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Behavioral Finance and Agent-Based Artificial Markets
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Behavioral Finance and gesimuleerde financiële markten

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MILAN LOVRIĆ

born in Zagreb, Croatia.
To My Family
The research reported in this thesis has been carried out in cooperation with SIKS, the Dutch Research School for Information and Knowledge Systems.
Preface

This thesis presents the work performed during my PhD studies. It is a result of a very exciting and stimulating period of my life that has surpassed my expectations in many different ways and markedly influenced how I reflect on the world and myself. Any PhD thesis is more than just a collection of research papers. To more practical students it might seem as a little brick that contributes to the world’s temple of scientific knowledge. On the other hand, to the more introspective ones it may seem as a mirror showing their strengths and weaknesses as researchers. Finally, the poetic ones may even consider it an ode to the eternal quest of the humankind to tackle the unknown unknowns. Irrespective of these personal attitudes, most of us would probably agree that a PhD thesis is above all a testament to the teamwork, a textbook that condenses the effort and support of a multitude of people. Here I would like to extend my gratitude to everyone who has been part of my PhD project in such a way.

First and foremost, I would like to thank my promoters, Jaap Spronk and Uzay Kaymak, for their guidance, focus, and patience throughout all these years. It has been a great pleasure working with Jaap, who is not only a person with a broad international experience, but also a person with a great charisma and an inspiring personality. His deep understanding and intuition in the field of finance has proven invaluable for this project. I would also like to thank my second promoter, Uzay, who has been a great role model and has had a big influence on the development of my research as well as teaching skills. I thank him for always having his door open to me. Also, I admire his ability to find the most pragmatic solutions to every challenge. I would also like to thank Jan van den Berg. While I have not had a chance to work with him, he indeed has to be credited for having initiated my PhD project. Thank you for believing in me and for giving me the opportunity to take part in such an exciting research area. I feel especially honored to have an inner Doctoral Committee composed of such distinguished researchers as Marno Verbeek, Dick van Dijk, and Sorin Solomon, and I appreciate their honest feedback which resulted in much improved thesis.

ERIM PhD programme is renown for its excellent and extensive support throughout the whole PhD trajectory. I would like to express my appreciation to all the members of the ERIM team who have supported me since the very first day. I would also like to thank the staff of the Tinbergen Institute, the Department of Business Economics, and especially the
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The ninth floor of the H-building (Erasmus School of Economics) has been my second home for the longest part of my PhD study. I believe it was this mixture of PhD students from all over the world and from various research fields that created such a unique research environment. I am grateful for having had the privilege to meet each and every one of these bright young people, and I wish them all the success in their careers, whether it be in academia or industry. I have to give special thanks to Merel and Rene for stimulating our interactions even more by organizing many gezellige evenings. My special appreciation goes as well to Diana and Nuno, with whom I shared the sunniest office on the floor for quite some time. Thank you for your true kindness and support. I am also thankful to Nalan, Rui, and Viorel for all our amusing and invigorating lunches.

This period of my life has proven to me that it is never too late to make new friends, especially those who are destined to stay with you forever. I would like to thank Ana, Igor, and Mateja for bringing a dash of home into Rotterdam as well as creating many memorable experiences in different corners of the world. Many thanks to my former roommates Eva, Kirsten, Bekim, and Yair for providing me with my very first Dutch home. Ana, Nikolina, Mila, and Božo, thank you for keeping me up to date with life in Zagreb and for always being there for me.

Finally, I am indebted to my mother, brother, late father, and other family members for their unconditional love and support. This little brick, mirror, ode and testament is dedicated to you.

Milan Lovrić
Rotterdam, February, 2011
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Chapter 1

Introduction

1.1 Motivation

This dissertation analyzes market-wise implications of behavioral finance by means of agent-based simulations of financial markets. The goal is to explore agent-based artificial markets as a tool for studying various behavioral biases of individual investors. We aim to show that such models can contribute to behavioral finance research by providing the link between micro and macro behavior that is often lacking in other research methodologies applied to behavioral finance. In addition, this thesis aims at bringing the topics of behavioral finance closer to the modeling community as well as at highlighting the challenges in this field.

Economics and financial theories have for long been dominated by the Efficient Markets Hypothesis (EMH), which posits that market prices fully reflect all available information (Fama, 1970). Efficient markets do not allow investors to earn above-average returns without accepting above-average risks (Malkiel, 2003). "It has been applied extensively to theoretical models and empirical studies of financial securities prices, generating considerable controversy as well as fundamental insights into the price-discovery process" (Lo, 2007). Financial theories and models have also been rooted in the representative agent framework, that rests on a formal representation of an individual who acts as a utility maximizer, given his preferences and constraints, and adheres to the axioms of rational choice theory. Over the past decades, however, psychologists and behavioral scientists have documented robust and systematic violations of principles of expected utility theory, Bayesian learning, and rational expectations - questioning their validity as a descriptive theory of decision making (De Bondt, 1998). Furthermore, Herbert Simon has emphasized the importance of bounded rationality, taking into account the limited ability of humans to adapt optimally, or even satisfactorily, to complex environments (Simon, 1991).
Chapter 1. Introduction

The idea of individual investors who are prone to biases in judgment, are frame-dependent, and use various heuristics, which might lead to anomalies on the market level, has been explored within the field of behavioral finance. Behavioral finance builds itself upon two pillars: (i) limits to arbitrage and (ii) psychology (Barberis and Thaler, 2003). While psychology lists a number of possible deviations from rationality, the limits to arbitrage argue that rational investors may not be able to exploit opportunities created by irrational investors. If irrational investors (noise traders) create dislocations in asset prices (departures from fundamental values), rational investors (arbitrageurs) should be able to correct this mispricing through the process of arbitrage. However, it has been recognized in literature (Barberis and Thaler, 2003), that arbitrage strategies in real financial markets (as opposed to the strict definition of arbitrage) can involve cost, risk, or various constraints, so the inefficiencies may persist for a long period of time.

Artificial financial markets are models for studying the link between individual investor behavior and financial market dynamics. They are often computational models of financial markets, and are usually comprised of a number of heterogeneous and boundedly rational agents, which interact through some trading mechanism, while possibly learning and evolving. These models are built for the purpose of studying agents’ behavior, price discovery mechanisms, the influence of market microstructure, or the reproduction of the stylized facts of real-world financial time-series (e.g. fat tails of return distributions and volatility clustering). A similar bottom-up approach has been utilized in agent-based computational economics (ACE) - the computational study of economies modeled as evolving systems of autonomous interacting agents (Tesfatsion, 2006). A methodology analogous to agent-based modeling has also been used in the physical sciences, for example the Microscopic Simulation - a tool for studying complex systems by simulating many interacting microscopic elements (Levy et al., 2000).

Since agent-based models can easily accommodate complex learning behavior, asymmetric information, heterogeneous preferences, and ad hoc heuristics (Chan et al., 1999), such simulations are particularly suitable to test and generate various behavioral hypotheses. Hence, the behavioral finance and agent-based computational economics can be considered to be complementary. This complementarity of behavioral finance research and the agent-based methodology has been recognized in the literature: "It is important to note that agent-based technologies are well suited for testing behavioral theories. They can answer two key questions that should be asked of any behavioral structure. First, how well do behavioral biases hold up under aggregation, and second which types of biases will survive in a coevolutionary struggle against others. Therefore, the connections between agent-based approaches and behavioral approaches will probably become more intertwined as both fields progress" (LeBaron, 2006).
1.2. Research Objective and Research Questions

So far, research in the combination of behavioral finance and agent-based methods has been sporadic. Some of the early studies that pursue the idea of explicit accounting for behavioral theories in agent-based financial market simulations are Takahashi and Terano (2003) and Hoffmann et al. (2007). In Takahashi and Terano (2003) the focus is on overconfidence and loss aversion, while Hoffmann et al. (2007) study social interaction between investors. Therefore, the confluence of behavioral finance and agent-based methods represents a nascent research area with a multitude of open questions and research opportunities.

1.2 Research Objective and Research Questions

In this thesis, we address the gap between behavioral finance and agent-based models of financial markets. We study various biases commented in the behavioral finance literature and propose novel models for some of the biases from the literature. Overall, the research objective is defined as follows.

Research Objective:

Contribute to the research areas of behavioral finance and agent-based artificial markets by providing mathematical representations for a number of behavioral phenomena and by studying their market-wise implications using agent-based simulations.

We consider four research questions for achieving the research objective.

Research Question 1

What are the relevant aspects of investor behavior that could be studied using an agent-based simulation approach?

Studying behavior of market participants is important because of its potential impact on asset prices and the dynamics of financial markets. Agent-based market models have been proposed as a tool for studying such impact of individual investor behavior. However, in order to build bottom-up models of financial markets we need to have an overview of the relevant aspects of investor behavior, particularly those studied in the behavioral finance literature.

Research Question 2

How can we implement various behavioral phenomena such as overconfidence, sentiment (optimism, pessimism), loss aversion, biased self-attribution, and recency and primacy effects, within existing agent-based artificial stock markets?
Behavioral finance approach is interesting for us as it focuses on realistic elements of the behavior of market participants, as opposed to normative financial theories that rely on very strong assumptions of investor rationality, which have been empirically questioned or disproved. Hence a lot of attention in this thesis has been given to various investor heuristics and psychological biases that constitute departures from such rational behavior. The challenging aspect of studying behavioral phenomena by means of agent-based simulations is how to devise their mathematical definitions and computational implementations. In this thesis we aim to provide implementations for a number of behavioral phenomena that have been proposed and studied in the behavioral finance literature.

Research Question 3

What is the influence of implemented behavioral phenomena on market dynamics and investor performance? Conversely, what is the influence of market dynamics or investor performance on investor behavior/psychology?

One of the most challenging questions in behavioral finance literature is how and to what extent individual behavior influences the market dynamics. Particularly, whether various investor irrationalities can cause anomalous market behavior. In addition, it is important to understand if heuristic and biased behaviors are detrimental to the investor performance, and if such behavioral phenomena can emerge or disappear over time, especially after taking into account feedback from the market and the success of investment strategies.

Research Question 4

Which models can be developed to study different definitions of the same behavioral phenomena, such as the overconfidence?

Another difficulty in studying the implications of behavioral biases is that their definitions and implementations can vary, which could lead to opposing results and different implications. In this thesis we study two distinct manifestations of investor overconfidence, miscalibration and better-than-average effect, and compare the consequences of those behavioral biases.

1.3 Research Methodology

To address the first research question we use literature review. By reviewing behavioral finance literature, including existing surveys of behavioral finance, we aim to identify important behavioral aspects of investment decisions, paying special attention to the behavioral biases of individual investors. The main findings are presented in a conceptual model, which provides an overall structure to consider specific im-
implementations of agent-based models for studying the influence of behavioral biases of investors.

To motivate the use of agent-based simulations for the rest of our research questions, we briefly discuss some alternative methodologies that can be used for studying the research topics of behavioral finance. Traditionally, behavioral finance had relied mostly on experiments, and there have been many studies in which subjects participated in various investment tasks. Furthermore, most of the behavioral biases and heuristics originate from experiments conducted by cognitive psychologists. One of the common critiques of these experimental studies is that they are often conducted only with student participants, rather than actual traders and investors. This critique is not always valid, since robust findings obtained with student participants can often be generalized to a broader population. One example are experiments with myopic loss aversion, in which professional traders exhibited this bias even to a greater extent than students (Haigh and List, 2005). Another disadvantage of experiments is that participants face such financial outcomes (almost exclusively gains), which cannot be compared to gains and losses that investors face in real financial markets. It is considered that shifting from laboratory to field experiments, where possible, would increase external and ecological validity.

A different methodology used in (behavioral) finance is the quantitative analysis of actual trading records. These databases are very difficult to obtain, but to a researcher who has acquired such a database they provide a compelling argument of working with actual trades made by investors. However, even though the performance of individual investors can be determined from the data, the explanatory variables used in such studies often do not go beyond simple demographics, such as age, gender, and education. However, we would like a richer description of investors, possibly about their strategies, psychological profiles, propensity to behavioral biases etc. This could be explored by using questionnaires with market participants to proxy for hardly observable psychological traits. Unfortunately, such information is difficult to obtain from real investors and traders, even if decoupled from the database of their actual trades.

Whereas experiments are good in capturing behavior in controlled environments, they might lack generalizability to real financial markets. Would similar behavior occur in financial markets, how would it affect the performance of individual investors, and how would it aggregate to the market level? Quantitative analysis of financial data may answer some of these questions, but it is not always clear what is the

\[\text{1Dorn and Huberman (2005) is an example of such a study, where survey responses are combined with trading records in order to give a better understanding of why investors fail to buy and hold a well diversified portfolio. In this study stated perceptions and self-assessments are used to develop proxies for psychological traits such as risk attitude and overconfidence.}
\[\text{2Given that most biases shown in these experiments come from hypothetical static risky choice problems, a great caution should be exercised when introducing such results into highly dynamic financial models (Chen and Liao, 2003).}
\]
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exact behavior of market participants that lead to it. Agent-based artificial financial markets are potentially able to fully bridge this gap between individual investor behavior and aggregate market phenomena, by allowing the modeler to specify the behavior of market participants, to implement various market mechanisms, and to analyze the resulting asset prices. In such a way, artificial financial markets could be used as a tool to generate and/or test various behavioral hypothesis and theories, to study scenarios for which empirical data do not exist, or are difficult to obtain.

In this thesis we develop and present models of various behavioral biases and study them within an existing agent-based financial market. We employ the incremental approach according to which an existing computational model is first replicated and then a new investor behavior, in our case a behavioral bias, is introduced into the model. By comparing the results of the original model with the results of the incremented model, we can study the implications of the newly introduced (biased) behaviors. The incremental approach is built upon the choice of an artificial market model that we selected to use in our studies. We have opted for an agent-based artificial financial model developed by Levy et al. (2000), also known as the LLS model.

Although there are many agent-based models of financial markets, at the moment there is no model which would be considered standard or the best. The reason for this is that agent-based financial markets include a wide variety of models, all of which have their own strengths and weaknesses, and which differ in many aspects, such as the representation of investor behavior and the market rules including the price generation mechanism. While some models may be more realistic in terms of the prices that they can reproduce, others may have better behavioral foundation of their agents, be more related to the theoretical concepts of the standard financial theory, or have more analytical tractability. An overview of some of the well-known agent-based models of financial markets is presented in Chapter 4.

The advantage of the LLS model is that it is a well-studied model with well-known results and properties. This enables us to make the comparison between the results of the original model and the results obtained after having included various behavioral biases. In addition, one investor type in the LLS model, the so-called Efficient Market Believers, uses an ex-post distribution of stock returns to predict future returns, which allowed us to introduce various behavioral biases by modeling their impact on the shape of this distribution. Another practical reason for choosing the LLS model is related to the issue of replicability, which often receives little attention in agent-based models. Since there are many implementation choices in computational studies, if left unreported, they can render replication of a study very difficult or impossible. LLS model has in that sense been described quite meticulously in Levy et al. (2000), which facilitated our replication of the original study.
1.4 Outline of the Thesis

The structure of the thesis is as follows. Chapter 2 presents a survey of behavioral finance literature with special emphasis on psychological biases of individual investors. In Chapter 3 we present a conceptual model of individual investor, which purpose is to give a structured representation of elements of investor behavior that could be interesting for implementation into an agent-based financial market. Chapter 4 gives an overview of some of the well-known artificial financial market models including a few agent-based studies which have focused explicitly on behavioral finance topics. In addition, the chapter discusses the modeling aspects of artificial financial markets and their statistical properties. Chapter 5 is a study of investor overconfidence within the financial market of Levy, Levy, and Solomon (2000; hereafter the LLS model), and Chapter 6 is a study of investor sentiment within the LLS model. Chapter 7 presents a general model of investor confidence and sentiment, and studies the influence of recency and primacy effects. Chapter 8 contains a study of a self-attribution bias and loss-aversion within the LLS model. Chapter 9 is a study of better-than-average overconfidence in the SSE model. Chapter 10 contains the summary and the conclusion of the thesis.

In addition to reading this thesis sequentially, it is possible to read the chapters independently. However, some chapters are best read in a sequence due to their dependencies (see Figure 1.1). For example, the conceptual model of Chapter 3 has been developed based on the insights from the literature review presented in Chapter 2, and Chapter 4 gives an overview of agent-based artificial markets by discussing their stylized representations of the elements of the conceptual model presented in Chapter 3. Chapter 5 through Chapter 8 are all based on the LLS model, so each of them starts with a brief introduction to the original model. Nonetheless, we suggest that a reader starts with Chapter 5 because it contains a more detailed description of the LLS model, including an appendix with implementation details. Furthermore, Chapter 7 presents a general sentiment-confidence model of investor behavior, which is based on Chapter 5 and Chapter 6. Chapter 8 is an extension of

Figure 1.1: Guideline for reading the chapters of this thesis.
the model presented in Chapter 7. A study of better-than-average overconfidence in Chapter 9 compares the results of the SSE model with the LLS model and also with the study of overconfidence developed in Chapter 5.
Chapter 2

Behavioral Biases in Investor Decision Making*

2.1 Introduction

This chapter focuses on individual investor decision making. It particularly draws on the heuristics and biases strand of the behavioral finance literature. By taking a descriptive point of view, we are mostly interested in how investors make their investment decisions in a real world setting, as opposed to rational/optimal behavior proposed by normative financial theories. This chapter incorporates results of the research on investment decisions from fields of behavioral finance and cognitive psychology, and is based on a review of existing studies, which themselves were conducted using various research methodologies.

In the rapidly growing field of behavioral finance it becomes more difficult to devise a unifying taxonomy of all the behavioral phenomena, as they arise from various mechanisms, manifest on different levels of behavior and cognition, and have been discovered using various methodologies (e.g. experiments vs. quantitative analysis of market data). Nonetheless, a unifying taxonomy that captures the majority of these biases would be useful from both a theoretical and a practical point of view. In this chapter we survey behavioral finance literature and present taxonomies proposed by different authors.

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2.2 Behavioral Finance

A research into individual investors and their behavior has received a lot of consideration during the past, and is increasingly in the focus of interest of many scientists, not being confined only to economists. However, the particular way of looking at individual investor has been subject to a great paradigmatic shift with the inclusion of the findings and the methodology of psychology into the financial studies\(^1\). Despite many ongoing debates, this has slowly led to the establishment of behavioral economics and behavioral finance as widely recognized subdisciplines.

"Behavioral finance, as a part of behavioral economics, is that branch of finance that, with the help of theories from other behavioral sciences, particularly psychology and sociology, tries to discover and explain phenomena inconsistent with the paradigm of the expected utility of wealth and narrowly defined rational behavior. Behavioral economics is mostly experimental, using research methods that are rarely applied in the traditional, mainstream finance literature." (Frankfurter and McGoun, 2000).

In social sciences, particularly economics, the term *Homo Economicus* has been used for a formal representation of an individual, who acts as a utility maximizer, given his preferences and other constraints. An economic man adheres to the axioms of rational choice theory. Even though this hypothetical construct has been useful in formulating economic theories and models, over the past decades psychologists and behavioral scientists have documented robust and systematic violations of principles of Expected utility theory, Bayesian learning, and Rational expectations - questioning their validity as a descriptive theory of decision making (De Bondt, 1998). Furthermore, Simon (1991), to whom the term *bounded rationality* is usually attributed, has emphasized "the limits upon the ability of human beings to adapt optimally, or even satisfactorily, to complex environments."

The efficient markets hypothesis (EMH), developed in the 1960’s independently by Paul Samuelson and Eugene F. Fama, posits that market prices fully reflect all available information. The EMH is much intertwined with the Random Walk Hypothesis (RWH) according to which securities prices are unpredictable. "It has been applied extensively to theoretical models and empirical studies of financial securities prices, generating considerable controversy as well as fundamental insights into the price-discovery process" (Lo, 2007). Meir Statman, a notable proponent of behavioral finance, pleaded for ”accepting market efficiency in the sense of beating the markets,” however, rejecting the definition in the sense of rationality, by which ”rational prices reflect only utilitarian characteristics, such as risk, not value-expressive characteristics, such as sentiment” (Statman, 1999). Burton G. Malkiel, one of the proponents

\(^1\)Statman (1999), however, gives another perspective: "Some people think that behavioral finance introduced psychology into finance, but psychology was never out of finance. Although models of behavior differ, all behavior is based on psychology." Yet another view is that economists distanced themselves from the psychological foundations of individual behavior during the development of neo-classical economics.
of the EMH, embraces the definition of market efficiency "that such markets do not allow investors to earn above-average returns without accepting above-average risks" (Malkiel, 2003). Also, "What I do not argue is that the market pricing is always perfect. After the fact, we know that markets have made egregious mistakes, as I think occurred during the recent Internet 'bubble.' Nor do I deny that psychological factors influence securities prices..." (Malkiel, 2003). Therefore, even though final opinions on EMH differ among academics, there appears to be at least some confluence of thought.

In order to reconcile the theory of efficient markets with behavioral finance, Lo (2005) proposes an alternative theory in which both can coexist - the Adaptive Markets Hypothesis. In this evolutionary model individual market participants adapt to changing environment by using heuristics. In other words, Lo provides us with a theoretical framework in which we could easily fit our conceptual model of the individual investor. Lo (2005) believes that if we were able to measure changes in investor population, changes in investor preferences, and changes in the investment environment, it might be possible to build actively managed portfolios that better suit an investor’s needs.

One of the most influential contributions to the theory of decision making under uncertainty and risk was made by Kahneman and Tversky (1979) with their Prospect Theory. The robustness and pervasiveness of this cognitive-psychological research have bolstered its impact on the economic theory, as well as finance (whose Modern Portfolio Theory is also based on the assumption of rational agents). Individual investors - who use heuristics, depend on framing of the problem, and are prone to biases, which in turn may lead to various anomalies at the market level - are subjects of research in the area of behavioral finance. "An empirical result qualifies as an anomaly if it is difficult to 'rationalize' or if implausible assumptions are necessary to explain it within the paradigm," as said by Thaler (1987) throughout his series of papers on anomalies.

Behavioral finance builds itself upon two blocks: limits to arbitrage and psychology (Barberis and Thaler, 2003). Psychology lists a number of possible deviations from rationality, while limits to arbitrage argue that rational investors may not be able to exploit opportunities created by irrational investors. If irrational investors (noise traders) create dislocations in asset prices (departures from fundamental values), rational investors (arbitrageurs) should be able to correct this mispricing through the process of arbitrage. However, the arbitrage can be too costly, too risky, or simply impossible due to various constraints, so the inefficiencies may persist for a longer period (Barberis and Thaler, 2003). Behavioral finance finds its applications on various levels of financial markets: on the aggregate market level, on the cross-section of average returns, on individual investor behavior, and on corporate finance (Barberis and Thaler, 2003).

One of the earlier comprehensive studies of individual investors who manage their
own equity portfolios, De Bondt (1998) identified four classes of anomalies on the level of individual investor behavior: Firstly, investors are prone to biases in the perception of asset price movements. In his 1987-1992 study, De Bondt conducted a mail survey among 125 investors affiliated with the American Association of Individual Investors (AAII), where he documented an extrapolation bias, that is, 'the expected continuation of past price changes.' Quite the opposite, in his 1991 study, by interpreting Livingston surveys data, he concluded that economic experts are contrarians in their predictions. Furthermore, investors predict too narrow confidence intervals in the subjective probability distributions of prices (Tversky and Kahneman, 1974). Secondly, the perception of asset’s value is largely dependent on popular models (Shiller, 1990), that is socially shared tips from peers, financial advisors, news in the media (and nowadays, especially, on Internet portals, forums, and news groups). De Bondt and Thaler (1985) found evidence that stock market overreacts, i.e. violates Bayes’ rule, as portfolios of prior losers outperform portfolios of prior winners. Thirdly, when managing risk and return, many investors do not diversify their portfolios. This may be due to false beliefs that the risk is defined at the level of an individual asset rather than the portfolio level, and that it can be avoided by hedging techniques, decision delay, or delegation of authority (De Bondt, 1998). Additionally, in their portfolios investors hold surprisingly large amounts of fixed-income securities (bonds), despite the empirical fact that stocks outperform them on the long run. Benartzi and Thaler (1995) have offered an explanation to this famous equity premium puzzle (Mehra and Prescott, 1985), by investors’ myopic loss aversion - a combination of high sensitivity to losses, and frequent monitoring of one’s wealth\(^2\). Finally, although traders are often precommitted to certain rules and techniques, even professionals seem to fail to maintain discipline and consistency (Slovic, 1972). Trading practices are highly influenced by two strong reference points - performance of the market index, and the price at which the asset was purchased.

A number of surveys and books on behavioral finance and behavioral economics have been published, including some popular books aimed at individual investors (Shefrin, 2000; Goldberg and von Nitzsch, 2001; Barberis and Thaler, 2003; Camerer et al., 2004; Altman, 2006; Peterson, 2007). An excellent review paper about neuroeconomics is written by Camerer et al. (2005). Neuroeconomics and neurofinance are yet another emerging fields, which aim to combine neuroscience methods with economic and financial theory. Neuroscience uses imaging of brain activity and other techniques to infer the details about how the brain (the ultimate ”black box”) works (Camerer et al., 2005). Neuroeconomists are particularly interested in neural evidence that can elucidate our assumptions on the economic constructs such as utility, risk attitude, time preference etc.

\(^2\)Haigh and List (2005) conducted an experimental analysis, and showed that not only undergraduate students exhibit myopic loss aversion, but also professional traders recruited from the Chicago Board of Trade (CBOT); the latter even to a greater extent.
2.3 Dimensions of Investment Decisions

The question of investor decision making is in financial literature often conceptualized as a process consisting of different stages. In this section we address a number of relevant (and most likely interdependent) sub questions, which correspond to the investment process often described in the financial literature. In addition, we discuss other relevant topics which are often studied in the area of behavioral finance, but sometimes neglected in traditional finance. Hence, this section presents a literature review on the following topics.

1. Profiling investors on their preferences and risk attitude.
2. Portfolio allocation in practice, diversification in practice, and the influence of various constraints.
3. Portfolio management, performance measurement, frequency of updating, employed strategies.
4. Information processing and learning.
5. Social interaction and peer influence.
6. The role of emotions
7. Heuristics, biases, and departures from rationality.

2.3.1 Risk Attitude

The crucial concept for investments, and decision making in general, is the concept of risk. Yet, there are many definitions of risk with its meaning varying across different domains. In standard decision theory, a risky prospect is expressed as a set of events and event-contingent outcomes, with probabilities assigned to each event. Knight (1921) made the distinction between decisions under risk and decisions under uncertainty, with risk being measurable (quantitative) and uncertainty non-measurable (non-quantitative). However, it is possible to conceive decision under risk as a special case of decision under uncertainty, where objective probabilities are known, and used in place of subjective probabilities. The most influential theories for decisions under risk and uncertainty are known as Expected Utility Theory (Bernoulli (1954), von Neumann and Morgenstern (1944)), Prospect Theory (Kahneman and Tversky, 1979), Rank-Dependent Utility Theory (Quiggin (1982), Schmeidler (1989)), and Cumulative Prospect Theory (Tversky and Kahneman, 1992). Traditional economics and finance have been dominated by these probabilistic models of uncertainty. However, other theories for dealing with uncertainty, ambiguity, or vagueness exist: e.g. Fuzzy Set Theory\(^3\) (Zadeh, 1965), Dempster-Shafer Theory (Shafer, 1976), and Rough Set Theory (Pawlak, 1991).

\(^3\)An example of a financial application is the article of Almeida and Kaymak (2009), which proposes a probabilistic fuzzy system (PFS) for Value-at-risk (VaR) estimation.
A decision maker’s attitude towards risk can be characterized as risk-aversion, risk-seeking (risk-tolerance, risk-taking, risk-loving), or risk-neutrality; and can be defined in a classical sense as a preference between a risky prospect and its expected value (the method of revealed preference). In these theoretical considerations risk attitude is usually captured through the curvature of utility function, or alternatively, through nonlinear weighting of probabilities. Another strong empirical phenomenon that is driving risk aversion to a large extent is known as loss aversion (Kahneman and Tversky (1979), Markowitz (1952)). “Losses loom larger than gains,” and while people are typically risk-averse for gains, they are risk-seeking in the domain of losses (Kahneman and Tversky, 1979). This highlights reference dependence, i.e. the importance of reference point against which outcomes are coded as losses or gains. Loomes (1999) suggests that the current evidence in literature is more in favor of the notion that individuals have only basic and fuzzy preferences, and that each decision problem triggers its own preference elicitation. This is in line with the claim that preferences are constructed (not elicited or revealed) as a response to a judgment or choice task (Bettman et al., 1998).

Slovic (1998) argues that although knowledge of the dynamics of risk taking is still limited, there is an evidence of little correlation in risk-taking preferences across different domains and situations. Only those tasks highly similar in structure and payoffs have shown any generality. Also, previous learning experiences in specific risk-taking settings seem more important than general personality characteristics. Furthermore, risk attitude can change depending on the outcomes of previous decisions. Thaler and Johnson (1990) found that previous gains increase risk-seeking behavior (house money effect), while in the presence of previous losses, those bets which offer a chance to break even seem particularly attractive (break-even effect). These are examples of what Thaler refers to as mental accounting. It is an important question whether investors have risk attitudes related to the gains and losses defined on the individual stock level (narrow framing), on parts of the portfolio, on the overall portfolio, or on their total wealth.

Risk in investments is usually considered as the standard deviation of asset returns. Next to volatility, other common measures of risk are downside risk, shortfall probability, and Value-at-Risk. Finance and econometrics literature are the source of other more sophisticated risk modeling techniques. In an early study of the judgmental processes of institutional investors, Cooley (1977) found that most investors perceive variance as a synonym or a large part of investment risk. A substantial number of investors, however, identified an additional dimension of risk in asymmetry (left skewness). Kurtosis, on the other hand, was perceived as risk-reducing. Investor perception of risk in security valuation can be biased. Shleifer (2000) illustrates this on an example of value and growth stocks, and conjectures that if this biased perception affects the demand for securities, while having nothing to do with the fundamental risk of a portfolio, it would still generate the same returns as we can
observe in the data. Ganzach (2000) found in experiments that judgments of risk and return for familiar financial assets were consistent with their ecological values (a positive relationship between risk and return). However, for unfamiliar assets he found that both judgments were based on a global attitude, which resulted in a negative relationship between risk and return (assets were perceived to have high risk and low return, or low risk and high return).

2.3.2 Portfolio Allocation

Behavioral finance takes a descriptive perspective, by studying how individual investors actually allocate their portfolios. Conversely, Modern Portfolio Theory (MPT) (Markowitz, 1959) lays the foundations of portfolio allocation from the normative point of view. In portfolio theory one of the crucial concepts is diversification, a risk-management technique where various investments are combined in order to reduce the risk of the portfolio. However, many investors do not (sufficiently) diversify their portfolios. This may be due to beliefs that the risk is defined at the level of an individual asset rather than the portfolio level, and that it can be avoided by hedging techniques, decision delay, or delegation of authority (De Bondt, 1998).

Benartzi and Thaler (2001) studied naive diversification strategies in the context of defined contribution saving plans. They found an evidence of $1/n$ heuristic, as a special case of diversification heuristic, in which an investor spreads his or her contributions evenly across available investment possibilities. As the authors convey, such a strategy can be problematic both in terms of ex ante welfare costs, and ex post regret (in case the returns differ from historical norms). Naive diversification does not imply coherent decision making. Although it may be a reasonable strategy for some investors, it is unlikely that the same strategy would be suitable for all investors, who obviously differ on their risk preferences and other risk factors, such as age. $1/n$ heuristic can produce a portfolio that is close to some point on the efficient frontier. However, the exact point might not match investors’ risk preferences - which can create a significant ex ante welfare costs, as exemplified by Brennan and Tourous (1999). Nevertheless, naive diversification portfolio strategy is actually a very strong benchmark, as shown by DeMiguel et al. (2007). By comparing out-of-sample performance of various optimizing mean-variance models, they found that no single model consistently beats the $1/n$ strategy in terms of the Sharpe ratio or the certainty-equivalent return. Poor performance of these optimal models is due to errors in estimating means and covariances (DeMiguel et al., 2007).

Benartzi and Thaler (2001) also found a support for mental accounting on the company stock: when company stock is in the array of available investment options, the total exposure to equities is higher than when it is not available. It seems that company stock is given a separate mental account different from the rest of equity classes. Home bias is another robust finding in portfolio allocation. Despite the
advantages of international portfolio diversification, the actual portfolio allocation of many investors is too concentrated in their domestic market (French and Poterba, 1991). So far, the literature has not provided a generally accepted explanation for the observed home bias. Huberman and Jiang (2006) argue that “familiarity breeds investment,” and that a person is more likely to invest in the company that she (thinks) she knows. Instances of this familiarity bias are investing in domestic market, in company stocks, in stocks that are visible in investors’ lives, and stocks that are discussed favorably in the media.

Goetzmann and Kumar (2001) examined the diversification of investors with respect to demographic variables of age, income, and employment. They found that low income and non-professional categories hold the least diversified portfolios. They also found that young active investors are overfocused and inclined towards concentrated, undiversified portfolios, which might be a manifestation of overconfidence.

2.3.3 Portfolio Management

Before discussing some of the main findings on how investors manage their portfolios, it is noteworthy mentioning that both EMH as well as much empirical evidence undermine practical relevance of active portfolio management. While financial literature on active portfolio management offers various techniques for beating the benchmarks, behavioral literature focuses on how individual investors manage (or make changes to) their portfolios. A common tendency to hold losers too long and sell winners to soon, has been labeled by Shefrin and Statman (1985) as the disposition effect. They attributed their findings to loss aversion, the issue of self-control, mental accounting, and the desire to avoid regret.

Odean (1998) found that a particular class of investors sell winners more readily than losers. Even when the alternative rational motivations are controlled for, these investors continue to prefer selling winners and holding losers. Their behavior is consistent with two behavioral hypothesis: the prospect theory, and a mistaken belief that winners and losers will mean revert. This investor behavior appears not to be motivated by a desire to rebalance portfolios or by a reluctance to incur the higher trading costs of low priced stocks. It is also not justified by subsequent performance, as, in fact, it leads to lower returns (Odean, 1998). Investors trade too much due to their overconfidence. For successful investors this overconfidence can be reinforced through self-attribution bias, i.e. belief that their trading success should be attributed mostly to their own abilities.

4 “Switching from security to security accomplishes nothing but to increase transactions costs and harm performance. Thus, even if markets are less than fully efficient, indexing is likely to produce higher rates of return than active portfolio management. Both individual and institutional investors will be well served to employ indexing for, at the very least, the core of their equity portfolio” (Malkiel, 2005).
While some investors may trade too much and often change their strategies, others may exhibit the tendency of "doing nothing or maintaining one's current or previous decision." This is how Samuelson and Zeckhauser (1988) defined the status quo bias. Explanations for the status quo bias fall into three main categories. The effect may be seen as the consequence of (1) rational decision making in the presence of transition costs and/or uncertainty; (2) cognitive misperceptions; and (3) psychological commitment stemming from misperceived sunk costs, regret avoidance, or a drive for consistency. The status quo is related to loss aversion (framing as gains and losses) in the sense that current position (status quo) is seen as the reference point. Other explanations, such as anchoring, sunk costs, regret avoidance, the drive for consistency, the avoidance of cognitive dissonance, and the illusion of control, may contribute to the perseverance of the status quo bias. It is also related to the influence of default option on choices.

2.3.4 Information Processing

Market participants are exposed to a constant flow of information, ranging from quantitative financial data to financial news in the media, socially exchanged opinions and recommendations. Processing all this information is a daunting task, so it would not be surprising that during this process people apply many heuristics. According to the representativeness heuristic people may overreact to a series of evidences, and see patterns where they do not exist. However, people can sometimes underreact to news, i.e. in the light of a new evidence they update their beliefs conservatively, and not in a fully Bayesian manner.

Processing information from the investment environment is important as it gives inputs for decision making and belief updating, gives feedback on investment strategies, and fosters learning. However, the confirmation bias may play an important role in how investors acquire and process this information. It suggests that people have a tendency to search for information that supports their current beliefs and decisions, while neglecting information that confronts those beliefs.

Oberlechner and Hocking (2004) studied information sources, news, and rumors in the foreign exchange market. In their study foreign exchange traders and financial journalists rated the importance of different information sources, such as wire services, personal contacts, analysts, daily newspapers, financial television etc. An interesting finding of this study is that the information speed, expected market impact, and anticipated market surprise are rated as more important than the reliability of the source, and the accuracy of information.

2.3.5 Social Interaction

Financial economists have borrowed more from the psychology of the individual than from social psychology (Hirshleifer, 2001). For example, they have examined how in-
Chapter 2. Behavioral Biases in Investor Decision Making

formation is transmitted by prices, volume, or corporate actions. However, person-to-person and media contagion of ideas and behavior also seems important (Hirshleifer, 2001).

Shiller (1990) has emphasized the importance of conversation in the contagion of popular ideas about financial markets. In a survey of individual investors, Shiller and Pound (1989) found that almost all of the investors who recently bought a particular stock had their attention drawn to it through direct interpersonal communication. The influence of conversation on trading may arise from individuals’ overconfidence about their ability to distinguish pertinent information from noise or propaganda (Hirshleifer, 2001).

