YURI PEERS

Econometric Advances in Diffusion Models

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Econometrische vooruitgang in diffusie modellen

Thesis

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Preface

For the reader these words are the beginning of the thesis, yet for me they are the last words I write down. An additional fact worth mentioning about the words in this preface is that they are written down over 18,000 km from Rotterdam. After ten years, of which five as PhD student, I will exchange the Erasmus University of Rotterdam for the University of Waikato, in Hamilton, New Zealand. This distance makes it possible to look back even more nostalgically at what for me were five unforgettable years as a PhD. In this preface I like to thank a few people for this period, because even if I am ultimately the one responsible for the journey towards 2 December, on my way there I got good company.

I like to start by thanking my promoter Philip Hans Franses and copromoter Dennis Fok. When in 2005 I asked Philip Hans for the possibilities of doing a PhD, he gave me the liberty to think about, and write down, my own research subject. In the years that followed I got this same freedom, that is not to say that he left me to my own. He always had time to meet, even when he became dean of the faculty. His enthusiasm in doing research is infectious, and his creative way of looking at things could help me a week further in only 30 minutes.

After a few months in my PhD, due to the busy schedule of Philip Hans, I got a daily supervisor. Dennis took the term daily very seriously, as he always found the time to help me. After four years I can confess that the first times I talked with Dennis I came back with more questions then I asked him, especially regarding programming. His excellent ways of explaining things contributed to the fact that the words "Econometric Advances" in the title of this thesis are justified, at least, that is what I hope. Even after I mastered the programming of models, the trips to Dennis were challenging, the speed with which he understands models and adapts them to practical applications is remarkable, and I learned a lot from it. I can't thank Dennis enough for the past few years.

Next to Philip Hans and Dennis, I worked together with two other coauthors for some of the chapters in this thesis. I want to thank Yvonne van Everdingen and Stefan Stremersch for this cooperation, that fortunately does not end after my defense. Stefan I also like to thank for evaluating this thesis as member of the small committee. In the same role I like to thank Marnik Dekimpe and Albert Bemmaor.

I have been very lucky to be able to do a PhD at Erasmus University. The Econometric Institute, as well as the research school ERIM, were great in facilitating my period as a PhD. Therefore, I like to thank my colleagues from the Econometric Institute. In particular, I like to thank Christiaan Heij, Jan Brinkhuis, Martyn Mulder and Wilco van den Heuvel, for the times I could assist them with teaching various econometric courses.

As said, I like to thank ERIM. Next to facilitating my courses and conferences, I also closely worked with ERIM as board member of the PhD council. There is one person at ERIM I like to thank in particular, without Marisa van Iperen this thesis before you would probably have not been there in time. She helped me a lot in getting all the things done for this book, as it proved to be difficult to arrange all things from the other side of the world.

Also, the colleagues from the Marketing Department I like to thank, as they often allowed me to join their seminars and other events.

During my PhD I worked together with some companies. Although I can't name the companies I like to thank some of the people for their insights and help for my thesis. I like to thank: Anette, Jens, Jin Young, Kasia, Selma, Simone, Stefanie and Vincent.

Now is the moment to thank my PhD colleagues. The list of PhD students I met the past five years, and whom I like to thank, is too long to do so. Therefore, I thank a few in particular. I will begin with the people I shared an office with, indeed plural. I want to believe that it were the circumstances that made that I have had 6 (4 at room H9-16 and 2 at H11-26) roommates, but probably I am just a difficult person to stay in one room with. In chronological order I like to thank Martijn, Eelco, Joris, Jonathan, Lanah and Morteza. There is actually a seventh person I shared an office with, in my last week at the Erasmus University Sjoerd accompanied me at the 11th floor. However, Sjoerd I like to thank as neighbor at the 9th floor, for the nice talks at the end of the week when we both realized the week had flown by again.

My two years in the board of the PhD council also contributed to the educational component, as well as the fun component of my period as a PhD. I thank Ivana, Joris and Manuel for the last six months of these two years. The first one and a half year I worked together with Mariano and Sebastiaan. I like to thank them both, I think we brought the PhD students closer together with a variety of social events and social drinks, during which we had a lot of fun.

I will continue this preface with thanking the "Lunchgroepje". These were the people I spend most of my lunches at the University with. However, it ended up as being much more than lunches, I will never forget the drinks, bowling, and the weekend in the Ardennes. With a personal twist I like to thank Mathijn for the many duo lunches, at the end of the "Lunchgroep" era. I thank Hans for the Baja. Arco and Kar Yin I thank for all the times I was welcome at H7-11 to talk about anything. Peter I thank for all the Friday nights which ended with some nice cozy dinners. Eelco I will thank later in this preface.

This preface mainly emphasizes on the people that I crossed paths with academically the past few years. However, also outside the walls of the University there are people that I like to thank for the past five years. The holidays and weekend trips with the "Oost-Europa groep" and "Bestuur 2003-2004" were welcome distractions, even the time I was stranded at Milan airport with Niels and Rosalie. I like to thank Kasper, Olrik and Thijs, and despite the lame jokes on doing a PhD, their interest and friendship helped me a lot. For example, the discussions at "mannenavond" have been a good exercise for 2 December.

We have arrived at the last paragraphs of this preface and I hope I did not yet lose all readers. I will start and end these last paragraphs with my paranymphs. To outsiders the concept of paranymphs always seems a bit strange, but I am happy to have these two people by my side when I defend my thesis. The first paranymph that will stand next to me is Eelco. It is difficult to put into words how important Eelco has been during my period as PhD student. A joint interest in research topics as well as other things in life made him an ideal sparring partner. Proposition six "Always being the best will not make you better, trying to be the best will." is partly inspired on the past years with Eelco. In many things we were evenly matched, and I think that this made me a better researcher, but certainly a better pool/snooker player. I thank Eelco for all the discussion about nothing and about everything, for the wines, playing pool, the Friday nights, etc.

Next to Rotterdam, maybe not the most beautiful but certainly the most fun city of the Netherlands, I also have a home base in Rijswijk. The same holds as before, that I like to mention everybody, but this would be impossible. I do like to give a special thanks to my sisters, Mascha and Lois. They are always there for me, and just like when we were kids they still can make me dress up in the most ridiculous costumes. Also the "aanhang" of my sisters, Joost and Erik, I like to thank. And for my grandparents a quick word in Dutch: Ik wil mijn opa en oma bedanken voor hun oprechte interesse in alles wat ik doe.

I would also like to seize the opportunity to thank my parents, Ingrid and Maurice. My parents always support me in whatever I do. Even if my research must often sound like gibberish to them they always show interest, and they always say the right words in the difficult times. I can't thank my parents enough for this and everything.

Finally, I like to thank my second paranymph. Daphne keeps up with me for over seven years now, and is even crazy enough to go with me to the other side of the world. In times when doing research didn't go as I wanted it to, for example if after three days of calculating it showed that I forgotten a parenthesis, I could always come home to Daphne and Tommie (our red tiger) and nothing seemed to matter that much anymore. Daphne I love you, and I couldn't think of a better person standing next to me during this important day, as I hope you will be standing next to me in many moments to come.

Yuri Peers
Rotterdam, December 2011

Contents

Chapter 1

Introduction

1.1 General motivation

This thesis discusses econometric advances in diffusion models. Diffusion models can be used to explain the pattern of the adoption of products, where adoption is the first time a customer buys a product. In particular, these models can be used to explain the first time purchase of a product or service, or the sales of a durable good, as customers often purchase these goods only ones. For example, in this thesis we consider the diffusion of consumer electronics (e.g. flat-screen television sets, DVDs, game consoles), cars, and pharmaceuticals.

The sales/adoption of a product typically starts slow with only a few customers, the so-called innovators. After that, due to social contagion, like word-of-mouth, the number of adopters grows rapidly until most customers adopted the product. When the product reaches its market potential the adoption process slows down as the last customers adopt the product. This results in the, for diffusion characteristic, bell curve for the sales/adoption and the S-curve for the cumulative sales/adoption.

Most diffusion models capture above mentioned patterns by modeling the three key aspects of these adoption patterns: (i) the innovation aspect, that is, the customers that are externally influenced to purchase a product; (ii) the imitation aspect, which captures the social contagion and other internal influences leading to purchase; and (iii) the market potential.

Diffusion models have been an important part of the marketing literature for over 40 years. The most well-known diffusion model, by far, is the Bass mixed-influence model (Bass, 1969). Since this paper was published a wide variety of extensions, critical notes and new estimation techniques have followed.

Proof of the interest from business is the increasing use of the Bass model in business school MBA and executive-level teaching (Godes and Ofek, 2004, 2007; Ofek, 2005a,b, 2008).

Also in academia, despite the critical notes, diffusion models are still an important part of the current marketing landscape. As said, the Bass diffusion model is the basis of a wide variety of extensions (see Peres *et al.*, 2010), for a review of extensions of the original diffusion model). For example, there are models that focus on the diffusion of products in various countries (so-called international diffusion models) and there are models that consider the competition across products (see, for example, Albuquerque *et al.*, 2007; Ganesh *et al.*, 1997; Kim *et al.*, 2000; Putsis *et al.*, 1997). Other scholars focus on the diffusion of different generations of the same products (Mahajan and Muller, 1996; Norton and Bass, 1987, 1992). All these models have the basic Bass diffusion model as the starting point. Further, models that allow the diffusion process to be more flexible, non-symmetric, time-varying, etc, also stem from the Bass model (Parker, 1993). As a final example, there are several papers that introduce hierarchical diffusion models (Ainslie *et al.*, 2005; Fok and Franses, 2007; Lenk and Rao, 1990; Talukdar *et al.*, 2002). These models pool information of diffusion data over products and/or countries in order to improve early or even pre-launch forecasts of new innovations.

Diffusion models remain an interesting research topic for marketing scholars. Despite the maturity of the diffusion literature in marketing this dissertation adds new and important insights. In particular, due to the increasing availability of diffusion data, there are new challenges to overcome. The challenges we deal with in this thesis are discussed next.

1.2 New challenges

Most current diffusion models are still based on a single diffusion series, that is, one studies the diffusion of one product in a single country. Often this series is observed at an annual frequency. In the past decade the focus has shifted from one diffusion series to the comparison of multiple series. Such a comparison allows one to learn from the differences and similarities across products and/or countries. Additionally, the most common observational frequency has shifted from annual to monthly or quarterly, and in some cases data is available at an even higher frequency. Both developments lead to new challenges in the modeling of diffusion processes.

In this thesis we provide solutions to these challenges, together with some new perspectives on known issues with diffusion modeling. In Chapter 2 we deal with the estimation of diffusion parameters for a single series. We start with an overview of existing estimation methods and we suggest a new method. Next we study the possible bias in the diffusion parameters, where we specifically consider the source of noise and observational frequency. Second, we look at the challenges arising due to using high-frequency diffusion data, that is, mixed-frequency diffusion data (Chapter 3) and seasonality (Chapter 4). Finally, in Chapter 5 we come back to the comparison of many diffusion series. Below, we briefly introduce these four studies.

Chapter 2: Parameter bias in Bass diffusion model estimation

This chapter considers the estimation of the diffusion parameters for a single series. We consider the main existing estimation methods, namely a least squares regression as proposed by Bass (1969), the nonlinear least squares method by Srinivasan and Mason (1986), and the generalization of the Bass regression by Boswijk and Franses (2005). It is well known that, using these methods, the Bass model parameters are sensitive to estimation bias (e.g. see van den Bulte and Lilien, 1997; van den Bulte and Stremersch, 2004). In this chapter we study the different estimation methods of the Bass model, and document that the bias that may result from the observational frequency and the source of the noise that is implicitly assumed in the estimation methods. Also, in this chapter, we propose a new and simple estimation method that suffers less from such bias.

We base our comparison on simulation experiments, and we validate these simulation results with empirical cases. We show that none of the existing methods performs well for all observational frequencies, while the new method does. We also show that the new method is most robust to different sources of noise, whereas for the existing methods the error structure needs to match with the actual source of noise. Our conclusion is that the new estimation method should from now on be used by diffusion scholars and analysts to avoid bias in diffusion parameters. This chapter is based on joint work with Dennis Fok and Stefan Stremersch.

Chapter 3: Estimating diffusion parameters on mixed-frequency data

The third chapter studies mixed-frequency data. Diffusion data is nowadays often available at the monthly or even weekly frequency. Such high-frequency data can be very useful for academic diffusion research and applied research. However, high-frequency diffusion data is often not available directly from the start of the diffusion process. In some cases, at the start data is collected at a low frequency. In practice one therefore usually has observations at mixed frequencies. For example, one may have two yearly observations followed by a number of monthly observations.

In this chapter we discuss model-based methods that allow the use of such mixed-frequency data. We adapt several estimation techniques for diffusion models to make them useful for mixed-frequency data. In particular, we consider the least squares regression as proposed by Bass (1969), the nonlinear least squares method by Srinivasan and Mason (1986), and the new method proposed in Chapter 2. We find that for some estimation methods it suffices to apply a simple correction for the different time intervals. In other situations, however, simulation methods are needed to appropriately use mixed-frequency data.

We compare the performance of our methods to several alternative approaches, such as, (i) aggregating data to the lowest available frequency; (ii) ignoring the initial low-frequency data; and (iii) using simple linear interpolation to get complete high-frequency data. The first alternative leads to a potentially large loss of information and therefore an efficiency loss. We show that the other two lead to biased estimates. The most appropriate model-based method depends on the chosen estimation procedure. This chapter is based on joint work with Dennis Fok.

Chapter 4: Modeling seasonality in new product diffusion

In Chapter 4 we consider another challenge that one may face when using high-frequency diffusion data, namely seasonality. The issue of seasonality is a logical consequence of using high-frequency diffusion data. However, very limited research has been done to model seasonality in diffusion models. In this third chapter we propose a method to include seasonality in any diffusion model that has a closed-form solution. The resulting diffusion model captures seasonality in a way that naturally matches the original diffusion model's pattern. The method assumes that additional sales at seasonal peaks are drawn from previous or future periods. This implies that the seasonal pattern does not influence the underlying diffusion pattern. The model is compared with alternative approaches through simulations and empirical examples. As alternatives we consider the Generalized Bass Model [GBM] and the basic Bass model, which ignores seasonality. One of our main findings is that modeling seasonality in a GBM generates good predictions, but gives biased estimates. In particular, the market potential parameter is underestimated. Ignoring seasonality, in cases where data of the entire diffusion period is available, gives unbiased parameter estimates in most relevant scenarios. However, when only part of the diffusion period is available, estimates and predictions become biased. We demonstrate that our model gives correct estimates and predictions even if the full diffusion process is not yet available. This chapter is based on joint work with Dennis Fok and Philip Hans Franses.

Chapter 5: Product-country interaction in new product diffusion

The first three chapters considered only a single diffusion series. Each of these chapters not only introduces new methods, but also points out situations where the existing models are still valid. In the final chapter of this thesis we extend this knowledge on estimation methods for one single diffusion series to an international diffusion case. We study diffusion series of 12 products in 82 countries. In this case the challenge is to pool the information across the different product-country combinations, and to find similarities and differences across these different diffusion series.

Historically, international diffusion models included only country and/or product characteristics. We show that the interactions between the country and product characteristics are an important part of the "story" behind the variation across diffusion series. As an example, the difference in diffusion speed between France and Norway may be very different for freezers than for cell phones.

In this chapter, we extend existing (multilevel) international diffusion models by explicitly including product-country interactions. These interactions can partly be captured using the standard approach of including the multiplication of product and country variables. However, a substantial part of the interactions remains unexplained. We use dimension reduction techniques to provide insight in this remaining variation. This technique reveals hidden correlation patterns in the unexplained product-country variation.

We show that interaction effects are indeed very important to include in international diffusion models. We present two potential applications for our results. First, the identification of the country, or countries, best suited for the launch of a new product. The second is early forecasting. The model can be used for a particular product-country combination to get better pre-launch and early forecasts, if the product has already been introduced in other countries. This chapter is based on joint work with Yvonne van Everdingen, Dennis Fok and Stefan Stremersch.

1.3 Outline of the thesis

Although diffusion models, and in particular econometric advances in diffusion models, are the common denominator in this thesis, each chapter is self-contained and can be read independently. Each chapter starts with its own introduction and ends with a conclusion and/or discussion of the major findings. Also, because each chapter can be read separately there is some overlap between the chapters, especially in describing the relevant literature. The thesis concludes with a short summary.

Chapter 2

Parameter bias in Bass diffusion model estimation: a new method and a comparison of existing methods

The Bass (1969) model has been one of the most influential models in marketing science and practice. However, Bass model parameters are sensitive to estimation bias (e.g. see van den Bulte and Lilien, 1997; van den Bulte and Stremersch, 2004). We document the bias that may result from observation frequency and the source of noise for three existing estimation methods, namely a least squares regression as proposed by Bass (1969), the nonlinear least squares method by Srinivasan and Mason (1986), and the generalization of the Bass regression by Boswijk and Franses (2005). We propose a new and simple estimation method that suffers less from such bias. We base the comparison in this chapter on simulation experiments, where we explicitly consider the impact of the observation frequency and the source and presence of noise. We validate the simulation results with empirical cases. We show that none of the existing methods perform well for all observation frequencies, while the new method does. We also show that the new method is most robust to different sources of noise, whereas for the existing methods the error structure needs to match with the actual source of noise. The conclusion is that the new estimation method should from now on be used by diffusion scholars and analysts to avoid bias in diffusion parameters.

2.1 Introduction

The Bass (1969) mixed-influence model has been one of the most influential models in marketing science and practice. Since the paper was published in 1969 there has been a wide variety of papers that presented extensions, critical notes, and new estimation techniques. The Bass model is now also increasingly used in business school MBA and executive-level teaching (Godes and Ofek, 2004, 2007; Ofek, 2005a,b, 2008). At the same time, several papers have shown that Bass model parameters are sensitive to estimation bias (e.g. see van den Bulte and Lilien, 1997; van den Bulte and Stremersch, 2004).

Apparently, this sensitivity to estimation bias has not prevented the spread of the Bass model, nor the development of extensions, which, in turn, in many cases suffer from estimation bias issues similar to the original Bass model (see Peres *et al.*, 2010, for a review of extensions of the original model). For instance, international and competitive diffusion models (Albuquerque *et al.*, 2007; Ganesh *et al.*, 1997; Putsis *et al.*, 1997) are often formulated as an extension of the regression estimation method of the Bass model. Models on generational diffusion of products (Mahajan and Muller, 1996; Norton and Bass, 1987, 1992) are usually based on the estimation method for the Bass model proposed by Srinivasan and Mason (1986). Further, allowing the diffusion process to be more flexible, non-symmetric, time-varying, etc, often leads to formulations similar to the Bass model (Parker, 1993), and are therefore subject to similar estimation issues. Other extensions, such as the frequently used flexible model proposed by Bemmaor (1994) and the two segment diffusion model of van den Bulte and Joshi (2007), often derive a deterministic diffusion curve, and therefore use similar estimation methods as used for the Bass model. Finally, there are several papers that introduce hierarchical diffusion models (Ainslie *et al.*, 2005; Lenk and Rao, 1990; Talukdar *et al.*, 2002). These models pool information of diffusion data over products and/or countries in order to improve early or even pre-launch forecasts. Also in these cases, the basic estimation technique is inspired on that for the standard Bass model.

The above not only shows the impact the Bass diffusion model has made on marketing science, but it also shows the proliferation in estimation methods. The differences between estimation methods mainly originate from the specification of the error term. As the Bass model itself is deterministic there is room for various error assumptions. In this chapter, we review the three main estimation methods [1]: (1) performing a least squares regression on the discrete time equation proposed by Bass (1969); (2) the nonlinear least squares method proposed by Srinivasan and Mason (1986); and (3) the generalization of the Bass regression by Boswijk and Franses (2005). We document that each of these estimation techniques has a different interpretation of the source of noise and that each of these

[1] Flexible forecasting models like the Kalman Filter suggested in Xie *et al.* (1997), Genetic Algorithm (Venkatesan *et al.*, 2004) or penalized splines (as in Stremersch and Lemmens, 2009; Sood *et al.*, 2009) are beyond the scope of this article. Note that most of these methods build on either the Bass regression equation or the method suggested in Srinivasan and Mason (1986).

methods may result in parameter bias, of which the magnitude depends upon source and presence of noise and observation frequency.

This chapter introduces a new estimation method that assumes the noise to be due to measurement error on the cumulative adoption, an assumption which is valid in many diffusion data. It is as easy to implement as the alternative estimation methods, but has the advantage that it is more robust to different sources of noise. The latter aspect results in approximately unbiased estimates across all observation frequencies.

In addition to estimation, we examine the performance of diffusion models for the purpose of data generation. While interesting in its own right to understand the origin of estimation bias, several papers have also developed simulations based on a data-generation process with diffusion models (f.e. van den Bulte and Lilien, 1997; Bemmaor and Lee, 2002; Kumar and Krishnan, 2002; Non *et al.*, 2003; Boswijk and Franses, 2005; Jiang *et al.*, 2006). While for the process of data generation the choice of the error structure is very important, a comparison of different error structures in terms of the properties of the generated data has not been presented, at least not to our knowledge.

We show that the new estimation method outperforms the other methods we review in most cases. In a simulation study, we show that the method leads to approximately unbiased and more efficient parameter estimates for all observational frequencies. Also, in this simulation study we show that the method is more reliable to use as data generation process. The method is also more robust than the other methods to different sources of noise. Other methods only perform well if noise is likely to match the error specification. Next, we confirm the estimation results of the simulation study in real-life empirical cases. We additionally show that the method is most robust regarding short term forecasts. The results of the simulation study and empirical cases support the importance of the error specification. The new estimation method and comparison of existing methods can help diffusion research in three ways: (i) better theoretical inference due to unbiased estimates; (ii) better short-term forecasts after the inflection point; and (iii) better pre-launch (analogical) forecasting.

The outline of the remainder of this chapter is as follows. First, we review the Bass model and present the four alternative estimation methods we consider in this chapter. In the following section, we consider pros and cons of each estimation method on theoretical grounds. Next, we report simulation results on all four estimation methods, where we compare the estimation methods in terms of bias and estimation uncertainty. Fourth, we confirm the results from the simulation study using several empirical cases, each with different types of noise and/or observational frequency. We additionally look at the short-term forecast performance of the different estimation methods. We conclude by discussing the implications of the findings for diffusion modeling, in academia and in practice.

2.2 Estimating diffusion parameters

The differential equation of the Bass mixed-influence model (Bass, 1969) is

$$\frac{f(t)}{1-F(t)} = p + qF(t), \text{ for } t \geq 0, \tag{2.1}$$

where $f(t) = dF(t)/dt$ and $F(t)$ denotes the fraction of adopters at time t relative to those who will ultimately adopt. The ultimate number of adopters is denoted by m and is usually referred to as the ceiling level. The number of adopters at time t, $N(t)$, is given by $mF(t)$. The instantaneous rate of change in adopters at time t therefore equals $n(t) = mf(t)$. The parameters p and q in equation (2.1) denote the coefficients of internal and external influence relative to the social system, respectively. The parameter q represents social contagion within a social system, either by imitation for social status considerations or word-of-mouth.[2]

It is common practice to interpret the model in terms of aggregates as we do above. However, the Bass model in (2.1) can also be seen as a model that describes the time until adoption of someone who will eventually adopt. In this interpretation $F(t)$ is the cumulative distribution function of the random time until adoption. If we aggregate this individual-level behavior over the population we again obtain $mF(t)$ as the expected fraction of adopters at time t.

The differential equation (2.1) has as closed-form solution

$$F(t) = \left[\frac{1-\exp(-(p+q)t)}{1+\frac{q}{p}\exp(-(p+q)t)}\right], \text{ and } f(t) = \left[\frac{p(p+q)^2\exp(-(p+q)t)}{(p+q\exp(-(p+q)t))^2}\right]. \tag{2.2}$$

This closed-form solution represents the diffusion process in continuous time. In practice, we do not observe the diffusion process in continuous time. In fact, we usually have observations at certain time intervals, say yearly, quarterly, or monthly. In other words, we observe $N(t)$ at $t = 0, \delta, 2\delta, 3\delta, \ldots$, where δ denotes the calendar time between observations. In what follows, we denote these observations by $N_i,\ i = 0, 1, 2, \ldots$, where the corresponding calendar time of observation i equals $i\delta$. Based on these observations the goal is usually to estimate the parameters p, q and m such that we recover the underlying continuous time diffusion curve $F(t)$. Next, the parameter estimates may be used to forecast the future diffusion of the product itself, or to forecast the diffusion process of other, analogous, products. In what follows, we assume that the latter is the goal of the forecasting. We therefore assume that the available observations span the complete diffusion curve. If this is not the case, that is, the observations are right censored many other problems arise for the existing estimation methods, see van den Bulte and Lilien

[2] Alternatively, the model can also be parameterized with p + q as a scale parameter and q/p as a shape parameter.

(1997). For now, we abstain from this issue in order to show that even in the ideal case that the complete diffusion has been observed, the currently available estimation methods may still perform poorly.

Given observations $N_i,\ i = 0, 1, 2, \ldots$ at time intervals δ, there are a number of ways to estimate p, q and m. However, an issue in this estimation is the "error term". In the specification above there is no error, but in practice the model will not fit the data perfectly. In the discussion below, we present all estimation methods and indicate how each method introduces the error term.

Estimation following Bass (1969)

The original idea of Bass (1969) to estimate p, q, and m is to look at a discretization of the original differential equation (2.1) in terms of N_i. The discretization that is used approximates $f(t)$ at $t = i\delta$ by $[N_{i+1} - N_i]/(m\delta)$ and $F(t)$ by N_i/m. The differential equation then is

$$\frac{[N_{i+1} - N_i]/(m\delta)}{(1 - N_i/m)} \approx p + q\frac{N_i}{m}, \tag{2.3}$$

which we can rewrite as follows

$$N_{i+1} - N_i \approx \delta(p + q\frac{N_i}{m})(m - N_i). \tag{2.4}$$

Denoting $X_i = N_i - N_{i-1}$ we obtain

$$X_i \approx \delta pm + \delta(q - p)N_{i-1} - \delta\frac{q}{m}N_{i-1}^2, \text{ for } i = 1, 2, \ldots. \tag{2.5}$$

Bass (1969) suggest estimating the parameters by applying OLS to the equation

$$X_i = \beta_1 + \beta_2 N_{i-1} + \beta_3 N_{i-1}^2 + \varepsilon_i, \tag{2.6}$$

and transforming the estimated parameters into estimates of p, q and m. Note that δ is given and does not have to be estimated. The error term in (2.6) captures the approximation error present in (2.3) and possible random deviations from the underlying diffusion curve.

If we take (2.6) literally, it specifies that the number of new adopters in the interval $[(i-1)\delta, i\delta]$, that is, X_i, can differ from its expected value. Furthermore, these "random" adopters will influence future adopters, that is, ε_i has a dynamic impact on future adoption.

Estimation following Srinivasan and Mason (1986)

Srinivasan and Mason (1986) propose to match $X_i = N_i - N_{i-1}$ directly with $F(t)$. The underlying idea is that the increment in the penetration must equal m times the change in $F(t)$. They propose to perform Nonlinear Least Squares [NLS] on

$$X_i = m(F(\delta i) - F(\delta(i-1))) + \varepsilon_i, \tag{2.7}$$

where $F(t)$ is of course a function of p and q. In this method, the error does not contain an approximation error, it should be seen as random deviations between the actual change in penetration versus the "predicted" change. In this specification the error ε_i does not affect future adoption. It does contribute to the cumulative adoption.

Estimation following Boswijk and Franses (2005)

Boswijk and Franses (2005) argue that the actual diffusion process can deviate from the underlying non-stochastic diffusion process, and that this deviation should be modeled. The natural assumption is that the actual diffusion process has the tendency to mean-revert to the underlying process. Therefore, Boswijk and Franses (2005) replace the (deterministic) differential equation in (2.1) by a stochastic differential equation. They specify

$$dn(t) = \alpha[n^*(t) - n(t)]dt + \sigma n(t)dW(t), \tag{2.8}$$

where $W(t)$ is a standard Brownian motion, $n^*(t)$ is the growth indicated by the deterministic differential equation, or *target path*, and the speed of mean-reversion is given by α. Given the current (stochastic) $N(t)$ the target path is specified as

$$n^*(t) = p(m - N(t)) + \frac{q}{m}N(t)[m - N(t)]. \tag{2.9}$$

The estimation method in Boswijk and Franses (2005) can be seen as a generalization of the Bass regression. Boswijk and Franses (2005) also apply a discretization and propose to apply OLS to the following equation

$$\Delta X_i = \beta_1 + \beta_2 N_{i-1} + \beta_3 N_{i-2} + \beta_4 X_{i-1} + X_{i-1}\varepsilon_i, \tag{2.10}$$

where $\beta_1 = pm\alpha\delta^2$, $\beta_2 = \alpha\delta^2(q-p)$, $\beta_3 = \frac{-q\alpha\delta^2}{m}$, and $\beta_4 = -\alpha\delta$. The error in (2.10) is the combination of the Brownian motion in (2.8) and a discretization error.

Estimation following the Newly Proposed method

The approach in this chapter is very close to the Srinivasan and Mason method. In this way, we do not introduce a discretization error. We propose to apply NLS to

$$N_i = mF(\delta i) + \varepsilon_i, \tag{2.11}$$

that is, we directly match the observed penetration levels with $F(t)$. Although the difference between this method and the Srinivasan and Mason method may seem to be minor, we will show below that this small difference has a large impact in terms of performance and the quality of this model as a data generating process. Note that in (2.11) the error term is specified relative to the cumulative adoption. As we will discuss below, the error term in this specification should be interpreted as measurement error.

2.3 A comparison of estimation methods

In this section, we first present a theoretical comparison of the estimation methods we presented above. We consider possible discretization errors and the substantial and statistical interpretation of the error structure in the different methods, where we specifically consider each method as Data Generating Process [DGP]. Next, we compare the performance of the estimation methods in two different simulation settings: a setting where the diffusion curve is observed perfectly, and in the more realistic setting where there is noise. For both settings we will explicitly focus on the impact of the observational frequency (δ).

2.3.1 Theoretical Comparison

The mixed-influence diffusion model is non-linear in its parameters. It is well known that non-linear least squares and maximum likelihood estimates are likely to show a (small-sample) bias, unless the variance of the errors is zero (see for example Shenton and Bowman, 1963; Box, 1971; Cook *et al.*, 1986). In this sense the estimation methods are all expected to be biased. However, the magnitude of the bias will largely depend on possible discretization errors and the assumed error structure. Below we assume that the full diffusion curve has been observed. Therefore, we do not consider the bias due to (right-hand) truncation as discussed in van den Bulte and Lilien (1997) and Bemmaor and Lee (2002).

Discretization error

An obvious difference between the estimation methods is that the Bass (1969) and Boswijk and Franses (2005) methods rely on a discretization of the underlying continuous time diffusion curve, whereas the other two do not. It is well-known that this discretization leads to a bias if the time between observations (δ) is relatively large (Schmittlein and Mahajan, 1982; Putsis, 1996; Non *et al.*, 2003). Such a bias will not occur in the method of Srinivasan and Mason (1986) or the newly proposed method. We will later show the magnitude of this bias.

Error structure

A less researched difference between the methods relates to how the errors are introduced in the methods. Most important is whether the error term correspond to actual adoption or not. If so, under the assumptions of the Bass model the error at time t will influence future adopters. The reason for this is that also these unexpected adopters will influence others. The error does not correspond to actual adoption when there is measurement error. This is especially the case when data is collected using a survey. At each point in time different individuals will be surveyed and we get a noisy measurement of the true adoption. Changes in the measured adoption over time are partly due to actual adoption and partly due to the measurement error in the surveys. In this case, the error at time t will not influence future adoption. Note that in the original Bass model (2.1) there is no error whatsoever.

In the Bass regression method (2.6) and in the Boswijk and Franses method (2.10) the error should be seen as a combination of discretization error and unexplained actual adoption. This is most clear for the Boswijk and Franses method, as in that method deviations from the underlying Bass diffusion curve are explicitly modeled.

The Bass regression (2.6) can be seen as a nonlinear dynamic model. In this model the error for observation i, ε_i, has an impact on N_i which in turn will impact N_{i+k}, for $k = 1, 2, \ldots$. In other words a positive error at a certain point leads to more adopters at that point in time, and these adopters will have an influence on the non-adopters. The result of this is that the diffusion will speed up. The relation between the shock ε_i and N_{i+k} is quite complex, for $k = 1$ one can use (2.6) to obtain

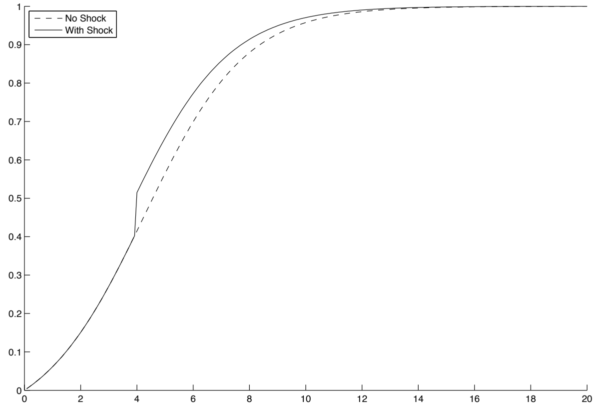
$$\frac{\partial N_{i+1}}{\partial \varepsilon_i} = \beta_2 + 2\beta_3 N_i.$$

This indicates that the impact of ε_i on the next observation is assumed to depend on the parameters and on N_i. In Figure 2.1 we give a graphical representation of the relation between a shock and the cumulative sales in the following periods. The figure is based on a simplified example where there is no noise and only one positive shock. It is clear that this shock speeds up the diffusion process. The diffusion series approaches $mF(t)$ only when the ceiling is reached.

The error structure in the newly proposed method (2.11) has the interpretation of measurement error. This method equates the observed penetration level to its “predicted” level under the continuous time diffusion model. The error for observation i does not impact future observations. In many practical cases, measurement error may be the main source of randomness in the observations. This is especially true if the data is obtained through a survey. The Euromonitor penetration database for consumer durables is a well-known, and often used, example of this data collection method.

To further develop the idea of measurement error in a survey, consider the following setting where there is a total population of size P and S randomly selected individuals are surveyed for each observation. Next we interpret (2.1) in terms of individual behavior. This means that from the total population a fraction m will finally adopt. For each potential adopter, the time to adoption is a random variable with cumulative distribution

Figure 2.1: Solutions generated by the Bass model (using $\delta = \frac{1}{12}$, $p = 0.05$, $q = 0.5$, and $m = 1$). The solid line includes a one time shock of 0.1 at year 4.



function $F(t)$. The fraction of adopters in the survey $(N(t))$ has a binomial distribution with

$$\begin{aligned} \mathrm{E}[N(t)] &= \frac{1}{S}\sum_{i=1}^{S} \Pr[\text{indiv } i \text{ adopts on or prior to } t] \\ &= \frac{1}{S}\sum_{i=1}^{S} mF(t) = \frac{m}{S}SF(t) = mF(t) \\ \mathrm{Var}[N(t)] &= \mathrm{Var}[\frac{1}{S}\sum_{i=1}^{S} I_{[\text{indiv } i \text{ adopts on or prior to } t]}] \\ &= \frac{1}{S^2}\sum_{i=1}^{S} \mathrm{Var}[I_{[\text{indiv } i \text{ adopts on or prior to } t]}] = \frac{mF(t)(1-mF(t))}{S}. \end{aligned} \tag{2.12}$$

For large S, $\mathrm{Var}[N_t] \approx 0$, see Boswijk and Franses (2005) for similar arguments. If the sample size of the survey is not that large some random variation remains.

The interpretation of the error structure in the Srinivasan and Mason method is a bit more complex. At first sight, the error also seems to have a measurement error interpretation. However, the measurement error is defined on the adoption instead of on the penetration. For the penetration level, the Srinivasan and Mason specification (2.7) can be rewritten as

$$N_i - N_0 = \sum_{k=1}^{i}(N_k - N_{k-1}) = m(F(\delta i) - F(0)) + \sum_{k=0}^{i-1} \varepsilon_{i-k}. \tag{2.13}$$

As $N_0 = F(0) = 0$, this implies

$$N_i = mF(\delta i) + \sum_{k=0}^{i-1} \varepsilon_{i-k}, \tag{2.14}$$

with independently distributed ε_i this means that the difference of the observed penetration and the underlying diffusion curve has a unit root.[3] The Srinivasan and Mason approach actually specifies a random walk for the deviation between N_i and the underlying diffusion curve. In fact we can even give the error term a continuous time interpretation and specify the error as a Brownian motion. The increments of the Brownian motion are homoscedastic with variance $\delta\sigma^2$. In the long run, the observations can deviate from $mF(t)$ unlimitedly! This is a natural consequence of specifying the measurement error on sales. The actual cumulative sales will equal the cumulative predicted sales plus the cumulative errors. Under this method we therefore cannot accurately estimate the ceiling level m. Most likely, this will also translate into difficulties in estimating p and q.

Data generating process

As part of the theoretical comparison we also consider the properties of the four methods when they are used as a data generating process [DGP] to generate stochastic diffusion curves. For the development of new complex diffusion models, such as diffusion models with cross-country effects or multi-generation diffusion models, it is important to be able to check the model's performance on simulated data. A good DGP is therefore crucial. It is well known that the error in diffusion curves tends to be heteroscedastic, that is, the variance of the error is small in the beginning of the curve, large in the middle, and converges to zero at the end. All methods therefore need to capture heteroscedasticity. To introduce this, we set the variance of ε_i equal to $\delta\sigma^2(N_{i-1} - N_{i-2})$ for the Bass regression, equal to $\delta\sigma^2 F(\delta i)(1 - F(\delta i))$ for the Srinivasan and Mason method, and equal to $\sigma^2 F(\delta i)(1 - F(\delta i))$ for the new method[4]. Note that in the DGP of the newly proposed

[3] Note that (2.11) can also be written as $N_i - N_{i-1} = m(F(\delta i) - F(\delta(i-1))) + \varepsilon_i - \varepsilon_{i-1}$. We could generalize this by introducing a new parameters to $N_i - N_{i-1} = m(F(\delta i) - F(\delta(i-1))) + \varepsilon_i - \varrho\varepsilon_{i-1}$. The resulting model encompasses the Srinivasan Mason model ($\varrho = 0$) and the newly proposed method ($\varrho = 1$). Given sufficient data, this specification would allow one to apply a test for both methods. We leave this issue for further research.

[4] This heteroscedasticity specification also reduces the probability of generating non-monotonic penetration curves. However, especially for the new method this is not ruled out. This corresponds to the practical situation where one also may have non-monotonic data. Decreasing penetration curves can be a result of (survey-based) measurement error.

model it makes no sense to scale the error variance with δ. The Boswijk and Franses specification (2.10) already contains a heteroscedastic error.

The figures in Appendix 2.A show examples of generated diffusion curves using the four methods, where we use $p = 0.05$, $q = 0.5$, and $m = 1$. For the Bass regression method and Boswijk and Franses method we generate the curves using a very small step size δ to avoid a discretization error in the DGP. We show one figure for each method. In each figure we show: (i) 10 generated diffusion curves; (ii) the spread and average over a large number of generated curves; (iii) the deviations from $mF(t)$ for 10 curves.

First we check whether the generated diffusion curves converge to m. If this does not happen, one can never consistently estimate m under that DGP. Only for the Srinivasan and Mason method do the curves *not* converge to m. This finding is consistent with the discussion above on the unit root behavior in the Srinivasan and Mason method. The Srinivasan and Mason model therefore does not seem to be a good candidate to generate stochastic diffusion curves. Note that the heteroscedasticity implies that the variance of ε_i goes to zero for large i. Therefore, all generated penetration curves do converge to some constant.

Next we look more closely at the differences between the generated curves and $mF(t)$. For most methods this deviation does not oscillate around 0, that is, a generated curve tends to be either permanently above or permanently below the underlying diffusion curve $mF(t)$. Only for the newly proposed method we do find such an oscillating pattern. This implies that under the other DGPs one cannot obtain a consistent estimator of p or q, see also Boswijk and Franses (2005) for a discussion of the consistency of estimates for the Bass regression and the Boswijk and Franses model. Only the newly proposed model generates diffusion series that cannot diverge from the underlying curve. With this DGP it can be possible to estimate the parameters consistently.

Another interesting finding is that for the Boswijk and Franses method (Figure 2.15) the deviations from $mF(t)$ seem to be mainly negative. This strange results is a consequence of the Boswijk and Franses model itself. Even if σ^2 is set to zero the Boswijk and Franses model does not exactly generate $mF(t)$. In the early part of the diffusion the generated curve tends to be below $mF(t)$, see Figure 2.2. The magnitude of this deviation depends on α, if $\alpha \to \infty$ the deviation disappears. This means that although the Boswijk and Franses model is designed to generalize the Bass model, it only matches it exactly for extreme parameter values.

