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Commentaries* on "Nonexpected utility as aversion of information"

Peter Wakker

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Comment by Jordon Howard Sobel
Scarborough College, University of Toronto, 1265 Military Trail, Scarborough, Ontario MIC 1A4, Canada

1. Peter Wakker's ingenious defense of the independence axiom is addressed to a particular imagined realization of a 'formal set-up' in which (i) 'lotteries are modelled as probability distributions over "prizes", ' every distribution over given 'prizes' being a lottery, and all lotteries being 'comparable'; and (I assume) in which (ii) a 'probability mix' of any two lotteries is the lottery determined by 'multiplying out and collecting probabilities for prizes' (p. 169). Wakker's 'imagined realization' of the formal mixing operation consists of a single 'mixing event' for each probability mix. For example, for (1/6, 5/6) the mixing event could be a particular throw of a fair die coming up 'snake eye' or not. This event is, in his imagined realization, to 'settle' partially every lottery determined by that probability mix. (In this way all 'compound' lotteries are 'settled down' to 'simple' ones, with subsequent 'chance experiments' settling these.) Contrast this with 'my realization' (in which I have no personal investment) of the mixing operation, that involves a distinct mixing event for each lottery, for example, for (1/6 A, 5/6 B) the throw of one fair die, and for (1/6 A, 5/6 C), where $B \neq C$, the throw of *another* fair die.

2. Wakker considers the lotteries L: [pL_1, (1-p) L*], and L': [pL_2, (1-p) L*], and supposes a violation of the independence axiom, specifically, that $L > L'$, though $L_2 > L_1$. He observes that (on certain natural assumptions) given a choice between selecting L or L' *with* 'information about ... *the* mixing event' (p. 172, emphasis added), or *without* this information: *'she will strictly prefer not to receive* [*this*] *additional information'* (p. 173). This, he argues, is because (here we have the main natural assumption) she is confident that an informed selection either would, with a chance of p, go by her preference $L_2 > L_1$ and thus result in L_2, or, would, with a chance of (1-p), be random and result in L*. Given this confidence: (i) she sees that the choice to be informed would, in terms of probabilities for L_2 and L* and in turn for prizes, be equivalent to the choice of L'; (ii) she realizes that the choice not to be informed is equivalent in term of probabilities for prizes to the choice of L; and (iii) it is given that she strictly prefers L to L'. [I note that Wakker's scenario has the decision to be informed or not coming before the execution of the mixing event, with different sequels depending on how this decision goes: a choice to be informed would be followed by the mixing event, and this by the selection of L or L'; a choice not to be informed would be followed by the selection of L or L', and this by mixing event. Wakker supposes a 'public' mixing event, with an immediately known-to-all result. For a simpler and perhaps even more provocative scenario, let the mixing event come first, and not be in its result automatically known to all; let the decision whether or not to be informed about this result come next; and let the selection of L or L' come last.]

3. The italicized conclusion is not necessarily forthcoming on 'my realization'. For suppose 'my realization', *and* suppose that $L^* > L_2$. Then there is by the mixing event for L, a (1-p) - chance that L will yield L*. So there is by the mixing event for L' a p-chance that L' will yield L_2 will yield. So there is a [(1-p) p]-chance that, given information concerning these mixing *events*, she would select L to get L*. By similar calculations, there is: a pp-chance that, given that information, she would select L' to get L*, a [p (1-p)])-chance that, given it, she would select L' to get L*, and a (1-p) (1-p)-chance that, given it, she would be indifferent between L and L' and get L* (having selected one of L and L' at random). But then a choice to be informed is, in terms of probabilities for prizes, equivalent to a choice, *not* of L': [p L_2, (1-p) L*] (see (ii) above), but of the lottery L": [p^2 L_2, (1-p^2) L*]. And she may strictly prefer L" to L, even though she strictly prefers L to L'. I note that we *expect* her strictly to prefer L" to L': consider that $p >$

*Commentaries co-ordinated by Paul Anand, Centre for Experimental Economics, University of York, York, YO1 5DD, England.

