A broadband vision of the development of the DAX over time

Winfried Hallerbach, Christoph Hundack,
Igor Pouchkarev, Jaap Spronk

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A broadband vision of the development of the DAX over time

Winfried Hallerbach[1] Christoph Hundack[2] Igor Pouchkarev[3] Jaap Spronk[4]

October 8, 2002

Abstract

We present an analysis of the performance of the DAX, German's major stock market index, over the last two years. Our analysis is broader than conventional benchmark approaches because we study the properties of all feasible portfolios, i.e. portfolios composed given the same investment opportunity set and also given the same constraints as implied by the definition of the DAX. We estimate the distribution of performance values of all feasible portfolios according to different performance measures and evaluate the position of the DAX with respect to this feasible set. As in existing approaches, our analysis describes the 'average' development of the market over time. In addition, our analysis provides an insight into the development of the dynamics of the market over time by following the dispersion of the performance distributions over time.

Keywords: Investments, Financial Markets, Market indexes, Performance Evaluation

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[1] Erasmus University Rotterdam, Rotterdam, Netherlands, hallerbach@few.eur.nl
[2] Erasmus University Rotterdam, Rotterdam, Netherlands and UBS AG, Zürich, Switzerland
[3] Erasmus University Rotterdam, Rotterdam, Netherlands, pouchkarev@few.eur.nl
[4] Erasmus University Rotterdam, Rotterdam, Netherlands, spronk@few.eur.nl

1 INTRODUCTION

The history of the description of financial markets by means of one concentrated gauge began in 1884 when Dow Jones & Co published their first index. This composite index described the development of shares of railroad companies. Since that time the use of various indexes as proxies for financial market dynamics has gained enormous popularity: if we want to see the development of a market (or market segment), we take a look at the appropriate market (or segment) index.

In this paper we present a new methodology for describing markets. To illustrate the new approach, we concentrate on the DAX, the major indicator of the German large capitalization companies segment. The organization of the paper is as follows. In Section 2 we review the DAX, its objective and selection constraints for stocks. Additionally, we present the conventional view on the index development over the last two years. In Section 3 we formulate the new methodology that enables us to broaden our view on the DAX and to better investigate its characteristics. In addition we describe our data set. Section 4 contains our empirical results. We apply our methodology and analyze the DAX and other benchmark portfolios over the last two years. In section 5 we focus on one particular benchmark portfolio, the equally weighted portfolio, and compare its development with the dynamics of the DAX. In addition we compare the performance of the small versus big caps components of the equally weighted portfolio. Section 6 concludes the paper and contains directions for future research.

2 CONVENTIONAL VIEW ON THE DAX

The DAX is the major index of the German stock market. It consists of the 30 largest German companies in 8 different industry sectors[5] that have the highest turnover on the Frankfurter Wertpapierbörse (FWB). The purpose of the index is to represent the financial capital dynamics of the largest German blue-chip companies. To achieve this goal, the following selection constraints and rules are imposed:

- Shares of companies traded in the Official Trading segment or in the Regulated Market segment are allowed for the selection only;
- Companies should be domestic, i.e. have Germany as their legal domicile;
- Companies should have at least 20% of share capital in free float;
- Each quarter the companies should prepare reports as well as hold analysts' meetings;
- The index contains 30 stocks that have the largest capitalization and the highest turnover on the Frankfurter Wertpapierbörse;
- The market capitalization of each stock is limited to 15% of the total index capitalization. (If the capitalization of a company exceeds the limit, then the number of shares is lowered to 15 percent of index capitalization.)

The DAX is a capitalization-weighted performance index.[6] It is based on the Laspeyres index formula, with base date December 30, 1987. The DAX is calculated every 15 seconds and its time series dates back to 1959. For the actual DAX formula, correction factors and the index

[5] As of August 30, 2002.
[6] Deutsche Börse uses free-float instead of market capitalization from June 24, 2002 on.

composition as of August 30, 2002 we refer to Appendix A. For a more detailed description of the DAX see [4], [5].

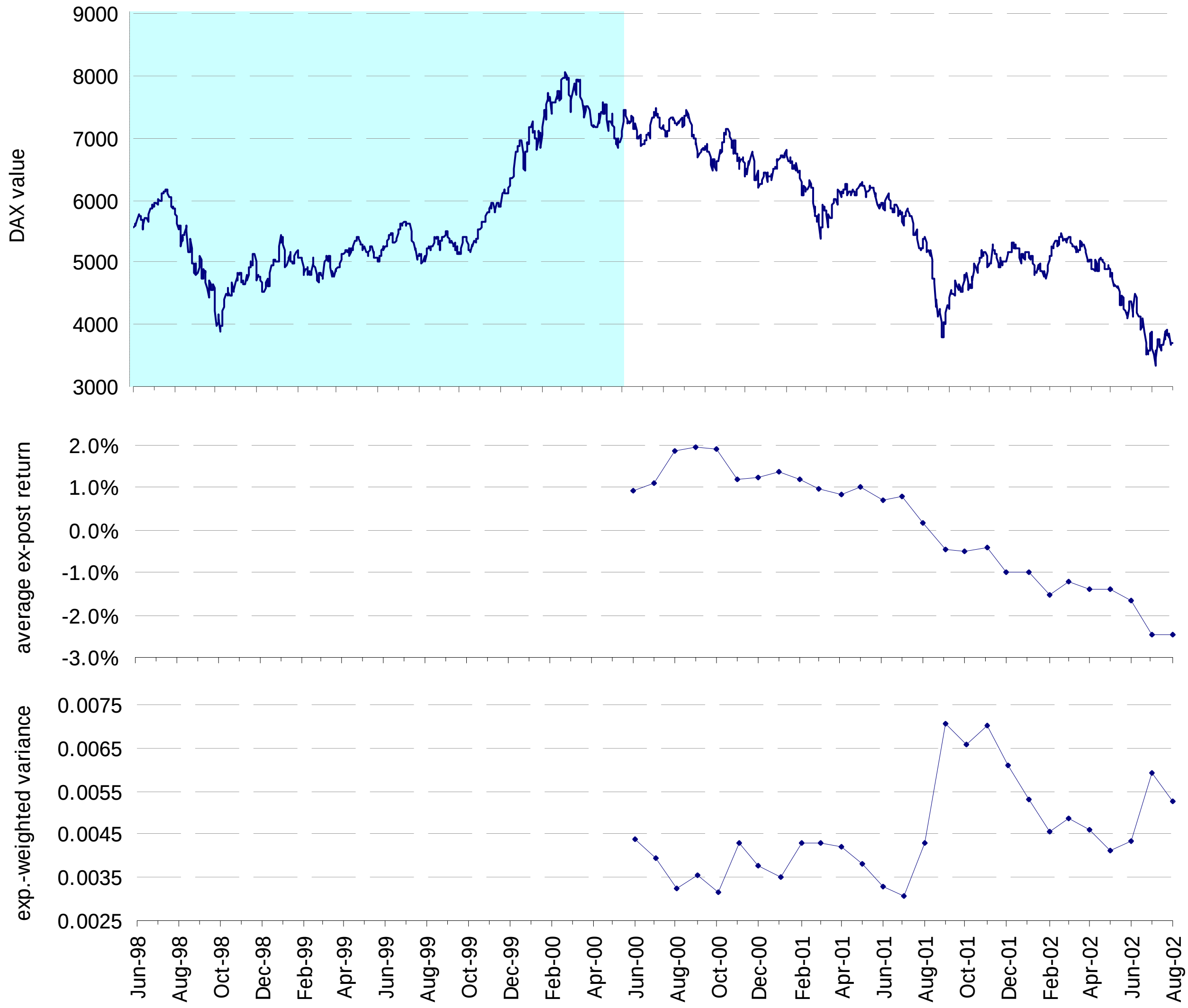

FIGURE 1 The development of the German DAX daily from June 1998 through August 2002 (top), two-year moving averages of monthly-realized returns (middle) and variance representing the index risk (bottom)

