [p_l^s, p_m]$ in equilibria where lower prices signal higher utility – see theorem 1.1. In our case we are also looking for such an equilibrium. Hence we also choose $p_h^s = p_m$. Recollect that

$$(p_m, q_m) = \arg\max_{p,q} \Pi(p,q) \quad \text{s.t.} \quad U(p,q) \geq U_R. \tag{A1.58}$$

The solution is attained when $U(p,q) = U_R$ and the necessary and sufficient conditions for this optimization problem are:

$$\begin{cases} \dfrac{\Pi_p(p,q)}{\Pi_q(p,q)} = \dfrac{U_p(p,q)}{U_q(p,q)}, \\ U(p,q) = U_R, \end{cases} \tag{A1.59}$$

where sufficiency follows from the strict convexity of the problem.

Suppose optimal $(p,q) \in D$. Then using (A1.49) and simplifying gives:

$$\begin{cases} 1 - a\hat{q}'(p) = 0, \\ \dfrac{\Pi_h}{p - aq} - \dfrac{\lambda_M}{\lambda_H + \lambda_M} \dfrac{\Pi_h}{p - a\hat{q}(p)} - \dfrac{\lambda_H}{\lambda_H + \lambda_M} = 0. \end{cases} \tag{A1.60}$$

Given that $a = \frac{1}{\hat{q}'(p_h)}$ and that $\Pi_h = p_h - a\hat{q}(p_h)$ it is straightforward to verify that point $(p_h, \hat{q}(p_h))$ satisfies (A1.60). So, $(p_m, q_m) = (p_h, \hat{q}(p_h))$ and $p_h^s = p_h$. From lemma 1.6

$$\hat{q}_s'(p) = -\frac{\lambda_H + \lambda_M}{\lambda_M} \frac{U_p(p, \hat{q}_s(p))}{U_q(p, \hat{q}_s(p))} - \frac{1}{a} \frac{\lambda_H}{\lambda_M}. \tag{A1.61}$$

The boundary condition comes from lemma A1.1: $\hat{q}_s(p)$ has to go through the point $(p_m, q_m) = (p_h, \hat{q}(p_h))$. For $p \in [p_l, p_h]$ we use (A1.51) and (A1.50) to rewrite (A1.61) as

$$\hat{q}_s'(p) = \frac{1}{a} - \frac{1}{a} \left(\frac{p - a\hat{q}_s(p)}{p - a\hat{q}(p)} \right) (1 - a\hat{q}'(p)). \tag{A1.62}$$

Clearly, for $p \in [p_l, p_h]$ $\hat{q}_s(p) \equiv \hat{q}(p)$ is a solution. Moreover, it is unique by the Picard's theorem. The lower bound p_l^s is implicitly defined by $U(p_l^s, \hat{q}_s(p_l^s)) = U_h$ and U_h comes from $F(U_h) = 1$. We now solve these equations. For $p \in [p_l, p_h]$

$$U(p, \hat{q}_s(p)) = U(p, \hat{q}(p)) = \frac{\lambda_H}{\lambda_H + \lambda_M} \left(\frac{\Pi_h}{p - a\hat{q}(p)} - 1 \right). \tag{A1.63}$$

From lemma 1.5

$$F(u) = \frac{1}{2} \frac{\lambda_L}{\lambda_H + \lambda_M} \left(\frac{\Pi_h}{\hat{\Pi}(u)} - 1 \right), \tag{A1.64}$$

where

$$\hat{\Pi}(u) = \tilde{p}(u) - a\hat{q}(\tilde{p}(u)) \tag{A1.65}$$

and $\tilde{p}(u)$ is implicitly defined by

$$U(\tilde{p}(u), \hat{q}(\tilde{p}(u))) = u. \tag{A1.66}$$

Substituting p with $\tilde{p}(u)$ in (A1.63) and the resulting $\tilde{p}(u) - a\hat{q}(\tilde{p}(u))$ with $\hat{\Pi}(u)$ and then comparing the outcome with (A1.64) gives

$$F(u) = \frac{1}{2} \frac{\lambda_L}{\lambda_H} \cdot u. \tag{A1.67}$$

Hence $U_h = 2\frac{\lambda_H}{\lambda_L}$. Suppose $p_l^s \in [p_l, p_h]$, then using (A1.46) and (A1.47) we can rewrite $U(p_l^s, \hat{q}(p_l^s)) = U_h$ as

$$p_l^s - a\hat{q}(p_l^s) = \Pi_l = p_l - a\hat{q}(p_l). \tag{A1.68}$$

Clearly then, $p_l^s = p_l$ is a solution. Moreover, it is unique because $U(p, \hat{q}(p))$ is strictly monotone in p (lemma A1.2).

