Modeling Dynamic Effects of the Marketing Mix on Market Shares

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Abstract

To comprehend the competitive structure of a market, it is important to understand the short-run and long-run effects of the marketing mix on market shares. A useful model to link market shares with marketing-mix variables, like price and promotion, is the market share attraction model. In this paper we put forward a representation of the attraction model, which allows for explicitly disentangling long-run from short-run effects. Our model also contains a second level, in which these dynamic effects are correlated with various brand and product category characteristics.

Based on the findings in for example Nijs *et al.* (2001), we postulate the expected signs of these correlations. We fit our resultant Hierarchical Bayes attraction model to data on seven categories in two geographical areas. This data set spans a total of 50 brands. Our main finding is that, in absolute sense, the short-run price elasticity usually exceeds the long-run effect. Moreover, we find that the longrun price effects are strongly correlated with relative price and coupon intensity of a brand.

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Modeling Dynamic Effects of the Marketing Mix on Market Shares

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Modeling Dynamic Effects of the Marketing Mix on Market Shares

Abstract

To comprehend the competitive structure of a market, it is important to understand the short-run and long-run effects of the marketing mix on market shares. A useful model to link market shares with marketing-mix variables, like price and promotion, is the market share attraction model. In this paper we put forward a representation of the attraction model, which allows for explicitly disentangling long-run from short-run effects. Our model also contains a second level, in which these dynamic effects are correlated with various brand and product category characteristics. Based on the findings in for example Nijs *et al.* (2001), we postulate the expected signs of these correlations. We fit our resultant Hierarchical Bayes attraction model to data on seven categories in two geographical areas. This data set spans a total of 50 brands. Our main finding is that, in absolute sense, the short-run price elasticity usually exceeds the long-run effect. Moreover, we find that the long-run price effects are strongly correlated with relative price and coupon intensity of a brand.

Key words: market shares; marketing mix; long-term effects; Hierarchical Bayes

1 Introduction

In recent literature on market structures it has been shown that marketing efforts, such as for example temporary price promotions, do not have permanent effects on sales or market shares. A prerequisite for permanent effects of temporary promotions is the non-stationarity of sales or market shares. Srinivasan *et al.* (2000), Nijs *et al.* (2001), and Pauwels *et al.* (2002), among others, have shown that almost all sales series for fast moving consumer goods are stationary. Lal and Padmanabhan (1995), DeKimpe and Hanssens (1995), and Franses *et al.* (2001) report similar results for market shares. Hence, to study dynamic effects of the marketing mix, one needs to examine the cumulative effect of a temporary promotion on current and future market shares. Only when the cumulative effect is positive a promotion is worthwhile.

In this paper we put forward a method which allows us to directly estimate the potentially differing short-run and long-run marketing-mix effects on market shares. The short-run effect is defined as the instantaneous effect of a promotion on current market shares. The long-run effect is defined as the cumulative effect of a temporary promotion on current and future market shares, see also Pauwels *et al.* (2002). If a promotion has positive carry-over effect, the long-run effect will exceed the short-run effect. The long-run effect will be zero if the positive direct effect of a promotion is exactly balanced by negative carry-over effects[1].

The long-run and short-run effects of the marketing mix usually differ across brands and markets. Differences in promotional intensities, price structures or market concentration may lead to different market structures, see also Mela *et al.* (1998), Bronnenberg *et al.* (2000) and Srinivasan *et al.* (2000). In this paper we aim to understand the potentially differing long-run and short-run marketing-mix effects on market shares. For that purpose, we link both the short-run and the long-run effects to brand-specific and category-specific characteristics in a second level of our model.

As market shares are in between zero and one, and also as they sum to unity, models for these dependent variables are a little more complicated than basic regression models. One might now be inclined to circumvent this complication by modeling own brand sales and

[1] Note that this definition differs from the usual approach in the marketing literature. There, the common definition of the long-run effect is the effect of a temporary promotion on market shares in the distant future. However, as discussed above, such a permanent effect is hardly found. Indeed, in case of stationarity only permanent changes of the marketing mix will effect market shares in the long run.

category sales, and simply dividing the outcomes. Fok and Franses (2001), however, show that this might complicate matters even more as these two components of market shares are not independent. Additionally, depending on the model specification for category sales, this approach would also not guarantee that shares will sum to one. Another motivation to consider market shares is that subsequent models allow us to link market share elasticities directly to model parameters.

A useful model for market shares, when measured at, say, the weekly level, is the so-called market share attraction model. This model has theoretically sound properties and it is also easy to analyze in practice, see Cooper and Nakanishi (1988) for an early introductory book on this model and Fok *et al.* (2002) for a recent review of its econometric aspects. In this paper we will also consider this model.

For the purpose of our paper, we introduce into the marketing literature two new modifications of the attraction model. The first modification amounts to explicit expressions of long-run and short-run effects for the reduced-form attraction model, which is typically used to estimate the relevant parameters. This may seem like a trivial issue, but as we will demonstrate in Section 2.2, it is not. The second modification concerns the introduction of a second level in our model. That is, we propose to simultaneously analyze the attraction model for I_c brands in category $c = 1, \ldots, C$. This can lead to a multitude of parameters. For parsimony, but also for interpretation purposes, we therefore correlate some of these parameters with brand-specific and category-specific variables. The resultant model is a Hierarchical Bayes Attraction model. As such, we extend on a similar route taken by the rigorous study in Nijs *et al.* (2001), who instead consider a two-step approach and focus on (category) sales, whereas we put everything into a single model and consider brands' market shares. We also use explicit measures of dynamic effects, instead of derivative measures such as the impulse-response function. That said, the empirical results obtained from our paper can be seen as adding to the knowledge base created by Nijs *et al.* (2001).

The outline of our paper is as follows. In Section 2, we put forward our new two-step attraction model in so-called error-correction format, which allows us to analyze dynamic marketing-mix effects across categories. In Section 3, we apply our Hierarchical Bayes Attraction model to weekly data for two to four brands in seven different categories at two locations. One of our conclusions is that only display and feature promotions are likely to have a higher long-run effect than short-run effect, while price elasticities are, in absolute value, higher for the short run. That is, price promotions often have negative carry-over effects, while display and feature tend to have positive carry-over effect. In Section 4, we conclude with a discussion of managerial and modeling implications.

2 Attraction models with dynamic effects

The basic attraction model contains two components. The first component is a specification for the (unobserved) attraction of a specific brand, which can depend on current and past marketing-mix instruments and past market shares or past attraction. The second component defines the market share by dividing own attraction by the sum of the attractions of all brands. Together this leads to a reduced-form attraction model with parameters that can be estimated using the relevant data.

In this section we put forward an attraction model specification with a reduced-form model that can be converted into error correction format. This error correction model [ECM] enables us to disentangle long-run from short-run effects of marketing-mix variables on market shares in a direct way. The derivation of these effects is discussed in Section 2.2. In Section 2.3, we discuss our Hierarchical Bayes specification which is used to summarize the information on long-run and short-run effects for a large number of categories. By considering multiple markets in a single model, we can provide empirical generalizations concerning the dynamic effects of elements of the marketing mix in an statistically efficient way.

Before we discuss our complete model, we first review some notational issues in Section 2.1, where we confine ourselves to a single category to save notation. In this section we furthermore consider various dynamic specifications of the attraction model.