In Appendix 2.A we also show generated diffusion curves according to survey measurement error, for the case where 500 respondents are surveyed (Figure 2.17). Note that the heteroscedastic error term for the newly proposed method can be seen as an normal approximation to the "survey noise". This figure is very similar to the one obtained when using the newly proposed method as the DGP, only in that case the variance of the error is larger.

Figure 2.2: Simulating diffusion curves using the BF method without noise.

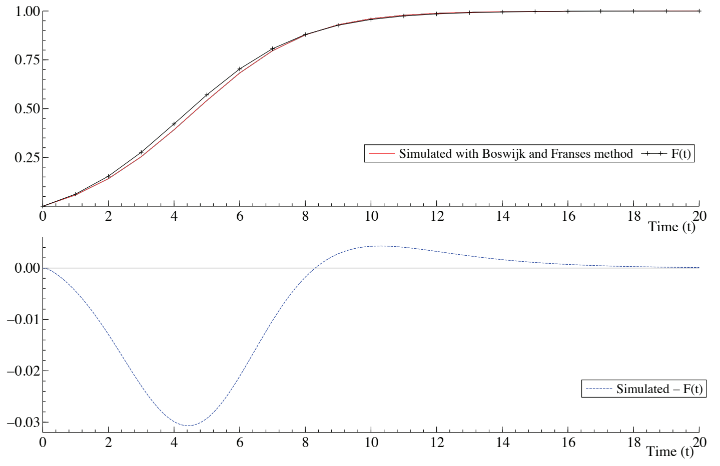


Table 2.1: Assumed properties of the error under different estimation methods

Assumption	Bass regression	Srinivasan and Mason	Boswijk and Franses	Newly Proposed Method
ε_t contains discretization error	Yes	No	Yes	No
Impact of ε_t on N_{t+1}	Yes	Yes	Yes	No
Impact of ε_t on $(N_{t+1} - N_t)$	Yes	No	Yes	Yes
ε_t as measurement error	No	No	No	Yes (on penetration)
Impact of ε_t on N_{t+k}, for $k \to \infty$	No	Yes	No	No
N_t fluctuates around $mF(t)$	No	No	No	Yes

Overview of theoretical comparison

We will come back to a number of these properties in the simulation experiments. For now we summarize the theoretical findings in Table 2.1. Overall, based on these theoretical properties the Srinivasan and Mason method is not a valid DGP, as the generated diffusion curves do not converge to m. The Bass regression is not a valid DGP as the shape of the generated curve differs permanently from the true curve $mF(t)$. The Boswijk and Franses method seems to have the same problem. For the newly proposed method these problems do not appear as the error can be seen as measurement error on the observed penetration.

In the sections below we consider how well the estimation methods are able to retrieve the parameters. First we consider the case without noise to be the DGP. Next, we consider each method as possible DGP.

2.3.2 Simulation performance without noise

In this section, we study the bias due to discretization error using a simple experiment. Given particular values of p, q, m and the time interval δ we calculate $N_i = mF(i\delta)$, for $i = 0, 1, \ldots, \frac{T}{\delta}$. We choose $p = 0.05$, $q = 0.5$, and $m = 1$. Furthermore we set $T = 20$. For this set of diffusion parameters, the diffusion process attains the ceiling value at about $t = 14$. Right-censoring is therefore no issue in our simulation experiment. Next, we apply all four methods to estimate the parameters. We experiment with different values of δ, which allows us to asses the extent to which a discretization bias is present. This complete process is deterministic so that only one iteration of this process is necessary. Note that using a smaller δ leads to more observations as we keep the time length of the time interval at $T = 20$.

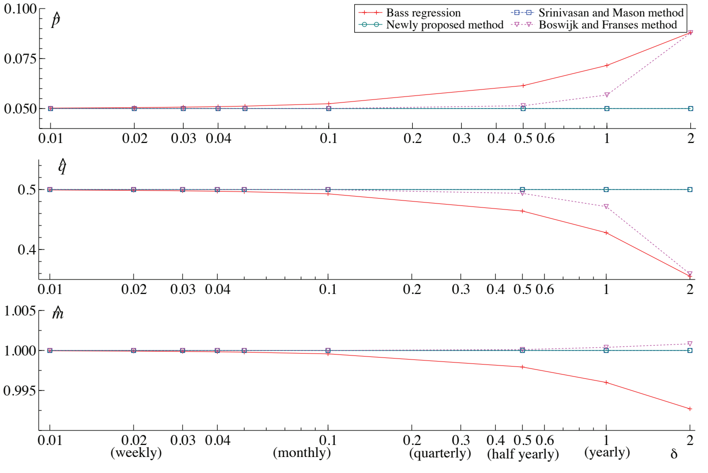
In Figure 2.3 we show the estimated parameter values as a function of δ. Note that we use a logarithmic scale for the horizontal axis (δ). The Srinivasan and Mason method and the new method both show perfect performance. Both methods recover the exact values for all parameters and all observational frequencies. However, the Bass regression and the Boswijk and Franses method do not perform equally well. Dependent on the time between observations, the discretization error can have a major influence on the results. The Bass regression performs worst for all δ. For this setting of the diffusion parameters the estimation results from the Bass regression and the Boswijk and Franses methods become biased when $\delta > 0.1$ (In other words, when data are at a periodicity lower than monthly). The most commonly used frequency for diffusion data is annual. The size of the bias these two estimation methods generate when used on annual data is very modest for m but large for p and q, with p being overestimated and q underestimated.

To further enlighten why Bass' and Boswijk and Franses' estimation methods do not perform well, Figure 2.4 shows the solution of $F(t)$ for the same parameter values as before. In the same figure, we plot the diffusion process that is generated by the (discrete time) difference equation that is implicitly assumed in the Bass regression equation for different values of δ. The difference equation is

$$N_i = \delta pm + (\delta(q - p) + 1)N_{i-1} - \delta \frac{q}{m} N_{i-1}^2, \text{ for } i = 1, 2, \ldots, \tag{2.15}$$

with $N_0 = 0$. The figure clearly shows that the shape of the diffusion process that is implied by the difference equation gets increasingly different from that of the underlying continuous time function $F(t)$, as δ becomes larger. In other words, for the true values of p, q and m the Bass regression does not match the true diffusion process. For large values of δ the diffusion curve, as generated by the Bass regression, underestimates the true curve. In order to fit the true curve for the Bass regression one should use values of p and q different than the true ones. This corresponds to an overestimation of p and an underestimation of q, exactly as shown in Figure 2.3 and found in Putsis (1996) and Non *et al.* (2003). The same reasoning applies to the Boswijk and Franses method. For

Figure 2.3: Parameter recovery without noise for different values of δ (on the horizontal axis).



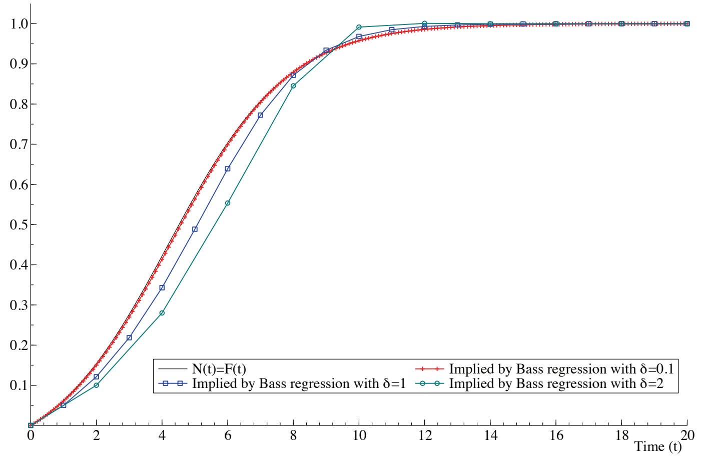
forecasting future adoption based on past adoption of the same product after the inflection point, this will have no impact as long as the forecasts are also generated with the Bass regression. However, the Bass model is also used as a tool for pre-launch forecasting based on diffusion parameters of analogous products and theory testing (e.g. on contagion).

2.3.3 Simulation performance with noise

The previous section shows that although discretization error can be a problem for the Bass regression and the Boswijk and Franses method, in the absence of noise all methods are unbiased if δ is small enough. In this section, we study how the methods perform in the presence of noise. Although a distinction between measurement error and true unexplained adoption may be possible, in practice one often does not know the true source of the error. In many cases, the true error will be a mix of measurement error and true unexplained adoption. Also, when we incorporate noise, there is not one "true" data generating process for diffusion models, as the underlying Bass diffusion model is deterministic. In the experiment, we will therefore use all four methods as a potential DGP. In this way we also investigate the robustness of the methods in case the DGP does not match the estimation method.

We consider each estimation method as a data generating process, that is, we see what happens if we would use equations (2.6), (2.7), (2.10), or (2.11) to generate data. To avoid the above-mentioned discretization error in the data generation, we generate the data with a small step size, that is, $\delta = 0.01$. In all cases below we generate data with heteroscedastic

Figure 2.4: $F(t)$ and generated solutions of the implied difference equation for various values of δ (using $p = 0.05$, $q = 0.5$, and $m = 1$).



errors, but estimate the parameters without a heteroscedasticity correction.[5] Main reason for the latter is that there are various choices for the heteroscedasticity structure and heteroscedasticity is often ignored in practice. Further, corrections for heteroscedasticity are expected to mainly impact the efficiency not a possible bias.

Bass regression noise as data generation process

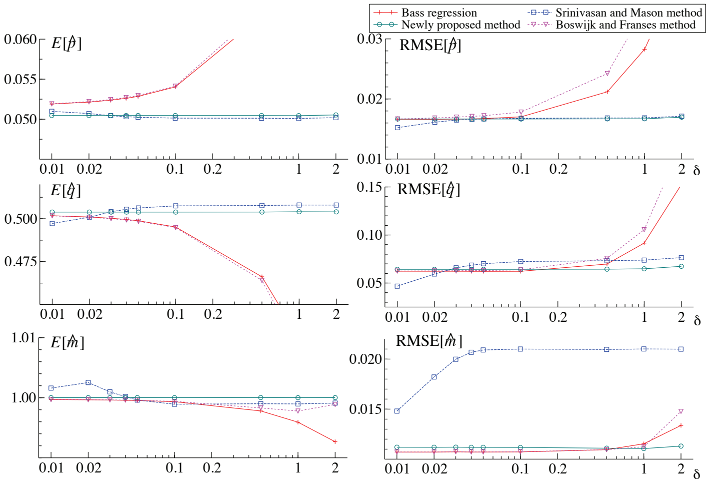
First, we look at the bias in the estimates for p, q, and m in case the Bass regression is the DGP. The left-hand column in Figure 2.5 shows the average value of the estimates over 10,000 replications for various values of δ.[6] It is interesting to see that under this DGP all estimation methods are biased for all values of δ. The bias in m is relatively small. However, for the Bass regression and the Boswijk and Franses method the bias in p and q is substantial even for small δ. In terms of bias, the Srinivasan and Mason method and the newly proposed method perform very similar. The bias for the new method is the same for all values of δ.

Next, the right-hand column in Figure 2.5 shows the Root Mean Squared Error [RMSE] for all estimation methods. Roughly speaking the RMSE gives the average distance between the estimated value and the true parameter value. First of all, the figure shows that for all methods the RMSE does not go to zero for small δ. This means that, under this

[5] For the Boswijk and Franses method this means that, for estimation, the error component $X_{i-1}\varepsilon_i$ in (2.10) is replaced with ε_i.

[6] The different δ's in this case refer to the observation/estimation frequency, not to the simulation where the δ used is fixed at 0.01.

Figure 2.5: Expected value and RMSE of estimators under DGP with Bass regression noise ($p = 0.05, q = 0.5, m = 1, \delta = 0.01, \sigma^2 = 0.05$), for varying levels of observation frequency (δ)



DGP, even if the diffusion is observed in continuous time, we cannot perfectly estimate the diffusion parameters.

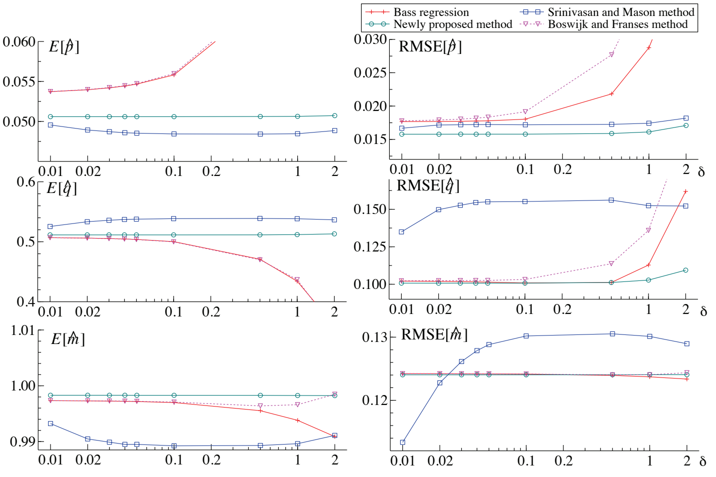
For p, the Bass regression and the Boswijk and Franses method have a high RMSE if $\delta > 0.1$, meaning that the bias we observed in the left-hand graphs of Figure 2.5 is not compensated by a small estimation uncertainty. The other two methods perform comparably on p. For q the situation is similar. The newly proposed and the Srinivasan and Mason methods work best and are comparable in performance. The Boswijk and Franses method and the Bass regression become very poor for $\delta \geq 1$. For m the Srinivasan and Mason method performs relatively bad for all values of δ, which corresponds to the assumed unit root behavior in this model. The Bass regression and the Boswijk and Franses method have relatively poor performance for $\delta = 2$, that is, observations every other year. The overall conclusion when the Bass regression model is used to generate the data is that the newly proposed method gives the best performance overall.

Srinivasan and Mason noise as data generating process

We repeat the same analysis as above for the case where the Srinivasan and Mason model is used to generate the data using a small step size. The bias and RMSE of the four estimation methods are shown in Figure 2.6.

In terms of bias we find the following. Again all methods are biased, where the bias for the newly proposed method tends to be small. The Srinivasan and Mason method has a relatively large bias for q. Again the discretization error causes the Bass regression

Figure 2.6: Expected value and RMSE of estimators under DGP with SM noise ($p = 0.05, q = 0.5, m = 1, \sigma^2 = 0.01$), for varying levels of observation frequency (δ)



method and the Boswijk and Franses method to have a large bias for large values of δ. However also for a very small value of δ we find a substantial bias for p with the Boswijk and Franses method.

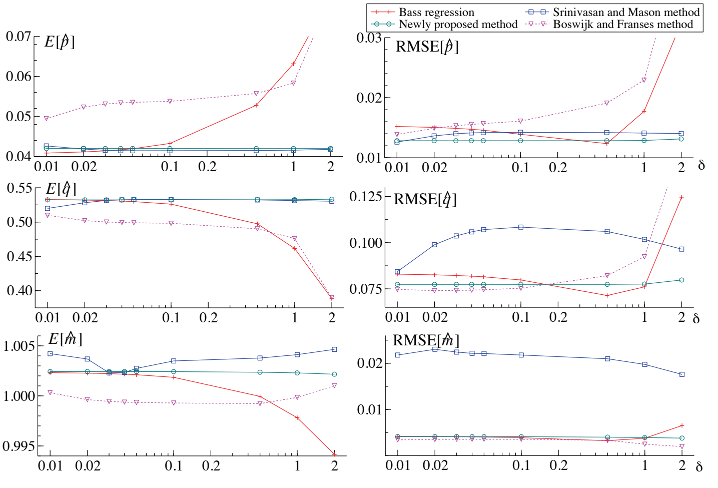
The RMSE is high for the Srinivasan and Mason method, even for relatively small δ. A surprising exception is the RMSE for m when δ is small. Here the Srinivasan and Mason method outperforms the others in RMSE. For the Bass regression and the Boswijk and Franses method we see good performance for small δ. In many cases the RMSE seems to be relatively independent of δ, this implies that observing the diffusion at a higher frequency does not add much information if the Srinivasan and Mason model is the DGP. For this DGP we again conclude that, overall, the newly proposed method performs best. It usually has the smallest bias and the smallest RMSE.

Boswijk and Franses model as data generating process

Figure 2.7 shows the estimation performance in case the DGP is the Boswijk and Franses model, where data is generated using a small step size. The figure shows that only the Boswijk and Franses model itself gives approximately unbiased estimates as long as δ is not too large. The other methods all have a bias. For p and q the bias is comparable for the Bass regression method, the Srinivasan and Mason method, and the newly proposed method up to $\delta = 0.1$. For larger δ the discretization error results in a larger bias for the Bass regression method.

For p, all methods give comparable RMSEs until the discretization error becomes too large. At that point, the Bass regression method and the Boswijk and Franses method

Figure 2.7: Expected value and RMSE of estimators under DGP with BF noise ($p = 0.05, q = 0.5, m = 1, \alpha = 5, \delta = 0.01, \sigma^2 = 1$), for varying levels of observation frequency (δ)

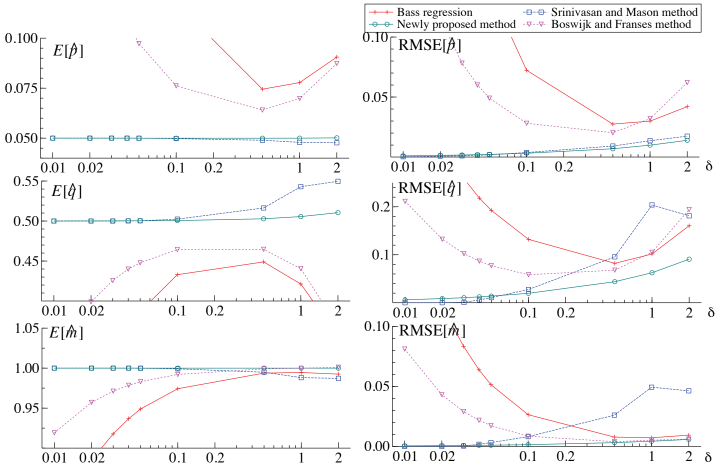


have a rather large RMSE. For every, δ the RMSE is smallest for the newly proposed method. For q, the Srinivasan and Mason method has a high RMSE, while the other methods are very comparable (for $\delta < 1$). For m the Srinivasan and Mason method also has a large RMSE for all δ, the RMSE is almost 4 times as high. In terms of bias one may prefer the Boswijk and Franses method in case the DGP is the Boswijk and Franses model and the data frequency is not too low. However, in terms of RMSE the newly proposed method again performs very well overall.

Noise according to the newly proposed method

If the newly proposed method is used as the DGP the error can be interpreted as pure measurement error on the penetration. Such measurement error corresponds to a setting where the data is obtained using surveys. Figure 2.8 gives the bias and RMSE for all estimation methods corresponding to this DGP. The results we get are remarkably different from the previous ones. The Bass regression method and the Boswijk and Franses method now also show a large bias when the time between observations is small. The bias for the Srinivasan and Mason method is small, and the bias for the newly proposed method is almost negligible. This holds for all three parameters. In terms of RMSE the Bass regression method and the Boswijk and Franses method do not perform better. The Srinivasan and Mason method and the newly proposed method are both consistent, in the sense that the RMSE converges to 0 for $\delta \to 0$. In terms of RMSE, the newly proposed method dominates all other methods. The main conclusion from this figure is that

Figure 2.8: Expected value and RMSE of estimators under DGP with measurement error (newly proposed method), $(p = 0.05, q = 0.5, m = 1, \sigma^2 = 0.01)$, for varying levels of observation frequency (δ)



the Bass regression and the Boswijk and Franses method are not robust to measurement error.

2.3.4 Summary of simulation results

We summarize the results across all DGPs in two tables. Table 2.2 summarizes the bias in percentages relative to the true parameter value. Table 2.3 summarizes the RMSE again in percentages relative to the true parameter value. In these tables, we only look at an extremely small time interval between observations, a time interval comparable with monthly data, and at yearly observations.

The ideal estimation method should work well under all forms of noise, that is, it should be robust. In case low-frequency data is used the method should not have a large discretization error. Table 2.2 shows that the Bass regression and Boswijk and Franses method always suffer from large discretization errors and if measurement error is involved even perform badly on high-frequency data. In terms of bias the Srinivasan and Mason method and the newly proposed method are very similar, where the Newly proposed method is slightly better.

If we include the RMSE in the comparison (see Table 2.3), the opinions on Bass regression and the Boswijk and Franses method do not change. In fact in terms of RMSE they even perform worse in some cases. The Srinivasan and Mason method and the newly

Table 2.2: Summary of bias in p, q, and m as a percentage of the true parameter value, bold values represent cases where the error structure of the estimation method matches the data generation process.

		Estimation method											
		p				q				m			
DGP	δ	Bass	SM	BF	New	Bass	SM	BF	New	Bass	SM	BF	New
No noise	0.01	0	0	0	0	0	0	0	0	0	0	0	0
Bass noise	0.01	**4**	2	4	1	**0**	0	0	1	**0**	0	0	0
SM noise	0.01	8	**-1**	8	1	1	**4**	1	2	0	**0**	0	0
BF noise	0.01	-18	-15	**-1**	-16	6	3	**1**	6	0	0	**0**	0
New noise	0.01	1525	0	465	**0**	-82	0	-35	**0**	-18	0	-8	**0**
No noise	0.1	5	0	0	0	-1	0	0	0	0	0	0	0
Bass noise	0.1	**8**	0	8	1	**-1**	1	-1	1	**0**	0	0	0
SM noise	0.1	12	**-3**	12	1	0	**7**	0	2	0	**-1**	0	0
BF noise	0.1	-13	-17	**8**	-16	5	6	**-1**	6	0	0	**0**	0
New noise	0.1	135	0	52	**0**	-13	1	-7	**0**	-3	0	-1	**0**
No noise	1	43	0	14	0	-14	0	-6	0	0	0	0	0
Bass noise	1	**45**	0	50	1	**-14**	2	-15	1	**0**	0	0	0
SM noise	1	48	**-3**	48	1	-13	**7**	-13	2	-1	**-1**	0	0
BF noise	1	26	-17	**17**	-16	-8	6	**-6**	6	0	0	**0**	0
New noise	1	56	-4	39	**0**	-16	8	-12	**1**	-1	-1	0	**0**

proposed method do differ on RMSE, where the newly proposed method usually performs best.

To conclude this section we give some important conclusions that follow from the simulations. First, for low-frequency data (quarterly, annual, etc.) the Bass regression and Boswijk and Franses method give biased results, especially for the p and q parameters.

Table 2.3: Summary of RMSE in p, q, and m as a percentage of the true parameter value, bold values represent cases where the error structure of the estimation method matches the data generation process.

		Estimation method											
		p				q				m			
DGP	δ	Bass	SM	BF	New	Bass	SM	BF	New	Bass	SM	BF	New
Bass noise	0.01	**32**	30	33	33	**12**	9	12	13	**1**	1	1	1
SM noise	0.01	35	**33**	35	31	20	**25**	20	20	12	**11**	12	12
BF noise	0.01	30	25	**28**	26	16	16	**15**	15	0	2	**0**	0
New noise	0.01	1549	0	469	**2**	101	0	42	**1**	18	0	8	**0**
Bass noise	0.1	**33**	33	35	33	**12**	14	13	13	**1**	2	1	1
SM noise	0.1	36	**34**	38	31	20	**29**	20	20	12	**13**	12	12
BF noise	0.1	28	28	**33**	26	16	21	**15**	15	0	2	**0**	0
New noise	0.1	144	7	56	**6**	26	5	11	**4**	3	1	1	**0**
Bass noise	1	**56**	33	70	33	**18**	15	21	13	**1**	2	1	1
SM noise	1	57	**34**	77	32	22	**30**	27	20	12	**13**	12	12
BF noise	1	36	28	**48**	26	15	20	**19**	15	0	2	**0**	0
New noise	1	60	27	64	**20**	20	40	21	**13**	1	5	1	**0**

Table 2.4: Empirical cases represented in a 2-by-2 framework

	Data with no or limited measurement error	Data with measurement error
Monthly Data	Sales of Toyota Prius (Los Angeles, San Francisco and New York) (See Figure 2.9)	Pharmaceuticals (Crestor and Caduet) (See Figure 2.11)
Annual Data	Sales of Toyota Prius (Los Angeles, San Francisco and New York) (See Figure 2.10)	Cumulative Adoption Euromonitor (CD-Canada, DVD-Canada, CD-UK and DVD-UK) (See Figure 2.12)

Increasing the frequency does not lead to unbiased results for all DGP's. If the proposed model is the DGP the bias and RMSE for the Bass regression and Boswijk and Franses method also increase when the observation frequency approaches zero. The reason for this is that the measurement error is misinterpreted as unexplained actual adoption. The Srinivasan and Mason method can have a high RMSE. This method is not really suitable as a DGP. Often the estimates are still unbiased, but the RMSE shows that there is much uncertainty in the estimates when the Srinivasan and Mason method is the DGP. Finally, the proposed method proves to be most robust to the specification of the error, that is, the bias and RMSE are in most cases smallest for the proposed method.

2.4 Empirical cases

In this section, we examine the estimation and forecasting performance of the four estimation methods on empirical data. In the simulation study, we showed that the origin of noise and the observational frequency can both result in bias or inefficiency in the parameter estimates. Therefore, in this section we build the cases around these two dimensions. In particular, the empirical cases can be represented in a 2-by-2 overview matrix, see Table 2.4. The dimensions of this matrix are annual versus monthly data and whether there is measurement error or not.

For confidentiality reasons we have to hide some details on the data. Therefore, in the figures to come below we will not show the scale of the adoption axis and in the estimation results we will show estimates of m relative to one of the models.

The case of monthly data without measurement error, is based on vehicle registration data for Toyota Prius, in three large cities in the US: Los Angeles, San Francisco and New York. The data is obtained from POLK, and runs from July 2000 until December 2008 (Figure 2.9)[7]. Replacements and measurement error are absent in this dataset.

[7] For confidentiality reasons we cannot reveal the scale of the data. The figures therefore do not show numbers on the vertical axis

Fluctuations in the sales can thus be classified as actual adoptions. Theoretically, this is in line with the noise of the Bass regression method and the Boswijk and Franses method.

For the cell with annual data without measurement error we use the same data, but aggregated over the calendar year. The data is given in Figure 2.10. The figure of course shows that this data has fewer fluctuations compared to the monthly data.

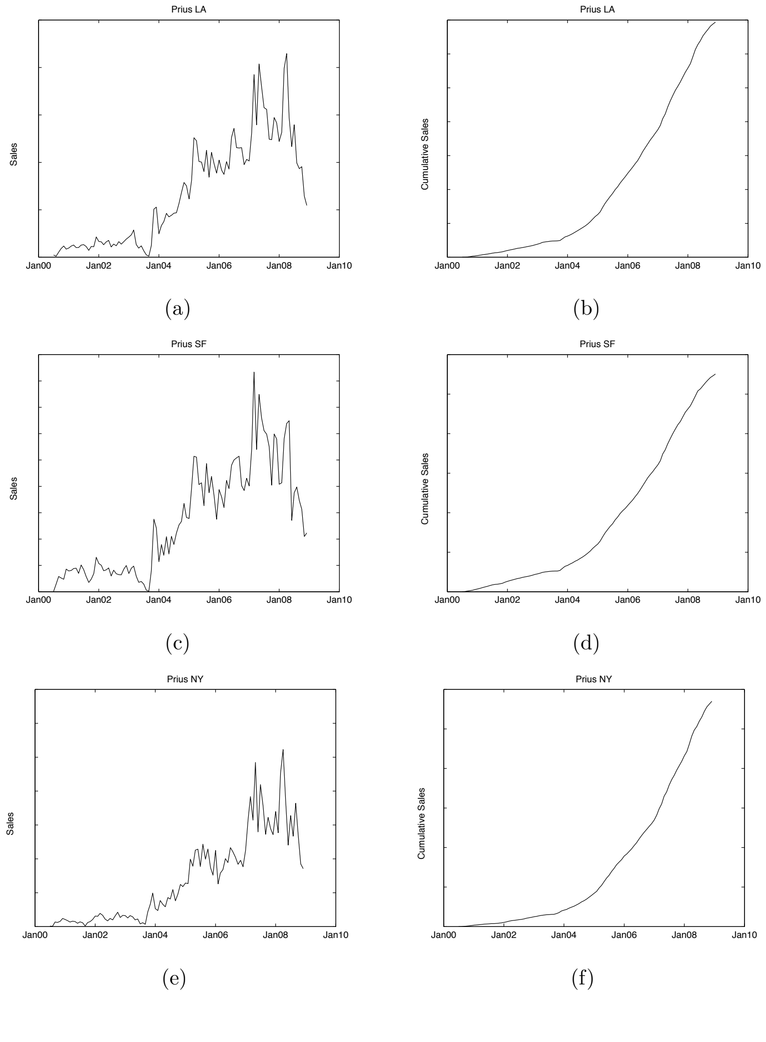
As an example of monthly data with measurement error, we use pharmaceutical adoption data based on a panel of physicians. The data is obtained from IMS Health. In this panel, a physician is classified as an adopter as soon as he/she made the first prescription of the drug. The panel consists of around 4000 physicians in the US. Although this is a large number, it is only a sample from the entire population of physicians. The observed diffusion process is therefore a noisy measurement of the true, country-wide adoption. We study the adoption of two medicines: the anti-cholesterol medicine Crestor, and the combination drug Caduet. For Crestor we look at the first two-and-a-half years after market introduction, that is, August 2003 until January 2006.[8] For Caduet, a combination drug for the treatment of cholesterol and high blood pressure, we take the sample from the introduction in April 2004 until July 2008. The data for both drugs is shown in Figure 2.11.

Annual diffusion data with measurement error is obtained from Euromonitor. Euromonitor supplies cumulative adoption data. The data is obtained through a survey-like method and therefore has potentially large measurement error. Changes in the procedure to correct for population coverage may results in additional noise. In some extreme cases these measurement errors can even lead to negative adoptions in some years. In this chapter we look at the adoption of DVD and CD in Canada and the UK. The data is presented in Figure 2.12. Note that the measurement error in the Euromonitor data is likely to be more severe then in the pharmaceutical data. The Euromonitor data is obtained using an annual cross-sectional survey with different respondents each year, whereas the pharmaceutical data is based on a (large) fixed panel.

The diffusion processes we study in this empirical section have not yet reached the ceiling level, i.e. we face a right-censoring problem. This especially holds for Toyota Prius data, as Figures 2.9 and 2.10 show. This right censoring may lead to estimation problems for some series and also complicates the comparison of the results to those in the simulation section. In a right-censored diffusion series the final observation has a relatively large impact (see van den Bulte and Lilien (1997) and Bemmaor and Lee (2002) for more discussion on right censoring). If, for some reason, the last observation is large compared to the underlying true diffusion curve, the diffusion curve is likely to be overestimated. As the estimation methods mainly differ in how they treat noise, the right censoring may have a different impact for each estimation method. Although this a downside of using

[8]In this period the cumulative adoption reaches its potential, that is, in the periods afterwards there are only a limited number of additional prescribers.

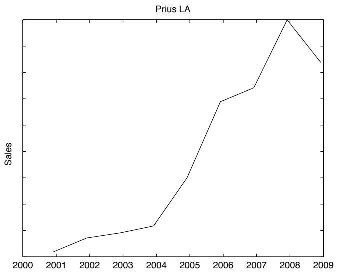
Figure 2.9: Monthly registration data (left) and cumulative registration data (right) of the Toyota Prius in Los Angeles, San Francisco and New York. The source of data is POLK.



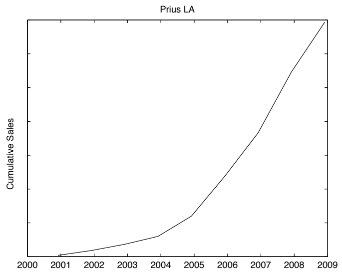
empirical data, we believe that a comparison of the estimation methods in the 2x2 setup is still very valuable.

In Table 2.5, we present the estimation and forecasting results. We structure the results according to the 2-by-2 framework. For each estimation method, we give the estimates of the diffusion parameters and their standard error. The forecasting performance is represented in the root mean squared prediction error [RMSPE] of the sales/adoption.

Figure 2.10: Annual registration data (left) and cumulative registration data (right) of the Toyota Prius in Los Angeles, San Francisco and New York. The source of the data is POLK



(a)



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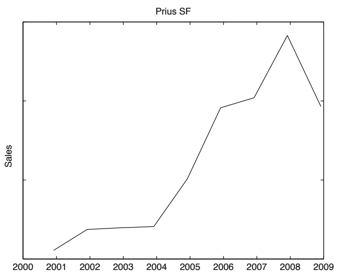

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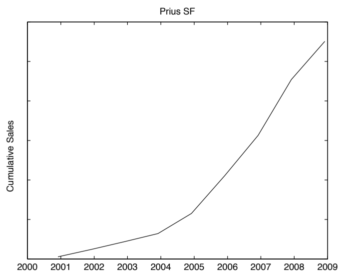
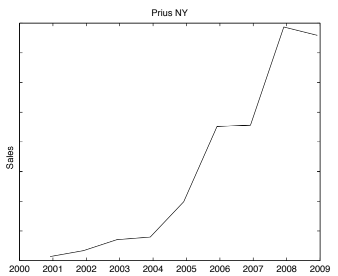
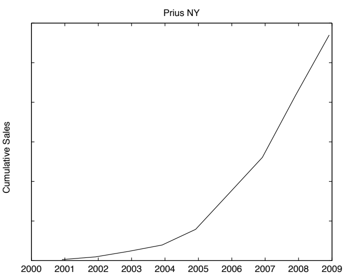


(d)



(e)

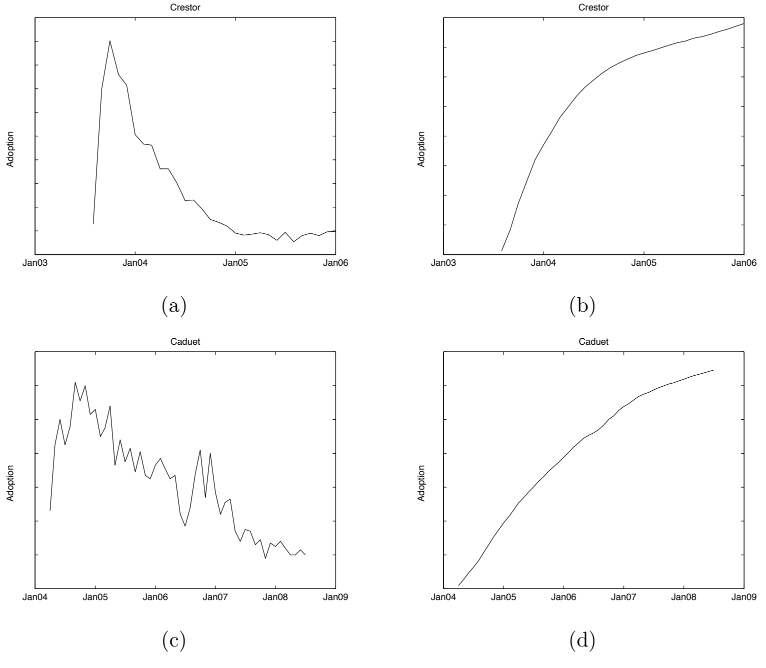


(f)

The RMSPE is based on one-step-ahead forecasts of every period after the inflection point. The number of forecasts taken into account differs across the diffusion series. In the table we mention the number of one-step-ahead forecasts used.

As discussed earlier we are not allowed to show all actual parameter estimates. We transform the estimates for m for each empirical case such that it is 1.0 for the Bass regression method. The estimates of the other methods, and all standard errors, are

Figure 2.11: Monthly adoption data (left) and cumulative adoption data (right) of the medicines Crestor and Caduet by US physicians. The source of the data is IMS Health.



transformed in the same way. Note that this transformation allows for an easy comparison of the estimates across estimation methods. To facilitate a comparison across the monthly and the annual Prius data, we transform the annual estimates for the Prius data by the same number as their monthly equivalent.

For the RMSPE we use a similar transformation, where we set the RMSPE of the Bass regression at 1.0 and scale the RMSPE of the other estimation methods by the same number. In this case we also set the RMSPE for annual Prius data to 1.0, using the RMSPE of the Bass regression.

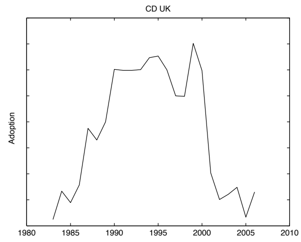
Table 2.5 shows that for the same dataset the results of the four estimation methods can differ substantially. Of course one cannot say which method is superior as there is no objective measure of the true underlying diffusion curve. The best one can do is a comparison of the face validity of the results across the estimation methods. However, we can confirm some conclusions from the simulation study.

Table 2.5: Estimation and forecasting results for the empirical cases represented in a 2-by-2 framework

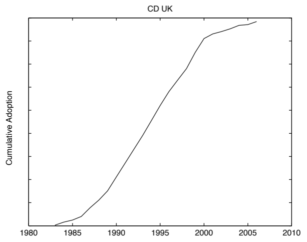
	Data with no or limited measurement error								Data with measurement error							
			p	q	m	α	♯ steps	RMSPE			p	q	m	α	♯ steps	RMSPE
Monthly Data	Prius LA	Bass	0.0012	0.766	1.000[a]		15	1.000[a]	Crestor	Bass	0.9355	1.185	1.000[a]		12	1.000[a]
		(StErr)	(0.0047)	(0.040)	(0.033)					(StErr)	(0.1129)	(0.407)	(0.029)			
		SM	0.0037	0.759	1.004		15	1.019		SM	0.8451	2.611	0.901		12	0.793
		(StErr)	(0.0001)	(0.005)	(0.006)					(StErr)	(0.0095)	(0.051)	(0.006)			
		BF	0.0122	0.478	1.189	3.8529	15	0.966		BF	1.6866	-1.687	1.267	4.1669	12	0.271
		(StErr)	(0.0069)	(0.132)	(0.507)	(0.9663)				(StErr)	(18.5150)	(8.826)	(0.528)	(2.1886)		
		New	0.0046	0.687	1.117		15	1.059		New	1.0112	1.059	0.997		12	0.643
		(StErr)	(0.0001)	(0.010)	(0.019)					(StErr)	(0.0608)	(0.229)	(0.012)			
	Prius SF	Bass	-0.0007	0.731	1.000[a]		15	1.000[a]	Caduet	Bass	0.3102	0.465	1.000[a]		24	1.000[a]
		(StErr)	(0.0055)	(0.042)	(0.033)					(StErr)	(0.0183)	(0.081)	(0.025)			
		SM	0.0047	0.749	0.976		15	0.995		SM	0.3079	0.484	1.000		24	0.991
		(StErr)	(0.0001)	(0.005)	(0.005)					(StErr)	(0.0013)	(0.006)	(0.003)			
		BF	0.0102	0.330	1.657	6.9389	15	01.125		BF	0.3557	0.303	1,022	7.2737	24	0.816
		(StErr)	(0.0096)	(0.098)	(1.315)	(1.8537)				(StErr)	(0.0349)	(0.112)	(0.025)	(1.0011)		
		New	0.0072	0.614	1.174		15	1.297		New	0.3144	0.414	1.020		24	01.075
		(StErr)	(0.0002)	(0.012)	(0.029)					(StErr)	(0.0042)	(0.031)	(0.011)			
	Prius NY	Bass	0.0014	0.755	1.000[a]		15	1.000[a]								
		(StErr)	(0.0046)	(0.047)	(0.051)											
		SM	0.0028	0.747	1.011		15	1.025								
		(StErr)	(0.0001)	(0.006)	(0.009)											
		BF	0.0038	0.402	2,638	6.1508	15	1.084								
		(StErr)	(0.0061)	(0.104)	(4.146)	(1.7187)										
		New	0.0033	0.680	1.150		15	1.006								
		(StErr)	(0.0001)	(0.010)	(0.029)											
Annual Data	Prius LA	Bass	0.0084	0.870	0.878		1	1.000[a]	CD Canada	Bass	0.0097	0.336	1.000[a]		12	1.000[a]
		(StErr)	(0.0076)	(0.075)	(0.050)					(StErr)	(0.0071)	(0.039)	(0.025)			
		SM	0.0030	0.721	1.046		1	27.300		SM	0.0040	0.339	1.018		12	1.011
		(StErr)	(0.0009)	(0.064)	(0.077)					(StErr)	(0.0019)	(0.047)	(0.104)			
		BF	0.0107	0.763	0.983	1.0585	1	47.412		BF	0.0032	0.394	1.018	1.3354	12	3.102
		(StErr)	(0.0090)	(0.209)	(0.317)	(0.7285)				(StErr)	(0.0015)	(0.148)	(0.022)	(0.5079)		
		New	0.0032	0.708	1.062		1	15.207		New	0.0038	0.343	1.012		12	0.864
		(StErr)	(0.0003)	(0.025)	(0.038)					(StErr)	(0.0004)	(0.011)	(0.009)			
	Prius SF	Bass	0.0062	0.829	0.883		1	1.000[a]	DVD Canada	Bass	0.0114	0.788	1.000[a]		5	1.000[a]
		(StErr)	(0.0094)	(0.080)	(0.049)					(StErr)	(0.0069)	(0.050)	(0.012)			
		SM	0.0034	0.741	0.987		1	1.227		SM	0.0004	0.801	1.025		5	0.324
		(StErr)	(0.0013)	(0.080)	(0.082)					(StErr)	(0.0001)	(0.031)	(0.031)			
		BF	0.0238	0.702	0.982	0.7002	1	2.189		BF	0.0006	0.832	1.022	1.4424	5	1.442
		(StErr)	(0.0298)	(0.326)	(0.518)	(0.6707)				(StErr)	(0.0002)	(0.085)	(0.007)	(0.3889)		
		New	0.0049	0.650	1.087		1	1.275		New	0.0004	0.812	1.019		5	0.220
		(StErr)	(0.0006)	(0.037)	(0.060)					(StErr)	(0.0000)	(0.012)	(0.004)			
	Prius NY	Bass	0.0075	0.866	0.874		1	1.000[a]	CD UK	Bass	0.0144	0.277	1.000[a]		12	1.000[a]
		(StErr)	(0.0086)	(0.104)	(0.097)					(StErr)	(0.0043)	(0.022)	(0.017)			
		SM	0.0024	0.684	1.117		1	2.107		SM	0.0097	0.275	1.029		12	0.950
		(StErr)	(0.0010)	(0.090)	(0.156)					(StErr)	(0.0020)	(0.024)	(0.057)			
		BF	0.0039	0.744	0.987	1.6572	1	0.741		BF	0.0120	0.274	1.009	1.1384	12	1.047
		(StErr)	(0.0015)	(0.094)	(0.199)	(0.5518)				(StErr)	(0.0027)	(0.034)	(0.012)	(0.2889)		
		New	0.0023	0.688	1.112		1	0.466		New	0.0087	0.287	1.014		12	1.068
		(StErr)	(0.0002)	(0.033)	(0.074)					(StErr)	(0.0005)	(0.009)	(0.009)			
									DVD UK	Bass	0.0177	0.889	1.000[a]		5	1.000[a]
										(StErr)	(0.0110)	(0.089)	(0.019)			
										SM	0.0001	0.944	1.028		5	0.195
										(StErr)	(0.0000)	(0.045)	(0.039)			
										BF	-0.0014	1.277	1.019	1.8338	5	3.765
										(StErr)	(0.0008)	(0.374)	(0.014)	(0.9958)		
										New	0.0001	0.971	1.014		5	0.255
										(StErr)	(0.0000)	(0.031)	(0.007)			

a = Scaled to 1 for confidentiality reasons

Figure 2.12: Annual adoption data (left) and cumulative adoption data (right) of DVDs and CDs in the UK and US. The source of the data is Euromonitor.