The development of the DAX over the last four years is shown in Figure 1. The top graph shows the daily DAX values over the period June 1998 through August 2002. The shadow area in the top graph indicates the data window that is used for calculating the first point (i.e. for June 2000) in the middle and bottom graphs. The graph in the middle plots the two-year moving average of monthly realized returns.[7] The bottom graph shows the variances of the monthly returns. To estimate variances we use the exponentially weighted moving average scheme (EWMA) with a decay factor of 0.87 and a tolerance level of 1%. This implies that the estimates are based on 24 monthly returns.[8] The EWMA scheme allows to register changes in the variance faster and to avoid clustering effects caused by shocks. The effect of September 11, 2001, for example, is clearly seen in the graph. We refer to [8] for further details about the EWMA procedure.

[7] The last business day of each month is used.

[8] J.P.Morgan´s [8] decay factor for monthly data is 0.97. In our analysis we use a slightly lower decay factor due to short time series of some DAX stocks.

In general, the DAX is considered to be a good representative of the complete dynamics of the large caps segment. The representativeness is sometimes not adequate due to the 15% capitalization restriction but exceeding occurs very rarely. The index is formulated very strictly and very clearly and can thus be easily "reproduced". For these reasons the index is widely used as underlying for derivatives such as options and index certificates (ETC).

Nevertheless the index represents only one of many possible alternatives to invest in the stocks that are represented in the DAX. From the perspective of an uninformed investor who has not enough information to discriminate between different stocks, the equally weighted portfolio represents a viable alternative to invest in these large-caps companies. This alternative portfolio can also be used to describe financial markets. Obviously, the equally weighted benchmark portfolio will yield a description that is different from the description provided by a constrained value-weighted index. Figure 2 depicts the development of the equally weighted portfolio against the development of the DAX, as measured by the two-year moving averages of the realized returns and the variances. Figure 2 uncovers an interesting result: the DAX performs worse than the equally weighted portfolio while at the same time having substantially higher risk for almost the entire period. In Section 5 we investigate the differences between the performance of the DAX and the equally weighted portfolio in more detail.

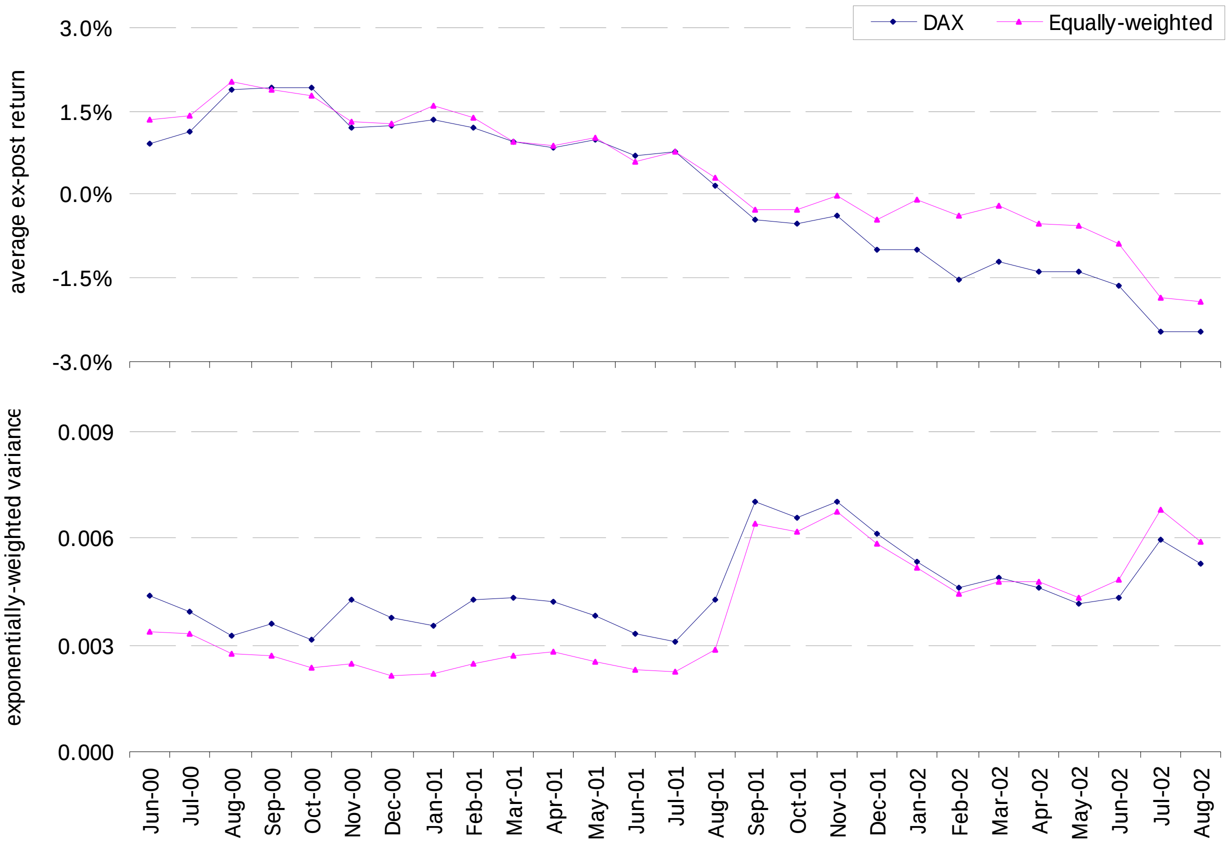

FIGURE 2 The two-year moving averages of monthly realized returns (top) and exponentially weighted variances (bottom) for the DAX and the equally weighted portfolio from June 2000 through August 2002.

Given the extraordinary performance of the equally weighted portfolio one may also wonder about the performance of alternative investment portfolios such as the 10 DAX stocks with the biggest or the smallest market capitalization. More generally, this leads us to the following question: *Given the stocks comprised in the DAX (or in the German large cap segment), what other opportunities do exist to compose portfolios and what is the performance of these alternative portfolios?* This issue is further explored in the next section.

