J Prod Anal (2012) 37:231–232
DOI 10.1007/s11123-011-0258-7

Imputing output prices for non-market production units: a comment

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Published online: 14 December 2011

1 Introduction

This note is a comment on Diewert (2011b), which is about productivity measurement for non-market production units. Diewert considered specifically the imputation of output prices when price and quantity data on output-specific inputs are available.

In this note I show that the two approaches offered by Diewert, the simple one in Sect. 2 and the seemingly more general one in the Appendix, are basically equivalent.

2 Two approaches

Consider a production unit producing $y_1^t, \ldots, y_M^t$ quantities of outputs (in the article called procedures) during period t. Each output m requires inputs from a set A_m, the vector of quantities during period t being x_m^t $(m = 1,\ldots, M)$. The corresponding vectors of input prices are w_m^t $(m = 1,\ldots, M)$. It is assumed that output quantities as well as output-specific input quantities and prices can be observed. Note that this is a rather strong assumption because usually only *aggregate* input quantities and prices can be observed.

Because the production unit is not operating on the market, there are no output prices. But given the input requirements, output prices can be imputed as being equal to unit costs; that is,

$$p_m^t \equiv w_m^t \cdot x_m^t / y_m^t = w_m^t \cdot a_m^t \quad (m = 1, \ldots, M), \tag{1}$$

where $a_m^t \equiv x_m^t / y_m^t$ is the vector of input quantities per unit of output $m (m = 1,\ldots, M)$ and the dot denotes inner product. The production unit's profitability, defined as its (total) revenue divided by its (total) cost, is then computed as

$$\frac{\sum_{m=1}^{M} p_m^t y_m^t}{\sum_{m=1}^{M} w_m^t \cdot x_m^t} \tag{2}$$

Substituting (1) into (2) leads immediately to the conclusion that profitability is identically equal to 1. *A fortiori*, when comparing two periods, profitability *change* is identically equal to 1. However, *productivity* change, defined as output quantity index divided by input quantity index (or, the quantity component of profitability change), is not necessarily equal to 1, unless the technical coefficients of the two periods are the same ($a_m^1 = a_m^0$, where 1 and 0 denote the two periods compared).

The above summarizes what is happening in Sect. 2 of Diewert's article. The Appendix generalizes this by assuming that the technology with which output m during period t is produced can be represented by a cost function $C_m^t(w_m, y_m)$. Instead of (1), the output prices can then be imputed as unit costs by setting them equal to

$$p_m^t \equiv C_m^t(w_m^t, 1) \quad (m = 1, \ldots, M). \tag{3}$$

Profitability is then computed as

$$\frac{\sum_{m=1}^{M} C_m^t(w_m^t, 1) y_m^t}{\sum_{m=1}^{M} w_m^t \cdot x_m^t}, \tag{4}$$

which is not necessarily equal to 1. Now, if each technology exhibits constant returns to scale (CRS) and the production unit acts cost minimizing, then

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$$C^t_m(w^t_m, 1)y^t_m = C^t_m(w^t_m, y^t_m) = w^t_m \cdot x^t_m \quad (m = 1, \ldots, M), \tag{5}$$

which implies that profitability is identically equal to 1. Rewriting expression (5) we see that

$$C^t_m(w^t_m, 1) = w^t_m \cdot x^t_m / y^t_m \quad (m = 1, \ldots, M), \tag{6}$$

which, when substituted into expression (3), leads to the same expression as (1). Thus the twin assumptions of CRS technologies and cost minimization essentially bring us back to the setup as discussed in Sect. 2. There is nothing gained here.

3 Conclusion

The basic assumption referred to in the sentence preceding expression (3) implies that the overall cost function has the following form

$$C^t(w_1, \ldots, w_M, y_1, \ldots, y_M) = \sum_{m=1}^{M} C^t_m(w_m, y_m). \tag{7}$$

If this assumption is dropped and replaced by the twin assumptions that the *overall* technology exhibits CRS and the production unit acts cost minimizing, then it appears that

$$\sum_{m=1}^{M} \frac{\partial C^t(w^t_1, \ldots, w^t_M, y^t_1, \ldots, y^t_M)}{\partial y_m} y^t_m = C^t(w^t_1, \ldots, w^t_M, y^t_1, \ldots, y^t_M) = \sum_{m=1}^{M} w^t_m \cdot x^t_m \tag{8}$$

Thus it makes sense to define output prices as marginal costs; that is,

$$p^t_m \equiv \frac{\partial C^t(w^t_1, \ldots, w^t_M, y^t_1, \ldots, y^t_M)}{\partial y_m} \quad (m = 1, \ldots, M). \tag{9}$$

Then, of course, profitability and profitability change are identically equal to 1.[1] Output quantity index divided by input quantity index, however, is not necessarily equal to 1, as in the simple situation considered before. However, expression (9) makes clear that for the computation of a productivity index knowledge of the cost function is essential.

References

Diewert WE (2011a) Measuring productivity in the public sector. J Prod Anal 36:177–191

Diewert WE (2011b) The measurement of productivity in the nonmarket sector. J Prod Anal. doi:10.1007/s11123-011-0247-x

[1] See also Diewert (2011a, Appendix), where the cost function additionally contains a vector of (output) quality characteristics.