(a)



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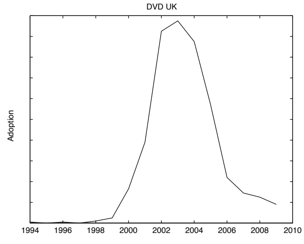

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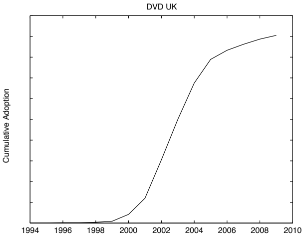

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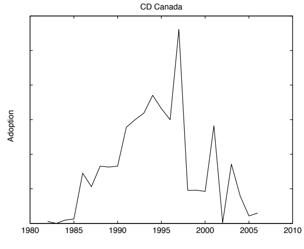

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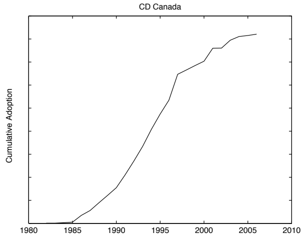
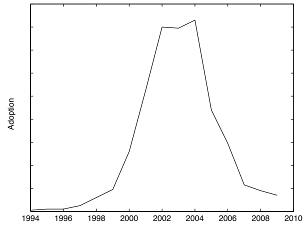
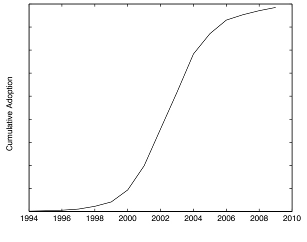


(f)



(g)



(h)

Monthly data without measurement error

For the case of monthly data without measurement error, the parameter estimates of the Srinivasan and Mason method and the Bass regression method are similar, where the Srinivasan and Mason method has smaller standard errors. However, the Bass regression yields a negative p estimate for San Francisco. Compared to the Srinivasan and Mason method and the Bass regression, the parameter estimates of the newly proposed method are slightly different. In particular, the market potential of the newly proposed method is larger. The Boswijk and Franses method has more extreme and quite different parameter estimates compared to the other methods.

For forecasting, we find that the Bass regression method and Boswijk and Franses method often have the lowest RMSPE. In general, the predictive performance in this scenario is similar across all methods.

Annual data without measurement error

For annual data without measurement error, the Srinivasan and Mason method and the newly proposed method yield similar parameter estimates. However, the new method has smaller standard errors and a smaller RMSPE, except for San Francisco in which case the RMSPE is slightly larger. The parameter estimates of the Bass regression model differ from that of the Srinivasan and Mason method and the new method, but the Bass regression method still performs well on short-term forecasting. The parameter estimates of the Boswijk and Franses method are more in line with the other models compared to the case with monthly data. Further, we notice that α is much smaller than for monthly data.

Monthly data with measurement error

For monthly data with measurement error, the newly proposed method and the Bass regression method have similar parameter estimates. Also, the Srinivasan and Mason method has similar estimates for the adoption of Caduet, but has more extreme estimates for the case of Crestor. The Boswijk and Franses method has trouble converging to plausible estimates, even with very specific starting values the “optimum” corresponds to a negative imitation parameter for Crestor. Despite the problems with the parameter estimates, the Boswijk and Franses model performs best on short-term forecasting. Although, similar to the cases with monthly data and no measurement error, differences in RMSPE are not very large.

Annual data with measurement error

For annual data with measurement error, the most typical case in past diffusion research, the parameter estimates of Srinivasan and Mason method and the newly proposed method are again comparable. Also in this case, the new method has smaller standard errors. The Bass regression method often has a larger estimate of p and a smaller estimate of q. A similar pattern in p and q is found for the Boswijk and Franses method, but there are again a few more extreme parameter estimates, especially for the adoption of DVDs in the UK. The value of α in the Boswijk and Franses method is similar to the yearly equivalent without measurement error.

The RMSPE in case of annual data with measurement error is often much larger for the Boswijk and Franses method. For the new method the RMSPE is often the smallest or close to the smallest.

Discussion of empirical results

For all empirical cases we do not know the true parameters to which we can compare the estimates. However, we can compare across methods and for Prius case we can also compare the results for the monthly data with those for annual data. Parameter estimates in both cases should be similar, as the interpretation of the parameters is independent of the observational frequency.

We conclude that, due to the discretization error, the Bass regression on annual data yields biased estimates. This is confirmed by the fact that the Srinivasan and Mason method and the new method find similar parameter estimates across the two frequencies. In line with the theoretical arguments, we find that the Srinivasan and Mason method yields larger standard errors, which is due to the assumed unit root in the error specification.

The conclusions from the monthly data without measurement error are a bit more difficult. Theory suggests that all methods should find similar results. The fact that this does not happen is probably due to the right censoring of the data. As we only have a limited number of periods after the inflection point, the noise around these data points seems to have a large impact on the estimates.

The monthly data with measurement error suffers less from right censoring, so we expect to find more similar estimates across the different methods. This is exactly what we find for the Bass regression method, the new method, and to a lesser extent the Srinivasan and Mason method. Regarding short-term forecasting these three methods also perform similarly. Note however, that the shape of the diffusion curves for the medicine data is far from the normal shape expected for the mixed-influence model.

Despite the discretization error when using annual data, the Bass regression method is among the best methods to use for forecasting if noise is likely to come only from actual shocks, that is, no measurement error. Only the newly proposed method is never substantially outperformed, and in one case is even better than the Bass regression model. In case of measurement error, the new method seems to outperform the other models in most cases.

The Boswijk and Franses method often yields very different results in all cases. Also, this model often has trouble converging to a plausible solution; starting values turned out to be crucial. In particular, it seems that for finding parameter estimates with monthly data, the Boswijk and Franses method is not suited. For annual data, the estimates are more in line with the other methods, although there are still some different outcomes and larger standard deviations. The differences are largest in cases with measurement error, this is in line with the simulation results. Moreover, in the simulation study we found that the Boswijk and Franses method had a large bias and RMSE not only for low-frequency data, but also for high-frequency data. In general, we can conclude that if the true noise is different from that specified in the Boswijk and Franses method, this method is not advisable. However, despite all issues and unlikely parameter estimates the Boswijk and Franses method seems to work relatively well in terms of forecasting.

In these empirical cases, we found that the source of the noise and the observational frequency are important factors for the performance of an estimation method. If noise comes from actual shocks with high-frequency data the Bass regression method is the best solution. The performance of the Srinivasan and Mason method is more difficult to interpret, the method sometimes performs well, but due to the unit root there also some less likely outcomes. The new method seems to be the best solution given data with measurement error. Also in other scenarios it is never very different from the best model. This especially holds for the forecasting performance. This is in line with the simulation study, which showed that the newly proposed method is most robust to different sources of noise.

2.5 Discussion

Although the Bass diffusion model has been researched extensively, this chapter adds new and important insights to the diffusion literature by thoroughly addressing the performance of estimation methods. We consider three existing methods: the original regression based approach suggested by Bass (1969), the Srinivasan and Mason (1986) NLS method, and the Boswijk and Franses (2005) aproach. We also propose a new estimation method, which follows directly from the closed-form solution of the Bass model, as does the Srinivasan and Mason method, but handles the errors differently. We show that each estimation method has its own accompanying interpretation of the error term. More importantly, the choice of the estimation method, and thereby the accompanying error process, has major consequences.

By using theory and a simulation study we showed the performance of the estimation techniques. First, we confirmed the discretization bias in case of low-frequency data for the Bass regression method and the Boswijk and Franses method. Additionally, we showed that increasing observational frequency has a negative effect as well. More specifically, this occurs when there is measurement error on penetration or cumulative sales data.

This has practical implications as these measurement errors will naturally occur when obtaining data through a survey, which is a common data source in diffusion research. The proposed method turns out to be most robust to different types of error. Also the interpretation of its error structure is similar to the case where data is obtained through a survey.

Second, we showed the performance of the models underlying the different estimations methods as DGP. In the diffusion literature there are several examples where simulations are used to test diffusion models or features of diffusion models. However, the literature lacks research of the quality of different methods as DGP. Although the method of Srinivasan and Mason is used often as DGP, we show that, due to an implicit unit root assumption in this method, the diffusion curve can deviate from the underlying curve unlimitedly. This unit root leads to increased estimation uncertainty, especially of the imitation parameter. If the Bass regression or Boswijk and Franses method is used as DGP caution is needed as well. In this case, the error structure is not so much the problem, but the simulation frequency is. In particular, if the frequency is low, the simulated curve is below the true underlying diffusion curve.

We also compared the estimation methods using empirical cases. Despite the fact that the estimates based on this data suffer from some well-known right censoring issues, we confirmed the theory and simulation results with the empirical data. Additionally, we found that the Boswijk and Franses method suffers from convergence problems. We find that the Bass regression method works well if the noise is likely to come only from actual adoption/sales shocks.

All these findings have implications for diffusion literature and practice. Theories are often build on estimated diffusion parameters. Systematic bias, as in the Bass regression model and Boswijk and Franses model, can lead to invalid theoretical inference. Increased estimation uncertainty, as in the Srinivasan and Mason method, can lead to weaker theoretical inference. The results of this chapter also have implications for forecasting. For short-term forecasting after the inflection point the empirical results show that if one knowns the origin of the noise, one should use the corresponding estimation method for forecasting. However, if one is uncertain of the source of the noise the new method is the safest option. For analogical forecasting models, there are mainly issues if one uses data on products with different observational frequencies. If data is available at different frequencies this can bias the predicted diffusion curve of a new product. This bias can occur due to: (i) estimation of the available products at different frequencies; and (ii) predicting the new diffusion pattern with a different frequency as the estimated diffusion curves. The new method does not suffer from these biases as it is robust to different frequencies.

This chapter shows that researchers as well as managers need to be careful in selecting the estimation method of a diffusion model. In particular, the match between the chosen error structure and the true error, as well as the data frequency can influence outcomes of the estimation and simulation of diffusion models. In case one has only low-frequency data available or if one is unsure about the error structure, the proposed method is most robust and hence the safest option.

2.A Using the estimation methods as DGP

This appendix shows examples of simulated diffusion curves using the models underlying the different estimation methods as data generating process.

Figure 2.13: Simulated diffusion series using the Bass regression with heteroscedasticity ($\sigma^2 = 0.05$), with small step size in the DGP.

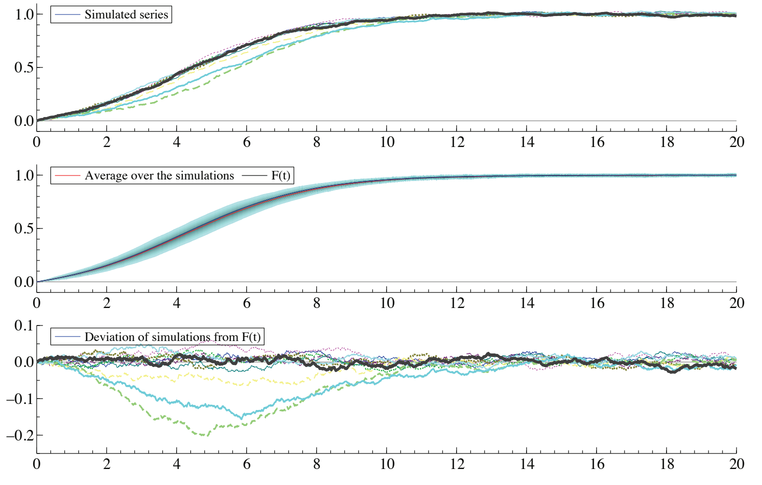


Figure 2.14: Simulated diffusion series with the SM approach, with heteroscedastic noise and $\sigma^2 = 0.01$

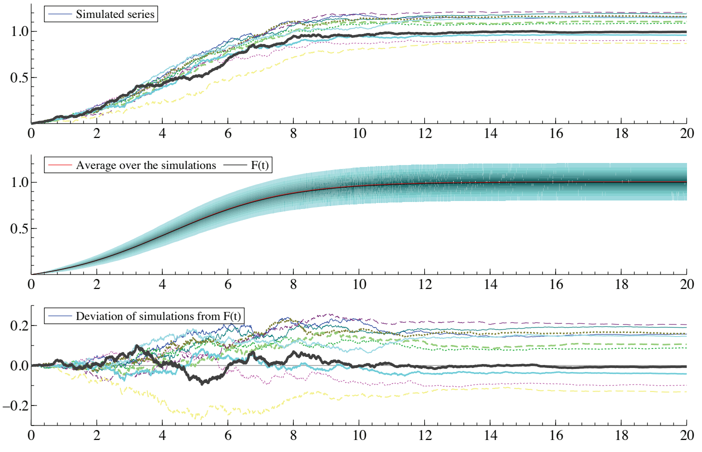


Figure 2.15: Simulating diffusion curves using the BF method with $\sigma^2 = 0.5$.

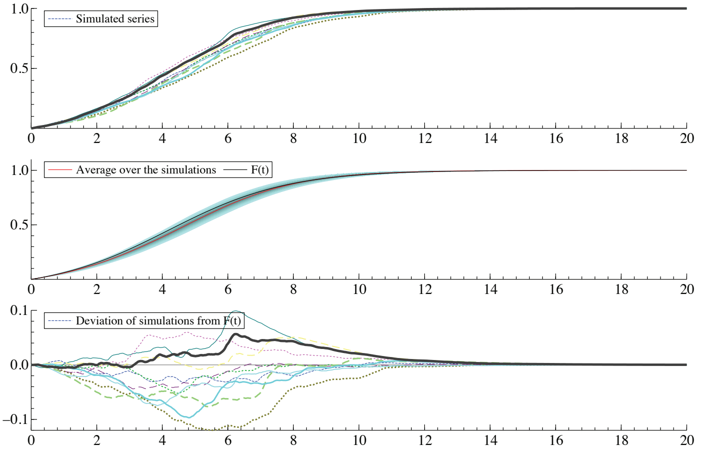


Figure 2.16: Simulated diffusion series with the new approach, with heteroskedasticity, observations at interval $\delta = 1$ and $\sigma^2 = 0.01$

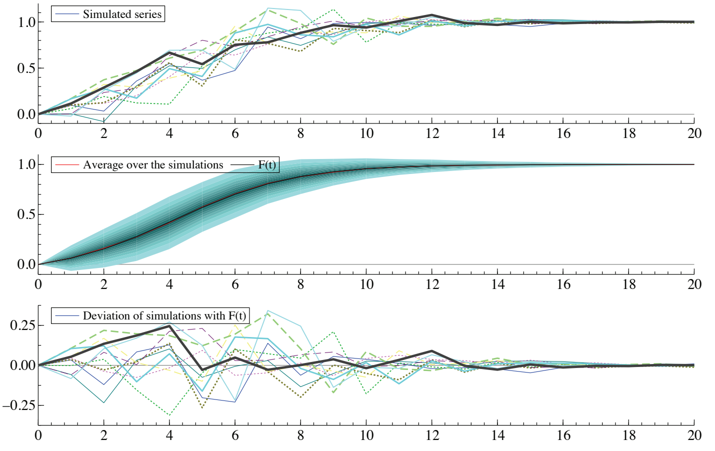
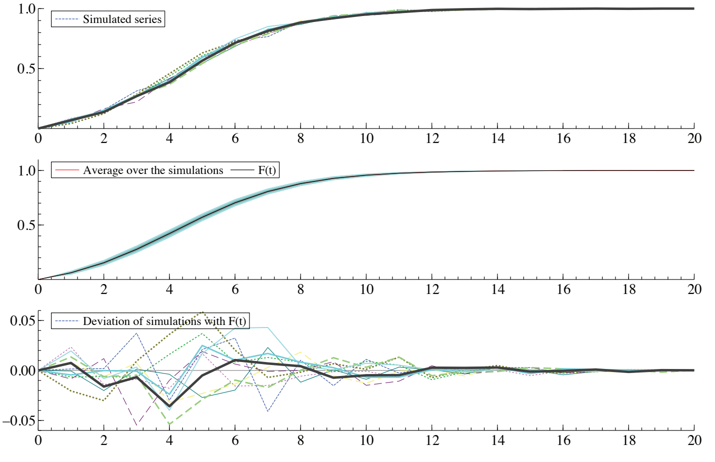


Figure 2.17: Simulated diffusion data under the assumption of an underlying survey with $S = 500$ respondents. Observations are plotted with $\delta = 1$.



Chapter 3

Estimating diffusion parameters on mixed-frequency data

Diffusion data is nowadays often available at the monthly or weekly frequency. Such high-frequency data can be very useful for academic diffusion research and applied research. Advantages of high-frequency data are: (i) a reduction of discretization errors and the prevention of ill-conditioning; (ii) that one can consider more complex diffusion models; and (iii) improvement of short term forecasting. However, high-frequency diffusion data is often not available from the start of the diffusion process. In some cases, at the start data is collected at a low frequency. In practice one therefore usually has observations at mixed *frequencies.*

In this chapter we discuss model-based methods that deal with such mixed-frequency data. We adapt a number of existing standard estimation techniques for diffusion models to make them useful for mixed-frequency data. We find that for some estimation methods it is enough to only correct for the different time intervals. In other situations, however, simulation methods are needed to appropriately use mixed-frequency data.

We compare the performance of the methods to several alternative approaches, such as, (i) aggregating data to the lowest available frequency; (ii) ignoring the initial low-frequency data; and (iii) using simple linear interpolation to get complete high-frequency data. The first alternative leads to a potentially large loss of information and therefore an efficiency loss. We show that the other two lead to biased estimates. The most appropriate model-based method depends on the chosen estimation procedure.

3.1 Introduction

Since the seminal paper of Bass (1969), published over forty years ago, the diffusion literature has grown at a rapid pace. At the same time data availability increased. Instead of having data on only a single product in one market, we now often have data on a large number of products for countries all over the world. Next to this richness in diffusion series, diffusion data has also become more detailed. An interesting development for academic researchers and practitioners, is the availability of high-frequency diffusion data. With the term "high frequency", we mean observations at a monthly or weekly interval. High-frequency diffusion data becomes more and more readily available. Durable goods companies often collect their sales data at a monthly frequency. For services, for example banking, the observational frequency is even higher. High-frequency data allows us to get better insights in the diffusion process. However, the actual use of such data comes with some challenges. In this chapter we deal with some of these challenges.

There is already some literature that implicitly or even explicitly discusses the importance of high-frequency data. High-frequency data is important in three aspects. First, current literature found that for the estimation of diffusion parameters it is important to have a sufficient number of data points. Many current applications and studies of diffusion models rely on annual data. In such studies it is clear that only a rather limited number of data points can be used. This often results in discretization errors (Putsis, 1996; Non *et al.*, 2003) and in the well-known ill-conditioning problem (van den Bulte and Lilien, 1997; Bemmaor and Lee, 2002). In turn this gives biased parameter estimates or very large standard errors. High-frequency data can help to solve both problems. Second, standard diffusion models are frequently used as a starting point for more complex models to answer a wide variety of questions (for a recent overview see Peres *et al.*, 2010). Detailed data may prove especially useful in such studies. For example, cross-country influences may be easier to identify with monthly data than with annual data. Finally, practitioners most often use diffusion models for short-term forecasting of sales. The term "short term" already suggests that annual data is perhaps not suitable, but that quarterly or monthly data is preferred.

Although using high-frequency diffusion data has obvious advantages and such data is nowadays often available, one cannot immediately use this data. There are some practical and modeling challenges to overcome. In contrast to the importance of high-frequency diffusion data less has been written on these challenges. One challenge is that high-frequency data is usually not collected immediately from the moment of product introduction. Oftentimes, a company will start to collect (sales) data at a high-frequency at the moment the product becomes successful. Furthermore, the collection of high-frequency diffusion data has only been done on a regular basis for the last decade. The result is that at the start of the diffusion process there is either no data, or only data at a lower frequency. In practice, one will therefore need to deal with *mixed-frequency data*. Inappropriate use of mixed-frequency data can lead to a bias in the diffusion parameters, as for diffusion models the start of the diffusion process holds crucial information.

Another challenge is seasonality. For some products high-frequency data will contain seasonality. Seasonal influences may for example lead to consistently higher sales for some periods of the year. Seasonality in diffusion models has recently been discussed in Peers *et al.* (2011). Seasonality and mixed-frequency data can in principle be treated independently, although in some practical cases one will need to deal with both. In this chapter we ignore seasonality and focus on the challenges that come with mixed-frequency data.

The problem of mixed frequencies is not new. In (macroeconomic) time-series models mixed frequencies are quite common (Seong *et al.*, 2007). However, most of these studies only take into account differences in frequencies across variables and not different frequencies within a single series. In marketing there are models, some of which are used for diffusion series as well, that handle mixed frequencies[1], for example Xie *et al.* (1997). These methods have the disadvantage that they can be too flexible. This flexibility may prove to be useful if the gap in the data lies in the middle of the time series. However, if the missing observations lie at the beginning of a diffusion series a more structured method is preferable.

There are a few studies that mention missing data in the first periods of the diffusion process. By far this is mostly in the context of left censoring, that is, the problem that no observations are available for the start of the diffusion process (e.g. Parker, 1994; Dekimpe *et al.*, 2000; Jiang *et al.*, 2006). Venkatesan *et al.* (2004) mention the practice of aggregating data to the annual level in order to get rid of the issues associated with high/mixed-frequency data. Finally, Albuquerque *et al.* (2007) mention that they use (cubic spline) interpolation to overcome the irregular spacing of observations in the beginning of their diffusion series. Further, most of these studies mention that these practices for high/mixed-frequency data are common in the field of diffusion modeling. The reason that we cannot present more references is due to the fact that researchers either do not mention the issues, or that they work around them, for example by using more general time-series models instead of diffusion models.

There are several ways to formally deal with mixed-frequency data. In this chapter we will present and compare a number of these methods in combination with different estimation methods. For ease of exposition we will base the discussion in this chapter on the Bass diffusion model. However, the discussion also applies to other diffusion models, such as for example the Gamma Shifted Gompertz Curve (Bemmaor, 1994). For the basic estimation methods we consider the regression approach proposed by Bass (1969), the method of Srinivasan and Mason (1986), and the method recently proposed by Fok *et al.* (2011). We consider the following mixed-frequency methods: (i) aggregating all data

[1]Most of the times it is called handling missing data. However, mixed frequencies can be seen as missing data.

to a low frequency; (ii) ignoring the start of the diffusion, leading to left censoring; (iii) ignoring the start in combination with dealing with the resulting left censoring; (iv) linear interpolation of missing high-frequency data; (v) an adjusted estimation method to use mixed frequencies; and (vi) model-based correction methods. The specific implementation of the model-based method depends on the particular estimation technique that is used. In combination with some estimation methods, the model-based correction method relies on simulation. We will compare the quality of the different methods on theoretical grounds and using simulation.

The remainder of this chapter is organized as follows. In the next section we describe data on flat-screen televisions as a typical example of high-frequency data on the diffusion of durables. This data section shows some common features of mixed-frequency data. With this example we highlight some challenges of using mixed-frequency data. In the third section we briefly review the different estimation techniques we apply in this chapter. Next, we discuss the different methods to use mixed-frequency data. Following, we use a simulation study to show which combinations of methods work and which do not. The chapter ends with a conclusion and discussion.

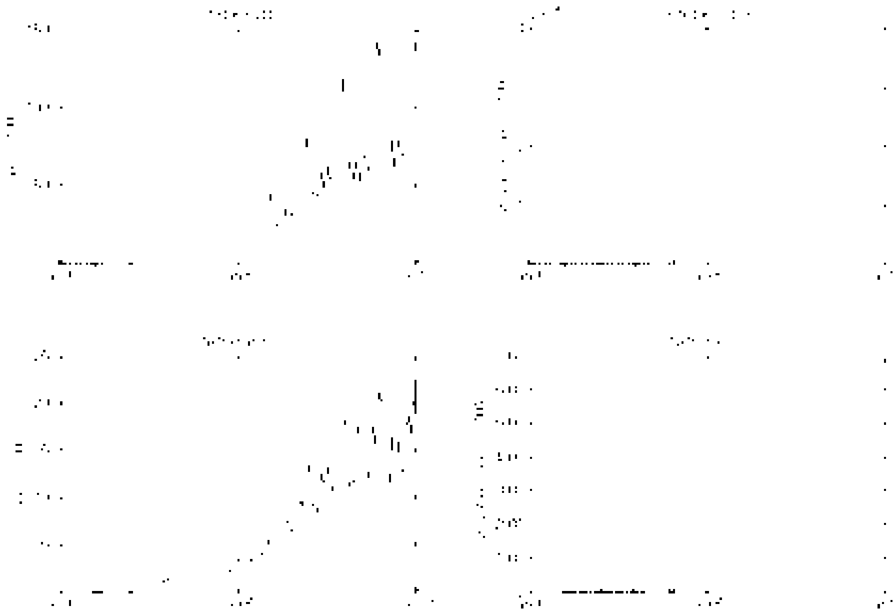
3.2 Examples of mixed-frequency data

In this section we present data on the sales of flat-screen television sets in the United Kingdom and in the Netherlands as an example of mixed-frequency diffusion data. We graphically show how some of the methods to overcome mixed frequencies affect the data. The actual consequences of these methods for parameter estimation is discussed later in this chapter.

We obtained the diffusion data from a large consumer electronics firm known worldwide. This firm bought in this data. We believe this represents how (large) firms often obtain diffusion data. From February 2004 until January 2010 the data is available for each month, for some countries the last month is not observed, in this case the data runs until December 2009. Before 2004 data is only available annually. In Figure 3.1 we show the sales and cumulative sales in the two countries. The yearly sales are represented by circles and a dotted line, the monthly sales are represented by the uninterrupted line.

From Figure 3.1 it becomes clear that the mixed frequencies do not affect the pattern of the *cumulative* sales much. This is the result of the fact that independent of how sales are distributed in a year, the observed cumulative sales at the end of the year is the same. For the sales this does not hold, as for the sales the observational frequency affects the scale of the observations. One annual data point is approximately twelve times as large as the monthly data points would be in the same year. If one wants to use the data as shown in Figure 3.1 directly for estimation, at least a correction for this scale difference is needed.

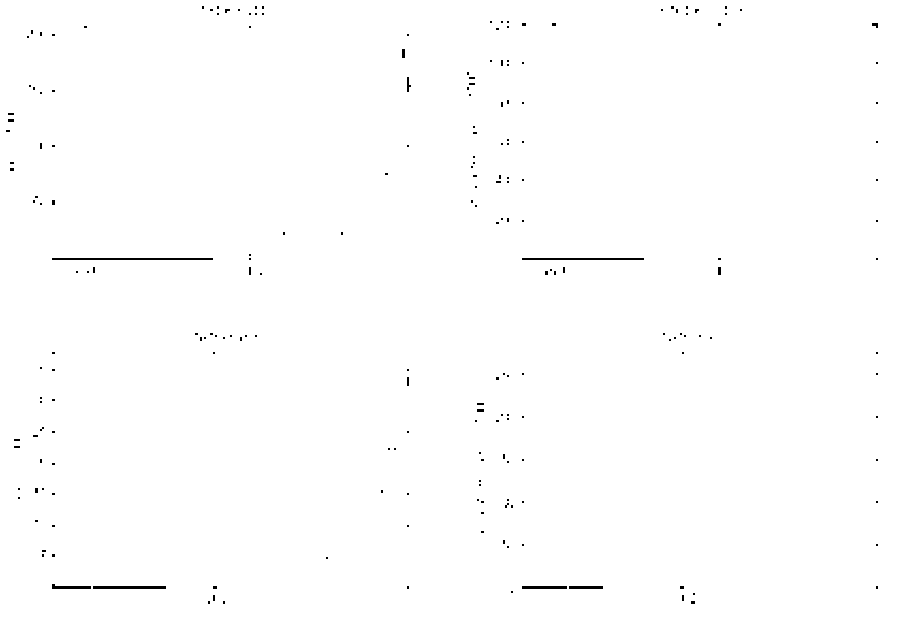
Figure 3.1: The sales (left) and cumulative sales (right) of flat-screen television sets in United Kingdom and the Netherlands. The dotted line with circular markers is annual data the uninterrupted line is monthly sales.



One method to make the scale of the annual and monthly data comparable is to divide the sales of one year over its twelve months by linear interpolation. Figure 3.2 shows how such interpolation affects the curves of the sales and cumulative sales. To clearly show the impact the figure only presents the first years of the diffusion. As expected the cumulative sales are not affected much. The scale of the sales over the different months is now comparable, but the linear interpolation creates a step like sales pattern. As we will show later, this step pattern affects parameter estimation. Preferably, we would like to interpolate the sales by a smooth curve matching the underlying diffusion pattern of the monthly sales. The latter is exactly what the novel interpolation method, which we present later, proposes to do.

An alternative procedure to allow for straightforward parameter estimation, is to aggregate the data to the annual frequency. The resulting curve is shown in Figure 3.3. The figure shows a very practical advantage of this aggregation. The aggregation gets rid of seasonality and other erratic monthly patterns. However, this most likely does not weigh against the resulting small number of data points. For many recently introduced

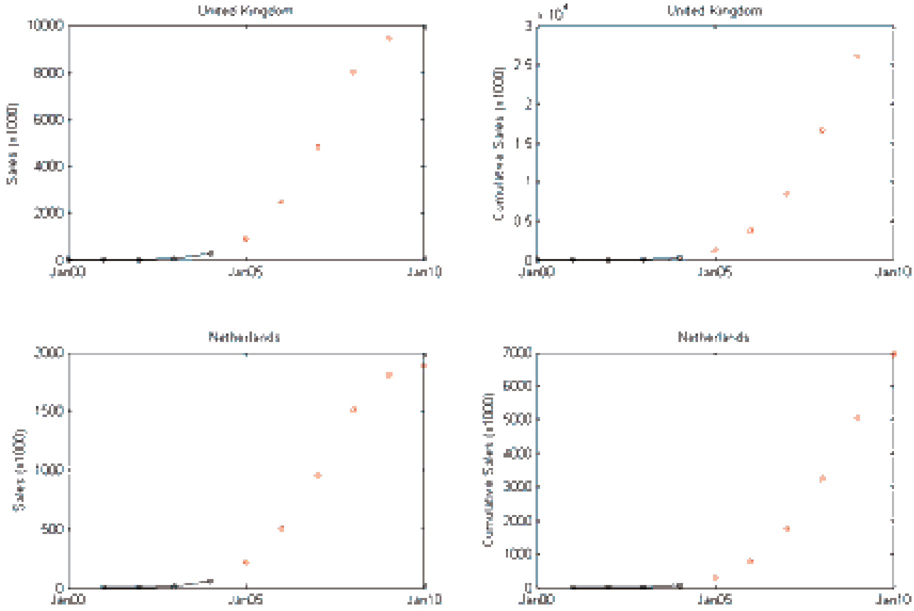
Figure 3.2: First years of the sales (left) and cumulative sales (right) of flat-screen television sets in United Kingdom and the Netherlands. The annual data points are divided over the months, resulting in a step-like function for the sales.



durable goods, using annual data results in too few data points for parameter estimation. The number of data points is further reduced if the underlying monthly data does not contain the end of the year. In that case one cannot obtain the annual aggregate for the final year. For the data the company defines the year to run from February to January. For the Netherlands we have data until January 2010 so we observe the full final year. However, for the United Kingdom we have data until December 2009, so we observe only eleven of the twelve months, and hence the last year is lost when aggregated data is used.

In this section we showed two examples of mixed-frequency data and highlighted the impact of some straightforward ways to deal with mixed-frequency data. In Section 3.3.2 we discuss the impact of these methods on parameter estimation in detail. We also present and discuss the new model-based method.

Figure 3.3: The sales (left) and cumulative sales (right) of flat-screen television sets in United Kingdom and the Netherlands. The line with circular markers shows the actual annual data; the dotted line corresponds to monthly data aggregated to the annual frequency.



3.3 Mixed-frequency data in diffusion models

In this section we present the details of a number of ways to deal with mixed-frequency data. We do this in the context of the standard Bass diffusion model (Bass, 1969) and we explicitly distinguish between different estimation methods. We consider (i) the regression estimation technique as discussed in the original paper of Bass (1969) [BM]; (ii) the nonlinear least squares [NLS] estimation technique discussed in Srinivasan and Mason (1986) [SM]; and (iii) the estimation technique recently introduced by Fok *et al.* (2011) [FPS]. The first two estimation techniques are both frequently used for standard diffusion models as well as for extensions of the basic model (see Peres *et al.*, 2010, for an overview). The regression technique is, for example, often the basis for global diffusion models and brand level models (for example Putsis *et al.*, 1997; Albuquerque *et al.*, 2007; Krishnan *et al.*, 2000), whereas the NLS technique is often used in multi-generation diffusion models (for example Norton and Bass, 1987, 1992; Mahajan and Muller, 1996; Danaher *et al.*, 2001). The FPS technique has been identified by Fok *et al.* (2011) as the estimation method that is most robust to various sources of error. The consequences of mixed frequencies and left censoring differ across the estimation methods. Moreover, the most appropriate way to deal with mixed frequencies depends on the estimation method.

Below, we first briefly review the different estimation techniques for the Bass model. Next, we describe the different methods to use mixed-frequency data.

3.3.1 Estimation methods

As usual we start with the differential equation that underlies the Bass diffusion model (Bass, 1969), that is,

$$\frac{f(t)}{1-F(t)} = p + qF(t), \tag{3.1}$$

where $F(t)$ denotes the fraction of adopters at time t relative to those who will eventually adopt and $f(t) = dF(t)/dt$. The parameters in this equation are p and q and these denote the coefficient of internal and external influence, respectively. The eventual number of adopters in the population is denoted by m. The number of adopters at time t, $N(t)$, is given by $mF(t)$. The instantaneous rate of change in adopters at time t equals $n(t) = mf(t)$.

The differential equation (3.1) has the closed-form solution

$$F(t) = \frac{1-\exp(-(p+q)t)}{1+\frac{q}{p}\exp(-(p+q)t)}, \text{ and } f(t) = \frac{p(p+q)^2\exp(-(p+q)t)}{(p+q\exp(-(p+q)t))^2}. \tag{3.2}$$

This closed-form solution gives the diffusion process in continuous time. In practice we do not observe the diffusion process in continuous time, we instead have observations at certain points in time, that is, we observe $N(t)$ at $t = 0, t_1, t_2, t_3, \ldots$. In what follows we denote these observations by N_i, $i = 0, 1, 2, \ldots$, where the corresponding calendar time of observation i equals t_i. We denote the time between observations at time t_i and t_{i-1} as $\delta_i = (t_i - t_{i-1})$. For example, $\delta_i = 1$ for an annual observation and $\delta_i = \frac{1}{12}$ for a monthly observation. In case of mixed frequencies, the time interval between the observations is not constant. For example, in case of 2 years of annual data and monthly data thereafter, we have observations at $t = 0, 1, 2, 2\frac{1}{12}, 2\frac{2}{12}, 2\frac{3}{12}, \ldots$.

There are several ways to estimate the model parameters. Below we give three estimation techniques for the Bass model[2]. The main difference across the methods is in how the error term is treated. We will explicitly take into account the time between observations (δ_i). This is not always done in the original presentations of these methods.

[2] A more thorough overview of the different estimation techniques is given in Fok *et al.* (2011). We will shortly describe the estimation techniques and focus on those parts relevant for the use of mixed-frequency data.

Bass regression

First we consider the original idea of Bass (1969). Here the starting point is to look at a discretization of the original differential equation in terms of N_i. The discretization that is used approximates $f(t_i)$ by $[N_{i+1} - N_i]/(m\delta_{i+1})$ and $F(t_i)$ by N_i/m. The differential equation than gives

$$\frac{[N_{i+1} - N_i]/(m\delta_{i+1})}{(1 - N_i/m)} \approx p + q\frac{N_i}{m}. \tag{3.3}$$

Rewriting this gives $N_{i+1} - N_i \approx \delta_{i+1}(p + q\frac{N_i}{m})(m - N_i)$. If we denote $X_i = N_i - N_{i-1}$ we obtain

$$X_i \approx pm\delta_i + (q - p)\delta_i N_{i-1} - \frac{q}{m}\delta_i N_{i-1}^2, \text{ for } i = 1, 2, \ldots. \tag{3.4}$$

Bass' suggestion comes down to adding an error term and performing OLS on the equation $X_i = \beta_1\delta_i + \beta_2(\delta_i N_{i-1}) + \beta_3(\delta_i N_{i-1}^2) + \varepsilon_i$. OLS estimates are next transformed to estimates of p, q and m. Note that if δ_i is constant, this can be treated as part of the parameters β_1, β_2, and β_3 and one exactly obtains the method as presented by Bass (1969). The explanatory variables in the regression will then be a constant, N_{i-1}, and N_{i-1}^2.

Srinivasan Mason method

Second, Srinivasan and Mason (1986) propose to match $X_i = N_i - N_{i-1}$ directly with $F(t)$. They perform NLS on

$$X_i = m(F(t_i) - F(t_{i-1})) + \varepsilon_i. \tag{3.5}$$

In this method no discretization error is introduced. Further, as is mentioned in Srinivasan and Mason (1986), the error term in the SM method can handle a variety of unobserved errors, including sampling errors. However, the random term implies that

$$N_i - N_0 = m(F(t_i) - F(0)) + \sum_{s=0}^{i-1} \varepsilon_{i-s}. \tag{3.6}$$

As $N_0 = F(0) = 0$, this gives

$$N_i = mF(t_i) + \sum_{s=0}^{i-1} \varepsilon_{i-s}, \tag{3.7}$$

with independently distributed ε_i this means that the difference of the observed penetration and the underlying diffusion curve has a unit root. This difference therefore behaves as a random walk and in the long run the observations can deviate from $mF(t)$ unlimitedly. As Fok *et al.* (2011) show this feature causes problems when estimating parameters. The unit root behavior is also a factor to take into account regarding mixed frequencies.

