Analysis of a 2-Phase Model for Optimization of Condition-Monitoring Intervals

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Summary & Conclusions — **Condition monitoring is a maintenance strategy where decisions are made depending on either continuously or regularly measured equipment states. It is often an efficient tool for cost-effective maintenance, since compared with time-based preventive maintenance, it reduces uncertainty with respect to actual states of equipment, and can thus avoid unnecessary repair or replacement. However, it involves capital expenditure and/or operational costs to perform measurements.**

This paper presents a basic model for the economic evaluation & optimization of the interval between successive condition measurements (also called inspections), where measurements are expensive and cannot be made continuously. It assumes that the technique can detect an intermediate state to failure for a failure mode of interest. The influence of competing risks is analyzed, leading to the conclusion that once the cost-effectiveness of the condition-monitoring has been established, competing risks need not be considered in determining the optimum condition monitoring interval. Inspection is cost-effective if the intermediate state has a: 1) non-decreasing hazard rate, and 2) shorter mean residence time than the good state (good-as-new condition), while costs of failure are high enough compared with inspection & repair costs in the intermediate state. Assuming that the distribution of the residence time in the second state is unimodal, estimation of the mean (or scale parameter) and standard deviation of this state, in many cases, provides enough information to make a good decision on the inspection interval. The most important model parameters are identified by sensitivity analyses; it is shown that the model can be simplified without seriously affecting optimal decision making.

1. INTRODUCTION

The main aim of inspections is to obtain useful information on the state of technical systems to be maintained. This paper focuses on regular inspections to check whether failures are impending, *eg*, vibration or oil-debris analysis. The value of inspections is twofold:

- less extensive repairs might be needed if a potential failure is detected before it creates follow-up damage,
- corrective actions might be deferred to a more opportune moment with less negative impact for the users of the system.

As such, inspections do not reduce the number of repairs. A good working inspection or condition monitoring scheme, however, does reduce the need to open and overhaul systems regularly, as this itself is sometimes a cause of failures. There is a tendency to apply condition monitoring techniques to more systems, and the question arises when condition monitoring in particular or inspection in general, is cost effective.

This paper analyzes a basic model for the economic evaluation & optimization of inspection techniques. The model assumes that for one type of failure mode a system passes through an intermediate state, which can be detected by the inspection. There might be other failure modes as well for which the inspection method cannot detect an intermediate state, the so-called competing risks. The definition & identification of the intermediate state depends on the inspection technique used, and indicates that a certain amount of deterioration has occurred. Usually, the mean time-to-failure from the intermediate state (phase-2) is much less than for a new system. Besides optimality, we focus on determining which model parameters (especially concerning phase-2) are essential for optimization. To this end we compare the optimum of the full model with optima of simplified versions of the model, in which some parameters have been left out or set at default values. In this way we list the key model-parameters. These results facilitate the parameter estimation problem.

This paper reviews only the immediately relevant literature. The PM-CD[1] can be considered as a variant of one in Mine & Kawai [4]; however, they: a) did not consider competing risks, and b) provided a criterion for existence of a finite optimal inspection strategy under a stronger assumption than we do. Our model is also similar to the delay-time model of Christer & Waller [1] in which the delay time corresponds to phase-2 in PM-CD and the competing risks are disregarded. Christer & Waller [2] applied their model with success, but did not present extensive sensitivity studies to determine which model parameters are essential (the main subject of this paper). The competing risks in PM-CD include all other failure modes which are not detectable by the condition monitoring technique and their occurrences lead to a renewal of the system. Hence preventive overhaul can be included in the competing risks. Valdez-Flores & Feldman [8] overview inspection models.

The 2-phase model is a special case of more general models in which a system passes through multiple states before failure, and in which state-dependent inspection policies are applied [6, 7]. Although for these general models the analysis & optimization results are not much more difficult, they are harder to apply in practice, because:

- data are needed to describe each phase, while implementation requires an on-line decision-support system;

[1] *Editors' note:* We have assigned this acronym PM-CD, for preventive maintenance - Coolen-Dekker (model) for simple, clear, unique reference to the concept.