2.1 Preliminaries

To model market shares we define the attraction of brand i, out of I brands in a single category, at time t by $A_{i,t}$. We assume that attraction can be described by

$$A_{i,t} = \exp(\mu_i + \varepsilon_{i,t}) x_{i,t}^{\alpha_i} x_{i,t-1}^{\beta_i} M_{i,t-1}^{\rho}, \qquad \text{for } i = 1, \ldots, I, \tag{1}$$

where $\varepsilon_t = (\varepsilon_{1,t}, \ldots, \varepsilon_{I,t})' \sim N(0, \Sigma)$, and where $M_{i,t}$ denotes the market share of brand i, and $x_{i,t}$ a marketing instrument. This dynamic attraction specification is often successfully applied to model market shares, see among others Leeflang and Reuyl (1984); Cooper and Nakanishi (1988); Kumar (1994) and Bronnenberg *et al.* (2000). It is however difficult to directly disentangle short-run effects and long-run effects of the marketing instruments on attractions and market shares using this parameterization. Another specification that has been suggested is based on including lagged attraction instead of lagged market share as an explanatory variable. However it turns out that such a specification is equivalent to (1) and has the same interpretation problems. Carpenter *et al.* (1988) and Hanssens *et al.* (1989) first transform the marketing instruments using an AR model to yield so-called effective advertising or promotion. The transformed instruments are then included in the attraction model. The main advantage of this approach is the reduction in parameters. The disadvantage is however that the short-run and long-run effects cannot be directly captured into single parameters. In the remainder of this section we therefore continue with specification (1) and show that it is possible to assign short-run and long-run effects of $x_{i,t}$ to separate parameters. To keep notation simple, we present our model assuming there is a single marketing instrument. An extension to more explanatory variables is straightforward and will be presented in Section 2.3.

The second component of an attraction model amounts to the definition of market share as the relative attraction of a brand in the market, that is,

$$M_{i,t} = \frac{A_{i,t}}{\sum_{j=1}^{I} A_{j,t}}. \tag{2}$$

Components (1) and (2) together lead to estimable reduced-form models.

The market share attraction model can be linearized by considering the natural logs of the market shares relative to a base brand. The model then reduces to

$$\begin{aligned} \ln \frac{M_{i,t}}{M_{I,t}} &= \ln M_{i,t} - \ln M_{I,t} = \ln A_{i,t} - \ln A_{I,t} \\ &= \mu_i - \mu_I + \alpha_i \ln x_{i,t} - \alpha_I \ln x_{I,t} + \beta_i \ln x_{i,t-1} - \beta_I \ln x_{I,t-1} \\ &\qquad + \rho(\ln M_{i,t-1} - \ln M_{I,t-1}) + \varepsilon_{i,t} - \varepsilon_{I,t}, \end{aligned} \tag{3}$$

for $i = 1, \ldots, I-1$ and where brand I is chosen as the base brand, see Fok *et al.* (2002). For parameter identification the choice of the base brand turns out to be arbitrary. The reduced-form specification in (3) shows that not all parameters in (1) can be identified. First, only the differences across the brand intercepts $\tilde{\mu}_i \equiv \mu_i - \mu_I$ are identified. Next, we can only identify the covariance structure of $\tilde{\varepsilon}_{i,t} \equiv \varepsilon_{i,t} - \varepsilon_{I,t}$, that is, the covariance matrix of $(\tilde{\varepsilon}_{1,t}, \ldots, \tilde{\varepsilon}_{I-1,t})'$ denoted by $\tilde{\Sigma}$. The parameters in the resulting system of $I-1$ equations can now easily be estimated.

2.2 Short-run and long-run effects

In this paper we consider the short-run and long-run effects that are implied by the dynamic structure of the model. This is in contrast with studies by, for example, Mela *et al.* (1998) and Jedidi *et al.* (1999) where dynamics enter through the model parameters. In these studies the preferences and marketing sensitivity of households may change as a consequence of (intensified) promotional activities. In this case the long-run effect is defined as the impact of a promotion on the future accounting for changes in individuals behavior. In this paper we take a different approach and consider (aggregated) household behavior to be constant. The dynamics in market shares are directly caused by feedback loops in household behavior.

It is well known that it is difficult to interpret the parameters in autoregressive distributed lag models as they combine short-run and long-run effects of the explanatory variables on the dependent variable. We will now show how to rewrite the attraction model in the interpretable error correction format. To our knowledge we are the first to use the error-correction model in the context of the market share attraction model. In the marketing literature the error-correction model, has been used by for example Franses (1994) and Paap and Franses (2000) for new product sales and brand choice, respectively.

Consider again the attraction specification in (1) which leads to the $I-1$ equations in (3). This equation is an Autoregressive Distributed Lag [ADL(1,1)] model for the variable $(\ln M_{i,t} - \ln M_{I,t})$. To determine the dynamic effects of lagged $x_{i,t}$ and $x_{I,t}$ on the market shares we solve (3) for $(\ln M_{i,t} - \ln M_{I,t})$ by repeated substitution until the first observation. The solution is

$$\begin{aligned}\ln M_{i,t} - \ln M_{I,t} &= \rho^t(\ln M_{i,0} - \ln M_{I,0}) \\ &+ \sum_{\tau=0}^{t-1} \rho^\tau(\tilde{\mu}_i + \alpha_i \ln x_{i,t-\tau} - \alpha_I \ln x_{I,t-\tau} + \beta_i \ln x_{i,t-\tau-1} - \beta_I \ln x_{I,t-\tau-1} + \tilde{\varepsilon}_{i,t-\tau}). \quad (4)\end{aligned}$$

The long-run market shares follow from (4) by taking $t \to \infty$. Under the stationary condition $|\rho| < 1$, the influence of the market shares at time 0 disappears over time as $\lim_{t\to\infty} \rho^t = 0$. If we further set the explanatory variables at fixed values over time, that is, $x_{i,t} = x_i$ and $x_{I,t} = x_I$ for all t, the long-run market shares are now given by

$$\ln M_i - \ln M_I = \frac{\tilde{\mu}_i}{1-\rho} + \frac{\alpha_i + \beta_i}{1-\rho} \ln x_i - \frac{\alpha_I + \beta_I}{1-\rho} \ln x_I + \sum_{\tau=0}^{\infty} \rho^\tau \tilde{\varepsilon}_{i,t-\tau}. \quad (5)$$

As $\mathrm{E}[\tilde{\varepsilon}_{i,t}] = 0$ for all t, the long-run expectation of $(\ln M_i - \ln M_I)$, given x_i and x_I, equals

$$\mathrm{E}[\ln M_i - \ln M_I | x_i, x_I] = \frac{\tilde{\mu}_i}{1-\rho} + \frac{\alpha_i + \beta_i}{1-\rho} \ln x_i - \frac{\alpha_I + \beta_I}{1-\rho} \ln x_I, \tag{6}$$

and the long-run conditional variance is given by

$$\mathrm{Var}[\ln M_i - \ln M_I | x_i, x_I] = \sum_{\tau=0}^{\infty} \rho^{2\tau} \mathrm{Var}[\tilde{\varepsilon}_{i,t-\tau}] = \frac{\mathrm{Var}[\tilde{\varepsilon}_{i,t}]}{1-\rho^2}. \tag{7}$$

This implies that, in the long-run, market shares are determined by the following attraction specification

$$A_i = \exp\left(\frac{\mu_i}{(1-\rho)} + \eta_i\right) x_i^{\gamma_i} \qquad \text{for } i = 1, \ldots, I, \tag{8}$$

where $\gamma_i = (\alpha_i + \beta_i)/(1-\rho)$ and $(\eta_1, \ldots, \eta_I)' \sim N(0, \frac{1}{(1-\rho^2)}\Sigma)$. Interestingly, as the long-run market shares correspond to an attraction model, we can use the standard results in Cooper and Nakanishi (1988) to compute long-run (cross)-elasticities. For example, the long-run elasticity of x_i is given by

$$\frac{\partial M_i}{\partial x_i} \frac{x_i}{M_i} = \gamma_i (1 - M_i), \tag{9}$$

which can provide useful information for managers who need to decide on the marketing mix.