Fok, Peers and Stremersch method

The estimation method proposed by Fok *et al.* (2011), see also Chapter 2 of this thesis, solves the issue of the SM method by applying NLS to

$$N_i = mF(t_i) + \varepsilon_i, \tag{3.8}$$

that is penetration levels are directly matched with $F(t)$. Under the assumption of identically distributed error terms we now have that the deviation of the observations with the underlying curve has mean zero at all times.

3.3.2 Dealing with mixed frequencies

Aggregation

One way to deal with mixed frequencies is to aggregate the data to a lower frequency. This use of aggregation is for example mentioned in the paper of Venkatesan *et al.* (2004). Aggregation naturally comes with a loss of information. The loss of information may in turn lead to ill-conditioning and a larger discretization bias. The impact of ill-conditioning will depend on the estimation method. The discretization bias latter will only be present in the Bass regression method.

Ignore low frequency data and left censoring

Another possible “solution” is to ignore the initial low-frequency data. However, contrary to standard time-series models, diffusion models critically depend on information on the full diffusion process. Therefore this data truncation may lead to estimation problems due to left censoring.

Although the problem of left censoring has been acknowledged in the literature (e.g. Parker, 1994; Dekimpe *et al.*, 2000), to our knowledge there is only one paper dealing with it, that is, Jiang *et al.* (2006). In this paper the authors use a method to correct estimates of the Bass diffusion model (Bass, 1969) in hindsight based on the correct date of introduction. In their paper they also show the prevalence of the problem in the diffusion literature: “Many researchers are aware that data series often suffer from left truncation, but only limited attempts have been made to address the issue. As a result, almost all published estimates of the Bass model suffer from left-hand truncation bias.” (Jiang *et al.*, 2006, page 93).

For some estimation methods for the Bass diffusion model, like the methods proposed in Srinivasan and Mason (1986) and Fok *et al.* (2011), only knowledge of the moment of introduction is needed to overcome left censoring. However, for the regression model left censoring cannot easily be solved. The only logical possibility to include the information of the low-frequency data points in this context, is to include the cumulative sales of the low frequency data points in the lagged cumulative sales.

In the comparison, we distinguish between two variants of ignoring low-frequency data. First, we consider the situation where both the lower frequencies and the information of the first periods are ignored. This assumes that the observed high-frequency data is actually the start of the diffusion process, this assumption leads to left censoring. This situation often arises in academics, because the moment of introduction is not known exactly for some products and services. For example, in Parker (1994) the author says: "Without recognition/acknowledgement, time series appear to be truncated on an ad hoc basis; the left hand tail is truncated, sometimes due to data unavailability, which masks the actual launch date, the years to the adoption or sales peak and the skew of the adoption curve." (Parker, 1994, page 373). Second, we consider the case where we again ignore the initial low-frequency data, but where the moment of introduction or the cumulative sales until that period is used in the model. In the context of the Bass regression model this has been suggested by Lilien *et al.* (2000), and for the SM method this has been suggested by Jiang *et al.* (2006).

Linear interpolation

A third option is to linearly interpolate the missing (high-frequency) data points. Linear interpolation, however, ignores the uncertainty and curvature of the diffusion process, as shown in Section 3.2. In particular, because the interpolated observations are relatively noise-free, it is as if this part can almost be explained perfectly. Therefore this practice will "down-weight" the higher frequency part of the diffusion curve. Hence, results are likely to be less efficient and perhaps biased. How severe this bias is, depends on the level of curvature and the noise in the higher frequency part of the data.

As we discussed in Section 3.2, it is important to note that even if the data contains sales data, or adoption per period, the linear interpolation should be done on the cumulative sales. To be more specific, when equally distributing the sales at the lower frequency to obtain high-frequency data, this results in a straight line for the cumulative sales. For the sales itself it will result in a step-like pattern, see Figure 3.2. Interpolating the sales instead will lead to an overestimation of the ceiling as the annual sales are "copied" to all the twelve months of the year.

Adjusting the estimation methods

If the data is available at different frequencies one could actually still directly use the estimation methods as presented in Section 3.3.1. In the presentation of these methods, we explicitly keep track of the time between observations (δ). Directly applying these methods is not possible using the formulation in the original studies (Bass, 1969; Srinivasan and Mason, 1986). In these papers the data frequency is implicitly assumed to be constant.

Although the specifications allows one to use the mixed-frequency data, if the part of the diffusion with lower frequencies is substantial this can still lead to discretization bias for the regression technique. The SM and FPS methods seem directly applicable. However, if one takes the error process in the SM method literally, the specification of the error term should be adapted in case of mixed-frequency data. Suppose that the data is a mix of annual and monthly observations. At the monthly frequency the SM model specifies

$$N(t_i) - N(t_{i-1}) = m(F(t_i) - F(t_{i-1})) + \varepsilon_i,$$

where $t_i - t_{i-1} = \frac{1}{12}$. A annual observation is in fact the sum of twelve monthly observations. For an annual observation we therefore get

$$\begin{aligned} N(t_i) - N(t_{i-12}) &= \sum_{k=0}^{11} (m(F(t_{i-k}) - F(t_{i-k-1})) + \varepsilon_{i-k}) \\ &= m(F(t_i) - F(t_{i-12})) + \sum_{k=0}^{11} \varepsilon_{i-k}. \end{aligned}$$

In other words, we still have the SM specification but the error term for the annual observations is different. Specifically, assuming homoscedasticity of ε_i on the monthly frequency, the variance of the error term for an annual observations is 12 times that of a monthly observation. Using mixed-frequency data in combination with the SM method therefore requires a correction for heteroscedasticity. In the discussion below we consider both variants of this procedure, with accounting for heteroscedasticity and without.[3]

Summarizing, for the regression technique as well as the FPS method one could just include the different δ's to correct for the time intervals. For the SM method we can also include a heteroscedastic weighting function for the error term.

Model-based data imputation using the EM Algorithm

The most involved method we consider uses the model to (stochastically) impute the missing high-frequency observations. In particular we consider the EM algorithm to explicitly deal with the missing data problem. Using this algorithm, we sample the missing data points between the lower frequency data points. Compared to the straightforward interpolation, this methods takes into account the curvature in the model and the uncertainty in the observations.

[3]Some may argue that annual data may actually be less noisy compared to monthly data. If this is true, this actually implies that the data is not generated by the Srinivasan and Mason specification. The specification by FPS may then be more appropriate, see also Chapter 2 for a discussion on the implied assumptions on the error term in different estimation methods.

This technique will be especially useful for the Bass regression technique. For the other two estimation method it will not give any added value. For the FPS method it will be exactly equivalent to the above mentioned adjusted method. For the SM method it is exactly equivalent to the adjusted method including the heteroscedasticity. However, for the Bass regression method this technique is also most complicated. For sake of exposition we therefore first present the details of the imputation technique for the FPS and SM method. This will also show why data imputation does not add anything for these two methods over the methods presented in Section 3.3.2.

This data imputation method is an iterative procedure. We need to alternate between the expectation and the maximization step of the algorithm. In the expectation step the expected mean of the missing data points is calculated, based on the data, the diffusion model, and estimation technique used. Then, in the second step these expected data points combined with the rest of the data is used to maximize the model fit in order to obtain new parameter estimates. These steps are repeated until the changes in the fit of the model and the expectation of the data points is below a certain threshold. A schematic overview of the iterative procedure is as follows:

1. *Obtain first guess of missing data* – In the first iteration there is no information about the parameters of the model to use for the expectation step. Therefore, start with an educated guess for the missing cumulative sales. For example, start with linear interpolation.

2. *Use imputed data to estimate* $\hat{p}, \hat{q}, \hat{m}$, (*and* $\hat{\sigma}^2$) – Estimate the diffusion model's parameters, given the data and the imputed missing (cumulative) sales.

3. *Impute missing data* – Use the obtained parameter estimates to find the expected value of the missing observations conditional on the observed data

4. *Iterate* – Repeat steps (2) and (3) until changes are below a certain threshold.

Given the imputed missing data points, the maximization step (2) is done using the normal estimation routine. The expectation step is less straightforward and depends on the chosen estimation method. We will separately discuss this step for each of estimation techniques. In this discussion we treat the case where annual data is available for the first year and monthly data afterwards. In this case we know $N_i = 0$ and have observed N_{12}, we therefore need to impute the observations $i = 1, 2, \ldots, 11$. Based on this discussion the methodology can straightforwardly be applied to other situations.

FPS method

We start with the method of Fok *et al.* (2011), because here the expectation step is easy. In this specification all cumulative sales observations are statistically independent. In other words the conditional expectation of N_k ($k = 1, 2, \ldots, 11$) given N_0 and N_{12} is equal to the unconditional expectation of N_k.

The unconditional expectation of the cumulative sales of period i for the FPS method is given by (3.8). As the expectation of the error term is zero, the expectation of the cumulative sales of a single period is:

$$E[N_i|\text{available data}] = E[mF(t_i) + \varepsilon_i] = mF(t_i) \tag{3.9}$$

In the expectation step of the EM algorithm we would therefore replace the missing observations by their predicted values which are precisely on the underlying diffusion curve. Actually, this implies that this model based imputation method will not add new information for parameter estimation. The performance of this method will therefore be exactly equal to that presented in Section 3.3.2.

SM method

In the SM method the (monthly) sales are assumed to be independent. This implies that the cumulative sales are not independent! In particular, the cumulative sales in period i, N_i, is part of $X_i = N_i - N_{i-1}$ and $X_{i+1} = N_{i+1} - N_i$. The expectation of period i given an earlier and a later observation can be obtained as the conditional expectation of a multivariate normal distribution.

Below we consider the simplified case where we derive the conditional expectation of N_i given N_{i-1} and N_{i+1}. In Appendix 3.A we show how this can be extended to multiple missing data points. The SM model implies that

$$\begin{pmatrix} N_{i-1} \\ N_i \\ N_{i+1} \end{pmatrix} = \begin{pmatrix} N_{i-2} & + & m(F(t_{i-1}) - F(t_{i-2})) \\ N_{i-1} & + & m(F(t_i) - F(t_{i-1})) \\ N_i & + & m(F(t_{i+1}) - F(t_i)) \end{pmatrix} + \begin{pmatrix} \varepsilon_{i-1} \\ \varepsilon_i \\ \varepsilon_{i+1} \end{pmatrix} \tag{3.10}$$

Similar to equation (3.7) we can rewrite this as

$$\begin{pmatrix} N_{i-1} \\ N_i \\ N_{i+1} \end{pmatrix} = \begin{pmatrix} mF(t_{i-1}) \\ mF(t_i) \\ mF(t_{i+1}) \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \sum_{j=1}^{i-1} \varepsilon_j \\ \varepsilon_i \\ \varepsilon_{i+1} \end{pmatrix}. \tag{3.11}$$

Assuming $\varepsilon_i \sim N(0, \delta\sigma^2)$ we get the following multivariate normal distribution,

$$\begin{bmatrix} N_{i-1} \\ N_i \\ N_{i+1} \end{bmatrix} \sim N\left(\begin{bmatrix} mF(t_{i-1}) \\ mF(t_i) \\ mF(t_{i+1}) \end{bmatrix}, \delta\sigma^2 \begin{bmatrix} (i-1) & (i-1) & (i-1) \\ (i-1) & i & i \\ (i-1) & i & (i+1) \end{bmatrix}. \right) \tag{3.12}$$

The conditional distribution of $N_i|N_{i-1}, N_{i+1}$ is now normal with mean

$$mF(t_i) + \begin{bmatrix} (i-1)\delta\sigma^2 \\ i\delta\sigma^2 \end{bmatrix}' \begin{bmatrix} (i-1)\delta\sigma^2 & (i-1)\delta\sigma^2 \\ (i-1)\delta\sigma^2 & (i+1)\delta\sigma^2 \end{bmatrix}^{-1} \begin{bmatrix} N_{i-1} - mF(t_{i-1}) \\ N_{i+1} - mF(t_{i+1}) \end{bmatrix}. \qquad (3.13)$$

The conditional variance is not important for the expectation step. The conditional mean can be seen as the value according to the underlying diffusion curve plus a correction based on a weighted average of the deviations of the observed data points, N_{i-1} and N_{i+1}, with the curve. Further, the weights turn out to be independent of δ_i, σ or i. Not very surprisingly, the expected value of N_i is,

$$E[N_i|N_{i-1}, N_{i+1}] = mF(t_i) + \frac{1}{2}(N_{i-1} - mF(t_{i-1})) + \frac{1}{2}(N_{i+1} - mF(t_{i+1})) \qquad (3.14)$$

In practice there are multiple missing data points between the observed data. For example, in case the lower frequency is yearly and the high frequency is monthly, there are eleven missing data points. In Appendix 3.A we show how the above expectation step of the interpolation method can be extended. The resulting conditional mean for multiple missing data is similar, only the weights are dependent on how far the missing data point is from the two observed data points.

Correcting for the error in the closest observed data points yields the same results as adjusted estimation with heteroscedasticity based on the time intervals as described in Section 3.3.2. Both methods estimate the parameters while accounting for the missing observations. However, the discussion above is very helpful to understand the equivalent procedure for the Bass regression method.

Regression method

For the regression technique the EM algorithm will make a difference. It will correct the discretization error that is introduced in the adjusted estimation method as presented in Section 3.3.2. The expectation step of the algorithm is however quite involved. In this case cumulative sales of different periods are also dependent on each other. However, due to the nonlinearity in the Bass regression the conditional distribution is not standard. Therefore, the expectation cannot be calculated directly. To implement the EM algorithm we use simulation methods.

Again we start with one missing data point and later come back to the more realistic case of a vector of missing data points. Suppose N_i is missing and N_{i-1} and N_{i+1} are observed. There are two data points dependent on N_i, $X_i = N_i - N_{i-1}$ and $X_{i+1} =$

$N_{i+1} - N_i$. Given equation (3.4) we have,

$$\begin{aligned} N_i - N_{i-1} &= \delta pm + \delta(q-p)N_{i-1} - \delta\frac{q}{m}N_{i-1}^2 + \varepsilon_i \\ N_{i+1} - N_i &= \delta pm + \delta(q-p)N_i - \delta\frac{q}{m}N_i^2 + \varepsilon_{i+1}, \end{aligned} \tag{3.15}$$

rewriting this gives

$$\begin{pmatrix} \delta pm + (1+\delta(q-p))N_{i-1} - \frac{\delta q}{m}N_{i-1}^2 \\ N_{i+1} - \delta pm \end{pmatrix} = \begin{pmatrix} 1 \\ 1+\delta(q-p-\frac{q}{m}N_i) \end{pmatrix} N_i + \begin{pmatrix} \varepsilon_i \\ \varepsilon_{i+1} \end{pmatrix}. \tag{3.16}$$

The reason to rewrite it such that all N_i terms are on the right-hand side is that this *almost* looks like an ordinary least squares regression with two observations and one coefficient (N_i). If the second vector did not contain N_i, the conditional mean of the N_i would be easy to derive. The fact that it does makes it impossible to directly get this conditional mean. We suggest to use a Markov Chain Monte Carlo [MCMC] method to evaluate the conditional mean. That is we iteratively sample from the conditional distribution and obtain the conditional mean from these samples. Of course, sampling N_i is also not straightforward. To this end we propose a Metropolis Hastings independence sampler. The candidate distribution is inspired on (3.16), we replace N_i in the second vector by something similar but observed, namely N_{i+1}. This results in the following candidate distribution

$$N\left(\frac{\delta pm + (1+\delta(q-p))N_{i-1} - \frac{\delta q}{m}N_{i-1}^2 + (1+\delta(q-p-\frac{q}{m}N_{i+1}))(N_{i+1}-\delta pm)}{1+(1+\delta(q-p-\frac{q}{m}N_{i+1}))^2}, \frac{\sigma^2}{1+(1+\delta(q-p-\frac{q}{m}N_{i+1}))^2}\right). \tag{3.17}$$

Given this candidate distribution the step by step sampling in the MCMC chain is

1. *Start with* N_i^0 - start the sampling method with a first guess for N_i, which we call N_i^0.

2. *Draw* N_i^* - draw a proposal (N_i^*) for N_i from the candidate function (3.17).

3. *Compute acceptance probability* - Calculate $\alpha = \min\left\{1, \frac{L(N_i^*, N_{i+1})}{L(N_i^m, N_{i+1})}\frac{P_{cand}(N_i^m)}{P_{cand}(N_i^*)}\right\}$ where $L(N_i, N_{i+1})$ is the likelihood function for the observations at t_i and t_{i+1}. P_{cand} is the density function of the candidate distribution, equation (3.17). Here N_i^m is current value in the m-th iteration.

4. *Accept or reject* - with probability α the proposal is accepted and N_i^{m+1} will be N_i^* otherwise it remains as N_i^m.

5. *Repeat* - repeat steps 2 to 4 many times.

After a long enough burn-in this routine samples from the true conditional distribution. Hence, taking the mean of these iterations will give the conditional expectation. If we call the burn-in period n^* and the number of iterations is n then the expected mean is approximated by

$$E(N_i)=\frac{1}{n-n^*+1}\sum_{k=n^*}^{n} N_i^k. \tag{3.18}$$

The above sampling method obtains the mean for one missing data point. However, most of the times there are more missing periods. The solution is to not only iterate over one period, but over all periods that are not observed. If data points N_{i-1} and N_{i+k+1} are known and N_i until N_{i+k} are missing the iterative sampling of the MCMC-chain looks like

1. *Start with* $N_i^0 \dots N_{i+k}^0$: start the sampling method with a first guess for N_i to N_{i+k}. For example using linear interpolation

2. *Sample* N_i^m *using* N_{i-1} *and* N_{i+1}^{m-1}: Use the observed data point N_{i-1} and the last iteration of N_{i+1} to obtain the m-th iteration of N_i. This step is the same as steps 2 to 4 for one missing data point.

3. *Sample* N_{i+s}^m *using* N_{i+s-1}^m *and* N_{i+s+1}^{m-1}, *for* $s = 1, \dots, k-1$: Use the last iteration of N_{i+s-1} and the last iteration of N_{i+s+1} to obtain the m-th iteration of N_{i+s}. This step is also the same as steps 2 to 4 for one missing data point.

4. *Sample* N_{i+k}^m *using* N_{i+k-1}^m *and* N_{i+k+1}: Use the last iteration of N_{i+k-1} and the observed data point N_{i+k+1} to obtain the m-th iteration of N_{i+k}. This step is the same as steps 2 to 4 for one missing data point.

5. *Repeat*: repeat steps 2 to 5 as many times as necessary.

The mean of the missing data points can be calculated in a same way as for one missing data point, using equation (3.18).

Overview and expected outcome

In Table 3.1 we give an overview of all the different ways to handle mixed frequencies. In this table we also indicate the expected outcome for the different estimation techniques. In the next section we take a closer look at these outcomes using a simulation study.

Aggregation will harm the estimation of the diffusion parameters through the problem of ill-conditioning. Of course this depends on how many data points remain after

Table 3.1: Expected bias in diffusion parameters using different mixed-frequency approaches for three estimation methods

	Estimation method		
	OLS	SM	FPS
Aggregation	Large	Small	Small
Ignore data			
Start at high freq.	Large	Large	Large
Deal with left censoring	Small	Small	No
Using all data			
Linear Interpolation	Small	Small	Small
Adjust Estimation	Large	Small/No*	No
Model-based correction using the EM algorithm	No	No	No

* = There is no expected bias if one corrects for the time interval in the error term, otherwise there is a small bias.

aggregation. Further, aggregation will lead to a discretization bias for the regression method.

Left censoring, incorrectly assuming that the diffusion starts at the moment one has available high-frequency data, harms all methods. The main reason is that the start of the diffusion process holds valuable information. Especially the innovation parameter will be biased. If one deals with this left censoring by using the information of the first periods, the Bass regression method is still expected to have a bias. The SM method is expected to have an underestimation of the uncertainty on the first observation, due to the unit root in the error term. This underestimation leads to inefficiency of the estimation technique, and possibly to a small bias. The FPS method in this case is expected not to be biased.

Linear interpolation is the easiest way to be able to use standard diffusion estimation routines with mixed-frequency data. More specifically, it does not need a correction for the time interval of the periods. The linear interpolation will however bring bias to the estimates. This bias can be small, but increases with increasing curvature and noise in the diffusion curve.

Adjustment of the estimation is done by correcting for the different time intervals δ_i in the estimation. For the SM method we look at two cases of adjusted estimation: (i) where we do not correct for heteroscedasticity due to the time interval; and (ii) where we do correct for this heteroscedasticity.

Finally, we look at the case of model-based correction, where we use the missing observations between the low-frequency data. Moreover, for this model-based correction method we use the EM algorithm. For the method of Fok *et al.* (2011) the results are an exact match to the adjusted estimation method. The same holds for the SM method, given that one also corrects for the time interval in the error variance. For the regression model we really need the model-based data imputation we propose in this chapter. This novel method is expected to result in unbiased parameter estimates for all estimation methods.

3.4 Simulation study

In this section we describe the results of a simulation study for all three estimation techniques of the Bass model and the different ways of dealing with mixed frequencies we have discussed. In each case we compare the estimated parameter values with the true values, that is, the values used to simulate the diffusion curve. The true diffusion curve is based on a simulation at the monthly frequency. From this true diffusion curve we construct a diffusion curve with mixed frequencies by aggregating the monthly observations to annual observations for a given period at the start of the diffusion curve. This mixed-frequency diffusion curve is then used for estimation.

To simulate the diffusion curve we use exactly the same functional form as for estimation. Thus, we use another specification for the regression method as for the SM method to simulate. To simulate with noise it is crucial to include heteroscedasticity in the simulation process. The heteroscedasticity makes sure that the fluctuations at the beginning and the end of the diffusion process are smaller, which prevents the curve from getting negative sales. For the estimation we do not include this heteroscedasticity, as there is no clear consensus on whether to do so in practice. Note that ignoring heteroscedasticity mostly affects the efficiency of the estimates (Boswijk and Franses, 2005). To make sure we do not attribute efficiency issues and bias of ignoring heteroscedasticity to the techniques handling mixed-frequency data, we also estimate the parameters based on the complete high-frequency diffusion curve. If there is bias or loss of efficiency due to heteroscedasticity this will also show in the parameter estimates on this full diffusion curve.

In total we consider 7 scenarios, one using the complete high-frequency diffusion curve, and six methods dealing with mixed-frequencies, that is, we have (1) *Full*: estimation is done with the full diffusion curve at the highest frequency; (2) *Aggregation*: estimation is done on annual data only; (3) *Start at high-frequency data*: ignore the entire low-frequency period; (4) *Dealing with left censoring*: all low-frequency data points are ignored, but the initial (low-frequency) period is included in the cumulative sales for the regression method and the period since introduction for the SM method and FPS method; (5) *Linear interpolation*: estimate using monthly data after interpolation; (6) *Adjusted estimation*: use δ_i to adjust the estimation, where for the SM method we look at two cases one with and one without correction for heteroscedasticity; and (7) *Using the EM algorithm*: stochastically impute the missing observations using the EM-algorithm. Note that these results of this algorithm for the FPS and SM method are identical to the outcomes of the adjusted estimation methods.

For the simulation we set the true parameters as $p = 0.03$, $q = 0.5$ and $m = 100$. These values are representative for the reported parameter estimates in the literature[4].

[4]We measure time in years. Therefore, these parameter values are directly comparable to previous studies on annual data. If one would measure time in months, the values of p and q would be smaller.

With these values the diffusion curve reaches the ceiling around 15 years. The moment of peak sales is between year 5 and year 6. We set the number of low frequency years at 4.

In Tables 3.2 and 3.3 we give the estimation results for all scenarios and models. Table 3.2 gives the results when no noise is added to the diffusion curves. Even in this case, some methods do not give correct estimates. Clearly, completely ignoring the low frequency information leads to biased estimates for all estimation methods. Linear interpolation performs surprisingly well. There is only a small bias. When we use the EM algorithm all estimation methods yield the correct parameter values. For the adjusted estimation as well as for other mixed-frequency approaches, the Bass regression leads to biased estimates. The SM and FPS methods have unbiased estimates for the adjusted estimation.

In Table 3.3 we consider the more realistic case of noisy data[5], where we set $\sigma = 0.2$. We report the percentage difference between the mean of the parameter estimates and the true value over 1000 runs and the Root Mean Squared Error [RMSE]. For the "Full" method, using all high-frequency data points, we show the absolute RMSE, for the other models we give the fraction of the RMSE compared to that of the Full Model.

Table 3.3 confirms that it is not wise to ignore the first period completely, that is, the left-censoring problem leads to an extremely large bias. Further, for all estimation methods, the model based data imputation (EM algorithm) results in (approximately) unbiased estimates.

The adjusted estimation performs the same as the model-based correction for the FPS method, as it should. For the SM method there is a small difference if it is not corrected for heteroscedasticity, and it gives the same results as the EM algorithm if one corrects for this heteroscedasticity. For the Bass regression the adjusted estimation performs worse, especially for the p parameter, this is due to the discretization error present in this method.

Overall, linear interpolation performs quite well. The bias is small, but the RMSE is larger compared to the model based correction. The linear interpolation affects the FPS method the most, especially for the innovation parameter. For SM and FPS, linear interpolation has a positive bias for the internal influence parameter and a negative bias for the external influence parameter. The market potential parameter seems unaffected by linear interpolation.

Dealing with left censoring by using the information of the low frequency observationsonly gives a small bias in the internal influence parameter, p, for the regression technique and the SM method. For FPS the bias is negligible.

For the regression method the use of aggregation gives severely biased results due to the discretization error. The bias is large for the internal influence parameter, moderate for the external influence parameter, and almost absent for the market potential. The other two estimation techniques do not yield biased results, but do have a higher RMSE, especially for FPS. This shows the difference between using high or low frequency data.

[5]Due to the heteroscedasticity this noise is scaled with the level of sales in each period

Table 3.2: Estimation performance of mixed-frequency approaches for three estimation methods. The parameter estimates are based on the simulation with no noise.

	p			q			m		
	Regression	SM	FPS	Regression	SM	FPS	Regression	SM	FPS
True Value	0.0300	0.0300	0.0300	0.500	0.500	0.500	100.0	100.0	100.0
Full	0.0300	0.0300	0.0300	0.500	0.500	0.500	100.0	100.0	100.0
Aggregated	0.0451	0.0300	0.0300	0.459	0.500	0.500	99.0	100.0	100.0
Start at High Freq.	0.1730	0.1766	0.1766	0.357	0.353	0.353	71.4	70.7	70.7
Dealing with LC	0.0306	0.0300	0.0300	0.499	0.500	0.500	100.0	100.0	100.0
Linear Interpolation	0.0300	0.0303	0.0305	0.497	0.496	0.496	100.1	100.0	100.1
Adjusted Estimation (Without Het.)	0.0402	0.0300	0.0300	0.514	0.500	0.500	99.6	100.0	100.0
Adjusted Estimation (With Het.)	-	0.0300	-	-	0.500	-	-	100.0	-
EM algorithm	0.0300	0.0300[a]	0.0300[b]	0.500	0.500[a]	0.500[b]	100.0	100.0[a]	100.0[b]

[a] results are identical to Adjusted Estimation with heteroscedasticity
[b] results are identical to Adjusted Estimation

Table 3.3: Estimation performance of mixed-frequency approaches for three estimation methods. We give the difference between the estimated parameter and the true value, in percentages. Also the RMSE is given, where for the Full method the absolute RMSE is given and for the other methods the RMSE relative to the RMSE of the Full method.

	p			q			m		
	Regression	SM	FPS	Regression	SM	FPS	Regression	SM	FPS
True Value	0.03	0.03	0.03	0.5	0.5	0.5	100	100	100
Full	-0.0094%	-0.1245%	0.0110%	0.0905%	0.1101%	-0.0091%	0.0009%	0.0068%	0.0008%
(RMSE)	0.0025	0.0017	0.0001	0.0182	0.0151	0.0007	0.2119	1.7880	0.0136
Aggregated	50.29%	-0.06%	0.02%	-8.17%	0.08%	-0.01%	-0.97%	0.01%	0.00%
(RMSE/RMSE$_{Full}$)	6.15	1.01	3.59	2.42	1.01	3.42	4.66	1.00	3.12
Start at High Freq.	478.78%	487.03%	488.87%	-28.85%	-28.79%	-29.34%	-28.54%	-29.35%	-29.32%
(RMSE/RMSE$_{Full}$)	57.98	84.02	1916.10	8.15	9.81	210.90	134.83	16.44	2151.90
Dealing with LC	4.77%	0.50%	0.01%	-0.40%	0.41%	-0.01%	0.02%	-0.01%	0.00%
(RMSE/RMSE$_{Full}$)	10.00	3.43	1.73	2.75	2.21	1.53	1.20	2.67	1.17
Linear Interpolation	-0.09%	0.87%	1.58%	-0.44%	-0.61%	-0.81%	0.04%	0.04%	0.07%
(RMSE/RMSE$_{Full}$)	1.02	1.03	6.66	1.00	1.02	6.22	1.02	1.00	5.35
Adjusted Estimation (Without Het.)	33.89%	-0.26%	0.01%	2.61%	0.36%	-0.01%	-0.42%	0.00%	0.00%
(RMSE/RMSE$_{Full}$)	4.18	1.03	1.61	1.67	1.42	1.44	2.73	1.06	1.14
Adjusted Estimation (With Het.)	-	-0.09%	-	-	0.09%	-	-	0.01%	-
(RMSE/RMSE$_{Full}$)	-	1.01	-	-	1.01	-	-	1.00	-
EM Algorithm	-0.16%	-0.09%	0.01%	0.13%	0.09%	-0.01%	0.00%	0.01%	0.00%
(RMSE/RMSE$_{Full}$)	1.00	1.01[a]	1.61[b]	1.01	1.01[a]	1.44[b]	1.00	1.00[a]	1.14[b]

[a] results are identical to Adjusted Estimation with heteroscedasticity
[b] results are identical to Adjusted Estimation

In this chapter we do not compare the different estimations techniques with each other, but Table 3.3 does show that the SM method has a fairly large RMSE, especially for the market potential parameter. This is due to the unit root assumption, for more details on this issue of estimation and simulating with the SM method we refer to Fok *et al.* (2011).

We repeated the simulation for a parameter setting with more curvature in the beginning of the curve, that is, $p = 0.01$ and $q = 0.8$. The moment to peak sales is almost similar in this setting as is the number of years until the market potential is reached. The results are very similar to those reported above. The main difference is that the bias and the RMSE in the linear interpolation model becomes slightly larger.

3.5 Conclusion

High-frequency diffusion data is nowadays readily available. Such data can prove to be very helpful in business and in academic research. This has actually been acknowledged in the literature, both implicitly as explicitly. However, high- frequency data is hardly used. This is most likely due to some challenges that come with high-frequency data. One of these challenges is that high-frequency data is usually not available for the start of the diffusion process. This leads to a situation where one has mixed-frequency data.

In this chapter we have suggested a number of ways to deal with mixed-frequency data. We have shown how these methods work and perform for three estimation methods: the Bass regression, the Srinivasan and Mason (1986) [SM] method and the estimation method proposed in Fok *et al.* (2011) [FPS]. For some estimation methods using mixed-frequency data is straightforward. This holds for the SM and FPS method. For the Bass regression, correctly accounting for mixed frequencies is quite involved. The analysis also showed that failing to correct for mixed frequencies can lead to a large bias in the parameter estimates. In this sense, this chapter is a useful summary of the do's and don'ts of using mixed-frequency data.

We compared six different methods to handle mixed-frequency data and tested these for the three estimation techniques of the Bass model. By comparing the results based on only high-frequency data and using only aggregated data, we show the importance of using high-frequency data, especially for the regression method. We find that ignoring the low-frequency data, without including information of these data points leads to a severe bias in all estimation techniques, due to left censoring.

For the regression method most correction methods get biased parameter estimates. This is largely due to a discretization error. Therefore, for the regression technique the EM algorithm is very useful. Only linear interpolation leads to parameter estimates with only a small bias, and hence can be seen as an easy alternative to the EM algorithm.

For the estimation methods of Srinivasan and Mason (1986) and Fok *et al.* (2011) most techniques yield the same parameter estimates, where linear interpolation leads to as small bias and dealing with left censoring by using the moment of introduction leads to a loss of information and efficiency. Further, aggregation only works if there remain enough data points. The EM algorithm is not necessary if one uses the mixed frequencies correctly, that is, correcting for the time interval between periods. For the SM method one also needs to correct for the time interval in the error term.

In this chapter we focussed on the benefits of high-frequency data regarding parameter estimation, where we argued that low-frequency data comes with a loss of information. We however did not look at the optimal data frequency for forecasting. Future research could study if a Bass model estimated on monthly data leads to more accurate forecasts of the future number of new adopters in a year than a Bass model estimated on annual data.

In sum, high-frequency data become more and more readily available. Further, models become more demanding of the data, which makes it important to use such high-frequency data. A downside of high-frequency data is that they often include mixed frequencies. In this chapter we showed that standard diffusion models still can be used if these mixed frequencies are dealt with properly. This not only helps managers, who frequently use these models for (pre-launch) sales forecasting, but also for academic researchers as the standard diffusion models are frequently used as basis for extensions.

3.A General case for the expectation step for the Srinivasan Mason method

In this appendix we show how the stochastic imputation method works for a situation where there are multiple missing data points between the lower frequency data. For example, in the case where there is a mix of annual and monthly data, there are eleven points to be interpolated.

In this appendix we assume there are $k+1$ points to be interpolated. The first period to be interpolated is i and the last period is $i+k$, this means we observe period $i-1$ and $i+k+1$. The multivariate formulation is,

$$\begin{pmatrix} N_{i-1} \\ N_i \\ \vdots \\ N_{i+k} \\ N_{i+k+1} \end{pmatrix} = \begin{pmatrix} N_{i-2} + m(F(t_{i-1}) - F(t_{i-2})) \\ N_{i-1} + m(F(t_i) - F(t_{i-1})) \\ \vdots \\ N_{i+k-1} + m(F(t_{i+k}) - F(t_{i+k-1})) \\ N_{i+k} + m(F(t_{i+k+1}) - F(t_{i+k})) \end{pmatrix} + \begin{pmatrix} \varepsilon_{i-1} \\ \varepsilon_i \\ \vdots \\ \varepsilon_{i+k} \\ \varepsilon_{i+k+1} \end{pmatrix} \tag{3.19}$$

Similar to equation (3.7) in Section 3.3.1 we can rewrite this as follows,

$$\begin{pmatrix} N_{i-1} \\ N_i \\ \vdots \\ N_{i+k} \\ N_{i+k+1} \end{pmatrix} = \begin{pmatrix} mF(t_{i-1}) \\ mF(t_i) \\ \vdots \\ mF(t_{i+k}) \\ mF(t_{i+k+1}) \end{pmatrix} + \begin{pmatrix} 1 & 0 & \dots & 0 & 0 \\ 1 & 1 & & & 0 \\ \vdots & & \ddots & & \vdots \\ 1 & & & 1 & 0 \\ 1 & 1 & \dots & 1 & 1 \end{pmatrix} \begin{pmatrix} \sum_{j=1}^{i-1} \varepsilon_j \\ \varepsilon_i \\ \vdots \\ \varepsilon_{i+k} \\ \varepsilon_{i+k+1} \end{pmatrix} \tag{3.20}$$

this results in the following multivariate normal distribution,

$$\begin{bmatrix} N_{i-1} \\ N_i \\ \vdots \\ N_{i+k} \\ N_{i+k+1} \end{bmatrix} \sim N\left(\begin{bmatrix} mF(t_{i-1}) \\ mF(t_i) \\ \vdots \\ mF(t_{i+k}) \\ mF(t_{i+k+1}) \end{bmatrix}, \right.$$

$$\left. \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 1 & 1 & & & 0 \\ \vdots & & \ddots & & \vdots \\ 1 & & & 1 & 0 \\ 1 & 1 & \dots & 1 & 1 \end{bmatrix} \begin{bmatrix} (i-1)\delta\sigma^2 & 0 & \dots & 0 & 0 \\ 0 & \delta\sigma^2 & & & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & & \delta\sigma^2 & 0 \\ 0 & 0 & \dots & 0 & \delta\sigma^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 1 & 1 & & & 0 \\ \vdots & & \ddots & & \vdots \\ 1 & & & 1 & 0 \\ 1 & 1 & \dots & 1 & 1 \end{bmatrix}' \right) \tag{3.21}$$

Multiplying the matrices results in the covariance matrix

$$\Sigma = \delta\sigma^2 \begin{bmatrix} (i-1) & (i-1) & (i-1) & \dots & (i-1) & (i-1) \\ (i-1) & i & i & \dots & i & i \\ (i-1) & i & (i+1) & \dots & (i+1) & (i+1) \\ \vdots & \vdots & & \ddots & & \vdots \\ (i-1) & i & (i+1) & \dots & (i+k) & (i+k) \\ (i-1) & i & (i+1) & \dots & (i+k) & (i+k+1) \end{bmatrix}. \tag{3.22}$$

The conditional mean of $N_i, \dots, N_{i+k}$ given N_{i-1} and N_{i+k+1} is now,

$$\begin{bmatrix} mF(t_i) \\ \vdots \\ mF(t_{i+k}) \end{bmatrix} + \Sigma_{c,-c}\Sigma_{-c,-c}^{-1} \begin{pmatrix} N_{i-1} - mF(t_{i-1}) \\ N_{i+k+1} - mF(t_{i+k+1}) \end{pmatrix}, \tag{3.23}$$

where the c subscript of Σ represents the rows or columns in the covariance matrix of the missing data periods and the subscript $-c$ represent the observed data periods, that is period $i-1$ and $i+k+1$. The matrices in the conditional mean can be seen as a weighted average of the deviation of the observed data points, N_{i-1} and N_{i+k+1}, to the underlying diffusion curve. The resulting weights are independent of δ, σ and i. The weights are, however, dependent on how far the missing data point is from the two observed data points. In particular the conditional expectation of the vector of missing data points becomes

$$\begin{bmatrix} mF(t_i) \\ mF(t_{i+1}) \\ \vdots \\ mF(t_{i+k-1}) \\ mF(t_{i+k}) \end{bmatrix} + \begin{bmatrix} \frac{k+1}{k+2} & \frac{1}{k+2} \\ \frac{k}{k+2} & \frac{2}{k+2} \\ \vdots & \vdots \\ \frac{2}{k+2} & \frac{k}{k+2} \\ \frac{1}{k+2} & \frac{k+1}{k+2} \end{bmatrix} \begin{pmatrix} (N_{i-1} - mF(t_{i-1})) \\ (N_{i+k+1} - mF(t_{i+k+1})) \end{pmatrix}. \tag{3.24}$$

The denominator of the weight is one more than the length of the vector, $k+2$. The numerator is the number of data points between the missing data point and the two observed data points. In the case of only one missing, $k=0$, the two weights both become $\frac{1}{2}$ as we found in Section 3.3.2.

Chapter 4

Modeling seasonality in new product diffusion

The authors propose a method to include seasonality in any diffusion model that has a closed-form solution. The resulting diffusion model captures seasonality in a way that naturally matches the original diffusion model's pattern. The method assumes that additional sales at seasonal peaks are drawn from previous or future periods. This implies that the seasonal pattern does not influence the underlying diffusion pattern. The model is compared with alternative approaches through simulations and empirical examples. As alternatives we consider the standard Generalized Bass Model [GBM] and the basic Bass model, which ignores seasonality. One of the main findings is that modeling seasonality in a GBM generates good predictions, but gives biased estimates. In particular, the market potential parameter is underestimated. Ignoring seasonality, in cases where data of the entire diffusion period is available, gives unbiased parameter estimates in most relevant scenarios. However, when only part of the diffusion period is available, estimates and predictions become biased. We demonstrate that the model gives correct estimates and predictions even if the full diffusion process is not yet available.

4.1 Introduction

Sales of new products typically follow a diffusion process that has an S-shaped pattern for cumulative sales whereas the corresponding pattern for sales is hump-shaped. There is a variety of models that can capture such a diffusion pattern. In marketing, the Bass (1969) model is most often used. The main application of diffusion models concerns forecasting sales. For new products one can use the parameter estimates based on data of similar products. And, after having observed the diffusion of a product for a while, one can also forecast the remainder of the diffusion pattern using the relevant parameter estimates.

Most diffusion models are set in a continuous-time context and assume a smooth development of sales. This smooth development often matches well with observed diffusion data at a yearly frequency. However, at a higher frequency the sales development tends to be less smooth. For example, within a year sales data are likely to show a strong seasonal pattern. Seasonality systematically generates periods with higher sales followed by periods with lower sales. For example, Christmas sales usually generate a sales spike in the month of December. In this chapter we present a model that allows for such seasonality while preserving the basic diffusion pattern.