3 Exploring the Set of DAX Portfolios

The prevalence of indexing for describing the dynamics of financial markets or market segments is based, among others, on the following grounds:

- *Indexes provide the ultimate summary of markets*: An index concentrates the dynamics of a financial market, a market segment or an industry into a single value development. In many cases, further analysis and modeling based on such "concentrated" value is much easier;
- *Standardization*: By indexing a market we "standardize" the market development. This allows for an easy comparison of different markets or market sectors. Standardization also leads to index based products, e.g. certificates, and derivatives such as futures and options;
- *Indexes are considered to be good substitutes for the market portfolio*: With the development of quantitative methods for optimal investment choice and asset pricing models, the concept of "the market portfolio" has gained importance. Often, a properly built index is used as a proxy for the market portfolio.

Of course, index measures are also exposed to several problems. For example, if we use an index for describing the development of financial markets, then the quality of index representativeness highly depends on the underlying calculation methodology. A performance index will replicate the total market changes more precisely than a price-weighted index or a pure Paasche index [9]. Another (and unavoidable) drawback of indexing is that the concentration of individual stock price dynamics into a single summary measure goes at the obvious cost of losing track of the components' dynamics. By applying the prescribed recipe for aggregating the stocks into the index, the broad and multifarious view on market developments is substituted by the index view. In the particular case of the DAX, investors view the index as a specific investment portfolio with predefined stock selection constraints and specific weightings. Obviously, investors are also interested in other opportunities to invest in the same market.

This observation motivated us to formulate a broadband vision of the DAX over time. In order to provide a broad view on the market development *we present the performance of all possible investment alternatives given the same investment opportunity set and also given the same constraints as implied by the definition of the DAX*. Instead of limiting ourselves to a single index we explore the whole set of portfolio formation opportunities (see for earlier work [7]). Instead of confining ourselves to evaluating the performance value of the index, we estimate the *distribution* of the performance values (e.g. realized returns, variance, mean absolute deviation etc.) of *all* feasible portfolios. The development of the location of these distributions yields a picture of the average development of the market over time, where the DAX stocks define the market. The development of the dispersion of these distributions provides a picture of the development of the market dynamics over time.

Of course, evaluating feasible portfolio alternatives when starting from the same opportunity set and imposing the same constraints as the DAX can be too restrictive. Consider for example the case in which we would like to investigate the representativeness of the DAX for the German large cap segment as a whole. In that case we would like to relax the constraints that we apply in defining the portfolio formation opportunity set. Nevertheless, the investigation of portfolios restricted by the DAX constraints can bring very interesting insights, for example for index tracking.

We illustrate our methodology by analyzing the performance of the DAX over the period from June 2000 through August 2002. The input data consist of monthly observations on DAX stocks from June 1998 through August 2002. We used closing prices at the last trading day of each month ignoring cash dividends.[9] The stocks are listed in Appendix A, Tables 1 and 2.

For our broadband view, we need to calculate the distribution of performance values measured over all possible portfolios consisting of DAX stocks (henceforth "DAX portfolios"). One of these portfolios is the DAX itself. In addition to the DAX and the DAX portfolios we consider the following benchmark portfolios:

- *The equally weighted portfolio*: The portfolio consists of all stocks in the DAX. The available capital is invested in equal proportions in each stock, i.e. w_i = 0.03(3), i=1,2,…,30;
- *The equally weighted big (small) caps portfolio*: It consists of 10 stocks from the DAX with the largest (smallest) market capitalization. The rating is produced according to the market capitalization at the last trading day of each month. Additional rules are applied to take account of IPO's and missing data;
- *The market capitalization weighted big (small) caps portfolio*: The methodology of the stock ranking is as described for equally weighted portfolios. However, the stocks are now weighed according to their market capitalization.

The crucial part is how to calculate the frequency distributions of the performance measures of all possible portfolios consisting of DAX stocks. First we give a more precise description of the set of feasible investment portfolios based on the DAX opportunity set and satisfying the DAX constraints. We call this set the *DAX portfolio opportunity set*.

By applying the rules and constraints used by the DAX to select stocks for the index (as described in the previous section), we reduce the universe of investment portfolios to one consisting of portfolios of specific German stocks only. For the DAX, the proportions of capital w_i, i=1,2,…,30, invested in each of the qualifying 30 stocks are defined by their relative market values and are constrained by $0 \leq w_i \leq 0.15$ and $\sum_{i=1}^{30} w_i = 1$.

The *DAX portfolio opportunity set* consists of all portfolios with the same German stocks as comprised in the DAX index itself with weights $0 \leq w_i \leq 0.15$, i=1,2,…,30 such that $\sum_{i=1}^{30} w_i = 1$. Even given these constraints, the number of the DAX portfolios is infinite. The DAX selection constraints determine the frequency distribution for any statistics or measure.[10]

There are several ways to calculate the frequency distribution(s) of the performance of all possible portfolios consisting of DAX stocks. We refer to [10] for further details. In this paper we use simulation to calculate the desired distributions. The procedure is as follows:

I. In each simulation step we sample ten millions feasible random portfolio weight vectors for DAX stocks. Each sampled weight vector defines a DAX portfolio. The sampled portfolios are uniformly distributed over the DAX portfolio opportunity set;

II. For these sampled DAX portfolios, as well as for the actual DAX and the benchmark portfolios, we calculate the average rates of return, equally and exponentially weighted variances, semi-variances and mean absolute deviations. These statistics are estimated

[9] Since DAX is a performance index and dividends are reinvested, this leads to a latent over-performance of the DAX *vis à vis* the DAX portfolios.

[10] The necessary conditions and the checking procedure for existence of the distribution are discussed in [10].

using 24 observations prior to the actual evaluation step. For example, by evaluating the market during June 2001, the stock prices from July 1999 to June 2001 are used;

III. We estimate the frequency distributions of the selected performance measures over the whole DAX portfolio set;

IV. The time window is shifted one month forward and the next simulation commences.

An important issue in this procedure is how to handle changes in the index composition. Regular changes are carried out yearly in September. Nevertheless, as Table 1 in Appendix A shows, the changes in the DAX are quite irregular due to mergers, new admissions, deletions, etc. which need to be reflected in the index shortly after their occurrence. The Deutsche Börse usually announces a forthcoming change in the structure of the DAX three to four weeks in advance. Therefore our strategy is to hold the security deleted from the DAX until the start of the replacement month and then replace it by the new one. For example, on July 23, 2001 Dresdner Bank was exchanged in the DAX against MLP. When we evaluate the performance of the DAX portfolios at the last trading day of June 2001, we have Dresdner Bank as one of the DAX stocks. For the evaluation month July 2001 the Dresdner Bank stock drops from the DAX stock set and, thus, from the DAX portfolios. Instead, MLP will be used as a new stock in the DAX stock set to form DAX portfolios.