It follows immediately from (4) that under stationarity ($|\rho| < 1$) the effect of a temporary change of x_i at time τ has no long-run impact on market shares as the term ρ^τ will be zero for large τ. Only a permanent change in the value of x_i will have a permanent long-run effect on the market shares. The long-run effect on the log relative market shares is measured by the parameter γ_i. A temporary change of x_i does however have a short-run effect on market shares. The direct short-run effect is measured by α_i. To disentangle the long-run effects from the short-run effects of x_i on market shares, that is, to allow for directly estimating these effects, it is convenient to rewrite (3) in error-correction format, see Hendry *et al.* (1984), that is,

$$\begin{aligned} \Delta(\ln M_{i,t} - \ln M_{I,t}) = \tilde{\mu}_i + \alpha_i \Delta \ln x_{i,t} - \alpha_I \Delta \ln x_{I,t} + \\ (\rho - 1)[\ln M_{i,t-1} - \ln M_{I,t-1} - \gamma_i \ln x_{i,t-1} + \gamma_I \ln x_{I,t-1}] + \tilde{\varepsilon}_{i,t}, \end{aligned} \tag{10}$$

where Δ denotes the first-differencing operator, that is, $\Delta y_t = y_t - y_{t-1}$. The short-run, or instantaneous, effects are given by α_i as

$$\frac{\partial \ln M_{i,t} - \ln M_{I,t}}{\partial \ln x_{i,t}} = \alpha_i. \tag{11}$$

The long-run relation between $x_{i,t}$ and $M_{i,t}$ is put in the so-called error-correction term and hence long-run effects of $\ln x_{i,t}$ on $\ln M_{i,t}$ are given by γ_i. That is, this parameter gives the marginal effect of a permanent change in $\ln x_{i,t}$ on the log relative market shares in the long-run. The parameter $(\rho - 1)$ is often called the adjustment parameter and determines the speed of convergence to the long-run relation. It can be shown that γ_i in error correction model (10) is also equal to the cumulative effect of a temporary change in $\ln x_{i,t}$ on current and future log relative market shares, that is, under stationarity the following property holds

$$\sum_{\tau=0}^{\infty} \frac{\partial(\ln M_{i,t+\tau} - \ln M_{I,t+\tau})}{\partial \ln x_{i,t}} = \gamma_i. \tag{12}$$

The error-correction model is nonlinear in some parameters. This is no problem as we can estimate (3) and transform the estimates to the parameters of (10). If one uses a Seemingly Unrelated Regression [SUR] estimator to estimate the parameters, standard errors can be obtained using the Delta method, see Greene (1993, p. 297). In this paper we will use Bayesian methods, and hence parameter uncertainty naturally follows from our sampling output, but we will return to this issue further below.

It is perhaps of interest to mention that the reduced-form model in (10) can also be derived from an alternative starting point. Consider the attraction component,

$$A_{i,t} = \exp(\mu_i + \varepsilon_{i,t}) x_{i,t}^{\alpha_i} x_{i,t-1}^{\beta_i} A_{i,t-1}^{\rho}, \tag{13}$$

where now lagged attraction instead of lagged market share enters the specification. If we take logarithms on both sides, we obtain a system of I equations

$$\ln A_{i,t} = \mu_i + \rho \ln A_{i,t-1} + \alpha_i \ln x_{i,t} + \beta_i \ln x_{i,t-1} + \varepsilon_{i,t}. \tag{14}$$

If we solve for $A_{i,t}$, the long-run expectation of the attraction for brand i is given by $\mathrm{E}[A_i|x_i] = (\mu_i + \gamma_i \ln x_i)/(1-\rho)$ and the long-run variance is $\mathrm{Var}[A_i] = \mathrm{Var}[\varepsilon_{i,t}]/(1-\rho^2)$. Hence, starting from (13) the long-run attractions also satisfy (8). The error-correction specification for the attractions is given by

$$\Delta \ln A_{i,t} = \mu_i + \alpha_i \Delta \ln x_{i,t} + (\rho - 1)(\ln A_{i,t-1} - \gamma_i \ln x_{i,t-1}) + \varepsilon_{i,t}, \tag{15}$$

which implies that we can make direct statements about the attractions, and not only on the market shares.

So far, we have only considered first order attraction model specifications. Extensions to higher order attraction models are straightforward. The resulting error-correcting specifications are similar to (10) but with extra lagged values of $\Delta \ln M_{i,t}$, $\Delta \ln M_{I,t}$, $\Delta \ln x_{i,t}$ and $\Delta \ln x_{I,t}$.

2.3 Hierarchical Bayes

Now we turn to an analysis of the error correction attraction model with a large number of categories. Let $A_{i,t}(c)$ and $M_{i,t}(c)$ denote the attraction and market share, respectively, of brand i in product category c in week t. In this section we consider multiple marketing instruments. Let $x_{i,k,t}(c)$ denote the k-th explanatory variable of brand i in category c in week t. An attraction specification of brand i in category c is given by

$$A_{i,t}(c) = \exp(\mu_{i,c} + \varepsilon_{i,t}(c)) M_{i,t-1}^{\rho_c} \prod_{k=1}^{K} \left(x_{i,k,t}(c)^{\alpha_{i,c,k}} x_{i,k,t-1}(c)^{\beta_{i,c,k}} \right) \tag{16}$$

for $i = 1, \ldots, I_c$, $t = 1, \ldots, T_c$ and $c = 1, \ldots, C$ with $\varepsilon_t(c) \sim N(0, \Sigma_c)$, where we now allow for multiple marketing instruments indexed by $k = 1, \ldots, K$. This attraction specification corresponds to a similar set of $I_c - 1$ linear equations as given in (3). We allow for the fact that categories may differ in the number of brands and the number of observed periods. The linear equations can be written in the error-correction model in a similar as (10), that is,

$$\begin{aligned} \Delta(\ln M_{i,t}(c) - \ln M_{I_c,t}(c)) = \tilde{\mu}_{i,c} + \sum_{k=1}^{K} (\alpha_{i,k,c} \Delta \ln x_{i,k,t}(c) - \alpha_{I_c,k,c} \Delta \ln x_{I_c,k,t}(c)) + \\ (\rho_c - 1) \Big(\ln M_{i,t-1}(c) - \ln M_{I_c,t-1}(c) + \\ \sum_{k=1}^{K} [-\gamma_{i,c,k} \ln x_{i,k,t-1}(c) + \gamma_{I_c,c,k} \ln x_{I_c,k,t-1}(c)] \Big) + \tilde{\varepsilon}_{i,t}(c), \end{aligned} \tag{17}$$

for $i = 1, \ldots, I_c - 1$ and $t = 1, \ldots, T_c$ and where $\gamma_{i,k,c} = (\alpha_{i,k,c} + \beta_{i,k,c})/(1 - \rho_c)$.

