Estimating Duration Intervals

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Estimating duration intervals

Abstract

Duration intervals measure the dynamic impact of advertising on sales. More precise, the p per cent duration interval measures the time lag between the advertising impulse and the moment that p per cent of its effect has decayed. In this paper, we derive an expression for the duration interval for a general dynamic model linking sales to advertising. Additionally, and this is the main novelty of the paper, we put forward a method to provide confidence bounds around the estimated duration interval. An illustration to real-life data emphasizes its usefulness.

Key words and phrases: Advertising effects, duration interval; simulation

1 Introduction

This research note concerns an analysis of the dynamic effects of current and past advertising on current and future sales. In the marketing literature there is a general agreement that advertising positively affects sales. Furthermore, this effect may last for multiple periods and is dependent on the product and market characteristics. A variety of econometric time series models have been put forward to describe this effect, see for example, Assmus, et al. (1984), Clarke (1976), Weinberg and Weiss (1982), and for a recent summary, see Leeflang, et al. (2000). An important and desired feature of properly specified models for describing advertising effects on sales is that they allow for a distinction between long-run and short-run effects of advertising.

An often applied model is the geometric distributed lag model, best known as the Koyck (1954) model. This model is rather easy to analyze, and also the so-called duration interval is easy to derive from this model. A p per cent duration interval measures the time lag between the advertising impulse and the moment that p per cent of its effect has decayed. This duration interval is a point estimate which follows from the estimated model parameters. Many academic studies report these estimates and evaluate the values across products or temporal aggregation levels, see for example Clarke (1976). However, we are not aware of studies where one also reports measures of uncertainty around the point estimates of the duration interval. One possible reason for this is that analytic expressions for these confidence bounds are not available, and hence one has to rely on simulation techniques.

The goal of this research note is to fill this gap in the literature. We propose a method to compute confidence bounds around p per cent duration intervals. In the end, one can then make statements like we make in our empirical section below, which is that "with 95 per cent confidence, the 95 per cent duration interval of advertising is between about 4.8 and 5.7 hours".

To achieve generality, we consider this method for a rather general econometric time series model, which is the autoregressive distributed lag (ADL) model of order (p,m), which goes back to at least Griliches (1967). Recently, this model has been successfully implemented in Tellis, et al. (2000). This model encompasses the Koyck model and various other models. A nice feature of this model, which is in contrast to the Koyck model, is that it allows the long-run effects to be smaller or to be larger than the short-run effects. The Koyck model implies that the long-run effects are always larger. We provide an alternative representation of the ADL model, which allows for direct inference on long-run and short-run effects. In fact, if the model parameters are estimated for this representation, one directly obtains estimated standard errors for the long-run effects, which is quite convenient.

The outline of our research note is as follows. In Section 2, we discuss the relevant features of the ADL model, including the method to compute confidence bounds around the p per cent duration intervals. Finally, in Section 3, we illustrate its relevance for one of the cases in Tellis, et al. (2000).

2 A general model

This section first deals with a discussion of the general ADL model. Next, we propose an alternative representation of this model, which allows for direct estimation of the long-run and short-run effects. Then we deal with the computation of the point estimate of the duration interval, as well as with its confidence bounds.

ADL(p,m)-model

Consider a sales variable S_t and an advertising variable A_t, with t running from 1 to n, where it can concern seconds, hours, days or even years. The general model we consider in this research note is an autoregressive distributed lags model of orders p and m, given by

$$S_t = \mu + \alpha_1 S_{t-1} + \ldots + \alpha_p S_{t-p} + \beta_0 A_t + \beta_1 A_{t-1} + \ldots + \beta_m A_{t-m} + \varepsilon_t, \quad (1)$$

which we will abbreviate as an ADL(p,m) model. It is assumed that ε_t is an uncorrelated variable with mean zero and common variance σ^2. This model is quite general as it also allows for delayed effects of advertising, when for example β_0 is 0, and it also allows for time gaps in these effects, which can occur when some β parameters are zero and others are not.

When m equals ∞, and all α parameters are zero and additionally $\beta_j = \beta_0 \lambda^{j-1}$, we have the familiar Koyck model. A nice feature of this Koyck model is that it can be rewritten as

$$S_t = \mu^* + \lambda S_{t-1} + \beta_0 A_t + \varepsilon_t - \lambda \varepsilon_{t-1}. \quad (2)$$

The short-run effect of advertising is β_0 and the long-run effect is $\frac{\beta_0}{1-\lambda}$. As $\lambda < 1$, the Koyck model implies that the long-run effect exceeds the short-run effect. The p per cent duration interval for this model can be estimated as $\frac{\log(1-p)}{\log \lambda}$. To compute confidence bounds around this point estimate, one needs to take account of the uncertainty around the estimate of λ. A natural way to compute these bounds is by means of simulation, and we will pursue this below for the general ADL(p,m) model.