Social psychology provides evidence of various social effects which might be important in the context of financial markets as well. Conformity effect, or the tendency of people to conform with the judgment and behavior of others, was studied by Asch (1956). Bond and Smith (1989) confirmed the conformity effect, showed its historical change, and emphasized its cultural dependence. Other effects found in the social context are fundamental attribution error and false consensus effect (see Hirshleifer, 2001). Herding behavior or mimetic contagion has been proposed as the source of endogenous fluctuations (bubbles and crashes) in financial markets (Kirman, 1991; Topol, 1991). This view is interesting as it suggests that such market fluctuations may arise endogenously, irrespective of exceptional news or other exogenous shocks to the market.

2.3.6 Emotions

Emotions have powerful effects on decisions, and decision outcomes have powerful effects on emotions (Mellers et al., 1998). Emotions can have both a predecision and postdecision effect. Most of the research focused on a unidimensional model in which a predecision emotion can be either positive or negative. However, a more detailed approach is needed given the variety and domain-specificity of emotions (Mellers et al., 1998). Positive emotions are shown to increase creativity and information integration, promote variety seeking, but also cause overestimation of the likelihood of favorable events, and underestimation of the likelihood of negative events. Negative emotions promote narrowing of attention and failure to search for alternatives. They promote attribute- vs. alternative-based comparisons (Mellers et al., 1998).

One of the most studied emotions that can follow a decision is the feeling of regret. Gilovich and Medvec (1995) showed that in the short run people experience more regret for actions rather than inaction, while in the long run they experience more regret for their inactions. Anticipated emotions, such as regret and disappointment, have drawn most attention of the economists, whereas immediate emotions (experienced at the moment of decision making) have been mainly studied by psychologists (Loewenstein, 2000). Loewenstein (2000) emphasizes that economists should also
2.3. Dimensions of Investment Decisions

pay attention to immediate emotions and a range of visceral factors which influence our decisions.

One evidence for the importance of emotions in decision making comes from patients with brain lesions in regions related to emotional processing. Shiv et al. (2005) made an experiment with 20 rounds of investment decisions (choosing whether to invest 1 dollar in a risky prospect with a 50-50 chance of winning 2.5 dollars or nothing), and found that target patients (with brain lesions in emotion-related areas of brain) made more investments than the normal participants and control patients, and thus earned more on average. Normal and control patients seem to have been more affected by the outcomes of previous decision - upon winning or losing they adopted a conservative strategy and less invested in subsequent rounds. However, the inability to learn from emotional signals (Somatic Marker Hypothesis, Damasio et al. (1996)) can also lead to unadvantageous decisions such as excessive gambling. Thus, emotion and cognition both play a crucial role in decision making.

Loewenstein et al. (2001) propose “risk-as-feelings” perspective on decisions under risk and uncertainty, which differs from classical cognitive-consequentialist perspective in the sense that feelings or affects play a crucial role in decision making: emotional evaluations of risky choices may differ from cognitive, and when such a divergence occurs, they often drive behavior. Both emotional and cognitive evaluations are influenced by expected outcomes (and expected emotions) and subjective probabilities. However, emotional evaluations are also influenced by a variety of factors, such as vividness of associated imagery, proximity in time, etc.

Goldberg and von Nitzsch (2001) describe a personal experience of a trader (market participant) who goes through various emotional states during profit-and-loss cycles. The feelings of hope and fear, depending on the success or failure on the market, can be transformed to the states of euphoria or panic (or even total recklessness).

\[
\begin{align*}
\text{Hope} & \rightarrow \text{pleasure} \rightarrow \text{elation(exuberance)} \rightarrow \text{euphoria} \\
\text{Fear} & \rightarrow \text{anxiety} \rightarrow \text{desperation} \rightarrow \text{panic/recklessness}
\end{align*}
\]

During these transitional states there is a selective perception of information - positive information is perceived and often exaggerated, while negative information is ignored. In final states of euphoria or panic, information has almost no role to play (Goldberg and von Nitzsch, 2001).

2.3.7 Heuristics and Biases

Much of the behavioral finance literature focuses on individual investor psychology, particularly the use of heuristics and various biases in judgment. Organizing and presenting these heuristics and biases is by no means an easy task, given that they arise from a wide range of mechanisms. However, a simple but comprehensive framework is important if one aims to use it for descriptive or prescriptive purposes.
Table 2.1 presents different frameworks in which most known judgment heuristics and biases from behavioral finance and behavioral economics literature can be organized and presented. The intention of these frameworks was most likely not to unify all possible heuristics and biases, but more to give an overview of behavioral phenomena that different authors deemed relevant. We can see that there is a significant overlap of heuristic and biases listed in various frameworks, although they are classified according to different criteria.

A traditional approach describes decisions as choices between risky prospects. A decision maker forms beliefs about probabilities of events and about values (utilities) of outcomes contingent on those events. Finally, he or she makes preferences between risky options. Biases can arise both in the process of forming beliefs and preferences. In the more general sense, a bias can be defined as a departure from normative, optimal, or rational behavior.

Heuristics are mechanisms (rules, strategies) for processing information to arrive at a quick (not necessarily optimal) result following little effort (Goldberg and von Nitzsch, 2001). In their seminal work, Tversky and Kahneman (1974) investigated heuristics that people often employ when making decisions under uncertainty. Heuristics are useful because they make the difficult task of assessing the probabilities related to uncertain events much easier. However, these heuristics can also lead to systematic biases in judgment. Three heuristics to which people are particularly susceptible are the following: Representativeness, Availability, and Adjustment and anchoring.

Hirshleifer (2001) proposed a unified explanation for most known judgment and decision making biases in the context of investor psychology and asset pricing. Shefrin (2000) distinguishes between heuristic-driven bias and frame-dependence. He considers heuristics as rules of thumbs, usually generated through a trial and error process, which can also lead to systematic biases. Frame dependence refers to the distinction between the substance and the form, and means that the way in which a decision problem is presented also matters. Kahneman and Riepe (1998) focus on biases in beliefs and preferences of which financial advisors should be particularly aware, and provide recommendations how to avoid them or mitigate their harmful effects. Goldberg and von Nitzsch (2001) divide heuristics into two major groups: heuristics for reducing complexity and quick judgments. Rabin (1998) discusses various behavioral aspects he finds important from the perspective of economics. Barberis and Thaler (2003) survey behavioral finance literature with a focus on the aggregate market level, on the cross-section of average returns, on individual investor behavior, and on corporate finance. Individual perspective is an application of psychology to financial markets. Barberis and Thaler (2003) list known biases that can arise when people form their beliefs and preferences.

Shefrin (2000) takes the so-called debiasing view, by which costly mistakes can be avoided if practitioners learn how to recognize mistakes, understand the underly-
### Table 2.1: Heuristics and biases in behavioral finance and behavioral economics literature

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<td>a. Representativeness bias</td>
<td>a. Simplifying the facts</td>
<td>a. Reference levels, adaptation and losses</td>
<td>a. Reference levels, adaptation and losses</td>
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<td>b. Narrow framing / Mental accounting / Reference effects</td>
<td>b. Gambler’s fallacy (the law of small numbers)</td>
<td>b. Overconfidence</td>
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<td>c. Representativeness heuristic</td>
<td>c. Overconfidence (overly narrow confidence intervals)</td>
<td>c. Optimism</td>
<td>ii. Endowment effect</td>
<td>ii. Endowment effect</td>
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<td>d. Belief updating: Combining effects</td>
<td>d. Anchoring and adjustment / Conservatism</td>
<td>d. Hindsight</td>
<td>iii. Status quo bias</td>
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<td>b. Mood, feelings and decisions</td>
<td>a. Loss aversion</td>
<td>c. Value function</td>
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<td>c. Time preference and Self-control</td>
<td>b. Mental accounting / Frame dependence</td>
<td>d. The shape and attractiveness of gambles</td>
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<td>3. Other biases</td>
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<td><strong>4. Social Interactions</strong></td>
<td>c. Hedonic editing</td>
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Chapter 2. Behavioral Biases in Investor Decision Making

In summary, Kahneman and Riepe (1998) find the ability to recognize situations in which one is likely to make a large error a useful skill for a decision maker. Critical or analytical thinking must be employed, whenever intuition cannot be trusted. Goldberg and von Nitzsch (2001) believe that individuals could increase their trading success by "knowing thyself," by being aware of these mechanisms, and by a little discipline. Many proponents of behavioral finance seem to share this view. However, some questions immediately arise, as it is not completely clear what the prerequisites for a successful implementation of this debiasing are. How do investors realize they are biased - is it through market feedback, their own introspection, or social interaction? How do they implement debiasing, and how do they evaluate this process?

"Why don’t people simply learn their way out of biased judgments? To some extent they do. One barrier is that learning is just too hard. The other barrier arises from self-deception" (Hirshleifer, 2001). Some biases may be avoided by careful framing of problems or by learning effects of repetition. However, there is no guaranty that financial decision problems will be presented in such a way which promotes an unbiased decision making process, as noted by Hirshleifer (2001). Inexperienced market participants should be particularly careful, as they have not been exposed to these positive learning effects. Similarly, Kahneman and Riepe (1998) identify three necessary conditions for a good calibration (i.e. avoiding overconfidence): (1) Facing the same problem every day; (2) Making explicit probabilistic predictions; (3) Obtaining swift and precise feedback on outcomes.

It could be argued, however, that individual biases are not that important, as individuals differ, so their biases should cancel out in the equilibrium. However, it is well-known that some biases can be systematic and persistent (Hirshleifer, 2001). Repeated patterns can even be used as a basis for prediction of behavior of others, which is very important in the context of financial markets (Goldberg and von Nitzsch, 2001).

Behavioral finance literature gets into the very heart of the debate about rationality and irrationality of market participants. Thaler (1991) makes an interesting remark about that: "If most individuals tend to err in the same direction, then a theory which assumes that they are rational also makes mistakes in predicting their behavior." A typical example in a game-theoretical setting is a game (reminiscent of beauty contest games) in which every participant must write down a number between 0 and 100. The winner of the game is the participant whose number is the closest to two thirds of the average value of all the numbers given by the participants. Even though the rational solution to the problem is 0, experiments show that winning

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5 The issue whether this could be exploited so as to create profitable opportunities is still rather controversial, and is related to a stance on the market efficiency. "I am skeptical that any of the 'predictable patterns' that have been documented in the literature were ever sufficiently robust so as to have created profitable investment opportunities, and after they have been discovered and publicized, they will certainly not allow investors to earn excess returns" (Malkiel, 2003).
numbers are usually larger. A proposed explanation for this out-of-the-equilibrium behavior involves subjects engaging in a finite-depth of reasoning about other subjects’ beliefs (Nagel, 1995). A player who wants to win the game should deliberately depart from the Nash equilibrium solution, by taking into account that not all players are fully rational (Thaler, 2000). Financial markets might be a real-world example where rational behavior does not necessarily imply success. While learning would lead players to the equilibrium in the previously mentioned repeated game, in the real market there may be not be such learning opportunities, and/or there might be an inflow of new irrational investors.

Importantly, heuristics and biases should not be treated as synonymous, even though we can often read about them together in behavioral finance literature. Not all biases arise from heuristics, and, also, not always the use of heuristics leads to biases. However, even when it does (with respect to some normative rational theory), a heuristic behavior may actually result in a successful performance compared to an optimal strategy, which was demonstrated in a seminal paper by Gigerenzer and Goldstein (1996). A financial example of a successful heuristic is naive diversification \((1/n)\) heuristic, which is costly ex ante, but ex post serves as a very strong benchmark (DeMiguel et al., 2007).

The following list presents some of the most known heuristics, biases and effects found in behavioral finance literature. It includes brief definitions, experimental examples, and financial examples or possible implications for investment decisions.

**Ambiguity Aversion**

Ambiguity aversion refers to the preference for known risks over unknown risks (uncertainty). "The emotional aspect of the aversion to ambiguity is the fear of unknown" (Shefrin, 2000).

**Anchoring and Adjustment**

Adjustment and anchoring is a heuristic which starts from an initial value (given by the problem formulation, or by some partial computation) and then adjusts it towards the final value. The problem with this heuristic is that the adjustment is often insufficient, which means that the final value will heavily depend on the initial value. The consequence of this heuristic is that people overestimate conjunction of events (with high individual probabilities), and underestimate a disjunction of events (with low individual probabilities). Anchoring effect is also present in the assessment of subjective probability distributions, resulting in overly narrow confidence intervals.

Overstrong anchoring to the status quo often results in underestimation of the probability of extreme movements (Lichtenstein et al, 1982). Setting confidence intervals overly narrow means that people get surprised more frequently than what they expect (Shefrin, 2000). In financial markets anchors can be based on opinions
or attitudes of friends or experts. Even data and forecast that initially appear to be unrealistic can still have an anchoring effect (Goldberg and von Nitzsch, 2001).

**Availability**

Availability is a judgmental heuristics in which a frequency/probability of a class/event is assessed on how easy it is to recall its instances (retrievability), how easy it is to mentally construct its instances (imaginability), or how easy it is to associate two instances (illusionary correlation), as studied by Tversky and Kahneman (1974).

In Tversky and Kahneman (1973), the authors analyze availability, a heuristic which uses the strength of association between instances to assess their probabilities. For example, in one study subjects were asked to compare the frequencies of words starting with letter ‘r’, and words that have letter ‘r’ in the third position. Even though the latter are more frequent, the participants opted for the first case, because it is much easier to mentally construct words by using their first letter. Similar results can be obtained when testing different mental operations. For retrievability, the frequency of instances which are more easily brought to mind (retrieved) looms larger. Another availability heuristic assesses the frequency of co-occurrences by the ease of making association between objects.

**Conditional Probability Fallacy**

Conditional probability fallacy means confusing conditional probability $p(a|b)$ with $p(b|a)$, and can be in a broader view considered as the fallacy of confusing cause and effect. Goldberg and von Nitzsch (2001) give a financial example where the probability of a stock market crash in October is overestimated based on the historical observation that most stock crashes occurred in October.

**Conservatism**

People respond too conservatively to new information. This is related to Anchoring and Adjustment heuristic. In a narrow sense conservatism means that people are not perfect Bayesian updaters.

**Endowment Effect**

Once a person comes to possess a good, the person immediately values it more (Thaler, 1980; Knetsch and Sinden, 1984).

**Frame Dependence**

A frame is the form used to describe a decision problem, and frame dependence means that the form is relevant for behavior (Shefrin, 2000).
Gambler’s Fallacy

The gambler’s fallacy is the false belief that if deviations from expected behavior are observed in repeated independent trials of some random process, then these deviations are likely to be evened out by opposite deviations in the future. In the case of a fair coin tosses, this fallacy would manifest as thinking that the coin is more likely to toss heads after having tossed tails a number of times in a row. See also the law of small numbers and representativeness.

Hedonic Editing

Hedonic editing means that people prefer some frames over others. It is the way people organize their mental accounts. For instance, "transferring assets" is an expression that emphasizes reallocation from one mental account to another, and can hide the fact that the first mental account was closed at loss. (Shefrin, 2000) People like to separate gains, and integrate losses. In financial markets investors who experience capital gains like to separate dividend payments, while in declining markets they can use dividends as silver linings which buffer capital losses. Thaler and Johnson (1990) study how prior gains or losses affect risk-taking. They find evidence of the house money effect (prior gains increase risk seeking), as well as break-even effect (under previous losses, gambles which offer a chance to break even seam very attractive).

Law of Small Numbers

Misapplication of the law of large numbers to small samples is also known as the law of small numbers or a gambler’s fallacy. The phenomenon is related to representativeness heuristic - people under-use base-rate information when forming their beliefs (Tversky and Kahneman, 1971). In the financial context, "we underestimate how often a good financial analyst will be wrong a few times in a row, and underestimate how often a clueless analyst will be right a few times in a row" (Rabin, 1998).

Loss Aversion

Loss aversion is a pervasive phenomenon in human decision making under risk and uncertainty, according to which people are more sensitive to losses than gains. It plays a crucial role in Prospect Theory (Tversky and Kahneman (1974), Tversky and Kahneman (1992)). A typical financial example is in investors’ difficulty to realize losses. Shefrin (2000) calls this phenomenon "get-evenitis," that is, people hope that markets will work in their advantage and that they will be able to terminate their investment without incurring any losses.
Money Illusion

A natural way for people to think about money is in terms of nominal rather than inflation-adjusted values (Shefrin, 2000). Thus, under hyperinflation people will view nominal wage increase more favorably than it really is.

Overconfidence

Overconfident people are not well calibrated. In their predictions they set confidence bands overly narrow, which means they get surprised more frequently than they anticipated (Shefrin, 2000). This type of overconfidence is known as miscalibration. A more general definition of overconfidence is the one by which people overestimate their own capabilities, usually with respect to capabilities of other people on average. This is also known as better-than-average overconfidence. In financial markets overconfident investors are considered those who actively trade in such a way that the difference between the stocks they buy and those they sell does not cover transaction costs (Odean, 1998).

Regret

Regret is the emotion experienced for not having made the right decision. It is the feeling of responsibility for loss (Shefrin, 2000). In a financial context the minimization of possible future regret plays an important role in portfolio allocation. It is also related with preference for dividends in financing consumer expenditures, because selling a stock that may rise in the future carries a huge potential for regret.

Representativeness

Representativeness bias occurs when it is required to assess the probability of an object A belonging/originating to/from a class/process B. The heuristic rule says that if object A is highly representative (highly similar to a stereotypical object) of class B, the probability of A originating from B is judged as high, and vice versa (Tversky and Kahneman, 1974). The problem with this heuristics is that it persists even when facts, which should affect the judgment of probabilities, are introduced. For instance, Tversky and Kahneman (1974) showed that representativeness is insensitive to the prior probability (base-rate frequency) of outcomes, when (uninformative) description is provided. Furthermore, it is insensitive to the sample size, when people estimate the probability related to the sample randomly drawn from a large population, based on the similarity with the population parameter. People also expect the global characteristics of a process to be present in each local part, which deviates from the chance expectations (i.e. expecting more randomness in a series of coin tosses; gambler’s fallacy). The representativeness heuristic often neglects different levels of predictability, and may lead to the illusion of validity when the predicted
outcome is highly representative for a given input. In this case people also neglect the phenomenon known as regression to the mean, which explains why people might overestimate the efficiency of a punishment, and underestimate the efficiency of a reward.

According to Shefrin (2000), representativeness heuristic is a judgment based on stereotypes. Representativeness is high when an observation fits the pattern (Goldberg and von Nitzsch, 2001). Some of the most important applications of this heuristic are in predicting the market, picking stocks, choosing mutual funds, selecting money managers, and investing in initial public offerings (IPOs) and seasoned equity offerings (SEOs), (Shefrin, 2000).

A financial example is the winner-loser effect documented in De Bondt and Thaler (1985). Investors who use the representativeness heuristic are too optimistic about past winners, and too pessimistic about past losers. This creates a temporary mispricing (overvaluation of past winners, and undervaluation of past losers), which is eventually reversed, as the portfolio of past losers outperforms the market, while the winners portfolio underperforms.

Another example is a misapplication of regression to the mean, which predicts that future returns will be closer to the historical average. However, practitioners often predict that after having a long period of high returns they are more likely to be below, which is a wrong prediction in positively autocorrelated financial markets. (Shefrin, 2000). Analogously, in an experiment where subjects could ostensibly buy an asset (whose price moved randomly), Maital (1986) found that the majority expected prices to rise after a long downward trend and consequently held on them longer on average.

Representativeness can cause illusory correlation, that is, overestimation of empirical relationships. Furthermore, empirical relationships are often turned into causal relationships, which may or may not be true (Goldberg and von Nitzsch, 2001).

Self-Control

The issue of self-control and hedonic editing underlies reasons for investors’ preference for portfolios that feature high dividends. To finance their consumer expenditures some investors prefer dividends rather then selling assets (the heuristic “don’t dip into capital”). This is due to framing/hedonic editing, because dividends are labeled as income, not as capital (Shefrin, 2000).

Status Quo Bias

People prefer status quo to changes that involve losing some goods, even when these losses are offset by gains (Knetsch and Sinden, 1984). It is related to the endowment effect and loss aversion.
2.4 Conclusion

This chapter has given an overview of behavioral biases in investor decision making. We have started with an introduction to behavioral finance and explained the main motivation and challenges in that field. After that, we have given an overview of the main dimensions of investment decisions, and summarized the main findings in behavioral finance related to those dimensions.

For the last dimension, namely the heuristics and biases, we have presented an overview of taxonomies that various authors used to classify heuristics and biases. As we can see, there are similarities between these framework in the sense that they contain similar heuristics and biases. Nonetheless, they are grouped according to different criteria. Some authors emphasize the distinction between quick judgments (that ease the mental effort) and framing effects (the way problems are presented), while others make distinction between effects resulting from cognitive processing and those resulting from emotional aspects of decision making. This motivated us for the incorporation of dual-processing theories into a conceptual model of individual investor.

While psychology studies behavioral biases in order to understand how they arise and manifest themselves, behavioral finance aims to understand how they aggregate across individuals, whether they have impact on the market dynamics and ramifications for investor performance. In order to study such implications of investor biases it is important to understand the most relevant aspects of investor behavior as well as the environment in which they are operating. Although there is a significant body of literature on those topics, surprisingly, not much attempt has been made to conceptualize and give more structure to this body of knowledge. This is what we aim in the following chapter.
Chapter 3

A Conceptual Model of Individual Investor*

3.1 Introduction

A conceptual model of the individual investor behavior presented in this chapter is built for the purpose of further implementation into an agent-based artificial financial market. It has already been acknowledge in the literature that agent-based models of financial markets can be useful for studying the topics of behavioral finance, but not much attempt has been made to conceptualize the knowledge of behavioral finance in a form that meets the needs of an agent-based modeler. One example in this direction is the paper by Chen (2008), where cognition, personality and culture are mentioned as the potential topics for agent-based studies in economics. Agent models inspired by cognitive ability, personality traits and culture are concerned with mechanisms which generate human decision and behaviors, and which have to a large extent and for a long time been considered as a black box (Chen, 2008).

Of course, individual behavior could be directly implemented into computational models of investors. However, building a separate conceptual model has a number of advantages. A conceptual model is a structured representation of the large diversity, complexity, and interdisciplinarity present in the field. By building a conceptual model we can become aware of the interrelations between components we may not otherwise see, we can indicate the strength of these relationships, and we can identify those areas which are still largely unknown, thus fostering future research. From a

modeler’s perspective, a conceptual model gives a structure that is easier to implement. On top of the same conceptual model various implementations can be made, for instance, using mathematical, or computational modeling techniques. Hence, we expect that this model will be particularly useful for a modeler of agent-based artificial markets who needs a short, yet rich description of behavioral phenomena that can play a role in investor behavior.

In this conceptual model, investment decisions are seen as an iterative process of interactions between the investor, as represented by the cognitive model (Figure 3.1) and the investment environment (Figure 3.2). This investment process is influenced by a number of interdependent variables and driven by dual mental processes. The interplay between these systems contributes to boundedly rational behavior in which investors use various heuristics and can exhibit biases. In the modeling tradition of cognitive science, the investor is seen as a learning, adapting, and evolving entity that perceives the environment, processes information, acts upon it (make changes to portfolio), and updates its internal states. Finally, the investor behavior is influenced by social interactions with his or her peers. This model can be used to build stylized representations of (classes of) individual investors, and further studied using the paradigm of agent-based artificial financial markets. Although investor agents could be built so that they incorporate many concepts presented in this model, in this thesis, we have focused primarily on heuristic and biases, for which we have conducted a number of studies and presented them in Chapter 5 through Chapter 9.

3.2 A Cognitive Model of the Investor

The central part of our cognitive model of the investor (Figure 3.1) is based on a two-dimensional framework of neural functioning proposed by Camerer et al. (2005). This is conceived as a central information processing and decision making unit that interacts with the investment environment through the perception-interaction-action interface, and is influenced by a number of variables, including risk attitude, time preference, strategies, goals, motivation, emotions, heuristics and biases, personality, demographics and other factors. For modeling purposes, it is possible to assume that all the variables have constant values, such as a constant risk aversion, constant strategies, constant degree of biases etc. However, it is also possible to consider the dynamics of those variables over time. For example, in Chapter 8 we have studied the dynamics of investor confidence and sentiment. Furthermore, the dependencies between those variables could also be taken into account. In the rest of the section, each element of the cognitive model is explained in more detail, except for the heuristics and biases, which have been explained in Chapter 2.
3.2. Dual-Process Systems

A two-dimensional framework of neural functioning by Camerer et al. (2005), distinguishes between affective and cognitive, and between controlled and automatic processes. A distinction between controlled and automatic processes can be found in psychology literature under various names of dual-processing theories (see Camerer et al., 2005). Controlled processes are serial (step-by-step), evoked deliberately, they cause the subjective feeling of effort, and are accessible by introspection. Automatic processes operate in parallel, they are relatively effortless, and are inaccessible to consciousness. A distinction between affect and cognition is also pervasive in contemporary psychology and neuroscience literature (Camerer et al., 2005). Most of the affective processing operates unconsciously. The central role of affective processing is in human motivation - affects address “go-no go” questions, while cognitive processes address “true or false” questions (Camerer et al., 2005). It is only when affect states reach a certain threshold level that the affective processing is associated with feeling states for which we have different names (Camerer et al., 2005). These feeling states include emotions such as happiness, sadness, anger, fear, greed etc, and in the cognitive model they are represented by the variable emotions. The role of some of these emotions has been discussed in Section 2.3.6.

In dual-process theories the choice is determined as the result of the interplay between the cognitive and affective system. This interaction can be collaborative (when both systems work in the same direction), or competing (in which one system wins and overrides the other system). A number of variables can influence the rela-
tive strength of these systems, e.g. cognitive load can undermine controlled cognitive processes. Much of the findings in behavioral economics and finance can be interpreted in the light of the dual-process theories (see Camerer et al., 2005). Hence, for a modeler, it might be just an appropriate level of abstraction, without the need to go into more complex details of cognitive mechanisms and neural functioning.

3.2.2 Risk Attitude

The conceptual model captures important variables in financial decision making, such as risk attitude and time preference. Risk attitude (risk aversion, risk neutrality, risk seeking) is influenced by the competition and collaboration between the cognitive and affective system (Loewenstein et al., 2001). Cognitive system is assumed to deal with risk in a probabilistic fashion, similar to traditional choice theories. Risk averse behavior is driven by fear and anxiety responses to risk and the stored pain of experienced losses (Camerer et al., 2005). Risk taking behavior is driven by the pleasure of gambling (Camerer et al., 2005). The feelings need not be mediated by the cognition. In the light of the Prospect Theory, Loewenstein and O'Donoghue (2004) propose that the affective system contributes to the risk attitude through loss aversion and nonlinear (usually S-shaped) probability weighting. However, the deliberative system responds to risk in a way predicted by Expected Utility Theory (or perhaps Expected Value). To support this interpretation, the authors point to research which suggests that emotional responses depend on mental images of outcomes, whereas they tend to be insensitive to probabilities.

Risk component of the model is related to risk factors such as gender and age. A meta-analysis study by Byrnes et al. (1999) confirmed a significantly higher propensity for risk taking in male participants. In addition, they found age-related shifts in this gender gap, particularly the tendency of the gender gap to decrease with age. Donkers et al. (2001) show that an individual’s risk attitude and probability weighting function is influenced by gender, education level, age and income.

Besides demographic factors, such as gender and age, an important variable influencing risk attitude is the investment horizon. "Most investment practitioners subscribe to the time diversification principle, which states that portfolio risk declines as the investment horizon lengthens. Accordingly, practitioners commonly advise younger clients to allocate a larger proportion of their retirement money to risky assets than older clients do. In contrast, many respected theorists argue that time diversification is a fallacy" (Jaggia and Thosar, 2000). In their opinion, the answer lies in the psychology of risk-taking, particularly as it relates to time horizon. In favor of the time diversification position, they "argue that risk perception is not only a function of age (and other cross-sectional idiosyncratic factors) but also of the temporal distance between the initial investment point and the cash-out point typically represented by the individual’s retirement" (Jaggia and Thosar, 2000).
Gilovich et al. (1993) have studied the effect of temporal perspective on subjective confidence, and they found that people tend to lose confidence in their prospects for success as they come closer to the "moment of truth," i.e. "the risk-assessment becomes more conservative with shorter temporal distance."

### 3.2.3 Time Preference

Time preference is in standard economic theory captured with the discount factor of the discounted utility (DU) model (Samuelson, 1937). However, a behavioral point of view suggests that modeling time preference with a constant discount rate may not be suitable for descriptive purposes. Hyperbolic discounting has been proposed to capture an empirical observation that between now and a point in the future people discount more than between two other temporally equidistant points in the far future, i.e. the discount rate is declining over time (Laibson, 1997). An opposing finding by Read (2001) suggests that an observed pattern of time preference could also be explained by subadditive discounting, i.e. the finding that the amount of "discounting over a delay is greater when the delay is divided into subintervals than when it is left undivided" (Read, 2001).

Frederick et al. (2002) review time discounting literature, and list the anomalies which contradict the DU model, namely: (1) the sign effect; (2) the magnitude effect; (3) the delay effect; (4) preference for improving sequences; and (5) violations of independence and preference for spread. They conclude: "we believe that economists’ understanding of intertemporal choices will progress most rapidly by continuing to import insights from psychology, by relinquishing the assumption that the key to understanding intertemporal choices is finding the right discount rate (or even the right discount function), and by readopting the view that intertemporal choices reflect many distinct considerations and often involve the interplay of several competing motives."

Time domain enters the conceptual model when choosing the investment horizon, the frequency of update, as well as in planning, forecasting, and discounting. An interplay between cognitive and affective mechanisms might give even more refinement in modeling time preference. Affective system is inherently myopic and impulsive, motivating behaviors that have short-term goals, whereas higher order cognitive functions of the prefrontal cortex can take long-term consequences and planning into account (Camerer et al., 2005; Shefrin and Thaler, 1988). According to Camerer et al. (2005) factors which strengthen or weaken an affective or cognitive system will influence people to behave more or less impulsively. Any factor which imposes cognitive load on the prefrontal cortex, i.e. the controlled cognitive system, will decrease the influence of this system on behavior. Other factors which can diminish the power of self-control are a previous exercise of self-control, alcohol, stress, and sleep deprivation. Analogously, "the activation of affective states should accentuate temporal myopia" (Camerer et al., 2005).
3.2.4 Strategies, Goals and Motivation

There is an abundant pool of strategies that investors use for the valuation of assets, for stock picking, and market timing. To obtain an overview of the basic groups of strategies, we looked at a standard investment book. In Sharpe et al. (1999) investment decision-making is described as an iterative process comprised of several steps: (1) Investment Policy refers to determining investor’s objectives and constraints, e.g. financial goals in terms of risk-return tradeoff, the amount of investable wealth, tax status, policy asset mix, investment benchmarks etc. (2) Security Analysis is the analysis of individual securities within previously identified broad asset classes, and can be either (a) technical (forecasting future price movements on the basis of historical data), or (b) fundamental (estimating the intrinsic price as the present value of future cash flows). (3) Portfolio Construction means determining in which assets to invest and what proportion of wealth. (4) Portfolio Revision is a periodic repetition of previous steps, as over time investors objectives and prices of securities can change. (5) Portfolio Performance Evaluation involves measuring the return and the risk of portfolio, as well as benchmarking.

The normative/prescriptive approach of standard investment books is essentially a top-down approach in which an important step is asset allocation, i.e. investment decision on the level of broad asset classes. Asset allocation can be strategic (based on long-term forecasts) or tactical (based on short-term forecasts). However, a fully bottom-up approach occurs in practice too. It focuses on specific assets that offer most attractive investment opportunities, without much concern for the resulting asset allocation, and as such may lead to a portfolio that is industry- or country-specific, or exposed to one source of uncertainty (Bodie et al., 2006). A descriptive model must be able to describe the behavior of investors who follow any of the two approaches to asset allocation, as well as various investment styles.

Investor behavior, like most human behaviors, can be conceptualized as goal-oriented, which means that investors make decisions in order to reach their various financial and non-financial goals. Goals are in the broad sense defined as mental (internal) representations of desired states (Austin and Vancouver, 1996). Here we study preferences, objectives, and constraints. Custers and Aarts (2005) summarize modern theories of motivation and goal-directed behavior: “the probability that a given goal state is set, adopted, and enacted depends on people’s ability (a) to mentally access the representation of the goal; (b) to subjectively assess the expected (or incentive) value of the goal state; (c) to activate, select, and execute instrumental actions; (d) to detect, assess, and reduce the discrepancy between the actual and desired state.” In this framework of Custers and Aarts (2005), positive affect linked to a goal representation is capable of directly feeding the motivation system, thus, propelling the goal pursuit behavior (aimed at attaining the desired state).

In a more standard framework, preferences (including risk attitude) are captured by an agent’s utility function, while the objective is the maximization of expected
utility. While economic agents are classically assumed to be self-interested, the list of motivators could be enriched by taking into account also social preferences, such as fairness, altruism, revenge, status seeking and survival.

3.2.5 Perception - Interaction - Action

Perception, interaction and action are processes which describe the way in which the investor interacts with the investment environment in which he or she is situated. Processes such as perception and action are commonly included in cognitive models (e.g. Sloman (2001)).

Perception is the process of acquiring information coming from various sources in the investment environment (classical media, such as television, radio, and paper news, as well as various Web applications). Through communication channels, other market participants (peers) can also serve as sources of information. Information can be quantitative or qualitative, and can include financial data, corporate news announcements, analysts recommendations, tips, etc.

Information processing of the acquired information could also be interpreted in the light of the dual-process systems. For instance, information could be used for controlled deliberative reasoning (e.g. whether it fits expectations), but also by an affective system (e.g. emotional response to price changes). Cognitive system is expected to deal with information processing by applying simplifying heuristics, while emotional system can be susceptible to hedonic editing and confirmation bias. These inputs from the environment are necessary to establish a feedback mechanism as the basis for learning. A dual-process view taken in this chapter suggests that learning processes are likely "a splice of cognitive and affective processes" (Camerer et al., 2005). Since literature on individual and social learning is vast, we also suggest a review paper by Brenner (2006), which presents different types of learning that could be implemented within intelligent economic agents.

Interaction is that part of the model which deals with peer influence and social factors, and can be introduced into the model as the formation of information cascades and herding type of behavior. In the study of Hoffmann et al. (2007), the agents are connected in a social network of two different topologies with different information diffusion characteristics. Such implementation also allows a study of agent’s (over)confidence, as each agent can give different weight to his or her own information relative to the aggregate information of other agents in the social network. Instead of information, agents can also exchange their strategies. This mimetic contagion of behavior (herding) can cause market fluctuations in the form of bubbles and crashes (Kirman, 1991; Topol, 1991). Agents can copy other agents’ strategies based on different reasons. For example, they can copy strategies of investors who are wealthier (this also captures the evolutionary idea that more successful strategies survive in the market). Switching between strategies can also involve some type of
uncertainty, as in the probabilistic switching mechanisms (Kirman, 1993). Finally, agents can differ in their personal propensity to imitate dominant behavior of people in their social network. Hoffmann et al. (2007) offer an explanation based on social psychology and people’s preference for simplifying or clarifying strategies, where simplifying means copying the behavior of others and clarifying means collecting information from others.

Action is performed when an investor wants to change his or her current portfolio. It can be characterized by the security that investors wants to buy or sell, a type of order, the size of the order, the timing, and other parameters characteristic to a particular type of order (e.g. price contingencies). The most common types of orders are market order and limit order. More about various types of orders and their properties can be found in Harris (2003). Our conceptual model can account for various underlying causes for a particular action, such as a trading strategy, peer influence, or an emotional response to financial news or price changes.

3.2.6 Personality

Psychological literature on personality has settled around a five-factor model (Digman, 1990): (1) Extraversion, (2) Agreeableness, (3) Conscientiousness, (4) Neuroticism, and (5) Openness. Recent studies have examined a possible influence of personality traits on financial decisions, particularly in the context of daily traders. Fenton-O’Creevy et al. (2004) conducted a study among 118 professional traders employed at investment banking institutions, and showed that successful traders tend to be emotionally stable introverts open to new experiences. Contrary to these results, Lo et al. (2005) found the lack of correlation between personality traits and trading performance. “This raises the possibility that different personality types may be able to function equally well as traders after proper instruction and practice. Alternatively, it may be the case that individual differences pertinent to trading success lies below the level that can be assessed through personality questionnaires, and may become visible only at deeper physiological and neuropsychological levels, or with a larger or more homogeneous sample of traders” Lo et al. (2005).

Given the current inconclusive results, the link between personality traits and investment performance might still be far-fetched. However, the relationship between personality and risk attitude, time preference, investment strategies, or susceptibility to particular behavioral biases might be relevant for practical investment purposes - especially given the availability of various batteries for testing personality types, and given the stability of personality traits during a long period of a lifetime.

The link between personality traits and risk propensity has been fairly studied in the literature. McCrae and Costa (1996) found sensation-seeking, a sub scale of the Extraversion dimension, to be highly correlated with most risk-taking domains, while overall risk propensity was higher for subjects with higher Extraversion and
Openness scores and lower for subjects with higher Neuroticism, Agreeableness, and Conscientiousness scores (Lo et al., 2005). Zuckerman and Kuhlman (2000) studied the relationship between personality types and risk-taking behavior in various domains: smoking, drinking, drugs, sex, driving, and gambling. They found impulsive sensation seeking, sociability, and aggression to be related to risk-taking behavior, whereas neuroticism/anxiety and activity were not related.