The importance of having a diffusion model that incorporates seasonality is amplified by the increasing availability of high-frequency data. Although high-frequency diffusion data is nowadays often available, researchers usually opt to aggregate such data to the yearly level. For example Venkatesan *et al.* (2004) mention this practice of aggregating to annual data in order to get rid of seasonal fluctuations. Although this aggregation of data reduces or even removes seasonality, it comes with a loss of information. Using only a small number of (annual) data points leads to the so-called ill-conditioning problem, which in turn leads to biased estimates, see van den Bulte and Lilien (1997) and Bemmaor and Lee (2002). Putsis (1996) and Non *et al.* (2003) find that the use of quarterly or monthly data significantly improves estimates of diffusion model parameters compared to only using annual data. The main reason for this improvement is the reduction in the data-interval bias that originates from the discrete time approximation of the underlying continuous-time diffusion model. Both studies, however, do not explicitly cover seasonality for monthly or quarterly data.

From a managerial point of view, seasonal patterns also contain valuable information. This information can be used to predict short-term demand as well as to support inventory management. Hence, filtering out seasonal effects, which is common practice in the literature of financial and macroeconomic time series, is not a preferable solution in case of diffusion models.

The conclusion is that seasonality must somehow be incorporated in a diffusion model. In this chapter we propose a natural way to incorporate seasonality in any diffusion model that has a closed-form solution. We build on the Bass model for expository purposes, but indicate that many other models can be considered. The main idea that underlies the approach in this chapter is that seasonality in a peak period is a result of consumers who delay or speed up their purchases. In other words, the model captures the pattern of intertemporal demand shifts that cause seasonal peaks. We treat the underlying diffusion model as the appropriate model for time-aggregated sales and study how these aggregates are distributed over, say, the months of a year.

We contrast the proposed model with other approaches on theoretical grounds and using empirical examples. The first alternative approach we consider is to include seasonal dummies in a way that matches with the Generalized Bass model [GBM] (Bass *et al.*, 1994). We show that for this model the estimates for the diffusion parameters are biased, where especially the market potential parameter gets underestimated. Estimates based on the proposed model are intuitively more appealing and do not lead to a bias. The second alternative approach is the traditional Bass model, which ignores seasonality even when it is present. The results give reassuring outcomes for the common practice of 'guessing by analogy', because for the case where the full diffusion series is available we demonstrate that the traditional Bass estimates are not biased.[1] However, if seasonality is ignored when the diffusion process is before its saturation level, the estimates as well as the forecasts are biased.

Next to the proposed model, we also propose another variation to the Generalized Bass Model with seasonality. In many practically relevant cases, this variation has the same nice statistical features as the proposed model, that is, the parameter estimates are unbiased. Contrary to the proposed model, this variation of the GBM is not based on intertemporal demand shifts. This results in biased estimates if the diffusion is fast.

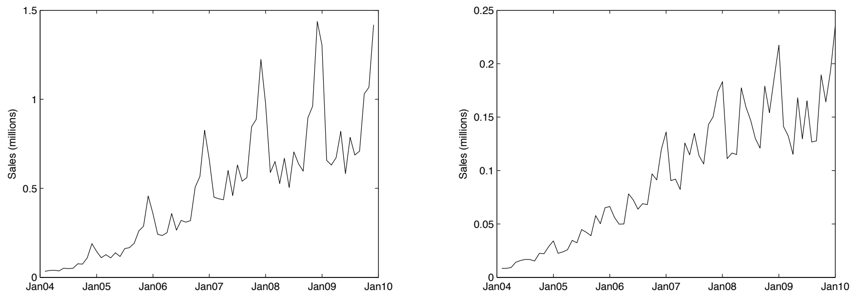
The outline of this chapter is as follows. In Section 4.2 we start by showing typical diffusion data, where we consider the monthly sales of flat-screen television sets (LCD and Plasma). In Section 4.3 we propose the model and theorize why the alternative approaches are less useful when seasonality is present. In Section 4.4 we extend the model such that the exact shape of the intertemporal demand shift can be estimated. In Section 4.5 we return to actual sales data and demonstrate that the new seasonal diffusion model fits naturally to these data and that it gives plausible forecasts. In Section 4.6 we conclude with some suggestions for further research.

4.2 An example of seasonality in diffusion data

Before we start modeling seasonality in diffusion models we first take a look at a typical example of seasonality in diffusion data. In particular, we have available monthly sales

[1] 'Guessing by analogy' is a popular method among researchers and managers to predict the diffusion parameters of a new product based on the diffusion parameters of earlier introduced similar products, see Ofek (2005a) and Lilien *et al.* (2000). Thus, if published or obtained estimates are biased by ignoring seasonality this affects the prediction of the diffusion of the new product as well.

Figure 4.1: Flat-screen television sales data of the United Kingdom (a) and the Netherlands (b).



(a) (b)

figures (in millions) of flat-screen televisions for ten countries in Europe[2]. The period covered by the data is from February 2004 until January 2010 or December 2009.

To give a graphical example of the feature we study in this chapter, we present in Figure 4.1 monthly sales of flat-screen televisions in two of the ten countries, that is, the United Kingdom and the Netherlands.

The observations in Figure 4.1 clearly show a systematic seasonal pattern. Furthermore, the graph suggests that the seasonal pattern is proportional to the speed and position of the diffusion process, that is, the seasonal peaks become larger closer to the moment of peak sales.

Notice also that the seasonal pattern is not similar across the United Kingdom and the Netherlands. For example, in the Netherlands the January peak is larger than the December peak, whereas in the United Kingdom it is the other way around. This shows that we need to allow for different seasonal structures across countries.

4.3 A diffusion model with seasonality

Although seasonality is a major issue for many market processes, the literature on seasonality in marketing is small. Shugan and Radas (1999) give an overview of the types of seasonality issues in the context of services marketing. They consider how to overcome these issues and how managers should react to them. Fok *et al.* (2007) look at weekly seasonality in sales in a panel of fast moving consumer goods. There are to our knowledge only two papers that focus on modeling seasonality in diffusion, and these are Radas and

[2]Austria, Belgium, France, Germany, Italy, Netherlands, Portugal, Scandinavia, Spain and the United Kingdom. The data concern aggregated sales figures for the four Scandinavian countries. For simplicity we refer to Scandinavia as a “country”.

Shugan (1998) and Einav (2007). Both papers consider the movie industry. Einav (2007) uses a structural model to distinguish between seasonal demand and supply effects. Sales could be higher because more people go to the movies in holiday seasons or because better movies are screened during these periods. The view on seasonality discussed in Radas and Shugan (1998) comes closest to the setting in this chapter. These authors interpret seasonality as a time transformation process, that is, it is as if the service or product ages more quickly along its life cycle in peak seasons. In off-peak seasons it is as if the product ages more slowly. The resulting diffusion model is to some extent in line with the Generalized Bass Model (Bass *et al.*, 1994). However, the seasonal structure, that is, the set of periods that corresponds to seasonal peaks and troughs, is imposed rather than estimated in their model. The proposed model, as well as the benchmark seasonal models, allow for a selection of the seasonal structure based on the observed diffusion process. In the model presented in Section 4.4 we even make the intertemporal demand shift pattern endogenous, that is, we estimate the proportion in which one month contributes to the seasonal peak in another month. Finally, the paper of Radas and Shugan (1998) does not consider the impact of seasonality on the estimation and interpretation of the standard diffusion parameters.

In the following subsection we build up to the proposed methodology. Next, in subsection 4.3.2, we discuss the alternative approaches, and indicate why these approaches are less satisfactory. Further, we deal with the consequences of ignoring seasonality.

4.3.1 The proposed model

The aim of this chapter is to create a seasonal model that is consistent with any (closed-form) diffusion model[3]. To emphasize this we first start with a general closed-form function for the cumulative diffusion curve, $F(t)$. $F(t)$ specifies the cumulative fraction of adopters relative to the market potential at time t. At the end of this subsection we will use the Bass (1969) model as the leading example for the functional form of $F(t)$.

We first discuss two general features of diffusion modeling: linking $F(t)$ to observed sales, and heteroscedasticity. Concerning the former, we adopt the non-linear estimation technique of Srinivasan and Mason (1986)[4]. In their paper the authors measure sales as the difference in the cumulative adopters between period t and $t-1$, multiplied by the eventual number of adopters. Although Srinivasan and Mason (1986) do this specifically for the Bass model, this estimation technique could be used for other closed-form solutions of $F(t)$ as well.

[3]In some cases, where there is no closed-form solution, the differential equation for the diffusion model can be calculated numerically. The numerical solution can then be used instead of the closed-form solution. In these cases the seasonal method we present in this chapter can also be used. In the rest of the chapter, however, we focus on the cases with a closed-form solution.

[4]The Srivivasan Mason method corresponds to non-linear least squares estimation. Under the assumption of normality of the error terms this is equivalent to Maximum Likelihood estimation.

The second issue is heteroscedasticity. Recently, Boswijk and Franses (2005) addressed possible deviations from the underlying S-shape. They introduced a specification that contains a heteroscedastic error process and a tendency for the diffusion to return towards its equilibrium growth path. The heteroscedasticity implies that larger fluctuations are more likely to occur around the moment of peak sales. Although the seasonal models introduced in this chapter can also be specified without heteroscedasticity, we believe that heteroscedasticity occurs in almost every empirical diffusion process. Heteroscedasticity affects the estimation of the seasonal structure. In particular, it helps to disentangle seasonality from random shocks.

The dependent variable in all the models in this chapter is the monthly sales[5] of a new product at month t, S_t. The basis for the proposed model, as well as for the alternatives presented later, is

$$S_t = m(F(t) - F(t-1)) + \varepsilon_t \qquad \varepsilon_t \sim N(0, f(t)^2\sigma^2), \tag{4.1}$$

where $F(t)$ is the fraction of cumulative adopters at time t, and $f(t)$ is the fraction of current adopters. $F(t)$ and $f(t)$ are the solutions of the differential equation associated with a continuous-time diffusion model. The m is the parameter capturing the market potential, that is, the ceiling of the typical S-shaped sales curve. The variance of the error term is proportional to $f(t)^2$. This variance specification is slightly different from that proposed by Boswijk and Franses (2005), as they scale the variance with the square of the sales of the previous period. The advantage of the specification in this chapter is that it leads to a smoother pattern for the variance, especially in case of seasonality[6].

To model seasonal peaks and troughs we need to increase or decrease the sales in some months relative to the general specification in (4.1). The seasonal effect should be proportional to the speed and position of the diffusion process (see also Figure 4.1). This proportionally additional effect of seasonality can be represented as

$$S_t = m(F(t) - F(t-1))(1 + \sum_{k \epsilon K} \delta_k D_{kt}^{01}) + \varepsilon_t \qquad \varepsilon_t \sim N(0, f(t)^2\sigma^2), \tag{4.2}$$

where D_{kt}^{01} represents a zero-one dummy for each month k in the set K, where K can consist of one or more months. To put it more formally,

$$D_{kt}^{01} = \begin{cases} 1 & \text{if observation } t \text{ is in month } k\text{, that is, } \kappa(t) = k \\ 0 & \text{otherwise,} \end{cases} \tag{4.3}$$

[5] In this chapter we take months as the frequency of the observed data, as the empirical data we use also has a monthly frequency. Further, in the diffusion literature a month is the most often used data interval for which seasonality is relevant. Of course, the model can be considered for any other data interval.

[6] At first sight this variance specification may seem to lead to an endogeneity problem. However, it does not as $f(t)$ only depends on the parameters, not on the data itself.

where $\kappa(t)$ gives the month number corresponding to observation t. In this formulation there is a maximum of eleven months that can be included in the set K, because otherwise the model parameters would not be identified.

In (4.2) the summation of the seasonal effects over a year is not necessarily equal to zero. This affects the interpretation of the diffusion parameters, and especially m. As we will show below Model (4.2) introduces a bias in the parameter estimates.

To avoid this bias we need to introduce the seasonal pattern in such a way that it does not interfere with the underlying S-shape. Thus, the added seasonal effect should have mean zero. This means that the additional sales at a seasonal peak should be compensated in other months. For monthly data we can define dummies such that the effect in the focal month is still 1, while the effect in the other months is minus 1/11. Over an entire year this results in a dummy that has mean zero. This zero-mean dummy is formally defined as

$$D_{kt}^{ZM} = \begin{cases} 1 & \text{if } \kappa(t) = k \\ -\frac{1}{11} & \text{otherwise.} \end{cases} \tag{4.4}$$

This zero-mean dummy (D^{ZM}) can replace the zero-one dummy (D^{01}) in (4.2). In Section 4.3.2 we will show that the resulting model has some preferable features. However there is a counter-intuitive feature as well. Consider again the case of a seasonal peak at some period t corresponding to month k. The additional sales equals $m\delta_k(F(t) - F(t-1))$. In other words, it is a fraction of the sales predicted by the underlying diffusion model. The "compensating" decrease in sales in the next month is $\frac{1}{11}m\delta_k(F(t+1) - F(t))$. The counter-intuitive aspect here is that the compensation is associated with the "predicted" sales for the *next* period. Intuitively it would be more appealing if the compensation equals $\frac{1}{11}m\delta_k(F(t) - F(t-1))$, that is, a fraction of the current increase itself. Stated differently, although the dummies have mean zero, the seasonal effect itself, $m(F(t) - F(t-1))\delta_k D_{kt}^{ZM}$, does not have mean zero. If the diffusion is relatively fast, this may have a substantial impact, as we will also show below.

In the final model we want to correct for this above-mentioned counter-intuitive feature. As a result we impose that a seasonal peak originates from consumers delaying or speeding up their purchases. In this case the additional sales during a seasonal peak is the summation of the postponed and accelerated purchases of the other months. Hence, the seasonal peak equals the sum of a fraction of the underlying adoption curve of all the months influencing the focal month.

First we define the set of months that influence a focal month k. We denote this set as H_k, the number of elements in H_k is denoted by $|H_k|$. For example, if $H_k = \{-3, -2, -1, 1, 2\}$ a fraction of the sales from up to three months before the focal month are delayed to the focal month, and a fraction of the sales from up to two months after are accelerated towards the focal month, and $|H_k| = 5$. For monthly data, one may also consider all the other months, that is, for example, $H_k = \{-6, \ldots, -1, 1, \ldots, 5\}$.

The seasonal diffusion model now becomes

$$S_t = m\left[F(t) - F(t-1) + \sum_{k\epsilon K} \delta_k |H_k| \left(D_{kt}^{01} \sum_{h\epsilon H_k} f(t+h) - D_{kt}^{OM} f(t)\right)\right] + \varepsilon_t, \qquad (4.5)$$

where still $\varepsilon_t \sim N(0, f(t)^2\sigma^2)$. The first dummy, D_{kt}^{01}, is a 0/1 dummy as used in equation (4.3). The second dummy is defined as

$$D_{kt}^{OM} = \begin{cases} 1 & \text{if period } t \text{ influences month } k\text{, that is, } (\kappa(t) - k) \in H_k \\ 0 & \text{otherwise.} \end{cases} \qquad (4.6)$$

The corresponding part of the specification concerns the decrease in sales at time t due to individuals delaying or speeding up their purchase. The first dummy, D_{kt}^{01} in (4.5) makes sure that the delayed and accelerated sales are added to the sales of the focal month. The summation $\sum_{h\epsilon H_k} f(t+h)$ sums the sales from the months influenced by the focal period. The seasonal effect parameter (δ_k) is multiplied by the number of elements in H_k, that is, $|H_k|$. This ensures that δ_k has a comparable interpretation to the previous models. More specifically, no matter which other months (hence the acronym OM) influence a seasonal peak, the resulting δ_k parameter is comparable to that in the other models.

An additional advantage of model (4.5) is that the parameters for all months are identified. In case the monthly effects are not strong, a model with twelve monthly dummies is of course not advisable, but the intuition behind the formulation is in this case still preferable.

Model (4.5) can be applied to any diffusion model with a closed-form solution. In Table 4.1 we give an overview of possibly relevant diffusion models.

For expository purposes we use the Bass model in the rest of the chapter. The closed-form solution of the Bass (1969) model is

$$F(t) = \frac{1 - \exp\{-(p+q)t\}}{1 + \frac{q}{p}\exp\{-(p+q)t\}}, \text{ and } f(t) = \frac{\frac{(p+q)^2}{p}\exp\{-(p+q)t\}}{(1 + \frac{q}{p}\exp\{-(p+q)t\})^2}. \qquad (4.7)$$

where p and q are the traditional Bass parameters capturing innovation and imitation, respectively.

4.3.2 Alternative approaches

In this subsection we briefly discuss some alternative approaches to the proposed model (4.5). We consider the (i) the standard Bass model without seasonality; (ii) the Bass model with additive dummies; (iii) the Generalized Bass model with zero-one seasonal dummies; and (iv) the Generalized Bass model with zero-mean seasonal dummies. In this section we give arguments for the strengths and weaknesses of these alternative approaches. All

Table 4.1: Diffusion models with a closed-form solution of the differential equation.

Model	Differential equation	Closed-form solution	Explanation parameters
Bass (1969)	$\frac{dF}{dt} = (p + qF)(1 - F)$	$F = \frac{1-\exp\{-(p+q)t\}}{1+\frac{q}{p}\exp\{-(p+q)t\}}$	p = innovation parameter q = imitation parameter
Gompertz Curve (Hendry, 1972; Dixon, 1980)	$\frac{dF}{dt} = qF\ln(\frac{1}{F})$	$F = \exp\{-\exp\{-(c + qt)\}\}$	c = constant q = imitation parameter
Mansfield (1961)	$\frac{dF}{dt} = qF(1 - F)$	$F = \frac{1}{1+\exp\{-(c+qt)\}}$	c = constant q = imitation parameter
Nelder (1962) (see also McGowan, 1986)	$\frac{dF}{dt} = qF(1 - F^{\Phi})$	$F = \frac{1}{[1+\Phi\exp\{-(c+qt)\}]^{1/\Phi}}$	c = constant q = imitation parameter Φ = shape parameter
Bertalanffy (1957) (see also Richards, 1959)	$\frac{dF}{dt} = \frac{q}{1-\theta}F^{\theta}(1 - F^{\theta})$	$F = [1 - \exp\{-(c + qt)\}]^{1/(1-\theta)}$	c = constant q = imitation parameter θ = shape parameter
Stanford Research Institute (e.g. Teotia and Raju, 1986)	$\frac{dF}{dt} = \frac{q}{t}F(1 - F)$	$F = \frac{1}{1+(\frac{T*}{t})^{q}}$	q = imitation parameter $T*$ = moment of peak sales
Flexible Logistic Growth (Bewley and Fiebig, 1988)	$\frac{dF}{dt} = q[(1 + kt)^{1/k}]^{\mu-k}$	$F = \frac{1}{1+\exp\{-(c+qt(\mu,k))\}}$	c = constant q = imitation parameter $t(\mu, k)$ = based on μ and k this model has different shapes
Gamma Shifted Gompertz Curve (Bemmaor, 1994)	-	$F = \frac{1-\exp\{-(p+q)t\}}{[1+\frac{q}{p}\exp\{-(p+q)t\}]^{\alpha}}$	p = innovation parameter q = imitation parameter α = shape parameter

Note: this overview is based on earlier overviews of Mahajan *et al.* (1990, 1993), complemented with the Gamma Shifted Gompertz Curve from Bemmaor (1994).

features of the alternative models discussed in this subsection are validated by a simulation study, which we include in a web appendix.

Of course, a first thought would be to ignore seasonality altogether. In general, if seasonality is not properly dealt with, the parameter estimates are biased. However, it turns out that if the full diffusion process is used for parameter estimation, this problem disappears to a large extent for almost all relevant cases. The seasonal fluctuations will then simply be seen as (large) errors. The symmetry in the diffusion curve helps to identify the underlying diffusion curve. Therefore, fitting a basic non-seasonal Bass curve to a completed diffusion series, that is, where sales have become zero at the end, yields sensible results. However, if the diffusion process is before its saturation level and seasonality is not explicitly modeled, parameter estimates are likely to be strongly biased.

One could also extend the Bass model with additive dummies, that is,

$$S_t = m(F(t) - F(t-1)) + \sum_{k \in K} \delta_k D_{kt}^{01} + \varepsilon_t. \tag{4.8}$$

However, in such a model the size of the seasonal effect is the same throughout the diffusion process. Furthermore, seasonal peaks will keep occurring even as $t \to \infty$. These two aspects rule out the practical use of this specification. Below we will therefore only consider models where the seasonality appears in a multiplicative form.

We compare the model with two seasonal alternatives. Both these alternatives are inspired on the Generalized Bass Model, where instead of marketing-mix variables we use seasonal dummies as explanatory variables. Both these seasonal GBM models are closely related to the proposed model.

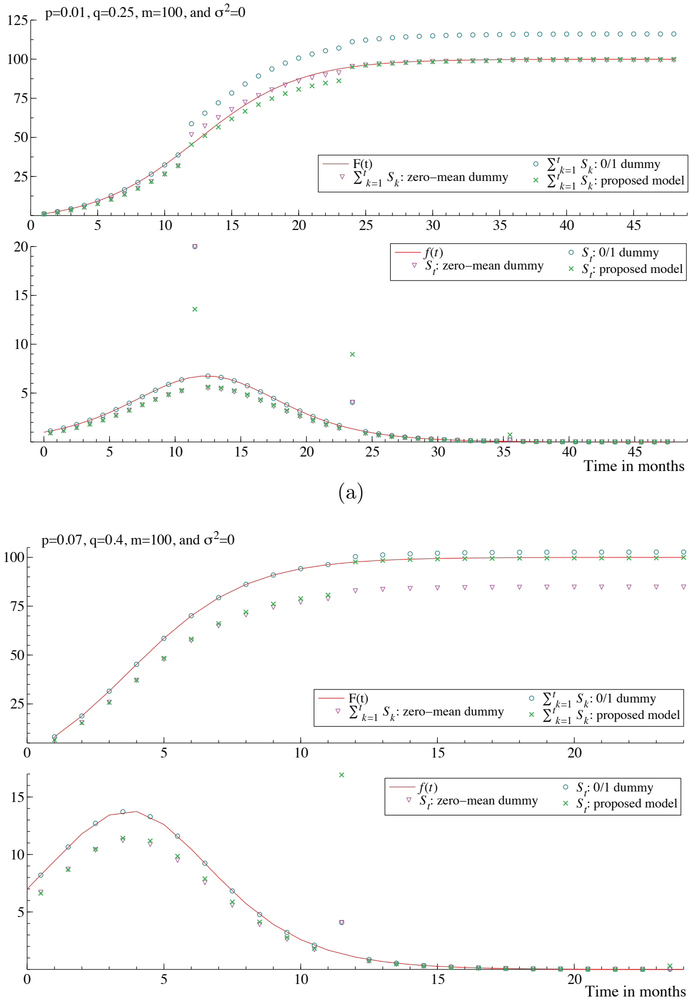
The first seasonal alternative is the GBM with a zero-one dummy, which we presented in equation (4.2), and which we call the SGBM_{01}. The second seasonal alternative is the GBM model with zero-mean dummies. This model is the same model as the SGBM_{01}, but now with a zero-mean dummy (4.4) instead of the zero-one dummy. This model we call the SGBM_{ZM}. Both seasonal models assume that seasonal effects are largest around the moment of peak sales.

In Figure 4.2 we give two examples in which the main differences between the proposed model and the two seasonal GBMs are illustrated. The top two graphs show the (cumulative) diffusion curves for the case $p = 0.01$, $q = 0.25$, $m = 100$. The other two graphs correspond to a speedy adoption, that is, $p = 0.07$, $q = 0.4$, and $m = 100$. In both cases a seasonal peak occurs every 12^{th} period, say each December. We set $\sigma^2 = 0$. For the proposed model, we specify the seasonal structure such that the additional sales in December are a result of postponed sales in earlier months of the same calendar year. Overall, a model should follow the basic shape that is implied by the Bass parameters. The figures therefore also show both $F(t)$ and $f(t)$.

From Figure 4.2a it is clear that SGBM_{01} is not appropriate. In this model the sales are equal to $m(F(t) - F(t-1)) \approx mf(t)$, but for each 12^{th} month the sales are higher. Therefore, the cumulative sales in this model eventually exceed m. In other words, the market potential parameter in this model does not have its natural interpretation. Note that this is a different kind of bias than the "traditional" estimation bias. In fact, if one generates data with the SGBM_{01}, and estimate it with the SGBM_{01} itself one will find the same parameter estimate as in the data generating process. In this sense there is no "traditional" bias. However, the actual ceiling of the cumulative sales will be higher than the market potential estimate, where this estimate should represent the ceiling. Hence, the SGBM_{01} model finds an inappropriate estimate of the market potential.

The difference between the SGBM_{ZM} and the proposed model Figure 4.2a is more subtle. As we assume that the additional sales are a result of postponed sales, the cumulative sales at the end of every last month of the year are exactly equal to $mF(t)$. This is not the case for the SGBM_{ZM}. The seasonal peaks also differ, which is also due to the fact that in the proposed model seasonal peaks are a result of postponed sales. If consumers postpone sales to the seasonal peak, seasonal peaks before the inflection point must be small relative to seasonal peaks after this point. Both curves in the end attain m and in that sense the differences between the two models are not extremely large.

Figure 4.2: Two examples of differences between the SGBM with 0/1 dummies, the SGBM with zero-mean dummies, and the proposed model (OM_{fixed}).



(b)

Figure 4.2b shows the case where the seasonal peak occurs after the inflection point of the underlying diffusion curve. As the zero-mean dummy is positioned relative to the current adoptions, the resulting seasonal peak in $SGBM_{ZM}$ is too low relative to the postponed sales. This implies that the cumulative sales do not reach m. In this situation the $SGBM_{ZM}$ will therefore also give misleading parameter estimates.

In Figure 4.2 we assumed that the seasonal peak only consists of postponed sales. This clearly shows the differences between the proposed model and the SGBM_{ZM}. If the seasonal peak originates from consumers delaying *and* speeding up their purchase, the differences between these two models, including the bias in SGBM_{ZM}, become smaller. Note that the problem in SGBM_{01} does not depend on the intertemporal pattern.

Summarizing, in this subsection we discussed four alternatives to the proposed seasonal model. The model with an additive dummy clearly is not a good option. Further, the standard Bass model, ignoring the seasonal fluctuations, gives biased results, unless the full diffusion process is used. The graphs in Figure 4.2 show that the SGBM_{01} is theoretically not appropriate. Furthermore, there are potentially substantive differences between SGBM_{ZM} and the proposed model. In some cases the overall shape of the diffusion process for SGBM_{ZM} closely resembles that generated by the proposed model and the real data.

4.4 Intertemporal structure of seasonality

In the previous section we assumed that the pattern of purchase acceleration or postponement is given. The model allowed to specify the moments of seasonal peaks through the set K, and allowed to specify the origin of seasonal peaks through the sets $H_k,\ k = 1, \ldots, K$. In particular, we assumed that the additional sales in the seasonal peak is proportionally drawn from all months in the set H_k. In this section we will relax this assumption and propose a method to estimate the pattern of the intertemporal shifts.

We suggest a functional form for the intertemporal demand shift pattern that is flexible and parsimonious. The two main features to capture are: (i) the percentage of sales due to purchase acceleration versus postponement; (ii) the extent to which months contribute to the sales of a focal month.

The specification we propose introduces three additional parameters. The parameter, $0 \leq \theta \leq 1$, indicates the relative influence on the focal month of the months before (purchase postponement). The relative influence of the months after the focal month (purchase acceleration) is given by $1 - \theta$. For the period before the focal month, the relative influence of l months prior to the focal month is λ_1^l. For l months after the focal month the relative influence is λ_2^l. Both parameters are restricted to the $(0, 1]$ interval. Finally, we normalize the weights by considering the relevant months before, denoted by the set H_k^-, and after the seasonal month, denoted by H_k^+. For example, $H_k^- = \{-2, -1\}$ indicates that the two months just before the seasonal peak contribute to the seasonal sales. Note that the setting $\theta = |H_k^-|/(|H_k^-| + |H_k^+|)$ and $\lambda_1 = \lambda_2 = 1$ corresponds to the model specification we had before, that is, equal weights for all months in $H_k = H_k^- \cup H_k^+$.

The combination of the weights gives the function

$$g_k^{flex}(t) = \begin{cases} \theta \frac{\lambda_1^{-(\kappa(t)-k)}}{\sum_{h\epsilon H_k^-} \lambda_1^{-h}} & \text{if } t \text{ is before the focal month } k\text{, that is, } (\kappa(t)-k) \in H_k^- \\ (1-\theta)\frac{\lambda_2^{(\kappa(t)-k)}}{\sum_{h\epsilon H_k^+} \lambda_2^{h}} & \text{if } t \text{ is after the focal month } k\text{, that is, } (\kappa(t)-k) \in H_k^+ \\ 0 & \text{otherwise.} \end{cases} \tag{4.9}$$

This function replaces the dummy D^{OM} in the proposed model (4.5). The parameter θ indicates the relative influence of the period before the focal month in determining the seasonal peak. If $\theta = 1$ only the period before the focal month determines the peak. Note that if θ equals 0 or 1, one of the two λ parameters is not identified. The fact that $0 < \lambda_i \leq 1,\ i = 1, 2$ makes sure that the effect for months further from the focal month does not increase. In Figure 4.3 we show the resulting seasonal pattern for different parameter settings, where we arbitrarily choose the focal month to be December. In these examples, as well as in the empirical section, we include all eleven months before and after this focal month in the set H_k. Note that this does not necessarily mean that all these months influence the focal month, as this depends on the shape of $g_k^{flex}(t)$. The figure shows that we can deal with a wide variety of patterns. The figure also shows that smaller λ parameters result in more mass in the months close to the focal month.

The second part of the seasonal effect defines the peak of the focal month, and this means that it should be a summation of "lost" sales in the months in H_k. The model is now as follows:

$$S_t = m\Bigg[F(t) - F(t-1) + \sum_{k\epsilon K} \delta_k \left(D_{kt}^{01} \sum_{h\epsilon H_k} g_k^{flex}(t+h) f(t+h) - g_k^{flex}(t) f(t) \right) \Bigg] + \varepsilon_t, \tag{4.10}$$

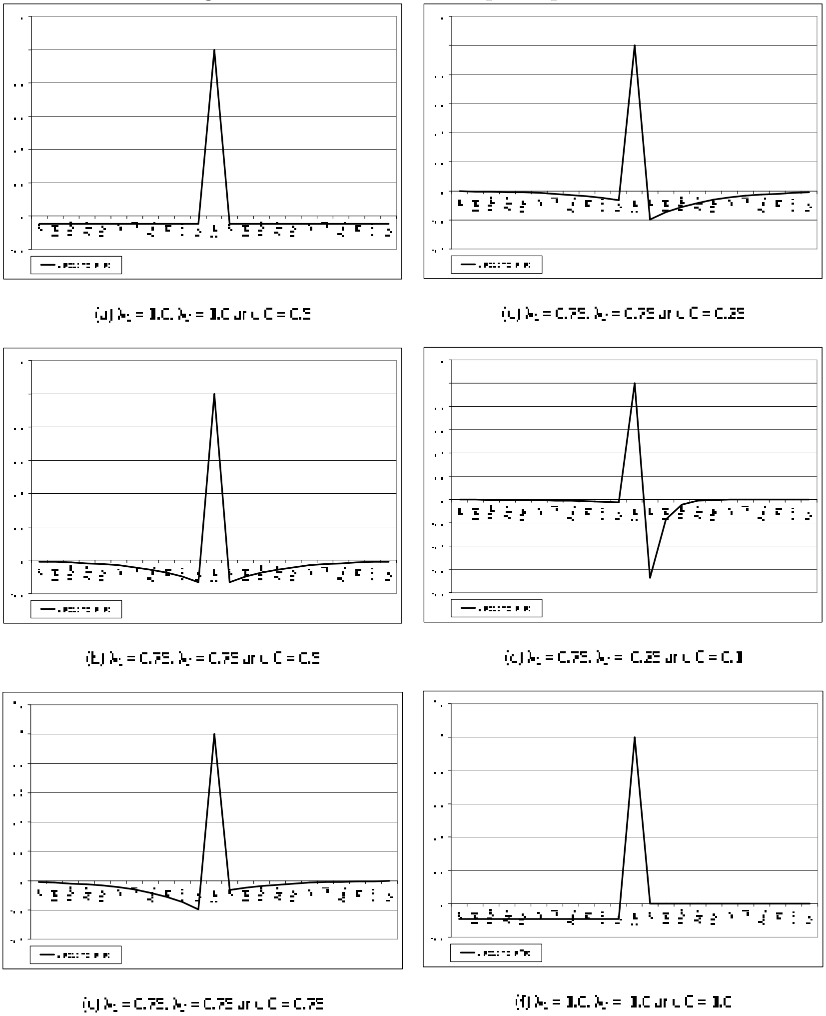
where $\varepsilon_t \sim N(0, f(t)^2\sigma^2)$.

In the next section we will use actual data to compare the proposed model to the benchmark models. We will consider models with fixed and with flexible seasonal patterns.

4.5 Empirical illustration

In this section we consider the performance of various models for the actual diffusion data, which we presented in Section 4.2. The data concern the sales figures (in millions) of flat-screen televisions for ten countries in Europe and cover February 2004 until January 2010. The first months of the diffusion process are not available. This has no major consequences for the estimation procedures, which is one of the benefits of the Srinivasan and Mason

Figure 4.3: Several intertemporal patterns.



Notes: λ_1 determines the pattern before the focal month, λ_2 determines the pattern after the focal month and θ determines the relative importance of these parts on the focal month. Note that smaller λ parameters result in more mass in the months near the focal month.

(1986) approach. With this approach it is only necessary to know the number of months since introduction (Jiang *et al.*, 2006).

We allow different countries to have different seasonal structures. Checking all model combinations for each country is a cumbersome and unnecessary task. In particular, we therefore limit the number of seasonal structures for each model. First we use *visual inspection*, as for some months it is clear that there is a seasonal peak each year. For example each country seems to have a January and December peak. Second, we *limit the number of seasonal dummies* we consider by excluding some months from further consideration. With a large number of seasonal dummies the model potentially suffers from identification issues or over-fitting. Third, we *consider models with various seasonal structures.* After the first two selection steps there are some 60 potential seasonal structures. Fourth, we *reject some seasonal combinations*, because for some of the seasonal structures the estimation routines do not converge, mainly when there are too many seasonal dummies. In other cases the model produces unlikely or even improbable parameter estimates. The final step is to *choose the best seasonal combination.* For this step we select from the remaining models the model with the smallest value of the Bayesian Information Criterion [BIC]. We use BIC as it penalizes the number of parameters more than other information criteria.

The models we compare are the two seasonal GBM models (SGBM$_{ZM}$ and SGBM$_{01}$), the standard Bass model (BM), and the two versions of the proposed model (to be labeled as OM$_{fixed}$ and OM$_{flex}$). The OM$_{fixed}$ has the given intertemporal demand shift pattern as in previous sections, that is $H_k = \{-6, \ldots, -1, 1, \ldots, 5\}$. In OM$_{flex}$ we estimate the intertemporal demand shift pattern using the model in (4.10).

The results are summarized in Tables 4.2 to 4.4 and in Figure 4.4. Table 4.2 gives the estimated moment of peak sales ($T^* = \frac{\log(q/p)}{(p+q)}$) and market potential based on the different models per country. Table 4.3 gives the in-sample performance of the models based on information criteria. To be more precise, this table gives the AIC and BIC values for the flexible model, whereas the other values represent the relative performance of the other models. A positive percentage indicates a higher information criterion and thus a worse in-sample fit. In Figure 4.4 we give a graphical insight in this in-sample fit for the United Kingdom and the Netherlands. Finally, in Table 4.4 a more complete overview of the results for the same two countries is shown[7].

The results in Table 4.2 show that the Bass model finds similar moments of peak sales, that is, the difference with the seasonal models is only 1 or 2 months. For the market potential the difference is often between 3% and 7%. If we look more closely at the estimation results for the Netherlands and the United Kingdom (Table 4.4), we see that parameters from the Bass model have larger confidence intervals, compared to the seasonal models. Based on in-sample fit (Table 4.3), the Bass diffusion model is outperformed by the seasonal models by a large margin.

If we now evaluate the seasonal models, we first find that the major difference between the seasonal models concerns the market potential (Table 4.2), where the proposed two models and SGBM$_{ZM}$ give a higher estimate than SGBM$_{01}$. This matches with the dis-

[7]The tables and figures of the other eight countries are excluded from the chapter, but can be obtained from the authors upon request.

Table 4.2: The moment of peak sales in months (T^*) and market potential in millions per country (m), as estimated by the different models.

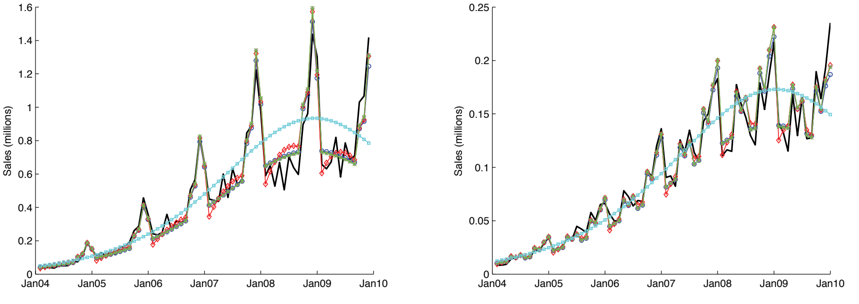
	T^*					Market Potential (m)				
	OM_{fixed}	OM_{flex}	$SGBM_{ZM}$	$SGBM_{01}$	BM	OM_{fixed}	OM_{flex}	$SGBM_{ZM}$	$SGBM_{01}$	BM
United Kingdom	111.48	110.91	111.33	111.39	111.98	49.81	49.23	49.68	40.39	50.90
France	117.87	117.33	117.70	117.74	119.74	38.20	37.79	38.05	31.31	40.94
Germany	117.24	116.72	117.09	117.13	118.35	41.11	40.80	40.98	35.12	43.02
Netherlands	100.16	99.69	100.02	100.08	100.94	10.00	9.95	9.98	8.22	10.30
Italy	124.02	123.57	123.86	123.89	125.40	37.90	37.56	37.78	32.30	40.17
Spain	114.98	114.50	114.88	114.90	115.96	24.55	24.32	24.50	22.11	25.53
Austria	91.81	91.40	91.64	91.68	92.41	3.96	3.93	3.95	3.40	4.08
Belgium	111.44	111.06	111.34	111.37	112.22	4.60	4.56	4.59	4.04	4.76
Portugal	105.26	104.64	105.07	105.11	106.99	3.92	3.86	3.90	3.28	4.20
Scandinavia	106.50	106.28	106.43	106.46	107.20	11.41	11.38	11.40	9.99	11.66

Table 4.3: In-sample performance (AIC and BIC) of diffusion models for flat-screen televisions in Europe.

	OM_{flex} (Absolute)		OM_{fixed}		$SGBM_{ZM}$		$SGBM_{01}$		BM	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
United Kingdom	-230.7	-221.7	0.5%	0.5%	0.5%	0.5%	0.4%	0.5%	57.5%	58.8%
France	-272.8	-263.7	0.2%	1.0%	1.2%	1.2%	1.2%	1.2%	42.0%	42.6%
Germany	-252.5	-241.1	-1.3%	-2.3%	-1.4%	-2.4%	-1.4%	-2.4%	29.4%	28.9%
Netherlands	-492.4	-476.5	0.9%	1.0%	0.8%	0.8%	0.7%	0.7%	20.3%	19.0%
Italy	-280.0	-270.9	0.7%	0.7%	0.6%	0.6%	0.6%	0.6%	29.1%	29.3%
Spain	-298.7	-291.9	1.7%	1.8%	1.6%	1.6%	1.6%	1.6%	16.4%	16.8%
Austria	-604.4	-590.9	-0.2%	-1.0%	-0.3%	-1.1%	-0.3%	-1.1%	16.3%	15.5%
Belgium	-522.1	-515.3	1.1%	1.1%	0.9%	0.9%	0.9%	0.9%	12.4%	12.5%
Portugal	-592.2	-583.2	0.3%	0.7%	0.2%	0.5%	0.2%	0.5%	16.6%	16.5%
Scandinavia	-343.4	-338.9	1.2%	1.9%	1.2%	1.9%	1.2%	1.9%	17.8%	18.7%

Note: The results for the proposed model with flexible intertemporal demand [OM_{flex}] are the absolute values of the information criteria, for the other models the results are given relative to the flexible model. A positive percentage represents a higher AIC or BIC. For example, the in-sample performance for the United Kingdom is best for the flexible model, because the information criteria are higher for the other models.

Figure 4.4: The in-sample fit for two countries (a) United Kingdom (b) The Netherlands.