Error correction representation

As for the Koyck model, for the general ADL(p,m) model it holds that the long-run and short-run effects of advertising on sales are functions of the parameters. In many cases, one would only be interested in these two effects, and perhaps test whether these are different. Indeed, the ADL model allows that the long-run effect might be smaller than the short-run effect. This is easily understood from an alternative representation of the ADL model, which is the so-called error correction representation. Denoting Δ_j as the j-th order differencing filter, that is, $\Delta_j y_t = y_t - y_{t-j}$, the error correction representation reads as

$$\begin{aligned}\Delta_1 S_t \quad &= \quad \mu + (\sum_{j=1}^{p} \alpha_j - 1)[S_{t-1} - \frac{\sum_{i=0}^{m} \beta_i}{1 - \sum_{j=1}^{P} \alpha_j} A_{t-1}] \\ &\quad + \beta_0 \Delta_1 A_t - \sum_{i=2}^{m} \beta_i \Delta_{i-1} A_{t-1} - \sum_{j=2}^{p} \alpha_j \Delta_{j-1} S_{t-1} + \varepsilon_t. \end{aligned} \tag{3}$$

This immediately shows that the long-run effect is equal to

$$\frac{\sum_{i=0}^{m} \beta_i}{1 - \sum_{j=1}^{p} \alpha_j}, \tag{4}$$

while the short-run effect is β_0. It is easy to understand that, given certain values of the β parameters, the long-run effect can be smaller than the short-run effect.

The parameters in the error correction model, when written as

$$\Delta_1 S_t = \mu + \rho[S_{t-1} - \gamma A_{t-1}] + \beta_0 \Delta_1 A_t - \sum_{i=2}^{m} \beta_i \Delta_{i-1} A_{t-1} - \sum_{j=2}^{p} \alpha_j \Delta_{j-1} S_{t-1} + \varepsilon_t, \tag{5}$$

can be estimated using non-linear least squares. This method provides direct estimates of the long-run effect γ and its associated standard error.

Duration interval

The interpretation of the ADL model can proceed by considering the long-run and short-run parameter estimates. These, however, do not convey the speed with which the effect of advertising decays. Hence, to be able to compute the p per cent duration interval, one needs explicit expressions for $\frac{\partial S_{t+k}}{\partial A_t}$ for all values of k running from 1 to, potentially, ∞. Given the general expression in (1), these expressions are easily derived as

$$\begin{aligned}
\frac{\partial S_t}{\partial A_t} &= \beta_0 \\
\frac{\partial S_{t+1}}{\partial A_t} &= \beta_1 + \alpha_1 \frac{\partial S_t}{\partial A_t} \\
\frac{\partial S_{t+2}}{\partial A_t} &= \beta_2 + \alpha_1 \frac{\partial S_{t+1}}{\partial A_t} + \alpha_2 \frac{\partial S_t}{\partial A_t} \\
&\vdots \\
\frac{\partial S_{t+k}}{\partial A_t} &= \beta_k + \sum_{j=1}^{k} \alpha_j \frac{\partial S_{t+(k-j)}}{\partial A_t}
\end{aligned}$$

where it should be noted that $\alpha_k = 0$ for $k > p$, and that $\beta_k = 0$ for $k > m$.

The p per cent duration interval is now based on the decay factor given by

$$p_k = \frac{\frac{\partial S_t}{\partial A_t} - \frac{\partial S_{t+k}}{\partial A_t}}{\frac{\partial S_t}{\partial A_t}}, \tag{6}$$

which can only be computed for discrete values of k, as these are only discrete time intervals. Again, this decay factor is a function of the model parameters. Through interpolation, one can decide on the time k it takes for the decay factor to be equal to some value of p, which typically is equal to 0.95 or 0.90. This estimated time k is then called the p per cent duration interval.

Computing confidence bounds

Of course, the p per cent duration interval is a stochastic variable. Next to its point estimate, one would also want to estimate its confidence bounds.

To determine these confidence bounds of p_k one should keep in mind that the decay factors are based on nonlinear functions of parameters. The problem that now arises, when determining the expected value of p_k, is that the expectation of this nonlinear function of parameters is not equal to the function applied to the expectation of the parameters, that is, $\mathrm{E}(f(\theta)) \neq f(\mathrm{E}(\theta))$. Hence, the values of p_k cannot analytically be determined and need to be simulated. For the general ADL model it holds that the OLS estimator is asymptotically normal distributed. We draw 100,000 parameter vectors from this multivariate normal distribution. According to (6) we calculate the values p_k. This gives us the full distribution of these decay factors. From this, we obtain the mean (expected) values as well as the 95 per cent confidence interval, or any other percentage.