p^2, and that, by our supposition, $L^* > L_2$. So possibly *she will strictly prefer to* ***receive*** *additional information.*

We have considered one case. In it $L^* > L_2$. There are four other cases: $L^* \approx L_2$, $L_2 > L^* > L_1$, $L^* \approx L_1$, and $L_1 > L^*$. But only of the second of these, where $L_2 > L^* > L_1$, can it be maintained that, if she is confident that her informed selection would go by her preferences for L_1, L_2, and L^*, choosing to be informed would be equivalent in terms of probabilities for prizes to choosing L', so that she will choose to be not informed.

4. Not every 'realization' of the formal set-up of utility theory harbors information that, if offered to a violator of the indpendence axiom would be declined. *But*, and Wakker can be read as intending no more than this, 'if someone [in his pairwise preferences for certain lotteries] ... violate[s] the independence condition, then, though his situation *need not* be one in which information-aversion is exhibited' (p. 169), it can be one in which he '[would] decline new information' and do so for reasons he could 'consider fully rational' (p. 172), given the attitudes of his towards 'risk' that are reflected in the violation in question. This deceptively complex insight challenges the rationality of persons whose preferences violate that axiom, and should move theorists who are inclined to think that such violations can be rational to scrutinize demonstrations to the effect that it cannot be reasonable to decline certain kinds of cost-free information.

It is noteworthy that Wakker's defense of the independence axiom does not suppose that ultimate 'prizes' in a formal set-up for utility theory must be 'completely specific' with respect to all things that matter to the decision-maker, or even with regard to all things of concern that are 'general' and not particular to certain lotteries, or pairs of lotteries. In this it may be compared with defenses produced by Leonard Savage and Howard Raiffa. And so it seems of particular importance to determine whether, and if so exactly how, demonstrations that, under certain conditions it is always reasonable to welcome certain kinds of information can be adapted to 'frameworks of preference' that feature total possible consequences or 'worlds' — 'states of nature where the human agent is taken to be part of nature and his acts are thus ingredients in states of nature [as are his subjective states]' (Jeffrey: 1974). According to fundamental principles erected in this framework, the rational object is to maximize in one's choice c, with total possible consequences w, something like Σ_w Probability (w given c). Value (w).

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Jeffrey, R. C. 'Frameworks for Preferences,' in M. Balch, *et. al* (eds.), *Essays on Economic Behavior under Uncertainty*, Amsterdam: North-Holland, 1974. [Jeffrey's is an 'evidential decision theory' in which 'Probability (w given c)' is the conditional probability of w on c. One can also erect 'causal decision theories' in 'possible worlds' frameworks: 'Probability (w given c)' can be taken to stand for something like the probability of the causal conditional that were it that c, then it would be in w.]

Maher, P. (forthomcing), 'Symptomatic Acts and the Value of Evidence in Causal Decision Theory,' *Philosphy of Science*. [This paper includes a demonstration 'that cost-free non-symptomatic observation never reduces' utility as understood in a 'causal decision theory', the fundamental rule for which theory 'is identical with Skryms' and is, in contrast with causal principles I have proposed, not a 'possible worlds' principle.]

McClennen, E. F. 'Sure-thing Doubts,' in P. Gärdenfors and N-E. Sahlin (eds), *Decision, Probability, and Utility: Selected Readings*, Cambridge: Cambridge University Press, 1988. [First published in 1983. Reprinted here with a '1986 Postscript'.]

Skryms, B. (forthcoming), 'The value of Knowledge' in C. W. Savage (ed.), *Justification, Discovery and the Evolution of Scientific Theories*, Minneapolis: University of Minnesota Press. [It is maintained that though 'Good's theorem fails for evidential decision theory ... [it] holds for the most general form of causal decision theory.' The 'most general form' considered does not take as fundamental a 'possible worlds' principle.]

Sobel, J. H. 'World Bayesianism: Comments on the Hammond/McClennen Debate,' in B. Munier (ed.), *Proceedings of FUR-III: Risk, Decision and Rationality*, Dordrecht: Reidel, 1988.

