# Macroeconomic Perspectives on the Equity Premium Puzzle

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# Macroeconomic Perspectives on the Equity Premium Puzzle

Macroeconomische perspectieven ten aanzien van de aandelenpremiepuzzel

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# Chapter 1

# Introduction and outline

## 1.1 Introduction

Suppose you were born on January 1, 1983. A few months later, your parents, wanting to set aside some money for your 18th birthday, were at a crossroads. They could either invest money in risk-free Dutch government bonds or in the newly created AEX index, investing in companies' shares. After careful consideration they decided to take a risk and invest (the equivalent of) € 100 in shares. On your 18th birthday your parents gave you an envelope and explained what they had done. When you opened the envelope you were flabbergasted to discover that this € 100 had grown to € 1.405,02; a staggering annual nominal return of slightly less than 16%. Inflation averaged at about 2%, so this translates into a real (i.e. corrected for inflation) return of 14%. But what would have happened if your parents had not taken a risk? Then the envelope would have contained € 260,01, a nominal and real return of 5.5% and 3.6%, respectively.

This example illustrates that over longer periods, stocks provide an investor with a higher return than bonds. In economics this return difference is called the equity premium: the premium stocks pay over bonds. Although stocks outperform bonds over longer timespans, this holds up less and less over shorter horizons. For example, during 2008 the AEX index lost more than half of its value. As stocks move up and down much more than bonds, they are considered more risky. The equity premium compensates an investor for bearing this risk.

Among asset pricing models the consumption-based Capital Asset Pricing Model is the workhorse model. This model assumes the existence of an investor who has an utility function defined over consumption. In other words, on the most fundamental level an investor is not interested in returns, but in the goods and services she can afford by trading stocks and bonds. Households need to be extremely risk averse for observed excess returns to be consistent with actual consumption behavior. This is, in a nutshell, the equity premium puzzle.

At the heart of the equity premium puzzle lies the low volatility (i.e. degree of variation) of observed consumption growth. To see this, notice that a risky asset commands a low rate of return if it provides poor insurance against consumption fluctuations by paying little in periods of high consumption growth and plenty in periods of low consumption growth. The underlying reason is that as such an asset hedges an individual against bad spells, it is in high demand and therefore only offers a low return. In contrast, an asset that pays little in tough times and plenty when times are good is less desirable. As a result, investors require a higher return to hold it in their portfolio. Data reveal that fluctuations in consumption growth are small. Stated differently, consumption growth is very smooth. This implies that high returns on risky assets can be supported only if one assumes that consumers highly dislike even tiny consumption fluctuations. In other words, one must assume that consumers are extraordinarily risk averse.

Typical values of risk aversion in the literature vary considerably, but values in excess of 50 are not uncommon. To see why many economists argue that this is excessive, consider the following lottery:

| | | | |
|---|---|---|---|
| Choice 1 | € | 50.000 | with probability 0.5 |
| | € | 100.000 | with probability 0.5 |
| Choice 2 | € | X | with probability 1.0 |

For which value of $X$ would you be indifferent between these two choices? That is, what is the least amount of money you would like to receive for certain in order to give up the gamble of the first choice?

Under the assumption of relative risk aversion, here is how your choice of $X$ corresponds to your risk aversion coefficient, denoted by $\gamma$:

| X | $\gamma$ |
|---|---|
| € 70,711 | 1 |
| € 63,246 | 3 |
| € 58,566 | 5 |
| € 53,991 | 10 |
| € 51,209 | 30 |
| € 50,712 | 50 |
| € 50,351 | 100 |

Notice that if one is risk neutral (i.e. does not care about risk), the exptected value of $X$ would be € 75.000. If someone is only slightly risk averse ($\gamma = 1$), she would already forgo € 4.289 compared to the situation of risk neutrality. As the value of $X$ decreases, the level of risk aversion increases. As mentioned, values of risk aversion exceeding 50 are commonplace. This translates into a value of $X$ of merely € 50.712. Would someone accept less than € 1000 to forgo the opportunity of having a 50% chance to gain an additional € 50.000? This seems implausible, hence most economists argue that the level of risk aversion necessary to generate the observed equity premium is too large to be believable — the equity premium puzzle.