Next, we have to verify c), i.e. we have to verify that the expected profits $\pi(p, q)$ attain their maximum over $\hat{q}(p)$. When the opponent is playing the equilibrium strategy

$$\pi(p, q) = \left(F(U(p, q)) \cdot \lambda_H + F(\hat{U}(p)) \cdot \lambda_M + \frac{\lambda_L}{2} \right) \Pi(p, q), \tag{A1.69}$$

where $\hat{U}(p)$ stands for the utility that partially informed consumers expect to receive given that the price is p. For $p \in [p_l, p_h]$ we have that

$$\hat{U}(p) = U(p, \hat{q}_s(p)) = U(p, \hat{q}(p)). \tag{A1.70}$$

Since $U(p, \hat{q}(p))$ is strictly decreasing in p for $p \in [p_l, p_h]$ we can choose and we choose such out-of-equilibrium beliefs that $\hat{U}(p)$ is decreasing in p for $p \in \mathbb{R}$.[9] Define

$$S_C = \{(p,q) : p \in [p_l, p_h], U_R \leq U(p,q) \leq U_h\}, \tag{A1.71}$$
$$S_B = \{(p,q) : U(p,q) < U_R\}, \tag{A1.72}$$
$$S_L = \{(p,q) : p < p_l, U_R \leq U(p,q) \leq U_h\}, \tag{A1.73}$$
$$S_R = \{(p,q) : p > p_h, U_R \leq U(p,q) \leq U_h\}, \tag{A1.74}$$
$$S_T = \{(p,q) : U(p,q) > U_h\}. \tag{A1.75}$$

Clearly, $\bigcup_x S_x = \mathbb{R}^2$. We consider $\pi(p,q)$ over each of these regions in turn.

Region $\mathbf{S_C}$. Suppose $(p,q) \in S_C$. For $p \in [p_l, p_h]$ it holds that $\Pi_l \leq p - a\hat{q}(p) \leq \Pi_h$. Then, using the definitions for $\lambda_H + \lambda_M$ and λ_L, it is straightforward to verify that

$$U(p,q) \geq U_R \Rightarrow \Pi(p,q) \leq \Pi_h, \tag{A1.76}$$
$$U(p,q) \leq U_h \Rightarrow \Pi(p,q) \geq \Pi_l. \tag{A1.77}$$

Consequently, $S_C \subseteq D$. But for $(p,q) \in D$ we have an explicit expression for $U(p,q)$ and $\hat{U}(p) = U(p, \hat{q}(p))$ for $p \in [p_l, p_h]$. Also, $F(u) = \frac{1}{2}\frac{\lambda_L}{\lambda_H} \cdot u$ for $U_R \leq u \leq U_h$. Expanding (A1.69) then gives:

$$\pi(p,q) = \left(F(U(p,q)) \cdot \lambda_H + F(U(p,\hat{q}(p))) \cdot \lambda_M + \frac{\lambda_L}{2} \right) \cdot (p - aq) =$$
$$\frac{\lambda_L}{2} \left(U(p,q) + \frac{\lambda_M}{\lambda_H} U(p,\hat{q}(p)) + 1 \right) \cdot (p - aq) = \frac{\lambda_L}{2} \Pi_h, \tag{A1.78}$$

where the last equality follows directly from the definitions of $U(p,q)$, $\lambda_H + \lambda_M$, λ_L and Π_h, Π_l, see equations (A1.49), (A1.46) and (A1.45). So, the expected profits are constant for $(p,q) \in S_C$.

Region $\mathbf{S_B}$. Suppose $(p,q) \in S_B$. But then $U(p,q) < U_R$, so no consumers buy the product and

$$\pi(p,q) = 0 < \frac{\lambda_L}{2} \Pi_h. \tag{A1.79}$$

[9] These are reasonable out-of-equilibrium beliefs as they depend upon p in the same direction as equilibrium beliefs do.

Region $\mathbf{S_L}$. Suppose $(p,q) \in S_L$. This implies $U_R \leq U(p,q) \leq U_h$. Let

$$q_u = \hat{q}(p_l), \tag{A1.80}$$
$$q_b = \frac{1}{a}\left(p_l - \frac{(\lambda_H + \lambda_M)\Pi_l\Pi_h}{\lambda_M\Pi_h + \lambda_H\Pi_l}\right). \tag{A1.81}$$

Then it directly follows from the definitions of $U(p,q)$, U_R, $\lambda_H + \lambda_M$, λ_L and Π_h, Π_l that $U(p_l, q_u) = U_h$ and $U(p_l, q_b) = U_R$. Moreover, $U(p_l, q)$ is continuous in q and therefore there exists q^* such that

$$U(p_l, q^*) = U(p,q). \tag{A1.82}$$

Consequently,

$$F(U(p,q)) = F(U(p_l, q^*)). \tag{A1.83}$$

Since $p < p_l$ and since $\hat{U}(p)$ is decreasing in p, $\hat{U}(p) \geq \hat{U}(p_l)$. But $\hat{U}(p_l) = U(p_l, \hat{q}(p_l)) = U_h$ and $F(u) = 1$ for $u \geq U_h$. So,

$$F(\hat{U}(p)) = F(\hat{U}(p_l)). \tag{A1.84}$$

Given (A1.82), we can take an iso-utility curve $\tilde{q}(p)$ going through points (p,q) and (p_l, q^*). From (A1.54) we have that

$$\tilde{q}'(p_l) = \frac{1}{a} - \frac{1}{a}\frac{\lambda_M}{\lambda_M + \lambda_H}\frac{(p_l - a\tilde{q}(p_l))^2}{\Pi_l^2}(1 - a\hat{q}'(p_l)). \tag{A1.85}$$