Sobel, J. H. (forthcoming), 'Machina and Raiffa on the Independence Axiom,' *Philosophical Studies*.

Weirich, P. 'Expected Utility and Risk,' *British Journal for the Philosophy of Science*, 1986. [It is observed that utility theory requires that 'prizes' be '*general* rather than particular' to certain lotteries, that they 'can be produced with arbitrary probabilities.']

Comment by Irving H. LaValle
A. B. Freeman School of Business, Tulane University, New Orleans, Louisiana 70118, USA

Despite its brevity, Wakker's careful demonstration of the undesirability of uncertainty-reducing information in the absence of Independence (Linearity, Substitutability, Sure-Thing, etc.) is profound. It deserves to be read and pondered by all who are interested in the logical foundations of normative choice.

One rather immediate implication of Wakker's argument is that LaValle and Wapman (1986) were

correct in asserting that the normal-form analysis, or *ex-ante* choice situation, is preferable to the extensive-form (rollback) analysis, or *ex-post* choice situation, when Independence fails. Shame on us for not suspecting that this would be tantamount to negative value of perfect information on the mixing event! Yet Hazen (1987), Sarin (1989), and perhaps others believe that when a failure of Independence causes the *ex-ante* and *ex-post* situations to be nonindifferent, then the *ex-post* situation *should* be preferable. I do not believe that such a position is tenable in light of Wakker's analysis.

Being forced to use *ex-ante*, normal-form analysis imposes much greater formulating and computational burdens on decision makers, however, since any reasonably stage-extended and bushy decision tree will lead to a very large *ex-ante*-strategies-*versus*-states-of-the world table even if the strategies and states are non-redundantly defined; see, e.g., La Valle and Fishburn (1987).

But in the absence of Independence there is even worse news for decision makers. When Independence fails, one cannot truncate a tree's descriptions of futures at convenient points and summarize subsequent acts and events in the definitions of outcomes. LaValle's (1989) discussion of Sarin (1989) furnishes an example of how such a replacement of subtree by certainty equivalent can affect preferences between initial acts. This means that if an Independence violator is to have any confidence in the implied preferences among the initial acts, then the normal-form table should be obtainable from a tree explicitly reflecting his or her complete views on all preference-relevant aspects of the future of the universe!

Other difficulties arise when Independence fails, one in particular being that preference theories which treat intra-state and inter-state uncertainty differently beg serious questions as to how a decision maker should allocate sources of uncertainty; see LaValle (1989) for a fuller exposition.

I sympathize with Wakker's concern about giving the impression of circularity but hasten to give assurance that his argument is valid. As the preceeding paragraph suggests, failure of Independence leads to a tangled mess of difficulties. Decision makers are better advised to *explicitly evaluate* temptations to violate *ceteris paribus* assumptions. See Raiffa (1985) on how to redefine outcomes so as to excompass anticipated feelings of ex-post regret, for example. Information evaluation when the very act of obtaining the information may affect the Bayesian prior probabilities is discussed in LaValle (1980).

Surely more remains to be discovered about the grim effects of Independence failures, but I daresay that motivations to violate Independence stem largely from inappropriately defined outcomes. To my way of thinking, Independence is *normatively* compelling – though I am sure I often violate it, *descriptively*, in casual choices. Much more normatively dubious to me is the Continuity axiom whenever there is an ethical aspect to the outcomes; see LaValle (1989) and Anand (1988) for hopefully provocative questions about this axiom.

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Comment by David Kelsey
Department of Ecnomics, Australian National University, GPO Box 4, Canberra ACT 2601 Australia

Peter Wakker has presented an important new defence for the controversial Von-Neumann Morgenstern independence axiom in this interesting paper. It is already known that dynamic consistency in decision trees implies the independence axiom (see Hammond (1988)). The author has shown that if the receipt of new information never makes the decision-maker worse off then the decision-maker must statisfy the independence axiom.