To relate the short- and long-run elasticity parameters to explanatory variables, we define K-dimensional vectors $\alpha_{i,c} = (\alpha_{i,1,c}, \ldots, \alpha_{i,K,c})'$ and $\gamma_{i,c} = (\gamma_{i,1,c}, \ldots, \gamma_{i,K,c})'$. The long-run and short-run effects of the marketing mix will obviously differ across brands and across categories. Some of these differences can be attributed to observable characteristics of the brand and the category, such as the size of a brand and the average use of a marketing instrument. Another part of the effects of the marketing mix cannot be explained, either by the fact that it is specific to the brand or that there are characteristics that we do not observe. In sum, we propose to describe the short-run and long-run effects parameters by

$$\alpha_{i,c} = \lambda_1' z_{i,c} + \eta_{i,c} \tag{18}$$

$$\gamma_{i,c} = \lambda_2' z_{i,c} + \nu_{i,c}, \tag{19}$$

where $z_{i,c}$ is an L-dimensional vector containing an intercept and $L-1$ explanatory variables for brand i in category c, like promotion frequency of brand i in category c, a market leader dummy and so forth. The $L \times K$ matrices λ_1 and λ_2 give the effects of the brand characteristics on the short-run and long-run parameters, respectively. The error terms $\eta_{i,c}$ and $\nu_{i,c}$ are assumed as uncorrelated across brands and normally distributed with mean 0 and covariance matrix Σ_η and Σ_ν, respectively. Note that there are $\sum_{c=1}^{C} I_c$ vectors $\alpha_{i,c}$ and $\gamma_{i,c}$.

To estimate the parameters in the model (17) with (18)–(19), we use a Bayesian approach. Bayesian estimation provides exact inference in finite samples. To obtain posterior results we use the Gibbs sampling technique of Geman and Geman (1984) which is an Markov Chain Monte Carlo [MCMC] technique. In the Appendix we derive the likelihood function of the model together with the full conditional posterior distributions which are necessary in the Gibbs sampler.

Another estimation strategy which is often applied in practice, is a two-step procedure in which, first, individual market-level models are estimated and, in a second stage regression, the parameters from the market-level models are related to brand and market characteristics, see for example Nijs *et al.* (2001). This method is however theoretically less elegant as the uncertainty in the first-level parameter estimates is not correctly accounted for in the second stage. In finite samples, this may lead to underestimation of the uncertainty in the parameter estimates in the second stage. At the end of our empirical section, we will briefly discuss the relevant differences between our Hierarchical Bayes approach and the two-step approach.

3 Empirical results

In this section we first discuss the data and the variables in the two components in our HB-ECM-attraction model. Then we elaborate on a few prior conjectures about the signs of the correlations in the second component of our model. Finally, we present the estimation results.

3.1 Data and variables

For the empirical part of this paper we consider the so-called ERIM database of the GSB of the University of Chicago. The data concern seven different categories in two geographical areas. So we have 14 different markets. For each category we have weekly observations of the market shares and of the marketing efforts of the major national brands and a rest category. On average, we have 123 weekly observations for each category. Two markets concerning sugar have just two brands. The tuna category has three brands and the remaining five categories (catsup, peanut butter, stick margarine, tube margarine and tissues) each have four brands. We model the market shares of all 50 brands simultaneously using our HB model.

As explanatory variables for the market shares in the first model component we use a dummy variable for coupon promotion, a dummy variable for the combination of feature and display promotion and the actual price. The price parameters therefore describe price elasticities and not promotional price elasticities. The dummy variables cannot directly enter our attraction specification (1), as in that case weeks with no promotion would by definition have zero market shares. Instead, we use an exponential transformation for these two 0/1 marketing instruments. Finally, we use a lag order of 1 to capture the dynamics in the markets, which effectively leads to the model discussed in Section 2.3. Previous studies show that this lag order is sufficient to capture the dynamics, see Fok *et al.* (2002, Table 1, p. 251) for an overview of lag orders used in the literature.

For the second model component, where we correlate the long-run and short-run effects of the marketing instruments with category- and brand-specific variables, we construct five covariates. Four of these covariates are brand-specific, these are, relative price, coupon intensity, display/feature promotion intensity and a 0/1 dummy variable for the market leader. The market leader is set as the brand having the largest market share averaged over time. The coupon and promotion intensity variables equal the observed weekly frequency of the use of coupons and promotions, respectively. Finally, the relative price is defined as the average price divided by the maximum average price in the market. The brand that, on average, has the highest price therefore has a relative price equal to one. Note that it is important to use a relative measure of price effectiveness as some categories are more expensive than others.

The fifth and final covariate is defined at the category level and it measures market concentration. As the concentration index, we take the so-called entropy measure, that is,

$$\mathrm{CI}_c = \sum_{i=1}^{I_c} \bar{M}_i(c) \ln \bar{M}_i(c), \tag{20}$$

where $\bar{M}_i(c)$ denotes the average market share of brand i in market c. If all market power is concentrated in one brand, the concentration index equals 0. The index decreases when power is spread over more brands.

3.2 Some a priori conjectures

As said, with our model we aim to provide empirical results that might add to the knowledge base, created in Nijs *et al.* (2001), see also Raju (1992) and Jedidi *et al.* (1999). Concerning the effects of price, we conjecture that a higher marketing-mix intensity has a positive effect on instrument effectiveness, see Nijs *et al.* (2001). Hence, for example, more promotions increase the effects of price changes. There are no strong theoretical reasons why these increases should differ across the long-run and short-run impact of the instruments.

Next, more market concentration would have a positive impact on the absolute price elasticity. Hence, the more concentrated is the market, the more negative is the price effect. Nijs *et al.* (2001) report a significant impact of market structure for short-run effects, and an insignificant effect for those in the longer run.

Finally, for the leading brands one would expect that marketing-mix elasticities are smaller in absolute sense. Evidence for this conjecture is found in Bolton (1989) and Srinivasan *et al.* (2001), where it is shown that brands with smaller market shares tend to have larger price elasticities.

Concerning the dynamic properties of display and feature promotion we are not aware of any direct evidence. However, Van Heerde *et al.* (2000) do find some dynamic effects of display and feature promotion. They report differences in the dynamic effects of price under four types of support (no support, feature only, display only, feature and display support). Indirectly this implies that display and feature also have dynamic effects. From the tables in Van Heerde *et al.* (2000) one can conclude that display and feature have positive carry-over effects. In our setting we therefore also expect the long-run effect of display and feature to be larger than the corresponding short-run effect.

3.3 Estimation results

We estimate our model using MCMC techniques, where we use 10,000 iterations as burn-in. Of the next 100,000 iterations, we retain each tenth draw to obtain an approximately random sample from the posterior distribution. Our posterior results are based on the resulting 10,000 draws.