4 EMPIRICAL RESULTS

We applied our methodology to our data set. This resulted in 243 distributions (27 periods times 9 types of distributions) of selected performance measures of the DAX portfolio. The selected performance measures are average returns, equally and exponentially weighted variances, semi-variances and mean absolute deviations. In addition we evaluated any return-risk combination of these statistics. Figure 3 shows box plots of the calculated frequency distributions of ex-post averages and exponentially weighted variances from June 2000 through August 2002. We also considered the summaries of semi-variance and mean absolute deviation distributions but these graphs are similar to the variance graph presented in Figure 3. For this reason we present the variance summaries graph only. For each month the box represents the interquartile range, containing the middle 50% of the performance measures of the sampled DAX portfolios. The vertical lines extending from the box represent the upper and lower of 22.5% quartiles. The extreme 2.5% on each side are not shown. In the same figure we also show the development of the DAX and the equally weighted portfolio.

PROPOSITION 4.1 *The return of the equally weighted portfolio* P_e *is equal to the average of returns of the portfolios, i.e.*

$$\overline{r}_{p_e} = E\left[\tilde{r}_p\right]$$

PROOF: see Appendix B.

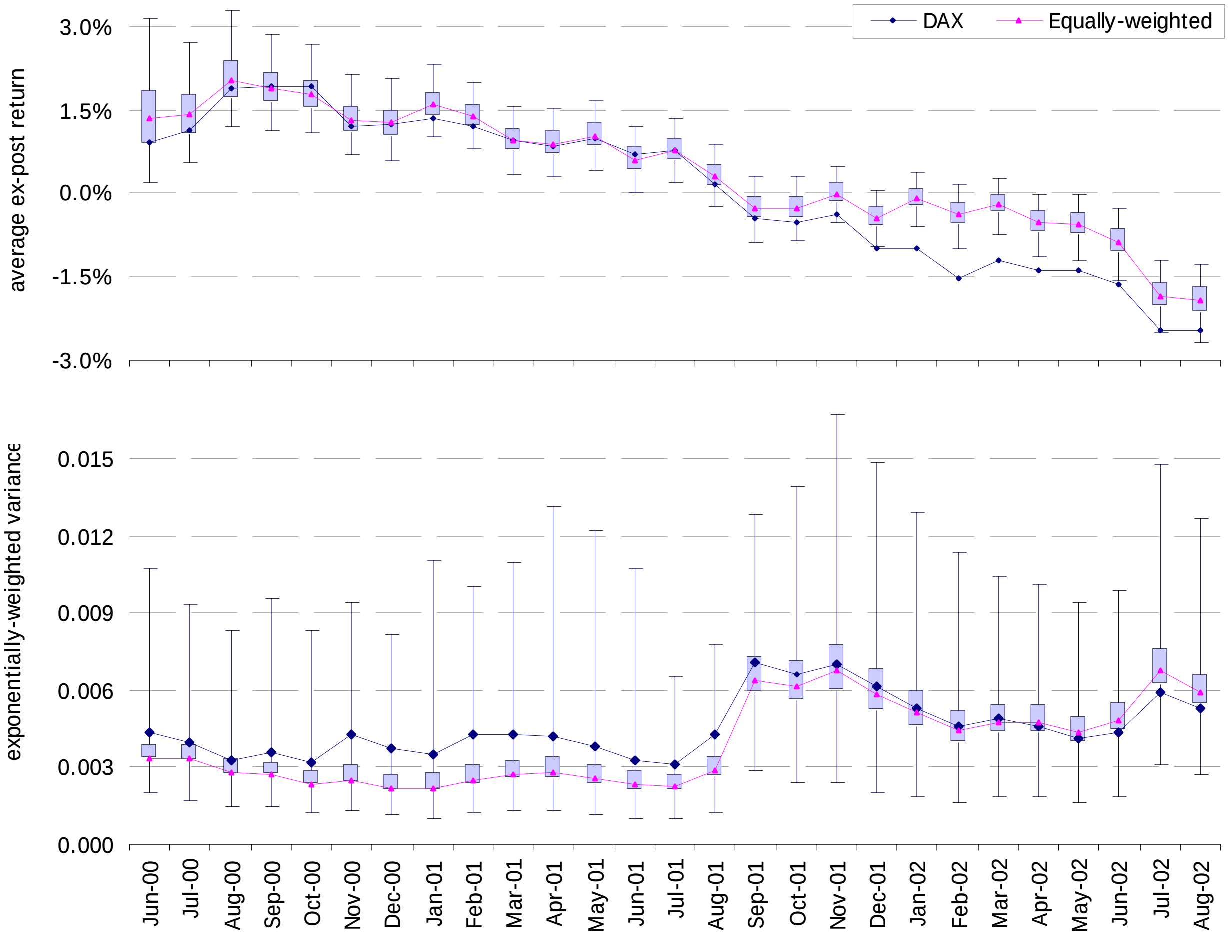

FIGURE 3 Summaries of the monthly frequency distributions of ex-post return averages (top) and variances (bottom) of the DAX portfolios. In addition, the time-series of the DAX and the equally weighted portfolio are plotted.

The broadband view on the DAX market is derived from:

- The shape and location of the distributions for each specific month, this is a cross-section (or broadwise) view;
- The development of these distributions over time, this is a time-series (or longwise) view.

The first type of view provides a picture of the average development of the market over a specific period of time, where the DAX stocks define the market. We have 27 of such market cross-sections or "cuts". Our primary result in this area is that the novel methodology substantially extends conventional single period descriptions. Figure 4 shows the derived frequency distributions of ex-post average returns plotted against exponentially weighted variances for June 2000 and June 2001.