As $\hat{q}'(p_l) < \frac{1}{a}$ it follows that $\tilde{q}'(p_l) < \frac{1}{a}$. Utility $U(p,q)$ satisfies assumption 1.1 and therefore $\tilde{q}''(p) > 0$. So, $\tilde{q}'(p) < \frac{1}{a}$ for all $p \leq p_l$. Consequently,

$$\frac{d}{dp}\Pi(p, \tilde{q}(p)) = 1 - a\tilde{q}'(p) > 0 \quad \text{for all} \quad p \leq p_l. \tag{A1.86}$$

As $p < p_l$ we then have that

$$\Pi(p,q) < \Pi(p_l, q^*). \tag{A1.87}$$

Bringing together (A1.83), (A1.84) and (A1.87) and noticing that $(p_l, q^*) \in S_C$ gives:

$$\pi(p,q) = \left(F(U(p,q)) \cdot \lambda_H + F(\hat{U}(p)) \cdot \lambda_M + \frac{\lambda_L}{2}\right)\Pi(p,q) <$$
$$\left(F(U(p_l,q^*)) \cdot \lambda_H + F(\hat{U}(p_l)) \cdot \lambda_M + \frac{\lambda_L}{2}\right)\Pi(p_l,q^*) = \frac{\lambda_L}{2}\Pi_h. \tag{A1.88}$$

Region $\mathbf{S_R}$. Suppose $(p,q) \in S_R$. This case is analogous to the previous one and we also get that

$$\pi(p,q) < \frac{\lambda_L}{2}\Pi_h. \tag{A1.89}$$

Region $\mathbf{S_T}$. Suppose $(p,q) \in S_T$, i.e. $U(p,q) > U_h$. Given that $U(p,q)$ satisfies assumption 1.1 there exists $q^* < q$ such that $U(p,q^*) = U_h$. As $F(u) = 1$ for $u \geq U_h$ we have that $F(U(p,q)) = F(U(p,q^*))$. Trivially, $\Pi(p,q) < \Pi(p,q^*)$. Therefore,

$$\pi(p,q) = \left(F(U(p,q)) \cdot \lambda_H + F(\hat{U}(p)) \cdot \lambda_M + \frac{\lambda_L}{2}\right)\Pi(p,q) < \\ \left(F(U(p,q^*)) \cdot \lambda_H + F(\hat{U}(p)) \cdot \lambda_M + \frac{\lambda_L}{2}\right)\Pi(p,q^*) = \pi(p,q^*). \tag{A1.90}$$

But $(p,q^*) \in S_L \cup S_C \cup S_R$, so $\pi(p,q^*) \leq \frac{\lambda_L}{2}\Pi_h$ and $\pi(p,q) < \frac{\lambda_L}{2}\Pi_h$. Given that $\pi(p,q) = \frac{\lambda_L}{2}\Pi_h$ for $(p,q) \in S_C$, that $\pi(p,q) < \frac{\lambda_L}{2}\Pi_h$ for $(p,q) \notin S_C$ and that the equilibrium curve $\hat{q}(p)$ belongs to S_C, we have that the profits attain their maximum over $\hat{q}(p)$. □

Theorem 1.4. *Consider a linear kinked utility function (eq. 1.44). Then an exact signalling equilibrium exists if and only if the following conditions are satisfied:*

$$(i) \quad -U_R - aq^* > 0, \tag{A1.91}$$

$$(ii) \quad b_2 < a, \tag{A1.92}$$

$$(iii) \quad \frac{b_1 - a}{a} > \frac{\lambda_M}{\lambda_H} \cdot \frac{2-\lambda_L}{\lambda_L}. \tag{A1.93}$$

If the conditions are satisfied, then the equilibrium is unique and is characterized by:

$$\hat{q}(p) = q^*, \quad p_l = \frac{\lambda_L(-U_R - aq^*)}{2-\lambda_L} + aq^*, \quad p_h = -U_R, \tag{A1.94}$$

$$F(u) = \frac{1}{2} \cdot \frac{\lambda_L}{1-\lambda_L}\left(\frac{U_R + aq^*}{u + aq^*} - 1\right), \quad U_l = -p_h, \quad U_h = -p_l. \tag{A1.95}$$