Figure 1 shows a histogram of the posterior means per brand of the short-run effects ($\alpha_{i,k,c}$), the long-run effects ($\gamma_{i,k,c}$) and the differences between these two effects ($\gamma_{i,k,c} - \alpha_{i,k,c}$) for each of the marketing instruments and for all brands. Note that, in the classical sense, these effects are not parameters of the model. They should be seen as latent variables. The histograms show the distribution of the expected values of these latent variables conditional on the market characteristics and the observed market shares. As expected, most of the mean price effects are negative and most of the mean coupon and display/feature promotion effects are positive. The posterior mean short-run effects over all brands are -3.127, 0.414, and 0.213 for price, coupon promotion, and feature/display, respectively. The long-run posterior mean effects equal -1.974 for price, 0.416 for coupon promotion and 0.415 for feature/display. Interestingly, the variation of the long-run effect of price is smaller than the corresponding short-run effect, while for feature/display we find the opposite outcome. For feature/display, it holds that the mean long-run effects tend to be larger than the mean short-run effects. For price, we find the opposite, that is, the short-run effects tend to be larger (in absolute size) than the long-run effects. On average, the long-run and short-run effects of coupon promotion seem to be equal in size. Whether these eyeball impressions stand a statistical test will be seen below.

Figure 2 shows how the short-run effects are related to the long-run effects. For all three variables, we notice a positive correlation between the short-run and long-run effects. That is, brands for which a marketing instrument has a large short-run effect (in absolute sense), the corresponding long-run effectiveness is also large, on average. This correlation seems strongest for price and the combination of feature and display.

Figure 3 shows scatter plots of the mean posterior effects for different combinations of marketing instruments. For some combinations we find strong correlations. It is noteworthy to mention that there is strong correlation between the effectiveness of feature/display and coupon at the short run and price and feature/display at the long run.

In Table 1, we present the posterior estimates of the effects of covariates on the marketing effectiveness. For the short-run effects, we find substantive interactions between the price effect and various brand characteristics. Higher priced brands and brands that more often issue coupons or are featured tend to have stronger price effects. Relative price also has an effect on the impact of coupons. Coupons of higher priced brands are less effective. Moreover, higher market concentration tends to lead to stronger price effects and higher coupon effectiveness. This corresponds to what we hypothesized above.

For the long-run parameters, we only find strong results for price effects. The signs of these effects are similar to those for the short-run. A high relative price or a high coupon intensity is correlated with a strong price effect. For the long-run effects we do not find substantive interactions with market concentration. This final result corresponds with the findings in Nijs *et al.* (2001).

In the final row of Table 1, we present the posterior probability that the absolute long-run effect of a marketing instrument exceeds the absolute value of the short-run effect. For price there is only a 28.3% probability that the long-run effect will exceed the short-run effect. For coupon this probability is 54.4% and for display/feature promotion it is 87.2%. These probabilities of course correspond well to the bottom row of graphs in Figure 1. Hence, price changes mainly impact the market shares in the short run, while promotions seem to have more of a longer-run impact.

Finally, when we compare our results with these obtained from the commonly used two-step procedure in which first individual market-level models are estimated and where the resulting parameters are then regressed on market and brand characteristics, we find that the signs of the estimated parameters in the second step are the same for both methods. However, the significance levels of the estimates differ substantially. As the uncertainty in market-level parameters in the two-step procedure is underestimated, we find more significant second-stage parameter estimates for the two-step method. Details can be obtained from the authors.

4 Conclusion

In this paper we have put forward a new and useful model for describing market shares. The first novelty was that we considered those attraction models which entail easy to estimate long-run and short-run effects of marketing-mix instruments. As a consequence, and that is the second novelty, we could explicitly link the long-run and short-run effects with category and brand characteristics in a second level. Our resultant error correction HB attraction model was applied to fifty brands covering seven product categories. The main results were that prices exercise mainly a short-run impact, while feature promotions have a larger long-run effect. Furthermore, which is line with the results in Nijs *et al.* (2001) who focused on category sales, we found that a more intensive use of marketing instruments, and also a higher level of market concentration, leads to stronger price effects, both in the long-run and in the short-run.

The model in this paper essentially is a rather natural, and statistically proper, framework to establish generalizing statements about dynamic effects of marketing instruments on market shares. The model resembles a logit structure, and hence one possible extension of our model could be in describing the choice between brands by households. Next, the model can also be used to analyze marketing-mix effectiveness of new to introduce brands, and also to determine optimal price levels. Finally the model we have proposed can provide the basis of a study of (optimal) competitive effects. The long-run and short-run effects we have derived in this paper are based on the assumption of no competitive reaction. In practice this will of course not be the case. Our model could be used to perform a scenario analysis of different competitive reactions.

Appendix: Bayes Estimation

Define $Y_{i,t}(c) = \ln M_{i,t}(c) - \ln M_{I,t}(c)$ and $X_{i,t}(c) = (\ln x_{i,1,t}(c), \ldots, \ln x_{i,K,t}(c))'$. Equation (17) can now be written as

$$\begin{aligned}\Delta Y_{i,t}(c) = \tilde{\mu}_{i,c} + \Delta X_{i,t}(c)'\alpha_{i,c} - \Delta X_{I_c,t}(c)'\alpha_{I_c,c} \\ + \delta_c(Y_{i,t-1}(c) - X_{i,t}(c)'\gamma_{i,c} + X_{I_c,t}(c)'\gamma_{I_c,c}) + \tilde{\varepsilon}_{i,t}(c), \quad (21)\end{aligned}$$

for $i = 1, \ldots, I_c - 1$, where $\delta_c = 1 - \rho_c$, $\alpha_{i,c} = (\alpha_{i,1,c}, \ldots, \alpha_{i,K,c})'$ and $\gamma_{i,c} = (\gamma_{i,1,c}, \ldots, \gamma_{i,K,c})'$ for $i = 1, \ldots, I_c$. Furthermore, define $\tilde{\mu}_c = (\tilde{\mu}'_{1,c}, \ldots, \tilde{\mu}'_{I_c-1,c})'$, $\alpha_c = (\alpha'_{1,c}, \ldots, \alpha'_{I_c,c})'$, $\gamma_c = (\gamma_{1,c}, \ldots, \gamma_{I_c,c})'$ and the error terms

$$\begin{aligned}\tilde{e}_{i,t}(c)(\tilde{\mu}_{i,c}, \delta_c, \alpha_c, \gamma_{i,c}) = \Delta Y_{i,t}(c) - \tilde{\mu}_{i,c} - (\Delta X_{i,t}(c)'\alpha_{i,c} - \Delta X_{I_c,t}(c)'\alpha_{I_c,c}) \\ - \delta_c(Y_{i,t-1}(c) - X_{i,t}(c)'\gamma_{i,c} + X_{I_c,t}(c)'\gamma_{I_c,c}). \quad (22)\end{aligned}$$

The vector of error terms is given by

$$\tilde{e}_t(c)(\tilde{\mu}_c, \delta_c, \alpha_c, \gamma_c) = (\tilde{e}_{1,t}(c)(\tilde{\mu}_{1,c}, \delta_c, \alpha_c, \gamma_c), \ldots, \tilde{e}_{I_c-1,t}(c)(\tilde{\mu}_{I_c,c}, \delta_c, \alpha_c, \gamma_{I,c}))', \quad (23)$$

and hence the likelihood of the model reads as

$$\prod_{c=1}^{C} \int_{\alpha_c, \gamma_c} \prod_{t=2}^{T_c} \phi\Big(\tilde{e}_t(c)(\tilde{\mu}_c, \delta_c, \alpha_c, \gamma_c); 0, \tilde{\Sigma}_c\Big) \prod_{i=1}^{I_c} \phi(\alpha_{i,c}; \lambda'_1 z_{i,c}, \Sigma_\eta)\phi(\gamma_{i,c}; \lambda'_2 z_{i,c}, \Sigma_\nu) d\alpha_c d\gamma_c, \quad (24)$$

where $\phi(x; \mu, \Sigma)$ is the density function of the multivariate normal distribution with mean μ and variance Σ evaluated at x.