This raises the question – is it rational always to

accept new information? One can easily think of kinds of behaviour where new information would be declined. For instance I hold a ticket in a multi-stage lottery where the combined odds of winning are equal to my lucky number. If I am informed of the outcome of the first state this will no longer be true hence I decline the information. I think it is reasonable to exclude such behaviour from a normative model of choice. There are reasons why a *rational* decision-maker may decline information. I may refuse to take a test for cancer because I know that I will not be able to purchase medical insurance if the result is positive This is also not relevant to Wakker's argument, since in the case the new information can reduce the strategies available, which is not the case in choices over decision trees.

The following example is more worrying. Suppose I learn that I may have a gene which will cause me to die around age 40. There is a test which will reveal whether I have the gene. I may decline the test, since I do not wish to spend the next few years knowing that I have a certain death sentence. I may have this preference even if the information does not affect the strategies open to me. Is such behaviour irrational? It seems essential to Wakker's argument that it is.

One might wonder how an expected utility maximiser would value information. The answer, somewhat surprisingly, is that it has no value whatsoever! Consider Wakker's exhibit 1. An expected utility maximiser will make the same choice whether or not he has the information. Further he has the same *ex ante* utility and the same *ex post* utility in all events, whether or not he rececives the information. Hence we may conclude that he had no value for information and would not be prepared to pay any positive price to receive it.

The notion, that an increase in information is always beneficial; is attractive. However, I do not think it should be taken as a primitive in a theory of rational choice, but should be derived from more basic assumptions. As yet no such basic assumption has been proposed. The principle does not imply expected utility maximisation. Wakker's results seem to suggest that there is no decision rule which gives a strictly positive value to new information.

Finally I wonder whether there are any further implications of the principle that a decision-maker should never object to receiving new information. For instance Hammond (1988) has shown that dynamic consistency implies the existence of an ordering over lotteries. Does, not rejecting information, have any similar implications? I think that this would be an interesting direction for future research.

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Reply by Peter Wakker*
University of Nijmegen, Nijmegen Institute for Cognition Research and Information Technology, Nijmegen, The Netherlands

It was a pleasant surprise to see the attention, new ideas, and insightful comments, of Kelsey, Sobel, and LaValle to my paper. Before turning to their comments, let me make some preparatory comments. The idea of the paper occurred to me when studying McClennen's (1983) criticism of independence (or, analogously, the sure-think principle). I appreciated his systematic way of cataloging the arguments in favor of independence. On page 118, lines 37/38, he wrote, concerning the commu(ta)tivity argument for independence:

... requiring the agent to be indifferent between the temporal order in which certain choice and chance events take place.

I added to this the observation that the agent will indeed not be indifferent, she will prefer the *ex ante* situation, and I added the observation of information-aversion. I considered this a marginal 'one-sentence'-supplement, the more so since there is triviality in the observation that an agent will *ex ante* prefer her *ex ante* preferences. I must confess that an important motivation for me to write the paper, apart from the notion of information aversion, was the simple fact that I could write the paper without using any formula. This also partly explains the length of the paper (rather than the 'brevity' mentioned by LaValle). In view of this the received comments are a nice surprise.

Now let me turn to the comments of Kelsey, and his interesting further observations concerning value of information. Concerning his example about the gene: normatively, if the test and the knowledge resulting from it do not affect any future action or utility (this idealized assumption will not be satisfied in reality) then I think one should be indifferent to taking or declining the test. I have the impression, though, that the *ex post* utilities, as described by Kelsey, are different from his *ex ante* utilities. I have problems with the notion that information is always beneficial, since information may

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be 'irrelevant' (at least for the expected utility maximizers in Exhibit 1 in my paper). With regard to the further implications of non-information-aversion, these will certainly not extend beyond expected utility maximization, as my paper shows. Probably, non-information-aversion on its own will not imply the existence of an ordering. My intuition is that one can let an agent prefer one lottery/continuation-of-decision-tree over another whenever the one can be considered to result form the other through the addition of information, and let the agent choose freely, in violation of completeness/transitivity, between the remaining pairs. I think from the theorem of Blackwell (1953) it will follow that there remain indeed sufficiently many 'free' choices to enable the agent to violate completeness and transitivity. A formal elaboration would require the nontrivial task of relating choices (and nonchoices?) to preferences when these preferences are no longer orderings, and of dealing with decision trees when independence, dynamic consistency etc. no longer hold.