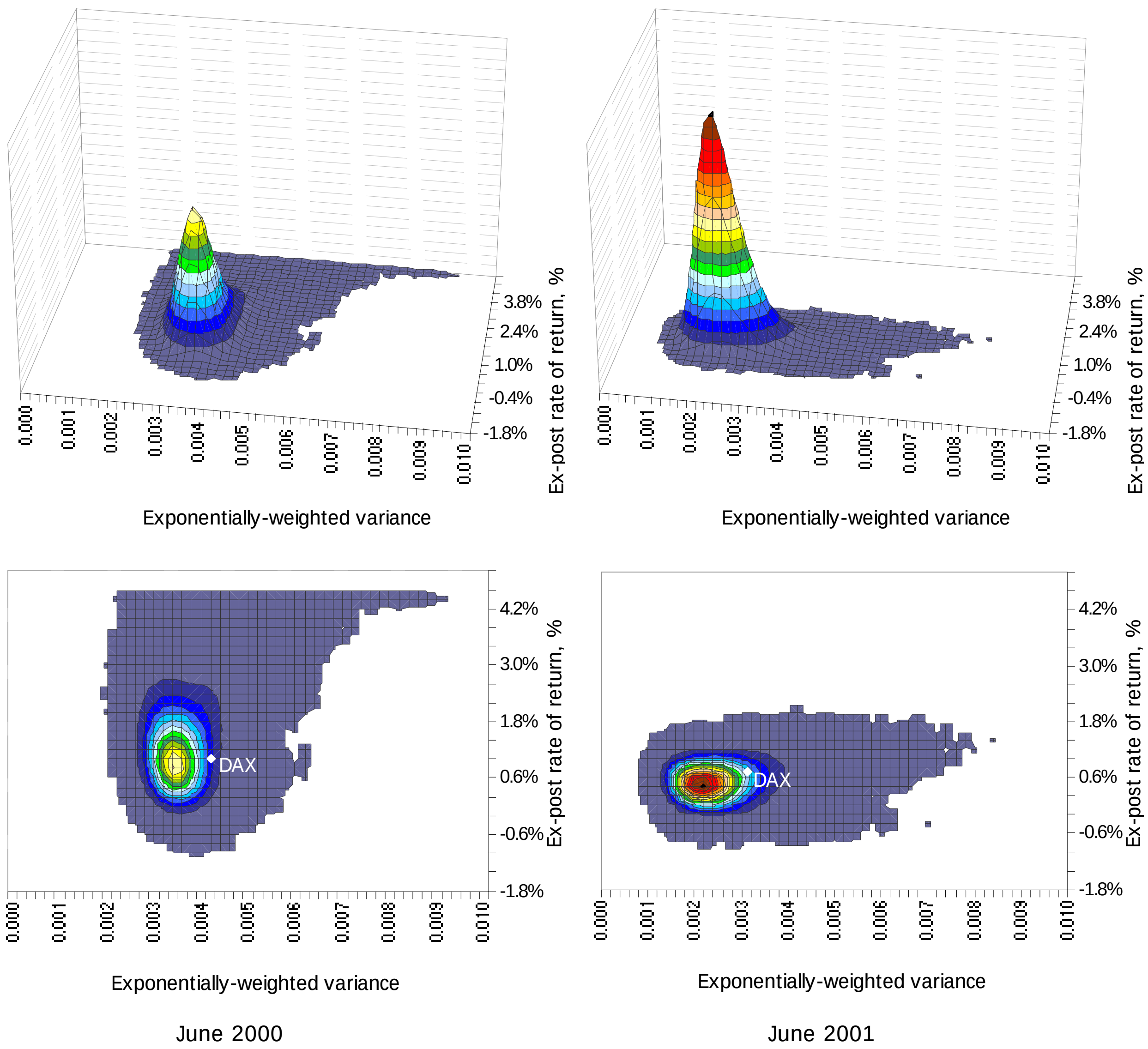

FIGURE 4 Average return — variance frequency distributions of DAX portfolios for June 2000 and June 2001. Lower diagrams show the projections of the upper diagrams on the standard return-variance space often used for portfolio analysis. The position of the DAX is also plotted in the lower diagrams.

The graphs in Figure 4 show very clearly some of the advantages of the new extended description of the DAX market:

- *The graphs show clearly the DAX portfolio opportunity set*: Between June 1998 and June 2000 the average monthly return by investing in a DAX portfolio was between -1.0% and 4.6%. Moreover, 95% of the portfolios earned an average return between 0.2% and 3.2%. Thus the probability to lose money by investing in a DAX portfolio over that time was very small. In June 2001 the bandwidth of return opportunities shrunk twice to the -0.6% to 1.8% range. The variance range expands 50% in that period. In the figure, this is illustrated by the iso-frequency ellipses;
- *The descriptions help to put the performance of the DAX index in perspective*: As the lower left diagram shows, in the period between June 1998 and June 2000 it was very easy to outperform the DAX. About 62% of the DAX portfolios dominated the index in terms of return and risk, and about 40% of the portfolios outperformed the index by

0.5% or more monthly return while having equal or lower variance. On the contrary, in June 2001 only 21% of the portfolios dominated the index when looking back 24 months.

Looking at the development of these monthly distributions over time gives a picture of the development of the dynamics of the market. In the particular case of the DAX such a summary picture is shown in Figure 3. The boxplots expose several interesting aspects of the DAX performance over the evaluated two-year period. Note that, as expected, the DAX is not a good indicator for the whole set of DAX portfolios. With a few exceptions, the DAX performs consistently lower than half of the DAX portfolios. Also, the index has a relatively high variance over the entire period under study. In other words, during this period there were apparently many opportunities to outperform the DAX index.

Figure 3 also gives an interesting insight in the behavior of the equally weighted portfolio: while the average return of the portfolio is median for the return frequency distribution, the risk of the equally weighted portfolio as measured by its variance is permanently below the average risk of the DAX portfolios. This observation motivated us to formulate the following proposition:

PROPOSITION 4.2 *The variance of the equally weighted portfolio* P_e *is always lower than the average variance of the portfolios, i.e.*

$$\sigma_{P_e}^2 < E\left[\tilde{\sigma}_p^2\right]$$

PROOF: see Appendix B.

The comparison of the DAX with the equally weighted portfolio leads to other observations, which are discussed in the next section.

5 THE DAX AND THE EQUALLY WEIGHTED PORTFOLIO

Figure 3 reveals that the equally weighted portfolio performs better than the DAX during almost the entire period under observation. A natural hypothesis for this outperformance is the size effect or "small"-caps effect [3],[6]. (The "small"-caps term is a little misleading. We use the term to denote the ten stocks out of the thiry in the index which have the smallest market capitalization. For that reason we use double quotes.)

To test this hypothesis we consider four additional benchmarks: the equally weighted "big"-caps and "small"-caps portfolios as well as the market capitalization-weighted "big"-caps and "small"-caps portfolios. The construction of these benchmarks is described in detail at the end of Section 3. Figure 5 shows the same frequency distributions as Figure 3, but now we have added the performance of the equally weighted "big"- and "small"-caps portfolios.