The puzzle is not that the return on stocks exceeds that of bonds. In this sense the puzzle is not of a qualitative nature, but quantitative instead. The excess return is too large to be explained by variation in actual consumption growth. It is arguably the single most prolific asset pricing puzzle and it has received a lot of attention from the academic community. The next Section discusses this existing literature by briefly reviewing some of the proposed resolutions.

## 1.2 Existing literature

As explained earlier, the equity premium puzzle, first documented by Mehra and Prescott (1985), holds that estimates of the coefficient of relative risk aversion ($\gamma$) from consumption data are excessively high. Another way to look at it is the following. Mehra and Prescott (1985) use U.S. data over a time span of ninety years where the return on the stock market is taken to be the return on the S&P 500 index and the risk-free rate is the 3-month U.S. Treasury Bill rate. Their calculations show an average equity premium over the period of 1889 - 1978 of a staggering 6.18%. While citing other articles, Mehra and Prescott (1985) argue that estimates of $\gamma$ exceeding 10 are implausible. Plugging in this value in the consumption based Capital Asset Pricing Model (henceforth c-CAPM) results in a theoretical equity premium of 0.35% which is, at least in economic terms, quite different from the empirical point estimate.

An alternative explanation is that consumption growth is too smooth. The expected excess return can be modeled as the quantity of risk times the price of risk. In the c-CAPM is the former is measured as the covariance of (per capita) consumption growth with the excess return while the coefficient of relative risk aversion denotes the price of risk. As mentioned above, the equity premium is high. Consumption is very smooth, as a result its covariance with the excess return is low. The only way for this equation to hold is through a high level of risk aversion.

The following will discuss a number of important contributions to the literature concerning this puzzle but is by no means exhaustive, as aptly put by Cochrane (2005, p. 22): 'The ink spilled on the equity premium would sink the Titanic.' The interested reader is referred to Kocherlakota (1996), Cochrane (1997), Campbell (2000), Campbell (2003) and Mehra and Prescott (2003) for excellent literature reviews. The purpose of this section is to provide the reader a flavor of the progress made so far. The voluminous literature concerning this topic can be separated into a number of different strands, some of which are more prominent than others. Moreover, the more recent literature on the equity premium puzzle usually combines several of these strands, making the ensuing distinction less clear.

Brown et al. (1995) argue that survival bias might resolve the puzzle. The focal point of this theory is that ex-post returns are used to compute the premium while an ex-ante expected premium is more appropriate. Specifically, although the U.S. stock market performed well over the last decades, this would have been hard to predict beforehand. The underlying reason is that it is a priori difficult to assess the survival probability of exchanges. However, for this argument to work the impact of financial crises (and other events) should be (much) smaller on bond markets compared to stock exchanges. This is not what real-life observations suggest. For example, it is well-known that in tough times governments all around the world have tried to inflate away part of their nominal government debt.

In a related manner, is the puzzle solely an U.S. phenomenon? This question is answered affirmatively by Jorion and Goetzmann (1999) and Aggarwal and Goodell (2008). The former paper collects data from 39 countries going as far back as the 1920s and concludes that the high equity premium found in the U.S. is the exception rather than the rule. Aggarwal and Goodell (2008) show that the equity premiums in 16 emerging markets are generally small. As a result they argue that there is no equity premium puzzle in these countries. However, given that their study spans a period of only 8 years, this conclusion is questionable. There is a larger body of research showing the existence of the puzzle in countries besides the U.S. For example, Campbell (2003) presents evidence that it is a feature of a large number of developed countries such as Canada, Japan, the UK and those located in Western Europe. Moreover, it also seems to be a feature of emerging markets, see e.g. Salomons and Grootveld (2003), Shackman (2006) and Erbaş and Mirakhor (2007).