The influence of personality traits has also been studied in various game-theoretical experiments, as summarized by Lo et al. (2005): “For example, higher extraversion and emotional stability—the opposite of neuroticism—appear to be related to a higher level of stability in intertemporal consumption patterns (Brandstätter and Güth, 2000). In Dictator and Ultimatum games, higher benevolence as a personality trait facilitated more equitable choices in offers to powerless opponents, and reciprocity orientation induces powerful recipients to set higher acceptance thresholds (Brandstätter and Güth, 2002). Greater internal locus of control, better self-monitoring ability, and higher sensation-seeking have all been linked to higher levels of cooperative behavior in Prisoner’s Dilemma experiments (Boone et al., 2002).”

3.2.7 Demographics and Other Factors

Demographics have been included in the conceptual model since they encompass variables which can have significant effects on investor behavior and cognition. Due to their availability, demographics variables are often used in research as explanatory variables. In the behavioral finance literature, for example, gender has been used as a proxy for overconfidence, since men have been found to exhibit this bias to a greater extent than women (see Odean (1999)). The research also suggest that an individual’s risk attitude can be explained by a number of demographic variables, including gender, education level, age and income (Donkers et al., 2001).

There are probably many other factors that can influence investor behavior which have not been explicitly and individually included in the conceptual model. For instance, they could include activities outside of the investment world or any exceptional events that can have impact one investors’ cognition and behavior. Under the assumption that such events happen randomly and independently, one could capture their effects by means of a Gaussian noise. Noise is sometimes included in computational models in order to account for variability in investor behavior caused by such events (for example, see the implementation of the so-called EMB investors in the model of Levy et al. (2000) used in Chapter 5 through Chapter 8).

3.3 Investment Environment

The investment environment in which investors operate (Figure 3.2) is based on the description of trading industry in Harris (2003). Harris defines traders as people who
trade, and who may either arrange trades for themselves, have other people arrange trades for them, or arrange trades for others. Accordingly, he distinguishes between proprietary traders who trade for their own account and brokerage traders (such as agency traders, commission traders, commission merchants) who arrange trades as agents for their clients. This distinction is particularly important as it can be expected that traders who trade with other people’s money have different incentives and goals than those who trade for their own account. Furthermore, trading industry can according to Harris be grouped into two sides of traders: Buy side - traders who buy exchange services, such as liquidity, or the ability to trade when they want (e.g. individuals, funds, firms, governments); and Sell side - traders who sell exchange services to the buy side (e.g. dealers, brokers, and broker-dealers).

According to Harris, investors are seen as traders who trade in order to move their wealth from the present to the future. Borrowers, hedgers, asset exchangers, and gamblers, on the other hand, have other incentives for trading. What is interesting in Harris’ taxonomy is that a trader is a generic term that encompasses all market participants. From another perspective, a trader, such as a day-trader, could be seen as a special type of investor who has a very short investment horizon. In such a context investors are usually market participants who have a longer investment horizon and employ some sort of portfolio management. Agent-based literature also does not always precisely define what is meant by investors or traders, so terms such as investors, traders, and agents are often used interchangeably. What we have in mind in this conceptual model is a broader definition of investor (which includes different types of traders), as we think that various classes of market participants could be represented by an appropriate instantiation of the elements of the model, particularly goals, strategies, and time horizons.

Trade facilitators are institutions that help traders trade (Harris, 2003). Exchanges provide forums for traders (dealers, brokers, and buy-side traders) to meet and arrange trades. At most exchanges only members can trade, while non-members trade by asking member-brokers to trade for them. Historically, traders used to meet at the trading floor, but now they meet via ECNs (electronic communication networks). At some exchanges traders arrange trades when they see fit, while other exchanges have order-driven systems that arrange trades by matching buy and sell orders according to a set of rules. OTC (over the counter) is trading that occurs outside exchanges, arranged by dealers and brokers. Trade facilitators which help traders settling their trades are known as clearing agents, settling agents, and depositories and custodians (Harris, 2003).

Trading instruments can be organized into asset classes, such as real assets, financial assets, derivative contracts, insurance contracts, gambling contracts, and hybrid contracts (Harris, 2003). Real assets include physical commodities, such as real estates, machines, or patents and other intellectual properties; Financial assets include instruments that represent ownership of real assets and the cash flows that they pro-
3.3. Investment Environment

Figure 3.2: The investment environment

Derivative contracts encompass instruments that derive value from the value of underlying instruments upon which they are based; Insurance contracts are instruments that derive value from future events related to losses; Gambling contracts are instruments that derive value from future events related to gains; and Hybrid contracts are instruments that embody elements of more types of instruments. These classes can further be divided into subclasses. For instance, financial assets can be divided into domestic and international, based on size (big, mid, small caps), based on style (value, blend, growth), etc. These asset classes have an important role in the process of asset allocation. By studying the behavioral finance literature, we can observe that most of the literature focuses on financial assets, particularly on equities. The same can be said about the agent-based literature, since most of the artificial financial markets are actually artificial stock markets. Nonetheless, modeling other types of financial assets could be relevant when the goal is a more realistic representation of the investment environment.

The investment environment (Figure 3.2) consists of a number of market participants, each of which can be represented by the cognitive model of an individual investor. One investor is represented inside the investment environment to show how he or she would interact with other elements of the environment. The investor is endowed with a portfolio that consists of a number of different assets. We can see three arrows coming to/from the investor which correspond to the three arrows of the perception-interaction-action interface of the cognitive model (Figure 3.1). The investor is perceiving information (e.g. about the market value of assets in his or her portfolio) and using that information to update believes and make investment deci-
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The investor is connected to peers with whom there is a bidirectional exchange of information or strategies (e.g. herding type of behavior). Actions the investor can take include different types of orders, which are submitted to the intermediaries and which, if executed, result in an updated portfolio.

3.4 Conclusion

Studying behavior of market participants is important because of its potential impact on asset prices. Agent-based simulation, as a methodology often used in studies of complex systems, can bridge this gap between the micro level of individual investor behavior and the macro level of aggregate market phenomena. Since agent-based simulation constitutes a bottom-up approach, we need to start from a realistic description of the behavior of market participants. For that purpose we looked at the behavioral finance and psychological literature with an aim to describe how individual investors behave in the markets. The conceptual model of individual investor behavior presented in this chapter aims to summarize a part of this vast knowledge on individual investor behavior, and gives the first level of structure necessary for the development of agent-based artificial financial markets.

The aim of this model is descriptive, that is, we want to be able to capture and describe observed behavioral phenomena. However, these behavioral phenomena stem from a wide variety of cognitive mechanisms and other sources, which makes it difficult to integrate them into a simple and parsimonious framework. It is a rather ambitious task to determine the nature of all these mechanisms and variables. Nonetheless, it is possible to indicate some of the relationships by finding evidence in the existing behavioral finance literature.

A descriptive model of individual investor behavior presented in this chapter sees investment decisions as an iterative process of interactions between the investor and the investment environment. This investment process is driven by dual mental processes (cognitive and affective), and the interplay between these systems contributes to boundedly rational behavior which manifests itself through various heuristics and biases. In the modeling tradition of cognitive science and artificial intelligence, the investor is seen as a learning, adapting, and evolving entity that perceives the environment, processes information, acts, and updates its internal states. Finally, the investor behavior is influenced by social interactions.

In the next chapter we will give an overview of some of the existing agent-based models of financial markets. We will look at them in terms of the proposed conceptual model in order to see what aspects of individual investor behavior and investment environment have been taken into account.
Chapter 4

An Overview of Agent-Based Artificial Financial Markets

4.1 Introduction

A novel bottom-up approach to studying and understanding stock markets comes from the area of computational finance as artificial financial markets (or, more specifically, as artificial stock markets). Agent-based artificial financial markets can be mathematical or computational models, and are usually comprised of a number of heterogeneous and boundedly rational agents, which interact through some trading mechanism, while possibly learning and evolving. These models are built for the purpose of studying agents’ behavior, price discovery mechanisms, the influence of market microstructure, the reproduction of the stylized facts of real-world financial time-series (e.g. fat tails of return distributions and volatility clustering). A number of reviews of studies with artificial financial markets are available, e.g. for computational models (LeBaron, 2006), and for mathematical models (Hommes, 2006).

A similar approach to studying economies has become known as agent-based computational economics (ACE) - the computational study of economies modeled as evolving systems of autonomous interacting agents (Tesfatsion, 2006). ACE researchers use computational laboratories to study the evolution of decentralized market economies under controlled experimental conditions. The construction of economy starts with an initial population of agents - both economic agents (e.g. traders, financial institutions, etc.) and socio-environmental agents (e.g. government, land, weather, etc.). The initial state of the economy is specified as initial attributes of the agents, such as type characteristics, internalized behavioral norms, internal modes of behavior (including modes of communication and learning), and internally stored information about itself and other agents. The economy then evolves over time and
arises from agent interactions, without further intervention from the modeler (Tesfatsion, 2006).

Analogous methodology to agent-based modeling comes from the physical sciences, namely the Microscopic Simulation. This methodology is a tool for studying complex systems by simulating many interacting microscopic elements, and has been also applied to financial markets by Levy et al. (2000). They believe that Microscopic Simulation models could be used to extend existing analytical models in finance by inspecting the role of their assumptions, or to build new models that could be as realistic as desired, e.g. models that incorporate various technical and fundamental strategies observed in experiments and real markets; dynamic models with heterogeneous investors that can learn and change strategies.

Despite many studies with artificial financial markets, not many attempts have been made to incorporate complex behavioral phenomena into agents’ behavior. These attempts may have been hindered by multiple reasons. For instance, complex behavior implies highly parameterized models which are difficult to examine and that often lie beyond analytical tractability. Some studies implement very simple behavior (e.g. zero-intelligence agents with budget constraints by Gode and Sunder (1993)) as they want to put more emphasis on other issues, such as the market microstructure. Also, sometimes it is interesting to examine what kind of complexity can emerge from very simple behavior, with the addition of some heterogeneity, interaction, and/or learning. The famous spatial proximity model by Schelling (1978) is an example of an early agent-based work where unexpected aggregate pattern of segregation appears on the macro level even though it was not coded as such in the micro-level behavior of agents. Sugarscape model (Epstein and Axtell, 1996) is another example of a simple local behavior that leads to interesting macro patterns. The idea of complexity which emerges from nonlinear interactions between heterogeneous components forms the foundation of Complex Adaptive Systems (CAS), which is closely related to the agent-based modeling approach (Figure 4.1).

However, an agent-based approach inspires us to seek for homeomorphic models (Harré, 1970), that not only reproduce the stylized facts of real-world markets, but also achieve them through processes that are grounded on reasonable (psychologically plausible) assumptions, and resemble actual human behavior and realistic market mechanisms. “Agent-based models can easily accommodate complex learning behavior, asymmetric information, heterogeneous preferences, and ad hoc heuristics” (Chan et al., 1999). It is far from the truth that everything is known about human behavior and cognition pertaining to the investment decisions, but as various fields (such as neuroeconomics and neurofinance) progress to open up these black boxes, a methodology that can utilize such knowledge may be given more opportunities in the future (see Chen, 2008).

Figure 4.1 compares the properties of analytical (or mathematical) and computational agent-based market models. Analytical models can be more related to
4.1. Introduction

Figure 4.1: Complex Adaptive Systems view on agent-based market models.

Figure 4.2: Distinction between analytical and computational agent-based artificial markets.
4.2 Modeling Aspects of Artificial Financial Markets

Even though the agent-based modeling approach allows for the implementation of arbitrary complex agent behaviors, perhaps such that would implement many aspects of the conceptual model proposed in Chapter 3, most agent-based artificial markets developed so far have focused on relatively simple agent behaviors. It seems that many artificial market models subscribe to the Complex Adaptive Systems view on systems modeling, according to which simple local behaviors with the addition of interaction and nonlinearity are sufficient to give rise to the emerging complexity in the system.

Artificial financial markets that are based on a small number of highly stylized behaviors have been labeled as few-type models (LeBaron, 2006). Typically, strategies (or agents who employ them) can be divided into two groups: fundamental who trade based on a perceived fundamental value of an asset, and technical who trade based on the past prices, e.g. some form of trend extrapolation. In addition, many models have been rooted in the zero-intelligence framework in which agents basically trade randomly, possibly subject to a budgetary constraint. Sometimes a small number of such agents is included into a few-type model in order to provide liquidity for other agents.

Depending on their type, agents will have access to different types of information. Typically, all agents will have information about the current market price, which will allow them to update their wealth status and possibly tune their strategies (in case learning mechanisms are employed). They will also receive news, such as dividends.
4.2. Modeling Aspects of Artificial Financial Markets

on risky assets and interest on risk-less assets. However, some agents will also have information about the determinants of the fundamental price, such as the dividend generating process. Depending on their strategies, agents will translate their price expectations or predictions into orders or demand functions, which will be cleared using a specific market mechanism, and finally a new market price will be formed.

In agent-based financial markets, the behavioral aspects of individual agents are not determined only through fixed strategies, but also through learning (evaluating and updating strategies based on the past performance) and social interaction (exchanging/combining information and/or strategies with other market participants, which can lead to herding type of behavior). Such aspects are sometimes conceptualized and modeled using the paradigm of genetic algorithms. This idea is explored in the many-type models, where the pool of strategies is co-evolving with market conditions in order to see which ones will survive and which ones will fail (LeBaron, 2006). Depending on their strategies, investors can use different information sources to assist their decision making and update their expectations. All investors are informed about the current market price which can be used to update the value of their portfolio and sometimes evaluate their strategies. Often there is an information asymmetry in the sense that some investors also know the fundamental price or the properties that allow them to estimate it (e.g. dividend process). When investors receive different news about the traded assets, they could also resort to their social connections in order to obtain information from their peers (see Hoffmann et al., 2007).

The investment environment presented in the conceptual model of Chapter 3 and showed on Figure 3.2 describes a number of asset classes existing in real markets. Asset classes modeled in agent-based markets are usually only a few, and they typically include risky assets (stocks), risk-less assets (bonds), and cash (Figure 4.3). In addition, many agent-based models generate and study time-series of only one risky asset. At first, this may seem as a very strong restriction, especially for models that aim to be realistic. However, that is not necessarily a strong restriction, if the focus is on modeling the financial market as a whole, in which case the risky asset could be interpreted as a market index. Furthermore, in many artificial markets generalizations to multiple assets would be straightforward. However, a more realistic approach with a multi-asset environment would introduce more intricacies into the model, which would stem from the correlations among those assets. For example, it would open the door to more complicated strategies and behaviors of agents.

Modeling trade facilitators shown on Figure 3.2 is a challenge itself, particularly if one wants to model financial exchanges as realistic as possible. Most existing agent-based studies, however, do not go into that level of detail, and instead use simple and more abstract market mechanisms. These mechanisms are pricing mechanism which translate the demand and supply of market participants (e.g. market orders, limit orders, demand and supply functions) into a price at which trade occurs, i.e.
Chapter 4. An Overview of Agent-Based Artificial Financial Markets

Figure 4.3: Investment environment in agent-based artificial markets

the market clears. Figure 4.3 shows four of such market mechanisms, which are described in LeBaron (2006). They include:

1. **Temporary market equilibrium** is a market clearing mechanism where the price is determined so that the total demand of agents equals the total number of shares in the market. Given that the demand of each agent typically has to be determined for different hypothetical prices until the equilibrium price is found, this mechanism can be computationally costly. This is particularly the case if the demand functions do not allow for an analytic solution, in which case a numerical solution needs to be sought.

2. **Price impact function** formalizes a price adjustment process in which the new price is determined from the past price and net order (the difference between demand and supply) scaled by a fixed parameter $\alpha$:

$$ p_{t+1} = p_t + \alpha(D(p_t) - S(p_t)). $$

(4.1)

This captures the basic intuition that excess demand raises the price, while excess supply lowers the price. The main advantage of this pricing mechanism is that it is computationally very fast, while some of the disadvantages are that the price changes are very sensitive to the choice of the liquidity parameter $\alpha$, and the issue of who fills in the excess demand or supply in the market (LeBaron, 2006). A variant of this mechanism is a log-linear price impact function (Farmer and Joshi, 2002), which additionally ensures that the price remains positive.
3. *Order book* is a pricing mechanism where actual *order book* is simulated, and buy and sell orders of agents are crossed using a certain well-defined procedure. This approach can be considered more realistic because it allows a detailed analysis of trading mechanisms (LeBaron, 2006). This particularly contrasts with the equilibrium-based mechanism where explicit trading is not modeled.

4. *Matching* is the type of mechanism where agents (randomly) meet, and, if it suits them, trade with each other. This mechanism may be appropriate for situations where no formal trading markets have been established, so there are no institutions to help buyers meet sellers in a less-than-random fashion (LeBaron, 2006).

### 4.3 Statistical Properties of Artificial Stock Markets

One way of examining artificial stock markets is by comparing their outputs (i.e. generated time-series) with real-world financial data. Assessing to what extent these artificial time-series are realistic is usually done by finding similarity between their statistical properties and the statistical properties of empirical data. A set of properties, found across many financial instruments, markets and time periods, has been observed by independent studies and classified as *stylized facts* (Cont, 2001). An overview of stylized facts observed in asset returns or prices can be found in Cont (2001), and they are listed below:

- **Heavy tails.** The unconditional distribution of returns tends to be non-Gaussian, sharp peaked and heavy tailed (see Figure 4.4).

- **Gain/loss asymmetry.** It is possible to observe large downward movements in stock prices and stock index values, but not equally large upward movements (an exception are exchange markets which are quite symmetrical).

- **Aggregational Gaussianity.** The shape of the distribution is not the same for all time scales; as the time scale is increased (returns are calculated over a longer period) the distribution becomes more normal.

- **Conditional heavy tails.** Returns that have been corrected for volatility clustering still exhibit some degree of heavy tails.

- **Absence of autocorrelations.** (Linear) autocorrelations in asset returns are often insignificant, except for very high-frequency data where microstructure starts playing a role.
• **Volatility clustering.** Nonlinear functions of returns (e.g. absolute or squared returns) exhibit significant positive autocorrelation. In time-series of asset returns this can be seen as periods of higher volatility clustered together (see Figure 4.5).

• **Slow decay of autocorrelation in absolute returns.** Autocorrelation function of absolute returns decays in time approximately as a power law.

• **Intermittency.** At any time scale, returns display a high degree of variability.

• **Leverage effect.** There is a negative correlation between the volatility of returns and returns themselves.

• **Volume/volatility correlation.** Trading volume is correlated with volatility measures.

• **Asymmetry in time scales.** Coarse-scale volatility measures predict fine-scale volatility measures better than the other way round.

Cont (2001) points out that those stylized facts should all be considered as mostly qualitative properties of asset returns since they may not be precise enough to distinguish between different quantitative models (for example, many distributions can be used to fit heavy-tailed data). Nonetheless, even as qualitative, these properties can be quite constraining, since it is difficult to come up with stochastic processes or models that can produce a lot of them at the same time Cont (2001).

The existing artificial financial markets differ in the stylized facts they can reproduce, and the most commonly reported ones are heavy tails of return distributions, volatility clustering in time-series of returns and/or the occurrence of bubbles and crashes (see Boer-Sorban, 2008). Some models are able to produce even more refined features, such as the power-law tails of the return distributions, thus concretizing the general notion of fat tails (see Samanidou et al., 2007).

Developing models of financial markets that are able to produce realistic outputs which are qualitatively similar to the empirical data is important, but it constitutes only one possible goal of these simulations. Different models can be focusing more on other aspects of agent-based modeling, such as realistic representations of agents’ behavior, their interactions, or realistic market mechanisms. Incorporating realistic elements of agents’ behavior can be achieved by implementing those behaviors that have been observed and documented in behavioral finance literature, which is the approach taken in this thesis.

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4.3. Statistical Properties of Artificial Stock Markets

Figure 4.4: The histogram of daily returns (1/7/1963 - 30/6/2010) on the US equity market (NYSE, AMEX, NASDAQ).

Figure 4.5: Time-series of daily returns (1/7/1963 - 30/6/2010) on the US equity market (NYSE, AMEX, NASDAQ).
Chapter 4. An Overview of Agent-Based Artificial Financial Markets

4.4 Examples of Artificial Financial Markets

In this section, we give an overview of a number of existing agent-based models of financial markets, and explain them in terms of the proposed conceptual model. We emphasize those elements of the conceptual model that have been taken into consideration in the models, and also summarize the main results of the simulations. Even though most agent-based models incorporate some behavioral aspects into the agents’ implementation, the last two examples given in this overview are interesting because they have explicitly accounted for a number of behavioral finance topics. Overviews of artificial financial markets from different perspectives can also be found in LeBaron (2006), Levy et al. (2000) and Boer et al. (2005).

1. Kim and Markowitz - Portfolio Insurers Model.

Although the model of Kim and Markowitz (1989) is not the first computational study in the area of finance, it is considered one of the first modern agent-based models of the financial market. The motivation for this model was the stock market crash of 1987, and the main focus of the study was exploring the link between portfolio insurance strategies and market volatility.

**Investment Environment.** In this model the market consists of two types of investors: Rebalancers and Portfolio Insurers. There are two *asset classes* in the market: a risky stock and cash with zero interest. At the beginning of the simulation all investors are endowed with the same value of the portfolio, allocated half in stock and half in cash. The pricing mechanism is based on the order book. Buy and sell orders are stored in the order book and executed in the case of a match (or kept until the end of the trading day).

Individual Investors. The strategy of Rebalancers is to keep the same proportion of their wealth in stocks (50%) and in cash (50%). The strategy of Portfolio insurers is based on the so-called Constant Proportion Portfolio Insurance of Black and Jones, according to which the proportions of wealth in stock is kept in a constant proportion to the cushion (the value of the portfolio above some minimal level of wealth called floor). *Timing* in the model is discrete with trading happening at random time points and with each agent reviewing his or her portfolio at random intervals. Investors do not interact directly amongst themselves. However, they can *perceive* all orders placed by other investors and use that information to form their prediction of the price. If their allocation according to a predicted price does not match their portfolio strategy, they will *act* by issuing a buy or sell order. Other exogenous influences on investors are modeled as randomly occurring withdrawals or deposits of random amounts of cash.

**Findings.** The main result of the simulations is that portfolio insurance strategy can have a destabilizing effect on the market, therefore, providing a possible explanation for the occurrence of the market crash.

This paper of Brock and Hommes (1989) investigates market dynamics in a simple present discounted value asset pricing model with heterogeneous beliefs. The authors investigate possible bifurcation routes to complicated asset price dynamics, by using a mixture of bifurcation theory and numerical methods. A few simple belief types are considered in the experiments.

**Investment Environment.** Investors can choose between two asset types: one risky asset and one risk free asset. The risky asset is paying a dividend that is exogenously given as a stochastic process. Price fluctuations in the market are driven by an evolutionary dynamics between different expectation schemes.

**Individual Investors.** Agents choose from a finite set of predictors of future prices of a risky asset and revise their beliefs in each period. Predictor selection (i.e. strategy selection) is based upon a fitness or performance measure such as past realized profits. Intensity of choice is a parameter measuring how fast agents switch between different prediction strategies. If intensity of choice is infinite, the entire mass of traders uses the strategy that has highest fitness. If intensity of choice is zero, the mass of traders distributes itself evenly across the set of available strategies. In terms of their *goal* all investors can be characterized as myopic mean variance maximizers with a homogeneous degree of risk aversion. Simple belief types that are characterized in experiments are: trend chasers, contrarians, upward biased type, downward biased type, fundamentalists (who believe prices revert to their fundamental value) and rational agents with perfect foresight (who not only know past prices and dividends, but also the market equilibrium equation and fractions of other types in the market).

**Findings.** Brock and Hommes (1989) present numerical evidence of chaotic attractors when the intensity of choice to switch prediction strategies is high. The paper shows how an increase in the intensity of choice to switch predictors can lead to market instability and the emergence of complicated dynamics for asset prices and returns. This includes irregular switching between phases where prices are close to the fundamental value, phases of optimism where traders extrapolate upward trends, and phases of pessimism where traders are causing a sharp decline in asset prices (Brock and Hommes, 1989).


Levy, Levy, Solomon model (LLS) is a prominent model of the financial market based on the microscopic simulation approach which has roots in physics. It is a numerical model developed in the framework of expected utility maximization. In this thesis we have focused on the variant of the model presented in Levy et al. (2000).

**Investment Environment.** In the LLS model, the market consists of two types of investors: Rational Informed Investors (RII) and Efficient Market Believers
There are two asset classes in the market: a risky stock that pays a dividend following a multiplicative random walk and has a finite number of outstanding shares; and a risk-less bond that pays a sure interest and has an infinite supply. At the beginning of the simulation all investors are endowed with the same amount of wealth that is comprised of cash and a number of shares. The trade facilitators and the sell side of financial services are not explicitly modeled in the LLS model. Instead, the pricing mechanism based on the temporary market equilibrium (LeBaron, 2006) is used to determine the price in such a way that the total demand for the risky asset equals the total number of outstanding shares.

Individual Investors. The goal of all the investors in the LLS model is the maximization of the expected utility of the next period wealth. The risk attitude of the investors is risk aversion, and is captured by the parameter of the utility function. In the LLS model of Levy et al. (2000), a myopic power utility function with DARA (Decreasing Absolute Risk Aversion) and CRRA (Constant Relative Risk Aversion) properties is used. Due to this myopia property of this utility function, it can be assumed that investors maximize their one-period-ahead expected utility, regardless of their actual investment horizon (Levy et al., 2000). An additional temporal characteristic of EMB investors is their memory length of past return realizations which are used in the prediction of future returns. Even though both RII and EMB investors have the same goal of expected utility maximization, their strategies are different because of the differences in information that they possess. RII investors know the properties of the dividend process, and can estimate the fundamental price of the risky asset as a discounted stream of future dividends. That fundamental price is used in their prediction of the next period return. EMB investors, however, do not know the dividend process, and must use ex post distribution of returns to estimate ex ante distribution. EMB investors use a rolling window of a fixed size, and in the original model are called unbiased if, in the absence of any additional information, assume that returns come from a discrete uniform distribution. In each period investors perceive information about the new price and new dividends, which they can use to update their wealth status. Both types of investors are expected utility maximizers, which is considered as the cornerstone of rationality. Nonetheless, with EMB investors some noise is added to the optimal proportion to account for other factors that could cause such departures from optimal behavior (Levy et al., 2000). In the LLS model, investors do not interact in the sense of the exchange information or strategies. If there is some volume (the change in portfolio holdings) of an individual investor, we can say that some trading occurred because the shares have exchanged hands. However, in this model we are not concerned with how exactly that happened. Actions, such as market orders, are not modeled explicitly,
since this is a model based on the temporary market equilibrium.

**Findings.** One of the main results of the simulations is that investors who use past information create cyclic bubbles and crashes which can be related to the size of their memory window. This happens in the case when investors are homogeneous with respect to their memory lengths. When investors are heterogeneous in memory lengths, the market dynamics becomes more realistic in the sense that it does not display such prominent and semi-predictable bubbles. Levy et al. (2000) have also applied this model to investors who are characterized by Prospect Theory type of preferences.

4. **Lux and Marchesi - Stochastic Interaction and Scaling Laws.**

The model of Lux and Marchesi (1999) is an agent-based model of the financial market that follows the tradition of earlier attempts to capture herding behavior by means of stochastic modeling. An example of this is the ant recruiting model of Kirman (1993), which has also been proposed as an analogy for herding behavior of investors in the financial markets.

**Investment Environment.** The market consists of two types of investors, fundamentalists and non-fundamentalists, and two types of investments, a risky stock and a risk-free asset. In addition, the risky asset pays out a stochastic dividend at the beginning of each period. The *market mechanism* is based on the price adjustment process which determines the market price based on the differences between the supply and the demand.

**Individual Investors.** According to their type of *strategy*, agents can be either fundamentalists or chartists (which are further divided into optimists (buyers) and pessimists (sellers)). In addition, there is a probabilistic switching between these groups of agents. The probabilities of switching between two types of chartists is based on the majority opinion and current price trend, while the probabilities of switching between chartists and fundamentalists is based on the observed differences in profits.

**Findings.** The model is able to generate the following properties of the market prices: unsystematic deviations of the market price from the fundamental price, heavy tails of return distributions, and volatility clustering.

5. **Takahashi and Terano - Investment Systems Based on Behavioral Finance.** Although various behavioral aspects of agents, including investor biases, have been studied in earlier literature, the model of Takahashi and Terano (2003) is to our knowledge one of the first agent-based models that explicitly studied a number of investor biases proposed in the behavioral finance literature.

**Investment Environment.** The market consists of two types of investors, fundamentalists and non-fundamentalists, and two types of investments, a risky stock and a risk-free asset. In addition, the risky asset pays out a stochastic
dividend at the beginning of each period. Market mechanism is based on the temporary market equilibrium, i.e. the traded price of the stock is derived so that the demand meets the supply.

*Individual Investors.* Fundamentalist investors use a *strategy* based on the fundamental value of the stock, which they estimate using a dividend discount model. Non-fundamental investors are trend predictors, and since they use past information to predict future prices, they could also be classified as technical traders or chartists. Both fundamental and non-fundamental investors predict next period price and dividend. Non-fundamental investors have additional *temporal* parameters, in the sense of short-term, medium, or long-term trend predictors. Two types of *biases* are studied within the model, overconfidence of investors and loss-aversion. There is no social *interaction* between various investor groups.

*Findings.* When the market consists of the same number of fundamental and technical traders, the market price agrees with the fundamental price. However, when there is a large fraction of technical traders in the market, the market price deviates largely from the fundamentals and fundamentalists are eventually eliminated from the market. There are also deviations from the fundamental price in the case of overconfident investors and when non-fundamentalists act asymmetrically towards losses.

6. **Hoffmann et al. - SimStockExchange Model.**

Following the tradition of Takahashi and Terano (2003), the paper of Hoffmann et al. (2007) is another study that combines a number of behavioral phenomena within an agent-based simulation of the financial market. Hoffmann et al. (2007) focus especially on the social aspects of investor behavior and study consequences of two distinct network topologies.

*Investment environment.* Asset classes available in the market are one risky stock and cash. The *market mechanism* is based on the order book. Using the rules of this mechanism, the limit orders submitted by agents are mutually crossed and executed. The market price is calculated as the average of the bid and ask prices, weighted by the number of asked and offered shares. The model of Hoffmann et al. (2007) also models the *news* arrival process, as a normally distributed noise around the current price.

*Individual Investors.* The investors are characterized by their level of confidence, which determines how much their private information (price expectation) is weighted compared to the expectations of their neighboring investors. *Strategy* used by investors is based on the comparison between the current market price and the expected market price of the stock: when the expected price is higher than the current price, it is attractive to invest in stock, and when the expected price is lower, it seems attractive to divest. The strategy also determines the proportion of cash to invest or the proportion of stock to divest. The
perception of risk depends on investors’ level of confidence. Investors who have high confidence perceive lower risk, while those investors who have lower levels of confidence perceive high risk and apply risk reducing strategies, which can be a simplifying strategy (heuristic) or a clarifying strategy (collecting more information). Time perspective of investors is myopic. They base their decisions only on the currently available information and expectations for the next period. Agents perceive information about the current market price, the news about the stock, and the price expectation of investors in their social network, which are all finally translated into their own market price expectation. The model of Hoffmann et al. (2007) is interesting as it pays special attention to the social interaction between agents. In different experiments the investors are connected in two different types of social networks which are used for the dissemination of market price expectations. Investors who exhibit social type of simplifying risk reducing strategy copy the behavior of other investors in the social network, while those who exhibit social type of clarifying risk reducing strategy ask other investors for more information. The investors act by sending limit orders, which consist of the number of shares that they want to buy or sell, and the limit price which is set to their expected price.

Findings. The results of the simulations indicate that the structure of the social network of investors influences the dynamics of the prices. When investors were forming a Barabasi and Albert scale-free network, there was no indication of volatility clustering in the market, but when they were forming a torus network, such evidence was found. The authors speculate that networks of investors may behave more like torus networks with respect to information diffusion, and that information may sometimes take longer to travel to distant parts of the networks, allowing the old shocks to influence the presence for a considerable period of time (Hoffmann et al., 2007).

4.5 Conclusion

In this chapter we have given an overview of a number of well-known agent-based artificial financial markets. Agent-based artificial financial markets are bottom-up models of financial markets which allow us to study their dynamics and emerging properties. They do so by focusing on the behaviors of individual market participants and well-defined market mechanism which are able to translate those behaviors into artificially generated asset prices. One of the aims of these models is to reproduce realistic asset prices which contain the stylized facts of real-world time-series, such as leptokurtic returns, volatility clustering, bubbles and crashes etc.

Since agent-based modeling constitutes a bottom-up approach, deciding on the elements of agents’ behaviors to implement and which market mechanism to use is of uttermost importance. Most agent-based models use very stylized market mech-
anisms, as well as highly stylized representations of investor behaviors (e.g. fundamental, technical, zero-intelligence agents). However, complexity can easily be introduced into the models by, for example, allowing the agents to learn, co-evolve with the market, or (stochastically) switch between different strategies. The work of Boer et al. (2007) is an extension of a discrete-time agent-based financial market (as most models are) into a continuous-time asynchronous model, and it shows that a continuous-time framework entails much more intricate market dynamics. Some of the newest developments in the field of agent-based financial markets are to look for inspiration into the behavioral finance literature, especially for various behavioral biases of investors. This thesis aims to contribute to this literature.

In the following chapters we will present models of various behavioral biases and study them within an existing agent-based financial market, namely the market of Levy et al. (2000), also known as the LLS model. We will employ the incremental approach according to which an existing computational model is firstly replicated and then a new behavior is introduced into the model. By comparing the results of the original model with the results of the incremented model, we can study the implications of the newly introduced (biased) behaviors of investors.
Chapter 5

Overconfident Investors in the LLS Financial Market*

5.1 Introduction

Overconfidence as a judgmental bias has received a lot of attention in financial studies. It has been proposed as an explanation for the observed high levels of trading in financial markets, as well one of the causes for poor investor performance: some investors trade too much, which might be a manifestation of their overconfidence (Odean, 1999). The behavioral foundation for including overconfidence in financial studies is very strong, given the evidences that most people exhibit overconfidence in many different contexts. However, the problematic aspect of early financial studies on overconfidence is that overconfidence was not directly measured (instead, a gender variable was used as a crude proxy), so it was difficult to establish the direct link between overconfidence and overtrading (Glaser and Weber, 2007).

In order to study the effects of overconfidence, one could measure investor overconfidence by means of psychometric tests (survey responses) and relate those measures to trading records of the same persons. Such studies have recently become available (e.g. the study of Glaser and Weber (2007)) and they shed important light on a number of behavioral phenomena, including overconfidence. On another note, they also show challenges in conducting empirical research on behavioral finance topics, which stem from the data collection efforts needed to close the gap between the micro-level investor behavior and the macro-level effects of those behaviors. Such

Chapter 5. Overconfident Investors in the LLS Financial Market

Micro-macro mapping is one of the main advantages of agent-based simulations of financial markets.

An important question for all studies on investor confidence is what type of overconfidence is under study. A type of overconfidence that has been prevalently considered in theoretical and computational studies is the *miscalibration*, which means overestimating the precision of own information, e.g. by setting too narrow confidence intervals in the assessment of the value of a financial asset. In the experimental market of Biais et al. (2005), miscalibration was found to have an effect of reducing trading performance. One of the first papers to explicitly study a number of behavioral biases in an agent-based model of the financial market (Takahashi and Terano, 2003), also models overconfidence as miscalibration, more specifically as underestimated variance of stock returns. In this chapter we make a similar implementation of overconfidence in the sense of miscalibration, on top of an existing artificial market model of Levy et al. (2000).

Agent-based artificial financial markets (or, more specifically, artificial stock markets) are models for studying the link between individual investor behavior and financial market dynamics. A similar bottom-up approach has been used in agent-based computational economics (ACE) - the computational study of economies modeled as evolving systems of autonomous interacting agents Tesfatsion (2006). A methodology analogous to agent-based modeling also comes from the physical sciences as the Microscopic Simulation - a tool for studying complex systems by simulating many interacting microscopic elements Levy et al. (2000).

Since agent-based models can easily accommodate complex learning behavior, asymmetric information, heterogeneous preferences, and ad hoc heuristics (Chan et al., 1999), it seems that such simulations could be particularly suitable to test and generate various behavioral hypotheses. This complementarity of behavioral finance research and the agent-based methodology has been recognized in the literature and it is a nascent research with many opportunities ahead. Rare examples of agent-based papers that pursue the idea of explicit accounting for behavioral theories in financial market simulations are Takahashi and Terano (2003) and Hoffmann et al. (2007). In Takahashi and Terano (2003) the focus is on overconfidence and loss aversion, while Hoffmann et al. (2007) focus on social dimensions of investor behavior.

The outline of the chapter is as follows. Section 5.2 explains the basics of the LLS model in which we implement the investor overconfidence. In Section 5.3 we show replicated results of the original LLS model (Levy et al., 2000). Section 5.4 presents the results of the simulation experiments. Section 5.5 concludes the chapter and discusses possible extensions for the future research. The appendix of this chapter explains some implementation details of our simulations, including the pseudo-code of the main program and the pricing mechanism. It also includes extended replication and simulation results.
5.2 Model Description

The model is based on the LLS microscopic simulation model of Levy et al. (2000), the variant with a small homogeneous subpopulation of EMBs (Efficient Market Believers) as described in Levy et al. (2000). LLS model is a well-known and early econophysics model, rooted in a standard utility maximization framework. Variants of the model have been published by the same authors in a number of articles and a book, and the model has also been critically evaluated in Zschischang and Lux (2001)\(^1\).

