Putting the cart before the horse.
A reply to Wagstaff on inequality measurement in the presence of binary variables

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Adam Wagstaff's (2011) recent paper sends a strong reminder that binary variables occur frequently in health inequality studies, and that it is important to examine whether the standard measurement tools can be applied without any modification when the health variable happens to be binary. In his note he reconsiders what he wrote previously on the subject (Wagstaff, 2005), in the light of recent work on bounded variables (Clarke *et al.*, 2002; Erreygers, 2009a,b; Wagstaff, 2009; Erreygers and Van Ourti, 2010). While Wagstaff's contribution undoubtedly enriches a much-needed debate, crucial aspects of his paper seriously misrepresent the positions and views set forth in Erreygers and Van Ourti (2010). In this note we would like to put the record straight, focusing on five specific points.

1. In Erreygers and Van Ourti (2010), we have tried to make a careful analysis of the issues involved in the measurement of socioeconomic inequality of health/health care/health expenditures by means of rank-dependent indices. This paper builds on the extensive literature on the Concentration Index, to which Adam Wagstaff has made many and distinctive contributions. We pay special attention to the nature of the health variable under consideration, an aspect which in our opinion had been relatively neglected. Since Wagstaff's (2011) recent paper contains a more detailed exploration of the case of binary variables, we welcome it as a valuable input to an ongoing investigation. Unfortunately, its starting point is an erroneous interpretation of our position.

According to Wagstaff (2011) we claim that because binary variables are ordinal, they are *ipso facto* unsuitable for *any* inequality analysis. This can only be called a gross distortion of what we maintain in the paper. In fact, after observing that "ordinal measurement scales do not allow differences between individuals to be

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compared" (Erreygers and Van Ourti, 2010), we immediately add the following qualification with regard to a variable of a categorical or binary nature: "If, however, such a variable can be transformed into or proxied by a cardinal variable, it becomes possible to compare these health differences." (ibidem) It is also instructive to reflect on the footnote which we append to this sentence: "For example, van Doorslaer and Jones (2003) have projected the ordinal self-assessed health categories upon the cardinal HUI-scale. In case of a binary 0/1 indicator, one might overcome the ordinal nature by assuming that it expresses the presence of a certain condition in percentage points, i.e. 100% or 0%. While this seems somewhat implausible at the individual level, it makes sense at the aggregate level (e.g. percentiles)." (ibidem) Put differently, we say that when we are dealing with binary variables we can *always* treat them as cardinal (even ratio-scale) variables, and when we are dealing with categorical variables we can *sometimes* do this. The suggestion that in our view binary variables can never be used for inequality analysis is almost exactly the opposite of what we have written.

2. In our paper we also make an explicit distinction between bounded and unbounded variables, and between cardinal, ratio-scale and absolute measurement scales. We cannot discuss in any detail the implications for rank-dependent inequality measurement in each of the separate cases as we do in the paper, but we have to stress that the distinction between bounded and unbounded variables – in particular, whether there is a finite upper bound or not – is crucial in our opinion. When the health variable is unbounded, the standard measurement apparatus seems to be fairly appropriate; when the variable is bounded, however, one needs to decide whether inequality rankings in attainments (i.e. health) or in shortfalls (i.e. illness = maximum health minus actual health) should convey a different message. The answer to this question depends on a value judgment and several ethical stances can be taken.

Erreygers (2009a,b) has argued that inequalities in shortfalls should 'mirror' inequalities in attainments, i.e. the magnitude of the two types of inequality should be identical. This mirror condition has recently been further underpinned and generalized by Lambert and Zheng (2011), albeit in a slightly different context.[2] In our paper we

[2] Lambert and Zheng (2011), as well as Erreygers (2009c), focus on one-dimensional inequality measurement (e.g. pure inequalities in health), but the arguments are straightforwardly transferred to the bivariate health-income rank case.

show that an entire class of rank-dependent indices satisfies the mirror condition. While Wagstaff's (2005) normalization belongs to this class, the standard concentration index does not. This is acknowledged by Wagstaff (2011) when he writes that "…if one feels attracted to the value judgment implied by the original concentration index, then my normalization is wrong and we face the problem that a ranking of countries can depend on which attribute is coded zero".