Proof. A linear kinked utility is defined as follows:

$$U(p,q) = b(q) \cdot (q - q^*) - p, \\ b(q) = \begin{cases} b_1 \text{ if } q \leq q^*, \\ b_2 \text{ if } q > q^*, \end{cases} \tag{A1.96}$$

with $b_2 < b_1$. A linear kinked utility does not fully satisfy assumption 1.1. However, most of the results still hold. Namely, lemmas 1.1 through 1.4, lemma A1.1, and lemma 1.5 are still valid without any modifications (the missing assumptions are not required for the respective proofs). Lemma A1.3 is valid for all p, where $U(p, \hat{q}(p))$ is differentiable, i.e. for all p such that $\hat{q}(p) \neq q^*$. The other results are not required in the current proof. We first prove that conditions (i)-(iii) are necessary conditions. Suppose that an exact signalling equilibrium exists. Lemma A1.1 tells us that point (p_m, q_m) defined as

$$(p_m, q_m) = \arg\max_{(p,q):U(p,q)\geq U_R} \Pi(p, q) \tag{A1.97}$$

belongs to the equilibrium curve $\hat{q}(p)$. In turn, this implies that (p_m, q_m) if finite and $\Pi(p_m, q_m) > 0$. Given that $\Pi(p, q) = p - aq$ and given (A1.97), it can be readily seen that for (p_m, q_m) to be finite it shall be that $b_1 > a$ and $b_2 < a$. In this case

$$(p_m, q_m) = (-U_R, q^*) \tag{A1.98}$$

and

$$\Pi(p_m, q_m) > 0 \;\Leftrightarrow\; -U_R - aq^* > 0. \tag{A1.99}$$

So, existence of an equilibrium implies (i), (ii) and $b_1 > a$. To show that condition (iii) is also a necessary condition, we need to solve the model fully. We start by solving for the equilibrium curve. Suppose $\hat{q}(p) \neq q^*$ for some $p \neq p_m$. Then lemma A1.3 applies, and

$$\begin{aligned}\frac{\partial^2 \pi(p, q)}{\partial^2 q}\Big|_{(p,\hat{q}(p))} &= \\ &\frac{a^2\lambda_L\lambda_M}{\lambda_H} \cdot \frac{\hat{\Pi}(U_R)}{\Pi(p, \hat{q}(p))}\left(\frac{1}{2}\frac{\hat{U}_{pq} - \frac{\hat{U}_p}{\hat{U}_q}\hat{U}_{qq}}{\hat{U}_q + a\hat{U}_p} + \frac{1}{\Pi(p, \hat{q}(p))}\right) = \\ &\frac{a^2\lambda_L\lambda_M}{\lambda_H} \cdot \frac{\hat{\Pi}(U_R)}{\Pi(p, \hat{q}(p))^2} > 0.\end{aligned} \tag{A1.100}$$

The second derivative is positive along $\hat{q}(p)$, so instead of a local maximum the profit function attains a local minimum along $\hat{q}(p)$, what contradicts the equilibrium requirements. Hence the supposition is wrong and it should be that $\hat{q}(p) = q^*$ for $p \neq p_m$. But we also know that $(p_m, q_m) = (p_m, q^*)$ belongs to the equilibrium curve, so $\hat{q}(p) = q^*$ for all p. Consider $U(p, \hat{q}(p))$. We have $U(p, \hat{q}(p)) = b(\hat{q}(p) - q^*) - p = -p$. This implies that the equilibrium curve $\hat{q}(p) = q^*$ can not extend to the right of p_m, because $U(p, \hat{q}(p)) < U_R$ for $p > p_m = -U_R$. So, $[p_l, p_h] = [p_l, p_m]$. We next consider the utility distribution.

By definition, $\tilde{p}(u) = \hat{U}^{-1}(u)$. We have $\hat{U}(p) = U(p, \hat{q}(p)) = -p$. Hence, $\tilde{p}(u) = -u$. Then $\hat{\Pi}(u) = \Pi(\tilde{p}(u), \hat{q}(\tilde{p}(u))) = -u - aq^*$, and lemma 1.5 gives

$$F(u) = \frac{1}{2} \cdot \frac{\lambda_L}{1-\lambda_L} \left(\frac{\hat{\Pi}(U_R)}{\hat{\Pi}(u)} - 1 \right) = \frac{1}{2} \cdot \frac{\lambda_L}{1-\lambda_L} \left(\frac{U_R + aq^*}{u + aq^*} - 1 \right). \quad \text{(A1.101)}$$

Lemma 1.2 gives $U_l = U_R$. As for U_h, it is implicitly defined by $F(U_h) = 1$. Solving the equation gives:

$$U_h = \frac{\lambda_L(U_R + aq^*)}{2 - \lambda_L} - aq^*. \quad \text{(A1.102)}$$