- the intermediate states are usually not observable, unless inspection is carried out.

These models are appropriate, however, if inspection yields a metric result, so a measurable quantity (*eg*, wall thickness) which has a quantitative relationship with reliability is required. In general, it is difficult to obtain such a quantitative relationship. For example, in vibration analysis the inspection results are usually put into two or three categories (normal conditions, some deterioration, and substantial deterioration urging immediate shutdown; the latter can be considered as a failure). The distinction between these categories is also difficult to make. It is clear that the 2-phase model, PM-CD, is simpler than the multi-phase models and requires less data. A fixed optimum inspection interval is also easy to implement. Refs [1, 2] show data can be obtained for such an approach.

All needed proofs & derivations are in the appendix.

Notation

G	good state: as-good-as-new
B	bad (or intermediate) state: the system is still functioning, but degradation is observable upon inspection
D1	down state due to the failure mechanism through the Bad state
D2	down state due to the competing risks
t	critical inspection time: inspections are made at time t after the most recent inspection or failure-repair
L	renewal-cycle length, a r.v.
$T_{i,j}$	time to transition from state i to j ($i=G,B$; $j=B$,D1,D2), a r.v.
C_G	cost of inspection in state G
C_B	cost of inspection & repair in state B
C_{Dj}	cost of repair in state Dj, $j=1,2$
$\Phi(t)$	mean long-term total cost-rate
$\Phi_{2PH}(t)$	the part of $\Phi(t)$ resulting from the 2-phase failure mode for which condition monitoring is applied
Φ_{CR}	the part of $\Phi(t)$ resulting from competing risks
$C(t)$	mean renewal cost per cycle
$L(t)$	$E\{L\}$
t^*	Optimal critical inspection time
$P_i(t)$, P_i	Pr{a cycle ends in state i}
λ	transition rate from G to B, $\lambda > 0$
η	transition rate from G or B, to D2, $\eta \geq 0$
F, f, h	Cdf, pdf, hazard-rate of $T_{B,D1}$
RL	relative loss
t_j^*	optimal critical inspection-time based on a simplified model j.

Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

2. THE 2-PHASE MODEL WITH COMPETING RISKS

The 2-phase model is semi-Markov representing the life & deterioration of a system, as demonstrated by figure 1.

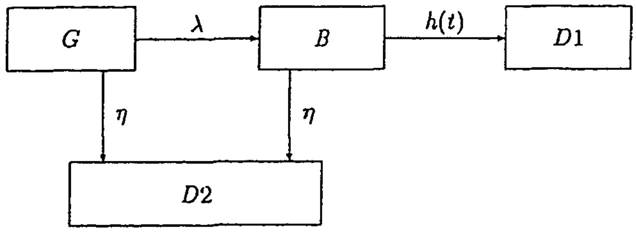

Figure 1. Schematic Representation of the Connections Between the Possible States

Assumptions

1. $T_{G,B}$ is exponentially distributed with constant transition rate λ.
2. $T_{G,D2}$ & $T_{B,D2}$ are both exponentially distributed with constant transition rate η.
3. $F(0)=0$, f is continuous, and $T_{B,D1}$ has finite 1st & 2nd moments.
4. $T_{G,B}$, $T_{G,D2}$, $T_{B,D2}$, $T_{B,D1}$ are mutually *s*-independent.
5. $C_{D1} > C_B > C_G > 0$.
6. States D1 & D2 reveal themselves immediately & correctly. Repair is to like-new (state G), immediately.
7. Inspection is the only way to determine whether the system-state is G or B.
8. Inspections & repairs are perfect, *eg*, instantaneous, correct, and harm nothing.

2.1 Analysis of Model

Our aim is to determine the critical inspection time that minimizes the mean total cost-rate.