To obtain posterior results, we use the Gibbs sampling technique of Geman and Geman (1984) with data augmentation, see Tanner and Wong (1987). An introduction into the Gibbs sampler can be found in Casella and George (1992), see also Smith and Roberts (1993) and Tierney (1994) Hence, the latent variables α_c and γ_c are sampled alongside the model parameters $\tilde{\mu}_c$, δ_c, $\tilde{\Sigma}_c$, λ_1, λ_2, Σ_η and Σ_ν. The Bayesian analysis is based on uninformative priors for the model parameters. To improve convergence of the MCMC sampler we impose inverted Wishart priors on the Σ_η and Σ_ν parameter with scale parameter $\kappa_1 \mathbf{I}_K$ and degrees of freedom κ_2. We set the value of κ_1 to $\frac{1}{1000}$ and κ_2 equal to 1 such that the influence of the prior on the posterior distribution is marginal, see Hobert and Casella (1996) for a discussion.

In the remainder of this appendix we derive the full conditional posterior distributions of the model parameters and α_c and γ_c. In deriving the sampling distributions we build on the results in Zellner (1971, Chapter VIII).

Sampling of $\tilde{\mu}_c$ and δ_c

To sample $\tilde{\mu}_c$ and δ_c we rewrite the model in (21) as

$$\Delta Y_{i,t}(c) - \Delta X_{i,t}(c)'\alpha_{i,c} + \Delta X_{I_c,t}(c)'\alpha_{I_c,c} = \tilde{\mu}_{i,c} + \delta_c(Y_{i,t-1}(c) - X_{i,t}(c)'\gamma_{i,c} + X_{I_c,t}(c)'\gamma_{I_c,c}) + \tilde{\varepsilon}_{i,t}(c) \quad (25)$$

for $i = 1, \ldots, I_c - 1$. Stacking the equations in (25) we obtain the multivariate regression model

$$W_t(c) = V_t(c)\beta + \tilde{\varepsilon}_t(c), \quad (26)$$

where $W_t(c)$ is a $I_c - 1$ dimensional vector containing the left-hand side of equation (25), $V_t(c)$ an $(I_c - 1)$ dimensional identity matrix extended with a $(I_c - 1)$ dimensional column vector of error-correction terms $(Y_{i,t-1}(c) - X_{i,t}(c)'\gamma_{i,c} + X_{I_c,t}(c)'\gamma_{I_c,c})$, and where $\beta = (\tilde{\mu}_{1,c}, \ldots, \tilde{\mu}_{I_c-1,c}, \delta_c)'$. The error term is normal distributed with mean 0 and variance $\tilde{\Sigma}$. Hence, the full conditional posterior distribution of β is a matrix normal distribution with mean

$$\left(\sum_{t=2}^{T_c} V_t(c)'\tilde{\Sigma}^{-1}V_t(c)\right)^{-1} \left(\sum_{t=2}^{T_c} V_t(c)'\tilde{\Sigma}^{-1}W_t(c)\right), \quad (27)$$

and variance

$$\left(\sum_{t=2}^{T_c} V_t(c)'\tilde{\Sigma}^{-1}V_t(c)\right)^{-1}. \quad (28)$$

Sampling of $\tilde{\Sigma}_c$

To sample $\tilde{\Sigma}_c$ we again consider the multivariate regression model (26). The full conditional posterior distribution of $\tilde{\Sigma}_c$ is an inverted Wishart distribution with scale parameter $\sum_{t=2}^{T_c}(W_t(c) - V_t(c)\beta)(W_t(c) - V_t(c)\beta))'$ and degrees of freedom $T_c - 1$.

Sampling of λ_1 and λ_2

To sample λ_1, we note that we can write (18) as

$$\alpha'_{i,c} = z'_{i,c}\lambda_1 + \eta'_{i,c}. \quad (29)$$

and hence it is a multivariate regression model with regression matrix λ_1. Hence, the full conditional posterior distribution of λ_1 is a matrix normal distribution with mean

$$\left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} z_{i,c}z'_{i,c}\right)^{-1} \left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} z_{i,c}\alpha_{i,c}\right), \quad (30)$$

and covariance matrix

$$\left(\Sigma_\eta \otimes \left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} z_{i,c} z_{i,c}'\right)^{-1}\right). \tag{31}$$

The derivation of the sampling distribution of λ_2 proceeds in the same manner. The full conditional posterior distribution of λ_2 is a matrix normal distribution with mean

$$\left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} z_{i,c} z_{i,c}'\right)^{-1}\left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} z_{i,c}\gamma_{i,c}\right), \tag{32}$$

and covariance matrix

$$\left(\Sigma_\nu \otimes \left(\sum_{c=1}^{C}\sum_{i=1}^{I_c} z_{i,c} z_{i,c}'\right)^{-1}\right). \tag{33}$$

Sampling of Σ_η and Σ_ν

To sample Σ_η we note that (18) is a multivariate regression model. Hence the full conditional posterior distribution of Σ_η is an inverted Wishart distribution with scale parameter $\kappa_1 \mathbf{I}_K + \sum_{c=1}^{C}\sum_{i=1}^{I_c}(\alpha_{i,c} - \lambda_1' z_{i,c})(\alpha_{i,c} - \lambda_1' z_{i,c})'$ and degrees of freedom $\kappa_2 + \sum_{c=1}^{C} I_c$. The κ terms results from the inverted Wishart prior on Σ_η which is used to improve convergence of our Gibbs sampler, see Hobert and Casella (1996) for a discussion.

The sampling of Σ_ν can be done in exactly the same manner. The parameter Σ_ν is sampled from an inverted Wishart distribution with scale parameter $\kappa_1 \mathbf{I}_K + \sum_{c=1}^{C}\sum_{i=1}^{I_c}(\gamma_{i,c} - \lambda_2' z_{i,c})(\gamma_{i,c} - \lambda_2' z_{i,c})'$ and degrees of freedom $\kappa_2 + \sum_{c=1}^{C} I_c$.