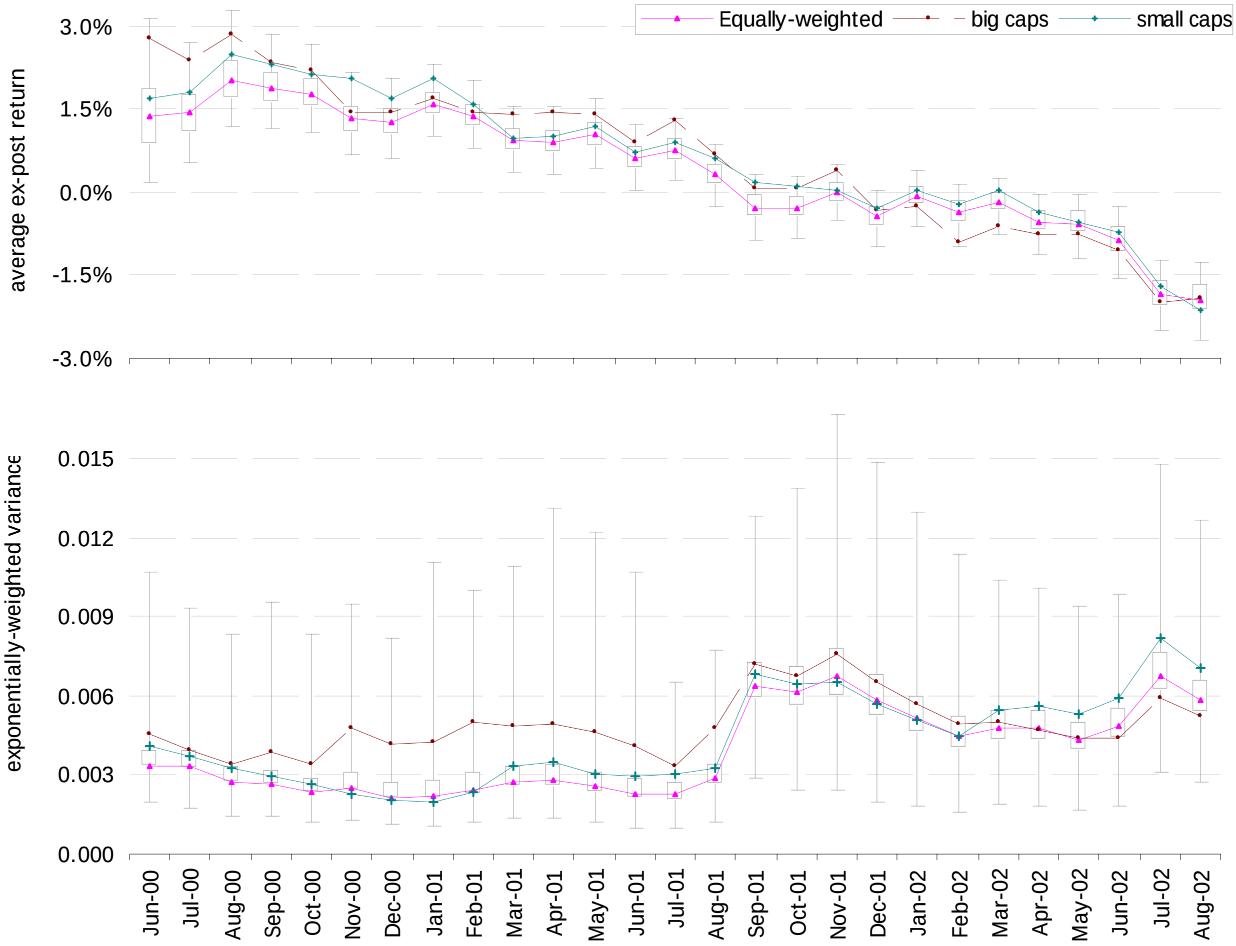

FIGURE 5 The two-year moving averages of monthly realized returns (top) and exponentially weighted variances (bottom) for the DAX "big"-caps and "small"-caps portfolios as well as for the equally weighted portfolio, from June 2000 through August 2002. Summaries of monthly frequency distributions are drawn in grey at the background.

As the top graph in Figure 5 shows, the movement of the equally weighted portfolio is really driven by the DAX "small"-caps in some periods. But in others it is not. For example, by looking at the period from June to July 2001 or the period from October to November 2001, we see that the equally weighted portfolio and the "small"-caps benchmark move in opposite directions. Thus, the extraordinary performance of the equally weighted portfolio can not be explained by the "small"-caps effect alone.

Another interesting aspect is the poor performance of the 10 "middle"-caps stocks from the DAX. Note that the average return on the equally weighted portfolio is equal to the sum of the average returns on the "big"-, "middle"- and "small"-caps portfolios. Hence, the return graph in Figure 5 implies that the equally weighted "middle"-caps portfolio lies well below the equally weighted portfolio for almost the entire period.

The variance graph of Figure 5 also points to some interesting results. One of them is that the "big"-caps stocks from the DAX are more volatile than the "small"- and "middle"-caps. Another is that at the end of the period we observe a huge increase in the market volatility after 11 September 2001, lasting for at least four months. This was accompanied by the phenomenon that the DAX portfolios (and possible the complete market) became less homogeneous. In Figure 5 both facts are reflected by a doubling of the interquartile range and by an upwards shifting of the complete variance ranges.

In addition to equally weighted "small" and "big" caps portfolios we also constructed market capitalization weighted sub-portfolios. The use of these portfolios showed a picture similar to the one presented in Figure 5.

6 SUMMARY AND CONCLUSIONS

The enhanced description of the DAX, combined with frequency distributions of performance metrics of all DAX portfolios, provides many advantages over the conventional view on the index:

- In the conventional view, the quality of market representation by an index (viz. the DAX) is assumed given, regardless of the performance attributes considered. The new methodology helps to evaluate the market index itself *vis à vis* the DAX portfolio opportunity set. In particular, the location of the market index may be plotted in the frequency distribution of the selected performance measure over the DAX portfolio opportunity set. The quantile in which the index plots indicates how many (feasible) portfolios have outperformed the index in terms of the selected performance measure (realized return, e.g.). In this way it can be judged whether an index is representative for the market under consideration or not. The adhered criterion for representativeness is not the degree of market coverage measured in terms of capitalization (the usual view) but the degree of coverage of the portfolio formation opportunity set;
- It provides a perspective on the *ex post* outcomes of the variety of portfolios that can be formed given some opportunity set and constraints;
- The analysis of the DAX and the index comparison with the equally weighted portfolio performance demonstrates the other powerful feature of the proposed methodology: It helps discovering promising investment strategies that comply with specific constraints and evaluating them comprehensively.

Finally, we point out some directions for further research. The first suggestions concern extending the data set. The presented analysis is carried out on non-dividend adjusted stock closing prices. On the other hand the DAX is a performance index, i.e. it reinvests the occurred dividends into the index. The dividend income of the DAX is huge – about 1.5 - 2% p.a. Therefore, we will extend our analysis with dividend data. Surely, incorporating dividends will magnify the effects already observed above. In addition we want to incorporate a longer history in our data set. This will allow us to perform statistical tests on the observed phenomena and to test the underlying hypotheses.

In addition to improve the description of financial markets, our methodology can be used to test alternative portfolio strategies. The performance of these strategies can be evaluated against the performance of all random strategies that comply with the assumptions behind and restrictions on the corresponding strategies.

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Appendix A SHORT DESCRIPTION OF THE DAX

The DAX based on the Laspeyres´ index formula and is calculated as follows:

$$\mathrm{DAX}_t = K_{t_1} \cdot \frac{\sum_{i=1}^{30}\left(p_{i(t)} \cdot q_{i(t_1)} \cdot ff_{i(t_1)} \cdot c_{i(t)}\right)}{\sum_{i=1}^{30}\left(p_{i(0)} \cdot q_{i(0)}\right)} \cdot 1000$$

where

0 – December 30, 1987

t_1 – day of last index chaining

K_{t_1} – chain index factor

$c_{i(t)}$ – actual adjustment factor of stock *I*

$ff_{i(t_1)}$ – free-float factor (from June 24, 2002)

$p_{i(0)}$ – price of individual stock *i* as at December 30, 1987

$q_{i(0)}$ – number of shares of individual stock *i* as at December 30, 1987

$p_{i(t)}$ – actual price of individual stock *I*

$q_{i(t_1)}$ – number of shares of individual stock *i* as at review date

Factors $c_i(\cdot)$ are used to adjust for dividends and equity capital changes between the last and the next chaining days. On the date the Eurex stock-index futures fall due, i.e. the third Friday of the quarter end month, the DAX is calculated for the last time using the actual factors $c_i(\cdot)$. This day is set-up a new chaining day and the Xetra closing prices are used for chaining procedure: all $c_i(\cdot)$ are set to 1 and the number of shares of each company q_i is updated. Simultaneously, the index-chaining factor K is adjusted in order to avoid an index breakup. (The factor K is used for adjustment after index composition change as well.)