(a) (b)

Note: The lines represent, the actual sales (black), OM_{flex} (red, diamonds), OM_{fixed} (blue, circles), $SGBM_{ZM}$ (magenta, pluses), $SGBM_{01}$ (green, x's) and the standard Bass model (light blue, squares)

cussion in Section 4.3.2. Another difference is that the seasonal parameters are estimated to be higher for the $SGBM_{01}$, which is partly due to the fact that this model uses a different (lower) underlying diffusion curve. For OM_{flex} we have sometimes selected a slightly different set of seasonal effects. The latter is partly explained by the difference across the models in the months influenced by a seasonal peak. To get a similar seasonal peak, the magnitude of the seasonal effect can be slightly different in OM_{flex} versus OM_{fixed}.

Based on the in-sample performance (Table 4.3), the flexible model outperforms the other seasonal models in eight of the ten countries, as the other models have higher values of the information criteria. For the other three seasonal models this fit is similar. Figure 4.4 shows that the seasonal models fit the actual in-sample data well. The standard Bass

Table 4.4: Results of the diffusion model of the flat-screen television in the United Kingdom and the Netherlands, standard errors between parentheses.

	United Kingdom					Netherlands				
	OM_{fixed}	OM_{flex}	$SGBM_{ZM}$	$SGBM_{01}$	BM	OM_{fixed}	OM_{flex}	$SGBM_{ZM}$	$SGBM_{01}$	BM
p	2.2E-05	2.2E-05	2.2E-05	2.2E-05	2.0E-05	7.9E-05	8.1E-05	8.1E-05	8.0E-05	7.6E-05
	(2.1E-06)	(2.2E-06)	(2.3E-06)	(2.3E-06)	(5.1E-06)	(5.1E-06)	(5.9E-06)	(5.4E-06)	(5.4E-06)	(1.0E-05)
q	0.073	0.073	0.073	0.073	0.073	0.067	0.067	0.067	0.067	0.067
	(0.001)	(0.002)	(0.001)	(0.001)	(0.004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)
m	49.81	49.23	49.68	40.39	50.90	10.00	9.95	9.98	8.22	10.30
	(1.49)	(5.12)	(1.34)	(1.30)	(3.94)	(0.26)	(0.94)	(0.23)	(0.25)	(0.54)
Jan	0.48	0.61	0.50	0.67	-	0.49	0.56	0.50	0.67	-
	(0.06)	(0.21)	(0.06)	(0.08)	-	(0.05)	(0.21)	(0.05)	(0.06)	-
May	-	-	-	-	-	0.23	0.20	0.23	0.31	-
	-	-	-	-	-	(0.05)	(0.06)	(0.04)	(0.06)	-
Jun	-	-	-	-	-	0.12	0.09	0.12	0.16	-
	-	-	-	-	-	(0.04)	(0.05)	(0.04)	(0.06)	-
Jul	-	-	-	-	-	0.18	0.13	0.18	0.24	-
	-	-	-	-	-	(0.04)	(0.06)	(0.04)	(0.06)	-
Oct	0.28	0.26	0.29	0.39	-	0.30	0.29	0.30	0.40	-
	(0.06)	(0.10)	(0.05)	(0.07)	-	(0.05)	(0.12)	(0.04)	(0.06)	-
Nov	0.37	0.39	0.38	0.51	-	0.19	0.23	0.20	0.26	-
	(0.06)	(0.12)	(0.05)	(0.07)	-	(0.05)	(0.12)	(0.04)	(0.06)	-
Dec	0.86	0.94	0.87	1.17	-	0.39	0.44	0.39	0.52	-
	(0.07)	(0.26)	(0.06)	(0.08)	-	(0.05)	(0.16)	(0.04)	(0.06)	-
λ_1	-	0.73	-	-	-	-	0.70	-	-	-
	-	(0.10)	-	-	-	-	(0.12)	-	-	-
λ_2	-	0.78	-	-	-	-	0.58	-	-	-
	-	(0.12)	-	-	-	-	(0.12)	-	-	-
θ	-	0.68	-	-	-	-	0.69	-	-	-
	-	(0.26)	-	-	-	-	(0.16)	-	-	-
RMSE	0.069	0.072	0.069	0.069	0.189	0.010	0.011	0.010	0.010	0.024
AIC	-229.6	-230.7	-229.7	-229.7	-98.1	-487.9	-492.4	-488.7	-488.9	-392.7
BIC	-220.6	-221.7	-220.6	-220.7	-91.4	-471.9	-476.5	-472.8	-472.9	-385.8

model only manages to roughly fit the underlying curve. The fitted curve does not match the seasonal months nor the non-seasonal months.

Table 4.4 and Figure 4.4 also show that seasonal patterns differ across countries. These differences are present in the seasonal components included in the model, as well as in the levels of the seasonal parameters. For example, the Netherlands have many seasonal fluctuations compared to the United Kingdom, respectively 7 and 4 peaks. However, the peaks in the Netherlands are relatively small compared to that of the United Kingdom. The seasonal models can capture these different seasonal patterns across countries. Additionally, from Figure 4.4 it seems that the flexible model we propose captures the periods between the peaks better than the other seasonal models.

Further, there are differences in the intertemporal demand shift parameters (mainly λ_2). In comparing the seasonal components, we note that the selected seasonal dummies can differ across the models we considered. However, for the United Kingdom and the Netherlands the optimal structure was similar across models.

Concerning the shape of the intertemporal demand shift, the λ parameters are between 0.57 and 0.85 for all countries. This means that the effect on the focal month becomes negligible after five to eight months from the focal month. Further, θ across countries is between 0.67 and 0.76, which means that more consumers postpone than speed up their purchase, which is an interesting and managerially relevant insight.

In Table 4.5 the models are compared on their out-of-sample performance, relative to the flexible model. In this table we compare the root mean squared prediction error (RMSPE) for up to 3 or 6 months ahead out-of-sample forecasts. The table gives the RMSPE for the flexible model. The values for the other models are relative to this model, where a higher percentage represents a higher RMSPE and thus worse out-of-sample performance. To obtain these RMSPE values we re-estimated the models, after leaving out the last three or six months of the sales figures. From Table 4.5 we find that in terms of out-of-sample forecasting the seasonal models OM_{fixed}, SGBM_{ZM} and SGBM_{01} perform almost equally well. This suggests that for short-term prediction all seasonal models can be useful. The Bass model is outperformed by all seasonal models.

The flexible model outperforms most models on the three-month-ahead prediction horizon. However, for six-month-ahead forecasting the flexible model performs less good, as it only outperforms the other seasonal models in 5 out of 10 cases. The reason for this might be that for this longer horizon there is less data left for estimation, and the flexible model by definition asks more from the data. This is supported by the fact that in these cases the flexible model also performs less good in sample. This shows that we can only accurately estimate the intertemporal pattern if there is enough data. The results show that in these cases the model with a fixed pattern is more suitable, as it outperforms the other models on fit.

Next to the increased fit and the better short-term forecasting, the model also allows to study the seasonal structure across countries. Of course the quality of generalizing

Table 4.5: Forecasting performance of diffusion models for flat-screen televisions in Europe, for up to three or six month ahead forecasts.

	OM_{flex} (Absolute)		OM_{fixed}		$SGBM_{ZM}$		$SGBM_{01}$		BM	
	Three	Six	Three	Six	Three	Six	Three	Six	Three	Six
United Kingdom	26.18	23.98	12.26%	17.54%	9.98%	16.33%	8.66%	17.30%	139.95%	93.10%
France	41.36	29.38	-34.10%	-3.81%	-35.06%	-5.83%	-35.37%	-6.23%	31.84%	51.93%
Germany	18.44	22.08	10.59%	3.40%	6.14%	1.44%	6.65%	0.82%	132.82%	69.79%
Netherlands	4.19	4.43	17.98%	8.00%	16.82%	7.75%	15.52%	8.06%	89.95%	53.61%
Italy	20.21	32.01	10.46%	-0.94%	8.99%	-1.89%	8.27%	-1.97%	98.99%	31.62%
Spain	21.11	19.74	5.97%	-7.06%	5.05%	-8.76%	5.49%	-7.89%	44.26%	22.23%
Austria	2.33	2.04	13.65%	14.91%	10.19%	13.78%	9.88%	14.23%	107.51%	75.13%
Belgium	5.10	4.23	-10.85%	-5.35%	-12.13%	-5.15%	-11.66%	-5.45%	25.80%	16.62%
Portugal	2.51	2.67	9.93%	12.59%	4.17%	11.61%	4.83%	12.44%	117.84%	59.43%
Scandinavia	8.93	11.09	7.97%	-0.30%	7.20%	-0.49%	7.53%	-0.56%	50.22%	17.12%

The results for the proposed with flexible intertemporal demand [OM_{flex}] are the absolute RMSPEs, for the other models the results are given relative to the flexible model. A positive percentage represents a higher RMSPE. For example, the out-of-sample performance for three months ahead in the UK are best for the flexible model, because all other models have a higher RMSPE. The RMSPEs are multiplied by 100 for convenience.

conclusions is limited by the number of countries in the empirical case. To give an example of differences across countries, we compare the seasonal peaks in December and January. In the Netherlands and Belgium the seasonal peaks are higher in January compared to December, where the reverse is true for the United Kingdom and other countries. This relatively higher January peak in the Netherlands and Belgium is probably due to the fact that in the Netherlands and Belgium the prices of flat-screen televisions are often reduced in January as product line extensions are often introduced in February. Another likely factor to influence seasonal peaks is the timing of additional income, like a new year's or vacation bonus, as consumers often use these for large expenditures such as a flat-screen television.

4.6 Conclusion

In this chapter we addressed seasonality in diffusion models. The current availability of high-frequency data makes this a subject of increasing importance. We developed a seasonal structure that can be used in combination with standard diffusion models. We based the proposed models on the classic Bass diffusion model using monthly data, but the seasonal structure works with any closed-form diffusion model. Also, extensions of diffusion models can be used in combination with the proposed method (for example cross-country diffusion models, generational diffusion models and multilevel diffusion models). Concerning the data frequency, the proposed method works for any interval as long as there is periodicity in the peaks. Further, because estimated diffusion parameters are often used for the practice of 'guessing by analogy', the extension should not influence the estimates and interpretation of the underlying diffusion pattern.

Through a detailed empirical case we showed that the proposed model lives up to all these goals. In contrast, the use of the Generalized Bass Model with seasonal dummies, which may seem a straightforward way to take seasonality into account, produces biased estimates. In particular the market potential is biased. Next to the proposed model we also have put forward a variation of this Generalized Bass Model, which uses a zero-mean dummy. This variation seems to give results similar to the proposed model in most practical cases, with the additional benefit that it can be used more straightforwardly in standard statistical software packages.

The proposed model allows to estimate the seasonal pattern. The basic structure is that the seasonal peak in a focal month consists of sales drawn from the months around it. In the proposed model we made a distinction between a flexible, estimated, intertemporal demand pattern and a fixed pattern. The flexible pattern allowed to study intertemporal demand shifts, and in most cases the corresponding model outperformed the other models on in-sample fit and short-term forecasting. However, with less data the flexible pattern is less suitable, and in these cases it is safer to set a fixed pattern. In this chapter we showed that the given pattern, six months before the focal months and five after influence a seasonal peak, works well for the empirical cases in this chapter, but in practice other underlying structures are possible as well. For, example it may be true that a focal month is influenced only by the months in the quarter surrounding it. Such structures are all possible in this setup.

An additional advantage of the proposed model is that it can be used to give managers a tool to handle challenges concerning seasonal fluctuations, such as inventory management. Also, the shape parameter (λ) and balance parameter (θ) of the intertemporal pattern hold useful information. In the set of countries, however, the differences are relatively small. In future research it would be interesting to compare more countries and products.

It is obvious that ignoring seasonality is not suitable for estimating and predicting seasonal peaks. However, for the estimation of the basic diffusion parameters the current practice is often to ignore seasonality. In this chapter we found the reassuring result that, if the completed diffusion series is available, this practice indeed finds to the underlying diffusion pattern. However, if the series is truncated, this does not hold anymore.

The implications for future empirical analysis of seasonal diffusion data are threefold. First, if the goal is to find a model that can be used for short-term forecasting, all three seasonal models described in this chapter can be used. Second, if the interest is to elicit the underlying diffusion, ignoring or adjusting for seasonality seems tempting. However, this only works for completed diffusion series. The seasonal GBM with zero-one dummy definitely does not work. Finally, if the interest lies with both the correct estimation as well as short-term forecasting the models proposed in this chapter are the way to go.

4.A Simulation results

In this section we use simulations to compare the proposed model with a fixed seasonal structure to the two versions of the Generalized Bass Model: SGBM_{01} and SGBM_{ZM}. We further compare the proposed model to the Bass model without seasonality, which we call BM.

We divide the simulation study in two parts. First, we consider completed diffusion processes. Here we only look at how well the parameters are recovered. Second, we consider what happens when only part of the process is used for parameter estimation, that is, in the presence of data truncation. In this case we consider parameter estimation and forecasting performance. For both parts we use a cross design for the four models. We simulate the diffusion process based on each model and each of these simulated series is analyzed using all models.

We vary the levels of the parameters for innovation (p) and imitation (q), as these two parameters define the curvature of the diffusion process. We use four values of p and four values for q, resulting in sixteen combinations for curvature. We use the moment of peak sales ($T^* = \frac{\log(q/p)}{(p+q)}$) to represent this curvature, see Table 4.6 for the different settings. The estimated market potential will be a key parameter when comparing the models. We set the level of market potential at 100 such that deviations can be interpreted as a percentage difference relative to the real market potential. The variance of the error for the models is set to $0.0001f(t)^2$, $25f(t)^2$, or $100f(t)^2$.

We use three different seasonal structures. The first has a single monthly peak, say December, which we give a parameter value (δ_{12}) of 0.6. For the second structure we add a month, say January, with a parameter value (δ_1) of 0.3. And for the third structure we add two more monthly dummies, say June with a parameter value (δ_6) of -0.2 and November with a parameter value (δ_{11}) of 0.1.

For the proposed model we use an intertemporal demand shift pattern that is almost symmetric in months before and after the focal month, that is $H_k = \{-6, \ldots, -1, 1, \ldots, 5\}$ for all k. As mentioned before, this generates smaller differences between the proposed model and the SGBM_{ZM}.

Table 4.6: Moment of peak sales in months (T^*) for combinations of parameters p and q used in the simulation design.

		q			
		0.05	0.10	0.15	0.25
p	0.01	27	21	17	14
	0.005	42	29	22	18
	0.001	77	46	33	26
	0.0005	91	53	38	30

We consider two levels of data truncation, that is one after and one before the inflection point. The exact number of observations depends on the level of curvature. The moment of peak sales (see Table 4.6) lies a little before 50% of the total diffusion process, and therefore we make the number of data points of the full diffusion process dependent on the number of data points until the moment of peak sales.[8] The two levels of truncation are 20% before the moment of peak sales and 20% after. So, if the sample until peak sales has 50 data points, the first truncated series contains 40 data points and the second 60 data points.

In sum, the design has 6912 cells. There are 4 simulated models, 4 estimated models, 16 curvature combinations, 3 noise levels, 3 truncation levels and 3 types of seasonality. In each cell we consider 100 replications. The results presented in the subsections below show the main conclusions of this simulation study. Other results are available upon request from the authors.

4.A.1 Completed Diffusion Process

In the first part of the simulation study we focus on the case where the complete diffusion process is observed. We consider the quality of the estimates of the moment of peak sales and the market potential. To measure the quality we consider the bias and the standard deviation of the estimates across the replications. To be able to summarize the results over the various cells in the design, we calculate the bias and the standard deviation in terms of percentages relative to the true value. More precisely, we report the mean and standard deviation of $\frac{100(\widehat{T^*}-T^*)}{T^*}$ and $\frac{100(\widehat{m}-m)}{m}$. In Table 4.7 we present these two criteria. The results in this table are aggregated over all levels of curvature, seasonal months, and noise. In case of the full diffusion the differences across the levels of noise and the number of seasonal months are minimal, and only the standard deviation increases with increasing noise.

Table 4.7 shows that almost all models on average find the correct moment of peak sales, that is there is no cell above 1%. The small difference in the moment of peak sales means that these models find the correct innovation and imitation parameters. This is especially remarkable for the case where the estimated model does not account for seasonality, whereas the generated data show seasonal fluctuations.

For the market potential, we obtain a very different picture. Here there is a large difference between the SGBM_{01} and the other models. All models overestimate the market potential by about 7% if the true model is the SGBM_{01}. If the SGBM_{01} is estimated when one of the other models is the true model the reverse effect of -7% appears. Earlier we already showed that for SGBM_{01} the final level of cumulative sales is higher than m, see

[8]The number of data points of the full diffusion process is not equal to two times the data points until peak sales, as the fraction of adopters at peak sales is actually less than 50%. How much less than 50% depends on the curvature.

Table 4.7: Parameter recovery results for the full diffusion series.

		Estimated Model							
		Mean and standard deviation (between parentheses) of $\frac{100(\widehat{T^*}-T^*)}{T^*}$				Mean and standard deviation (between parentheses) of $\frac{100(\widehat{m}-m)}{m}$			
		OM_{fixed}	$SGBM_{ZM}$	$SGBM_{01}$	BM	OM_{fixed}	$SGBM_{ZM}$	$SGBM_{01}$	BM
Simulated Model	OM_{fixed}	0.00 (0.44)	-0.01 (0.45)	0.10 (0.46)	0.35 (0.57)	0.00 (0.90)	-0.10 (0.91)	-6.61 (1.43)	-0.43 (1.15)
	$SGBM_{ZM}$	0.01 (0.44)	0.00 (0.42)	0.11 (0.43)	0.36 (0.52)	0.09 (0.92)	0.00 (0.91)	-6.36 (1.41)	-0.25 (1.08)
	$SGBM_{01}$	-0.10 (0.43)	-0.11 (0.41)	0.00 (0.40)	0.23 (0.48)	6.46 (1.40)	6.37 (1.37)	-0.01 (0.94)	6.12 (1.52)
	BM	0.00 (0.43)	0.00 (0.43)	0.00 (0.43)	0.00 (0.43)	0.00 (0.90)	0.00 (0.90)	0.00 (0.95)	0.00 (0.90)

Note: The cells show the percentage difference of real moment of peak sales compared to the estimated moment of peak sales ($\frac{100(\widehat{T^*}-T^*)}{T^*}$) and the percentage difference of real market potential compared to the estimated market potential ($\frac{100(\widehat{m}-m)}{m}$), averaged over all combinations of curvature, number of months and noise. For example the average difference in moment of peak sales across all combinations with OM_{fixed} being the true model as well as the simulated model is 0.00%. The standard deviation (given between parentheses) of this percentage difference for this combination is 0.44 percentage points.

Figure 4.2. So if $SGBM_{01}$ is used to generate data, it *seems as if* the other models produce biased estimates of m.

In case the full diffusion process is available, the main goal of estimation is to find the correct estimates. The $SGBM_{01}$ fails in this objective. The standard Bass model does find the correct diffusion parameters. If the goal is to find the underlying diffusion process and not the seasonal structure, the standard Bass model can be used. Note that the use of the standard Bass model will lead to a loss of precision in the estimates relative to a model with seasonality.

4.A.2 Truncated Diffusion Process

In this subsection we look at the case of incomplete information. We consider the situation where the truncation is just before the moment of peak sales, or where it is just after. As estimation is done on only a part of the diffusion curve, we encounter the usual estimation problems of the Bass diffusion model. This occurs mainly if only data before the moment of peak sales is used and/or the noise level is large. If the estimation procedure does not converge we do not take the estimates into account in the averages given in the tables below.

Table 4.8 shows the results based on the truncated series. In this table we again look at the difference in moment of peak sales and market potential, averaged over all levels of curvature, number of seasonal months and noise. Although it is not shown in the table, we find that the bias and the uncertainty grow with increasing levels of noise and number of seasonal dummies.

Table 4.8: Parameter recovery results for the truncated diffusion series.

		Estimated Model							
		Mean and standard deviation (between parentheses) of $\frac{100(\widehat{T^*}-T^*)}{T^*}$				Mean and standard deviation (between parentheses) of $\frac{100(\widehat{m}-m)}{m}$			
		OM_{fixed}	$SGBM_{ZM}$	$SGBM_{01}$	BM	OM_{fixed}	$SGBM_{ZM}$	$SGBM_{01}$	BM
Simulated Model (Before peak sales)	OM_{fixed}	-0.13 (8.93)	-2.19 (8.94)	-2.18 (9.29)	-12.17 (13.51)	1.44 (19.58)	-3.18 (19.51)	-9.74 (18.74)	-19.15 (24.76)
	$SGBM_{ZM}$	2.40 (10.28)	-0.08 (8.87)	-0.14 (9.20)	-11.05 (13.49)	7.62 (23.20)	1.54 (19.84)	-5.17 (19.10)	-16.97 (25.04)
	$SGBM_{01}$	2.29 (9.89)	-0.10 (8.62)	-0.11 (8.99)	-9.85 (13.26)	14.32 (23.36)	8.11 (19.91)	1.47 (19.38)	-8.91 (27.15)
	BM	0.03 (8.72)	0.00 (8.68)	-0.19 (9.08)	-0.09 (8.62)	1.79 (19.75)	1.72 (19.44)	1.32 (20.01)	1.39 (19.04)
Simulated Model (After peak sales)	OM_{fixed}	0.17 (3.25)	-0.32 (3.02)	-0.21 (3.03)	-1.57 (6.27)	0.27 (4.49)	-0.54 (4.26)	-6.97 (4.18)	-2.59 (9.25)
	$SGBM_{ZM}$	0.69 (3.51)	0.13 (3.19)	0.25 (3.19)	-0.96 (6.96)	1.14 (4.90)	0.21 (4.44)	-6.19 (4.29)	-1.49 (10.33)
	$SGBM_{01}$	0.56 (3.41)	0.04 (3.02)	0.14 (3.03)	-1.00 (6.53)	7.56 (5.21)	6.64 (4.58)	0.25 (4.19)	4.90 (10.48)
	BM	0.08 (3.08)	0.08 (3.09)	0.08 (3.09)	0.06 (2.90)	0.17 (4.31)	0.17 (4.32)	0.18 (4.32)	0.13 (4.05)

Note: The cells show the percentage difference of real moment of peak sales compared to the estimated moment of peak sales ($\frac{100(\widehat{T^*}-T^*)}{T^*}$) and the percentage difference of real market potential compared to the estimated market potential ($\frac{100(\widehat{m}-m)}{m}$), averaged over all combinations of curvature, number of months and noise. The table shows both the results for the estimation of the series truncated before and after the moment of peak sales. For example the average difference in moment of peak sales across all combinations with OM_{fixed} being the true model as well as the simulated model for series until 20% after the moment of peak sales is 0.17%. The standard deviation (given between parentheses) of this percentage difference for this combination is 3.35 percentage points.

For the seasonal models, the results in Table 4.8 are in line with the earlier findings. A main difference is that the standard Bass model does not produce correct estimates. Not only are the results biased, but the large standard deviation is also an indication of misspecification. Further, the bias and variation increase substantially with the complexity of the seasonality in the simulated process.

In Table 4.9 we compare the models based on the out-of-sample forecasting performance. The comparison is based on the Root Mean Squared Prediction Error (RMSPE) for 12 months after each (truncated) estimation period. These results show that, despite the wrong estimates, the forecasts are comparable for all seasonal models. In particular, it shows that the $SGBM_{01}$ generates similar forecasting results as the other two seasonal models. Hence, for short-term forecasting the three seasonal models perform equally well. In the case of no seasonality in the true model, the difference between the seasonal models and the standard Bass version is small. If the diffusion shows seasonality, the RMSPE of the standard Bass model is more than two times as large compared to any seasonal model. Despite this, the magnitude of the difference indicates that, although the estimates of the

Table 4.9: Forecasting performance on the truncated diffusion series: mean and standard deviation (between parentheses) of RMSPE.

			Estimated Model			
			OM_{fixed}	$SGBM_{ZM}$	$SGBM_{01}$	BM
Simulated Model	**(Before peak sales)**	OM_{fixed}	33.95 (52.01)	42.32 (52.76)	46.26 (57.38)	93.66 (63.43)
		$SGBM_{ZM}$	43.88 (58.74)	34.98 (53.42)	39.61 (60.71)	91.44 (59.78)
		$SGBM_{01}$	44.51 (59.15)	35.31 (53.72)	39.35 (59.44)	90.88 (60.55)
		BM	35.29 (50.77)	35.72 (50.82)	41.09 (71.55)	35.47 (52.93)
Simulated Model	**(After peak sales)**	OM_{fixed}	12.92 (15.87)	14.46 (15.14)	14.54 (15.15)	46.19 (25.13)
		$SGBM_{ZM}$	14.83 (15.54)	13.1 (16.08)	13.17 (16.05)	48.29 (27.01)
		$SGBM_{01}$	14.81 (15.59)	13.15 (16.13)	13.1 (16.20)	47.98 (26.73)
		BM	12.79 (15.29)	12.84 (15.37)	12.84 (15.37)	12.22 (14.33)

Note: The cells show the Root Mean Squared Prediction Error for 12 months after the truncated period, which is 20% before or after the moment of peak sales. The RMSPE is averaged over all levels of curvature, levels of noise and the number of months. For example the average RMSPE where OM_{flex} is the true model and the estimated model, and the truncation is after peak sales, is 12.92 with a standard deviation of 15.87. The RMSPEs are multiplied by 100 for convenience.

standard Bass models are incorrect, the direction of the short-term forecast is still correct. Of course the seasonal peaks are underestimated for the standard Bass model.

Summarizing, the simulations show that apart from the difficulties all models encounter with parameter estimation in case of truncated diffusion series, the standard Bass model is not suited for estimating the underlying diffusion pattern when seasonality is present. For short-term forecasting all models are able to predict the overall direction of sales. However, if the objective is to forecast the precise sales in each month then the seasonal structure should be taken into account as well. For forecasting it does not matter which seasonal model is used. Even the $SGBM_{01}$, which has a bias in the market potential, provides good forecasts.

Chapter 5

Product-country interaction in new product diffusion

The Bass model has triggered a very rich research stream on the diffusion of new products. With globalization an increasing interest among scholars and practitioners emerged on the international diffusion of new products. International new product diffusion models can generate valuable insights for companies' international launch strategies, and their decisions on marketing instruments across countries.

Historically, these models included only country and/or product characteristics. We show that the interactions between the country and product characteristics are an important part of the "story" behind the variation across diffusion series. As an example, the difference in diffusion speed between France and Norway may be different for freezers than for cell phones.

In this chapter, we extend existing (multilevel) international diffusion models by explicitly including product-country interactions. These interactions can partly be captured using the standard approach of including the multiplication of product and country variables. However, a substantial part of the interactions remains unexplained. We use dimension reduction techniques to provide insight in this remaining variation. This technique reveals hidden correlation patterns in the unexplained product-country variation.

We show that interaction effects are indeed very important to include in international diffusion models. We present two potential applications for the results. First, the identification of the country, or countries, best suited for the launch of a new product. The second is early forecasting. The model can be used for a particular product-country combination to get better pre-launch and early forecasts, if the product has already been introduced in other countries.

5.1 Introduction

The Bass (1969) model has triggered a very rich research stream on the diffusion of new products. With globalization an increasing interest among scholars and practitioners emerged on the international diffusion of new products (Gatignon *et al.*, 1989; Helsen *et al.*, 1993, etc.). International new product diffusion models can generate insights that are valuable to companies for their international launch strategy, their decisions on marketing instruments across countries, etc. It has, therefore, become increasingly important to learn from previous diffusion processes, not just within a country, but especially across countries. In this chapter we use a truly global dataset of 82 countries and a set of twelve products to give a better understanding of where the variation in diffusion processes across product-country combinations comes from.

There are several types of international diffusion models, for example cross-country models (e.g. Putsis *et al.*, 1997; Kumar and Krishnan, 2002), multilevel diffusion models (e.g. Lenk and Rao, 1990; Talukdar *et al.*, 2002), and non/semi-parametric diffusion models (e.g. Stremersch and Lemmens, 2009; Sood *et al.*, 2009). Some of these papers explicitly try to find factors that explain the variation between different products and countries. The focus is mainly on country characteristics, such as GDP, GINI, and cultural dimensions (Hofstede Dimensions). For example, a typical result is that in countries with higher GDP per capita new products diffuse faster and have more potential.

Less research has been done on the product characteristics, whereas Sood *et al.* (2009) find that products are a more dominant source of variation than countries. Studies regarding aspects directly related to the diffusion process (e.g. take-off, growth) found some key product characteristics (Tellis *et al.*, 2003; van den Bulte and Stremersch, 2004; Golder and Tellis, 2004). The important characteristics these authors find are: (i) whether a product's main purpose is for fun or for utility, (ii) if products have a competing standard, and (iii) if using a product is a new concept or builds on existing products.

Another aspect that has been ignored by many studies is the interaction effect between products and countries. Talukdar *et al.* (2002) find that, in their model, there is a substantial part of the variation that cannot be explained by, either observed, or unobserved product and country characteristics. The latter shows that product-country are an important part in explaining the variation in diffusion series. As an example, the difference in diffusion speed between France and Norway may be very different for freezers than for cell phones. As this variance is left unexplained by existing models, this may bias their (theoretical) inferences on the main effects of country (and product) covariates and may impact their forecasting accuracy.

In this chapter, we focus on the product variation and most importantly on the interaction effects. The basic starting point of the modeling efforts is a multilevel diffusion model, similar to that of Talukdar *et al.* (2002). We add to the literature in four main ways. First, we consider a more comprehensive set of product and country variables to explain differences in diffusion processes. Second, we explicitly consider the impact and relevance of product-country interactions. This way we try to more fully explain the story behind the differences in diffusion curve across countries and products. Third, despite the inclusion of interaction variables, we show that there is still a substantial amount of variation left unexplained. This variation can be explained in terms of unobserved interactions. These unobserved interactions translate into particular correlation patterns, that is, the deviation from the expected diffusion parameters in country A may be positively correlated to the deviation in country B. So, if the diffusion in country A is faster than expected based on product and country characteristics, it is likely that the diffusion in country B will also be relatively fast. The reasons behind these correlations are by definition difficult to give, instead we will visualize them. More technically, we use dimension reduction techniques, that is, principal component analysis [PCA], on the remaining, unexplained, variation between the product-country pairs. The components coming from this PCA are represented graphically to give insights in the correlation structure. By using property fitting lines we relate the correlations to country-specific factors. Fourth, we extend the multilevel diffusion model in such a way that it is robust against cross-country influences. In contrast to the traditional cross-country models (Putsis *et al.*, 1997; Kumar and Krishnan, 2002), we do not model cross-country influences by changing the diffusion parameters. Instead, in the second level model for the main diffusion parameters we include an additional variable, which represents the installed base of a product in “neighboring” countries at the moment the product is introduced in the focal country. This variable captures the kick-start the diffusion has in a country in which the product launch occurs relatively late.

The combination of the multilevel model and the PCA is very useful for marketing management. First, the multilevel model can be used as an indication where a new product needs to be introduced first for a successful rollout. If there is managerial insight that the diffusion will go faster than predicted by the model in some country, the graphical representation of the PCA can give insights in which countries will behave similarly, and thus should be considered for early introduction as well. Finally, a combination of the multilevel model and the graphical results of the PCA can be used for pre-launch forecasting for a product new to a country, but already available in other countries.

The outline of the remainder of the chapter is as follows. In the next section we present the multilevel model and explain how we avoid a possible bias of ignoring cross-country spillover. In the third section we describe the data we use. Next we give the results of the multilevel diffusion model. In the fifth section we show the hidden structure in the remaining variation, that is, we study the correlations between product-country pairs using PCA. We give the formulations and results of the PCA. In the sixth section we describe how the results in this chapter can be used for managerial purposes. Finally, we conclude with a discussion and some limitations.

5.2 Multilevel new product diffusion model

In this section we describe the multilevel diffusion model, which consists of two levels. In the first level, we specify a diffusion model for each product-country combination. In the second level we relate the (first-level) diffusion parameters to observed product, country, and product-country variables and to (unobserved) product and country random effects.

To quantify the (relative) effect sizes of different characteristics of the product-country combinations, we experiment with different sets of variables in the second level. All these variants of the second level are nested in what we call the "Full Model". We first present the "Full Model" and next look at the different restricted forms.

The first level model explains the cumulative penetration of product i in country j, in period t (N_{ijt}), using the Bass mixed-influence model (Bass, 1969), that is,

$$N_{ijt} = m_{ij}F_{ij}(t) + \varepsilon_{ijt}, \qquad \varepsilon_{ijt} \sim N(0, \sigma_{ij}^2), \tag{5.1}$$

where m_{ij} gives the ceiling level for this product-country pair and $F_{ij}(t)$ gives the closed-form solution of the Bass diffusion. This closed form-solution is given by

$$F_{ij}(t) = \frac{1 - \exp(-(p_{ij} + q_{ij})t)}{1 + \frac{q_{ij}}{p_{ij}} \exp(-(p_{ij} + q_{ij})t)}, \ t \geq 0. \tag{5.2}$$

There are alternative ways of relating the observed diffusion to the Bass model. In (5.1) we relate the closed-form solution directly to the observed cumulative penetration. This is in line with Chapter 2 of this thesis. In Chapter 2 we show that this specification is most robust to various forms of noise, and can be interpreted as a measurement error on the cumulative penetration/sales. From the data section that follows, it will become clear that the data is subject to substantial measurement error, especially since it is obtained through a survey-like method.

The first level model (5.1) contains three parameters for each product-country combination, that is, p_{ij}, q_{ij} and m_{ij}. Transformations of these diffusion parameters are used as dependent variables in the second level[1]. These transformations are,

$$\begin{aligned} \theta_{ij}^{p} &= \ln\left(\tfrac{p_{ij}}{1-p_{ij}}\right), \\ \theta_{ij}^{q} &= \ln\left(\tfrac{q_{ij}}{2-q_{ij}}\right), \text{ and} \\ \theta_{ij}^{m} &= \tfrac{m_{ij}}{100}. \end{aligned} \tag{5.3}$$

[1]We use slightly different transformations than used in previous multilevel diffusion models (e.g. Talukdar *et al.*, 2002), where the transformation only restricts the parameters to be positive. We believe that our specification better captures the range of values one expects p and q to take.

These transformations ensure that p_{ij} remains between 0 and 1, q_{ij} between 0 and 2, and for convenience we scale m_{ij} such that it becomes comparable in size to the other parameters.[2]

As explanatory variables in the second level we consider: product variables, country variables and product-country variables. The latter variables differ per country as well as per product. An example of such a variable is the year of introduction of a product in a specific country. Further, we consider the interactions between the product variables, on the one hand, and the country and product-country variables, on the other hand. Finally, we allow for product- and country-specific random effects. In this way, we can study the origin of the unobserved variation. In the Full Model, all variables and random effects are included. The second level then becomes

$$\theta_{ij}^d = \beta_0^d + X_i'\beta_1^d + Z_j'\beta_2^d + W_{ij}'\beta_3^d + (X_i \otimes [Z_j, W_{ij}])'\beta_4^d + \eta_{ij}^d, \tag{5.4}$$

where d refers to one of the diffusion parameters p, q or m and X, Z, W are product, country and product-country variables, respectively. $X \otimes [Z, W]$ indicates the interaction terms between the product variables and the combination of country and product-country variables, that is, the cross products of the variables in X with those in Z and W. All variables in this second level are standardized, such that β_0^d gives the average for the (transformed) diffusion parameter d. Furthermore, the standardization allows for a straightforward comparison of the effect sizes of the variables. The error term η_{ij}^d is decomposed into a product-specific, a country-specific, and an idiosyncratic part, that is,

$$\eta_{ij}^d = \gamma_{1i}^d + \gamma_{2j}^d + \nu_{ij}^d. \tag{5.5}$$

The variance components can be correlated across the transformed diffusion parameters p, q, and m. We therefore specify

$$\begin{pmatrix} \gamma_{1i}^p \\ \gamma_{1i}^q \\ \gamma_{1i}^m \end{pmatrix} \sim N(0, \Omega_1), \quad \begin{pmatrix} \gamma_{2j}^p \\ \gamma_{2j}^q \\ \gamma_{2j}^m \end{pmatrix} \sim N(0, \Omega_2), \text{ and } \begin{pmatrix} \nu_{ij}^p \\ \nu_{ij}^q \\ \nu_{ij}^m \end{pmatrix} \sim N(0, \Lambda). \tag{5.6}$$

Restricted models are easily obtained by setting parameter restrictions. For example, we obtain a model without unobserved product- and country-specific effects by setting $\Omega_1 = 0$ and $\Omega_2 = 0$. A model without interaction effects is obtained by setting $\beta_4^d = 0$. We will simultaneously estimate the parameters of the first- and the second-level model using (Hierarchical) Bayesian estimation, in particular Markov Chain Monte Carlo [MCMC] techniques. Details on the estimation procedure are given in Appendix 5.A.

[2]The upper limit on q is motivated by the type of data we consider in the empirical part of this paper. In our application q is always below 1.33. One can easily relax the upper limit or remove it completely.

To study the impact of different model components we look at seven restricted models. The differences between these models are only in the second level. Although the first level is similar for all models, a different second level influences the results of the first level as we consider them jointly when estimating the model parameters. We consider four models with only one (general) error term, that is, $\Omega_1 = 0$ and $\Omega_2 = 0$. These four models only differ with respect to the variables used. Model 1 only includes the product variables. Model 2 only includes the country variables. Model 3 includes the product and country variables and adds product-country variables. Model 4 extends model 3 with the interaction variables. To study the impact of using the error-components structure suggested in (5.5) we extend Model 4. This results in: Model 5, which includes a product-specific error term, next to a general error term ($\Omega_1 \neq 0$ and $\Omega_2 = 0$); Model 6 with a country-specific error term, next to a general error term ($\Omega_1 = 0$ and $\Omega_2 \neq 0$). Finally, Model 7 includes a product- and a country-specific error term, next to a general error term ($\Omega_1 \neq 0$ and $\Omega_2 \neq 0$). We will refer to Model 7 as the "Full Model", as it includes all variables and error terms.

To analyze and explain differences between diffusion curves across countries, it is important to obtain unbiased estimates of the diffusion parameters. One possible disturbing factor that we ignored in the above discussion is cross-country effects. We incorporate these in the second level of the multilevel model. The philosophy of the model in this chapter is that the variation in the diffusion parameters of a product-country pair can be explained by observed and unobserved factors that define the products and countries. The cross-country effects can be seen as one such factor. Therefore, we propose to add cross-country effects in the second level, as a product-country variable.

The variable we include is the installed base of a product in "neighboring" countries at the moment of introduction of the product in the focal country, where neighbors can be defined in terms of geography or in terms of other country characteristics. The motivation for this variable is that a major cross-country effect is that the diffusion in lagging countries gets a kick-start, and often diffuses faster building on the success of the product in neighboring countries that already introduced the product. In other words, the installed base of the product of the neighbors at the moment of introduction gives a good instrument for the total cross-country effect between the countries. We will use similar rules as in Albuquerque *et al.* (2007) to define the neighbors of a country.

5.3 Data

For the analysis we use annual penetration data of 867 product-country combinations, coming from twelve different products and 82 different countries. The data is obtained from the Global Market Information Database of Euromonitor. We choose the products such that we have data available from the introduction of the product in most countries. However, as the data is truncated at 1977 there are still several product-country

Table 5.1: Overview of the countries per region

Africa		Algeria, Cameroon, Egypt, Kenya, Morocco, Nigeria, South Africa, Tunisia
America	Middle America	Costa Rica, Dominican Republic, Mexico
	Northern America	Canada, USA
	South America	Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Peru, Uruguay, Venezuela
Asia	Developing Asia	Azerbaijan, India, Indonesia, Kazakhstan, Malaysia, Pakistan, Philippines, Thailand, Turkmenistan, Vietnam
	East Asia	China, Hong Kong, Japan, Singapore, South Korea, Taiwan
	West Asia	Bahrain, Iran, Israel, Jordan, Kuwait, Qatar, Saudi Arabia, United Arab Emirates
Europe	Eastern Europe	Belarus, Bosnia-Herzegovina, Bulgaria, Croatia, Czech Republic, Estonia, Georgia, Hungary, Latvia, Lithuania, Macedonia, Poland, Romania, Russia, Slovakia, Slovenia, Ukraine
	Western Europe	Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom
Oceania		Australia, New Zealand

combinations with left-hand truncation. For these combinations we use the moment of introduction to control for this truncation. Estimation of Bass parameters using the closed-form solution, such as in (5.1), still produces unbiased estimates in the presence of left-hand truncation (Jiang *et al.*, 2006). Further, we only include countries that have data available for at least four of the twelve products. Finally, we excluded product-country combinations with less than ten observations in the diffusion series.

Table 5.1 provides an overview of the countries and shows how they are balanced over different geographic clusters. In Table 5.2 we show the twelve products and three variables based on which we can classify the products. These variables have been used in other international diffusion studies (e.g. Tellis *et al.*, 2003; van den Bulte and Stremersch, 2004; Golder and Tellis, 2004) and are distinctive in classifying the products. We will therefore use these characteristics as product variables in the second-level model. The three variables capture, (i) whether a product's main purpose is for fun or for utility (Fun vs Utility), (ii) if products have a competing standard (Competing Standards), and (iii) if using a product is a new concept or builds on existing products (New Use), respectively.