5.2.1 Asset Classes

As in the LLS model, there are two investments alternatives: a risky stock (or market index) and a riskless asset (bond). This is in line with many of the agent-based artificial financial markets, which typically do not deal with portfolio selection in multi-asset environments. The risky asset pays at the beginning of each period a dividend which follows a multiplicative random walk

\[
D_{t+1} = D_t (1 + \tilde{z}), \tag{5.1}
\]

where \(\tilde{z}\) is a random variable uniformly distributed in the interval \([z_1, z_2]\). The bond pays interest with a rate of \(r_f\).

5.2.2 Agent Behavior

Many early agent-based artificial financial markets were based on a small number of relatively simple strategies. Such markets have been labeled as few-type models in LeBaron (2006). Typically, strategies (or agents who employ them) could be divided into two groups: fundamental (based on a perceived fundamental value) and technical (based on the past prices, e.g. some form of trend extrapolation). Zero-intelligence framework in which agents trade randomly, might be useful for studying the influence of market microstructure, and sometimes a small number of such agents can be included into a few type model to provide liquidity for other agents. LLS model is a two-type model based on two groups of investors, named as Rational Informed Investors (RII) and Efficient Market Believers (EMB).

\(^1\)Zschischang and Lux (2001) have studied how initial conditions of the model influence the emerging wealth distribution among agents. For example, they have studied the influence of different types of utility functions and different degrees of risk aversion, and found that less risk averse investors tend to dominate the market. Also, when a group of investors characterized by a fixed proportion of shares in their portfolio is included, these investors over time take up the majority of the available wealth. The authors have also examined the sensitivity of the model’s long term outcomes to the random price history given at the start of the simulation.
LLS model follows a more standard framework where preferences (and risk attitude) are captured by an agent’s utility function, and the objective is the maximization of expected utility\(^2\). However, even in such a framework there are many possibilities for the functional form of the utility, which differ both in their descriptive validity and analytical tractability. Levy et al. (2000) explain that, when empirical support is taken into account, most evidence suggests DARA (Decreasing Absolute Risk Aversion) and CRRA (Constant Relative Risk Aversion), which motivates their choice of power (myopic) utility function:

\[
U(W) = \frac{W^{1-\alpha}}{1-\alpha}.
\] (5.2)

LLS model contains two types of investors: (1) RII (Rational Informed Investors) and (2) EMB (Efficient Market Believers).

RII investors

(1) RII investors are ‘informed’ about the properties of the dividend process. They do not know exact future dividend realizations, but by knowing the distribution from which they are generated, they can estimate the fundamental value as a discounted stream of future dividends, according to the Gordon model:

\[
\tilde{P}_{f,t+1} = \frac{D_t(1 + \tilde{z})(1 + g)}{k - g},
\] (5.3)

where \(k\) is the discount factor of the expected rate of return demanded by the market for the stock, and \(g\) is the expected growth rate of the dividend. RII investors assume that the price will converge to the fundamental value in the next period, i.e. \(\tilde{P}_{t+1} = \tilde{P}_{f,t+1}\).

In each period RII investor \(i\) chooses the proportion of wealth to invest in stocks and bonds so that he or she maximizes the expected utility of wealth in the next period, given by the following equation from Levy et al. (2000)\(^3\):

\[
EU(\tilde{W}_{i,t+1}) = EU\left(W_h^i[(1 - x)(1 + r_f) + x\tilde{R}_{t+1}]\right),
\] (5.4)

where \(W_h^i\) represents a hypothetical wealth of investor \(i\) in period \(t\) given that the price in period \(t\) is some hypothetical price \(P_h\). \(W_f^i\) consists of the previous period wealth \(W_{i,t-1}\), interest and dividend accumulated from the last period, and capital gains or losses incurred on the difference between \(P_h\) and \(P_{t-1}\).

\(^2\)Not all agent-based artificial financial markets endow their agents with a utility function. In some models agents are simply characterized by their wealth (see overviews of agent-based markets in LeBaron (2006) and Boer-Sorban (2008)).

\(^3\)Rationality in the name Rational Informed Investors refers to the maximization of the expected utility of next period wealth. RII will never depart from this optimal investment proportion.
5.2. Model Description

Next period return is a random variable influenced by the uncertainty in dividend realizations:

$$\tilde{R}_{t+1} = \tilde{P}_{t+1} + \tilde{D}_{t+1} = \frac{D_t(1+z)(1+g)}{P_t} + D_t(1+\tilde{z}). \quad (5.5)$$

Therefore, the expected utility of next period wealth can be calculated by integrating over all possible values of the random variable $\tilde{z}$:

$$EU(\tilde{W}_{t+1}^i) = \left(\frac{W^i}{(1-\alpha)}\right) \int_{z_1}^{z_2} \left[(1-x)(1+r_f)\right]^{1-\alpha}$$

$$+ x \left(\frac{D_t(1+z)(1+g)}{k-g} + D_t(1+\tilde{z})\right) \frac{1}{P_t} f(z)dz,$$

which for a uniform distribution of $z$ on the interval $[z_1, z_2]$ results in:

$$EU(\tilde{W}_{t+1}^i) = \left(\frac{W^i}{(1-\alpha)}\right) \frac{1}{(1-\alpha)(2-\alpha)(z_2-z_1)} \frac{k-g}{k+1} \frac{P_h}{xD_t} \left(\begin{array}{c}
(1-x)(1+r_f) + \frac{x}{P_h} \left(\frac{k+1}{k-g} D_t(1+z_2)\right) \\
(1-x)(1+r_f) + \frac{x}{P_h} \left(\frac{k+1}{k-g} D_t(1+z_1)\right)
\end{array}\right)^{(2-\alpha)} \quad (5.7)$$

Based on the optimal proportion, $x$, which maximizes the expected utility of next period wealth, RII investors determine the number of stocks demanded by multiplying their wealth with this optimal proportion. The rest of their wealth is invested in the risk-less asset. Since all RII investors are assumed to have the same degree of risk aversion (parameter $\alpha$), they will all have the same optimal proportion $x$. The actual number of demanded shares might differ only if investors differed in their wealth. However, as in the experiments of Levy et al. (2000) we assume that they all start with the same initial wealth.

EMB investors

(2) **EMB investors** believe that the price accurately reflects the fundamental value. However, since they do not know the dividend process, they use *ex post* distribution of stock returns to estimate the *ex ante* distribution.

---

4 Notice that in Levy et al. (2000) the return is defined as a fraction between the prices, instead of the usual relative difference. Hence, the investment proportion $x$ in Equation 5.4 directly multiplied by $\tilde{R}_{t+1}$. If actual rate of return is needed, one can subtract one or take logarithm.

5 According to Levy et al. (2000), the constraint of no borrowing and no short-selling is imposed, i.e. $0 \leq x \leq 1$, "in order to avoid problems of negative wealth, bankruptcy, and negative prices".
EMB investor $i$ uses a rolling window of size $m^i$, and is in the original model of Levy et al. (2000) said to be unbiased if, in absence of additional information, he or she assigns the same probability to each of the past $m^i$ return observations. Hence, the original, unbiased EMBs assume that returns come from a discrete uniform distribution:

$$\Pr^i(\tilde{R}_{t+1} = R_{t-j}) = \frac{1}{m^i}, \text{ for } j = 1, ..., m^i. \quad (5.8)$$

The expected utility of EMB investor $i$ is given by

$$EU(\tilde{W}_{t+1}^i) = \left(\frac{W_h}{1-\alpha}\right) \sum_{j=1}^{m^i} \Pr^i(\tilde{R}_{t+1} = R_{t-j}) \times \left[(1-x)(1+r_f) + xR_{t-j}(1-\alpha)\right]. \quad (5.9)$$

In accordance with the LLS model, for all EMB investors an investor specific noise is added to the optimal investment proportion $x^*$ (that maximizes the expected utility) in order to account for various departures from the rational optimal behavior:

$$x^i = x^{*i} + \tilde{\varepsilon}^i. \quad (5.10)$$

$\tilde{\varepsilon}^i$ is for each investor drawn from a normal distribution with mean 0 and standard deviation of 0.2. When needed, the value of noise is truncated so that $0 \leq x^i \leq 1$ is satisfied, imposing the constraint of no borrowing and no short-selling (see also footnote 5).

**New Agent Behavior: Normal EMB and Overconfident EMB**

In our model we create two new EMB types: normal EMBs and overconfident EMBs.

(a) Normal EMBs assume that returns come from a stable normal distribution, and in each period estimate the mean $\hat{\mu}$ and standard deviation $\hat{\sigma}$ using the rolling window of size $m^i$. Based on the estimates of this distribution, they assign probabilities to each of the past $m^i$ returns observations by calculating the values of the probability density function (pdf) of the estimated normal distribution at each observed return, and by normalizing these values so that they add up to one. In such a way we obtain the probability mass function (pmf) for each investor $i$:

$$\Pr^i(\tilde{R}_{t+1} = R_{t-j}) = \frac{\text{pdf}(R_{t-j}|\hat{\mu}, \hat{\sigma})}{\sum_{k=1}^{m^i} \text{pdf}(R_{t-k}|\hat{\mu}, \hat{\sigma})}, \quad (5.11)$$

$$\text{pdf}(x|\mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \quad (5.12)$$

(b) Overconfident EMBs also estimate normal distribution from the sample, but they underestimate the standard deviation of the distribution, making it more peaked around the mean: $\sigma = oc \times \hat{\sigma}$, where $oc$ is the overconfidence coefficient, $0 < oc < 1$. 

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5.2. Model Description

The probabilities are calculated and normalized using the pdf of that peaked normal distribution:

$$\text{pdf}(x|\mu = \hat{\mu}, \sigma = oc \cdot \hat{\sigma}) = \frac{1}{oc \cdot \hat{\sigma}\sqrt{2\pi}} e^{-\frac{(x-\hat{\mu})^2}{2(oc \cdot \hat{\sigma})^2}}. \quad (5.13)$$

In experiments we study different levels of overconfidence ($oc = 0.75$, $oc = 0.5$, $oc = 0.25$). In the special case of the full overconfidence ($oc = 0$), EMBs predict with certainty that the return will be equal to the mean of the sample, so the expected utility of wealth is given by:

$$EU(\hat{W}^{i}_{t+1}) = \left(\frac{W^i_h}{1 - \alpha}\right) \left[(1 - x)(1 + r_f) + x\hat{\mu}\right]^{(1-\alpha)}. \quad (5.14)$$

Figure 5.1 shows an example of obtained probability mass functions for a specific sample of observed returns. The case of uniform distribution represents the original, unbiased EMBs from the LLS model. As the overconfidence increases (overconfidence coefficient decreases) observed returns that are closer (further) to the mean are given a higher (lower) probability, so that the distribution becomes more peaked. The special case $oc = 0$ is the full overconfidence where all the probability mass is given to the sample mean.

Figure 5.1: Probability mass functions of observed past returns for different levels of overconfidence.
5.2.3 Market Mechanism

LeBaron (2006) describes four types of market mechanisms used in agent-based artificial financial markets. (1) The first mechanism is based on a price adjustment process, where new price is determined from the past price and the net order scaled by a factor called liquidity. This captures the basic intuition that excess demand raises the price, while excess supply lowers the price. A variant of this mechanism is a log-linear price impact function (Farmer and Joshi, 2002), which additionally ensures that the price remains positive. The main advantage of this mechanism is that it is computationally very fast, while some of the disadvantages are that the price changes are very sensitive to the choice of the liquidity parameter, and the issue of who takes the excess demand or supply. (2) The second mechanism is clearing in a temporary market equilibrium, where the price is determined so that the total demand equals the total number of shares in the market. Given that the demand of each agent has to be determined for different hypothetical prices, this mechanism can be computationally costly, particularly if there are no analytical solutions for demand functions. (3) The third mechanism is a more realistic one, where actual order book is simulated, and buy and sell orders are crossed using a certain well-defined procedure. (4) The final mechanism is the one in which agents (randomly) meet, and, if it suits them, trade with each other.

In this chapter, as in the LLS model, we use clearing by using the temporary market equilibrium. RII and EMB investors determine optimal proportion in the stock so as to maximize the expected utility of their wealth in the next period. However, expected utility is the function of the future price that is in the current period unknown. Investors therefore determine optimal proportions for various hypothetical prices \( P_h \) and respective demands for shares, and the equilibrium price is set to that price for which the total demand of all investors in the market equals the total number of outstanding shares, such that

\[
N^i_h(P_h) = \frac{x^i_h(P_h)W^i_h(P_h)}{P_h}.
\]  

(5.15)

This market mechanism is discussed in greater detail in Section 5.6.

5.3 Replication

In the first experiment we have replicated the LLS benchmark model with only RII investors present in the market. The Figure 5.2 shows the obtained price series. In this market there is no trade, the log prices follow random walk and there is no excess volatility (Levy et al., 2000). Since all RII investors are identical, they agree on the optimal investment proportion. This means that they achieve the same returns on their portfolios and that their wealth levels are always in the same proportion. Since
5.3. Replication

Figure 5.2: Price dynamics in the benchmark model with only RII.

Figure 5.3: Price dynamics with 95% RII and 5% unbiased EMB.
Chapter 5. Overconfident Investors in the LLS Financial Market

Table 5.1: Parametrization of the model used for experiments with investor overconfidence

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>950</td>
<td>Number of RII investors</td>
</tr>
<tr>
<td>$M_2$</td>
<td>50</td>
<td>Number of EMB investors</td>
</tr>
<tr>
<td>$m$</td>
<td>10</td>
<td>Memory length of EMB investors</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.5</td>
<td>Risk aversion parameter</td>
</tr>
<tr>
<td>$N$</td>
<td>10000</td>
<td>Number of shares</td>
</tr>
<tr>
<td>$r_f$</td>
<td>0.01</td>
<td>Riskless interest rate</td>
</tr>
<tr>
<td>$k$</td>
<td>0.04</td>
<td>Required rate of return on stock</td>
</tr>
<tr>
<td>$z_1$</td>
<td>-0.07</td>
<td>Maximal one-period dividend decrease</td>
</tr>
<tr>
<td>$z_2$</td>
<td>0.10</td>
<td>Maximal one-period dividend growth</td>
</tr>
<tr>
<td>$g$</td>
<td>0.015</td>
<td>Average dividend growth rate</td>
</tr>
</tbody>
</table>

they keep the same number of stocks throughout the simulation, there is no trade in the market. The market price closely follows the fundamental price which is entirely driven by the stochastic dividend process.

The second experiment is the replication of the LLS model with a small fraction of homogeneous and unbiased EMB investors (with parametrization given in Table 5.1). All our experiments are conducted with the same parametrization as in the original study (Levy et al., 2000) in order to enable direct commensurability with our results. This concerns also the 5% fraction of EMB investors and the 95% majority of RII investors, which according to Levy et al. (2000) demonstrates the power of irrational behavior, since even a small fraction of irrational investors can have a marked impact on the dynamics of the market prices. EMB investors are homogeneous with respect to their memory length and they use a uniform ex-post distribution of returns to predict future returns. Figure 5.3 shows typical market dynamics for this model setup, including semi-predictable booms and crashes, substantial trading in the market, and excess volatility (Levy et al., 2000). Excess volatility is used to quantify the behavior of the market which exhibits more volatility than what fundamentals would suggest, and for this experiment it is reported in Table 5.2 (the ‘Uniform’ column).

An interesting feature of the LLS model, although not entirely realistic, is that undervaluation never occurs, i.e. the market price never falls below the fundamental price, and this feature pervades all our experiments. The explanation is as follows. When market consists only of RII investors (or when EMB are investing only in the risk-less asset), the market price closely follows the fundamental price, since
RII investors assume price is converging to the fundamental price in the next period. When EMB are investing in the risky asset, the share is overvalued, because the total demand of RII and EMB in equilibrium has to equal the fixed amount of available shares, which can be achieved by raising the price. Undervaluation does not occur as the LLS model does not allow short selling, which is a common assumption in many models. In an earlier literature, e.g. Miller (1977), a lack of short selling was proposed as an explanation for why excess valuation occurs commonly and undervaluation does not. According to Levy et al. (2000) this constraint is included ”in order to avoid problems of negative wealth, bankruptcy, and negative prices”. For a more realistic case, these constraints could be relaxed to limited short-selling and limited borrowing (Levy et al., 2000).

The occurrence of bubbles and crashes can be related to the memory length of investors. When a very high dividend is realized, EMB investors switch to the risky asset, which creates a surge in the market price. Such a high capital gain entices them into further high exposure to the risky asset. However, as the memory window moves so that the initial jump in the price is forgotten and a low dividend is realized, they can shift back to the risk-free asset, which in turn causes a sudden drop in the price. As long as this market crash remains in their memory window, it reminds EMB investors to stay invested in the bond. After the crash is forgotten, there is an opportunity for a new bubble to start.

The silent periods after the crashes are not a realistic finding of this experiment, since in real markets crashes usually mark a period of higher volatility. Furthermore, although the very existence of market upheavals can be considered a realistic feature of the model, such periodic exchange of bubbles and crashes generated by the second experiment is not. In order to address these shortcomings, further experiments reported in Levy et al. (2000) introduced heterogeneity in the memory lengths of the EMB investors, and the findings showed that the resulting price became smoother and more realistic, in the sense that the bubbles and crashes were less regular and without the obvious predictability (observed in the case of EMB investors homogeneous with respect to their memory length). The smoother price of the heterogenous experiment can be understood as the superimposition of market prices obtained for different memory lengths, so that the bubbles and crashes of different periodicity smooth out each other. Nonetheless, Hellthatler (1995) has shown that as the number of investor increases, these stock price developments are replaced by periodic motion again. Also, as noted in Zschischang and Lux (2001) the model is still not able to reproduce certain stylized facts of the financial markets, such as the power-law behavior of both large returns and the time-dependence in various powers of absolute returns.
Chapter 5. Overconfident Investors in the LLS Financial Market

In our studies we conduct experiments mostly with EMB investors homogeneous with respect to the memory length, except when stated otherwise. This is because we are interested in the main effect of implemented behavioral biases, and we do not want to mask or smooth out that effect by introducing additional heterogeneity in the memory lengths. However, as the results of our experiments will show, the introduction of behavioral biases can create varied and surprising market dynamics (which in many cases eliminate the silent period after a crash and the obvious periodicity of bubbles) even when all the investors are homogeneous with respect to their memory length.

5.4 Results

Figure 5.4 through Figure 5.8 show the results of our extension of the model with normal and overconfident EMBs investors with various degrees of overconfidence. The simulations show that as the degree of overconfidence increases the booms and crashes become more extreme and less frequent than in the case of unbiased Efficient Market Believers of the original model. The results reported in Table 5.2 are the volatility of detrended market price $\sigma(p)$, the volatility of detrended fundamental price $\sigma(p^f)$, the excess volatility of the market, and average (per period) volume of trade in the market (expressed as the percentage of the total number of outstanding shares $N$).

The results presented in Table 5.2 are averaged over 100 simulations, each conducted for 1000 periods, and with a different seed of the random number generator each time. Following the methodology of Levy et al. (2000), the excess volatility is here calculated from the average values of $\sigma(P)$ and $\sigma(P^f)$. A more detailed presentation of the results including descriptive statistics is given in Table 5.3 (there the excess volatility is calculated per each simulation). It can be seen that more overconfident investors increase the excess volatility of the market, reduce average (per period) volume of trade in the market (this is evident only for higher levels of overconfidence), but also take a better position in terms of relative wealth compared to RII investors (Figure 5.9).

---

\[ \sigma(p) - \sigma(p^f) \]

As in Levy et al. (2000), excess volatility is calculated from the volatility of detrended market price $\sigma(p)$ and the volatility of detrended fundamental price $\sigma(p^f)$.
5.4. Results

Figure 5.4: Price dynamics with 95% RII and 5% normal EMB ($oc = 1$).

Figure 5.5: Price dynamics with 95% RII and 5% overconfident EMB ($oc = 0.75$).
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Figure 5.6: Price dynamics with 95% RII and 5% overconfident EMB (oc = 0.5).

Figure 5.7: Price dynamics with 95% RII and 5% overconfident EMB (oc = 0.25).
5.4. Results

Figure 5.8: Price dynamics with 95% RII and 5% fully overconfident EMB ($oc = 0$).

Figure 5.9: Relative wealth dynamics of RII against unbiased (uniform), normal, and overconfident EMB.
Table 5.2: Results of the experiments with various levels of investor overconfidence.

<table>
<thead>
<tr>
<th></th>
<th>Uniform</th>
<th>Normal</th>
<th>oc = 0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(p)$</td>
<td>11.4369</td>
<td>12.1090</td>
<td>13.1782</td>
</tr>
<tr>
<td>$\sigma(p^f)$</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
</tr>
<tr>
<td>excess volatility %</td>
<td>100.09</td>
<td>111.85</td>
<td>130.55</td>
</tr>
<tr>
<td>mean volume p.p. %</td>
<td>9.92</td>
<td>10.59</td>
<td>10.65</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>oc = 0.5</th>
<th>oc = 0.25</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(p)$</td>
<td>15.6394</td>
<td>21.3250</td>
<td>27.4616</td>
</tr>
<tr>
<td>$\sigma(p^f)$</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
</tr>
<tr>
<td>excess volatility %</td>
<td>173.61</td>
<td>273.08</td>
<td>380.44</td>
</tr>
<tr>
<td>mean volume p.p. %</td>
<td>9.46</td>
<td>6.10</td>
<td>2.82</td>
</tr>
</tbody>
</table>

5.5 Conclusion

In this chapter we have replicated the LLS model of the stock market (Levy et al., 2000), and adapted it to study investor overconfidence. Our experiments demonstrate how by using an incremental approach, a well known agent-based model such as LLS, can be easily adapted to study behavioral phenomena, in this case investor overconfidence. We show how small changes in behavior (the assumption of the equal probability weighting of past observations) can have a marked impact on the price dynamics.

In this study we have focused on overconfidence defined as miscalibration and implemented as underestimated variance. Overconfident investors predict future returns with more peaked distribution than what would be implied by the historical variance. In financial literature a popular supposition is that overconfident people trade too much, which in combination with transaction costs, leads to loss of their money (Odean, 1999). Our findings are contrary. We found that highly overconfident investors traded less because they tended to stick more with the risky asset. They also performed better against the rational informed investors in terms of the dynamics of relative wealth.

7 These results are also related to how the volume of trade is in the LLS model defined. Since LLS is a model based on the temporary market equilibrium where trading is not modeled explicitly, the volume is calculated as the difference in portfolio holdings between each two consecutive periods. We know that shares exchanged hands between investors, but we do not know how exactly that happened.

8 These findings are also due to the fact that LLS model is a rising market, so strategies that invest aggressively in the risky stock are more profitable.
Evidence in empirical studies is also contradictory. In the experimental market of Biais et al. (2005), miscalibration was found to have an effect of reducing trading performance. However, in a study which combines psychometric measures of judgment biases (overconfidence scores) and field data (trading records), Glaser and Weber (2007) could not relate measures of miscalibration to measures of trading volume, whereas they could do so with the "better-than-average" overconfidence. Glaser and Weber (2007) suggest that, despite being widely used, miscalibration may not be the best proxy for overconfidence. The inconclusive results of empirical studies motivate us to study a different type of overconfidence in Chapter 9.

Since our current experiments with overconfidence show results only for the case of homogeneous EMB investors, it would be interesting to conduct experiments in the case of heterogeneous EMB investors with various memory lengths (which in the original LLS model produced more realistic market dynamics, with booms and crashes not periodic or predictable). It could also be interesting for future work to extend our analysis to the interplay of investors with various degrees of overconfidence within the same market.

Another important issue in an agent-based financial market is the choice of the market mechanism, i.e. the price formation mechanism. LLS model focuses on clearing by temporary market equilibrium, which is a mechanism with well-known advantages and disadvantages (LeBaron, 2006). However, models with different market mechanisms, such as the price impact function or a simulated order book, could be used in a further study to evaluate the robustness of the results. The model studied in Chapter 9 uses a market mechanism based on an order book.

5.6 Appendix: Implementation Details

The model has been implemented using the numerical computing environment MATLAB and associated fourth-generation programming language. Since the LLS model is a numerical model and the pricing mechanism can be characterized as a temporary market equilibrium, we found the MATLAB environment to be suitable for the simulations. Using a dedicated agent-based simulation toolkit and implementing investors as fully developed agents (i.e. objects in the paradigm of object-oriented programming) would probably constitute a programming overhead. We have also used MATLAB to analyze the results and draw graphs.

Replicating computational models can sometimes be challenging, because a lot of implementation details needs to be reported in order for replication to be successful or even possible. We found the book of Levy et al. (2000) very detailed in that sense and providing the necessary implementation details. The only detail we could not
find was information on how EMB investors operated in the first few periods of the simulations. EMB investors need past prices from \( m \) previous periods in order to make their investment decisions. However, this information is not available in the first few periods of the simulation. There were two possible solutions that we could think of. One was to generate prices in negative periods (i.e. before the first period of the simulation) by using some stochastic process, and then to ‘feed’ agents with this information. We have opted, however, for a simpler solution. EMB investors can use a fixed portfolio strategy as an optimal strategy during the first few periods until they have enough prices to fill in their memory window. This fixed strategy was to allocate half of their wealth into the risky asset and the other half into the bond. The final investment proportion \( x \) was obtained by adding noise to this fixed proportion, according to Equation 5.10.

### 5.6.1 Pricing Mechanism

The pricing mechanism of the LLS model is based on the temporary market equilibrium, according to the classification by LeBaron (2006). RII and EMB investors determine optimal proportion in the stock so as to maximize the expected utility of their wealth in the next period. However, expected utility is the function of the price, which is in the current period still unknown. Investors therefore need to determine optimal proportions of wealth to invest in the risky asset \( x^i_h(P_h) \), and respective demands for shares \( N^i_h(P_h) \), for various hypothetical prices \( P_h \) (see Figure 5.10). The equilibrium price \( P_t \) is set to that hypothetical price for which the total demand of all investors in the market equals the total number of outstanding shares, according to Equation 5.16.

\[
\sum_i N^i_h(P_t) = \sum_i \frac{x^i_h(P_t)W^i_h(P_t)}{P_t} = N.
\] (5.16)
To find the equilibrium price, we define a function \( f(P_h) = \sum_i N_i^h(P_t) - N \). This is a decreasing function of \( P_h \) that changes sign in the point where \( \sum_i N_i^h(P_t) = N \), which is the market price we are looking for. To find that zero point, we use an existing method in MATLAB called \textit{fzero}. This method implements an algorithm originated by T. Dekker that uses a combination of bisection, secant, and inverse quadratic interpolation methods in order to find the root of a continuous function (or, more precisely, the point where the function has a value near zero, with a given tolerance).

### 5.6.2 Pseudo-code

The Main Program

1. set parameters and initial conditions

\[
\begin{align*}
N &= 10000 \quad \text{number of stocks issued} \\
M &= 950 \quad \text{number of RII investors} \\
M_2 &= 50 \quad \text{number of EMB investors} \\
r_f &= 0.01 \quad \text{risk-free rate} \\
T &= 1000 \quad \text{number of time steps} \\
k &= 0.04 \quad \text{required rate of return on stock} \\
z_1 &= -0.07 \quad \text{maximal one-period dividend decrease} \\
z_2 &= 0.10 \quad \text{maximal one-period dividend growth} \\
g &= 0.015 \quad \text{average dividend growth rate \((z_1+z_2)/2\)} \\
alpha &= 1.5 \quad \text{degree of risk aversion} \\
P_0 &= 50 \quad \text{initial price} \rightarrow P(1) \\
D_0 &= 0.5 \quad \text{initial quarterly dividend} \rightarrow D(1) \\
N_{i0} &= 10 \quad \text{initial stock holdings of RII investors} \\
W_0 &= 1000 \quad \text{initial wealth of RII investors} \\
x_0 &= 0.5 \quad \text{initial proportion of wealth invested in stocks} = P_0 N_{i0} / W_0 \\
m_2 &= 10 \quad \text{memory length of EMB investors} \\
N_{2i0} &= 10 \quad \text{initial stock holdings of EMB investors} \\
W_20 &= 1000 \quad \text{initial wealth of EMB investors} \\
x_{20} &= 0.5 \quad \text{initial proportion of wealth invested in stocks} = P_0 N_{2i0} / W_20 \\
sigma &= 0.2 \quad \text{standard deviation of noise added to the optimum solution (see Equation 5.10)} \\
\end{align*}
\]

2. set seed of the pseudo-random number generator

3. preallocate all variables and store initial values
4. for $t = 2$ to $T+1$

4a. calculate dividend
   \[ z = \text{rand} \text{uniform}_{[0,1]} \times (z_2 - z_1) + z_1; \]
   \[ D(t) = D(t-1) \times (1+z); \]

4b. update wealth of each agent $i$ with realized dividends and bond interest
   \[ W(t,i) = W(t-1,i) + N(t-1,i) \times D(t) + (W(t-1,i) - N(t-1,i) \times P(t-1)) \times rf; \]
   \[ W_2(t,i) = W_2(t-1,i) + N_2(t-1,i) \times D(t) + (W_2(t-1,i) - N_2(t-1,i) \times P(t-1)) \times rf; \]

4c. generate random departures from optimal behavior for each EMB investor $i$
   and store these values for later usage (see Equation 5.10)
   \[ x_2(t,i) = \text{rand} \text{normal}_{[0,1]} \times \sigma; \]

4d. determine the market clearing price ($P(t-1)$ is the starting search value)
   \[ \text{surplus} = \lambda(Ph) \text{AggregateDemand}(Ph, \text{<listOfParameters>}); \]
   \[ P(t) = \text{fzero}(\text{surplus}, P(t-1)); \]

4e. calculate return
   \[ R(t) = (P(t) + D(t)) / P(t-1); \]

4f. calculate fundamental value
   \[ P_f(t) = D(t) \times (1+g) / (k-g); \]

4g. update wealth of each agent with capital gains
   \[ W(t,i) = W(t,i) + N(t-1,i) \times (P(t) - P(t-1)); \]
   \[ W_2(t,i) = W_2(t,i) + N_2(t-1,i) \times (P(t) - P(t-1)); \]

4h. determine new holdings of investors from the optimal proportions
   calculated in equilibrium (for EMB investors use optimal proportion + noise, if needed truncated so that it falls into the $[0,1]$ interval)

end

5. store simulation results

6. repeat experiment for different seeds

Aggregate Demand Function

function [surplus] = AggregateDemand(Ph, <listOfParameters>)

1. calculate optimal investment proportion $x_{opt}$ for RII investors that maximizes their EU

2. calculate demand in shares $N(t,i)$ for all RII investors
5.6. Appendix: Implementation Details

3. if (t>m2) there is enough history for the rolling window

3a. if (oc > 0) overconfidence is not full

- estimate mean and standard deviation
  \[ m_{ju} = \text{mean}(R(t-1:t-m2)); \]
  \[ \text{sig} = \text{std}(R(t-1:t-m2)); \]

- calculate new overconfident distribution
  \[ \text{prob} = \text{normpdf}(R(t-1:t-m2), m_{ju}, \text{sig} \cdot \text{oc}); \]
  \[ \text{prob} = \text{prob}/\sum(\text{prob}); \]

elseif (oc = 0) overconfidence is full

- use one value (the mean) as prediction
  \[ \text{mean}(R(t-1:t-m2)) \]

3b. find optimal proportion \( x_{opt2} \) for EMB that maximizes their EU

else there is not enough history for the rolling window

3a. use 50%-50% rule
  \[ x_{opt2} = 0.5; \]

4. add noise to the optimal proportion and fit into \([0,1]\)
  \[ x_{opt2}\text{star} = x_{2}(t,i) + x_{opt2}; \]
  if \( x_{opt2}\text{star} < 0 \) \( x_{opt2}\text{star} = 0; \) end
  if \( x_{opt2}\text{star} > 1 \) \( x_{opt2}\text{star} = 1; \) end

5. calculate demand in shares \( N_{2i}(t,i) \) for all EMB investors
  \[ N_{2i}(t,i) = x_{opt2}\text{star} \cdot Wh_{i}/Ph; \]

6. return the surplus between the total demand (of RII and EMB) and \( N \)
  \[ \text{surplus} = N_{i}(t,i) + N_{2i}(t,i) - N; \]

5.6.3 Extended Replication Results

Figure 5.3 shows the replicated results of the LLS model with a small subpopulation of EMB investors who predict future returns using a uniform distribution, and who are homogeneous with respect to their memory length \( m \). The dynamics of the market price shows periodic bubbles and crashes to the fundamental value, which can
be linked to the size of their memory window. Even though bubbles and crashes can be considered as stylized facts of financial markets, this particular type of periodic behavior is not very realistic. Autocorrelation function of logarithmic returns (Figure 5.11) shows negative correlation for lags somewhat larger than 10, and positive correlation for lags somewhat larger than 20, which is the consequence of the homogeneous memory length of 10 periods. Figure 5.12 shows the histogram of returns with a superimposed normal density. The returns distribution has fat tails, which is another robust stylized fact of the financial time series, but the shape of the distribution is not entirely realistic. Its shape reflects the market dynamics which consists of a few large movements and small intermediate movements, but not as many medium movements as expected for a bell-shaped distribution of returns (see Figure 5.3).

The finding that large movements of the market price are followed by very little movements (during which market prices follows the fundamental price) is not in line with the stylized fact of volatility clustering. According to this empirical fact, large movements should be followed by more large movements and vice versa. LLS model is not able to replicate this stylized fact, neither in the case of homogeneous nor heterogeneous memory lengths of EMB investors, although heterogeneity does lead to a more realistic looking price dynamics. Some agent-based financial market models have focused specifically on the reproduction of volatility clustering, for example the model of Lux and Marchesi (1999).

In this section we have replicated the case of heterogeneous memory lengths demonstrated in Levy et al. (2000). Figure 5.13 shows the price dynamics of a market that consists of 95% RII and 5% EMB with their memory length $m^i$ normally distributed around mean value of 10 with a standard deviation of 10 (the distribution is truncated, so that the minimum memory length is 1). Figure 5.14 shows the price dynamics of a market with 95% RII and 5% EMB with their memory length uniformly distributed on the interval $[1, 50]$ (the distribution is forced by assigning each investor a different value). Although we were not able to reproduce as smooth of a price as in Levy et al. (2000), in both of these experiments the market showed more irregular prices than in the case of homogeneous memory, without the obvious periodicity related to the memory length of 10 (although some autocorrelation in simple returns could still be observed for small lags, see Figure 5.15 and Figure 5.16). Distribution histograms (Figure 5.17 and Figure 5.18) still contain fat tails, but the overall shape is more bell-shaped and therefore more realistic than the distribution shown on Figure 5.12.
5.6. Appendix: Implementation Details

Figure 5.11: Autocorrelation function of returns with 5% uniform EMB with homogeneous memory lengths.

Figure 5.12: Histogram of logarithmic returns and normal density plot (5% uniform EMB with homogeneous memory lengths).
Chapter 5. Overconfident Investors in the LLS Financial Market

Figure 5.13: Price dynamics with 5% uniform EMB with heterogeneous memory lengths (normally distributed).

Figure 5.14: Price dynamics with 5% uniform EMB with heterogeneous memory lengths (uniformly distributed).
5.6. Appendix: Implementation Details

Figure 5.15: Autocorrelation function of returns with 5% uniform EMB with heterogeneous memory lengths (normally distributed).

Figure 5.16: Autocorrelation function of returns with 5% uniform EMB with heterogeneous memory lengths (uniformly distributed).
Figure 5.17: Histogram of logarithmic returns and normal density plot (5% uniform EMB with heterogeneous memory lengths (normally distributed)).

Figure 5.18: Histogram of logarithmic returns and normal density plot (5% uniform EMB with heterogeneous memory lengths (uniformly distributed)).
## 5.6. Appendix: Implementation Details

Table 5.3: Descriptive statistics for the results of the experiments with various levels of investor overconfidence.