Date of change	*Date of announcement*	*Companies*	
		Deleted	*New*
03.09.90	22.05.90	Feldmühle Nobel Nixdorf	Metallgesellschaft Preussag
15.09.95	18.07.95	Deutsche Babcock	SAP
22.07.96	06.01.96	Kaufhof*	Metro
23.09.96	16.07.96	Continental	Münchener Rück
18.11.96	16.07.96	Metallgesellschaft	Deutsche Telekom
19.06.98	26.05.98	Bay. Vereinsbank* Bay. Hypo-und Wechsel-Bank	Bay. Hypo- und Vereinsbank Adidas-Salomon
21.12.98	05.11.98	Daimler* Thyssen*	DaimlerChrysler Thyssen-Krupp
01.01.99	22.10.98	*switched from DEM to Euro*	
22.03.99	03.02.99	Degussa*	Degussa-Hüls
20.09.99	20.07.99	Hoechst	Fresenius Med. Care
14.02.00	10.02.00	Mannesmann	Epcos
19.06.00	10.05.00	Veba Viag	EON Infineon
18.12.00	14.11.00	Degussa-Hüls*	Degussa (Fusion with SKW)
19.03.01	14.02.01	Karstadt Quelle	Deutsche Post

23.07.01	26.06.01	Dresdner Bank	MLP
23.09.02	13.08.02	Degussa	Altana

TABLE 1 REVIEW OF THE DAX OVER JANUARY 1990 – SEPTEMBER 2002. THE STAR SIGN ´*´ MARKS THE MERGER COMPANIES. SOURCE: DEUTSCHE BÖRSE AG.

Share	*ISIN*	*Market cap*	*Weight*	*Sector*
Adidas-Salomon	DE0005003404	3,299.15	0.88%	Retail & Consumer
Allianz	DE0008404005	22,914.95	6.14%	Insurance
BASF	DE0005151005	22,222.13	5.96%	Chemicals & Pharma
Bayer	DE0005752000	16,648.57	4.46%	Chemicals & Pharma
BMW	DE0005190003	11,242.14	3.01%	Automobile & Transportation
Commerzbank	DE0008032004	4,478.68	1.20%	Banks & Financial Services
Deutsche Bank	DE0005140008	39,276.91	10.53%	Banks & Financial Services
DaimlerChrysler	DE0007100000	35,800.22	9.59%	Automobile & Transportation
Degussa	DE0005421903	2,151.94	0.58%	Chemicals & Pharma
Deutsche Post	DE0005552004	3,832.59	1.03%	Automobile & Transportation
Deutsche Telekom	DE0005557508	27,348.69	7.33%	Utilities & Telecommunication
E.ON	DE0007614406	33,579.95	9.00%	Utilities & Telecommunication
Epcos	DE0005128003	775.23	0.21%	Software & Technology
Fresen. Med. Care	DE0005785802	1,002.20	0.27%	Chemicals & Pharma
Henkel	DE0006048432	4,145.26	1.11%	Retail & Consumer
Bay. Hypo-Vereinsbank	DE0008022005	7,395.56	1.98%	Banks & Financial Services
Infineon	DE0006231004	4,787.37	1.28%	Software & Technology
Lufthansa	DE0008232125	4,421.05	1.18%	Automobile & Transportation
Linde	DE0006483001	3,426.40	0.92%	Machinery & Industrials
M.A.N.	DE0005937007	1,472.79	0.39%	Machinery & Industrials
Metro	DE0007257503	3,650.74	0.98%	Retail & Consumer
MLP	DE0006569908	914.94	0.25%	Banks & Financial Services
Münchener Rück	DE0008430026	20,669.91	5.54%	Insurance
RWE	DE0007037129	13,786.93	3.69%	Utilities & Telecommunication
SAP	DE0007164600	15,697.27	4.21%	Software & Technology
Schering	DE0007172009	10,052.26	2.69%	Chemicals & Pharma
Siemens	DE0007236101	40,017.73	10.72%	Software & Technology
ThyssenKrupp	DE0007500001	5,094.08	1.37%	Machinery & Industrials
TUI	DE0006952005	2,871.23	0.77%	Automobile & Transportation
VW	DE0007664005	10,189.61	2.73%	Automobile & Transportation

TABLE 2 THE DAX CONSTITUTING STOCKS AND THEIR WEIGHTING IN THE INDEX AS OF AUGUST 30, 2002. SOURCE: DEUTSCHE BÖRSE AG.

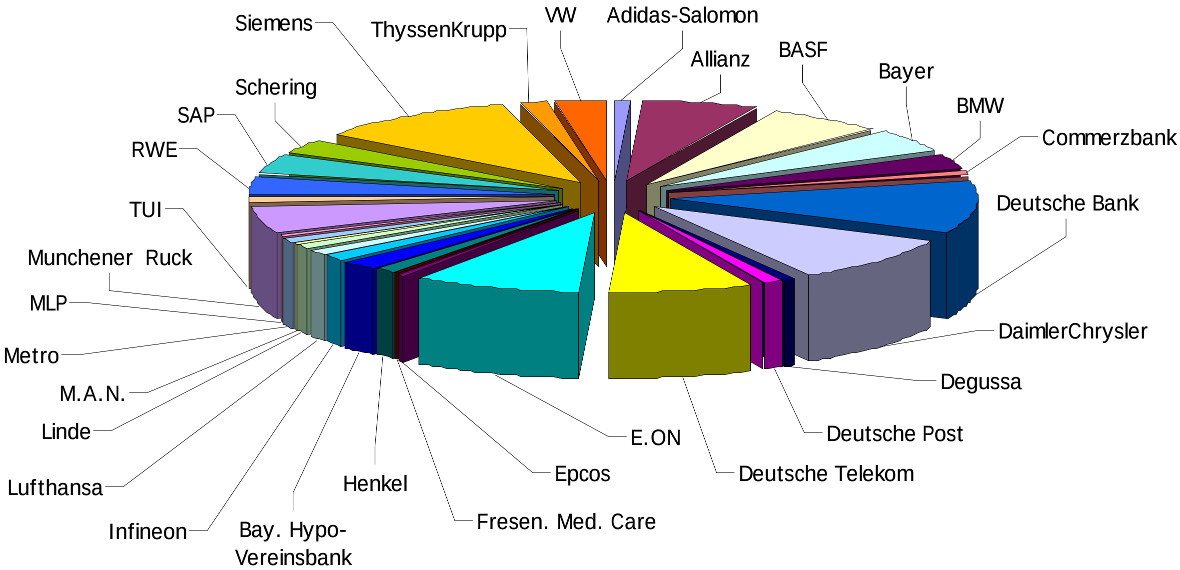

FIGURE 6 The DAX constituting stocks and their weighting in the index as of August 30, 2002.