Next to the three product variables, we include seven country characteristics: GDP per capita[3], GINI, and four Hofstede Cultural Dimensions (Uncertainty Avoidance, Individualism, Power Distance and Masculinity/Femininity). Finally, we also include a dummy indicating if the Hofstede dimension is available or not, as the cultural variables are not available for all countries. Without loss of generality we set the cultural dimensions to zero if they are missing.

[3]We take the average over time, as variables in the second level need to be time invariant.

Table 5.2: Overview of products and product variables

	Fun vs Utility	Competing standards	New Use
1. CD	1	0	1
2. Gameconsole	1	1	0
3. Video camera	1	0	1
4. DVD	1	0	1
5. Microwave	0	0	1
6. Dishwasher	0	0	0
7. Internet	1	0	0
8. Cable TV	1	1	0
9. VCR	1	0	0
10. Satellite TV	1	1	1
11. CityplaceMobile (subscriptions)	0	1	0
12. PC	0	1	0

We also use product-country variables. One of these is the average installed base in the five countries with closest geographic proximity[4] to the focal country at the time of product launch. As described earlier we use this variable to control for cross-country effects. We also include two variables to describe the entry-timing effect. The first entry variable describes the lag of a product-country combination to the first introduced combination across all products. Actually, this is a general time trend that captures the effect that the diffusion process has become faster over time. The second entry variable gives the lag compared to the first country in which a specific product is introduced. The latter captures the fact that the diffusion process regarding a product becomes faster over time, due to, for example, technological improvements. Note that this entry variable is not similar to the installed based variable; the entry variable captures a general trend for all countries regarding a product, whereas the latter deals with the effect of entry in a limited set of neighbors.

5.4 Results

he results of the multilevel model presented in this section are based on (Hierarchical) Bayes estimation (see Appendix 5.A for details) of the models in Section 5.2, with 100,000 iterations, of which 50,000 were used as burn in, and a thinning value of 10.

Table 5.3 provides some general statistics of the first-level results to facilitate a comparison of the models. In particular, some summary statistics of the first-level parameters are given, that is, the mean, standard deviation and minimum and maximum of the diffusion parameters across all product-country pairs as well as two fit statistics.

[4]Albuquerque *et al.* (2007) also define some other measurements to obtain the neighbors of a country (f.e. economic or cultural distances), in this case these turned out not to hold additional information given the set of other variables. This is probably due to the fact that this information is already to a large extent captured by the geographic "neighbors" and the country-specific variables.

Table 5.3: Some summary statistics of the first–level results for seven models.

	Model 1.	**Model 2.**	**Model 3.**	**Model 4.**	**Model 5.**	**Model 6.**	**Model 7. (Full model)**
mean p	0.016	0.016	0.016	0.016	0.016	0.016	0.016
stdev p	0.031	0.032	0.031	0.031	0.029	0.031	0.029
min p	0.000	0.000	0.000	0.000	0.000	0.000	0.000
max p	0.539	0.553	0.519	0.526	0.444	0.529	0.447
mean q	0.323	0.325	0.326	0.327	0.328	0.327	0.328
stdev q	0.222	0.224	0.219	0.220	0.220	0.220	0.220
min q	0.008	0.007	0.009	0.010	0.006	0.006	0.007
max q	1.269	1.321	1.311	1.273	1.216	1.269	1.243
mean m	45.703	44.976	44.352	44.025	43.863	44.002	43.888
stdev m	32.823	32.485	32.495	32.387	32.204	32.447	32.124
min m	0.222	0.222	0.223	0.223	0.224	0.222	0.224
max m	139.606	123.273	126.795	123.374	115.257	122.701	116.728
mean RMSE	1.298	1.297	1.299	1.302	1.304	1.301	1.303
stdev RMSE	1.416	1.416	1.418	1.424	1.427	1.422	1.427
min RMSE	0.017	0.017	0.017	0.018	0.017	0.017	0.017
max RMSE	12.384	11.818	12.225	12.323	12.302	12.467	12.655
mean R^2	0.987	0.987	0.987	0.987	0.987	0.987	0.987
stdev R^2	0.033	0.029	0.034	0.032	0.033	0.032	0.033
min R^2	0.481	0.562	0.443	0.528	0.464	0.524	0.482
max R^2	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5.3 shows that the first-level results are stable over the different models. The only noteworthy findings are the small differences in the maximum and minimum of the market potential across the seven models. The models with a product-specific error term have somewhat less extreme market potential estimates. In these models there seems to be more shrinkage for the first-level parameters, that is, the second-level model gives a narrower prior for these parameters. For product-country combinations with few data points or many fluctuations, this prior has more influence on the posterior first-level results, especially for the market potential parameter.

In Tables 5.4 and 5.5 the posterior means of the second-level parameters are given together with the estimated covariance matrices. For each model there are three columns, indicating the three transformed diffusion parameters: p, q and m. Bold and underlined coefficients are significantly different from zero, that is, zero is not in the 95% highest posterior density interval. The models in Tables 5.4 and 5.5 can be compared based on differences in the parameter estimates in terms of sign, size, and/or significance.

Table 5.4 shows that adding variables has little influence on the significance and sign of the parameters. For example, going from Model 1 to Model 3, that is, adding country variables, only results in small deviations in the parameters of the product-specific variables, for example, the estimate for "Fun vs Utility" for the innovation parameter goes from 0.844 to 1.082. Only when adding the interaction variables (Model 4) there are shifts in the significance and sign of the parameters. In particular, interactions with entry variables have a significant effect on the diffusion parameters. An important remark is that the error covariance matrix is a lot smaller for Model 4 than for Models 1-3.

If we now look at Table 5.5, we see that much of this unobserved variance returns when a product random effect is added, that is, the size of the covariance terms of the combined error (η_{ij}) in Models 5-7 is now again comparable to the error variance for Models 1-3. For example, the combined variance term of the transformed innovation parameter in Model 5 is 1.090 + 1.397 = 2.487, which is similar to the variance term in Models 1-3; 2.650, 3.341, and 2.246, respectively. Also, when we compare Models 5-7 (Table 5.5) to Model 4 (Table 5.4) we find that the significance of many interaction terms disappears when product random effects are considered. This most likely occurs because the interaction is based on three product variables and only twelve observed products. Thus, it seems that if product random effects are ignored there is some over-fitting in terms of the impact of observed product characteristics. Including product random effects removes these spurious results.[5]

Table 5.6 gives the percentage of the total variance of the transformed diffusion variables that can be explained using observed characteristics versus the percentage that is unexplained. These percentages are obtained as the posterior mean of:

$$\begin{aligned} &100 * \frac{var(\beta_0^d + X_i'\beta_1^d + Z_j'\beta_2^d + W_{ij}'\beta_3^d + (X_i \otimes [Z_j W_{ij}])'\beta_4^d)}{var(\beta_0^d + X_i'\beta_1^d + Z_j'\beta_2^d + W_{ij}'\beta_3^d + (X_i \otimes [Z_j W_{ij}])'\beta_4^d) + var(\eta_{ij}^d)}, \text{ and} \\ &100 * \frac{var(\eta_{ij}^d)}{var(\beta_0^d + X_i'\beta_1^d + Z_j'\beta_2^d + W_{ij}'\beta_3^d + (X_i \otimes [Z_j W_{ij}])'\beta_4^d) + var(\eta_{ij}^d)}, \end{aligned} \tag{5.7}$$

where the variance is taken over all product-country pairs, see also (5.4). Note that some of the coefficients are restricted to zero in some models.[6] In case η_{ij} includes product- and/or country-specific random effects, we present how the unexplained variance is divided over the components, that is,

$$\begin{aligned} &\frac{var(\gamma_{1i})}{var(\gamma_{1i}) + var(\gamma_{2j}) + var(\nu_{ij})}, \frac{var(\gamma_{2j})}{var(\gamma_{1i}) + var(\gamma_{2j}) + var(\nu_{ij})}, \text{ and} \\ &\frac{var(\nu_{ij})}{var(\gamma_{1i}) + var(\gamma_{2j}) + var(\nu_{ij})}, \end{aligned} \tag{5.8}$$

where the variance components are defined in (5.5).

Table 5.6 gives insights in which type of variables explain most of the variance in each of the diffusion parameters. For example, Model 1, which only contains product-specific variables (β_2 to β_4 are zero), explains 21.1%, 2.3% and 15.0% of the variation in the innovation, imitation and market potential parameter, respectively. Further, we see that, as expected, the error decomposition does not affect the explanation power of the variables much, that is, the percentage explained and unexplained stay approximately the

[5] We also tested three models with different sets of the error components in a model without interaction terms, that is, extensions of Model 3. The same pattern of “gaining” and “losing” significance, as in the presented models with interaction terms seems to be present when adding additional error terms. As these models do not give any additional insights we do not include these results in the chapter.

[6] By using the formulation in (5.7) the percentages always add up to 100 %. However, because we use Bayesian methodology, the denominator does not always exactly matches with var(θ_{ij}^d). Especially in the case of multiple errors terms there are some deviations, these are however small. Only for least squares estimation it is guaranteed that the total variance equals the explained variance plus the unexplained variance.

Table 5.4: The second-level results for four models: 1: Only Product Variables, 2: Only Country Variables, 3: Product, Country and Product-Country Variables, and 4: Product, Country, Product-Country variables and their interaction terms. θ^p, θ^q, and θ^m are transformations of the innovation, imitation and market potential parameter of the Bass model, respectively.

		Model 1. (Products)			Model 2. (Countries)			Model 3. (Prod. & Countries)			Model 4. (All variables)		
		θ^p	θ^q	θ^m	θ^p	θ^q	θ^m	θ^p	θ^q	θ^m	θ^p	θ^q	θ^m
Constant	Constant	**-5.433**	**-1.955**	**0.457**	**-5.431**	**-1.959**	**0.450**	**-5.399**	**-1.919**	**0.443**	**-5.123**	**-2.030**	**0.503**
	FunVsUtil	**0.844**	**-0.117**	**-0.120**				**1.082**	**-0.278**	**-0.159**	**1.353**	**-0.366**	**-0.232**
	CompStand	**0.161**	**0.092**	-0.007				**0.226**	-0.007	-0.001	-0.114	**0.112**	**-0.084**
	NewUse	-0.043	0.023	**-0.026**				0.072	-0.052	**-0.035**	0.019	-0.051	-0.002
Entry All	Constant							**-0.580**	**0.700**	**0.026**	**-0.313**	**0.606**	**0.025**
	FunVsUtil										**-0.571**	**0.227**	**-0.072**
	CompStand										**0.699**	**-0.179**	**-0.060**
	NewUse										-0.010	**0.077**	0.013
Entry Product	Constant							**0.477**	**-0.170**	**-0.129**	**0.414**	**-0.136**	**-0.083**
	FunVsUtil										**0.521**	**-0.179**	**0.149**
	CompStand										**-0.994**	**0.334**	**0.045**
	NewUse										**-0.399**	**0.155**	**0.034**
Installed Base Geographic	Constant							**0.235**	-0.057	0.006	0.139	-0.056	-0.019
	FunVsUtil										0.031	**-0.091**	-0.011
	CompStand										0.108	0.035	**-0.023**
	NewUse										0.160	-0.049	**-0.029**
Culture PowerDist.	Constant				-0.058	0.032	**-0.029**	-0.046	-0.017	**-0.023**	-0.096	0.007	**-0.025**
	FunVsUtil										0.120	-0.006	0.003
	CompStand										-0.049	0.001	0.015
	NewUse										0.118	-0.058	-0.007
Culture UncAvoid.	Constant				-0.060	**0.077**	0.001	-0.030	0.044	0.003	-0.041	0.044	0.000
	FunVsUtil										0.069	-0.010	-0.007
	CompStand										-0.070	0.021	0.006
	NewUse										0.005	0.006	0.002
Culture Individ.	Constant				-0.081	0.027	**0.036**	-0.103	0.054	**0.031**	-0.053	0.020	**0.025**
	FunVsUtil										-0.063	0.005	0.004
	CompStand										-0.095	-0.037	-0.012
	NewUse										-0.012	0.020	0.004
Culture MasFem	Constant				0.013	-0.016	-0.009	-0.010	0.009	-0.009	-0.019	0.016	-0.008
	FunVsUtil										0.024	-0.016	0.006
	CompStand										-0.024	0.007	0.016
	NewUse										0.015	0.025	0.006
Culture NoInfo	Constant				-0.053	-0.011	**-0.040**	0.012	**-0.139**	**-0.030**	-0.043	**-0.119**	**-0.022**
	FunVsUtil										0.020	-0.044	0.005
	CompStand										0.036	-0.049	**0.035**
	NewUse										0.058	-0.030	0.017
Gini	Constant				0.117	**-0.097**	-0.006	**0.173**	**-0.096**	-0.014	**0.205**	**-0.113**	-0.013
	FunVsUtil										0.039	-0.043	0.004
	CompStand										0.080	**-0.092**	0.000
	NewUse										0.113	-0.061	0.004
GDP	Constant				-0.089	0.033	**0.112**	-0.061	**0.152**	**0.084**	-0.024	**0.146**	**0.089**
	FunVsUtil										-0.001	0.051	**0.026**
	CompStand										**-0.151**	0.072	-0.020
	NewUse										-0.050	-0.008	0.015
	Unobs. Prod Effects												
	Unobs. Country Effects												
	Unobs. Prod-Country Effects	2.650	-1.028	-0.164	3.341	-1.141	-0.233	2.246	-0.706	-0.095	1.764	-0.536	-0.080
		-1.028	1.040	0.051	-1.141	1.102	0.061	-0.706	0.596	0.034	-0.536	0.517	0.032
		-0.164	0.051	0.097	-0.233	0.061	0.084	-0.095	0.034	0.058	-0.080	0.032	0.045

same for Models 5-7, compared to Model 4. Note, that these models still hold valuable information as they all treat the random effects differently.

Table 5.5: The second-level results for four models, all including Product, Country, Product-Country variables and their interaction terms, but differing in their error structure: 4: No additional error term, 5: additional Product specific error term, 6: additional Country specific error term, and 7: Both an additional Product and Country specific error term. θ^p, θ^q, and θ^m are transformations of the innovation, imitation and market potential parameter of the Bass model, respectively.

		Model 4. (All Variables)			Model 5. (Product Error)			Model 6. (Country Error)			Model7. (Full model)		
		θ^p	θ^q	θ^m	θ^p	θ^q	θ^m	θ^p	θ^q	θ^m	θ^p	θ^q	θ^m
Constant	Constant	**-5.123**	**-2.030**	**0.503**	**-5.010**	**-2.160**	**0.491**	**-5.125**	**-2.028**	**0.500**	**-5.008**	**-2.156**	**0.486**
	FunVsUtil	**1.353**	**-0.366**	**-0.232**	**1.391**	-0.368	-0.232	**1.350**	**-0.364**	**-0.233**	**1.388**	-0.352	-0.226
	CompStand	-0.114	**0.112**	**-0.084**	-0.320	0.252	-0.054	-0.114	**0.106**	**-0.082**	-0.319	0.251	-0.057
	NewUse	0.019	-0.051	-0.002	0.001	-0.079	-0.002	0.017	-0.051	0.000	0.001	-0.068	-0.001
Entry All	Constant	**-0.313**	**0.606**	**0.025**	-0.046	**0.470**	-0.017	**-0.308**	**0.619**	**0.027**	-0.050	**0.485**	-0.015
	FunVsUtil	**-0.571**	**0.227**	**-0.072**	-0.550	0.277	-0.080	**-0.570**	**0.226**	**-0.076**	-0.557	0.272	-0.084
	CompStand	**0.699**	**-0.179**	**-0.060**	**0.882**	-0.302	-0.094	**0.695**	**-0.174**	**-0.058**	**0.867**	-0.289	-0.089
	NewUse	-0.010	**0.077**	0.013	0.067	0.041	-0.007	-0.010	**0.076**	0.012	0.064	0.034	-0.007
Entry Product	Constant	**0.414**	**-0.136**	**-0.083**	**0.648**	-0.252	-0.119	**0.417**	**-0.126**	**-0.079**	**0.659**	-0.237	-0.112
	FunVsUtil	**0.521**	**-0.179**	**0.149**	0.801	-0.372	0.119	**0.521**	**-0.171**	**0.144**	0.807	-0.369	0.114
	CompStand	**-0.994**	**0.334**	**0.045**	**-1.082**	0.387	0.038	**-0.979**	**0.332**	**0.050**	**-1.078**	0.372	0.043
	NewUse	**-0.399**	**0.155**	**0.034**	-0.150	0.016	-0.016	**-0.390**	**0.164**	**0.037**	-0.165	0.026	-0.008
Installed Base Geographic	Constant	0.139	-0.056	-0.019	0.048	-0.015	-0.014	0.145	-0.051	-0.023	0.040	-0.029	-0.019
	FunVsUtil	0.031	**-0.091**	-0.011	-0.030	-0.060	-0.007	0.032	**-0.088**	-0.011	-0.033	**-0.065**	-0.008
	CompStand	0.108	0.035	**-0.023**	0.078	0.041	-0.008	0.095	0.029	**-0.022**	0.080	0.044	-0.007
	NewUse	0.160	-0.049	**-0.029**	-0.022	0.024	0.003	0.149	-0.055	**-0.033**	-0.021	0.016	-0.003
Culture PowerDist.	Constant	-0.096	0.007	**-0.025**	**-0.125**	0.021	**-0.020**	-0.094	0.003	**-0.027**	**-0.130**	0.015	**-0.023**
	FunVsUtil	0.120	-0.006	0.003	0.089	0.011	0.008	0.121	-0.008	0.002	0.087	0.009	0.006
	CompStand	-0.049	0.001	0.015	-0.042	-0.001	0.013	-0.052	0.001	0.014	-0.042	-0.003	0.013
	NewUse	0.118	-0.058	-0.007	0.109	-0.049	-0.007	0.118	-0.056	-0.009	0.113	-0.053	-0.007
Culture Unc.Avoid.	Constant	-0.041	0.044	0.000	-0.052	**0.048**	0.002	-0.042	0.043	0.000	-0.052	**0.052**	0.001
	FunVsUtil	0.069	-0.010	-0.007	0.049	-0.009	-0.005	0.064	-0.012	-0.007	0.052	-0.006	-0.007
	CompStand	-0.070	0.021	0.006	-0.067	0.021	0.006	-0.070	0.021	0.005	-0.063	0.019	0.006
	NewUse	0.005	0.006	0.002	-0.005	0.010	0.003	0.001	0.006	0.001	0.002	0.008	0.003
Culture Individ.	Constant	-0.053	0.020	**0.025**	-0.057	0.021	**0.026**	-0.051	0.022	**0.028**	-0.051	0.028	**0.030**
	FunVsUtil	-0.063	0.005	0.004	-0.063	-0.002	0.002	-0.062	0.003	0.002	-0.056	-0.005	0.001
	CompStand	-0.095	-0.037	-0.012	-0.067	-0.059	-0.019	-0.094	-0.036	-0.013	-0.062	-0.061	**-0.020**
	NewUse	-0.012	0.020	0.004	-0.006	0.020	0.002	-0.010	0.020	0.002	-0.005	0.011	0.001
Culture MasFem	Constant	-0.019	0.016	-0.008	-0.005	0.006	-0.010	-0.018	0.016	-0.009	-0.006	0.004	-0.011
	FunVsUtil	0.024	-0.016	0.006	0.031	-0.015	0.004	0.020	-0.013	0.005	0.030	-0.017	0.004
	CompStand	-0.024	0.007	0.016	-0.004	-0.001	0.011	-0.021	0.007	**0.016**	-0.007	0.001	0.012
	NewUse	0.015	0.025	0.006	0.032	0.020	0.003	0.019	0.026	0.006	0.027	0.021	0.004
Culture NoInfo	Constant	-0.043	**-0.119**	**-0.022**	-0.081	**-0.094**	**-0.015**	-0.051	**-0.122**	**-0.028**	**-0.091**	**-0.099**	**-0.020**
	FunVsUtil	0.020	-0.044	0.005	0.003	-0.041	0.007	0.016	-0.044	0.008	0.011	-0.034	0.008
	CompStand	0.036	-0.049	**0.035**	-0.011	-0.022	**0.045**	0.038	-0.047	**0.034**	0.000	-0.019	**0.042**
	NewUse	0.058	-0.030	0.017	0.037	-0.028	**0.020**	0.056	-0.034	**0.018**	0.044	-0.029	**0.019**
Gini	Constant	**0.205**	**-0.113**	-0.013	**0.224**	**-0.124**	**-0.015**	**0.203**	**-0.116**	-0.012	**0.229**	**-0.113**	-0.016
	FunVsUtil	0.039	-0.043	0.004	0.044	-0.049	0.003	0.036	-0.046	0.002	0.049	-0.045	0.002
	CompStand	0.080	**-0.092**	0.000	0.022	**-0.067**	0.010	0.074	**-0.096**	0.001	0.020	**-0.065**	0.010
	NewUse	0.113	-0.061	0.004	0.069	-0.051	0.009	0.106	-0.065	0.005	0.068	-0.052	0.009
GDP	Constant	-0.024	**0.146**	**0.089**	0.087	**0.085**	**0.071**	-0.024	**0.148**	**0.085**	0.075	**0.087**	**0.065**
	FunVsUtil	-0.001	0.051	**0.026**	0.036	0.032	**0.021**	-0.006	0.047	**0.020**	0.034	0.025	0.010
	CompStand	**-0.151**	0.072	-0.020	-0.106	0.044	**-0.033**	**-0.161**	0.070	-0.021	-0.111	0.039	**-0.032**
	NewUse	-0.050	-0.008	0.015	-0.022	-0.031	0.004	-0.056	-0.008	0.016	-0.015	-0.032	0.008
	Unobs. Prod Effects				1.090	-0.191	-0.066				1.087	-0.188	-0.065
					-0.191	0.449	0.043				-0.188	0.447	0.045
					-0.066	0.043	0.182				-0.065	0.045	0.184
	Unobs. Country Effects							0.007	0.001	0.001	0.008	0.002	0.001
								0.001	0.008	0.002	0.002	0.011	0.003
								0.001	0.002	0.003	0.001	0.003	0.004
	Unobs. Prod-Country Effects	1.764	-0.536	-0.080	1.397	-0.369	-0.029	1.753	-0.532	-0.082	1.400	-0.373	-0.031
		-0.536	0.517	0.032	-0.369	0.411	-0.005	-0.532	0.513	0.028	-0.373	0.395	-0.008
		-0.080	0.032	0.045	-0.029	-0.005	0.031	-0.082	0.028	0.044	-0.031	-0.008	0.029

Table 5.6: The percentage Explained and Unexplained Variance of seven models, together with the source of the error.

		Percentage of Total Variance		Percentage of total unobserved variance		
		Explained	Unexplained	Product	Country	
1. Products	θ^p	21.1	79.0			100.0
	θ^q	2.3	97.7			100.0
	θ^m	15.0	85.0			100.0
2. Country	θ^p	1.2	98.8			100.0
	θ^q	1.8	98.2			100.0
	θ^m	25.3	74.7			100.0
3. Prod & Countries	θ^p	34.1	65.9			100.0
	θ^q	40.3	59.8			100.0
	θ^m	50.0	50.2			100.0
4. All Variables	θ^p	50.4	49.6			100.0
	θ^q	51.1	49.0			100.0
	θ^m	62.2	37.8			100.0
5. All Variables & Product Error	θ^p	49.5	50.5	43.8		56.2
	θ^q	48.1	51.9	52.2		47.8
	θ^m	61.2	38.8	85.5		14.5
6. All Variables & Country Error	θ^p	50.5	49.5		0.4	99.6
	θ^q	51.5	48.9		1.5	98.5
	θ^m	61.6	38.4		6.4	93.6
7. Full Model	θ^p	49.6	50.4	43.6	0.3	56.1
	θ^q	48.4	51.6	52.4	1.3	46.3
	θ^m	60.2	39.8	85.1	1.6	13.3

The results of Table 5.6 also show that country-specific variables explain almost nothing of the innovation and imitation parameters. This is a remarkable finding, as for the imitation parameter we do find some significant estimates. This result is similar to the findings of Sood *et al.* (2009), namely that product characteristics are far more important than country characteristics in explaining the shape of the diffusion curve. The international diffusion literature tends to focus on the significance of the (country–specific) variables and not on how much they explain. The results show that this may overstate the real importance of some variables. For the market potential the country-specific variables do hold valuable information as they actually explain most of the variation.

Differences in the innovation parameters are best explained by product characteristics; differences in imitation parameters by the product-country variables. The added value of the interaction variables is quite substantial. These variables increase the percentage of explained variance with 10 percentage points for each diffusion parameter.

With twelve different products, the study in this chapter has a lot of variety in products, compared to other studies in the international diffusion literature. Despite these twelve products, the large portion of unexplained product variation (i.e., up to 85.5 % for the market potential parameter) shows that the differences across all product-country diffusion patterns is only partly captured by the three product constructs. However, the results of Model 4 show that ignoring unobserved product variation seems to influence the parameter estimates of the country and product-country variables, due to over-fitting.

The multilevel model, especially the “Full Model”, explains more of the variation than published results in diffusion literature. For example, Talukdar *et al.* (2002) find that around 80% of the variation in the innovation and imitation parameter of the diffusion model comes from unobserved interaction or unobserved product effects, where in the “Full Model” we only have respectively 50.4% and 51.6% unexplained variation. However, even though we do better, 50% is still a substantial amount of variation that is unexplained. In the next section, we take a closer look at the remaining variation, and put forward a method to reveal the hidden structure in this variation. In particular, this variation can be explained in terms of correlations. For the remaining sections we use the results of the Full model, as this is most complete, and does not suffer from over-fitting as for example Model 4 does.

5.5 Graphical representation of the remaining variation

In this section, we show how the remaining variation from the multilevel models can be explained in terms of correlations, and how these can be revealed and visualized by using Principal Component Analysis [PCA].

The multilevel model thus far accounts for product, country, and product-country variation in the diffusion parameters. By including these characteristics we tried to explain the variation in the diffusion parameters. The product-country interactions are important in explaining the variation in the transformed diffusion variables, θ, but the multilevel model finds that a relatively large portion of the variation in θ is left unexplained. However, there may still be some structure in the unexplained variation. For example, deviations from the expected diffusion parameters in country A may be positively correlated to those in country B. So, if the diffusion in country A for product 1 is faster than expected based on the unobserved product and country characteristics, it is likely that the diffusion of product 1 in country B will also be faster. The reasons behind these correlations are difficult to give and they cannot be captured by the “traditional” use of product and country variables, or interactions between the two. In this section we propose a method to visualize the correlations.

We use dimension reduction techniques, that is, principal component analysis [PCA], on the remaining, previously unexplained, variation across the product-country combinations, that is, ν_{ij}^{d}. The extracted components can be represented graphically to give insights in the hidden structure in the unexplained variation. The PCA can also be used to generate predictions of the expected deviation in the actual diffusion parameters from the expected parameters based on the second-level model (5.4). Therefore this methodology can also be used for pre-launch forecasting. We further discuss this application in a later section.

5.5.1 Weighted PCA

In our case, a traditional PCA is not possible as we have missing data in the observation matrix, that is, we do not observe all products for all countries. A “missing observation” might also correspond to a country in which a product has not been launched yet. We will present a variant of PCA that deals with missing data by iteratively generating predictions for the missing cases, such that next a standard PCA can be performed.

A traditional PCA starts with an observation matrix. In our case we use the residuals ν_{ij}^d of the second level (5.6). We take the residuals of the “Full Model” as these are cleaned for observed and random product and country effects. We treat the residuals of each transformed diffusion parameter separately. The procedure for each diffusion parameter is the same, hence we present it for $\nu^{\mathbf{d}}$ in general, where d can be p, q or m.

The 867-by-1 vector of residuals $\nu^{\mathbf{d}}$ can also be represented as a 12-by-82 matrix, that is, a matrix with the twelve products on the rows and the 82 countries as the columns. We denote this matrix by $\boldsymbol{\Upsilon}^{\mathbf{d}}$. In our case $\boldsymbol{\Upsilon}^{\mathbf{d}}$ has several missing values, as we included countries that do not have information on all the products. For a standard PCA the objective is to find a subspace of the data with rank k, that accounts for as much variation as possible. To be able to graphically represent the subspace we set k at 2. The PCA then results in a 12-by-2 matrix of object (product) scores $\mathbf{X}^{\mathbf{d}}$ and a 82-by-2 matrix of component (country) loadings $\mathbf{A}^{\mathbf{d}}$. $\mathbf{X}^{\mathbf{d}}$ and $\mathbf{A}^{\mathbf{d}}$ can be obtained by minimizing

$$L_{PCA}(\mathbf{X}^{\mathbf{d}}, \mathbf{A}^{\mathbf{d}}) = ||\boldsymbol{\Upsilon}^{\mathbf{d}} - \mathbf{X}^{\mathbf{d}}\mathbf{A}^{\mathbf{d}\prime}||^2 = \sum_{i=1}^{12}\sum_{j=1}^{82}(\nu_{ij}^d - x_i^{d\prime}a_j^d)^2, \tag{5.9}$$

where x_i^d and a_j^d are columns of $\mathbf{X}^{\mathbf{d}}$ and $\mathbf{A}^{\mathbf{d}}$, respectively.

Minimization of (5.9) is only possible in the absence of missing values for ν_{ij}^d. To deal with the missing values, we use a Weighted Majorization Algorithm (Kiers, 1997; Groenen *et al.*, 2003). In this case the missing observations are iteratively estimated based on the result of the PCA. However, this does not work well if there is a block of missing data in the matrix, that is, if there is a group of countries that have a missing value for the same group of products. Therefore, for the PCA we exclude 11 countries having missing data for 5 or more products. For the remaining 12-by-71 matrix there are still missing entries. The objective function to minimize for the iterative majorization is

$$L_{WPCA}(\mathbf{X}^{\mathbf{d}}, \mathbf{A}^{\mathbf{d}}) = \sum_{i=1}^{12}\sum_{j=1}^{71} w_{ij}(\nu_{ij}^d - x_i^{d\prime}a_j^d)^2, \tag{5.10}$$

where w_{ij} is a vector of weights, which is 1 if the entry in matrix $\mathbf{\Upsilon^d}$ is observed and 0 if it is missing. Details on the implementation of this iterative majorization can be found in Groenen et al. (2003), and a short overview is given in Appendix 5.B. From these iterations we get two matrices $\mathbf{X^d}$ and $\mathbf{A^d}$, each containing two columns with respectively, the object scores of the twelve products and the component loadings of the 71 remaining countries. Because the dimension is two, these component loadings and object scores can be plotted together in a graph. Note that in the iterative majorization we use a scaling factor to make $\mathbf{X^d}$ and $\mathbf{A^d}$ comparable in scale (see Appendix 5.B).

By multiplying $\mathbf{X^d}$ and $\mathbf{A^d}$ we obtain an approximation, or fit, of the original residuals in $\mathbf{\Upsilon^d}$. In particular if the k dimensions (in our case 2) would explain 100% of all the variation, a multiplication of $\mathbf{X^d}$ and $\mathbf{A^d}$ will result in an exact match with $\mathbf{\Upsilon^d}$. The latter is normally not the case, but in general the multiplication results in a good approximation. The approximation of an element of $\mathbf{\Upsilon^d}$ equals the inner product of the corresponding product and a country vector. In Figure 5.1 we illustrate how the magnitude of this inner product can be obtained geometrically. First, project a product (represented by a dot, x in Figure 5.1) onto a country vector (a in Figure 5.1). Next, calculate the length of the projection vector (vector p) times the length of the country vector, in the example this product is 8.5. The inner product is positive if the product and country vectors point in the same direction, and negative otherwise. Note that this procedure also can be used for the missing data points. For example, if we do not observe the diffusion of product X in country A, but we do observe the diffusion of product X in other countries and of country A for other products, we obtain an estimate of the relative performance of that X in country A.

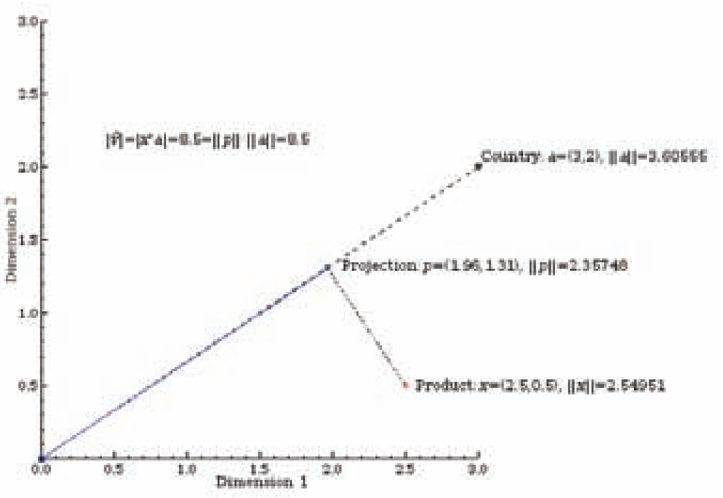
In the next section we further explain how, in combination with the results from the multilevel model, this can be very helpful for pre-launch and early forecasts.

Besides a weighted PCA on the residuals of separate diffusion parameters, we can also perform a weighted PCA on the combination of the three. In this case, the observation matrices of the separate diffusion parameters are stacked together resulting in a single observation matrix $\mathbf{\Upsilon}$, which is a 36-by-71 matrix. In order to do this, the residuals of the separate diffusion parameters should be similar in size, which is obtained by standardizing the complete residual vector with then estimated covariance matrix Λ from the multilevel model, see (5.6).

5.5.2 Graphical results

In Figures 5.3 to 5.5 of Appendix 5.C we give the graphical results of the weighted PCA on the residuals of the three transformed diffusion parameters, while Figure 5.6 shows the graphical results of the combined PCA on all residuals. In these figures the object scores ($\mathbf{X^d}$), representing the products, are given as black dots. The component loadings ($\mathbf{A^d}$), representing the countries, are given as gray vectors. We give each geographical

Figure 5.1: Illustrative example to graphically obtain the fitted residuals in the PCA.



region a different marker. The percentage of the variation explained by each dimension is given in parentheses, thus the first dimension for the residuals of the innovation parameter (Figure 5.3) explains 17% of the total variation in these residuals. The second dimension explains another 15.5%. In total these two dimensions explain over 30% of the remaining variation. Note that this 30% only explains the idiosyncratic part of the error in the multilevel model. For the innovation parameter this is 56% of the remaining 50% of the total variation, see Table 5.6.

Additionally, we relate the PCA dimension to country characteristics. It could be that countries that are highly correlated, so located close to each other, are similar in terms of some characteristic, for example, GDP per capita. To analyze this we use so-called property fitting lines. These lines can be obtained by regressing the component (country) loadings ($\mathbf{A^d}$) on economic variables. In the figures in Appendix 5.C we also show these property fitting lines for six variables. On the label next to the line we give the attribute, and how much of this attribute is explained by these component loadings. The direction of these lines indicates how they relate to the component loadings, that is, traveling along the line corresponds with higher values of the attribute. For example, consider Figure 5.3 for the innovation parameter. Countries with a high GDP should, on average, be located in the left-hand part of the figure and slightly near the top. In the opposite direction should be the countries that have a relatively low GDP. This is exactly the case, as on the left side are mostly western European countries, such as Denmark and the Netherlands, and the bottom-right shows the African and Developing (Asian) countries, such as Vietnam and Morocco.

In the next section, we will show how the results of this graphical representation of the remaining variation, in combination with the multilevel model, can be used for managerial purposes.

5.6 Managerial implications

The combination of the multilevel model and the PCA can be used for managerial purposes. We present two important applications. First, the information can be used for a global roll-out strategy of a new product. Second, the results can be used for pre-launch or early forecasts of products in a particular country. The results are particularly useful if the product has already been introduced in other countries.

5.6.1 Global roll-out strategy of a new product

The success of a new product is in part dependent on the global roll-out strategy. A successful introduction in a country A often has beneficial effects in other countries as well. The model provides managers with a tool to identify the best country to first introduce a new product.

There are two aspects to this tool, which can be used independently or together. First, one can obtain a general indication of which country is likely to have the fastest diffusion for the new product. Second, the roll-out strategy can be fine-tuned by combining external managerial information with the graphical representation of the correlations between products and countries.

We first discuss how to get a general indication of the diffusion of a new product. For this aspect one only uses the results of the multilevel model. Managers will probably introduce the product first in countries with a fast predicted diffusion. These predictions can easily be obtained from the multilevel model by multiplying the product and country variables with the estimated parameters.

As an example, we re-estimated the full multilevel model without the videogame console data. We used the results from this model to predict the diffusion parameters for this product for all countries. Note, that because we included entry variables in the multilevel model we corrected for any historical pattern due to the moment of introduction. Candidate countries for the initial launch should ideally have an early moment of peak sales as well as a high market potential. We found many Latin American countries amongst the countries with an early moment of peak sales, whereas based on market potential Asian countries where on top of the list. However, a combination of the two is preferred. In Table 5.7 we give the countries that score highest on a combination of the moment of peak sales (T^*) and market potential (m), where we give an equal importance to the two.

The tool described thus far only gives a general indication, as the estimates coming from the multilevel model are averages, meaning that there may be substantial uncer-

Table 5.7: Countries that have an early moment of peak sales and a high market potential for the market of Video Game Consoles.

Country	T*	m
Norway	12.2	48.1
Denmark	13.1	53.6
Slovenia	11.8	39.1
Portugal	13.6	55.2
Sweden	13.6	54.6
Netherlands	13.7	56.0
Austria	13.5	52.2
Israel	12.3	40.1
Finland	13.5	50.2
Singapore	14.7	74.1

tainty. The second aspect is used to fine-tune the roll-out strategy. For this part one can use the results from the first step, but managers can also use their own beliefs on which country is best for the introduction of the new product. In the end, the manager may predict that the new product will diffuse faster in country A than predicted by the multilevel model. In this case, the weighted PCA can be used to build on this belief. In particular, countries near country A in the figures in Appendix 5.C will also have a better performance than the average given by the multilevel model. For this tool it is best to look at the graphical results of the diffusion parameters together, because the nearby countries may differ across the diffusion parameters. As an example, if a manager believes that a new product in the U.S. diffuses faster than predicted by the multilevel model, then the same holds for Canada, New Zealand and Sweden as well (see the magenta circle in Figure 5.6). In that case he might consider an early launch in these countries as well.

This use of our methodology requires the skills of a manager, but hands him the opportunity to utilize much more information in his decision making process.

5.6.2 Pre-launch forecasts of a specific product-country combination

This second use of the results is to generate pre-launch or early forecasts of a particular product-country combination. In this subsection we discuss how to predict the diffusion parameters of a product that is new to a country, but has been introduced in other countries.

The first step is to use the multilevel model to get a first estimate for the diffusion parameters based on the variables of the product and country (e.g., GDP, Fun vs. Utility, etc.). Second, these first suggestions can be improved by including the estimated random effects of the specific country and product. Finally, because this product-country combination is missing in the observation matrix in the PCA, one can use the estimate of this missing observation coming from the weighted PCA, as described in Section 5.5.1.

Table 5.8: The out-of-sample case of DVDs in Venezuela. The θ is build up from different parts.

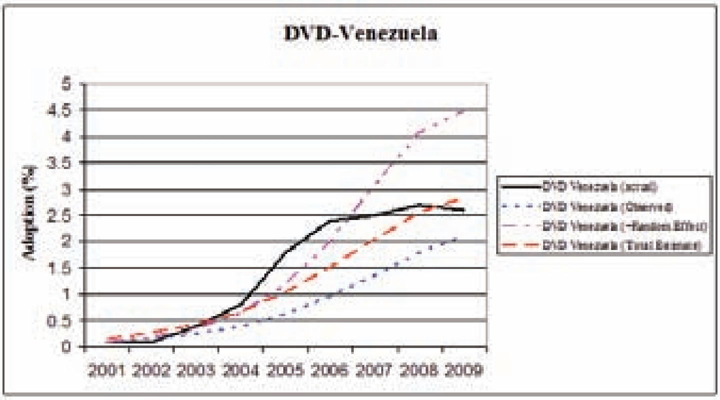
	DVD Venezuela		
	θ^p	θ^q	θ^m
observed characteristics	-5.483	-1.049	0.167
random product effect	-0.519	0.331	0.112
random country effect	0.033	-0.009	-0.003
estimate PCA	0.806	-0.376	-0.051
Total θ	-5.163	-1.103	0.226
	p	q	m
Estimate based on characteristics	0.0041	0.519	16.7
Estimate on obs. char. + Random Effects	0.0025	0.652	27.6
Estimate on Total θ	0.0057	0.498	22.6

In this case, it is most beneficial to look at the PCA results of the diffusion parameters separately.