<table>
<thead>
<tr>
<th></th>
<th>Uniform</th>
<th>Normal</th>
<th>oc = 0.75</th>
<th>oc = 0.5</th>
<th>oc = 0.25</th>
<th>Mean</th>
</tr>
</thead>
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<tr>
<td>mean($\sigma(p)$)</td>
<td>11.4369</td>
<td>12.1090</td>
<td>13.1782</td>
<td>15.6394</td>
<td>21.3250</td>
<td>27.4616</td>
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<tr>
<td>std($\sigma(p)$)</td>
<td>5.1692</td>
<td>5.4447</td>
<td>6.3535</td>
<td>7.6695</td>
<td>11.5467</td>
<td>16.2047</td>
</tr>
<tr>
<td>median($\sigma(p)$)</td>
<td>10.3719</td>
<td>11.2145</td>
<td>11.6131</td>
<td>14.6779</td>
<td>18.8685</td>
<td>23.9397</td>
</tr>
<tr>
<td>mode($\sigma(p)$)</td>
<td>4.8933</td>
<td>5.0317</td>
<td>5.5169</td>
<td>5.8334</td>
<td>7.4943</td>
<td>9.0271</td>
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<tr>
<td>skewness($\sigma(p)$)</td>
<td>1.1953</td>
<td>1.2956</td>
<td>1.2280</td>
<td>1.5678</td>
<td>1.6708</td>
<td>1.8115</td>
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<tr>
<td>kurtosis($\sigma(p)$)</td>
<td>4.3689</td>
<td>4.4500</td>
<td>4.2629</td>
<td>6.4211</td>
<td>7.1379</td>
<td>6.8294</td>
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<tr>
<td>mean($\sigma(p^f)$)</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
</tr>
<tr>
<td>std($\sigma(p^f)$)</td>
<td>1.9581</td>
<td>1.9581</td>
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<td>5.4220</td>
<td>5.4220</td>
<td>5.4220</td>
<td>5.4220</td>
<td>5.4220</td>
</tr>
<tr>
<td>mode($\sigma(p^f)$)</td>
<td>3.2044</td>
<td>3.2044</td>
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<td>3.2044</td>
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<td>skewness($\sigma(p^f)$)</td>
<td>1.1854</td>
<td>1.1854</td>
<td>1.1854</td>
<td>1.1854</td>
<td>1.1854</td>
<td>1.1854</td>
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<tr>
<td>kurtosis($\sigma(p^f)$)</td>
<td>4.1405</td>
<td>4.1405</td>
<td>4.1405</td>
<td>4.1405</td>
<td>4.1405</td>
<td>4.1405</td>
</tr>
<tr>
<td>mean(excess volatility) %</td>
<td>95.1607</td>
<td>107.1606</td>
<td>124.5575</td>
<td>165.9632</td>
<td>262.9995</td>
<td>358.5883</td>
</tr>
<tr>
<td>std(excess volatility) %</td>
<td>34.5865</td>
<td>33.8699</td>
<td>51.2809</td>
<td>57.9288</td>
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<tr>
<td>median(excess volatility) %</td>
<td>90.5133</td>
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<td>mode(excess volatility) %</td>
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<td>0.7278</td>
<td>0.5526</td>
<td>1.3787</td>
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<tr>
<td>kurtosis(excess volatility)</td>
<td>3.6452</td>
<td>2.2884</td>
<td>2.8499</td>
<td>3.2734</td>
<td>6.6708</td>
<td>3.6049</td>
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<tr>
<td>std(mean volume p.p.) %</td>
<td>0.4811</td>
<td>0.5926</td>
<td>0.60918</td>
<td>0.6479</td>
<td>0.7289</td>
<td>0.3759</td>
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<tr>
<td>skewness(mean volume p.p.)</td>
<td>0.2881</td>
<td>-0.2392</td>
<td>0.4046</td>
<td>0.1688</td>
<td>0.4318</td>
<td>0.0115</td>
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<tr>
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<td>3.1110</td>
<td>3.4045</td>
<td>3.0765</td>
<td>2.9275</td>
<td>2.4301</td>
</tr>
</tbody>
</table>

### 5.6.4 Extended Simulation Results

Table 5.3 presents descriptive statistics for the results of the experiments with various levels of investor overconfidence. Variable of interests are volatility of detrended market price $\sigma(p)$, volatility of detrended fundamental price $\sigma(p^f)$, excess volatility and mean volume per period, and these are calculated across 1000 time periods. Descriptive statistics for those four variables (mean, standard deviation, median, mode, skewness and kurtosis) are calculated across 100 simulations with different seeds of the pseudo-random number generator. It can be seen that the variables of interest are not exactly normally distributed across the simulations with different seeds. However, the same results and trends can be inferred by comparing either means, medians or mods.
Chapter 6

Modeling Investor Optimism with Fuzzy Connectives

6.1 Introduction

One of the key characteristics that govern investor behavior is the optimism or pessimism of the investors. The link between asset valuation and investor sentiment has been the subject of considerable debate in the finance, and has been studied in the context of mispricing (departures from the fundamentals) (Brown and Cliff, 2005), the limits of arbitrage (De Long et al., 1990), as well as the underreaction and overreaction of stock prices (Barberis et al., 1998). Two methodological approaches can be found in the finance literature. One is concerned with finding adequate proxies for the aggregate investor sentiment, and using them in statistical analysis to explain the variation of stock prices and the occurrences of mispricing, such as bubbles and crashes. The other one is a bottom-up approach that aims at modeling individual investor optimism and pessimism by using the insights from psychological theories. For these theories, it is important to have a flexible framework that can be adapted to capture the complexity of human decision making behavior.

In fuzzy decision theory, a wide range of connectives (aggregation operators) has been proposed and studied in order to model the flexibility of human decision making. In this sense, the use of fuzzy connectives for modeling elements of behavioral finance is promising, since the wide range of behaviors documented in the behavioral finance

literature necessitates the use of a flexible framework for aggregating information. In this chapter, we make a step in this direction by proposing a model of investor optimism based on fuzzy aggregation.

In probabilistic decision theory, such as the Prospect Theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) and Rank-Dependent Utility Theory (Quiggin, 1982), optimism and pessimism are modeled using the probability weighting function. If, for example, the decision under risk is considered, a decision problem is presented using risky prospects, i.e. a set of possible outcomes and their probabilities. Because of the probability weighting, the decision weights associated with the outcomes are not equal to their probabilities (as would be in the case of Expected Value Theory or Expected Utility Theory). To model optimism we would need to specify and parameterize such a probability weighting function that gives more decision weight to good outcomes and less decision weight to bad outcomes. However, an empirically observed probability weighting function is usually S-shaped, which means that when dealing with such prospects, people are at the same time optimistic about the best outcomes, pessimistic about the worst outcomes, and insensitive to middle outcomes (Weber, 1994).

A decision maker’s optimism or pessimism has also been studied within a fuzzy decision making setting. Various fuzzy connectives studied in this context have parameters that denote explicitly the optimism or pessimism degree of a decision maker. Apart from the well-known Hurwicz operator (Hurwicz, 1951), the grade of compensation in Zimmermann–Zysno operator (Zimmermann and Zysno, 1980) can also be interpreted as an index of optimism. All these operators view the decision as a mixture of conjunctive and disjunctive behavior, and the degree of optimism determines which aggregation type dominates and to which degree. Another optimism–pessimism index was proposed in van Nauta Lenke et al. (1983), where the parameter of the generalized averaging operator (Dyckhoff and Pedrycz, 1984) is interpreted as the decision maker’s characteristic degree of optimism. This is an intuitive way of modeling the degree of optimism, since optimism is now modeled as the disposition of the decision maker to believe or give importance to positive events compared to his/her disposition to consider negative events (Kaymak and van Nauta Lenke, 1998). An application of this operator in the risk management of power networks has been considered in Kaymak et al. (1998).

Optimism and pessimism of investors have also been studied in the context of agent-based simulations of financial markets. For example, the model of Lux and Marchesi (1999) divides chartist traders into two groups, optimists and pessimists, which, depending on the group they belong to, either buy or sell a risky stock. In that model, optimists and pessimists could also have been labeled as buyers and sellers.
In our model, on the other hand, the optimism of investors is explicitly related to the investors’ opinions on the future market returns. Whether these opinions are going to be translated into exclusive buying or selling behavior is something that is not imposed by the definition. The implications of different levels of optimism and pessimism are going to be tested by market simulations.

In this chapter, we propose a model of investor optimism based on the generalized averaging operator. The advantage of the proposed approach is that the influence of different levels of optimism can be studied by varying a single parameter. We study the effects of investor optimism in an artificial financial market based on the Levy, Levy, Solomon (LLS) model (Levy et al., 2000).

The outline of the chapter is as follows. Section 6.2 explains the basics of the LLS model in which we study the investor optimism and pessimism. Section 6.3 describes the setup of the conducted experiments. Section 6.4 presents the simulation results, and Section 6.5 presents the results of the extension for investors’ heterogeneity in memory lengths. Section 6.6 concludes the chapter and discusses possible extensions for the future research.

### 6.2 Model Description

The proposed model of investor optimism is based on the LLS microscopic simulation model with a small homogeneous subpopulation of efficient market believers (EMBs) as described in Levy et al. (2000). LLS model is a well-known and early econophysics model, rooted in a utility maximization framework. Variants of the model have been published in a number of articles and a book, and the model has also been critically evaluated in Zschischang and Lux (2001). The LLS model has been explained in more detail in Chapter 5. Here we present only the basic setup of the model, before moving on to the new agent behavior, namely the sentiment EMBs.

#### 6.2.1 Asset Classes

As in the original LLS model, there are two investments alternatives: a risky stock (or market index) and a risk-free asset (bond). This is in line with many of the agent-based artificial financial markets, which typically do not deal with portfolio selection in multi-asset environments. The risky asset pays at the beginning of each period a dividend which follows a multiplicative random walk according to

\[
\tilde{D}_{t+1} = D_t (1 + \tilde{z}),
\]  

(6.1)
where $\tilde{z}$ is a random variable distributed uniformly in the interval $[z_1, z_2]$. The bond pays interest with a rate of $r_f$.

### 6.2.2 Agent Behavior

LLS model contains two types of investors: (1) Rational Informed Investors (RII) and (2) Efficient Market Believers (EMB). Both investor types are expected utility maximizers characterized by a power (myopic) utility function with DARA and CRRA properties (Levy et al., 2000):

$$U(W) = \frac{W^{1-\alpha}}{1-\alpha}. \quad (6.2)$$

**RII investors** know the dividend process, and therefore can estimate fundamental value as the discounted stream of future dividend, according to the Gordon model

$$\tilde{P}_{t+1}^f = \frac{D_t(1 + \tilde{z})(1 + g)}{k - g}, \quad (6.3)$$

where $k$ is the discount factor of the expected rate of return demanded by the market for the stock, and $g$ is the expected growth rate of the dividend. RII investors assume that the price will converge to the fundamental value in the next period. In each period RII investor $i$ chooses the proportion of wealth to invest in stocks and bonds so that he or she maximizes the expected utility of wealth in the next period, given by the following equation from Levy et al. (2000):

$$EU(\tilde{W}_{t+1}^i) = \frac{(W_{h}^i)^{1-\alpha}}{(1-\alpha)(2-\alpha)} \frac{1}{(z_2 - z_1)} \left( \frac{k - g}{k + 1} \right) \frac{P_h}{x D_t} \times \left\{ \left. \left[ (1 - x)(1 + r_f) + \frac{x}{P_h} \left( \frac{k + 1}{k - g} \right) D_t(1 + z_2) \right] \right\}^{(2-\alpha)} - \left. \left[ (1 - x)(1 + r_f) + \frac{x}{P_h} \left( \frac{k + 1}{k - g} \right) D_t(1 + z_1) \right] \right\}^{(2-\alpha)} (6.4)$$

Based on the optimal proportion, they determine the number of stocks demanded by multiplying this optimal proportion with their wealth. Since all RII investors are assumed to have the same degree or risk aversion (parameter $\alpha$), they will all have the same optimal proportion $x$. The actual number of demanded shares might differ only if investors differ in their wealth. However, as in the experiments of Levy et al. (2000) we assume that they all start with the same initial wealth.
EMB investors

*EMB investors* believe that the price accurately reflects the fundamental value. However, since they do not know the dividend process, they use *ex post* distribution of stock returns to estimate the *ex ante* distribution. EMB investor *i* uses a rolling window of size *m* *i*, and is in the original model of Levy et al. (2000) said to be *unbiased* if, in absence of additional information, he or she assigns the same probability to each of the past *m* *i* return observations Levy et al. (2000). Hence, the original, unbiased EMBs assume that returns come from a discrete uniform distribution

\[
\Pr^i(\tilde{R}_{t+1} = R_{t-j}) = \frac{1}{m^i}, \text{ for } j = 1, ..., m^i. \quad (6.5)
\]

The expected utility of EMB investor *i* is given by Levy et al. (2000)

\[
EU(W^i_{t+1}) = \frac{(W^i_h)^{1-\alpha}}{(1-\alpha)} \sum_{j=1}^{m^i} \Pr^i(\tilde{R}_{t+1} = R_{t-j}) \times [(1-x)(1+r_f) + xR_{t-j}]^{(1-\alpha)}, \quad (6.6)
\]

In accordance with the LLS model (Levy et al., 2000), for all EMB investors an investor specific noise is added to the optimal investment proportion *x* *i* * (that maximizes the expected utility) in order to account for various departures from rational optimal behavior (\(\tilde{\varepsilon}^i\) is truncated so that 0 ≤ *x* *i* ≤ 1, imposing the constraint of no borrowing and no short-selling), i.e.

\[
x^i = x^* + \tilde{\varepsilon}^i. \quad (6.7)
\]

**New Agent Behavior: Sentiment EMBs**

In this chapter we create a new EMB type, called the *sentiment EMBs* by using a fuzzy set connective. *Sentiment EMBs* use generalized aggregation operator to estimate future returns, using the rolling window of size *m* *i*. The prediction of the next period return for each investor *i* is given by

\[
\tilde{R}_{t+1} = \left( \frac{1}{m^i} \sum_{j=1}^{m^i} (R_{t-j})^s \right)^{1/s}. \quad (6.8)
\]

The higher the parameter *s*, the higher the estimate of the return (more closer to the maximum value from the sample), and vice versa. In such a way, we use parameter *s* to capture the phenomena of investor optimism and pessimism. Here we are also exploiting the fact that according to the Equation 5.5 returns are always positive, so the Equation 6.8 can always be calculated.
In our experiments we consider several values of the parameter $s$ which also coincide with the special cases of the generalized mean:

- $s \rightarrow -\infty$, the minimum of the sample;
- $s = -1$, the harmonic mean;
- $s \rightarrow 0$, the geometric mean;
- $s = 1$, the arithmetic mean;
- $s = 2$, the quadratic mean;
- $s \rightarrow \infty$, the maximum of the sample.

Since there is only one value for the expected return, instead of a probability distribution, the expected utility of sentiment EMB investor $i$ is given by

$$EU(\tilde{W}_i^{t+1}) = \left(\frac{W_i}{1-\alpha}\right)^{1-\alpha} \left[(1-x)(1+r_f) + x\tilde{R}_{t+1}\right]^{(1-\alpha)}.$$  \hspace{1cm} (6.9)

The investors will maximize this expected utility if in each period they invest all their wealth either in the stock or in the bond, depending on the actual comparison between the expected return on the stock $\tilde{R}_{t+1}$ and the return on the riskless bond $(1+r_f)$.

### 6.2.3 Market Mechanism

LeBaron (2006) describes four types of market mechanisms used in agent-based artificial financial markets. In this paper, as in the original LLS model, we use clearing by temporary market equilibrium. RII and EMB investors determine optimal proportion in the stock so as to maximize the expected utility of their wealth in the next period. However, expected utility is the function of the future price, which is in the current period unknown. Investors therefore need to determine optimal proportions, and respective demands for shares, for various hypothetical prices. The equilibrium price $P_t$ is set to that hypothetical price for which the total demand of all investors in the market equals the total number of outstanding shares according to

$$\sum_i N_{h}^{i}(P_t) = \sum_i \frac{x_{h}^{i}(P_t) W_{h}^{i}(P_t)}{P_t} = N.$$  \hspace{1cm} (6.10)
Table 6.1: Parametrization of the model used for experiments with investor optimism

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>950</td>
<td>Number of RII investors</td>
</tr>
<tr>
<td>$M_2$</td>
<td>50</td>
<td>Number of EMB investors</td>
</tr>
<tr>
<td>$m$</td>
<td>10</td>
<td>Memory length of EMB investors</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.5</td>
<td>Risk aversion parameter</td>
</tr>
<tr>
<td>$N$</td>
<td>10000</td>
<td>Number of shares</td>
</tr>
<tr>
<td>$r_f$</td>
<td>0.01</td>
<td>Riskless interest rate</td>
</tr>
<tr>
<td>$k$</td>
<td>0.04</td>
<td>Required rate of return on stock</td>
</tr>
<tr>
<td>$z_1$</td>
<td>-0.07</td>
<td>Maximal one-period dividend decrease</td>
</tr>
<tr>
<td>$z_2$</td>
<td>0.10</td>
<td>Maximal one-period dividend growth</td>
</tr>
<tr>
<td>$g$</td>
<td>0.015</td>
<td>Average dividend growth rate</td>
</tr>
</tbody>
</table>

6.3 Experimental Design

In the benchmark model where only RII investors are present in the market, there is no trade, the log prices follow random walk, and there is no excess volatility of the market price (Levy et al., 2000). In the experiment with a small fraction of homogeneous (with respect to memory length) and unbiased EMB investors (of the original model), the market dynamics show semi-predictable (unrealistic) booms and crashes, with substantial trading in the market and excess volatility (Levy et al., 2000). The resulting market dynamics of the benchmark experiment and the experiment with a small fraction of EMB investors can be seen in Chapter 5. This experimental setup of Levy et al. (2000) is also the basis for the experiments in this chapter.

In our new model we conduct six experiments for six different levels of optimism of EMB investors that correspond to the special cases of the parameter $s$, representing the minimum of the sample, harmonic mean, geometric mean, arithmetic mean, quadratic mean, and the maximum of the sample. In each experiment the market consists of 95% RII investors and 5% EMB investors, with the parametrization given in Table 6.1. We run 100 independent 1000-period-long simulations, with different initial seeds of the random number generators. The results in the Table 6.2 are averaged over these 100 simulations.
Table 6.2: Results of the experiments with different levels of investor optimism

<table>
<thead>
<tr>
<th></th>
<th>$s \to -\infty$</th>
<th>$s = -1$</th>
<th>$s \to 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(p)$</td>
<td>6.0249</td>
<td>12.8370</td>
<td>17.8668</td>
</tr>
<tr>
<td>$\sigma(p_f)$</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
</tr>
<tr>
<td>excess volatility %</td>
<td>5.41</td>
<td>124.59</td>
<td>212.58</td>
</tr>
<tr>
<td>mean volume p.p. %</td>
<td>0.48</td>
<td>9.04</td>
<td>6.40</td>
</tr>
<tr>
<td></td>
<td>$s = 1$</td>
<td>$s = 2$</td>
<td>$s \to \infty$</td>
</tr>
<tr>
<td>$\sigma(p)$</td>
<td>27.4739</td>
<td>28.8751</td>
<td>25.0327</td>
</tr>
<tr>
<td>$\sigma(p_f)$</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
</tr>
<tr>
<td>excess volatility %</td>
<td>380.66</td>
<td>405.18</td>
<td>337.95</td>
</tr>
<tr>
<td>mean volume p.p. %</td>
<td>2.82</td>
<td>1.18</td>
<td>0.12</td>
</tr>
</tbody>
</table>

6.4 Results

Figure 6.1 shows a typical price dynamics from the first experiment with pessimistic EMB investors. The market price closely follows the fundamental price which is driven by the random dividend process. Hence, this experiment resembles the benchmark model in which there are only RII investors in the market (see Chapter 5). Pessimistic investors predict next period return with the minimum return in the sample of past returns. The minimum return is almost always below the risk-less return, so the optimal investment for pessimistic EMB investors is to invest everything in the bond. The actual investment proportion will slightly vary due to the error term in Equation 6.7. Only in rare occasions when there is a series of returns higher than the risk-less return, the EMB investors will invest in the risky asset. The results in Table 6.2 show that for this experiment the volatility of the detrended market price $\sigma(p)$ is similar to the volatility of the detrended fundamental price $\sigma(p_f)$, which means that there is a low excess volatility. The relative mean volume per period shows that there is very little trading in the market, i.e. from period to period the investors do not change much their portfolio holdings.

Figure 6.2 shows the price development for the second experiment with slightly more optimistic investors that predict future return using the harmonic mean. The results of this experiment qualitatively and quantitatively resemble the results of the original model with a small fraction of unbiased EMB investors (which predict future returns using a uniform discrete distribution over the observed returns). The market exhibits cyclical booms and crashes to the fundamental value. According to Table 6.2, the market is more volatile, and there is also more trading. This exchange
6.4. Results

Figure 6.1: Price dynamics with 95% RII and 5% minimum sentiment EMB ($s \to -\infty$).

Figure 6.2: Price dynamics with 95% RII and 5% harmonic sentiment EMB ($s = -1$).
of risky assets between RII and EMB investors occurs mostly when the booms begin and when they crash.

Figure 6.3, Figure 6.4, and Figure 6.5 depict the market dynamics when EMB investors are even more optimistic. As the index of optimism increases the market shows more extreme (longer lasting) booms, followed by very sharp crashes. During these bubbles the EMB investors aggressively invest in the risky asset, while the RII investors divest expecting that the overvalued asset would fall to its fundamental value. The crash occurs when there is a series of low returns, due to low dividend realizations, so the EMB investors suddenly shift toward a risk-less asset. However, as soon as a better return is realized, EMB investors invest in the risky asset and a new boom starts. From Table 6.2 it is also evident that the more optimistic EMB investors are, the more volatile market price is. However, the trading is reduced because the booms are longer lasting, i.e. the cycles of booms and crashes appear less frequently.

In the case of full optimism, there is an ongoing market bubble, as shown in Figure 6.6. The market does not crash because the maximum return in the rolling window of past returns is always above the risk-less return, so the EMB investors are always highly invested in the risky asset. The trading in this experiment is even more reduced, but the volatility of the market price is also somewhat reduced. The reason for the latter is that the crash does not occur within the experiments.
6.4. Results

Figure 6.4: Price dynamics with 95% RII and 5% arithmetic sentiment EMB ($s = 1$).

Figure 6.5: Price dynamics with 95% RII and 5% quadratic sentiment EMB ($s = 2$).
Figure 6.6: Price dynamics with 95% RII and 5% maximum sentiment EMB ($s \rightarrow \infty$).

Figure 6.7: Relative wealth dynamics of RII against sentiment EMBs with various levels of optimism.
Figure 6.7 shows the development of the relative wealth of RII investors over time. At the beginning, RII investors possess 95% of all the wealth in the market. In the case of extremely pessimistic EMB investors, RII investors end up asymptotically dominating the market. This is because the LLS market is a growing market, and only RII investors are investing in the risky asset and exploiting that growth. Conversely, in the case of extreme optimism, EMB investors are highly invested in the stock, and eventually dominate the market. In non-extreme cases of optimism, both types of investors coexist in the market.

6.5 Extended Results

In this extension we study the influence of investor heterogeneity with respect to their memory length. The experiments conducted in this chapter until now have focused on the case of homogeneous EMB investors in that respect. However, even from the results of these experiments we can infer something about the effects of the memory length. We expect that the choice of the memory length has a great impact on the occurrence of booms and crashes, particularly in the extreme cases of optimism and pessimism. The larger the window of past returns is, the less likely it is that all returns are below (or all returns are above) the risk-less return. Hence, investors will stick even more with their preferred investment choices, which is the risky asset in the case of optimism and the risk-less asset in the case of pessimism. When investors have very short memory lengths, it is more likely that those few recent observations they use for prediction point them in the direction of switching their investment alternative.

To test for the impact of heterogeneity in memory lengths, we conduct the experiments with EMB investors having memory lengths uniformly distributed on the interval [1, 50] (which is actually achieved by assigning each of the 50 EMB investors a different memory length). Figure 6.8, Figure 6.9, Figure 6.10, Figure 6.11, Figure 6.12, and Figure 6.13 show price dynamics for the case of fully pessimistic EMB (minimum of the sample), harmonic EMB, geometric EMB, arithmetic EMB (neutral sentiment), quadratic EMB, and fully optimistic EMB investors (maximum of the sample), respectively. We can see that the main effects of optimism and pessimism found in Section 6.4 are still present: optimism leads to a large bubble, while pessimism pushes the market price towards the fundamentals. However, the market dynamics shows some differences from the homogeneous case.

It can be seen that in the case of neutral (arithmetic) sentiment (Figure 6.11) market price shows much more departures from the fundamental price than in the homogeneous case. This is the consequence of investors with larger memory lengths.
Chapter 6. Modeling Investor Optimism with Fuzzy Connectives

Figure 6.8: Price dynamics with 95% RII and 5% minimum sentiment EMB ($s \to -\infty$) with memory length $m_i$ uniformly distributed on [1,50].

Figure 6.9: Price dynamics with 95% RII and 5% harmonic sentiment EMB ($s = -1$) with memory length $m_i$ uniformly distributed on [1,50].
6.5. Extended Results

Figure 6.10: Price dynamics with 95% RII and 5% geometric sentiment EMB ($s \to 0$) with memory length $m^i$ uniformly distributed on [1,50].

Figure 6.11: Price dynamics with 95% RII and 5% arithmetic sentiment EMB ($s = 1$) with memory length $m^i$ uniformly distributed on [1,50].
Figure 6.12: Price dynamics with 95% RII and 5% quadratic sentiment EMB ($s = 2$) with memory length $m^i$ uniformly distributed on $[1,50]$.

Figure 6.13: Price dynamics with 95% RII and 5% maximum sentiment EMB ($s \to \infty$) with memory length $m^i$ uniformly distributed on $[1,50]$.
who require more negative returns or bad dividend realizations in order to switch towards the risk-less asset. Due to investors’ heterogeneity, we do not observe many coordinated actions which cause full price drops to the fundamental value, as can often be seen in experiments with homogeneous memory (e.g. Figure 6.4). In the cases of extreme optimism and extreme pessimism, we can see the effect of investors with very short memory lengths. Those investors are driven by only few recent observations, which could be so surprising (in comparison with the risk-less rate) that they cause them to change their portfolio allocation regardless of how optimistic or pessimistic they are. For this reason, fully pessimistic (Figure 6.8) and fully optimistic (Figure 6.13) EMB investors with heterogenous memory lengths cause some small price deviations that do not appear in the homogenous case (Figure 6.6 and Figure 6.1). Nevertheless, it can be seen that the main effects of investor sentiment are still present, and they cannot be eliminated by introducing such additional heterogeneity into the model.

Perhaps the most interesting market dynamics are obtained for the harmonic EMBs \((s \to 0)\), which are shown in Figure 6.10. Market price has very volatile dynamics, but on top of this volatility contours of larger bubbles can be observed. In order to get a better understanding of the market dynamics, we also analyze the returns on the risky asset. Figure 6.14 shows the time series of logarithmic returns, which are obtained by taking logarithm of returns calculated by Equation 5.5. We

Figure 6.14: Logarithmic returns for a market with 95% RII and 5% geometric sentiment EMB \((s \to 0)\) with memory length \(m^i\) uniformly distributed on [1,50].
can see that large changes tend to be followed by large changes, while small changes tend to be followed by small changes, which looks like a manifestation of volatility clustering. In order to further explore if this stylized fact has been obtained, we plot autocorrelation function of returns (Figure 6.15) and absolute value of returns (Figure 6.16). For returns, the autocorrelation is largely not significant (except some negative autocorrelation for lags 2-4). However, for absolute returns, the autocorrelation function is positive, significant, and slowly decaying, which is consistent with the stylized fact of volatility clustering. In addition, as in most other experiments, the stylized fact of fat tails is observed in return distribution (see Figure 6.17).

The finding of volatility clustering in one of the experiments is intriguing because in the original LLS model (with EMB investors who make prediction using uniform distribution over past return observations) it could not be achieved neither in the case of homogeneous nor heterogeneous memory lengths. In this case we have heterogeneous memory lengths, but the behavior of EMB investors is even simpler, such that instead of a uniform distribution over past returns only one value is used for prediction and it is calculated by aggregating past information using a geometric mean\(^1\).

6.6 Conclusion

In this chapter we have used a fuzzy connective to study investor optimism in the modified LLS model of the stock market by Levy et al. (2000). We show how changes in the formation of expectations by EMB investors can have a marked impact on the price dynamics. The levels of investor optimism have been related to the occurrences of market booms and crashes, as well as to the measures of excess volatility and trading volume. In the first set of experiments we have focused on the case of homogeneous EMB investors with the same memory length. Additional analysis with heterogeneous memory lengths shows that market dynamics becomes more intricate, although the main sentiment effect is still present in the sense that optimism increases bubbles, while pessimism reduces market price departures from the fundamental price. The extension for heterogeneous memory is also interesting because we were able to obtain volatility clustering in the case of geometric sentiment, which is a stylized fact that has not been reproduced in the original LLS model (neither with

\(^1\)A geometric mean represents a sentiment that is not far from neutral, i.e. it is not substantially pessimistic nor optimistic. We have assumed that an arithmetic mean represents this neutral sentiment. However, according to the literature, e.g. Kaymak and van Nauta Lemke (1998), a geometric mean could have been a viable choice too.
Figure 6.15: Autocorrelation function of logarithmic returns (5% geometric sentiment EMB with heterogeneous memory length uniformly distributed on [1,50]).

Figure 6.16: Autocorrelation function of absolute logarithmic returns (5% geometric sentiment EMB with heterogeneous memory length uniformly distributed on [1,50]).
Figure 6.17: Histogram of logarithmic returns and normal density plot (5% geometric sentiment EMB with heterogeneous memory length uniformly distributed on [1,50]).

homogeneous nor heterogeneous memories). This results demonstrates how small behavioral changes can have a large impact on the obtained market dynamics and its statistical properties.

Since we have used the same model to study investor overconfidence, a distinct although related behavioral phenomenon, it would also be interesting to study both phenomena at the same time. The overconfidence in the model presented in Chapter 5 refers to the peakedness of the return distribution around the mean of return observations, while optimism in the model presented in this chapter determines how that mean is chosen (ranging from the minimum observation to the maximum observation in the sample of past returns). This combined study of investor confidence and sentiment is presented in Chapter 7. Furthermore, instead of focusing only on fixed levels of investors optimism or pessimism we would like to implement an updating mechanism by which the level of investor optimism changes based on the past performance. This study is presented in Chapter 8.

This chapter demonstrates the advantage of using a fuzzy connective for modeling investor optimism, as we were able to control investor optimism by varying only a single parameter. The results of our experiments show that this parameter was a valid choice for an index of optimism in the context of financial markets. In future research other fuzzy set connectives could be investigated for agent decision making.
Chapter 7

A General Model of Investor Confidence and Sentiment

7.1 Introduction

In Chapter 5 and Chapter 6 we have studied separate effects of investor overconfidence and investor sentiment. This chapter presents a general model which allows a combined study of both investor optimism and investor overconfidence. Each behavioral phenomena is modeled by only a single parameter and influences either the mean (sentiment) or the standard deviation (confidence) of the return distribution investors use to predict future returns. The contribution of this chapter is to study the interaction of overconfidence and investor sentiment in the same model, as we expect that overconfidence will have different consequences for optimistic and pessimistic investors. In addition, in this chapter we also study recency and primacy effects, which concern investors who assign more importance to either more recent or older return observations.

The outline of the chapter is as follows. Section 7.2 explains the basics of the LLS model in which we study investor overconfidence and sentiment. Section 7.3 describes the setup of the experiments we have conducted. Section 7.4 presents the results of the simulations regarding the general confidence-sentiment model, and Section 7.5 presents the results of the simulations on recency and primacy effects. Section 7.6 concludes the chapter and discusses possible extensions for the future research.

7.2 Model Description

The proposed model of investor sentiment and overconfidence is based on the LLS microscopic simulation model with a small homogeneous subpopulation of efficient market believers (EMBs) as described in Levy et al. (2000). It is a generalization of our models presented in Chapter 5 and Chapter 6. LLS model is a well-known and early econophysics model, rooted in a utility maximization framework. Variants of the model have been published in a number of articles and a book, and the model has also been critically evaluated in Zschischang and Lux (2001). A more detailed description including the implementation details can be found in Chapter 5. Here we present only the basic setup of the original model, before moving on to the extension.

7.2.1 Asset Classes

As in the original LLS model, there are two investments alternatives: a risky stock (or market index) and a risk-free asset (bond). This is in line with many of the agent-based artificial financial markets, which typically do not deal with portfolio selection in multi-asset environments. The risky asset pays at the beginning of each period a dividend which follows a multiplicative random walk according to

\[ \tilde{D}_{t+1} = D_t (1 + \tilde{z}), \]  

(7.1)

where \( \tilde{z} \) is a random variable distributed uniformly in the interval \([z_1, z_2]\). The bond pays interest with a rate of \( r_f \).

7.2.2 Agent Behavior

LLS model contains two types of investors: (1) Rational Informed Investors (RII) and (2) Efficient Market Believers (EMB). The model is rooted in the framework of expected utility maximization, and agents’ preferences are captured by a power (myopic) utility function with DARA and CRRA properties (Levy et al., 2000):

\[ U(W) = \frac{W^{1-\alpha}}{1-\alpha}. \]  

(7.2)

RII investors

RII investors know the dividend process, and therefore can estimate fundamental value as the discounted stream of future dividend, according to the Gordon model

\[ \tilde{P}_{t+1}^f = \frac{D_t (1 + \tilde{z})(1 + g)}{k - g}, \]  

(7.3)
where $k$ is the discount factor of the expected rate of return demanded by the market for the stock, and $g$ is the expected growth rate of the dividend. RII investors assume that the price will converge to the fundamental value in the next period. In period $t$ RII investor $i$ chooses the proportion of wealth to invest in stocks and bonds so that he or she maximizes the expected utility of wealth in the next period, given by the following equation from Levy et al. (2000):

$$
EU(\tilde{W}_{t+1}^i) = \frac{(W_h^i)^{1-\alpha}}{(1-\alpha)(2-\alpha)} \frac{1}{(z_2 - z_1)} \frac{(k - g)}{(k + 1)} \frac{P_h}{xD_t} \times \left\{ \left[ (1 - x)(1 + r_f) + \frac{x}{P_h} \left( \frac{k + 1}{k - g} \right) D_t(1 + z_2) \right]^{(2-\alpha)} - \left[ (1 - x)(1 + r_f) + \frac{x}{P_h} \left( \frac{k + 1}{k - g} \right) D_t(1 + z_1) \right]^{(2-\alpha)} \right\},
$$

(7.4)

where $P_h$ represents a hypothetical price of the risky asset in period $t$. $W_h^i$ represents hypothetical wealth of investor $i$ in period $t$, which consists of the previous period wealth, interest and dividend accumulated from the last period, and capital gains or losses incurred on the difference between $P_h$ and $P_{t-1}$.

Based on the optimal proportion, they determine the number of stocks demanded by multiplying this optimal proportion with their wealth. Since all RII investors are assumed to have the same degree of risk aversion (parameter $\alpha$), they will all have the same optimal proportion $x$. The actual number of demanded shares might differ only if investors differ in their wealth. However, as in the experiments of Levy et al. (2000) we assume that they all start with the same initial wealth.

**EMB investors**

**EMB investors** believe that the price accurately reflects the fundamental value. However, since they do not know the dividend process, they use ex post distribution of stock returns to estimate the ex ante distribution. EMB investor $i$ uses a rolling window of size $m^i$, and is in the original model of Levy et al. (2000) said to be unbiased if, in absence of additional information, he or she assigns the same probability to each of the past $m^i$ return observations. Hence, the original, unbiased EMBs assume that returns come from a discrete uniform distribution

$$
Pr^i(\tilde{R}_{t+1} = R_{t-j}) = \frac{1}{m^i}, \text{ for } j = 1, ..., m^i.
$$

(7.5)

The expected utility of EMB investor $i$ is given by Levy et al. (2000)
Chapter 7. A General Model of Investor Confidence and Sentiment

\[ EU(W_{t+1}^i) = \left( \frac{W_h^i}{1 - \alpha} \right) \sum_{j=1}^{m^i} \Pr^i(\tilde{R}_{t+1} = R_{t-j}) \]
\[ \times \left[ (1 - x)(1 + r_f) + xR_{t-j} \right]^{(1-\alpha)}. \]  

In accordance with the LLS model of Levy et al. (2000), for all EMB investors an investor specific noise is added to the optimal investment proportion \( x^* \) (that maximizes the expected utility) in order to account for various departures from rational optimal behavior (\( \tilde{\varepsilon}^i \) is truncated so that \( 0 \leq x^i \leq 1 \), imposing the constraint of no borrowing and no short-selling), i.e.

\[ x^i = x^* + \tilde{\varepsilon}^i. \]  

In Chapter 5 we have created two new EMB types: normal EMBs and overconfident EMBs.

(a) Normal EMBs assume that returns come from a stable normal distribution, and in each period estimate the mean \( \hat{\mu} \) and standard deviation \( \hat{\sigma} \) using the rolling window of size \( m^i \). Based on the estimates of this distribution, they assign probabilities to each of the past \( m^i \) returns observations by calculating the values of the probability density function (pdf) of the estimated normal distribution at each observed return, and by normalizing these values so that they add up to one. In such a way we obtain the probability mass function (pmf) for each investor \( i \):

\[ \Pr^i(\tilde{R}_{t+1} = R_{t-j}) = \frac{\text{pdf}(R_{t-j} | \hat{\mu}, \hat{\sigma})}{\sum_{k=1}^{m^i} \text{pdf}(R_{t-k} | \hat{\mu}, \hat{\sigma})}, \]

\[ \text{pdf}(x | \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \]  

(b) Overconfident EMBs also estimate normal distribution from the sample, but they underestimate the standard deviation of the distribution, making it more peaked around the mean: \( \sigma = oc \times \hat{\sigma} \), where \( oc \) is the overconfidence coefficient, \( 0 < oc < 1 \). The probabilities are calculated and normalized using the pdf of that peaked normal distribution:

\[ \text{pdf}(x | \mu = \hat{\mu}, \sigma = oc \cdot \hat{\sigma}) = \frac{1}{oc \cdot \hat{\sigma} \sqrt{2\pi}} e^{-\frac{(x-\hat{\mu})^2}{2(oc \hat{\sigma})^2}}. \]  

In the experiments of Chapter 5 we have studied different levels of overconfidence (\( oc = 0.75, oc = 0.5, oc = 0.25 \)). In the special case of the full overconfidence
7.2. Model Description

![Probability mass functions of observed past returns for different levels of overconfidence.](image)

Figure 7.1: Probability mass functions of observed past returns for different levels of overconfidence.