$\hat{U}(p) = -p$, therefore $p_l = -U_h$. So, if there is an exact signalling equilibrium, then $F(u)$ is given by (A1.101), $U_l = U_R$, U_h is given by (A1.102), $\hat{q}(p) = q^*$, $p_l = -U_h$ and $p_h = -U_l = -U_R$. Hence, it is unique by construction. Given $\hat{q}(p)$ and given $F(u)$ we can write down an explicit expression for the profits:

$$\pi(p,q) = \left(F(U(p,q)) \cdot \lambda_H + F(U(p,\hat{q}(p))) \cdot \lambda_M + \frac{\lambda_L}{2} \right) \cdot \Pi(p,q) =$$
$$\frac{1}{2} \cdot \frac{\lambda_L(-U_R - aq^*)}{1-\lambda_L} \cdot (p - aq) \left(\frac{\lambda_H}{p - b(q-q^*) - aq^*} + \frac{\lambda_M}{p - aq^*} \right) \quad \text{(A1.103)}$$

for all (p,q) such that $p \in [p_l, p_h]$ and $U_l \leq U(p,q) \leq U_h$. Here $b = b_1$ for $q \leq q^*$ and $b = b_2$ for $q > q^*$. Along the equilibrium curve we have

$$\pi(p, \hat{q}(p)) = \frac{\lambda_L(-U_R - aq^*)}{2}. \quad \text{(A1.104)}$$

For there to be an equilibrium it should be that $\pi(p,q) \leq \pi(p, \hat{q}(p))$ for all $(p,q) \in \mathbb{R}^2$. Consider (p_0, q_0) with $p_0 = p_l$ and q_0 defined by $U(p_0, q_0) = U_R$. More explicitly,

$$(p_0, q_0) = \left(\frac{\lambda_L(-U_R - aq^*)}{2-\lambda_L} + aq^*,\ \frac{2 - 2\lambda_L}{2 - \lambda_L} \cdot \frac{U_R + aq^*}{b_1} + q^* \right). \quad \text{(A1.105)}$$

It should be that $\pi(p, \hat{q}(p)) \geq \pi(p_0, q_0)$. We have:

$$\pi(p, \hat{q}(p)) - \pi(p_0, q_0) = \frac{-U_R - aq^*}{b_1(2-\lambda_L)}(\lambda_H \lambda_L(b_1 - a) - a\lambda_M(2 - \lambda_L)). \quad \text{(A1.106)}$$

Therefore,

$$\pi(p, \hat{q}(p)) \geq \pi(p_0, q_0) \ \Leftrightarrow\ \frac{b_1 - a}{a} \geq \frac{\lambda_M}{\lambda_H} \cdot \frac{2 - \lambda_L}{\lambda_L}. \quad \text{(A1.107)}$$

So, we have shown that conditions (i)-(iii) are necessary conditions and we have shown that if there is an exact signalling equilibrium, then it is unique.

We next turn to sufficiency. Suppose conditions (i)-(iii) are satisfied. Consider an equilibrium candidate with $F(u)$, $[U_l, U_h]$, $\hat{q}(p)$ and $[p_l, p_h]$ as defined in the preceding paragraphs. The beliefs are consistent by construction, i.e. $\hat{U}(p) = U(p, \hat{q}(p))$ for all $p \in [p_l, p_h]$. What remains is to show that the profit function attains its maximum over the equilibrium curve. Define

$$S_C = \{(p, q) : p \in [p_l, p_h], U_R \leq U(p, q) \leq U_h\}. \tag{A1.108}$$

Consider $(p, q) \in S_C$. The profits $\pi(p, q)$ are then given by (A1.103). Consequently,

$$\pi(p, \hat{q}(p)) - \pi(p, q) = \frac{1}{2} \cdot \frac{\lambda_L(-U_R - aq^*)}{1 - \lambda_L} \left(\lambda_H \cdot \frac{(a-b)(q-q^*)}{p - b(q-q^*) - aq^*} + \lambda_M \cdot \frac{a(q-q^*)}{p - aq^*} \right). \tag{A1.109}$$

So, $\pi(p, \hat{q}(p)) \geq \pi(p, q)$ if and only if

$$\lambda_H \cdot \frac{(a-b)(q-q^*)}{p - b(q-q^*) - aq^*} \geq -\lambda_M \cdot \frac{a(q-q^*)}{p - aq^*}. \tag{A1.110}$$

From $U(p, q) \leq U_h$, from the definition of U_h, and from condition (i) it immediately follows that $p - b(q - q^*) - p > 0$. Also, from the definition of p_l as well as from condition (i) it follows that $p_l - aq^* > 0$. Therefore $p - aq^* \geq p_l - aq^* > 0$. Suppose $q > q^*$. Then $b = b_2 < a$ and (A1.110) is always satisfied (the LHS is positive, while the RHS is negative). If $q < q^*$, then $b = b_1 > a$ and inequality (A1.110) can be rewritten as

$$\frac{b_1 - a}{a} \geq \frac{\lambda_M}{\lambda_H} \left(1 + b \cdot \frac{q^* - q}{p - aq^*} \right). \tag{A1.111}$$

The RHS is strictly decreasing in p and q. Consider (p_0, q_0) as defined by (A1.105). Trivially, $(p_0, q_0) \in S_C$ and $(p, q) \geq (p_0, q_0)$ for all $(p, q) \in S_C$. Therefore

$$\frac{\lambda_M}{\lambda_H} \left(1 + b \cdot \frac{q^* - q}{p - aq^*} \right) \leq \frac{\lambda_M}{\lambda_H} \left(1 + b \cdot \frac{q^* - q_0}{p_0 - aq^*} \right) = \frac{\lambda_M}{\lambda_H} \cdot \frac{2 - \lambda_L}{\lambda_L}. \tag{A1.112}$$

From this inequality and from condition (iii) it immediately follows that inequality (A1.111) is satisfied. Consequently, the profit function attains its maximum along the equilibrium curve for all $(p, q) \in S_C$. It is straightforward to show that for $(p, q) \notin S_C$ the profits can not be greater than for $(p, q) \in S_C$. The reasoning is similar to that in theorem 1.3 and we do not repeat it here. □

Appendix to Chapter 2

Here we give a detailed derivation of (2.19), (2.20).