From assumption 6, inspection or repair terminates a renewal cycle. From the renewal reward theorem [5]:

$$\Phi(t) = C(t)/L(t), \quad (1)$$

$$C(t) = P_G \cdot C_G + P_B \cdot C_B + P_{D1} \cdot C_{D1} + P_{D2} \cdot C_{D2}, \quad (2)$$

$$P_G = \exp[-(\lambda + \eta)\cdot t], \quad (3)$$

$$P_B = \exp(-\eta \cdot t)\cdot[1-\exp[-\lambda \cdot t]-F(t)] + (\lambda + \eta)\cdot I_1(t), \quad (4)$$

$$P_{D1} = I_2(t) - \lambda \cdot I_1(t), \quad (5)$$

$$P_{D2} = 1 - \exp(-\eta \cdot t)\cdot[1-F(t)] - \eta \cdot I_1(t) - I_2(t)$$
$$= 1 - P_G - P_B - P_{D1}, \quad (6)$$

(for justification see appendix),

$$I_1(t) \equiv \frac{\exp(-\eta \cdot t)}{\lambda + \eta} \cdot \int_0^t f(u)\cdot \exp(-\lambda \cdot (t-u))\, du, \quad (7)$$

$$I_2(t) \equiv \frac{\lambda}{\lambda+\eta}\cdot\int_0^t f(u)\cdot\exp(\{-\eta\cdot u)\ du. \qquad (8)$$

These equalities also hold for $\eta = 0$.

Lemma 1.

$$L(t) = P_{D2}/\eta, \text{ for } \eta > 0. \qquad (9)$$

$$L(t) = \int_0^t (1-F_{D1}(u))\ du, \text{ for } \eta = 0,$$

$$F_{D1}(u) \equiv \int_0^u F(u-y)\cdot\lambda\cdot\exp(-\lambda\cdot y)\ dy. \blacktriangleleft$$

Calculating $\Phi(t)$ needs specification of $f(\cdot)$ only from 0 to t.

$$\Phi(t) = \Phi_{2PH}(t) + \Phi_{CR}, \qquad (10)$$

$$\Phi_{2PH}(t) \equiv \eta\cdot(P_G\cdot C_G + P_B\cdot C_B + P_{D1}\cdot C_{D1})/P_{D2}, \qquad (11)$$

the cost resulting from the 2-phase part, and

$$\Phi_{CR} \equiv \eta\cdot C_{D2} \qquad (12)$$

the cost resulting from competing risk failures (independent of t).

$P_B/(P_B+P_{D1})$ indicates the relative number of bad states detected before a failure occurs in a cycle. Hence it can be used as an effectiveness measure of the condition monitoring technique; it depends on the competing-risk rate.

Without inspections ($t \to \infty$) a cycle can end only in states D1 & D2, with probabilities

$$P_{D1}(\infty) = I_2(\infty), \qquad (13)$$

$$P_{D2}(\infty) = 1 - I_2(\infty). \qquad (14)$$

The corresponding mean average costs are:

$$\Phi(\infty) = \Phi_{2PH}(\infty) + \Phi_{CR}, \qquad (15)$$

$$\Phi_{2PH}(\infty) = \eta\cdot I_2(\infty)\cdot C_{D1}/(1-I_2(\infty)). \qquad (16)$$

The C_{D2} do not influence the position of t^*, but η does. The optimization problem is similar to that of [4].

Theorem 1. If $C_{D1} > \gamma$, and if,

$$\Psi(\lambda) \equiv \int_0^\infty f(u)\cdot\exp(\lambda\cdot u)\ du < \infty, \qquad (17)$$

then $\Phi(t)$ has a minimum for a finite t;

$$\gamma \equiv \frac{1-I_2(\infty)}{\lambda/(\lambda+\eta) - I_2(\infty)}\cdot[C_G y + C_B\cdot(1-y)]$$

$$= \left[1 + \frac{\eta}{\lambda - (\lambda+\eta)\cdot I_2(\infty)}\right]\cdot[C_G y + C_B\cdot(1-y)], \qquad (18)$$

$$1/y \equiv \lim_{t\to\infty} [(\lambda+\eta)\cdot\exp[(\lambda+\eta)\cdot t]\cdot I_1(t)] = \Psi(\lambda). \blacktriangleleft(19)$$

Eq (17) is satisfied if $h(t)$ —

a. has a limiting value that exceeds λ; or
b. is non-decreasing and $E\{T_{B,D1}\} < 1/\lambda$.