\( oc = 0 \), EMBs predict with certainty that the return will be equal to the mean of the sample, so the expected utility of wealth is given by:

\[
EU(\tilde{W}_t+1^i) = \frac{(W_t^i)^{1-\alpha}}{(1-\alpha)} [(1-x)(1+r_f) + x\hat{\mu}]^{(1-\alpha)}.
\] (7.11)

Figure 7.1 shows an example of obtained probability mass functions for a specific sample of observed returns. The case of uniform distribution represents the original, unbiased EMBs from the LLS model. As the overconfidence increases (overconfidence coefficient decreases) observed returns that are closer (further) to the mean are given a higher (lower) probability, so that the distribution becomes more peaked. The special case \( oc = 0 \) is the full overconfidence where all the probability mass is given to the sample mean.

In Chapter 6 by using a fuzzy set connective we have created a new EMB type, called the sentiment EMBs:

(c) Sentiment EMBs use the generalized aggregation operator to estimate future returns, based on the rolling window of size \( m^i \). The prediction of the next period return for each investor \( i \) is given by
Chapter 7. A General Model of Investor Confidence and Sentiment

\[ \tilde{R}_{t+1} = \left( \frac{1}{m^i} \sum_{j=1}^{m} (R_{t-j})^s \right)^{1/s}. \quad (7.12) \]

The higher the parameter \( s \), the higher the estimate of the return (more closer to the maximum value of the sample), and the lower the parameter \( s \) the lower estimate of the return (more closer to the minimum value of the sample). In this way, the parameter \( s \) can be used to capture the phenomena of investor optimism and pessimism. The following values of the parameter \( s \) represent special cases of the generalized mean, and have been studied in Chapter 6:

- \( s \to -\infty \), the minimum of the sample;
- \( s = -1 \), the harmonic mean;
- \( s \to 0 \), the geometric mean;
- \( s = 1 \), the arithmetic mean;
- \( s = 2 \), the quadratic mean;
- \( s \to \infty \), the maximum of the sample.

**New Agent Behavior: General EMB Investors**

In this chapter we propose a new, generalized behavior of EMB investors:

(d) *General EMB type*, which combines investor confidence and optimism to determine the discrete distribution of returns. Optimism determines the mean of distribution, while confidence determines the peakedness of the distribution.

The prediction of the next period mean return \( \tilde{\mu}_{t+1} \) is calculated as follows.

\[ \tilde{\mu}_{t+1} = \left( \frac{1}{m^i} \sum_{j=1}^{m} (R_{t-j})^s \right)^{1/s}. \quad (7.13) \]

Based on the level of optimism (or pessimism), a general EMB investor \( i \) centers his or her prediction of the next period return around a value which can range from the minimum to the maximum value in the rolling window \( R_{t-1}, ..., R_{t-m^i} \).

The predicted deviation of the next period return \( \tilde{\sigma}_{t+1} \) is calculated from the standard deviation \( \hat{\sigma} \) of the sample of past returns \( R_{t-1}, ..., R_{t-m^i} \) and the level of confidence \( c \):

\[ \tilde{\sigma}_{t+1} = c \times \hat{\sigma}. \quad (7.14) \]

\(^1\)Value \( R_t \) is not included in the calculation because return in period \( t \) is yet to be determined based on this prediction of the next period return.
7.2. Model Description

In this general model, we use the coefficient of confidence \( c \in [0, \infty) \) instead of the coefficient of overconfidence \( oc \) (see Figure 7.2), as we want to study both the case of investor overconfidence \( c \in [0, 1) \) and investor underconfidence \( c > 1 \). Overconfident EMBs are too confident about their predictions; they underestimate the standard deviation of the distribution, making it more peaked around the generalized mean of the sample. Underconfident EMBs are less certain about future returns; they overestimate the standard deviation of returns making the distribution broader around the generalized mean.

In each period of the simulation EMB investor \( i \) predicts next period return by the following discrete probability distribution that incorporates the effects of investor sentiment and confidence:

\[
Pr^i(\tilde{R}_{t+1} = R_{t-j}) = \frac{\text{pdf}(R_{t-j} | \tilde{\mu}_{t+1}, \tilde{\sigma}_{t+1})}{\sum_{k=1}^{m^i} \text{pdf}(R_{t-k} | \tilde{\mu}_{t+1}, \tilde{\sigma}_{t+1})}, \tag{7.15}
\]

where pdf is the probability density function of a normal distribution. Since probability mass function assigns probabilities only to the returns of the rolling window, for pronounced levels of optimism or pessimism (i.e. high offset of the generalized mean from the arithmetic mean) the resulting discrete probability distribution will be skewed.
Chapter 7. A General Model of Investor Confidence and Sentiment

This chapter presents a general model which captures the behavior of EMB investors studied in Levy et al. (2000) and Chapter 5 and Chapter 6 of this thesis. The study of investor overconfidence presented in Chapter 5 can be considered a special case of the general model where overconfidence is varied but sentiment is set to the special case of $s = 1$, or the arithmetic mean. The study of investor sentiment in Chapter 6 is a special case of the general model where optimism level is varied, but confidence is set to the special case of $c = 0$, or the full overconfidence, where only one value of return is given as a prediction. In the case of extreme underconfidence, $c \to \infty$, the distribution becomes uniform, which represents the so-called unbiased EMBs of the original LLS model Levy et al. (2000). These original EMBs are not influenced by the level of sentiment because the uniform distribution does not depend on the mean.

7.2.3 Market Mechanism

The market mechanism used in the LLS model is the clearing by a temporary market equilibrium, according to the taxonomy of LeBaron (2006). RII and EMB investors determine optimal proportion in the stock so as to maximize the expected utility of their wealth in the next period. However, expected utility is the function of the future price, which is in the current period unknown. Investors therefore need to determine optimal proportions, and respective demands for shares, for various hypothetical prices. The equilibrium price $P_t$ is set to that hypothetical price for which the total demand of all investors in the market equals the total number of outstanding shares, according to

$$\sum_i N_h^i(P_t) = \sum_i x_h^i(P_t)W_h^i(P_t) = N. \tag{7.16}$$

7.3 Experimental Design

Table 7.1 shows various combinations of investor optimism (pessimism) and overconfidence (underconfidence) of EMB investors, that have been studied in this chapter, in previous chapters, and in the original model of Levy et al. (2000). In this chapter we conduct experiments for combinations marked by X in Table 7.1, which we deem important to disentangle the effects of overconfidence and optimism. In each experiment the market consists of 95% RII investors and 5% EMB investors, with the parametrization given in Table 7.2. We run 100 independent 1000-period-long simulations, with different initial seeds of the random number generators. The results in the Table 7.3 are averaged over these 100 simulations. The reported values are
Table 7.1: Combinations of investor confidence \( c \) and sentiment \( s \) that have been studied, and information where those experimental results are reported.

<table>
<thead>
<tr>
<th></th>
<th>uniform</th>
<th>underconfidence</th>
<th>normal</th>
<th>overconfidence</th>
<th>full</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \infty )</td>
<td>1.75</td>
<td>1.5</td>
<td>1</td>
<td>0.75</td>
</tr>
<tr>
<td>full pessimism</td>
<td>( -\infty )</td>
<td>LLS</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>pessimism</td>
<td>-5</td>
<td>LLS</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>pessimism</td>
<td>-1</td>
<td>LLS</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>neutral</td>
<td>1</td>
<td>LLS, Ch.5</td>
<td>Ch.5</td>
<td>Ch.5</td>
<td>Ch.5</td>
</tr>
<tr>
<td>optimism</td>
<td>2</td>
<td>LLS</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>optimism</td>
<td>5</td>
<td>LLS</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>full optimism</td>
<td>( \infty )</td>
<td>LLS</td>
<td>*</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

standard deviation of the detrended market price \( \sigma(p_t) \), standard deviation of the detrended fundamental price \( \sigma(p^f_t) \), excess volatility, and mean trading volume per period (as a percentage of the total number of outstanding shares \( N \)). As in Levy et al. (2000), the excess volatility is calculated from the volatility of detrended price \( \sigma(p_t) \) and the volatility of detrended fundamental price \( \sigma(p^f_t) \) as

\[
\frac{\sigma(p_t) - \sigma(p^f_t)}{\sigma(p^f_t)}.
\] (7.17)

The experiment marked by * in Table 7.1 is one simulation made to illustrate the effect of underconfidence on market dynamics in the case of full optimism (Figure 7.8).

In the benchmark model of the original study (Levy et al., 2000), where only RII investors are present in the market, there is no trade, the log prices follow random walk, and there is no excess volatility of the market price (Levy et al., 2000). In Figure 7.3 it can be observed that in the benchmark model the market price closely follows the fundamental price of the risky asset.

In the experiment with a small fraction of homogeneous (with respect to memory length) and unbiased EMB investors (of the original model), the market dynamics (Figure 7.4) show semi-predictable booms and crashes, with substantial trading in the market and excess volatility (Levy et al., 2000). The occurrence of these cyclic bubbles can be related to the size of the memory length of EMB investors. This experimental setup of Levy et al. (2000) is the basis for the experiments in our paper.
Table 7.2: Parametrization of the model used for experiments with investor confidence and sentiment.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>950</td>
<td>Number of RII investors</td>
</tr>
<tr>
<td>$M_2$</td>
<td>50</td>
<td>Number of EMB investors</td>
</tr>
<tr>
<td>$m$</td>
<td>10</td>
<td>Memory length of EMB investors</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.5</td>
<td>Risk aversion parameter</td>
</tr>
<tr>
<td>$N$</td>
<td>10000</td>
<td>Number of shares</td>
</tr>
<tr>
<td>$r_f$</td>
<td>0.01</td>
<td>Riskless interest rate</td>
</tr>
<tr>
<td>$k$</td>
<td>0.04</td>
<td>Required rate of return on stock</td>
</tr>
<tr>
<td>$z_1$</td>
<td>-0.07</td>
<td>Maximal one-period dividend decrease</td>
</tr>
<tr>
<td>$z_2$</td>
<td>0.10</td>
<td>Maximal one-period dividend growth</td>
</tr>
<tr>
<td>$g$</td>
<td>0.015</td>
<td>Average dividend growth rate</td>
</tr>
</tbody>
</table>

Table 7.3: Results of the experiments with various combinations of investor confidence and sentiment.

<table>
<thead>
<tr>
<th>sentiment</th>
<th>confidence</th>
<th>$s = 5$</th>
<th>$s = 5$</th>
<th>$s \to \infty$</th>
<th>$s \to \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(p_t)$</td>
<td>$c = 1.5$</td>
<td>15.5752</td>
<td>25.7670</td>
<td>25.8876</td>
<td>25.0226</td>
</tr>
<tr>
<td>$\sigma(p_t^f)$</td>
<td>$c = 0.25$</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
</tr>
<tr>
<td>excess volatility %</td>
<td></td>
<td>172.49</td>
<td>350.80</td>
<td>352.91</td>
<td>337.77</td>
</tr>
<tr>
<td>mean volume p.p. %</td>
<td>$c = 1.5$</td>
<td>9.60</td>
<td>0.41</td>
<td>0.26</td>
<td>0.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>sentiment</th>
<th>confidence</th>
<th>$s = -5$</th>
<th>$s = -5$</th>
<th>$s \to -\infty$</th>
<th>$s \to -\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(p_t)$</td>
<td>$c = 1.5$</td>
<td>9.4627</td>
<td>8.3804</td>
<td>7.4034</td>
<td>6.0249</td>
</tr>
<tr>
<td>$\sigma(p_t^f)$</td>
<td>$c = 0.25$</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
</tr>
<tr>
<td>excess volatility %</td>
<td></td>
<td>65.55</td>
<td>46.62</td>
<td>29.52</td>
<td>5.41</td>
</tr>
<tr>
<td>mean volume p.p. %</td>
<td>$c = 1.5$</td>
<td>10.82</td>
<td>11.16</td>
<td>10.90</td>
<td>0.56</td>
</tr>
</tbody>
</table>
7.3. Experimental Design

Figure 7.3: Price dynamics in the benchmark model of RII.

Figure 7.4: Price dynamics with 95% RII and 5% unbiased (uniform) EMB of the original LLS model.
7.4 Results

Figure 7.5 shows the price development for the first experiment with optimistic investors and some underconfidence. The results of this experiment qualitatively resemble the results of the original model with a small fraction of unbiased EMB investors (which predict future returns using a uniform discrete distribution over the observed returns). The market exhibits cyclical booms and crashes to the fundamental value.

However, as we increase investor confidence for that same level of optimism (Figure 7.6), we see a large bubble in the market that eventually crashes by falling sharply to the fundamental value. According to excess volatility in Table 7.3, the market is more volatile than the fundamentals, but there is less trading on average. This exchange of risky assets between RII and EMB investors occurs mostly when the booms begin and when they crash. During the bubble the EMB investors invest aggressively in the risky asset, while the RII investors divest expecting that the overvalued asset would fall to its fundamental value. The crash occurs when there is a series of low returns, due to low dividend realizations, so the EMB investors suddenly shift toward a risk-less asset.

Nonetheless, as soon as a better return is realized, EMB investors invest in the risky asset and a new boom starts. In the original model, the point where a new

![Figure 7.5: Price dynamics with 95% RII and 5% EMB with optimism s = 5 and underconfidence c = 1.5.](image)

Figure 7.5: Price dynamics with 95% RII and 5% EMB with optimism $s = 5$ and underconfidence $c = 1.5$. 
bubble can start (after a crash has happened) is very much related to the size of
the rolling window $m$ - a new bubble can start only when the rolling window moves
outside of the crash so that investors ”forget” about the very bad return experienced
during the crash. When investor optimism is taken into account this is not the case,
because investors can calculate higher mean of the returns even when the bad return
of the crash is included in the window used for estimation. This is a consequence of
the generalized mean that can give more weight to higher (or lower) returns depending
on its parameter $s$.

Figure 7.7 and Figure 7.8 depict the market dynamics when EMB investors are
fully optimistic, i.e. they base return distribution around the maximum value of the
sample of past returns. For both experiments of underconfidence and overconfidence
the market shows an ongoing bubble. The market does not crash during experiment
because the maximum return in the rolling window of past returns is always above
the risk-less return, so the EMB investors are always highly investing in the risky
asset. It is possible, however, to break this bubble by reducing investor confidence,
i.e. moving into the direction of underconfidence so that the return distribution
broadens away from the maximum value. For example, in one simulation we found
that underconfidence of $c = 1.75$ was already enough to break the bubble into a few
smaller bubbles (Figure 7.9).
Figure 7.7: Price dynamics with 95% RII and 5% EMB with full optimism and underconfidence $c = 1.5$.

Figure 7.8: Price dynamics with 95% RII and 5% EMB with full optimism and overconfidence $c = 0.25$. 
7.4. Results

Figure 7.9: Price dynamics with 95% RII and 5% EMB with full optimism and underconfidence $c = 1.75$.

Figure 7.10 depicts the case of pessimistic investors with some underconfidence. Compared to the optimistic cases, the market now exhibits a moderate degree of bubbles in the market. As we increase overconfidence the estimated return distribution tightens around the lower (pessimistic) values observed in the rolling window (Figure 7.11). The result is that the market is more stabilized: the bubbles appear somewhat reduced and the market price is closer to the fundamental price, which is also reflected in a reduced excess volatility.

In the case of extreme pessimism where investors build distribution around the minimum value of the past returns (Figure 7.12), by increasing the overconfidence the market reduces to the benchmark model of only RII investors where market price follows the fundamental price (Figure 7.13). This is an interesting finding that gives a new perspective to our results in Chapter 5, where overconfidence was shown to increase the bubbles and destabilize the market. Now, we can see that overconfidence has such effect only for optimistic investors and those close to neutral sentiment. In the case of high pessimism investor overconfidence stabilizes the market. Thus, overconfidence enhances the effects of investor sentiment, and we can conclude that it is more relevant what investors are overconfident about rather than the mere fact they are overconfident.
Figure 7.10: Price dynamics with 95% RII and 5% EMB with pessimism $s = -5$ and underconfidence $c = 1.5$.

Figure 7.11: Price dynamics with 95% RII and 5% EMB with pessimism $s = -5$ and overconfidence $c = 0.25$. 
7.4. Results

Figure 7.12: Price dynamics with 95% RII and 5% EMB with full pessimism and underconfidence $c = 1.5$.

Figure 7.13: Price dynamics with 95% RII and 5% EMB with full pessimism and overconfidence $c = 0.25$. 
Figure 7.14 shows the development of the relative wealth of RII investors over time. At the beginning, RII investors possess 95% of all the wealth in the market. In the case of extremely pessimistic EMB investors, RII investors end up asymptotically dominating the market. This is because the LLS market is a growing market, and only RII investors are investing highly in the risky asset and exploiting that growth. Conversely, in the case of extreme optimism, EMB investors are investing highly in the stock, and eventually dominating the market. These two cases can be seen for overconfident investors, because underconfidence seems to reduce the effects of extreme sentiments, particularly in the case of full optimism (for full optimism more underconfidence is needed to observe that difference). In non-extreme cases of optimism, both types of investor coexist in the market, as is the case with the uniform distribution used in the original model.

We have conducted a robustness check by varying the proportion of EMB investors from 5% to 10% and 20%, and we have observed that only the starting point on the y-axis has changed, but the behavior thereafter was the same. Figure 7.15 shows the development of relative wealth of RII investors for different fractions of EMB investors who exhibit full pessimism. It can be seen that different percentages of EMB investors influence only the initial fraction of their wealth, whereas the asymptotic behavior is the same, since in all cases RII investors acquire all the wealth in the market,
7.4. Results

Figure 7.16 shows wealth dynamics for different percentages of EMB investors which use a uniform distribution (as in the original LLS model). This is the case when both investors coexist in the market, and again, the percentage of EMB investors influences the initial fraction of wealth, whereas the asymptotic behavior thereafter is similar.

![Figure 7.15: The dynamics of relative wealth of RII vs. EMB for various fractions of (fully pessimistic) EMB.](image1)

Figure 7.16: The dynamics of relative wealth of RII vs. EMB for various fractions of uniform (original) EMB.

![Figure 7.16: The dynamics of relative wealth of RII vs. EMB for various fractions of uniform (original) EMB.](image2)
7.5 Recency and Primacy Effects

In previous experiments, we have studied the effects of investors confidence and sentiment. They represent two psychological phenomena which are influencing the shape of the probability distribution that investors use to predict future returns. For both of these phenomena, the important aspect is the order of observed past returns based on their magnitude. Sentiment, in the case of optimism, gives more weight to higher returns observed in the memory window, while in the case of pessimism, sentiment gives more weight to the lower returns. Confidence determines the weight based on the departures from the mean value which is determined by the sentiment. In this model timing does not play a role, in the sense that it does not matter whether a particular return observation occurs at the beginning or the end of the memory window.

In the following experiments, we want to explicitly take into account the timing aspects of observed returns. In psychological literature two inverse effects have been observed in the way people give salience to received stimuli or observations depending on their serial position (Miller and Campbell, 1959). These cognitive biases are known as recency and primacy effects. While primacy refers to the tendency to give more weight to the first received piece of information (the oldest one), recency describes the tendency to give more weight to the last received piece of information (the most recent one). Recency effects have been studied in the financial literature in relation with the overreaction hypothesis (De Bondt and Thaler, 1985). "De Bondt and Thaler attribute overreaction to the psychological phenomenon of recency. When processing information, people tend to overweight recent information compared with their prior belief. Thus, traders who are not sure of the intrinsic value of a stock will be too optimistic about its value when the firm is winning and too pessimistic when it is losing" (Offerman and Sonnemans, 2009). The empirical finding that a portfolio composed of past losers eventually beats a portfolio composed of past winners is considered an evidence for such overreaction.

Recency and primacy in our model refer exclusively to the importance given to observation based on their time stamp. It is not relevant whether a particular observation was winning or losing, because those effects have already been captured with our experiments on investors optimism and pessimism. In such a way, we are disentangling between the effects of timing and the effects of the magnitude of return observations.
7.5.1 EMB Investors with Recency Effects

Here we conduct additional experiments in the LLS model in order to study recency effects among the EMB investors. The recency effect refers to the tendency of EMB investors to give more weight to more recent return observations compared to those farther in the past. This is modeled by assigning exponentially decaying probability mass towards the older return values in the rolling window of size $m_i$. Figure 7.20 shows probability mass functions for four different levels of recency effect represented by parameter $\mu \in [0, 1]$. The case of $\mu = 1$ represents full recency effect where only the most recent observation is taken into account (this is equivalent to the memory length of size $m = 1$). The case of $\mu \rightarrow 0$ represents no presence of recency effect, where each observation is given the same probability mass (this is the case of a uniform distribution, which is studied in the original model of Levy et al. (2000)).

The calculation of the probability mass function is carried out as follows. The most recent return observation is initially given weight of $\mu$ and the weights of older observations are iteratively reduced by factor $(1 - \mu)$. These initial weights are then normalized to give the probability mass function:

$$w(R_{t-1}) = \mu$$  \hspace{1cm} (7.18)

$$w(R_{t-j}) = w(R_{t-j+1})(1 - \mu), \ j = 2..m_i$$  \hspace{1cm} (7.19)

$$\Pr^i(\bar{R}_{t+1} = R_{t-j}) = \frac{w(R_{t-j})}{\sum_{k=1}^{m_i} w(R_{t-k})}. \hspace{1cm} (7.20)$$

We experiment with two different models of the recency effect. In the first variation, EMB investors aggregate returns from their memory window by calculating a weighted average of returns, and they use only this value as their prediction. The portfolio choice is then based on the comparison between that aggregated value and the risk-free return (similarly to the study of investor optimism in Chapter 6). In the second variation, the investors do not aggregate returns, but they plug the entire probability mass function (pmf) into the formula for the expected utility of next period wealth (Equation 7.6. The difference between these two implementations stems from the fact that the utility function is nonlinear (namely power function).

Figure 7.18 and Figure 7.19 show the market dynamics for different levels of recency effects under aggregation and no aggregation. In the case of full recency effect ($\mu = 1$), we see much smaller departures from the fundamental value than in the original model of uniform EMBs (see Chapter 5). However, the market price is still deviating from the fundamental value. Since only the most recent return is used, the nonlinearities in expected utility do not play a role, and two variations
Figure 7.17: Probability mass function for different levels of the recency effect.
Figure 7.18: Market dynamics when EMB investors show recency effects (implementation with aggregation).
Figure 7.19: Market dynamics when EMB investors show recency effects (implementation with no aggregation).
7.5. Recency and Primacy Effects

in implementation (aggregation and no aggregation) produce the same result. For other values of recency \((\mu = 0.7 \text{ and } \mu = 0.4)\) we can see that the departures from the fundamentals are somewhat increasing, but there is no substantial difference between the two implementations. In the case of low recency effect \((\mu = 0.1)\), we can see much more prominent departures from the fundamental value, and we can also see a distinction between the two implementations. For no recency effects \((\mu \to 0)\), the case of aggregation becomes the case of neutral sentiment (arithmetic mean) and full overconfidence (see Chapter 6), and the case of no aggregation becomes the original market model of uniform EMB investors (see Chapter 5).

7.5.2 EMB Investors with Primacy Effects

Now we focus on experiments with EMB investors exhibiting primacy effects, which captures the tendency of investors to give more importance to the return observations that they encountered first, i.e. the oldest observations in their memory window. This is modeled by assigning exponentially decaying probability mass from the oldest return observation towards the most recent return observation in the rolling window of size \(m^i\). Probability mass functions for four different levels of primacy effect represented by parameter \(\eta \in [0,1]\) are shown in Figure 7.20. In the case of full primacy effect \((\eta = 1)\) only the oldest observation is taken into account, whereas in the case of no primacy effect \((\eta \to 0)\) each observation is given the same probability mass. The latter is the case of uniform distribution, which is studied in the model of Levy et al. (2000)). Hence, both in the case of recency and primacy effects, as those effects diminish, the resulting behavior becomes the original behavior of the EMB investor.

The probability mass function which captures the primacy effect is calculated as follows. First, the oldest return observation is given weight of \(\eta\), and then the weights of more recent observations are iteratively reduced by factor \((1 - \eta)\). Finally, all the weights are normalized:

\[
\begin{align*}
    w(R_{t-m^i}) &= \eta \\
    w(R_{t-j}) &= w(R_{t-j-1})(1 - \eta), j = 1 .. (m^i - 1) \\
    Pr^i(\tilde{R}_{t+1} = R_{t-j}) &= \frac{w(R_{t-j})}{\sum_{k=1}^{m^i} w(R_{t-k})}.
\end{align*}
\]

Analogously to the experiments with recency effect, we show results for two variations in implementation, with and without aggregation. In the first variation (with aggregation), EMB investors calculate a weighted average of returns and use it for
Figure 7.20: Probability mass function for different levels of the primacy effect.
7.5. Recency and Primacy Effects

Figure 7.21: Market dynamics when EMB investors show primacy effects (implementation with aggregation).
Figure 7.22: Market dynamics when EMB investors show primacy effects (implementation with no aggregation).
their prediction, while in the second variation (without aggregation) the investors use the entire probability mass function (pmf) for their prediction. Figure 7.21 and Figure 7.22 show the market dynamics for different levels of primacy effects under aggregation and no aggregation. In the case of full primacy effect ($\eta = 1$), we can see much smaller departures from the fundamental value than in the original model of uniform EMBs (see Chapter 5). As the primacy effect becomes lower, the market dynamics show more excess volatility and becomes similar to the behavior of the original model with uniform EMB investors (the variation with no aggregation) and to the case of full overconfidence and arithmetic (neutral) sentiment (the variation with aggregation).

As can be seen from the figures with market dynamics, both primacy and recency effects are reducing the excess volatility, when compared to the original behavior of uniform EMB investors. In cases of full primacy and full recency effects, the investment choice is in each period based on the comparison between only one return observation and the risk-less return. When a high return on the stock is observed, it entices investors into buying the stock. However, as opposed to the original model where such a high return is influencing investors as long as it is in their memory window (creating a bubble in such a way), in this case the high return is not relevant in the next time step because only the new return observation is taken into account. Although, the effects of primacy and recency seem to be similar, there are still some differences in their impact on market dynamics. By comparing the market dynamics of full recency effect and full primacy effects, we can see that extrapolating the most recent return observation (recency effect) causes somewhat higher excess volatility than extrapolating the return observation with a larger time lag (primacy effect). Hence, timing differences in strategies do matter, but their effects are not as prominent as the effects of magnitude captured by the investor sentiment (optimism or pessimism).

Figure 7.23 shows the time-series of actual rate of return for a market with EMB investors who exhibit a medium degree of primacy effect ($\eta = 0.6$)$^2$. When compared to the original experiment with EMB investors who use uniform distribution to predict future returns (Figure 7.24), this time-series looks more realistic, since large movements are not followed by calm periods as in the original model (see also Figure 7.4). In the original model, more realistic price dynamics was achieved only after adding heterogeneity in memory lengths (see the appendix of Chapter 5), while in our models investors are homogeneous both with respect to their memory lengths and the type and amount of a behavioral bias they exhibit. This illustrates that variations in behavior can have impact and add to the realism of the market dynamics. However,

$^2$These returns can be calculated by subtracting 1 from $R_t$ or similarly as $\log(R_t)$.

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this still was not enough to obtain volatility clustering (which we checked qualitatively by plotting autocorrelation functions of raw returns and absolute/squared returns, and quantitatively by means of Ljung-Box-Pierce Q-test and Engle’s ARCH test). It appears that heterogeneity in memory lengths used in Section 6.5 was one of the key components in obtaining volatility clustering. Nonetheless, the heterogeneity itself is not sufficient, since the original LLS model with heterogeneous memory lengths does not reproduce that stylized fact.

Figure 7.23: Returns on the risky asset for a market with 5% EMB investors exhibiting primacy effects ($\eta = 0.6$).

Figure 7.24: Returns on the risky asset for a market with 5% uniform EMB investors of the original model.
7.6 Conclusion

In this chapter we have studied investor sentiment and investor overconfidence in the modified LLS model of the stock market by Levy et al. (2000). The overconfidence in our model refers to the peakedness of the return distribution around the mean of return observations, while sentiment in the model determines how that mean is chosen (ranging from the minimum observation to the maximum observation in the sample of past returns).

We show how changes in the formation of expectations by EMB investors can have a marked impact on the price dynamics. However, this study is not just a robustness test of the original LLS model, by which we relax the assumption of uniform distribution used for EMB investors. Here we also give a meaningful behavioral interpretation for these moments of probability distribution, and show how conveniently an existing agent-based model can be extended to study various behavioral phenomena.

The levels of investor optimism and confidence are related to the occurrences of market booms and crashes, as well as to measures of excess volatility and trading volume. We find that optimism increases the intensity of market bubbles, while pessimism stabilizes the market in the sense of reducing the departures from the fundamental value. Overconfidence is found to amplify the effects of investor sentiment, both in the case of optimism and pessimism. Underconfidence reduces the effects of the sentiment, since in the case of extreme underconfidence return distribution becomes uniform, i.e. independent of the mean. Overconfidence also decreases the mean trading volume per each period, as more overconfident investors, depending on their optimism or pessimism, tend to stick more with one investment alternative, which is risky asset in case of optimism and risk-less asset in case of pessimism.

The interaction between confidence and sentiment shows us the importance of studying these two behavioral biases at the same time, because the study of overconfidence in Chapter 5 gave us only a partial view on overconfidence for a given (neutral) level of investor sentiment (the arithmetic mean of the sample was taken as the center of the distribution). When overconfidence is modeled in the sense of miscalibration, the results suggest it is more relevant what investors are overconfident about (the mean of the distribution) than the mere fact they are overconfident (the distribution is peaked). Being overconfident about a correct view is beneficial, but when investors have wrong views on the market (e.g. they are pessimistic in a rising market) it is better to be underconfident in order to spread probability mass away from this wrong opinion).

In this chapter we have also introduced the recency and primacy effect and mod-
eled it by shifting probability mass from older observations towards the more recent ones, or the other way round. We have shown how limit cases of recency effect become equivalent to some special cases of confidence and sentiment, as well as to the original experiment of the LLS model Levy et al. (2000). We can conclude that (under two different variations in implementation) the recency and the primacy effects in the LLS model reduce the departures of the market price from the fundamental price, compared to the case of equal probability weighting used in the original model of Levy et al. (2000). Both primacy and recency effects are in that sense interesting as an example of a psychological bias that can stabilize the market dynamics. The results also suggest that it is not the overweighing of return observations from the past with a given time lag (e.g. the most recent one) that causes particularly high levels of excess volatility, but the overweighing of the highest observation in the sample, which is consistent with the optimism bias (as demonstrated earlier in this chapter and in Chapter 6). Hence, investors who remember extreme events are expected to have the most impact on the market dynamics.
Chapter 8

Self-Attribution Bias and Loss Aversion*

8.1 Introduction

In Chapter 5 and Chapter 7 we have modeled investor overconfidence as miscalibration and studied its impact on market dynamics. However, it seems that the market can also influence the confidence of investors: the overconfidence of successful investors can be reinforced through self-attribution bias, i.e. a belief that their trading success should be attributed mostly to their own abilities (Odean, 1999). In Lovric et al. (2009b) we have studied emerging overconfidence due to self-attribution bias. That study demonstrates the advantages of agent-based approach, because we can easily model the dynamics of investor attitudes based on some feedback from the market. In this chapter we propose a somewhat different update mechanism using a transformation function.

In Chapter 6 and Chapter 7 we have also modeled investor sentiment using a generalized average operator. It allowed us to conveniently model optimism and pessimism using just one parameter. In this chapter we propose an updating mechanism for investor sentiment, so that it depends on the market performance. Investors who increase their wealth subsequently increase their optimism, while those who lose their wealth decrease their optimism (i.e. increase their pessimism). However, loss

*A preliminary version of this chapter has been published in M. Lovric, U. Kaymak, and J. Spronk. Modeling loss aversion and biased self-attribution using a fuzzy aggregation operator. In Proceedings of the World Congress on Computational Intelligence (WCCI), pages 2297-2304, Barcelona, Spain, July 2010b.
as a robust finding of human psychology and decision making, suggests that people can react differently towards loses and gains ("Losses loom larger than gains." (Kahneman and Tversky, 1979)). In our model we are able to incorporate this asymmetry and study its impact on the dynamics of investor sentiment. Hence, loss aversion in our model operates through a different mechanism than in the Prospect Theory, where it is incorporated in the shape of the value function.

The outline of the chapter is as follows. Section 8.2 explains the basics of the LLS model in which we study self-attribution bias and loss aversion. This is the general sentiment-confidence model presented in Chapter 7, so a familiarized reader can jump to the parts of Section 8.2.2 that describe new agent behavior for updating investor sentiment and investor confidence. Section 8.3 describes the experimental design. Section 8.4 presents the results of the simulations and Section 8.5 concludes the chapter and discusses possible extensions for the future research.

8.2 Model Description

The proposed model of investor sentiment and overconfidence is based on the LLS microscopic simulation model with a small homogeneous subpopulation of efficient market believers (EMBs) as described in Levy et al. (2000). LLS model is a well-known and early econophysics model, rooted in a utility maximization framework. Variants of the model have been published in a number of articles and a book, and the model has also been critically evaluated in Zschischang and Lux (2001). This model is an extension of the general sentiment-confidence model presented in Chapter 7.

8.2.1 Asset Classes

As in the original LLS model, there are two investments alternatives: a risky stock (or market index) and a risk-free asset (bond). This is in line with many of the agent-based artificial financial markets, which typically do not deal with portfolio selection in multi-asset environments. The risky asset pays at the beginning of each period a dividend which follows a multiplicative random walk according to

\[ \tilde{D}_{t+1} = D_t (1 + \tilde{z}), \]  

(8.1)

where \( \tilde{z} \) is a random variable distributed uniformly in the interval \([z_1, z_2]\). The bond pays interest with a rate of \( r_f \).
8.2. Model Description

8.2.2 Agent Behavior

LLS model contains two types of investors: (1) Rational Informed Investors (RII) and (2) Efficient Market Believers (EMB). Both of these investor groups are expected utility maximizers, and the model uses power (myopic) utility function

\[ U(W) = \frac{W^{1-\alpha}}{1-\alpha}. \]  

(8.2)

RII investors

RII investors are informed about the dividend process, and therefore they can estimate fundamental value as the discounted stream of future dividend, according to the Gordon model

\[ \tilde{P}_{t+1}^f = \frac{D_t(1 + \tilde{z})(1 + g)}{k - g}, \]  

(8.3)

where \( k \) is the discount factor of the expected rate of return demanded by the market for the stock, and \( g \) is the expected growth rate of the dividend. RII investors assume that the price will converge to the fundamental value in the next period. In period \( t \) RII investor \( i \) chooses the proportion of wealth to invest in stocks and bonds so that he or she maximizes the expected utility of wealth in the next period, given by the following equation from Levy et al. (2000):

\[
EU(\bar{W}_{t+1}^i) = \frac{(W_{h}^i)^{1-\alpha}}{(1-\alpha)(2-\alpha)(z_2 - z_1)} \left( \frac{k - g}{k + 1} \right) \frac{P_h}{xD_t} \\
\times \left\{ \left[ (1 - x)(1 + r_f) + \frac{x}{P_h} \left( \frac{k + 1}{k - g} \right) D_t(1 + z_2) \right]^{(2-\alpha)} \right. \\
- \left. \left[ (1 - x)(1 + r_f) + \frac{x}{P_h} \left( \frac{k + 1}{k - g} \right) D_t(1 + z_1) \right]^{(2-\alpha)} \right\},
\]  

(8.4)

where \( P_h \) represents a hypothetical price of the risky asset in period \( t \). \( W_{h}^i \) represents hypothetical wealth of investor \( i \) in period \( t \), which consists of the previous period wealth, interest and dividend accumulated from the last period, and capital gains or losses incurred on the difference between \( P_h \) and \( P_{t-1} \).

Based on the optimal proportion, they determine the number of stocks demanded by multiplying this optimal proportion with their wealth. Since all RII investors are assumed to have the same degree or risk aversion (parameter \( \alpha \)), they will all have the same optimal proportion \( x \). The actual number of demanded shares might differ only if investors differ in their wealth. However, as in the experiments of Levy et al. (2000), we assume that they all start with the same initial wealth.
EMB investors

EMB investors believe that the price accurately reflects the fundamental value. However, since they do not know the dividend process, they use *ex post* distribution of stock returns to estimate the *ex ante* distribution. EMB investor $i$ uses a rolling window of size $m^i$. As in the original model by Levy et al. (2000), the investor is said to be *unbiased* if, in absence of additional information, he or she assigns the same probability to each of the past $m^i$ return observations (Levy et al., 2000). Hence, the original, unbiased EMBs assume that returns come from a discrete uniform distribution

$$\Pr^i(\tilde{R}_{t+1} = R_{t-j}) = \frac{1}{m^i}, \text{ for } j = 1, ..., m^i. \quad (8.5)$$

The expected utility of EMB investor $i$ is given by Levy et al. (2000)

$$EU(\bar{W}_{t+1}^i) = \frac{(W^i_h)^{1-\alpha}}{(1-\alpha)} \sum_{j=1}^{m^i} \Pr^i(\tilde{R}_{t+1} = R_{t-j}) \times [(1-x)(1+r_f) + xR_{t-j}]^{(1-\alpha)}. \quad (8.6)$$

In accordance with the LLS model, for all EMB investors an investor specific noise is added to the optimal investment proportion $x^*$ (that maximizes the expected utility) in order to account for various departures from rational optimal behavior ($\tilde{\varepsilon}^i$ is truncated so that $0 \leq x^i \leq 1$, imposing the constraint of no borrowing and no short-selling), i.e.

$$x^i = x^* + \tilde{\varepsilon}^i. \quad (8.7)$$

In Chapter 7 we have proposed *general EMB type*, which combines investor confidence and optimism to determine the discrete distribution of expected returns. Sentiment of investors determines the mean of that distribution, while confidence determines the peakedness of the distribution.