Let $z_i = x_i - \frac{1}{3}$. As $x_1 + x_2 + x_3 = 1$, so $z_1 + z_2 + z_3 = 0$. Next we derive $\pi_i(\hat{y}(z))$.

First,

$$\hat{y}_{ij}(x) = \frac{x_i + c(x_k - x_j)}{2} = \frac{z_i + c(z_k - z_j)}{2} + \frac{1}{6}. \tag{A2.1}$$

Then (using $\sum_i z_i = 0$ where appropriate)

$$\begin{aligned}\pi_i(\hat{y}(z)) = (a - b(\hat{y}_{ij} + \hat{y}_{ji}))\hat{y}_{ij} + (a - b(\hat{y}_{ik} + \hat{y}_{ki}))\hat{y}_{ik} = \\ \left(a - b\left(\frac{z_i + c(z_k - z_j)}{2} + \frac{z_j + c(z_k - z_i)}{2} + \frac{1}{3}\right)\right) \cdot \\ \left(\frac{z_i + c(z_k - z_j)}{2} + \frac{1}{6}\right) + \\ \left(a - b\left(\frac{z_i + c(z_j - z_k)}{2} + \frac{z_k + c(z_j - z_i)}{2} + \frac{1}{3}\right)\right) \cdot \\ \left(\frac{z_i + c(z_j - z_k)}{2} + \frac{1}{6}\right) = \\ \left(a - \frac{b}{3}\right)\left(z_i + \frac{1}{3}\right) - \frac{b(3c-1)}{2}\left(z_k\left(\frac{z_i + c(z_k - z_j)}{2} + \frac{1}{6}\right) + \right. \\ \left. z_j\left(\frac{z_i + c(z_j - z_k)}{2} + \frac{1}{6}\right)\right) = \\ \frac{b(3c-1)}{4}z_i^2 + \frac{12a + b(3c-5)}{12}z_i + \frac{3a-b}{9} - \frac{bc(3c-1)}{4}(z_k - z_j)^2. \end{aligned} \tag{A2.2}$$

Let $m = 3k(c+1)/2$, then $z_i(t) = z_i e^{mt}$. So,

$$\begin{aligned}V_i(z) = \int_0^\infty e^{-rt}\pi_i(\hat{y}(z(t)))dt = \\ \int_0^\infty e^{-rt}\left(\frac{b(3c-1)}{4}\left(z_i e^{mt}\right)^2 + \frac{12a + b(3c-5)}{12}z_i e^{mt} + \right. \\ \left. \frac{3a-b}{9} - \frac{bc(3c-1)}{4}\left(z_k e^{mt} - z_j e^{mt}\right)^2\right) dt = \\ \frac{b(3c-1)}{4}\frac{1}{r-2m}z_i^2 + \frac{12a + b(3c-5)}{12}\frac{1}{r-m}z_i + \\ \frac{3a-b}{9}\frac{1}{r} - \frac{bc(3c-1)}{4}\frac{1}{r-2m}(z_k - z_j)^2. \end{aligned} \tag{A2.3}$$

Plugging in $z_i = x_i - \frac{1}{3}$ and $m = 3k(c+1)/2$ gives precisely (2.19) and (2.20).

Appendix to Chapter 3

Here I show that if $a > 0$ and $a + |b| < 1$, then demand system (3.1) is rationalizable.

Suppose there are 5 goods: $x_1, \ldots, x_5$, where x_5 is a numeraire good. Suppose there is a representative consumer, whose income is w and whose demand is as follows:

$$\begin{aligned} x_1 &= 1 - \frac{p_1}{p_5} + a\frac{p_3}{p_5} + b\frac{p_2}{p_5}, \\ x_2 &= 1 - \frac{p_2}{p_5} + a\frac{p_4}{p_5} + b\frac{p_1}{p_5}, \\ x_3 &= 1 - \frac{p_3}{p_5} + a\frac{p_1}{p_5} + b\frac{p_4}{p_5}, \\ x_4 &= 1 - \frac{p_4}{p_5} + a\frac{p_2}{p_5} + b\frac{p_3}{p_5}, \\ x_5 &= \frac{1}{p_5}\left(w - \sum_{i=1}^{4} p_i x_i\right). \end{aligned} \tag{A3.1}$$

Let

$$\begin{aligned} &x_{11} = x_1, \quad x_{12} = x_2, \quad x_{21} = x_3, \quad x_{22} = x_4, \\ &p_{11} = p_1, \quad p_{12} = p_2, \quad p_{21} = p_3, \quad p_{22} = p_4. \end{aligned} \tag{A3.2}$$

Good x_5 is a numeraire good, hence $p_5 = 1$. Therefore, given definitions (A3.2), demand system (A3.1) implies demand system (3.1).