While our paper draws attention to the fact that an *additional* value judgment must be made when dealing with a bounded variable (such as a binary variable), we leave it to the reader to decide what value judgment he or she prefers. It is the whole set of preferred value judgments and choices which ultimately points in the direction of a particular normalization.[3] We therefore disagree with Wagstaff's (2011) conclusion that "…criticisms of the normalization I proposed, and indeed of the use of the binary variable for inequality analysis, stem from a misrepresentation of the properties of the binary variable…".The objections we have raised to Wagstaff's normalization are based on the implied properties of his index, and have little to do with the binary nature of the health variable. Our paper clarifies that the choice of one normalization rather than another is not simply a matter of convenience: it has wide-ranging implications, as shown by the fact that the standard concentration index applied to binary variables violates the mirror condition. If that property is deemed unattractive, then our analysis provides guidelines on which class of indices – including the one proposed by Wagstaff (2005) – could be used alternatively.

3. Wagstaff also suggests on the first page of his paper that "…criticisms of the normalization I proposed…stem from … a switch of focus away from relative inequality to absolute inequality". Again, this is a topic which we treat at length in our paper. We believe that Wagstaff's claim is misleading for two reasons.

[3] It may be useful to point out that what Wagstaff (2011) writes on the issues of scale invariance and the principle of transfers is in accordance with what we have affirmed in Erreygers and Van Ourti (2010). On scale invariance he notes: "one can reasonably argue that in the case of a binary outcome the issue of scale independence is simply moot". Indeed, if one follows our logic/argument to rescale a bounded variable to the 0-1 range, the axiom of scale independence is satisfied. Wagstaff also writes that "with a binary outcome we can still talk meaningfully about the principle of transfers". Indeed, all rank-dependent inequality indices considered in our paper satisfy the "principle of transfers".

First, one cannot speak of relative inequality and absolute inequality in the traditional sense in the case of bounded variables. “The difficulty when trying to apply these notions to bounded variables is that some of the changes are infeasible, because the bounds of the variables act as constraints” (Erreygers and Van Ourti, 2010). By developing the notions of ‘quasi-relativity’ and ‘quasi-absoluteness’, which take this infeasibility into account and are applicable to a wide range of bounded variables, we manage to resolve the difficulty. Wagstaff, however, restricts the discussion to the special case of binary variables and sees no need to go beyond the standard notions of relative and absolute inequality. We believe this is a missed opportunity to point out that bounded variables are of a special nature.

Second and much more importantly, Erreygers and Van Ourti (2010) state an impossibility result showing that a rank-dependent inequality index cannot at the same time have the property of quasi-relativity and satisfy the mirror condition. Since the normalization proposed by Wagstaff (2005) satisfies the mirror condition, it does not have the property of quasi-relativity. This also means that Wagstaff’s justification for his normalization – that it ensures that the index has fixed bounds “…and tries to separate out inequality from the mean” – is incompatible with quasi-relativity. In other words, dividing the standard concentration index by it maximum bound makes the resulting normalization satisfy the mirror condition, but fail quasi-relativity. The standard concentration index, on the other hand, allows for quasi-relativity, but does not meet the mirror condition. Hence, also the normalization proposed by Wagstaff (2005) shifts the focus away from relative inequality. Wagstaff seems to be aware of this issue when he concedes that “…while the binary variable has some unusual properties, it shares many of the properties of the ratio-scale variable and hence lends itself to both relative and absolute inequality analysis, albeit with some qualifications”. We believe that this statement is misleading, since the implied qualification is that the normalized index does not satisfy quasi-relativity.