Next I turn to the interesting comments of LaValle. Comments on the brevity of my paper were given above. The first paragraph of LaValle's comments concerning my paper applies as well to the paper of LaValle & Wapman (1986). LaValle refers to two papers which claim that agents who violate independence should not prefer the *ex ante* situation. My paper shows that the latter implies independence. It escapes me why LaValle then considers my paper to be reason for considering the above-mentioned claim as untenable. Summing up, the claims says: 'Agents who violate independence should satisfy independence'. I sympathize with that claim!

Of course I also sympathize with LaValle's viewpoint that violations of independence should lead to redefinitions of consequences/outcomes: compare the last sentence in section 3 of my paper. We should however not underestimate the argument, put forward for instance in a lecture by Machina (1988), that consequences thus defined will be intractable, and make acts assigning all kinds of consequences to all kinds of states even more illusionary.

Next I turn to the thoughtful comments of Sobel. The scenario called 'more provocative' by Sobel has as the 'chance' option, choose in advance whether the mixing event obtains, and without the decision maker being informed. I am with Sobel on this point, I personally like it more than the one sketched by Sobel before. The terminology *'for example* ... in any case' in the last paragraph of page 170 of my paper deliberately leaves open the provocative scenario. Unfortunately, the scenario has the disadvantage that nondeterministic readers will object that the mixing event is then no longer 'really random' but is deterministic-but-only-unknown; they may object against the assignment of probabilities to it. With these views I usually write, somewhat reluctantly, 'the mixing event *obtains*', whereas I personally prefer (even more than the provocative scenario) the atemporal terminology 'the mixing event *is true*'. Indeed, sometimes I use, for simplicity of exposition, the temporal terminology, such as on page 171 of my paper. In Wakker (1989) I consequently use the true-formulation, and nowhere use the obtain-formulation.

Concerning Sobel's formulation '*But*, and Wakker can be read as intending no more than this', that should be changed into '*But*, and Wakker **should** be read as intending no more than this'; see for example the term 'existence' in my abstract, and the first paragraph of section 3. Indeed, I strongly symphathize with the view that consequences do not need to be 'completely specific' with respect to all things that matter to the decision maker. I don't know if Sobel inferred that from my paper. Anyway, on this point I disagree with Savage's views (see Wakker, 1989, p. 25/26). However, I do symphathize with the view that the descriptions of consequences should be complete enough to contain all things that matter to the decision maker *considered in the analysis*, and to justify consequentialism (this is formulated in Wakker, 1989, p. 29 but unfortunately in a way that may be confusing as regards the disagreement with Savage just mentioned). My work was not at all intended for the approach to decision making under uncertainty of R. C. Jeffrey. Analogous results for that approach would be very interesting.

Finally, let me give some references in addition to the several ones given by LaValle and Sobel. Only after completion of my paper I was informed about several related papers discussing the positive/negative value of information in all kinds of special models. Keasy (1984) gives a discussion for regret theory as do Loomes & Sugden (1984). Hilton (1988) shows that among the nonexpected utility maximizers who exhibit the 'fanning-out' phenomenon, as described in works of Machina, there exist some who will exhibit what I called information aversion. Hilton shows the implications of this for the applicability of the theorem of Blackwell (1953). Schlee (1988) considers the value of information for anticipated utility theory. Also, I would like to mention the work in Burks (1977, Chapter 5), where decision trees and invariances in them are used to derive conditions such as the sure-thing principle. This work is related to the works of LaValle & Wapman (1986) and Hammond (1988). Many further references to conditions equivalent to the sure-thing principle are given in Wakker (1989, section II.5).

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