Ref [4] shows that the criterion $C_{D1} > \gamma$ is necessary & sufficient for the existence of a finite optimum, if the hazard rate increases monotonically to infinity.

If the 2-phase failure mode is relatively rare compared with the competing risks (η/λ is large), then,

$$C_{D1}/[y\cdot C_G + (1-y)\cdot C_B]$$

should be large enough to justify inspection.

2.2 Example

A bearing wears out (leading to play in the bearing) and the wear can be measured by monitoring & analyzing its vibration. Table 1 gives the values of the model parameters, with costs C_G for a single inspection, C_B for a realignment (including inspection), and $C_{D1}=C_{D2}$ related to failure of the bearing (including all repair & downtime costs). Section 3 shows that the final result is rather insensitive to changes in these figures, and to changes in η; so these figures do not need to be very accurate.

TABLE 1
Model Parameters

C_G	= \$20
C_B	= \$120
C_{D1}	= \$2000
C_{D2}	= \$2000
λ	= 1/(30 months)
η	= 1/60
$E\{T_{B,D1}\}$	= 2 months
$StdDev\{T_{B,D1}\}$	= 1 month

Three distributions for $T_{B,D1}$ have been used (Weibull, Gamma, and truncated s-normal); all of them led to virtually the same result, *viz*, an optimal inspection interval of about 1.5 month. The long-run mean total cost-rate is about \$57/month. This value can be compared with the following two cases:

1. If neither preventive maintenance nor inspection is applied, then the mean total cost-rate is \$93.9/month (including \$33.3/month caused by competing risks).

2. If a preventive realignment is done at fixed intervals (adjusted for failures, so according to an age-replacement model), and not based upon vibration inspections, then minimal mean total cost-rate is \$93.5/month, for an optimal interval of about 5 months. Of these, \$21.4/month relates to preventive realignment, \$33.3/month to competing risks, and \$38.8/month to failures caused by the failure mode through the intermediate state. ◄

Condition monitoring can substantially reduce the mean total cost-rate. Approx 93.5% of the bad states occurring in a cycle are detected before they lead to failure; this number is approx 79.5% when fixed age-replacement intervals are applied. These comparisons include only operational costs. Deciding to introduce condition-monitoring also requires a balancing of the necessary equipment costs with the operational savings (\$200 for this failure mode).

Let the costs for failure of the bearing (\$2000 in table 1) are not know accurately, *eg*, it could be off by a factor of 2. Similarly C_B (\$120 in table 1) might not be known within a factor of 2. Table 2 shows the optimal critical inspection times for all combinations of these uncertain costs, using Cdf$\{T_{B,\mathrm{D2}}\}$ = weif$(t_{B,\mathrm{D2}}/2.25;\ 2)$. Table 2 also provides the mean cost-rate per month if the optimal inspection times are applied according to the assumed cost figures, but in the situation where the true, unknown, values of C_B, C_{D1}, C_{D2} are \$120, 2000, 2000 respectively. Table 2 shows that the loss caused by assuming wrong costs are within reasonable limits.

TABLE 2
Example, Sensitivity Results
[the body of the table gives t^* and (mean cost)]

	$C_{\mathrm{D1}}=C_{\mathrm{D2}}$		
C_B	1000	2000	4000
60	1.984 (59.25)	1.454 (57.77)	1.108 (59.11)
120	2.052 (59.62)	1.476 (57.76)	1.116 (59.07)
240	2.221 (60.59)	1.522 (57.78)	1.131 (58.97)

An algorithm to compute t^* has 2 parts: 1) evaluate $\Phi(t)$, and 2) optimize $\Phi(t)$. Evaluating $\Phi(t)$ requires calculating the integrals $I_1(t)$ & $I_2(t)$, which can be solved with usual integration routines, *eg*, Laguerre-integration. For optimization we used a search and 10-point-section procedure, which determined t^* with an accuracy up to 4 decimal places.