4. In our paper, we use the concepts ‘quasi-absoluteness’ and ‘quasi-relativity’ to show the different value judgments underlying rank-dependent indices. In the conclusion of our paper, we express a preference for one particular member of the subclass that satisfies the mirror condition, i.e. the index proposed by Erreygers (2009a), which has the property of quasi-absoluteness. Two things deserve to be noted at this stage: (a) we mention explicitly that one need not agree with our preference for an index which

satisfies quasi-absoluteness; and (b) we also clarify the value judgments underlying the other indices belonging to the general class. As we say in the paper: "If one does not want to impose [quasi-absoluteness], one could use the other members of the class… (including the Wagstaff index) that satisfy the Mirror property. In this case, one implicitly agrees that for some distributions inequality increases in magnitude when there is a ceteris paribus decrease of relative differences, which we think is unacceptable." This is precisely the reason why we do not favour the normalization of Wagstaff, and not – as suggested by Wagstaff – because we misrepresent the properties of the binary variable.

5. Finally, let us consider Wagstaff's own criticism of his normalization: "…I also concede in this paper that the normalization I proposed is contentious in a way that has not previously been noted." As a result, he seems to advocate a return to the use of the standard concentration index when dealing with binary variables. This is what can be inferred from the following quotation, in which Wagstaff (2011) discusses an example given by Erreygers (2009b): "…given the mean, the two distributions are maximally and hence equally pro-rich. But it could be argued—and contrary to Erreygers' suggestion—that the second distribution is less pro-rich than the first, on the grounds that the privilege of being healthy is not quite so dramatically associated with being rich as in the first. That is, of course, what the concentration index concludes. I have some sympathy with this view." In his discussion Wagstaff refers to the fact that some normalizations imply that the pivotal individual is not necessarily the individual with median income rank. The pivotal individual is the individual for which a change in health has no effect on measured inequality; "giving the attribute to someone who does not have it" changes the distribution in a pro-poor direction when the person in question has a lower rank than the pivotal individual, and in a pro-rich direction when the person has a higher rank.

Once again, this shows the intimate connection between normalizations and value judgments, in this case concerning the type of changes considered to be pro-poor or pro-rich. There is bound to be a diversity of views about which set of value judgments is best. In our paper we follow the strategy of letting explicit value judgments decide what normalization should be adopted. Moreover, we point out that some choices are incoherent: for instance, no rank-dependent index can satisfy both the mirror condition and the property of quasi-relativity. Wagstaff seems to do the opposite:

by first choosing a particular normalization, and then exploring what it implies, he has put the cart before the horse. When realizing that "the normalization I proposed is contentious in a way that has not previously been noted", Wagstaff's fall-back option is to revert to the standard concentration index. Since this is equivalent to turning a blind eye to the crucial mirror issue we think it is a particularly unconvincing choice.

To sum up, while we certainly appreciate Wagstaff's (2011) efforts to come to grips with the issue of health inequality measurement in the presence of binary health variables, we reject both his flawed interpretation of our views on binary health variables, and his misleading suggestions with regard to our criticism of the normalization which he proposed in Wagstaff (2005). In this note we have concentrated on five specific points, which can be summarized as follows.

1) We have never maintained, and do not maintain now, that binary health variables are unsuitable for any form of inequality analysis.
2) When dealing with bounded variables, additional value judgments have to be made, and we insist that these should be made explicit. Our stance is that the mirror condition is of central importance, but we accept that this view may not be shared by all.
3) We argue that the notions of quasi-absoluteness and quasi-relativity are useful devices to explore the properties of indices in the case of bounded variables.
4) The main reason why we have criticized Wagstaff's (2005) normalization is that it has neither the property of quasi-relativity nor that of quasi-absoluteness.
5) In our opinion, Wagstaff's (2011) apparent plea to revert to the standard concentration index to measure inequality with binary variables, is a poorly justified position.

The message which runs through all of our comments is that we need to be fully aware of the underlying value judgments, and that the choice for a specific normalization should be guided by these value judgments, not the other way around.

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