Appendix B THE MEAN VALUES OF THE PORTFOLIO VARIANCE AND RETURN DISTRIBUTIONS (AND THE RETURN AND THE VARIANCE OF THE AVERAGE PORTFOLIO)

PROOF OF THE PROPOSITION 4.1: The random-generated portfolio return is:

$$\tilde{r}_p = \sum_{i=1}^{N} \tilde{w}_i r_i \tag{A.1}$$

where $\tilde{w}_i$ is the weight of security *i* in this random portfolio and r_i is the return of security *i* in the evaluated period, $i=1,2, \ldots, N$.

The expected value of the portfolio return is:

$$E\left[\tilde{r}_p\right] = E\left[\sum_{i=1}^{N} \tilde{w}_i r_i\right] = \sum_{i=1}^{N} E\left[\tilde{w}_i\right] \cdot r_i \tag{A.2}$$

The portfolio weights are uncorrelated but related through the budget constraint:

$$\sum_{i=1}^{N} \tilde{w}_i = 1 \tag{A.3}$$

Due to uniformity of portfolios over the feasible set, expected values of weights for random portfolios are:

$$E\left[\tilde{w}_i\right] = E\left[\tilde{w}_j\right] = \frac{1}{N} \qquad \forall i, j \in N \tag{A.4}$$

Using (A.4) we can rewrite (A.2) as

$$E\left[\tilde{r}_p\right] = \sum_{i=1}^{N} \frac{1}{N} \cdot r_i \tag{A.5}$$

where the right side of (A.5) identifies the return of the equally weighted portfolio.

■

PROOF OF THE PROPOSITION 4.2: The random-generated portfolio variance is:

$$\tilde{\sigma}_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{w}_i \tilde{w}_j \sigma_{ij} \tag{A.6}$$

where $\sigma_{ij} \equiv \operatorname{cov}\left(\tilde{r}_i, \tilde{r}_j\right)$ is the return covariance of securities *i* and *j*.

The expected value of the portfolio variance is:

$$
\begin{aligned}
E\left[\tilde{\sigma}_p^2\right] &= E\left[\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{w}_i\tilde{w}_j\sigma_{ij}\right] \\
&= \sum_{i=1}^{N}\sum_{j=1}^{N} cov\left(\tilde{w}_i,\tilde{w}_j\right)\sigma_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{N} E\left(\tilde{w}_i\right)E\left(\tilde{w}_j\right)\sigma_{ij} \qquad \text{(A.7)} \\
&= \sum_{i=1}^{N}\sum_{j=1}^{N} cov\left(\tilde{w}_i,\tilde{w}_j\right)\sigma_{ij} + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{N^2}\sigma_{ij}
\end{aligned}
$$

The second term in the last equality of Equation (A.7) represents the variance of the average (i.e. equally weighted) portfolio. The first term in this equality is somewhat cumbersome to analyse: the portfolio weights are not uncorrelated but related through the budget constraint:

$$
\sum_{i=1}^{N}\tilde{w}_i = 1 \qquad \text{(A.8)}
$$

Using (A.8) we can rewrite the first term in the last equality of (A.7) as:

$$
\begin{aligned}
\sum_{i=1}^{N}\sum_{j=1}^{N}\operatorname{cov}\left(\tilde{w}_i,\tilde{w}_j\right)\sigma_{ij} &= \sum_{i=1}^{N-1}\sum_{j=1}^{N-1}\operatorname{cov}\left(\tilde{w}_i,\tilde{w}_j\right)\sigma_{ij} \\
&\quad + 2\sum_{i=1}^{N-1}\operatorname{cov}\left(\tilde{w}_i, 1-\sum_{j=1}^{N-1}\tilde{w}_j\right)\sigma_{iN} \\
&\quad + \operatorname{cov}\left(1-\sum_{j=1}^{N-1}\tilde{w}_i, 1-\sum_{j=1}^{N-1}\tilde{w}_j\right)\sigma_N^2 \qquad \text{(A.9)} \\
&= \sum_{i=1}^{N-1}\operatorname{var}\left(\tilde{w}_i\right)\sigma_i^2 - 2\sum_{i=1}^{N-1}\operatorname{var}\left(\tilde{w}_i\right)\sigma_{iN} + \sum_{i=1}^{N-1}\operatorname{var}\left(\tilde{w}_i\right)\sigma_N^2 \\
&= \sum_{i=1}^{N-1}\operatorname{var}\left(\tilde{w}_i\right)\left[\sigma_i^2 - 2\sigma_{iN} + \sigma_N^2\right]
\end{aligned}
$$

In the term between the square brackets we recognize the variance of the return differential $\left(\tilde{r}_i - \tilde{r}_N\right)$, hence:

$$
\sum_{i=1}^{N}\sum_{j=1}^{N}\operatorname{cov}\left(\tilde{w}_i,\tilde{w}_j\right)\sigma_{ij} = \sum_{i=1}^{N-1}\operatorname{var}\left(\tilde{w}_i\right)\cdot\operatorname{var}\left(\tilde{r}_i - \tilde{r}_N\right) \qquad \text{(A.10)}
$$

The first variance term in the summand is strictly positive. When not all securities i are perfectly positively correlated (i.e. when the covariance matrix of the security returns is of full rank) then the second term in the summand is strictly positive too. Hence:

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\operatorname{cov}\left(\tilde{w}_i,\tilde{w}_j\right)\sigma_{ij}>0 \tag{A.11}$$

This in turn implies that:

$$\begin{aligned}
E\left[\tilde{\sigma}_p^2\right] &= \sum_{i=1}^{N}\sum_{j=1}^{N}\operatorname{cov}\left(\tilde{w}_i,\tilde{w}_j\right)\sigma_{ij}+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{N^2}\sigma_{ij} \\
&= \sum_{i=1}^{N-1}\operatorname{var}\left(\tilde{w}_i\right)\cdot\operatorname{var}\left(\tilde{r}_i-\tilde{r}_N\right)+\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{N^2}\sigma_{ij} \qquad \text{(A.12)} \\
&> \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{N^2}\sigma_{ij}
\end{aligned}$$

We conclude that the average portfolio variance is greater than the variance of the average portfolio (i.e. the equally weighted portfolio).

■

NOTE: On the basis of the last formulation one might be tempted to simply invoke Jensen´s Inequality. Since the variance is a convex function, the average portfolio variance is greater than the variance of the average portfolio. However, the first averaging is over the re-samplings in the simulation whereas the second averaging is over the securities in the portfolio. This “shortcut” is thus invalid.

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