Sampling of α_c

To sample $\alpha_c = (\alpha_{1,c}, \ldots, \alpha_{I_c,c})'$ we rewrite (17) as

$$\begin{aligned}\Delta Y_{i,t}(c) - \tilde{\mu}_i - \delta_c(Y_{i,t-1}(c) - X_{i,t}(c)'\gamma_{i,c} + X_{I_c,t}(c)'\gamma_{I_c,c})& \\ = \Delta X_{i,t}(c)'\alpha_{i,c} - \Delta X_{I_c,t}(c)'\alpha_{I_c,c} + \tilde{\varepsilon}_{i,t}(c),&\end{aligned} \tag{34}$$

for $i = 1, \ldots, I_c - 1$ which can be written in matrix notation

$$W_t(c) = V_t(c)\alpha_c + \tilde{\varepsilon}_t(c), \tag{35}$$

where $W_t(c)$ is a $(I_c - 1)$ dimensional vector containing $\Delta Y_{i,t}(c) - \tilde{\mu}_i - \delta_c(Y_{i,t-1}(c) - X_{i,t}(c)'\gamma_{i,c} + X_{I_c,t}(c)'\gamma_{I_c,c})$ and

$$V_t(c) = \begin{pmatrix} \Delta X_{1,t}(c)' & 0 & \dots & 0 & -\Delta X_{I_c,t}(c)' \\ 0 & \Delta X_{2,t}(c)' & \dots & 0 & -\Delta X_{I_c,t}(c)' \\ \vdots & \vdots & \ddots & \vdots & -\Delta X_{I_c,t}(c)' \\ 0 & \dots & 0 & \Delta X_{I_c-1,t}(c)' & -\Delta X_{I_c,t}(c)' \end{pmatrix}. \tag{36}$$

Furthermore, we write the I_c equations of (18) as

$$-U_c = -\mathbf{I}_{KI_c}\alpha_c + \eta_c, \tag{37}$$

where U_c is a (KI_c) dimensional vector containing the terms $\lambda_1' z_{i,c}$ and where $\mathbf{I}_{KI_c}$ is a (KI_c) dimensional identity matrix. The error term η_c is normal distributed with mean 0 and covariance matrix $(I_c \otimes \Sigma_\eta)$. To sample α_c, we combine (35) and (37)

$$\begin{aligned} \tilde{\Sigma}^{-1/2} W_t(c) &= \tilde{\Sigma}^{-1/2} V_t(c)\alpha_c + \tilde{\Sigma}^{-1/2}\tilde{\varepsilon}_t(c), \\ -(\mathbf{I}_c \otimes \Sigma_\eta^{-1/2})U_c &= -(\mathbf{I}_c \otimes \Sigma_\eta^{-1/2})\alpha_c + (\mathbf{I}_c \otimes \Sigma_\eta^{-1/2})\eta_c. \end{aligned} \tag{38}$$

Hence, the full conditional posterior distribution of α_c is normal with mean

$$\left((\mathbf{I}_c \otimes \Sigma_\eta^{-1}) + \sum_{t=2}^{T_c} (V_t(c)'\tilde{\Sigma}^{-1} V_t(c)) \right)^{-1} \left((\mathbf{I}_c \otimes \Sigma_\eta^{-1})U_c + \sum_{t=2}^{T_c} (V_t(c)'\tilde{\Sigma}^{-1} W_t(c)) \right), \tag{39}$$

and covariance matrix

$$\left((\mathbf{I}_c \otimes \Sigma_\eta^{-1}) + \sum_{t=2}^{T_c} (V_t(c)'\tilde{\Sigma}^{-1} V_t(c)) \right)^{-1}. \tag{40}$$

Sampling of γ_c

To sample $\gamma_c = (\gamma_{1,c}, \dots, \gamma_{I_c,c})'$, we rewrite (17) as

$$\begin{aligned} \Delta Y_{i,t}(c) - \tilde{\mu}_i - \Delta X_{i,t}(c)'\alpha_{i,c} + \Delta X_{I_c,t}(c)'\alpha_{I_c,c} - \delta_c Y_{i,t-1}(c) &= \\ - \delta_c X_{i,t}(c)'\gamma_{i,c} + \delta_c X_{I_c,t}(c)'\gamma_{I_c,c} + \tilde{\varepsilon}_{i,t}(c), \end{aligned} \tag{41}$$

for $i = 1, \dots, I_c - 1$ which can be written in matrix notation

$$\tilde{\Sigma}^{-1/2} W_t(c) = \tilde{\Sigma}^{-1/2} V_t(c)\gamma_c + \tilde{\Sigma}^{-1/2}\tilde{\varepsilon}_t(c), \tag{42}$$

where now $W_t(c)$ is a $(I_c - 1)$ dimensional vector containing $\Delta Y_{i,t}(c) - \tilde{\mu}_i - \Delta X_{i,t}(c)'\alpha_{i,c} + \Delta X_{I_c,t}(c)'\alpha_{I_c,c} - \delta_c Y_{i,t-1}(c)$ and

$$V_t(c) = \begin{pmatrix} -\delta_c X_{1,t-1}(c)' & 0 & \dots & 0 & \delta_c X_{I_c,t}(c)' \\ 0 & -\delta_c X_{2,t}(c)' & \dots & 0 & \delta_c X_{I_c,t}(c)' \\ \vdots & \vdots & \ddots & \vdots & \delta_c X_{I_c,t}(c)' \\ 0 & \dots & 0 & -\delta_c X_{I_c-1,t}(c)' & \delta_c X_{I_c,t}(c)' \end{pmatrix}. \tag{43}$$

Again, we write the I_c equations of (19) as

$$-(\mathbf{I}_c \otimes \Sigma_\nu^{-1/2})U_c = -(\mathbf{I}_c \otimes \Sigma_\nu^{-1/2})\gamma_c + (\mathbf{I}_c \otimes \Sigma_\nu^{-1/2})\omega_t, \tag{44}$$

where U_c is a (KI_c) dimensional vector containing the terms $\lambda_2' z_{i,c}$. The distribution of the error term ω_t is normal with mean 0 and covariance matrix $(I_c \otimes \Sigma_\nu)$. If we combine (42) with (44) it is easy to see that the full conditional posterior distribution of γ_c is normal with mean

$$\left((\mathbf{I}_c \otimes \Sigma_\nu^{-1}) + \sum_{t=2}^{T_c} (V_t(c)'\tilde{\Sigma}^{-1} V_t(c)) \right)^{-1} \left((\mathbf{I}_c \otimes \Sigma_\nu^{-1})U_c + \sum_{t=2}^{T_c} (V_t(c)'\tilde{\Sigma}^{-1} W_t(c)) \right), \tag{45}$$

and covariance matrix

$$\left((\mathbf{I}_c \otimes \Sigma_\nu^{-1}) + \sum_{t=2}^{T_c} (V_t(c)'\tilde{\Sigma}^{-1} V_t(c)) \right)^{-1}. \tag{46}$$