3. SENSITIVITY ANALYSES

The main aim of this paper is to assess the importance of model parameters, thus to check whether some parameters can harmlessly be set at default values without appre on optimality; thereby simplifying the paramet Recall that the entrance into the bad state is not c we only get censored information with respect

Sensitivity analyses were performed, compar of the complete model over a wide range of para The base values for the parameters, used in ste

$\lambda=1,\ \eta=1,$

$C_G=1,\ C_B=6,\ C_{\mathrm{D1}}=100,\ C_{\mathrm{D2}}=100,$

Weibull distribution with mean = 0.2 and shape pa (StdDev = 0.07) for $T_{B,\mathrm{D1}}$.

The justification for these parameter values ing. Inspection makes sense only when C_{D1} is Furthermore, C_B should be much less than C_{D1}, t C_G. The $T_{B,\mathrm{D1}}$ should also be substantially less t it should have an increasing failure rate, which is The other choices are more or less arbitrary.

3.1 Results From Sensitivity Analyses

1. t^* is very insensitive to changes in η, an in C_B.

2. An increase of E$\{T_{B,\mathrm{D1}}\}$ leads to a pro crease of t^*, while t^* increases slowly when the St decreases.

3. An increase of λ or of C_{D1}, both lead decrease of t^*.

Another important aspect is sensitivity towards tion of $T_{B,\mathrm{D1}}$. We compared four unimodal distri the same mean & standard deviation (Weibull, G cated *s*-normal, and lognormal), and found that proximately the same t^*. These results suggest th the mean and standard deviation of $T_{B,\mathrm{D1}}$ while fitt distribution, is reasonable. Only the first part of t a role. Hence not the mean of $T_{B,\mathrm{D1}}$ is important ing (which is directly related to the mean fo distributions).

3.2 Comparison of Models Through Sensitivity

We compare the complete 2-phase model models, all representing simplifications obtained ding certain aspects. The objective is to verify th timal results are still guaranteed with a min acquisition effort. The models are:

1. PM-CD complete model.
2. $T_{B,\mathrm{D1}}$ deterministic, with same mean.
3. $\eta=0$.
4. $\eta=0,\ C_{\mathrm{D1}}=100$.
5. $\eta=0,\ C_{\mathrm{D1}}=100$, $T_{B,\mathrm{D1}}$ exponential, with
6. $\eta=0,\ C_{\mathrm{D1}}=100$, $T_{B,\mathrm{D1}}$ exponential, $\lambda=1/$

Model 2 was incorporated, because its $t_2^* = \mathrm{E}\{T_{B,\mathrm{D1}}\}$; so a comparison of model 2 with model 1 gives insight into the performance of using $\mathrm{E}\{T_{B,\mathrm{D1}}\}$ as approximation for t^*. Models 3 - 6 avoid successively, competing risks, estimation of failure costs, estimation of the distribution of $T_{B,\mathrm{D1}}$, and rate-of-occurrence of the bad state.

Several values for each parameter are considered.

Assumption

The final results hold between the extreme parameter values for input data. ◄

The data are:

$C_G = 1$ (scaling);

$C_B = 1, 2, 6, 10$;

$C_{\mathrm{D1}} = 2, 6, 10, 50, 100, 1000$, with $C_{\mathrm{D1}} > C_B$;

$C_{\mathrm{D2}} = C_{\mathrm{D1}}$;

$\mathrm{E}\{T_{B,\mathrm{D1}}\} = 1$ (scaling);

$\mathrm{StdDev}\{T_{B,\mathrm{D1}}\} = 0.25, 0.5, 1$;

$T_{B,\mathrm{D1}}$ has Weibull distribution (in the complete model);

$\lambda = 1, 0.5, 0.2, 0.1, 0.04, 0.01, 0.001$ (per month);

$\eta = 1, 0.2, 0.1, 0.01, 0.001$.