From (A3.1) it immediately follows that 1) Walras' law is satisfied, and 2) demand x(p,w) is homogeneous of degree zero. We next check whether the Slutsky matrix is symmetric and negative semi-definite.

Let S be the Slutsky matrix. By definition, its elements s_{lk} are given by

$$s_{lk} = \frac{\partial x_l}{\partial p_k} + \frac{\partial x_l}{\partial w} x_k. \tag{A3.3}$$

Given (A3.1) it is straightforward to check that $s_{lk} = s_{kl}$. As for the semi-definiteness, let Δ_k denote the k^{th} leading principal minor of matrix S (the determinant of the upper-left $k \times k$ submatrix of matrix S). Then we have

$$\begin{aligned}
\Delta_1 &= -\frac{1}{p_5}, \\
\Delta_2 &= \frac{1-b^2}{p_5^2}, \\
\Delta_3 &= -\frac{1-(b^2+a^2)}{p_5^3}, \\
\Delta_4 &= \frac{(1-(b+a)^2)(1-(b-a)^2)}{p_5^4}, \\
\Delta_5 &= 0.
\end{aligned} \tag{A3.4}$$

From $a > 0$ and $|b| + a < 1$ it follows that

$$\Delta_1 < 0, \quad \Delta_2 > 0, \quad \Delta_3 < 0, \quad \Delta_4 > 0. \tag{A3.5}$$

Therefore, by Sylvester's criterion, the Slutsky matrix S is negative semi-definite.

So, Walras' law is satisfied, the demand is homogeneous of degree zero and the Slutsky matrix is symmetric and negative semi-definite. Therefore, there exist preferences that rationalize demand system (A3.1), which in turn implies demand system (3.1).

Appendix to Chapter 4

Proposition 4.3.

$$\mathbb{E}\pi_i^{ts} \gtrless \mathbb{E}\pi_i^{rjv} \Leftrightarrow \frac{\alpha}{d} \lessgtr \begin{cases} \frac{1}{8} & \textit{if} \quad 0 < r \leq \frac{1}{4}, \\ f(r) & \textit{if} \quad \frac{1}{4} < r \leq \frac{\sqrt{13}-3}{2}, \\ 0 & \textit{if} \quad \frac{\sqrt{13}-3}{2} < r < 1 \end{cases} \tag{A4.1}$$

with

$$f(r) = \frac{r(1-3r-r^2) + (2+r)\sqrt{r(1-r)(1-3r-r^2)}}{4-r}. \tag{A4.2}$$

Proof. Let us first consider the case when $\frac{\alpha}{d} \geq \frac{r}{2}$. In figure 4.1 the corresponding region is the one above the dotted line. In this case, according to (4.10) and (4.17),

$$\mathbb{E}(\hat{\pi}_i^{ts}) = \frac{4\alpha^2 + 8\alpha rd + 9rd^2 - 5r^2d^2}{36b}, \tag{A4.3}$$

$$\mathbb{E}(\hat{\pi}_i^{rjv}) = \frac{\alpha^2 + 4\alpha rd + 2rd^2 - 2\alpha r^2 d - r^2d^2}{9b}. \tag{A4.4}$$

Therefore $\mathbb{E}(\hat{\pi}_i^{ts}) \gtreqless \mathbb{E}(\hat{\pi}_i^{rjv})$ if and only if

$$4\alpha^2 + 8\alpha rd + 9rd^2 - 5r^2d^2 \gtreqless 4\alpha^2 + 16\alpha rd + 8rd^2 - 8\alpha r^2 d - 4r^2d^2$$
$$\Leftrightarrow \quad (d - 8\alpha)(1 - r) \gtreqless 0 \quad \Leftrightarrow \quad \frac{\alpha}{d} \lesseqgtr \frac{1}{8}. \qquad \text{(A4.5)}$$

Hence, when $\frac{\alpha}{d} \geq \frac{r}{2}$, we have TS and RJV regions as depicted in figure 4.1. Notice that the line $\frac{\alpha}{d} = \frac{1}{8}$ intersects the line $\frac{\alpha}{d} = \frac{r}{2}$ at $r = \frac{1}{4}$.