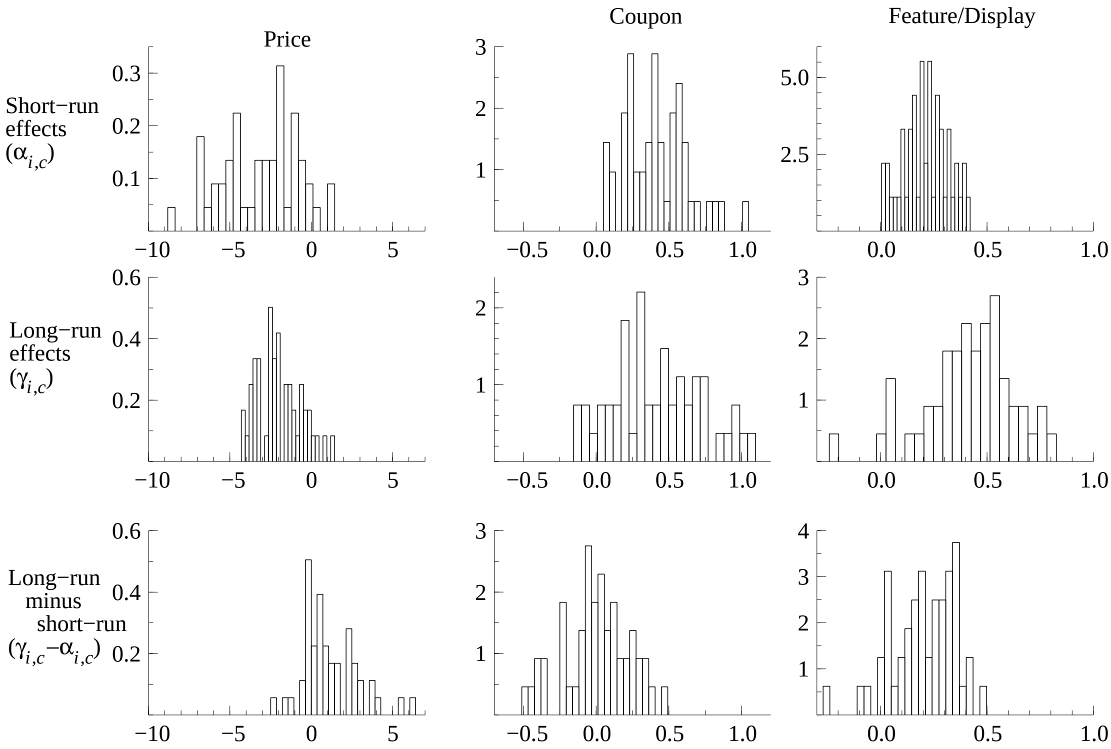

Figure 1: Histograms of posterior means of marketing-mix effectiveness, for all fifty brands

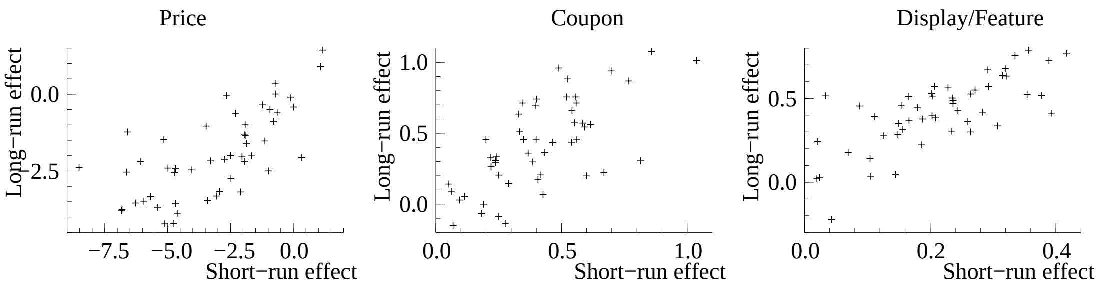

Figure 2: Scatter plots of long-run effects versus short-run effects (posterior means per brand), for all fifty brands. Short-run effects are given by $\alpha_{i,c}$, long-run effects equal $\gamma_{i,c}$, see (17)

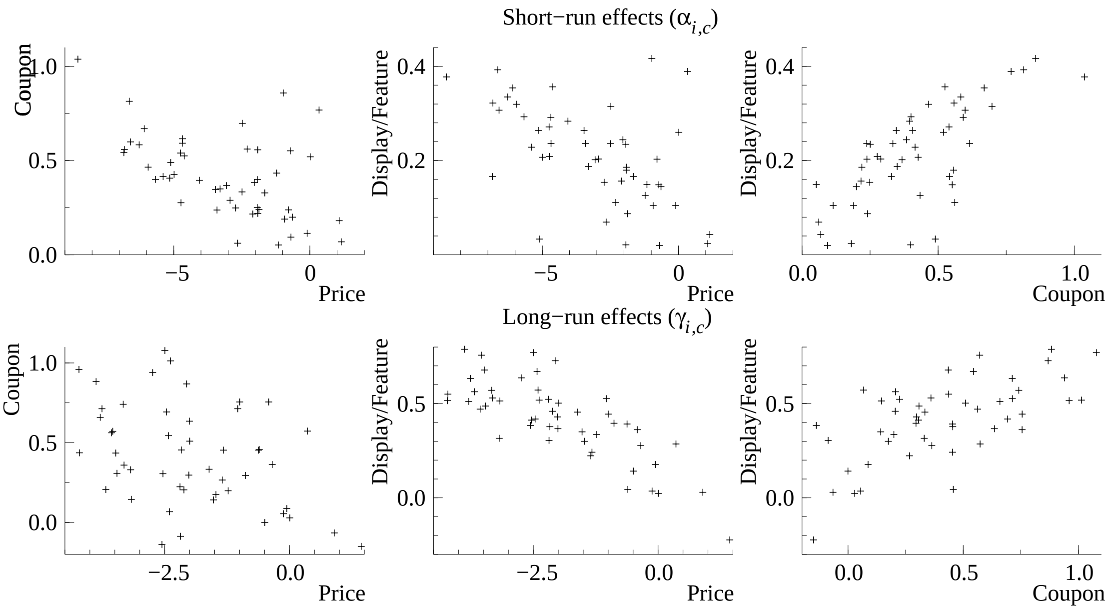

Figure 3: Scatter plots of posterior means of marketing-mix effectiveness

Table 1: Posterior means of the effects of covariates on short-run and long-run effects of the marketing mix (λ_1 and λ_2 in (18) and (19)), posterior standard deviation between brackets.

	Marketing-mix effectiveness					
	Price		Coupon		Display/Feature	
	Short-run effects (λ_1)					
Intercept	-5.952***	(2.010)	0.918***	(0.233)	0.173	(0.158)
Feature/Display intensity	-3.138*	(1.566)	-0.090	(0.210)	-0.069	(0.129)
Coupon intensity	-3.210**	(1.378)	-0.045	(0.169)	0.054	(0.112)
Relative price	-0.222**	(0.115)	-0.027*	(0.015)	0.004	(0.009)
Market concentration	-5.242**	(1.979)	0.372*	(0.227)	-0.062	(0.159)
Market leader	1.030	(0.836)	-0.004	(0.103)	-0.074	(0.065)
	Long-run effects (λ_2)					
Intercept	-3.241**	(1.592)	1.185***	(0.340)	0.472*	(0.263)
Feature/Display intensity	-0.756	(1.107)	-0.435	(0.301)	-0.194	(0.213)
Coupon intensity	-2.632***	(0.960)	-0.062	(0.232)	0.257	(0.183)
Relative price	-0.215***	(0.072)	-0.032	(0.021)	0.005	(0.016)
Market concentration	-2.462	(1.567)	0.384	(0.326)	0.051	(0.267)
Market leader	0.813	(0.534)	-0.127	(0.131)	-0.087	(0.111)
Pr[|Long run| > |Short run|]	0.283		0.544		0.872	

*, **, *** Zero not contained in 90%, 95% or 99% highest posterior density region, respectively

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