There are 1890 combinations for parameter values. For comparison, we calculated the relative loss:

$\mathrm{RL} = [\Phi(t_j^*) - \Phi(t^*)]/\Phi(t^*)$,

t_j^* = optimal strategy for model $j = 2,\ldots,6$.

It is obvious that RL is non-negative, because t^* is the point where $\Phi(\cdot)$ is minimal. Table 3 shows the maximum, mean, and standard deviation of the RL values per model. These values are skewed to the right.

TABLE 3
Maximum, Mean, StdDev of RL for Alternative Models

model	max RL	mean RL	StdDev RL
2	17.74	0.303	1.14
3	0.02164	0.000151	0.000923
4	13.45	0.359	1.05
5	15.43	0.745	1.51
6	459.8	3.16	21.9

The main conclusions from this comparison are:

- Model 3 instead of model 1 leads to negligible loss.
- Models 2, 4-6 yield worse results. Therefore, it is not possible, within the range of parameter values considered, to avoid estimating C_{D1}, $\mathrm{StdDev}\{T_{B,\mathrm{D1}}\}$, λ. The necessary accuracy of these estimates can be determined by analyzing the sensitivity for a desirable application.

Model 6 yields the maximum RL values for cases where there is no finite maximum RL value for the full model. This occurs when,

$\lambda = \eta = 0.001$,

$C_B = C_G = 1,\ C_{\mathrm{D1}} = C_{\mathrm{D2}} = 2$,

$\mathrm{StdDev}\{T_{B,\mathrm{D1}}\} = 1$ (with almost equal results for $\mathrm{StdDev}\{T_{B,\mathrm{D1}}\} = 0.25$ or 0.5). This case leads to $t^* \to \infty$ (no inspection at all) based on the full model as well as models 2 - 5, while model 6 yields $t^* = 0.54$. As the mean time to occurrence of state B is 1000, far too many inspections are carried out in model 6. This example also shows, that although a failure mode is preventable, inspection still might not be cost-effective, because of the low failure frequency.

Table 4 compares t^* and $\mathrm{E}\{T_{B,\mathrm{D1}}\}$. Out of the 1890 combinations, 777 lead to $t^* \to \infty$; thus inspection is not cost-effective. In most of the remaining 1113 cases, t^* is smaller than $\mathrm{E}\{T_{B,\mathrm{D1}}\}$. Table 4 shows, for these 1113 cases, the distribution of the ratio $t^*/\mathrm{E}\{T_{B,\mathrm{D1}}\}$ over several intervals.

TABLE 4
Distribution of $t^*/\mathrm{E}\{T_{B,\mathrm{D1}}\}$

interval	number
0.0 - 0.2	140
0.2 - 0.4	231
0.4 - 0.6	207
0.6 - 0.8	202
0.8 - 1.0	145
1.0 - 2.0	165
2.0 - 5.0	22
5.0 - ∞	1

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APPENDIX

A.1 Derivation of (3) - (6)

Eq (3) - (6) are, by elementary calculus (convolution-integrals and partial integration), derived from:

$$P_G = Pr\{T_{G,B} > t \wedge T_{G,D1} > t\},$$

$$P_B = \Pr\{T_{G,B} \le t \wedge T_{G,D2} > T_{G,B} \wedge T_{G,B}+T_{B,D1} > t \wedge T_{G,B}+T_{B,D2} > t\},$$

$$P_{D1} = \Pr\{T_{G,B}+T_{B,D1} \le t \wedge T_{G,D2} > T_{G,B} \wedge T_{G,B}+T_{B,D2} > T_{G,B}+T_{B,D1}\},$$

$$P_{D2} = \Pr\{[T_{G,D2} \le t \wedge T_{G,B} > T_{G,D2}] \vee [T_{G,B}+T_{B,D2} \le t \wedge T_{G,D2} > T_{G,B} \wedge T_{G,B}+T_{B,D1} > T_{G,B}+T_{B,D2}]\}.$$