3 Illustration

We illustrate the above results for one of the cases (Miami) studied in Tellis, et al. (2000). This case deals with a firm that offers a referral service for medical care in the city of Miami. There is information on the number of referrals [R_t] and advertisements [A_t] on an hourly basis. Furthermore, as the variable R_t is truncated we use the dummy variable O_t indicating when the service is available, see also Tellis, et al. (2000). We use the same model for these Miami data as in Tellis et al. (2000), that is,

$$R_t = \Big(\mu + \sum_{j=1}^{3} \alpha_j R_{t-j} + \sum_{i=0}^{4} \beta_i A_{t-i}\Big) O_t + \varepsilon_t, \tag{7}$$

with one modification, which is that we multiply all variables (including the intercept) with the dummy variable O_t.

The parameter estimates and their associated standard errors are shown in Table 1. When we estimate the model in error correction format, we obtain an estimate of the long-run effect equal to 2.619, with an estimated standard error of 0.481 for this long-run effect. Table 1 conveys that the immediate short-run effect is 1.156 with standard error 0.050. Hence, these two effects are clearly distinct.

As discussed earlier, the duration intervals are not easy to derive from the parameter estimates in Table 1. For that purpose, we use the recursion formula above. We depict these in Figure 1, where we also plot the 95 per cent confidence bounds around these duration intervals. The 95 per cent duration interval appears to be around 5.2 hours. A closer look around this time, as in Figure 2, reveals that with 95 per cent confidence, the 95 per cent duration interval of advertising is between about 4.8 and 5.7 hours.

Table 1: OLS estimation results Miami market

Parameter	Estimate	st.error
μ	0.654	0.028
α_1	0.200	0.015
α_2	0.098	0.015
α_3	0.001	0.014
β_0	1.156	0.050
β_1	0.347	0.053
β_2	0.064	0.053
β_3	0.173	0.053
β_4	0.106	0.051

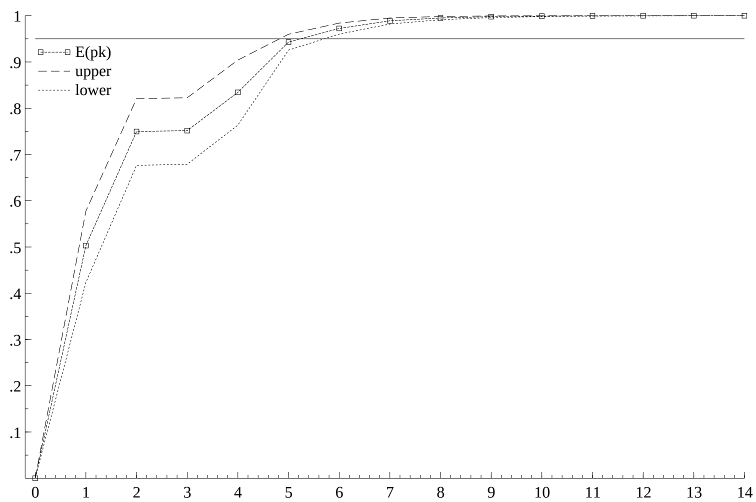

Figure 1: p per cent duration interval against hours

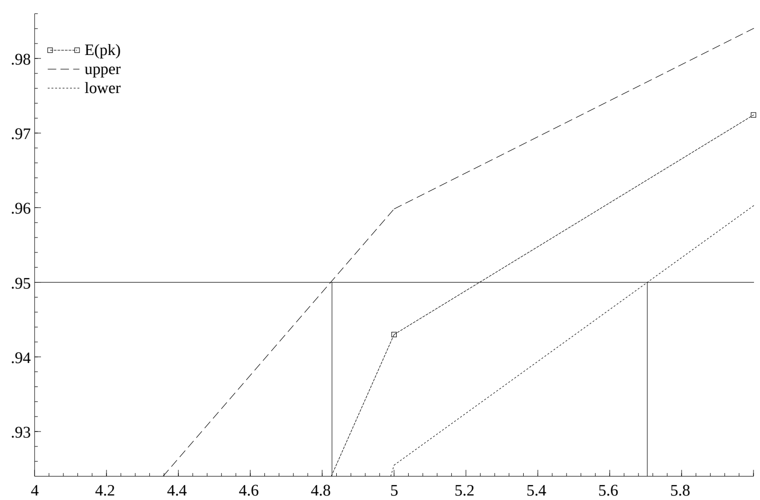

Figure 2: p per cent duration interval against hours, a closer look

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