The prediction of the next period mean return $\tilde{\mu}_{t+1}$ is calculated as follows.

$$\tilde{\mu}_{t+1} = \left( \frac{1}{m^i} \sum_{j=1}^{m^i} (R_{t-j})^s \right)^{1/s}. \quad (8.8)$$

Based on the level of optimism (or pessimism), a general EMB investor $i$ centers his or her prediction of the next period return around a value which can range from
the minimum to the maximum value in the rolling window $R_{t-1}, ..., R_{t-m}$. This is achieved by using the generalized aggregation operator, which for higher values of the parameter $s$ gives higher estimate of the return (closer to the maximum value of the sample), and for lower values of the parameter $s$ gives lower estimate of the return (closer to the minimum value of the sample). In such a way, the parameter $s \in (-\infty, \infty)$ captures the phenomena of investor optimism $s > 1$ and investors pessimism $s < 1$, and we call it a sentiment index. The following values of the parameter $s$ represent well-known special cases of the generalized mean, which have been studied in Chapter 6:

- $s \to -\infty$, the minimum of the sample;
- $s = -1$, the harmonic mean;
- $s \to 0$, the geometric mean;
- $s = 1$, the arithmetic mean;
- $s = 2$, the quadratic mean;
- $s \to \infty$, the maximum of the sample.

The predicted standard deviation of the next period return $\tilde{\sigma}_{t+1}$ is calculated from the standard deviation $\hat{\sigma}$ of the sample of past returns $R_{t-1}, ..., R_{t-m}$ and the level of confidence $c$:

$$\tilde{\sigma}_{t+1} = c \times \hat{\sigma}. \quad (8.9)$$

In this general model, we use the confidence coefficient $c \in [0, \infty)$ which captures both the case of investor overconfidence ($c \in [0, 1)$) and investor underconfidence ($c > 1$). Overconfident EMBs are too confident about their predictions. They underestimate the standard deviation of the distribution, making it more peaked around the generalized mean of the sample. Underconfident EMBs are less certain about future returns. They overestimate the standard deviation of returns making the distribution broader around the generalized mean.

Figure 8.1 shows an example of obtained probability mass functions for a specific sample of observed returns. The case of a uniform distribution represents the original, unbiased EMBs from the LLS model. For confidence coefficient $c = 1$ EMB investors use the standard deviation of past returns from the rolling window. As the overconfidence increases (confidence coefficient decreases below 1) observed returns that are closer (further) to the mean are given a higher (lower) probability, so the

$^{1}$Value $R_t$ is not included in the calculation because return in period $t$ is yet to be determined based on this prediction of the next period return.
distribution becomes more peaked. The special case of $c = 0$ is the full overconfidence where all the probability mass is given to the sample mean, so only one value represents the expected returns. Values $c > 1$ represent underconfidence, as investor expectations have more spread than what the past observations would suggest. In the limit case of extreme underconfidence, the distribution becomes uniform, as in the case of the original LLS model.

![Figure 8.1: Probability mass functions of observed past returns for different levels of confidence.](image)

In each period of the simulation, EMB investor $i$ predicts the next period return by the following discrete probability distribution that incorporates the effects of investor sentiment and confidence:

$$
\Pr_i(\tilde{R}_{t+1} = R_{t-j}) = \frac{\text{pdf}(R_{t-j}|\tilde{\mu}_{t+1}, \tilde{\sigma}_{t+1})}{\sum_{k=1}^{m'} \text{pdf}(R_{t-k}|\tilde{\mu}_{t+1}, \tilde{\sigma}_{t+1})},
$$

(8.10)

where pdf is the probability density function of a normal distribution. Since probability mass function assigns probabilities only to the returns of the rolling window, for pronounced levels of optimism or pessimism (i.e. high offset of the generalized mean from the arithmetic mean) the resulting discrete probability distribution will be skewed.
8.2. Model Description

New Agent Behavior (1): Updating Investor Sentiment

In this chapter we model a change in investor sentiment based on the market performance of investors. The investors look at their return on investment in the last period (the relative change in their own wealth), and based on that change they update their index of optimism. If the relative return is higher than one (their wealth has increased), they increase their optimism (increase parameter $s$), and if the relative return is lower than one (their wealth has decreased), they decrease their optimism (decrease parameter $s$). In order to make this update, the parameter $s \in (-\infty, \infty)$ is first mapped into $S \in (0, 1)$, for which purpose we use the logistic function $L$ (see also Figure 8.2):

$$L(s^i_t) = \frac{1}{1 + e^{-(s^i_t - 1)}} = S^i_t \quad (8.11)$$

The index of optimism is mapped using the logistic function in order to simplify the update equations (Equation 8.12 and Equation 8.13). Optimism can now easily be increased (decreased) by multiplying with a value that is higher (lower) than one. In addition, the logistic function is translated horizontally so that the neutral sentiment, i.e. the border between optimism and pessimism ($s = 1$) corresponds to the middle point of the transformed interval ($S = 0.5$). This will be convenient when analyzing the graphs of the dynamics of investor sentiment, because the upper half of the graph represents investor optimism and lower half of the graph represents investor pessimism (see Figure 8.5, Figure 8.8 and Figure 8.6).

After the transformation, the index $S$ is modified based on the recent performance. If the wealth has increased, the optimism of EMB investors is increased by a factor $\bar{l}$:

$$\text{If } \left(\frac{W^i_t}{W^i_{t-1}} > 1\right) \text{ then } S^i_{t+1} = S^i_t \cdot \bar{l}, \quad (8.12)$$

and if the wealth has decreased, the optimism is decreased by a factor $\underline{l}$:

$$\text{If } \left(\frac{W^i_t}{W^i_{t-1}} < 1\right) \text{ then } S^i_{t+1} = S^i_t \cdot \underline{l}, \quad (8.13)$$

where $\bar{l} > 1$ and $0 < \underline{l} < 1$. Finally, the index $S$ is mapped back into the original interval $(-\infty, \infty)$, using an inverse logistic function

$$s^i_{t+1} = L^{-1}(S^i_{t+1}) = \ln \left(\frac{S^i_{t+1}}{1 - S^i_{t+1}}\right) + 1. \quad (8.14)$$

We study two types of updates, a symmetric update where investors are equally sensitive to losses and gains:

$$1 - \underline{l} = \bar{l} - 1, \text{ e.g. } \bar{l} = 1.01 \text{ and } \underline{l} = 0.99, \quad (8.15)$$

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and an \textit{asymmetric} update in which investors are more sensitive to losses than gains:

\[
1 - \ell = \lambda(\overline{R} - 1), \lambda > 1, \text{ e.g. } \lambda = 2, \overline{R} = 1.01 \text{ and } \ell = 0.98.
\] (8.16)

This can be seen as a way of modeling loss aversion, since investors are more influenced by negative returns than positive returns. By increasing pessimism, loss aversion decreases the mean of expected returns. Compared to the Prospect Theory, our mechanism may be seen as more similar to the effects of nonlinear probability weighting (which influences the weights given to negative outcomes) rather than value function (which influences the utility of negative outcomes). In Prospect Theory the disutility of a loss is about twice the utility of the same gain.

The inspiration for this type of model we found in literature that studied the emotional aspects of decision under risk, particularly the effects of experienced (as opposed to expected) losses and gains. In a study by Shiv et al. (2005), patients with brain lesions in emotion-related parts of brain participated in an experiment with investment decisions. At the beginning of the experiment the subjects were endowed with 20 dollars and in each of 20 rounds they could choose whether to invest 1 dollar in a risky prospect that has a 50%-50% chance of winning 2.5 dollars or winning nothing. The results of the study showed that patients with brain lesions made more investments than the normal participants and control patients, and thus earned more on average. Normal and control patients seem to have been more affected by the outcomes of previous decision - upon winning or losing they adopted a conservative strategy and invested less in subsequent rounds. Compared with the target patients, who invested in 85.2% of rounds following losses, normal participants invested in only 40.5% of rounds following losses. Similarly, target patients invested in 84% of rounds following gains, while normal participants invested in only 61.7% of rounds following gains. Hence, normal participants showed considerably more risk aversion following losses than following gains.

\textbf{New Agent Behavior (2): Updating Investor Confidence}

Here we also implement un updating mechanism for investor confidence based on the success of return predictions. Similarly to Lovric et al. (2009b) and Daniel et al. (1998), we add shifts in investors’ confidence as a function of the investment outcomes. Let $\hat{\mu}_t$ and $\hat{\sigma}_t$ be the generalized mean (fuzzy aggregation operator) and standard deviation of the sample $R_{t-1}, ..., R_{t-m}$, respectively, and $c_i^t$ the confidence of investor $i$ in period $t$. For good predictions (the outcome is within two standard deviations around the generalized mean) the overconfidence increases ($c$ decreases), while for bad predictions the underconfidence increases ($c$ increases). First we map the confidence
8.2. Model Description

Figure 8.2: Logistic function used for mapping the index of optimism. The inflection point is at \( s = 1 \) which means that the arithmetic mean is considered as a neutral sentiment on the border between optimism and pessimism.

coefficient from \( c \in [0, \infty) \) into a more convenient interval \( C \in <0, 1> \), for which purpose we use the following transformation function \( T \) (see also Figure 8.3):

\[
T(c_i^t) = 1 - 2^{-c_i^{1/4}} = C_i^t
\]

(8.17)

Transformation function is chosen so that the normal level of confidence \( (c = 1) \) is mapped into the middle point of the transformed interval \( (C = 0.5) \). This means that in the graphs which depict the dynamics of investor confidence (see Figure 8.4, Figure 8.8 and Figure 8.6), the upper half of the graph represents investor overconfidence, while the lower half of the graph represents investor underconfidence.

The updating rule is given by:

\[
\text{If } |\hat{\mu}_t - R_t| < 2(c_i^t \cdot \sigma_t) \text{ then } C_{t+1}^i = a \cdot C_t^i,
\]

\[
\text{Otherwise, } C_{t+1}^i = \alpha \cdot C_t^i,
\]

(8.18)

where \( \alpha > 1 \) and \( 0 < a < 1 \). After the update, \( C \) is mapped back into the original interval \([0, \infty)\):

\[
c_i^{t+1} = T^{-1}(C_{t+1}^i) = \left( \frac{\ln(1 - C_{t+1}^i)}{\ln 0.5} \right)^4.
\]

(8.19)

For a biased self-attribution there is an asymmetry in update: \( 1 - \alpha < \bar{a} - 1 \) (e.g. \( \bar{a} = 0.75 \) and \( a = 1.05 \)), which means that overconfidence increases for good predictions relatively more than it decreases for bad predictions. This is because people tend to attribute success to themselves more so than they do with unsuccessful
outcomes, which they rather attribute to some external effects or bad luck. We define an *unbiased self-attribution* as the case of a symmetric update: $1 - \bar{a} = \bar{a} - 1$ (e.g. $\bar{a} = 0.75$ and $a = 1.25$).

### 8.2.3 Market Mechanism

LeBaron (2006) describes four types of market mechanisms used in agent-based artificial financial markets. In this chapter, as in the original LLS model, we use clearing by *temporary market equilibrium*. RII and EMB investors determine optimal proportion in the stock so as to maximize the expected utility of their wealth in the next period. However, expected utility is the function of the future price, which is in the current period unknown. Investors therefore need to determine optimal proportions, and respective demands for shares, for various hypothetical prices. The equilibrium price $P_t$ is set to that hypothetical price for which the total demand of all investors in the market equals the total number of outstanding shares, according to

$$
\sum_i N_h^i(P_t) = \sum_i \frac{x_h^i(P_t) W_h^i(P_t)}{P_t} = N. \quad (8.20)
$$
8.3 Experimental Design

In the benchmark model of the original study Levy et al. (2000), where only RII investors are present in the market, there is no trade, the log prices follow random walk, and there is no excess volatility of the market price. The dynamics of the price in the benchmark model is showed in Chapter 5, where we replicate the original study. In the experiment with a small fraction of homogeneous (with respect to memory length) and unbiased EMB investors (of the original model), which is also replicated in Chapter 5, the market dynamics show semi-predictable booms and crashes, with substantial trading in the market and excess volatility. The occurrence of these cyclic bubbles can be related to the size of the memory length of EMB investors. This experimental setup of Levy et al. (2000) is the basis for the experiments in this chapter too.

In Chapter 7 we have analyzed market dynamics for various combinations of investor confidence and sentiment. The general finding is that the more optimistic EMB investors are, the higher the market bubbles (the departures of the market price above the fundamental price) are. Very pessimistic EMB investors push the market price toward the fundamental price (since they invest mostly in the riskless asset). Overconfidence of investors augments the impact of investor sentiment, while investor underconfidence pushes the market dynamics towards the original LLS model of unbiased (uniform) EMB investors.

In this chapter we study the dynamics of investor psychology (confidence and

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Table 8.1: Parametrization of the model used for experiments with self-attribution and loss aversion.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>950</td>
<td>Number of RII investors</td>
</tr>
<tr>
<td>$M_2$</td>
<td>50</td>
<td>Number of EMB investors</td>
</tr>
<tr>
<td>$m$</td>
<td>10</td>
<td>Memory length of EMB investors</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.5</td>
<td>Risk aversion parameter</td>
</tr>
<tr>
<td>$N$</td>
<td>10000</td>
<td>Number of shares</td>
</tr>
<tr>
<td>$r_f$</td>
<td>0.01</td>
<td>Riskless interest rate</td>
</tr>
<tr>
<td>$k$</td>
<td>0.04</td>
<td>Required rate of return on stock</td>
</tr>
<tr>
<td>$z_1$</td>
<td>-0.07</td>
<td>Maximal one-period dividend decrease</td>
</tr>
<tr>
<td>$z_2$</td>
<td>0.10</td>
<td>Maximal one-period dividend growth</td>
</tr>
<tr>
<td>$g$</td>
<td>0.015</td>
<td>Average dividend growth rate</td>
</tr>
</tbody>
</table>
sentiment) based on the success of market predictions and market performance. The aim of the first two experiments is to study the effects of sentiment update and confidence update separately. Our first experiment focuses on the difference between biased (self-attribution) and unbiased update in confidence for a fixed investor sentiment (neutral, i.e. arithmetic mean $s = 1$). Our second experiment studies the difference between biased and unbiased update in sentiment for a given level of confidence (normal confidence $c = 1$).

The next two experiments focus on the combined effects of sentiment update and confidence update. The third experiment consists of unbiased updates, both with the confidence (unbiased self-attribution) and with the optimism (unbiased sentiment update). The fourth experiment consists of biased updates, both with the confidence (biased self-attribution) and with the optimism (loss aversion). In each experiment the market consists of 95% RII investors and 5% EMB investors, with the parametrization given in Table 8.1. The observed variables are: transformed confidence coefficient $T(c)$, transformed sentiment index $L(s)$, market price and fundamental price, and relative wealth of RII investors compared to EMB investors.

### 8.4 Results

Figure 8.4 shows the dynamics of confidence for biased and unbiased self-attribution, under a constant neutral sentiment modeled by the arithmetic mean. We can observe that with unbiased update investors are mostly overconfident, since the transformed value of confidence coefficient varies around value $T(c) = 0.4$, which corresponds to the value of $c = 0.3$. However, there is a substantial variation in confidence level, which sometimes even crosses into underconfidence. This depends on the success of investors predictions and how often they are surprised by return observations. When update is biased due to self-attribution bias, the investors become more overconfident, since variation around value $T(c) = 0.3$, corresponds approximately to variations around the value of $c = 0.1$. Hence, the simulations confirm the expectation that self-attribution bias leads to stronger overconfidence.

With a symmetric update of sentiment, investor sentiment is very optimistic throughout the whole simulation (Figure 8.5). When the update is asymmetric due to loss aversion, investor sentiment drops down, but it is also in the optimistic domain. The reason for the optimistic sentiment in both unbiased and biased update is that from period to period EMB investors’ wealth is more times increasing than decreasing. This is especially the case at the beginning of the simulation where market price simply rises with the fundamental price before it reaches the point where more interesting dynamics starts unfolding. The simulation also demonstrates the
8.4. Results

Figure 8.4: The dynamics of confidence for unbiased and biased self-attribution, with a fixed neutral sentiment (arithmetic mean).

The expected effect of loss aversion, since loss aversion decreases optimism level when compared to the case of unbiased sentiment update.

Figure 8.6 shows the dynamics of investor sentiment and confidence when both updates are unbiased. We can see that investors are very optimistic since the transformed value of sentiment, $L(s)$, varies closely to the value of one which represents full optimism. Again, the explanation for this is that there are more positive than negative changes in investors’ wealth from period to period. However, their confidence varies greatly and is more in the area of overconfidence. An interesting finding is that even though investors are highly optimistic, which is expected to lead to bubbles in the market price, the high volatility in confidence seems to prevent that from happening (Figure 8.7). A possible explanation for this observation is that when a good return happens such that it falls outside of the investor’s confidence interval, the investor is surprised and becomes consequently less confident. By increasing the standard deviation of the return distribution used for next period prediction, the probability mass is pulled away from the high optimistic mean. This may be enough to break down the self-reinforcing mechanism which would normally create bubbles.

Figure 8.8 shows the dynamics of investor sentiment and confidence when both updates are biased. Throughout the simulation, investors exhibit overconfidence, but
Chapter 8. Self-Attribution Bias and Loss Aversion

Figure 8.5: The dynamics of sentiment for unbiased sentiment update and loss aversion, under a constant normal confidence level.

Figure 8.6: The dynamics of sentiment and confidence for unbiased sentiment update and unbiased self-attribution.
Figure 8.7: Price dynamics with 95% RII and 5% EMB with unbiased sentiment update and unbiased self-attribution.

their sentiment at a certain point shifts from optimism to pessimism. Market price in Figure 8.9 shows similar behavior as in the biased case, with some bubbles being more prominent. It is important to note that the same patterns have been observed in experiments with different seed values of the pseudo-random number generator.

Finally, Figure 8.10 shows the dynamics of relative wealth of RII investors with respect to EMB investors when they are biased in updates of both sentiment and confidence, and when they are unbiased in both of these updates. The figure shows the dynamics that is averaged across 100 simulations in order to smooth out the volatilities of individual simulations and allow easier observation of trends. It can be seen that in both cases RII investors and EMB investors split the total wealth approximately in half. Interestingly, the difference between biased and unbiased EMBs is not that prominent, with biased investors even having a slight advantage at the beginning. This is a surprising result, as we were expecting much better performance from unbiased investors due to their high levels of optimism. However, market dynamics also shows a lack of bubbles which EMB investors could ride in order to gain advantage. It seems that biased investors are better able to exploit the combination of optimism and higher overconfidence at the beginning of the simulation to gain some advantage, although this advantage slowly dissipates throughout the simulation as they start decreasing their optimism levels.
Figure 8.8: The dynamics of sentiment and confidence for loss aversion and biased self-attribution.

Figure 8.9: Price dynamics with 95% RII and 5% EMB with loss aversion and biased self-attribution.
8.5 Conclusion

In this chapter we have studied the dynamics of investor sentiment and investor overconfidence in the modified LLS model of the stock market presented in Chapter 7. The overconfidence in the general sentiment-confidence model Chapter 7, refers to the peakedness of the return distribution around the mean of return observations, while optimism in this model determines how that mean is chosen (ranging from the minimum observation to the maximum observation in the sample of past returns), which is achieved by using a fuzzy aggregation operator. Besides the unbiased updates in sentiment and confidence, we have also studied biased updates in investor sentiment (due to loss aversion) and investor confidence (due to self-attribution bias). The simulations show that overconfidence can emerge through biased self-attribution and that loss aversion can decrease investor optimism. The simulations with combined updates exhibit interesting interactions between modeled psychological effects, which stem from the fact that the confidence update and the sentiment update are interrelated. Overconfidence is updated based on the success of predictions, which is determined by checking if the return realization falls outside or within a given interval around the mean. However, that same mean is also shifted through the sentiment
update, which is based on the investor performance, i.e. the changes in wealth.

This study demonstrates the capabilities of computational agent-based approach for studying not only various behavioral phenomena, but also the dynamics of changing investor psychology based on some market feedback. In order to facilitate this, it is very useful if behavioral phenomena can be modeled by only one parameter. There are, however, additional implementation choices which concern the updating mechanism. For example, updates can be achieved with multiplicative increments (as in our experiments), or with additive increments (when small values are added or subtracted). It is possible that by choosing different nonlinear transformation functions and different types of updates the sentiment (and confidence) levels would in our experiments end up fluctuating around different values. However, this is not of our primary concern, because we are not interested in absolute values of these parameters as much as in their comparison i.e. the relative dynamics of unbiased and biased updates. Since the line representing sentiment dynamics for biased update falls below the line representing unbiased update, we can conclude that loss aversion decreases investor optimism. Similarly, we can see that with a biased self-attribution investors become more confident than with an unbiased self-attribution.

If in the future we would change the rules of updating mechanisms, it would be as follows. For optimism, we would like to incorporate a better measure of investor performance into the updating rule. In the current implementation, to give an illustration, an investor who won one euro twice and then lost one hundred euros would become more optimistic since there were two gains and only one loss. It would be more realistic to update sentiment based on how much the wealth has increased or decreased, instead of only taking into account whether the wealth increase or decrease occurred. For confidence, instead of a discrete rule which determines whether a prediction was good or bad (by checking if the observation falls within or outside the confidence interval), we would like to incorporate the magnitude of the prediction error, i.e. how far the return observation fell from the mean of the return distribution used for prediction.

An important feature of the LLS model is a positive trend, which means that strategies with higher exposure to the risky asset will be more lucrative. It would be interesting, however, to investigate the role of different market motions, for example a declining market trend, so that we could possibly draw a more general conclusion about the role of loss aversion and self-attribution bias.

Finally, the finding that biased types of behavior can not only survive in the market with rational investors, but also at times outperform them, as well as other non-biased investors, is thought provoking. It suggests that what is considered a bias with respect to some theoretical norm, may not necessarily be detrimental to the
performance of investors. This is because specific market conditions and the structure of the market (including the very presence of biased investors) influence what type of behavior is successful, and such behavior may not always be in accordance with the assumptions of normative theories and narrowly defined rational behavior.
Chapter 9

Better-Than-Average
Overconfidence in the SSE Model

9.1 Introduction

In previous chapters we have focused on overconfidence defined as miscalibration and implemented as underestimated variance. The results of our study in Chapter 7 suggest that it is difficult to say whether overconfidence is bad or good for investors. When overconfidence is understood and modeled in the sense of miscalibration, our model shows that it is more relevant what investors are confident about than the mere fact they are overconfident. Optimistic investors benefited from their overconfidence because optimistic views were correct for the rising market of the LLS model. However, pessimistic investors suffered from overconfidence because it enhanced their wrong views on the market.

Importantly, psychologists have documented other kinds of overconfidence, which can be conceptually, as well as empirically, distinguished from miscalibration (Biais et al., 2005). They refer to "positive illusions" such as overestimating one’s own skills and abilities, particularly in comparison with the skills and abilities of other people on average. Glaser and Weber (2007) suggest that, even though widely used, miscalibration may not be the best proxy for overconfidence. In a study which combines psychometric measures of judgment biases (overconfidence scores) and field data (trading records), they could not relate measures of miscalibration to measures
of trading volume, whereas they could do so with the "better-than-average" overconfidence Glaser and Weber (2007)\(^1\).

In this chapter we are interested in modeling a different type of overconfidence, namely the better-than-average overconfidence. This type of overconfidence influences how investors compare their own opinions to the average opinion of other market participants. Overconfident investors think they are better than other investors on average, so they give more importance to their own opinions. However, since the average opinion of market participants is expected to mainly influence the formation of prices in the market, perhaps departing from that opinion will have measurable consequences for overconfident investors. In this chapter we provide a definition of a better-than-average overconfidence and run experiments using an executable version of an existing artificial stock market model - the Sim Stock Exchange (SSE) by Hoffmann et al. (2007)\(^2\). After that, we propose a way of modeling better-than-average overconfidence in the original LLS model of Levy et al. (2000).

### 9.2 Better-Than-Average Overconfidence

Better-than-average overconfidence is a type of positive illusion in which people think they are better than other people on average. In the context of financial markets it could mean that people think they have better investment skills or have better information than other market participants. From the implementation point of view, better-than-average overconfidence means that each agent needs to have some information about other agents (at minimum some aggregate measure of other agents’ information). These agents could be all other agents in the model, or only agents to which an agent is connected in a network.

In order for an agent-based market to be a good candidate for studying better-than-average confidence, it is important that the model contains some mechanisms of social interaction between agents. For example, the agents could be placed in a type of network structure, or there could be a random mechanism that explains how agents meet and interact with each other (for example randomly matching two agents as in Kirman (1993). In the paper by Lux and Marchesi (1999), an important parameter

\(^1\)A presumption in the financial literature is that investor overconfidence increases trading volume because overconfident investors trade "too much" (Odean, 1999). Our findings in Chapter 7 are contrary. We find that overconfident investors decrease trading volume as they tend to stick with one trading alternative, depending on their sentiment (see Chapter 7). In the LLS model, the volume is calculated as the change in portfolio holdings between successive periods, since trading is not modeled explicitly.

\(^2\)We would like to thank the authors of this model for making the executable version available online.
of the model was the propensity of agents to switch to another strategy. This variable could be interpreted in the light of the investor confidence. Those investors who are more confident about their skill are less likely to switch to a neighbor’s strategy, while less confident investors are more likely to imitate other investors’ strategies. The problem with this approach is that it is not clear what would be an average strategy, especially if a strategy cannot be represented by a numerical value. Also, it is not clear what level of confidence would be considered as the neutral point (possibly a normative choice) against which overconfidence and underconfidence biases would be defined.

When investors are exchanging a numerical value, such as price expectations, it is meaningful to calculate the average opinion of neighboring agents, as well as to value this average opinion against an agent’s own opinion. The SSE model of Hoffmann et al. (2007) proposes a model of investor confidence according to which an agent’s private information signal is weighed against the aggregate signal of other agents’ in the network:

\[
E_{st} = (E_{st} \times \text{Conf}) + (NE_{st} \times (1 - \text{Conf})),
\]

where \(E_{st}\) is an agent’s expected price for stock \(s\) at time \(t\), and \(NE_{st}\) is the aggregated expected price from agent’s neighbors. \(\text{Conf}\), such that \(0 \leq \text{Conf} \leq 1\) is the agent’s confidence level.

In the paper by Hoffmann et al. (2007) different levels of confidence have been studied, yet there is no reference which levels of confidence would constitute overconfidence. For our simulations we would like to designate the following values of Conf parameter as:

- \(\text{Conf} = 0\) - full (better-than-average) underconfidence.
- \(0 \leq \text{Conf} < 1/2\) - better-than-average underconfidence.
- \(\text{Conf} = 1/2\) - normal confidence.
- \(1/2 < \text{Conf} \leq 1\) - better-than-average overconfidence.
- \(\text{Conf} = 1\) - full (better-than-average) overconfidence.

The choice of equal weighting of the personal opinion and the average opinion of others is taken as the neutral point or the normal level of confidence. This is to signalize that the investor does not think that his or her opinion is better or worse than the average opinion of others. Better-than-average overconfidence in that case constitutes giving more than 50% of the weight to the investor’s own opinion and less than 50% to the average opinion of others. One justification for this threshold
Chapter 9. Better-Than-Average Overconfidence in the SSE Model

level is that it semantically fits the meaning of "better than average". However, it is still just an assumption that we make. Hoffmann et al. (2007) have in their study measured confidence levels $Conf$ of a group of investors and obtained a mean value of 0.60 and a standard deviation of 0.17. In terms of our definition of better-than-average overconfidence, those empirical measurements (fully reported in the Appendix 3 of Hoffmann et al. (2007)) indicate that 44% of investors have normal levels of confidence ($Conf = 0.5$), 38% of investors exhibit medium better-than-average overconfidence ($Conf = 0.63$ or $Conf = 0.75$), 10% have very high better-than-average overconfidence ($Conf = 0.88$ or $Conf = 1$), and 8% have better-than-average underconfidence ($Conf \in [0.38, 0.25, 0.13, 0]$). Hence, according to the chosen threshold level, the majority of people are either overconfident or normal, while only few are underconfident. This is consistent with our knowledge on the prevalence of human overconfidence and rarity of underconfidence, so it can be considered as a basic validation check for our choice of the threshold level.

9.3 SimStockExchange Model

SimStockExchange (SSE) is an agent-based model of a stock market with a goal to replicate real-world stock market dynamics by investigating possible key elements affecting stock traders’ behavior (Hoffmann et al., 2007). The novelty of the SSE model is the inclusion of a number of behavioral theories that explain investor behavior in the social context. The full model specification and original simulation results can be found in Hoffmann et al. (2007) and Hoffmann (2007).

SSE model has been developed using an agent-based toolkit Repast. The advantage of using Repast over a general high-level programming language (usually object-oriented) is that it provides libraries specifically aimed at agent-based models, which may include a time scheduler, a mechanism for agent communication, flexible interaction topologies, facilities for storing and displaying agent states etc. SSE model has also exploited Repast’s support for different network topologies, such as a torus network and a Barabasi and Albert scale-free network. By comparing the results for these two networks, Hoffmann et al. (2007) found evidence of volatility clustering only in the case of torus network. The authors speculate that, with regard to the information diffusion capacities, the investment population is more likely to behave like a torus network in which information can take longer to travel to remote corners, thus influencing the prices for a considerable amount of time (Hoffmann et al., 2007). In our experiments we have used a torus type of network, which is a network where every agent is connected to four neighboring agents (see Figure 9.1).
9.3. SimStockExchange Model

Figure 9.1: A torus type of network.

9.3.1 Agent Behavior

The SSE can simulate markets with an arbitrary number of stocks and investors who trade among themselves. The agents’ behavior is determined by a number of parameters, such as wealth, confidence level and risk reducing strategies. The study of Hoffmann et al. (2007) includes only one group of agents that differ in the values of the parameters. That means that the SSE model does not have heterogeneity of agents in the sense of few-type models (e.g. fundamental, technical, zero intelligence traders), but a parametric heterogeneity. However, some of the parameters are causing substantial changes in the behavior, for example, the parameter $R$ is determining which combination of risk-reducing strategies is going to be used.

In each step $t$, an agent needs to decide how much of its wealth to invest in the stock and how much to keep in cash (on which there is no interest received). The decision is made based on a simple rule that compares the Current market price of the stock at period $t$ ($P_{st}$) and Expected (next period) price of the stock (expected) at period $t$ ($E_{st}$). If the expected price is higher than the current price, an agent will invest the following proportion of cash in the stock: $x_i = (E_{st} - P_{st})/P_{st}$, and if the expected price is lower than the current price, an agent will divest the following proportion of stocks in its portfolio $x_d = (P_{st} - E_{st})/P_{st}$. The restrictions of no
borrowing \((x_i \leq 1)\) and no short-selling \((x_d \leq 1)\) are imposed. In the SSE model it is possible for agents to go bankrupt, and they can be either replaced with similar new agents or left in the model not participating in market activities. In the simulation experiments of this chapter, bankrupt agents are not replaced.

The expected price is based on the previous period market price and news that enters the market. This news is the main source of heterogeneity in the model as each agent gets different piece of news \(N_{st}\). Nonetheless, \(N_{st}\) are for all agents pooled from the same distribution (either normal or uniform):

\[
E_{st} = P_{st-1} + (P_{st-1} \times N_{st}).
\] (9.2)

After that, agents’ expectations are changed based on their levels of confidence Conf. An agent’s private information signal is weighed against the aggregate signal of the neighboring agents in the network \(NE_{st}\):

\[
E_{st} = (E_{st} \times \text{Conf}) + (NE_{st} \times (1 - \text{Conf})),
\] (9.3)

The SSE model introduces some more sophistication in how \(NE_{st}\) (the aggregated expected price from agent’s neighbors) is calculated. The agents who exhibit informational conformity behavior use the so-called clarifying risk-reducing strategy, which means that they take average expected prices of all other agents to which they are connected in the network. The agents who exhibit normative conformity behavior use the so-called simplifying risk-reducing strategy. Those agents copy the majority behavior (either buying or selling) of the people in the network. If the majority of agents are buying then they take average expected price of those neighbors, and if the majority of them are selling then they take average expected price of those neighbors (in case the number of buying and selling agents is equal, the average expected price of all neighbors is taken). The actual behavior of agents is a combination of the clarifying and the simplifying strategy, which is determined by the parameter \(R\).

\[
NE_{st} = (\text{Simpl}NE_{st} \times R) + (\text{Clar}NE_{st} \times (1 - R)).
\] (9.4)

The value of 0 means that only clarifying strategy is used, and the value of 1 means that only simplifying strategy is used. The study of Hoffmann et al. (2007) uses values of \(R\) empirically obtained from a group of investors. In our experiments, we will use only clarifying strategy, which means that the average opinion of all neighboring investors in the network will be taken into account.
9.3.2 Market Mechanism

The agents receive news at the same time, but the order in which they can decide whether to buy or sell and submit their limit orders is determined randomly. The market mechanism of the SSE model is based on the order book. This mechanism of the SSE model can be considered a more realistic feature than the market mechanism based on the temporary equilibrium, as with the LLS model by Levy et al. (2000). Order book mechanism is more suitable for models in which a more detailed trading activity is observed, whereas temporary equilibrium models usually represents a longer period within which we are not interested in detailed trading activity, but only at the equilibrium at which demand meets supply. In the SSE model agents submit limit orders. In the case of buying, they submit the maximum price and the number of assets to buy, and in the case of selling they submit the minimum price and the number of assets to sell. The market price is set to the average of the bid and ask prices, weighted by the size of the limit orders.

9.4 Experimental Design

The aim of these experiments is to show the effects of different levels of confidence in the better-than-average sense. Since we are not interested in the effects of different risk-reducing strategies (simplifying vs. clarifying) we have chosen the clarifying strategy, i.e. agents use the expectation of all the agents from the network (instead of only those who do the majority action). Also, we have chosen not to replace the bankrupt agents, not to weigh neighboring agents according to their wealth status, and, not to update confidence level based on the performance. These choices were made to exclude any additional effects that may influence the results, but are not of our main interest. The experiments of Hoffmann et al. (2007) have been run for 500 time steps (“to allow eventual early transients to die out”) plus additional 929 steps for which the returns and results were reported. One period in the simulation corresponds to one week in actual data, for which there were 929 observations. We run experiments for 1500 time periods and conduct analysis for all the time steps, since the resulting time-series appear to be stationary. Table 9.1 shows the parametrization for the experiments.
Table 9.1: Parametrization of the model used for experiments with better-than-average overconfidence.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>100</td>
<td>Number of agents</td>
</tr>
<tr>
<td>$N$</td>
<td>1</td>
<td>Number of stocks</td>
</tr>
<tr>
<td>$T$</td>
<td>1500</td>
<td>Number of time steps</td>
</tr>
<tr>
<td>$Ni$</td>
<td>10</td>
<td>Initial number of stocks in portfolio</td>
</tr>
<tr>
<td>$P_0$</td>
<td>10</td>
<td>Initial stock price</td>
</tr>
<tr>
<td>$W_0$</td>
<td>100</td>
<td>Initial wealth of agents</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0</td>
<td>News average</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.020</td>
<td>News standard deviation</td>
</tr>
<tr>
<td>Every time step</td>
<td>Normal</td>
<td>Type of news distribution</td>
</tr>
<tr>
<td>Torus</td>
<td>Linear</td>
<td>Loss aversion type</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>Replace bankrupt agents</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>Update confidence based on the performance</td>
</tr>
<tr>
<td>No</td>
<td>No</td>
<td>Weight friends with links</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>Seed to generate network</td>
</tr>
</tbody>
</table>

Table 9.2: Results of the experiments with different levels of better-than-average confidence in the SSE market.

<table>
<thead>
<tr>
<th></th>
<th>Conf = uniform[0, 1]</th>
<th>Conf = 0</th>
<th>Conf = 0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(P)$</td>
<td>1.2103</td>
<td>1.4571</td>
<td>1.2648</td>
</tr>
<tr>
<td>$\sigma(R)$</td>
<td>0.0696</td>
<td>0.0768</td>
<td>0.0691</td>
</tr>
<tr>
<td></td>
<td>Conf = 0.5</td>
<td>Conf = 0.75</td>
<td>Conf = 1</td>
</tr>
<tr>
<td>$\sigma(P)$</td>
<td>1.0958</td>
<td>0.9473</td>
<td>0.9067</td>
</tr>
<tr>
<td>$\sigma(R)$</td>
<td>0.0602</td>
<td>0.0568</td>
<td>0.0581</td>
</tr>
</tbody>
</table>
9.5 Results

Figure 9.2 shows the dynamics of the market price for different levels of confidence: the first graph is the dynamics for confidence level uniformly distributed on the [0, 1] interval, the second graph is full underconfidence (Conf = 0), the third graph is (better-than-average) underconfidence (Conf = 0.25), the fourth graph is normal level of confidence (Conf = 0.5), the fifth graph is better-than-average overconfidence (Conf = 0.75), and the last graph is the case of full overconfidence (Conf = 1). Similarly, Figure 9.3 shows return distributions for the same levels of underconfidence and overconfidence. We can see that compared to the normal level of confidence, the market with underconfident investors is more volatile, while the market with overconfident investors is less volatile. This is quantified in Table 9.2 which reports the volatility of the market price, as well as the standard deviation of the returns.