A.2 Proof of Lemma 1

The length of a renewal cycle is t unless it stops prematurely by arrival in states D1 or D2. Let F_{D1} & F_{D2} denote the Cdf's of the times-to-arrival in states D1 & D2 respectively, then $L(t) = \mathrm{E}\{\min(l,t)\}$ where l represents the lifetime without inspections, so $\mathrm{Sf}\{l\} = (1-F_{D1})\cdot(1-F_{D2})$. This implies that $L(t)$ is the mean life restricted to t, leading to [3: page 97]:

$$L(t) = \int_0^t (1-F_{D1}(u))\cdot(1-F_{D2}(u))\, du,$$

$$F_{D1}(u) \equiv \int_0^u F(u-y)\cdot\lambda\cdot\exp(-\lambda\cdot y)\, dy,$$

$$F_{D2}(u) \equiv 1-\exp(-\eta\cdot u).$$

Straightforward calculation leads to lemma 1. *Q.E.D*

A.3 Proof of Theorem 1

A sufficient condition for the cost-effectiveness of the condition monitoring technique, or in other words, for the existence of a finite t with $\Phi(t) < \Phi(\infty)$ is obtained by considering the sign of $\Phi'(t)$ as $t \to \infty$.

For simplicity we define,

$$G(t) \equiv \frac{P_{D1}(\infty)\cdot C_{D1}\cdot\Phi_{2PH}(t)}{P_{D2}(\infty)\cdot\Phi_{2PH}(\infty)} = \frac{1}{P_{D2}}[P_G\cdot C_G + P_B\cdot C_B + P_{D1}\cdot C_{D1}];$$

so $G(t) = c\cdot\Phi_{2PH}(t)$ with c a positive constant, and $G(t)$ & $\Phi(t)$ behave similarly as $t \to \infty$. The following results are straightforward:

$$\lim_{t\to 0}[G(t)] = \infty,$$

$$\lim_{t\to 0}[G'(t)] = -\infty,$$

$$\lim_{t\to\infty}[G(t)] = P_{D1}(\infty)\cdot C_{D1}/P_{D2}(\infty),$$

$$\lim_{t\to\infty}[G'(t)] = 0.$$

A sufficient condition for the existence of a finite t^* is obtained if $G'(t)\to 0$ as $t\to\infty$. Taking the derivative of the terms of $G(t)$ yields

$$P'_G = -(\lambda+\eta)\cdot\exp(-(\lambda+\eta)\cdot t),$$

$$P'_B = (\lambda+\eta)\cdot\exp(-(\lambda+\eta)\cdot t) - \eta\cdot\exp(-\eta\cdot t)\cdot(1-F(t)) - (\lambda+\eta)^2\cdot I_1(t),$$

$$P'_{D1} = \lambda\cdot(\lambda+\eta)\cdot I_1(t),$$

$$I_1'(t) = \frac{\exp(-\eta\cdot t)}{\lambda+\eta}f(t) - (\lambda+\eta)\cdot I_1(t),$$

$$I_2'(t) = \frac{\lambda}{\lambda+\eta}\exp(-\eta\cdot t)\cdot f(t).$$

After some calculus we arrive at: $G'(t) > 0$ for $t\to\infty$ (with $G'(½)=0$) iff

$$C_{D1}\cdot[B\cdot(\lambda - (\lambda+\eta)\cdot I_2(\infty)) - \eta\cdot A\cdot I_2(\infty)] > [1-I_2(\infty)]\cdot[C_G\cdot(\lambda+\eta) + C_B\cdot[\eta\cdot A + (B-1)\cdot(\lambda+\eta)]]$$

$$A \equiv \lim_{t\to\infty}[\exp(\lambda\cdot t)\cdot(1-F(t))],$$

$$B \equiv \lim_{t\to\infty}[(\lambda+\eta)\cdot\exp((\lambda+\eta)\cdot t)\cdot I_1(t)] = \int_0^\infty f(u)\cdot\exp(\lambda\cdot u)\, du.$$

$B < \infty$ implies that $A=0$. *Q.E.D.*

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