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Determining economic maintenance frequency of a transport fleet

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Goyal and Gunasekaran (1992) presented a model and an algorithm to determine the economic maintenance frequency of a transport fleet. The authors suggest that the algorithm is optimal, but is is not; it is often stuck in a local optimal solution. Even the example provided by the authors is not solved to optimality. We explain why the algorithm is not optimal and give the optimal solution of the example obtained by global optimization.

Notation

- m number of groups of vehicles
- S fixed cost incurred in each basic maintenance cycle
- a_i fixed operating cost per unit of time
- b_i increase in the operating cost per unit of time
- s_i fixed cost of maintenance for a vehicle
- n_i number of vehicles in the group
- X_i time required for maintenance work on the vehicle
- Y_i utilization factor of a vehicle on the road
- T basic maintenance cycle time
- k_i an integer which when multiplied by the basic maintenance cycle time T gives the maintenance cycle time of the vehicles in group i

Goyal and Gunasekaran (1992) presented the following model to determine the economic maintenance strategy of a transport fleet:

$$\min_{T,k_i} Z(T, k_i) = \frac{S}{T} + \sum_{i=1}^{m} n_i \frac{s_i + \int_0^{Y_i(Tk_i - X_i)} (a_i + b_i t)\,\mathrm{d}t}{Tk_i}$$

$$T \in \mathbb{R}^+ \quad \text{and} \quad k_i \in \{1, 2, 3, \dots\} \tag{1}$$

The authors present an algorithm to find the optimal solution to this problem. This algorithm is also given by Goyal and Kusy (1985), who developed a similar model. Since the function to be minimized is convex, they suggest that the algorithm will easily find the global optimum. The algorithm is based on two equations that are derived by setting the first derivative of $Z(T, k_i)$ with respect to the decision variables to zero:

$$T(k_1, k_2, \dots, k_m) = \left\{ 2\left[\frac{S \sum_{i=1}^{m} (n_i(s_i - X_i Y_i(a_i - \frac{1}{2} b_i X_i Y_i))/k_i)}{\sum_{i=1}^{m} n_i b_i k_i Y_i^2} \right] \right\}^{1/2} \tag{2}$$

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$$k_i(T) = \frac{1}{TY_i}\left[2\left(\frac{s_i - X_i Y_i(a_i - \frac{1}{2}b_i X_i Y_i)}{b_i}\right)\right]^{1/2} \tag{3}$$

The algorithm they use to solve the model is as follows.

Step 1. For the first iteration assume $k_i = k_i^{(0)} = 1$, for $i = 1, 2, \ldots, m$, and obtain the first estimate of $T = T^{(1)}$ from (2). At $T = T^{(1)}$ determine $k_i = k_i^{(1)}$ from (3), for $= 1, 2, \ldots, m$. If $k_i^{(1)}$ values are not integers, then select the nearest non-zero integer.

Step 2. At $k_i = k_i^{(1)}$, for $i = 1, 2, \ldots, m$, obtain $T = T^{(2)}$ from (2) and then $k_i = k_i^{(2)}$ from (3) using $T = T^{(2)}$. Repeat the process until the rth iteration and stop when $k_i^{(r)} = k_i^{(r-1)}$, for $i = 1, 2, \ldots, m$. The economic policy will be obtained at $T^* = T^{(r)}$ and $k_i^* = k_i^{(r)}$.

However, three problems now arise. Firstly, even though the function to be minimized is in fact convex, the state space is not convex, since the k_i need to be integers. This implies that the determination of the global optimum will not be as easy as the authors suggest.

Secondly, the authors round (3) to the nearest non-zero integer value. However, this is not necessarily the k_i minimizing Z. This is owing to the fact that the term of Z that contains k_i is equal to

$$z_i = Y_i(a_i - b_i X_i Y_i) + \frac{s_i - X_i Y_i(a_i - \frac{1}{2}b_i X_i Y_i)}{Tk_i} + \tfrac{1}{2}b_i Y_i^2 Tk_i \tag{4}$$

and this expression is not symmetric in k_i.

Thirdly, the algorithm will often stop after its first iteration without yielding an optimal solution. To see this, note first that the optimal T is an element of the following set:

$$\Phi = \{T \mid \exists\, k_i \in \{1, 2, 3, \ldots\}, i = 1, \ldots, m \text{ such that } T = T(k_1, k_2, \ldots, k_m)\} \tag{5}$$

Since, for given k_i, (2) gives the T that minimizes the objective function, and the set Φ contains the minimizing T for all k_i, the globally minimizing T is also an element of Φ. It can easily be seen from (2) that

$$\max \Phi = T(1, 1, \ldots, 1) = T^{(1)} \tag{6}$$

By taking $k_i^{(0)} = 1$ in the algorithm of Goyal and Gunasekaran, $T^{(1)}$ will become the largest T that can possibly be optimal.

Deviating k_i from 1 will cause the individual maintenance intervals to become quite large. This means that usually $k_i^{(1)}$ will also be 1, which implies that the algorithm will stop.

The three problems given above explain why the example given by Goyal and Gunasekaran was not solved optimally. In the Table the data for this example are given. The algorithm of Goyal and Gunasekaran stops after one step and gives a strategy of $T = 14{\cdot}988$ (the original paper erroneously mentions $1{\cdot}4988$) and $k_i = 1$, $i = 1, 2, \ldots, 5$, which yields an average cost of 8498·66. However, the strategy

Variable	Value
m	5
S	800
a_i	80, 50, 90, 85, 95
b_i	3, 2, 1, 1·5, 2·5
s_i	198, 192, 193, 205, 204
n_i	10, 24, 30, 16, 12
X_i	0·8, 0·6, 0·4, 0·6, 0·5
Y_i	0·9, 0·95, 0·85, 0·95, 0·94

Data for the example.

$T = 12{\cdot}78$ and $k = (1, 1, 2, 1, 1)$ yields an average cost of 8472·72. This strategy has been obtained by minimizing

$$C(T) = \min_{k_i} Z(T, k_i) \tag{7}$$

which is a non-convex function. However, local optima can be found with standard minimization algorithms. To find the global optimum, different starting values have to be tried. Since it is known that the global optimum will be between 0 and $T^{(1)}$, finding the global solution will, in practice, not be a problem.

References

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