These findings can be explained as follows. The heterogeneity of expectations in the SSE model is generated at the beginning of each period when agents are given their estimated price. The estimated price will differ among agents since every agent is given different news information (sampled from either uniform or normal distribution). From that point on, the weighting of one’s own expectation against the neighboring expectations (through confidence and through risk-reducing strategies, if applicable) is essentially the averaging of expectations, which results in more similar expectations across the agents. Similar expectations create coordinated actions in the form of limit orders, which in turn causes more prominent price changes. When investors exhibit higher degrees of better-than-average overconfidence, they tend to stick more with their own estimates, which preserves the heterogeneity of opinions. Heterogeneous expectations create more heterogeneous limit orders, which result in a smoother price dynamics.

Since similar opinions in case of underconfidence were found to increase the price volatility, we were interested to see how far this averaging of opinions can go. Given that in a torus network each investor has only four neighboring agents, we have decided to conduct a robustness check by changing the network structure into a network where information diffusion is very fast. A scale-free network shown on Figure 9.4 is a very extreme type of network where every investor is connected to the same central investor. In the case of full better-than-average overconfidence, we have obtained the same market dynamics as in the case of torus network and full better-than-average overconfidence. This is not surprising, since in the case of full overconfidence there is no exchange of information between investors (except through the market), so the network topology does not play a role. However, in the case of full underconfidence when investors used the average opinion of neighboring investors, we
found that the market experienced two peaks, but at a certain point it simply crashed (see Figure 9.5). Hence, there is a limit to how homogeneous opinions of investors can be. Also, we can conclude that the results of the previously conducted experiments were not only driven by the effects of the better-than-average overconfidence, but also by the specificities of the network topology and the market mechanism.
Figure 9.3: Return distributions for different levels of better-than-average confidence in the SSE model (torus network).
Chapter 9. Better-Than-Average Overconfidence in the SSE Model

Figure 9.4: A scale-free type of network.

Figure 9.5: Market price dynamics for the full better-than-average underconfidence in the SSE model (scale-free network).
9.6 Better-Than-Average Overconfidence in the LLS Model

In Chapter 5 through Chapter 8 we have implemented and studied a number of behavioral phenomena using an existing agent-based financial market, namely the LLS market of Levy et al. (2000). The reason why the same model was not our first choice for a study of better-than-average overconfidence is that the LLS model is missing one of the key ingredients needed for the implementation of such a bias, and that is the notion of social interaction between investors. As opposed so the SSE model, in the original LLS model there is no exchange of information or strategies between investors of different types, nor between the investors of the same type. Although it would be possible to implement a network structure along which the individual investors in the LLS model could exchange information, we do not expect that such a model would yield interesting or unexpected results, especially taking into account the limited heterogeneity of investors in the LLS model. LLS model is a two-type artificial market model whereby RII investors are completely homogeneous and EMB investors are homogeneous up to a noisy variable influencing their optimal strategies. For that reason, in the LLS model we will only look at the exchange of opinions between the two major groups of investors, and not on the individual agent level.

Another issue for the LLS model with respect to modeling better-than-average effect is what type of information could be exchanged between the investors. Since RII and EMB investors do not use the same input for their strategies, instead of exchanging information about the risky asset, these investors could exchange their strategies. A strategy is in the LLS model represented by an optimal proportion of wealth invested in the risky asset \( x \in [0, 1] \), and since the average between two strategies also represents a strategy, this variable seems a good candidate for the implementation of better-than-average overconfidence.

In the following experiments, we give an implementation of better-than-average overconfidence in the LLS model in terms of mixing the strategies between the two major groups of investors. The strategy of the RII investors is kept intact as it represents the benchmark of rationality. Hence, only EMB investors will update their strategies based on their levels of better-than-average confidence. After having calculated their optimal strategy, EMB investors will create a new optimal strategy.

---

3In Chapter 5 through Chapter 8 we have assumed that EMB investors are also homogeneous with respect to their memory length \( m \). In their study, Levy et al. (2000) have also made experiments with EMB investors heterogeneous with respect to their memory length. They showed how a market model with such heterogeneity leads to a more realistic (smoother) time-series without obvious pro-cyclic price behavior. Such heterogeneity was not included in our models in order to avoid masking the intrinsic effects of the behavioral phenomena under study.
as a weighted average between their original optimal strategy $x^*_{EMB}$ and the optimal strategy of RII investors $x^*_{RII}$:

$$x^{**}_{EMB} = \text{Conf} \cdot x^*_{EMB} + (1 - \text{Conf}) \cdot x^*_{RII},$$

where $\text{Conf} \in [0, 1]$ represents the coefficient of confidence in the better-than-average sense. More confident EMB investors will give more weight to their personal optimal strategy, while less confident investors will give more weight to the strategy of RII investors. As in the experiments with the SSE model, we define the following values of parameter $c$ as the cases of:

- $\text{Conf} = 0$ - full (better-than-average) underconfidence.
- $0 \leq \text{Conf} < 1/2$ - better-than-average underconfidence.
- $\text{Conf} = 1/2$ - normal confidence.
- $1/2 < \text{Conf} \leq 1$ - better-than-average overconfidence.
- $\text{Conf} = 1$ - full (better-than-average) overconfidence.

The final strategies of EMB investors are calculated from the confidence-adjusted optimal strategy $x^{**}_{EMB}$ and a small amount of noise (consistent with the experiments in previous chapters).

The results of the simulations with various levels of better-than-average confidence are shown in Figure 9.6 and reported in Table 9.3. It can be seen that depending on the level of confidence, the resulting market dynamics shows the mixture of the price that would have been generated by the original strategy of EMB investors and the fundamental price. Hence, for all strategies that results in a behavior different than the benchmark model (which is true for most strategies, except for those that are very pessimistic or too risk-averse), increasing the level of overconfidence will also increase the departures from the fundamental value (up to the behavior of the original strategy).

### 9.7 Conclusion

Motivated by empirical studies in behavioral finance that emphasize the distinction between various types of overconfidence, in this chapter we have presented a study of better-than-average type of overconfidence. We have given a definition of better-than-average overconfidence based on the mathematical formulation of confidence in a social network of investors (Hoffmann et al., 2007), and studied market-wise
9.7. Conclusion

Figure 9.6: Market price dynamics in the LLS model for different levels of better-than-average confidence.
Chapter 9. Better-Than-Average Overconfidence in the SSE Model

Table 9.3: Results of the experiments with different levels of better-than-average confidence in the LLS market.

<table>
<thead>
<tr>
<th></th>
<th>EMB uniform</th>
<th>Conf = 0</th>
<th>Conf = 0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(p)$</td>
<td>11.4369</td>
<td>6.0250</td>
<td>5.9122</td>
</tr>
<tr>
<td>$\sigma(p^f)$</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
</tr>
<tr>
<td>excess volatility %</td>
<td>100.0892</td>
<td>5.4087</td>
<td>3.4352</td>
</tr>
<tr>
<td>mean volume p.p. %</td>
<td>9.9153</td>
<td>1.3765</td>
<td>4.6672</td>
</tr>
<tr>
<td></td>
<td>Conf = 0.5</td>
<td>Conf = 0.75</td>
<td>Conf = 1</td>
</tr>
<tr>
<td>$\sigma(p)$</td>
<td>5.9450</td>
<td>6.9973</td>
<td>11.4356</td>
</tr>
<tr>
<td>$\sigma(p^f)$</td>
<td>5.7159</td>
<td>5.7159</td>
<td>5.7159</td>
</tr>
<tr>
<td>excess volatility %</td>
<td>4.0094</td>
<td>22.4190</td>
<td>100.0682</td>
</tr>
<tr>
<td>mean volume p.p. %</td>
<td>7.3034</td>
<td>8.1441</td>
<td>9.9201</td>
</tr>
</tbody>
</table>

Implications of various levels of confidence in SSE (Hoffmann et al., 2007) and LLS (Levy et al., 2000) models of the financial market.

In order to study better-than-average overconfidence in an artificial market model, it is useful to have the possibility to implement a network structure, since better-than-average overconfidence takes into account the average opinion of neighboring investors in the network. The advantage of using an agent-based modeling toolkit such as Repast is the existence of libraries which enable straightforward implementation of various types of networks. Another advantage of Repast is that it separates model development from model execution. This allowed us to conduct various experiments in terms of changing parameter values, by having only the executable version of the SSE model. On the other hand, having no access to the actual code prevented us from having insight into important implementation details. Furthermore, it prevented us from making changes or additions to the model, e.g. changing the behavior of the agents outside the scope of behaviors provided by the model developers, changing the rules of the market mechanism etc.

In the case of the LLS model, the exchange of information did not happen through a network of connections between individual investors, but between the two major groups of investors. Also, as opposed to the SSE model, the investors did not exchange their estimations of the prices, but they exchanged their strategies. Confidence-based exchange of strategies is interesting as a possible psychological explanation for the diversity of opinions on the one hand and herding types of behavior on the other hand. This makes our study interesting because most market models that investigated switching strategies between investors, focused either on proba-
9.7. Conclusion

blistic or evolutionary mechanisms of switching strategies, or on switching based on more objective, deliberative reasons, such as the comparisons between wealth levels or strategy performance. In the LLS model, better-than-average overconfidence has been implemented in the sense of mixing the strategies of two investor types. More confident EMB investors give more weight to their own optimal strategy, while less confident investors give more weight to the optimal strategy of RII investors. Better-than-average overconfidence in the sense of weighted average between one’s own opinion and the opinion of others results in more diversified opinions, while better-than-average underconfidence results in a more uniform (averaged) opinion among investors. In the LLS model full underconfidence results in the market dynamics with very low trading where market price closely follows the fundamentals. Overconfident investors in the better-than-average sense trade more than the underconfident or normal investors, provided that they follow a strategy which does not generate a market price that follows the fundamentals (e.g. very pessimistic investors who invest only in the risk-less asset). This finding is in line with the assumption in the financial literature that investor overconfidence increases trading volume, and in the LLS model it has been achieved for better-than-average overconfidence, but not for overconfidence as miscalibration (see Chapter 5).

The findings for the SSE model can be explained as follows. The heterogeneity of expectations in the SSE model is generated at the beginning of each period when agents are given their estimated price. The estimated price will differ among agents since every agent is given different news information (sampled from either uniform or normal distribution). From that point on, the weighting of one’s own expectation against the neighboring expectations (through confidence and through risk-reducing strategies, if applicable) is essentially the averaging of expectations, which results in more similar expectations across the agents. Similar expectations create coordinated actions in the form of limit orders, which in turn causes more prominent price changes. When investors exhibit higher degrees of better-than-average overconfidence, they tend to stick more with their own estimates, which preserves the heterogeneity of opinions. Heterogeneous expectations create more heterogeneous limit orders, which result in a smoother price dynamics, since the market price is calculated as the average of bid and ask prices, weighted by the size of orders.

Better-than-average overconfidence can be seen as a mechanism that diminishes the exchange of information or the weight given to information or strategies of other investors. In such a way, overconfidence ensures the diversity of opinions and heterogeneity of strategies. Underconfidence, on the other hand, gives more weight to the

opinions of other investors which results in more averaged opinions and more similar behaviors, or even herding. Other factors that have influence on the diversity of opinions are the initial distribution of opinions among agents before the confidence is taken into account, and also the network structure that determines the neighboring agents, since the averaging of opinions happens faster when agents are connected to more people in the network. The implications of more or less diverse opinions on the dynamics of the market price are also dependent on the market mechanism used, as can be seen from different implications of overconfidence in the SSE and the LLS model.

In comparison with overconfidence in the sense of miscalibration, better-than-average type of overconfidence has a number of differences. Miscalibration is related with the precision of individual investor’s opinion. The more confident the investor is, the more precise his or her opinion is. Underconfidence ensures the diversity of opinions which can be characterized by a broader distribution. Better-than-average overconfidence is a different phenomena defined in a social context, where investor’s individual opinion is weighed against the average opinion of other investors in the network. As opposed to miscalibration, better-than-average overconfidence ensures the diversity of opinions, since overconfident investors give less weight to the opinions of others and stick with their own views. Underconfidence in the better-than-average sense results in more equal opinions among investors. Since most people are found to exhibit overconfidence to some extent, perhaps better-than-average effect is one of its manifestations that contributes to the diversity of opinions as well as the observed levels of trading in the markets. In such a way, our simulation results also seem to support the empirical study of Glaser and Weber (2007), which distinguishes between the effects of better-than-average overconfidence and miscalibration, and their potential implications for finance.

explain high levels of trading volume and that a higher degree of differences of opinion leads to a higher degree of trading volume. However, as pointed by Glaser and Weber (2007), the models are usually silent about the reason why there are differences of opinion in the first place. Glaser and Weber (2007) argue that "an investor who regards himself as above average is more likely to maintain a specific opinion about the future performance of an asset even though he knows that other investors or the market hold a different opinion."
Chapter 10

Conclusions

10.1 Summary

The aim of this dissertation is to contribute to the research areas of behavioral finance and agent-based artificial markets by providing mathematical representations for a number of behavioral phenomena and by studying their market-wise implications using computational simulations. The main premise for this thesis was that agent-based models of financial markets are suitable for studying behavioral finance topics since they are able to link the micro behavior of market participants and the aggregate fluctuations of market prices.

Since an agent-based simulation constitutes a bottom-up approach, we need to start from a realistic description of the behavior of market participants. For that purpose we looked at the behavioral finance and psychological literature with an aim to describe how individual investor behave in the markets. In Chapter 2 we have given an overview of the main dimensions of investment decisions and summarized the findings in behavioral finance related to those dimensions, particularly with regards to various psychological biases of investors. The conceptual model of individual investor behavior presented in Chapter 3 aims to summarize a part of this vast knowledge on individual investor behavior, and gives the first level of structure necessary for the development of agent-based artificial financial markets. Chapter 4 gives an overview of agent-based artificial financial markets, focusing on the relevant aspects of individual investor behavior represented in such models and relating them to our conceptual model.

In Chapter 5 through Chapter 9 we have provided mathematical definitions and implementations for a number of behavioral phenomena, including overconfidence...
(miscalibration and better-than-average effect), investor sentiment (optimism and pessimism), biased self-attribution, loss aversion, and recency and primacy effects. In Chapter 5 we have proposed a model of investor overconfidence in the sense of probabilistic miscalibration. In Chapter 6 we have proposed a model of investor sentiment based on a fuzzy aggregation operator. Chapter 7 presents a general sentiment-confidence model in which sentiment (ranging from optimism to pessimism) determines the mean of predictions, while confidence (ranging from overconfidence to underconfidence) determines the standard deviation of predictions. In Chapter 8 we have studied the dynamics of investor psychology as a consequence of market feedback and investor performance. Chapter 9 presents a study of better-than-average overconfidence, which is a different manifestation of overconfidence than miscalibration studied in previous chapters.

10.2 Conclusion

In this thesis we have presented a conceptual model of individual investor behavior bringing together various behavioral and cognitive elements that play a role in the behavior of market participants. We think this model is useful for bridging the gap between the findings in behavioral finance literature and stylized behaviors and market representations often found in the literature on agent-based artificial financial markets. Chapter 2 and Chapter 3 provide answers to our first research question on identifying the relevant aspects of investor behavior that could be studied using an agent-based simulation approach. The review of behavioral finance literature results in a collection of heuristics and biases, which operate through different cognitive and emotional mechanisms, and manifest on various aspects of investment decision making. The major focus of behavior literature on heuristics and biases might seem as an implication of their utmost importance in investor behavior and the dynamics of financial markets. However, this is not necessarily the case. From our perspective, the usefulness of behavioral finance lies in offering a richer description of investor behavior than those captured by fully rational expected utility maximizers with limited heterogeneity (e.g. in risk preferences), by giving a collection of possible heuristics and biases, which have been documented in a financial, or sometimes a more general decision-making setting. The relevance of each behavioral phenomenon should be treated as a research question on its own and addressed using appropriate techniques, whether empirical or in our case agent-based simulations.

The research strategy we have used to develop our models is an incremental approach. We have used existing agent-based models and adapted them for the purpose of our study. The advantage of the incremental approach is that the existing
models first need to be replicated to reproduce the original results (which enforces the validity of both our research and replicated studies). Then the behavior of agents in the original model is changed, so that the agents exhibit a behavioral phenomenon under study. Each existing agent-based model usually has restrictions on what type of behavior can be studied and how it can be implemented. However, when we are able to change the behavior of individual market participants in a desired way, we can demonstrate what changes in the simulation results are due to the implemented behavioral phenomena.

With regards to our second research question, we have provided mathematical definitions and implementations for a number of behavioral phenomena, including overconfidence (miscalibration and better-than-average effect), investor sentiment (optimism and pessimism), biased self-attribution, loss aversion, recency and primacy effect. From a modeling perspective, our overall conclusion for these models is that behavioral biases should be modeled with fewest parameters possible, ideally with only one, so as to ease the manipulation and study of different levels of these biases. This has been particularly demonstrated in our sentiment-confidence model, where each phenomenon is conveniently modeled using only one real-valued parameter. Confidence refers to the precision of investor predictions, while sentiment determines the mean of those predictions. When overconfidence is modeled in the sense of miscalibration, i.e. too narrow confidence intervals, it is difficult to say whether overconfidence is good or bad for investor performance. If predictions are correct it is good to be overconfident about it, but if they are wrong it is better to be underconfident in order to spread the probability mass away from the wrong prediction. In addition, theoretically expected relationship between overconfidence and overtrading has not been observed. Quite the contrary, overconfident investors were found to trade less throughout the simulations as they chose to stick with their current portfolio choices. We found that sentiment (optimism and pessimism) had much more impact on the market dynamics in the studied agent-based model. Bubbles and crashes were caused by investors who remember (and extrapolate) extreme past events, i.e. highly positive returns in case of optimism or highly negative returns in case of pessimism. Overconfidence is only found to enhance those effects of investor sentiment.

In this thesis we have confirmed the added value of agent-based financial market models in studying the topics of behavioral finance. The usefulness of agent-based simulations stems from their ability to relate micro-level behavior of individual market participants (agents) and macro behavior of the market (aggregate market dynamics) as well as the consequences of such macro behavior (e.g. investor performance). This micro-macro mapping of agent-based methodology is particularly
useful for behavioral finance, since that link is often broken when studying behavioral finance topics using different methodological approaches. The levels of these behavioral biases have been related to various stylized features of market dynamics, such as the bubbles and crashes, the excess volatility of the market price, the fat tails of return distributions, and volatility clustering. Also, the impact of behavioral biases on the performance of investors has been studied. Hence, we have provided answers to the third research question on market-wise implications of investor biases.

Another contribution of this research and the advantage of computational agent-based models can be found in the link between investor behavior and his or her market performance or some other changing value in the simulation. In such a way, agents can change their behavior according to the market movements, or as in our experiments, the strength of a behavioral bias can be influenced by the market dynamics. Agent-based models are particularly useful for studying complex adaptive systems in which such feedback loops occur, and in which interaction between micro elements can lead to emerging phenomena that cannot be easily deducted from the behavior of individual components. For example, we have modeled how investors update their confidence level based on the success of their predictions. In case this update is asymmetrical (biased self-attribution), investors can become more overconfident. We have also modeled loss aversion as an asymmetry in sentiment update when investors wealth is decreasing or increasing. The results of our sentiment-confidence model as well as our models of self-attribution bias and loss aversion suggest that studying only individual effects of several biases is not sufficient. This is because of the possible interaction between those effects. That means that we should study more complex implementations of agents behavior that contain a number of such behavioral phenomena. These findings are increasing the importance of having a richer conceptual model of individual investor behavior that can capture the complexity of investor behavior, such as the one presented in this thesis.

In relation with the fourth research question, this thesis has also demonstrated the importance of definitions of behavioral phenomena under study. Unless an explicit definition of a bias, such as overconfidence, is given, the claims for possible effects of such behavior cannot be proven or falsified. Agent-based modeling approach forces us to provide operational definitions of behavioral phenomena, since such behavior needs to be represented and implemented in a computational model. This is particularly helpful for distinguishing between behavioral phenomena that seem related in their psychological effects and possible implications on investor behavior, for example overconfidence, optimism and risk taking. Moreover, one particular behavioral phenomena can have multiple definitions and manifestations, and in this thesis we have presented two distinct implementations of investor overconfidence. Our results
suggest that overconfidence in the sense of miscalibration and overconfidence in the better-than-average sense are indeed different phenomena with different implications for the market participants and the market dynamics. Hence, the results of our simulations are confirming the empirical research which clearly distinguishes between these two types of overconfidence and suggests more modeling efforts should be made in the direction of the better-than-average effect.

10.3 Future Research

We think that the future research of agent-based financial markets should develop in several directions. Firstly, agent-based research methodology should be seen as a complementary methodology to other approaches for studying behavioral finance topics, such as experiments and empirical research. In order to make this possible, it is important to develop flexible toolkits that will enable easy construction of agent-based models as well as easy implementation of agents behaviors. An example of such framework is ABSTRACTE environment developed at the Econometrics department of Erasmus School of Economics, Erasmus University Rotterdam and described in the PhD thesis of Boer-Sorban (2008).

Constructing an agent-based model can be a time consuming process, whether the model is implemented using a general high-level (possibly object-oriented) programming language, or an existing toolkit for agent-based simulations (such as Repast). Specialized toolkits should provide additional features which are specific to artificial markets, such as the choice of various market mechanisms, a library of simple stylized agents behaviors (e.g. zero intelligence, fundamental, technical), as well as demos for a number of well-known market models. Since the results can very depending on the model used, to the researchers interested in this area, we would suggest modeling and testing the same behavioral phenomena using a couple of agent-based models, based on different market mechanisms and market conditions. Also, we would suggest trying one of the newer models that are able to replicate the more advanced stylized facts of financial markets.

The agent-based models could also be strengthened and validated by using the experimental measurements of behavioral biases and parametric values that represent them. When biases are represented by parametric values, by a comparison of these values it can be easily determined which investor is exhibiting a bias to a larger extent. However, it is more difficult to interpret these parametric values in absolute terms. For this purpose, fuzzy logic and its ability to express expert knowledge using linguistic variables and natural language could be useful. For example, fuzzy membership functions could be used to model which values of confidence coefficient
represent a "very high degree of overconfidence" or a "medium underconfidence".

Another direction in which agent-based models should advance, so that they cross the border between theoretical market models and practical tools for financial practitioners, is the inclusion of empirical financial data into agent-based models. Empirical data can be used not only to calibrate the parameters of the model, but also as the source of information that agents can utilize in their decision making. We expect that the increased availability of fine-granularity financial data and improved capabilities of distributed computing will only further the collaboration between empirical and simulation research to the benefit of both behavioral finance and other financial areas.
Samenvatting

Het doel van deze dissertatie is om bij te dragen aan de onderzoeksgebieden 'behavioral finance' en 'agent-based financiële markten' door wiskundige representaties te verstrekken voor een aantal gedragsgerelateerde verschijnselen en door de marktgerelateerde effecten hiervan te bestuderen door gebruik van computationele simulaties. De voornaamste voorafgaande stelling van dit proefschrift is dat op agenten gebaseerde modellen geschikt zijn voor het bestuderen van 'behavioral finance'-onderwerpen, aangezien deze modellen geschikt zijn om het micro-gedrag van deelnemers aan de markt en de gezamenlijke fluctuaties van marktprijzen te verbinden.

Aangezien op agenten gebaseerde simulaties onderdeel zijn van een 'bottom-up' benadering, moeten we beginnen bij een realistische beschrijving van de marktdeelnemers. Hiervoor hebben wij gekeken naar de 'behavioral finance' en psychologieliteratuur met als doel het beschrijven van het gedrag van investeerders in de markten. In hoofdstuk 2 hebben wij een overzicht gegeven van de belangrijkste dimensies van investeringsbeslissingen en hebben wij de bevindingen gerelateerd aan deze dimensies samengevat vanuit het perspectief van 'behavioral finance', met speciale aandacht voor de verschillende psychologische neigingen van investeerders. Het conceptuele model van het gedrag van investeerders dat in hoofdstuk 3 werd gepresenteerd richt zich op het samenvatten van deze uitgebreide kennis over het gedrag van investeerders en verstrekt een basisstructuur voor het ontwikkelen van agent-based kunstmatige financiële markten. Hoofdstuk 4 geeft een overzicht van agent-based kunstmatige financiële markten, met aandacht voor relevante aspecten van het gedrag van investeerders in dit soort modellen, dit tegelijkertijd in verband brengende met ons conceptueel model.

In de hoofdstukken 5 tot en met 9 hebben wij wiskundige definities en implementaties geïntroduceerd voor een aantal gedragsgerelateerde verschijnselen, namelijk 'overconfidence' ('miscalibration' en 'better-than-average effect'), het sentiment van investeerders (optimisme en pessimisme), 'biased self-attribution', 'loss aversion', 'recency effect' en 'primacy effect'. In hoofdstuk 5 hebben wij een model van investor
'overconfidence' voorgesteld in de zin van 'probabilistic miscalibration'. In hoofdstuk 6 hebben wij een model van het sentiment van investeerders geïntroduceerd, gebaseerd op een 'fuzzy aggregation operator'. Hoofdstuk 7 introduceert een algemeen sentiment-vertrouwen model waarin sentiment (variërend van optimisme tot pessimisme) het gemiddelde van voorspellingen bepaalt, terwijl vertrouwen (variërend van 'overconfidence' tot 'underconfidence') de variantie van de voorspellingen bepaalt. In hoofdstuk 8 hebben wij de dynamiek van de psychologie van investeerders bestudeerd als gevolg van de markt feedback en de prestaties van investeerders. Hoofdstuk 9 presenteert een studie over 'better-than-average overconfidence', een ander soort 'overconfidence' dan de in voorgaande hoofdstukken bestudeerde 'miscalibration'.

In dit proefschrift hebben wij een conceptueel model van het gedrag van investeerders gepresenteerd dat de verschillende gedragsgerelateerde en cognitieve elementen samenbrengt die een rol spelen in het gedrag van marktdeelnemers. Wij denken dat dit model nuttig kan zijn bij het overbruggen van de kloof tussen bevindingen in de 'behavioral finance' literatuur en gestileerde feiten en marktrepresentaties die vaak worden gevonden in de literatuur over agent-based kunstmatige financiële markten. De hoofdstukken 2 en 3 geven een antwoord op onze eerste onderzoeksvraag over het identificeren van de relevante aspecten van het gedrag van investeerders die bestudeerd kunnen worden door het gebruik van een benadering gebaseerd op agent-based simulaties.

Met betrekking tot onze tweede onderzoeksvraag hebben wij wiskundige definities en implementaties verstrekt voor een aantal gedragsgerelateerde verschijnselen, inclusief 'overconfidence' ('miscalibration' en 'better-than-average effect'), het sentiment van investeerders (optimisme en pessimisme), 'biased self-attribution', 'loss aversion', 'recency effect' en 'primacy effect'. Onze algemene conclusie voor deze modellen is dat gedragsgerelateerde neigingen gemodelleerd moeten worden met zo weinig mogelijk parameters, in het meest ideale geval met alleen een parameter, om de manipulatie en de studie van de verschillende niveaus van deze neigingen de vergemakkelijken.

De onderzoekstrategie die wij hebben gebruikt om onze modellen te ontwikkelen bestaat uit een incrementele benadering. Wij hebben bestaande agent-based modellen gebruikt en deze aangepast voor het doel van onze studie. Het voordeel van onze incrementele benadering is dat bestaande modellen eerst gerepliceerd moeten worden om de oorspronkelijke resultaten weer te geven (wat de validiteit van onze studie waarborgt, zowel als die van de oorspronkelijke studies). Vervolgens is het gedrag van agenten in het oorspronkelijke model aangepast, zodat agenten het gedragsgerelateerde verschijnsel vertonen dan bestudeerd wordt. Iedere bestaande agent heeft meestal beperkingen wat betreft het type gedrag dat bestudeerd en geïmplementeerd
kan worden. Echter, wanneer we in staat zijn om het gedrag van de deelnemers aan de markt aan te passen op een gewenste manier, kunnen we de verschillen in de resultaten van de simulaties relateren aan de geïmplementeerde gedragsgerelateerde verschijnselen.

De resultaten van onze sentiment-vertrouwen model en onze modellen van ‘self-attribution bias’ en ‘loss aversion’ suggereren dat het bestuderen van alleen individuele effecten van verschillende neigingen niet genoeg is. Dit komt door de mogelijke interactie tussen deze effecten. Dit betekent dat we meer complexe implementaties van het gedrag van agenten moeten bestuderen, die dit soort gedragsgerelateerde verschijnselen bevatten. Deze bevindingen vergroten het belang van een conceptueel model van investeerders die de complexiteit van het gedrag van investeerders kan bevatten.

In dit proefschrift hebben wij de waarde van agent-based modellen van financiële markten voor het bestuderen van ‘behavioral finance’-onderwerpen bevestigd. Het nut van agent-based simulaties vindt zijn oorsprong in het vermogen om het gedrag op microniveau van de marktdeelnemers te relateren aan het macro-gedrag van de markt (‘aggregate market dynamics’) en aan de gevolgen van dergelijk macro-gedrag (e.g. ‘investor performance’). Dit micro-macro mapping van agent-based methodologie is met name bruikbaar voor ‘behavioral finance’, aangezien deze link vaak wordt verbroken bij het bestuderen van ‘behavioral finance’ onderwerpen gebruikmakend van verschillende methodologische benaderingen. De niveaus van deze gedragsgerelateerde neigingen werden gerelateerd aan de kenmerken van de markt dynamiek, zoals ‘bubbles en crashes’, en de overmatige volatiliteit van de marktprijs. Tevens, hebben wij de invloed van gedragsgerelateerde neigingen op de prestatie van investeerders bestudeerd. Op deze manier hebben wij antwoorden op de derde onderzoeksvraag verstrekt.

Een andere contributie van dit onderzoek en het voordeel van computational agent-based modellen kan gevonden worden in de link tussen het gedrag van investeerders en zijn of haar marktprestatie of een andere veranderende waarde in de simulatie. Op deze manier kunnen agenten hun gedrag aanpassen afhankelijk van de bewegingen in de markt of, zoals in onze experimenten, kan de sterkte van een gedragsgerelateerde neiging beïnvloed worden door marktdynamiek. Agent-based modellen zijn vooral bruikbaar voor het bestuderen van complexe adaptieve systemen waarin zulke feedbacklussen voorkomen, en waarin de interactie tussen de microelementen kan leiden tot opkomende verschijnselen die niet makkelijk afgeleid kunnen worden uit het gedrag van de individuele componenten. Zo wij hebben bijvoorbeeld de manier waarop investeerders hun vertrouwen niveaus aanpassen gemodelleerd aan de hand van het succes van hun voorspellingen. Als deze bijwerking asymmetrisch is
Samenvatting

(‘biased self-attribution’), kunnen investeerders ‘overconfident’ worden. We hebben ook ‘loss aversion’ gomodelleerd als een asymmetrie in het bijwerken van sentiment wanneer de waarde van de investeerde toeneemt of afneemt.

In verband met de vierde onderzoeksvraag heeft dit proefschrift het belang van definities van de gedragsgerelateerde verschijnselen die bestudeerd worden gedemonstreerd. Tenzij een expliciete definitie van een neiging, zoals ‘overconfidence’, wordt gegeven, kunnen de beweringen over mogelijke effecten van zulk gedrag niet worden bewezen of gefalsificeerd. De agent-based benadering dwingt ons om operationele definities te verstrekken voor gedragsgerelateerde verschijnselen, aangezien dit soort gedrag geregistreerd en geïmplementeerd moet worden in een computatiemodel. Onze resultaten suggereren dan ‘overconfidence’ als ‘miscalibration’ en ‘overconfidence’ in de zin van ‘better-than-average’ zijn inderdaad verschillende verschijnselen met verschillende gevolgen voor de marktdeelnemers en de marktdynamiek.

Wij denken dat toekomstig onderzoek van agent-based financiële markten zich kan ontwikkelen in verschillende richtingen. Ten eerste, agent-based onderzoeksmethodologie zou gezien moeten worden als aanvullende methodologie op andere benaderingen voor het bestuderen van ‘behavioral finance’-onderwerpen, zoals experimenten en empirisch onderzoek. Om dit mogelijk te maken, is het belangrijk om flexibele toolkits te ontwikkelen die het makkelijk bouwen van agent-based modellen in staat zullen stellen evenals het makkelijk implementeren van het gedrag van agenten. Een voorbeeld van zo een framework is de ABSTRACTE omgeving ontwikkeld door het Econometrisch Instuut van de Erasmus School of Economics, Erasmus University Rotterdam, en beschreven in het proefschrift van Boer-Sorban (2008).

Het bouwen van een agent-based model kan tijdrovend zijn, hetzij het model geïmplementeerd is gebruikmakend van een high-level (mogelijk objectgeoriënteerde) programeertaal, of een bestaande toolkit voor agent-based simulaties (zoals Repast). Gespecialiseerde toolkits zouden aanvullende kenmerken moeten verstrekkend die specifiek zijn voor artificiële markten, zoals de keuze van verschillende markt mechanismen, een library van eenvoudige gestileerde voorbeelden van het gedrag van agenten (zoals zero intelligence, fundamenteel, technisch), evenals demo’s voor een aantal bekende marktmodellen. Een andere richting waarin agent-based modellen voortgezet kunnen worden, zodat ze de kloof tussen theoretische marktmodellen en praktische gereedschappen toepassingen kunnen overbruggen, is de introductie van empirische data in agent-based modellen. Empirische data kan niet alleen gebruikt worden, voor het kalibreren van de parameters in het model, maar ook als bron van informatie die agenten kunnen gebruiken bij het nemen van beslissingen.


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Milan Lovrić was born on October 17, 1980 in Zagreb, Croatia. He studied computer science at University of Zagreb, Faculty of Electrical Engineering and Computing (FER), from which he obtained his degree of a graduate engineer of Computing (MSc) in 2005. The same year he also obtained a Diploma Study in Management postgraduate certificate from University of Zagreb, FER. In 2006 Milan joined Erasmus University Rotterdam, Erasmus School of Economics as an ERIM PhD Candidate in the area of finance. He worked with the Finance section of the Business Economics department until 2010, when he joined the Econometrics department. During his PhD project, Milan was involved in a number of courses at Erasmus School of Economics and Rotterdam School of Management, as a lecturer, a teaching assistant or a practical coordinator. He has presented his work at a number of international conferences and meetings, and part of his work has been published in Human Systems Management journal and as a book chapter in Advances in Cognitive Systems. His research interests include agent-based modeling, artificial financial markets and behavioral finance. Milan is currently pursuing a postdoctoral research at Rotterdam School of Management, the Department of Decision and Information Sciences.


BEHAVIORAL FINANCE AND AGENT-BASED ARTIFICIAL MARKETS

Studying the behavior of market participants is important due to its potential impact on asset prices and the dynamics of financial markets. The idea of individual investors who are prone to biases in judgment and who use various heuristics, which might lead to anomalies on the market level, has been explored within the field of behavioral finance. In this dissertation, we analyze market-wise implications of investor behavior and their irrationalities by means of agent-based simulations of financial markets. The usefulness of agent-based artificial markets for studying the behavioral finance topics stems from their ability to relate the micro-level behavior of individual market participants (represented as agents) and the macro-level behavior of the market (artificial time-series). This micro-macro mapping of agent-based methodology is particularly useful for behavioral finance, because that link is often broken when using other methodological approaches. In this thesis, we study various biases commented in the behavioral finance literature and propose novel models for some of the behavioral phenomena. We provide mathematical definitions and computational implementations for overconfidence (miscalibration and better-than-average effect), investor sentiment (optimism and pessimism), biased self-attribution, loss aversion, and recency and primacy effects. The levels of these behavioral biases are related to the features of the market dynamics, such as the bubbles and crashes, and the excess volatility of the market price. The impact of behavioral biases on investor performance is also studied.

ERIM

The Erasmus Research Institute of Management (ERIM) is the Research School (Onderzoekschool) in the field of management of the Erasmus University Rotterdam. The founding participants of ERIM are Rotterdam School of Management (RSM), and the Erasmus School of Economics (ESE). ERIM was founded in 1999 and is officially accredited by the Royal Netherlands Academy of Arts and Sciences (KNAW). The research undertaken by ERIM is focused on the management of the firm in its environment, its intra- and interfirm relations, and its business processes in their interdependent connections.

The objective of ERIM is to carry out first rate research in management, and to offer an advanced doctoral programme in Research in Management. Within ERIM, over three hundred senior researchers and PhD candidates are active in the different research programmes. From a variety of academic backgrounds and expertises, the ERIM community is united in striving for excellence and working at the forefront of creating new business knowledge.