Consider now the case when $\frac{\alpha}{d} < \frac{r}{2}$. The corresponding region lies below the dotted line in the figure. In this case $\mathbb{E}(\hat{\pi}_i^{rjv})$ is still give by (A4.4) and

$$\mathbb{E}(\hat{\pi}_i^{ts}) = \frac{r(\alpha + d)^2}{(r+2)^2 b}. \qquad \text{(A4.6)}$$

So, $\mathbb{E}(\hat{\pi}_i^{ts}) \gtreqless \mathbb{E}(\hat{\pi}_i^{rjv})$ if and only if

$$\frac{r(\alpha + d)^2}{(r+2)^2 b} \gtreqless \frac{\alpha^2 + 4\alpha rd + 2rd^2 - 2\alpha r^2 d - r^2d^2}{9b}. \qquad \text{(A4.7)}$$

Let $x = \frac{\alpha}{d}$. Using this substitution and rearranging the terms gives

$$\left(9r - (2+r)^2\right)x^2 + \left(9r - r(2-r)(2+r)^2\right)(2x+1) \gtreqless 0. \qquad \text{(A4.8)}$$

This a quadratic expression in x. The roots are

$$x_{1,2} = \frac{\mp r(1 - 3r - r^2) + (2+r)\sqrt{r(1-r)(1-3r-r^2)}}{4-r}. \qquad \text{(A4.9)}$$

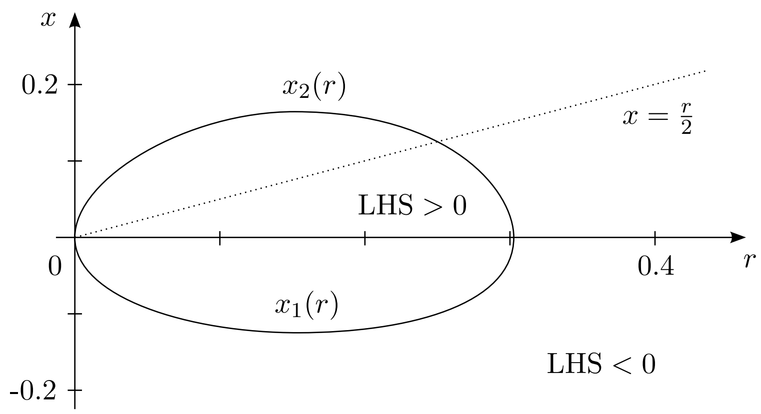
The coefficient at x^2 is always negative for $0 < r < 1$, hence the LHS of (A4.8) is positive if and only if $x_1 < x < x_2$. Figure A4.1 plots x_1 and x_2 as functions of r. For $r > \frac{\sqrt{13}-3}{2} \approx 3.03$ the roots are imaginary, so the LHS < 0. Otherwise the LHS is positive within the "circle" and negative outside. Of course, we are only interested in the region, where $0 < x < \frac{r}{2}$.

To combine this case with the earlier case, where $\frac{\alpha}{d} \geq \frac{r}{2}$, recollect that the boundary delimiting TS and RJV crossed the line $\frac{\alpha}{d} = \frac{r}{2}$ at point $\left(\frac{\alpha}{d} = \frac{1}{8}, r = \frac{1}{4}\right)$. It is straightforward to verify that this point belongs to $x_2(r)$, hence the boundary is continuous when crossing $\frac{\alpha}{d} = \frac{r}{2}$. Consequently, the regions for TS and RJV are as they are depicted in figure 4.1. Combining figure 4.1 with the above analytical expressions gives the proposition.

□

Proposition 4.5. $\mathbb{E}(CS^{rjv}) > \mathbb{E}(CS^{ts})$ *and* $\mathbb{E}(TW^{rjv}) > \mathbb{E}(TW^{ts})$ *for any* r *such that* $0 < r < 1$.

Figure A4.1: Contour plot for the LHS of (A4.8)



Proof. Suppose $\frac{\alpha}{d} \geq \frac{r}{2}$. In this case

$$\mathbb{E}(CS^{rjv}) - \mathbb{E}(CS^{ts}) = \frac{rd(1-r)(7d+16\alpha)}{36b} \tag{A4.10}$$

and

$$\mathbb{E}(TW^{rjv}) - \mathbb{E}(TW^{ts}) = \frac{rd(1-r)(5d+32\alpha)}{36b}. \tag{A4.11}$$

Clearly then, $\mathbb{E}(CS^{rjv}) > \mathbb{E}(CS^{ts})$ and $\mathbb{E}(TW^{rjv}) > \mathbb{E}(TW^{ts})$ given that $0 < r < 1$.

Suppose $\frac{\alpha}{d} < \frac{r}{2}$. In this case

$$\mathbb{E}(CS^{rjv}) - \mathbb{E}(CS^{ts}) = \frac{(1-r)(8\alpha^2 + 7\alpha^2 r + 14\alpha rd + 12\alpha r^2 d + 4\alpha r^3 d + 7rd^2 + 6r^2d^2 + 2r^3d^2)}{9b(r+2)^2} \tag{A4.12}$$

and

$$\mathbb{E}(TW^{rjv}) - \mathbb{E}(TW^{ts}) = \frac{(1-r)(16\alpha^2 + 5\alpha^2 r + 10\alpha rd + 24\alpha r^2 d + 8\alpha r^3 d + 5rd^2 + 12r^2d^2 + 4r^3d^2)}{9b(r+2)^2}. \tag{A4.13}$$

As before, these differences are clearly positive for $0 < r < 1$, so $\mathbb{E}(CS^{rjv}) > \mathbb{E}(CS^{ts})$ and $\mathbb{E}(TW^{rjv}) > \mathbb{E}(TW^{ts})$.

□

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