We illustrate this tool using the diffusion of DVDs in Venezuela. The reason to choose this case is that it was left out of the multilevel estimation, because it only has 9 data-points. We give the forecasted diffusion curve of the first nine periods and compare it to the actual adoption. In Table 5.8 we present the different components that make up the diffusion parameter estimates. First, the multilevel model provides a part of the θ's based on the observed product and country characteristics for DVDs in Venezuela. Second, we include the product and country random effects, also coming from the multilevel model (i.e. the Full Model in Table 5.5). Note, that the product-specific effect is quite large, this is due to the large variance in the product random effects as shown in Section 5.4. Finally, there is a product-country specific part coming from the PCA. In particular, we use PCA results to estimate the residual effect. These four parts add up to a total estimate for all three θ's, which need to be transformed back to diffusion parameters. To show the added value of each of these components, we distinguish three cases: (i) only using predictions based on observed characteristics; (ii) a combination of the observed characteristics and the random effects; and (iii) the total θ's.

In Figure 5.2 we show the diffusion patterns that correspond to the three sets of predicted diffusion parameters in Table 5.8, and we compare them to the actual sales of DVDs in Venezuela. The figure shows that only using the observed part leads to an underestimation of the diffusion curve. The additional part of the random effects leads to an overestimation, which is mainly due to the product-random effect. Finally, including the PCA estimate of the residual leads to the best estimated diffusion curve.

Figure 5.2: The actual and estimated sales for DVDs in Venezuela, where we look at the estimated diffusion pattern based on several information sources: Observed Characteristics, Random effects and PCA-estimates.



5.7 Conclusion

In this chapter we studied the international diffusion of products using a truly global dataset of 82 countries and, compared to other international diffusion studies, a large set of products. We find that the existing models explain only a part of the story behind the variation in the diffusion across the different product-country pairs. In these models a substantial amount of variation remains unexplained. The hardest part in explaining this variation is the interaction between products and countries. For each combination there is a unique story why a particular product diffuses the way it does in a particular country.

We extended the multilevel diffusion model of Talukdar *et al.* (2002) to capture the product, country, product-country, and interaction effects. We are able to explain more variation than previous studies. However, even in the most complete version of the model we explain less than 50% of the total variation in the internal and external diffusion parameters of the diffusion curve and 60% of the market potential parameter. However, the remaining variation is not completely random. We used dimension reduction techniques to show the hidden correlation patterns in the "unexplained variation". We suggest PCA to graphically represent these correlations. If two countries are close to each other in the PCA map, they are positively correlated in how they differ from their average diffusion patterns. In other words, they tend to have an above average diffusion for the same set of products. We show that there are substantial correlations and the PCA map has a high face validity.

The results from this chapter can be useful for managers. First, the multilevel model can be used as an indication where a new product needs to be introduced first for a successful international roll-out. Second, based on this first indication and the manager's beliefs, the graphical representation can give insights in which countries behave similarly, and thus should be considered for early introduction as well. Finally, a combination of the multilevel model and the graphical results of the PCA can be used for pre-launch forecasts of a product new to a country, but which has already been introduced in other countries. In an illustration we showed that the final adjustment given by the PCA results give a substantial increase in predictive performance.

The suggested methodology is new and most complete in capturing product-country variation, and with the PCA we can give insights in the remaining variation. In this way this chapter tries to give a new direction to the international diffusion literature. However, this is a first step as this chapter also shows that there is substantial unexplained variation across products. Although we have twelve products, a larger and more diverse set of products will probably make it possible to explain more of the product variation. Additionally, this may lead to more explanatory power of the interaction terms. Furthermore, a challenge for further research is to integrate the correlations structure (revealed by our PCA) into the hierarchical diffusion model.

5.A The Hierarchical Bayes MCMC estimation steps

In this appendix we give a schematic overview of the MCMC sampler used for the multi-level diffusion model. Steps 2 and 3 presented below are iterated. After a burn-in period this sampling scheme will produce draws from the posterior distribution of the parameters. Using these draws one can calculate statistics of the model and parameter estimates.

1. Initialization
We start with some starting values for all parameters, these are given by ($\theta_{ij}^{p,0}$, $\theta_{ij}^{q,0}$, $\theta_{ij}^{m,0}$, σ_{ij}^{0}, $\beta_k^{p,0}$, $\beta_k^{q,0}$, $\beta_k^{m,0}$, Ω_1^0, Ω_2^0, Λ^0).

2. Sampling of first-level parameters
In the s-th iteration of the chain, we first sample all first-level parameters. We do this for each product-country combination and all (transformed) diffusion parameters separately. That is we sample

$$\begin{aligned}&\theta_{ij}^{p,s}|N_{ijt},X_i,Z_j,W_{ij},\theta_{ij}^{q,s-1},\theta_{ij}^{m,s-1},\sigma_{ij}^{s-1},\beta_k^{p,s-1},\beta_k^{q,s-1},\beta_k^{m,s-1},\Lambda^{s-1},\\&\theta_{ij}^{q,s}|N_{ijt},X_i,Z_j,W_{ij},\theta_{ij}^{p,s},\theta_{ij}^{m,s-1},\sigma_{ij}^{s-1},\beta_k^{p,s-1},\beta_k^{q,s-1},\beta_k^{m,s-1},\Lambda^{s-1},\text{ and}\\&\theta_{ij}^{m,s}|N_{ijt},X_i,Z_j,W_{ij},\theta_{ij}^{p,s},\theta_{ij}^{q,s},\sigma_{ij}^{s-1},\beta_k^{p,s-1},\beta_k^{q,s-1},\beta_k^{m,s-1},\Lambda^{s-1},\end{aligned} \tag{5.11}$$

for all i and j. For all three parameters, we use a Metropolis-Hastings Random Walk algorithm. First we draw a candidate value:

$$\theta_{ij}^{d,+} \sim N(\theta_{ij}^{d,s-1}, a^2), \tag{5.12}$$

where d represents the diffusion parameters p, q or m. Further, a^2 is a scale parameter which is set such that we obtain a good acceptance rate. The candidate is now accepted with probability:

$$\psi = \min\left(\exp\left(q(\theta_{ij}^{d,+}) - q(\theta_{ij}^{d,s-1})\right), 1\right) \tag{5.13}$$

if the candidate is rejected we set $\theta_{ij}^{d,s} = \theta_{ij}^{d,s-1}$. The function $q()$ contains the fit of the diffusion model and the contribution of the hierarchical prior, that is, the second level model. Formally, $q()$ is given by

$$q(\theta_{ij}^d) = -\frac{1}{2} r^d (\Lambda_{d,d} - \Lambda_{d,-d}\Lambda_{-d,-d}^{-1}\Lambda_{-d,d})^{-1} r^d - \frac{1}{2\sigma_{ij}^2}\sum_t [N_{ijt} - mF_{ij}(t)]^2, \tag{5.14}$$

where the subscript d denotes the row/column position in Λ corresponding to the focal diffusion parameter and $-d$ the row/column positions in Λ of the other two diffusion parameters. $F_{ij}(t)$ is given in equation (5.2) and r^d is given by:

$$r^d = \theta_{ij}^d - \left(f(X_i, Z_j, W_{ij}|\beta_k^d)'\beta_k^d + \Lambda_{c,-c}\Lambda_{-c,-c}^{-1}\left[\theta_{ij}^{-d} - f(X_i, Z_j, W_{ij}|\beta_k^{-d})'\beta_k^{-d}\right]\right), \tag{5.15}$$

where $f(X_i, Z_jW_{ij}|\beta_k^d)$ represents the fitted value for parameter d in the second-level model (5.4).

We use a Gibbs sampler for the errors variance, that is, we sample

$$\sigma_{ij}^s|N_{ijt}, \theta_{ij}^{p,s}\theta_{ij}^{q,s}, \theta_{ij}^{m,s}. \tag{5.16}$$

The conditional distribution of σ_{ij}^s is an inverted gamma distribution with parameters n_{ij} and $\sum_t[N_{ijt} - mF_{ij}(t)]^2$, where n_{ij} is the number of observations in N_{ijt}.

3. Sampling the second-level parameters

To sample the second-level parameters we write (5.4) and (5.5) as

$$\begin{pmatrix} \theta_{ij}^p & \theta_{ij}^q & \theta_{ij}^m \end{pmatrix} = [(1, X_i', Z_j', W_{ij}', (X_i \otimes [Z_jW_{ij}])', C_i', P_j')] \begin{pmatrix} \beta_0^p & \beta_0^q & \beta_0^m \\ \beta_1^p & \beta_1^q & \beta_1^m \\ \beta_2^p & \beta_2^q & \beta_2^m \\ \beta_3^p & \beta_3^q & \beta_3^m \\ \beta_4^p & \beta_4^q & \beta_4^m \\ \gamma_1^p & \gamma_1^q & \gamma_1^m \\ \gamma_2^p & \gamma_2^q & \gamma_2^m \end{pmatrix} + \begin{pmatrix} \nu_{ij}^p & \nu_{ij}^q & \nu_{ij}^m \end{pmatrix}, \tag{5.17}$$

where C_i is a dummy vector for country i and P_j a dummy vector for product j. We summarize this equation as

$$\theta_{ij} = V_{ij}\beta + \nu_{ij}. \tag{5.18}$$

The model in (5.18) is a multivariate regression with Λ as the covariance matrix of the error and (5.6) specifies priors on γ_1 and γ_2. Stacking all equations we obtain $\theta = V\beta + \nu$, where $\text{vec}(\nu \sim N(0, \Lambda \otimes I))$. Next to the priors on the random effects γ_1 and γ_2 we need to define priors on the intercepts (β_0^d). This is necessary as we use transformations of the diffusion parameters, using a diffuse prior can lead to improper distributions of the diffusion parameters itself. The priors on the constants have the following form:

$$\begin{aligned}
\beta_o^{p,prior} &= N\left[\frac{1}{2}\left(\ln\left(\frac{p^{\max}}{1-p^{\max}}\right) + \ln\left(\frac{p^{\min}}{1-p^{\min}}\right)\right), \frac{1}{16}\left(\ln\left(\frac{p^{\max}}{1-p^{\max}}\right) + \ln\left(\frac{p^{\min}}{1-p^{\min}}\right)\right)^2\right], \\
\beta_o^{q,prior} &= N\left[\frac{1}{2}\left(\ln\left(\frac{\frac{q^{\max}}{2}}{1-\frac{q^{\max}}{2}}\right) + \ln\left(\frac{\frac{q^{\min}}{2}}{1-\frac{q^{\min}}{2}}\right)\right), \frac{1}{16}\left(\ln\left(\frac{\frac{q^{\max}}{2}}{1-\frac{q^{\max}}{2}}\right) + \ln\left(\frac{\frac{q^{\min}}{2}}{1-\frac{q^{\min}}{2}}\right)\right)^2\right], \\
\beta_o^{m,prior} &= N\left[\frac{m^{prior}}{100}, 10\right].
\end{aligned} \tag{5.19}$$

where p^{min}, p^{max} and q^{min} and q^{max} are prior beliefs on the interval of p and q, and m^{prior} is a prior belief on the location of the m parameters.

We can now use a Gibbs sampling step to draw vec(β) as follows

$$\text{vec}(\beta^s) = \begin{pmatrix} \beta^{p,s} \\ \beta^{q,s} \\ \beta^{m,s} \end{pmatrix} \sim \\ N\left[((\Lambda^{-1} \otimes (V'V)) + \Xi^{-1})^{-1}(\text{vec}(V'\theta^s\Lambda^{-1}) + \Xi^{-1}\mu), ((\Lambda^{-1} \otimes (V'V)) + \Xi^{-1})^{-1}\right] \quad (5.20)$$

where μ is the vector containing the mean of the prior distribution, that is, on the entry of the constants we have the mean of the distributions (5.19). Similarly, Ξ is the covariance of the prior distribution, where on the diagonal entry of the constants we have the variance of the distributions (5.19). For the locations corresponding to the random effects we have Ω_1 or Ω_2.

Next we sample the covariance matrices of the second level, that is,

$$\begin{array}{l} \Lambda^s | X_i, Z_j, W_{ij}, \theta_{ij}^{q,s}, \theta_{ij}^{q,s}, \theta_{ij}^{m,s}, \beta_k^{p,s}, \beta_k^{q,s}, \beta_k^{m,s}, \kappa_1, \mathrm{K}_2, \\ \Omega_1^s | \beta_k^{p,s}, \beta_k^{q,s}, \beta_k^{m,s}, \pi_{1,1}, \Pi_{1,2}, \text{and} \\ \Omega_2^s | \beta_k^{p,s}, \beta_k^{q,s}, \beta_k^{m,s}, \pi_{2,1}, \Pi_{2,2}. \end{array} \quad (5.21)$$

All three conditional distributions are Inverted Wishart. For all covariance matrices we use a prior. For Λ we use the prior distribution $IW(\kappa_1, K_2)$ where κ_1 is set at 15 and K_2 is set at 2 I_3. For Ω_1 and Ω_2 we use $IW(\pi_{1,1}, \Pi_{1,2})$ and $IW(\pi_{2,1}, \Pi_{2,2})$, respectively. For the product random effects the prior parameters need to be set carefully, because there is a limited number of products available. We set $\pi_{1,1}$ at the number of products, and $\Pi_{1,2}$ as a diagonal matrix with elements [1 0.5 0.25]. For the country specific random effects the prior is less influential and do we set $\pi_{2,1}$ at 6, and $\Pi_{1,2}$ at 2 I_3. In the s-th iteration we now drawn the matrices from

$$\Lambda^s \sim IW(n + \kappa_1, \\ \textstyle\sum_{ij} \begin{pmatrix} \theta^{p,s} - f(X_i; Z_j; W_{ij} | \beta^{p,s}) \\ \theta^{q,s} - f(X_i; Z_j; W_{ij} | \beta^{q,s}) \\ \theta^{m,s} - f(X_i; Z_j; W_{ij} | \beta^{m,s}) \end{pmatrix}' \begin{pmatrix} \theta^{p,s} - f(X_i; Z_j; W_{ij} | \beta^{p,s}) \\ \theta^{q,s} - f(X_i; Z_j; W_{ij} | \beta^{q,s}) \\ \theta^{m,s} - f(X_i; Z_j; W_{ij} | \beta^{m,s}) \end{pmatrix} + \mathrm{K}_2), \quad (5.22)$$

$$\Omega_1^s \sim IW(n_1 + \pi_{1,1}, \textstyle\sum_i \gamma_{1i}^{s'} \gamma_{1i}^s + \Pi_{1,2}), \text{and}$$
$$\Omega_2^s \sim IW(n_2 + \pi_{2,1}, \textstyle\sum_j \gamma_{2j}^{s'} \gamma_{2j}^s + \Pi_{2,2})$$

5.B PCA with iterative majorization

In this appendix we give a short schematic overview of the iterative majorization procedure, a more complete overview is given in Groenen *et al.* (2003). In this appendix we ignore the superscript representing the diffusion transformation, because the iterative scheme is similar for each vector of residuals coming from the multilevel model.

We start with a matrix $\boldsymbol{\Upsilon}$ which is built from the vector of residuals ν. Additionally, we have a vector of weights $\mathbf{w}$, where element w_{ij} is 1 if entry (i, j) in $\boldsymbol{\Upsilon}$ is observed and 0 if it is missing. The goal is to find two matrices $\mathbf{X}$ and $\mathbf{A}$ that have a lower dimension k and explain as much of the variation in $\boldsymbol{\Upsilon}$.

1. Initialize $\mathbf{X_0}$ and $\mathbf{A_0}$. At the start of the iterative scheme one needs a first guess of the missing values. For example, by performing standard PCA as in (5.9), but by setting the missing residuals at zero.

2. Compute matrix $\mathbf{R}$. Matrix $\mathbf{R}$ is the weighted matrix of the observed entries from $\boldsymbol{\Upsilon}$ and the estimated missings based on $\mathbf{X_{s-1}}$ and $\mathbf{A_{s-1}}$, where $\mathbf{X_{s-1}}$ and $\mathbf{A_{s-1}}$ are given by step 3 of the previous iteration (or by $\mathbf{X_0}$ and $\mathbf{A_0}$ in case it is the first iteration). The entries of $\mathbf{R}$ are given by $r_{ij} = (1 - w_{ij})x_i^{s-1\prime}a_j^{s-1} + w_{ij}\nu_{ij}$.

3. Compute $\mathbf{X_s}$ and $\mathbf{A_s}$ by using a singular value decomposition [SVD] on $\mathbf{R}$. The SVD on $\mathbf{R}$ gives three matrices for which it holds that $\mathbf{R} = \mathbf{VDU}'$, from these three matrices we can obtain $\mathbf{X_s}$ and $\mathbf{A_s}$, namely

$$X_s = \left(\frac{r}{c}\right)^{1/4} \mathbf{VD}^{1/2} \text{and} \mathbf{A_s} = \left(\frac{\mathrm{c}}{\mathrm{r}}\right)^{1/4} \mathbf{UD}^{1/2}, \tag{5.23}$$

   where r and c are the number of rows and columns of $\boldsymbol{\Upsilon}$, respectively. We multiply the matrices with the scale factor to make the values in the matrices of a similar size. The latter does not affect the SVD, but makes it easier to make plots.

4. Repeat steps 2 and 3 until $L_{WPCA}(\mathbf{X_{s-1}}, \mathbf{A_{s-1}}) - L_{WPCA}(\mathbf{X_s}, \mathbf{A_s})$ is smaller than a certain threshold, where $L_{WPCA}()$ is defined in (5.10).

5.C Two-dimensional plots of the PCA on residuals

Figure 5.3: The graphical representation based on a PCA on the residuals of the innovation diffusion parameter

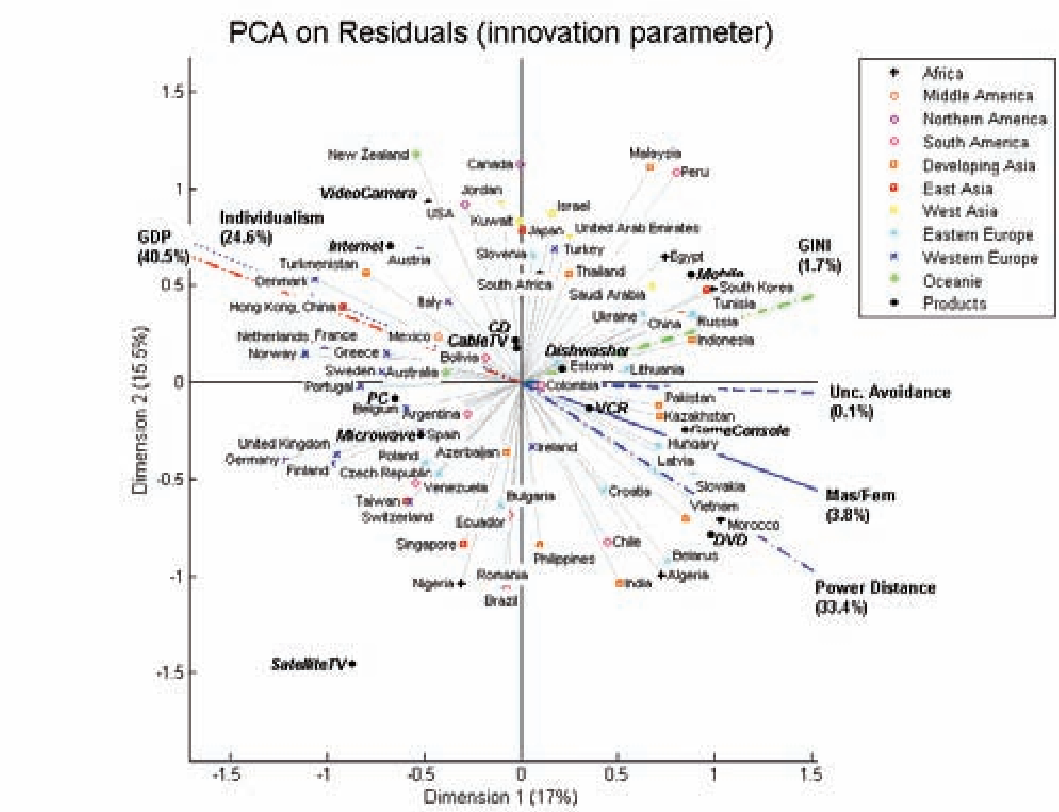


Figure 5.4: The graphical representation based on a PCA on the residuals of the imitation diffusion parameter

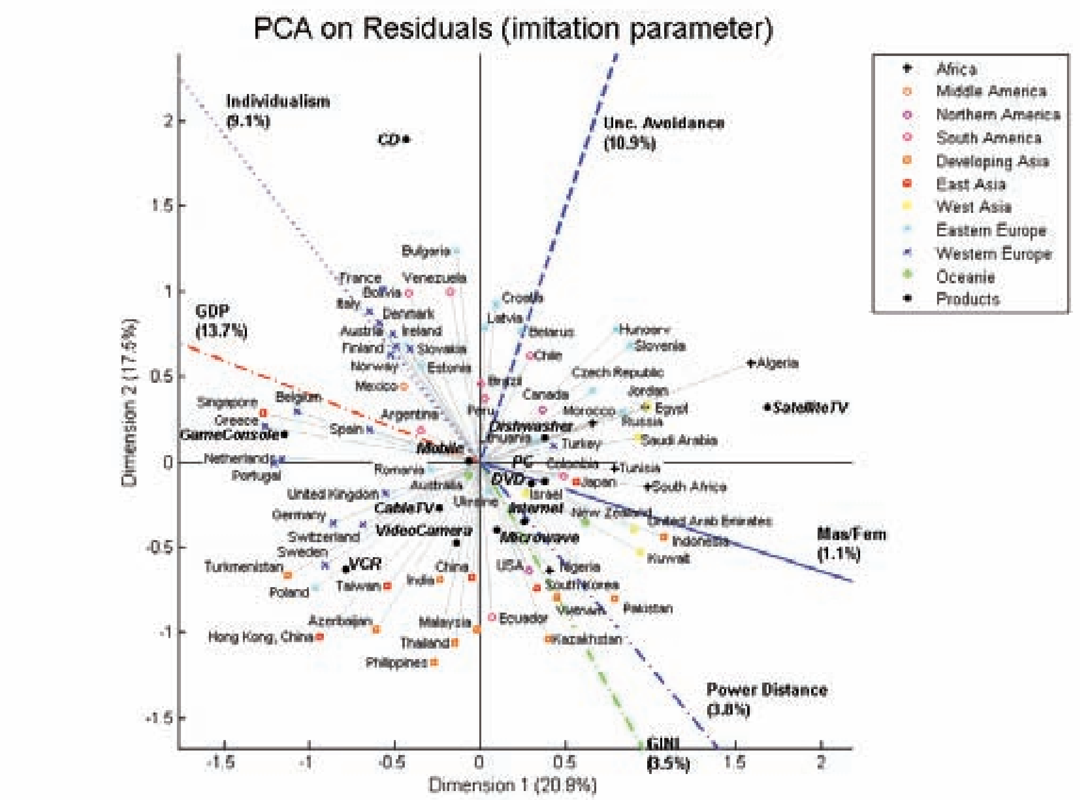


Figure 5.5: The graphical representation based on a PCA on the residuals of the market potential diffusion parameter

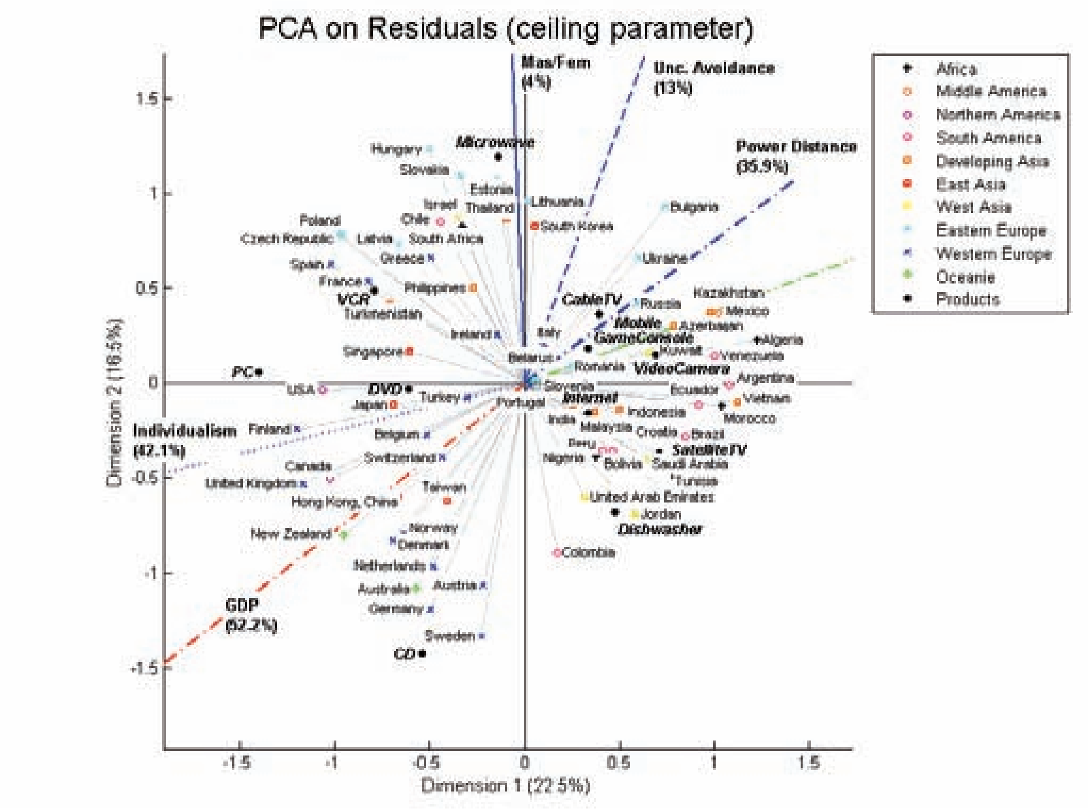
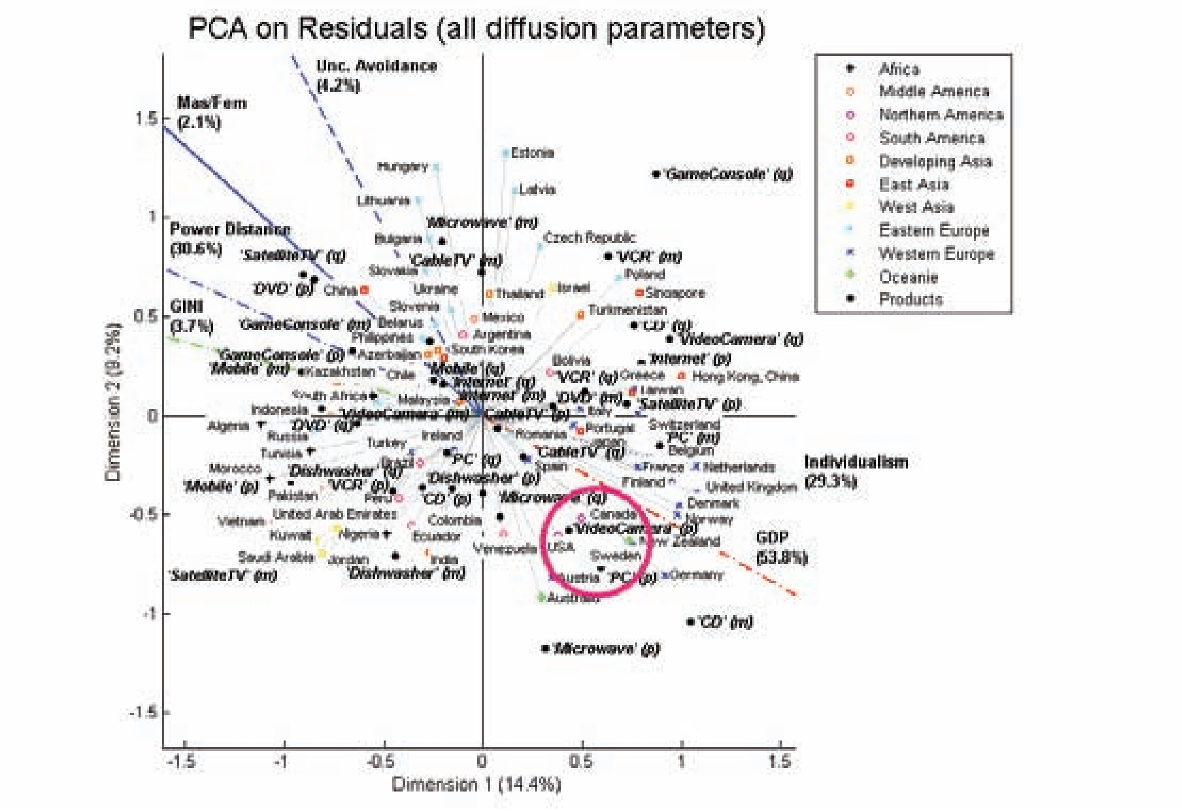


Figure 5.6: The graphical representation based on a PCA on the combined residuals of the diffusion parameters



Chapter 6

Summary

In this chapter we give a summary of the main findings of this dissertation. Although diffusion models have been studied extensively, this thesis gives new and important insights in modeling diffusion. In particular, all chapters in this thesis showed that the recent increase in availability of diffusion data is associated with new challenges for modeling the diffusion pattern. This increase in data availability has two sources: (i) in the past decade the focus has shifted from one diffusion series to the analysis of multiple series, and (ii) the most common observational frequency has shifted from annual to monthly or quarterly, and in some cases data is available at an even higher frequency.

In Chapter 2 we considered three current estimation methods and one new estimation method for the Bass Diffusion model. Although estimation bias in the Bass model has been studied before, this chapter gives new insights. We show that each estimation method has its own assumption on the error term. More importantly, the choice of the estimation method, and thereby the associated error process, can have major consequences. Also, the observational frequency may be a source of estimation bias. Next to the estimation performance of the different estimation techniques, we also consider the performance of each method for data generation.

This chapter shows that researchers as well as managers need to be careful in selecting the estimation method of a diffusion model. In particular, the match between the chosen error structure and the true error as well as the data frequency influence the estimation and simulation of diffusion models. In case one has only low-frequency data available or if one is unsure about the error structure, the newly proposed method in this chapter is most robust and hence the safest option.

Chapters 3 and 4 consider challenges arising from using high-frequency diffusion data, namely: (i) mixed frequencies, which occur due to irregular observation at the start of the diffusion process, and (ii) seasonality, which is a logical consequence of using high-frequency data.

In Chapter 3 we have suggested a number of ways to deal with mixed-frequency data. We have shown how these methods work and perform for different estimation methods. For some estimation methods, using mixed-frequency data is straightforward. However, for the Bass regression method, correctly accounting for mixed frequencies is quite involved. The analysis also showed that failing to correct for mixed frequencies can lead to a large bias in the parameter estimates. In this sense, this chapter is also a useful summary of the do's and don'ts of using mixed-frequency data.

In Chapter 4 we addressed seasonality in diffusion models, where we developed a seasonal structure that can be used in combination with standard diffusion models. The model proposed in this chapter allows to estimate the seasonal pattern, without interfering with the underlying diffusion pattern. The basic structure is that the seasonal peak in a focal month consists of sales drawn from the months around it. In the proposed model we made a distinction between a flexible, estimated, intertemporal demand pattern and a fixed pattern. The flexible pattern allows to study intertemporal demand shifts, and in most cases the corresponding model outperformed the other models on in-sample fit and short-term forecasting. Further, we showed that these other seasonal diffusion models lead to biased estimates. Ignoring seasonality, that is, using a standard diffusion model, only works for completed diffusion series.

In the final chapter (Chapter 5) we consider the case of increased data availability due to multiple series, rather than more data points in one series. We studied the international diffusion of products using a truly global dataset of 82 countries and, compared to other international diffusion studies, a large set of products. We find that the existing models explain only a part of the story behind the variation in the diffusion across the different product-country pairs. In these models a substantial amount of variation remains unexplained. The hardest part in explaining this variation is the interaction between products and countries. For each combination there is a unique story why a particular product diffuses the way it does in a particular country. A multilevel diffusion model still shows much unexplained variation. Therefore, we used Principal Component Analysis [PCA] to graphically represent the hidden structure in the remaining variation. The suggested methodology in this chapter is new and most complete in capturing product-country variation, and with the PCA we can give insights in the remaining variation. In this way this chapter tries to give a new direction to the international diffusion literature.

In sum, using high-frequency diffusion data, and comparing multiple series can prove to be very helpful in academic research and in practice. This has actually been acknowledged in the literature, both implicitly as explicitly. However, thus far the data availability has not been used to its full potential. This is most likely due to the challenges that are associated with this increasing data availability. This dissertation gives a few solutions.

Nederlandse samenvatting (Summary in Dutch)

Dit proefschrift beschrijft de econometrische vooruitgang in diffusie modellen en deze Nederlandse samenvatting geeft een overzicht van de belangrijkste bevindingen van dit proefschrift. Diffusie modellen kunnen gebruikt worden om het patroon van de adoptie van producten te verklaren, waarbij adoptie de eerste keer is dat een klant een bepaald product koopt. In het bijzonder kunnen deze modellen gebruikt worden om deze eerste aankoop te beschrijven. Ook kunnen deze modellen gebruikt worden om de verkoopcijfers van een duurzaam product, dat wil zeggen een product dat iemand maximaal één keer koopt, te verklaren. In dit proefschrift kijken we bijvoorbeeld naar de diffusie van elektronische producten (flat-screen televisies, Dvd's, spelcomputers, etc.), auto's en medicijnen.

De verkoop/adoptie van een product begint vaak langzaam met een paar klanten. Voor succesvolle producten groeit het aantal klanten daarna snel, doordat mensen elkaar beïnvloeden. Op het moment dat het marktpotentieel bijna is bereikt en dus bijna iedereen die het product wil aanschaffen dat heeft gedaan, neemt de groei af. Dit resulteert in een "berg-curve" voor de verkoopcijfers en een "S-curve" voor de cumulatieve verkoopcijfers.

Diffusiemodellen zijn al vaak en grondig onderzocht in de marketingliteratuur. Desondanks geeft dit proefschrift nieuwe en belangrijke inzichten in het modelleren van diffusiepatronen van de adoptie van producten. Alle hoofdstukken in dit proefschrift laten zien dat de recente groei in beschikbaarheid van diffusiedata enkele nieuwe uitdagingen met zich meebrengt. Deze groei in beschikbaarheid van data heeft twee oorzaken: (i) in de afgelopen jaren is de focus verschoven van het modelleren van één reeks naar het modelleren van meerdere reeksen tegelijkertijd, en (ii) de meest voorkomende datafrequentie is verschoven van jaar naar kwartaal of maand, en soms zelfs nog preciezer.

In hoofdstuk 2 kijken we naar drie veelgebruikte schattingsmethoden en een nieuwe schattingsmethode voor het Bass diffusiemodel. De problemen van schattingsfouten in het Bass model zijn al vaak onderzocht, maar dit hoofdstuk biedt nieuwe inzichten. We laten zien dat elke schattingsmethode zijn eigen interpretatie voor de onverklaarde schokken heeft. Een belangrijke bevinding is dat de keuze van de schattingsmethode, en dus de keuze van de onverklaarde schokken, grote consequenties kan hebben. Ook de frequentie van de data kan een bron zijn van schattingsfouten. Naast de prestatie van de schattingsmethodes, kijken we ook naar de prestatie van de methodes voor het gebruik van het genereren van de data.

Dit hoofdstuk laat zien dat onderzoekers en managers voorzichtig moeten zijn in het kiezen van een schattingsmethode van een diffusiemodel. In het bijzonder geldt dat de overeenkomst tussen de gekozen structuur van de onverklaarde schokken en de echte schokken, en de frequentie van de data invloed hebben op zowel de schattingen als het genereren van data met diffusiemodellen. We laten zien dat als je alleen data hebt met grote intervallen of als je geen informatie hebt over de structuur van de echte schokken, dat je dan het beste het model kan gebruiken dat we in dit hoofdstuk voorstellen.

Hoofdstukken 3 en 4 kijken naar de uitdagingen die ontstaan door het gebruik van hoge frequentie data (bijvoorbeeld maandelijkse data), namelijk: (i) verschillende frequenties in een dataset, dit ontstaat omdat diffusie vaak onregelmatig wordt geobserveerd in het begin van de diffusiereeks, en (ii) seizoeneffecten, omdat dit een logisch gevolg is van hoge frequentie data.

In hoofdstuk 3 stellen we enkele manieren voor om een datareeks met verschillende frequenties te gebruiken. We laten zien hoe deze methodes werken voor verschillende schattingstechnieken. Voor sommige schattingstechnieken is het gebruik van verschillende frequenties makkelijk op te lossen. Echter, voor de regressie methode van het Bass model is het op een juiste manier meenemen hiervan een stuk lastiger. De resultaten uit dit hoofdstuk laten ook zien, dat als je op een verkeerde manier omgaat met deze verschillende frequenties in één datareeks dat dit kan leiden tot schattingsfouten. Dit hoofdstuk is daarom ook een bruikbaar overzicht van wat je wel en niet kan doen met betrekking tot verschillende datafrequenties.

In hoofdstuk 4 kijken we naar seizoeneffecten in diffusie modellen. Hiervoor is een seizoenstructuur ontworpen die gebruikt kan worden in combinatie met standaard diffusiemodellen. Het model dat we in dit hoofdstuk laten zien kan gebruikt worden om seizoeneffecten te verklaren zonder dat het invloed heeft op de geschatte onderliggende diffusiepatronen. De structuur is gebaseerd op het idee dat de extra verkopen in een bepaald seizoen veroorzaakt worden door klanten die anders het product eerder of later hadden gekocht. We maken onderscheid tussen twee modellen. In het ene leggen we de seizoenstructuur op en in de andere schatten we deze mee. De variant waar we seizoenstructuur meeschatten schat en voorspelt de verkoopcijfers beter dan de andere seizoenmodellen die we bekijken. Deze andere modellen kunnen ook schattingsfouten tot gevolg hebben. Ook kijken we naar het model dat seizoenfluctuaties negeert, met andere woorden, een standaard diffusiemodel, en we vinden dat dit model alleen gebruikt kan worden als je alle data van de volledige diffusie hebt.

In het laatste hoofdstuk (Hoofdstuk 5) kijken we naar de grotere beschikbaarheid van diffusie data door meerdere reeksen te bekijken, in plaats van meer data punten in één reeks. We bekijken hier de internationale diffusie van producten. We hebben data voor 82 landen en 12 producten tot onze beschikking, wat meer is dan in vergelijkbare studies. We vinden dat de beschikbare modellen slechts een deel van de variatie tussen producten en landen verklaren. Het moeilijkste gedeelte om te verklaren is de variatie die hoort bij de interactie tussen producten en landen. Voor elke combinatie van een product en een land is er namelijk een uniek verhaal waarom de diffusie van dat product in dat land een gegeven diffusiepatroon heeft. Zelfs na het gebruik van een meerlaags diffusie model blijft er nog een groot deel van de variatie onverklaard. Daarom gebruiken we Principale Componenten Analyse [PCA] om het overgebleven deel van de variatie te verklaren. De in dit hoofdstuk voorgestelde methode is nieuw in de manier van verklaren van de product-land variatie, en door het gebruik van de PCA kunnen we ook inzicht geven in deze overgebleven variatie. Dit hoofdstuk probeert hiermee een nieuwe richting te geven aan de internationale literatuur.

Samenvattend, het gebruik van reeksen met hogere frequenties en het vergelijken van meerdere reeksen kan erg nuttig zijn voor het bedrijfsleven en academisch onderzoek. Dit is ook aangegeven in de huidige literatuur, zowel impliciet als expliciet. Echter, tot nu toe is deze beschikbaarheid aan data niet volledig gebruikt. Dit laatste is waarschijnlijk het gevolg van de uitdagingen die het gebruik van deze extra data met zich meebrengt. Dit proefschrift helpt bij enkele van deze uitdagingen.

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ECONOMETRIC ADVANCES IN DIFFUSION MODELS

This thesis gives new and important insights in modeling diffusion data in marketing. It addresses modeling multiple series instead of just one series such that one can learn from the differences and similarities across products and countries. Additionally, this thesis addresses the current availability of higher frequency diffusion data. The two issues provide challenges for modeling of diffusion processes.

In this thesis we provide solutions to these challenges, and we also suggest another look at some older issues with a particular focus on parameter estimation. In the first chapters we deal with the estimation of diffusion parameters for a single series. We start with an overview of currently used estimation methods and we suggest that a new method is needed. In fact, our new method does not suffer from bias as it properly incorporates the source of noise and the observational frequency. In the next chapters we focus on modeling high-frequency diffusion data, where we specifically address mixed-frequency diffusion data and seasonality. Finally, we propose a new approach to jointly modeling many diffusion series, where we allow for cross effects between products and countries.

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