Other papers introduce the concept of disaster risk as a possible remedy to the puzzle. Rietz (1988) adds a third state to the two-state world of Mehra and Prescott (1985) that represents a possible, though highly unlikely, crash state. However, in order to match the empirical premium, he needs to assume that a possible crash evaporates at least half of the output in one year. To put this into perspective, this is equivalent to the fall in output during the first three years of the Great Depression. Even in today's times, such an event seems highly improbable. In addition, notice that for this explanation to have a bite, such a crash must affect shares much more than bonds for if it influences both asset classes equally it would not have an effect on the equity premium. Finally, given the cross-country evidence substantiating this puzzle, such a catastrophe should hit all major asset markets simultaneously.

Another line of research adapts the preferences of investors. The standard model uses the power utility function which is very convenient but suffers from one major flaw: the elasticity of substitution is restricted to be the inverse of the relative risk aversion coefficient. The former governs the aversion to intertemporal variation (i.e. growth), while the latter indicates an individual's aversion to risk (i.e. different states of nature). In other words, this utility function ties risk to time preferences. If an agent is very risk averse, she will also dislike consumption growth. As a fundamental economic reason for this restriction is lacking, the suitability of this type of preferences is questioned. This strand can be further separated into three broad categories.

The first group is based on the work of Epstein and Zin (1989) and Epstein and Zin (1991). They introduce a generalization of the standard preferences which has one important advantage. Their preferences class decouples the elasticity of substitution from risk aversion such that both parameters can be estimated/calibrated independently. Its main complication is that it depends on future utility which is unobservable. Epstein and Zin (1991) rewrite the utility function in such a way that it depends on the return to the representative agent's entire portfolio of assets (hence including human capital, housing etc.). They use the value-weighted return to the NYSE as a proxy, but this of course understates the true level of diversification. More detrimental, it may overstate the covariability of consumption growth and the excess return. Hence although this alternative preference class leads to lower levels of risk aversion, there are serious doubts whether it offers a true resolution or that smaller risk aversion coefficients are merely the result of a favorable empirical implementation.

Second, as some economists became more and more critical towards the assumption of rational agents and introduced concepts from psychology into the debate, there are now also a number of papers that deal with the equity premium puzzle from a behavioral point of view. Benartzi and Thaler (1995) is probably the best known paper taking this perspective, advocating an explanation they call myopic loss aversion; also see Barberis et al. (2001) and Rabin and Thaler (2001). This concept is based on the concepts of loss aversion (the tendency to be more sensitive to losses than to equal-sized gains) and mental accounting (the myopic heuristics people use to organize and evaluate their financial positions introduced by Kahneman and Tversky (1979) in their prospect theory). The latter reflects the notion that investors pay too much attention to the substantial short-term risks associated with equity. The results in Benartzi and Thaler (1995) reveal that the observed equity premium is consistent with this theory if investors evaluate their portfolio annually. Although individual investors might not be aware of these biases, institutional investors are without a doubt mindful of them. Given that these agents have a dominant position in trading stocks, it seems unlikely that this line of research provides a true resolution.

Finally, besides the generalization and behavioral modification to the utility function, others have argued that introducing a habit term into this function constitutes a promising resolution strategy. The basic idea is that consumers become accustomed to certain levels of consumption such that they (highly) dislike drops in consumption, much more so than in the standard model. To see why this has the potential to solve the puzzle notice that habit-forming consumers dislike variations in habit-adjusted consumption rather than variations in consumption itself. Moreover, a given percentage change in consumption produces a much larger percentage change in habit-adjusted consumption. Hence small fluctuations in consumption growth generate larger variations in habit-adjusted consumption growth. As a result, excess returns can be explained using more moderate risk aversion coefficients.

Research in this field introduces either an internal habit or an external habit. The former is based on an individual's own past consumption and is studied by, among others, Sundaresan (1989) and Constantinides (1990). In such a setup, an individual requires a high level of consumption just to survive. In other words, she will pay a lot to avoid small consumption gambles. External habit formation, known as 'keeping up with the Joneses' postulates that an individual's utility is a function not just of her own consumption but also of societal's level of consumption. If someone sees a neighbor with a nice new car, she is more inclined to follow suit. This resolution is taken up by Abel (1990), Gali (1994), Abel (1999) and Campbell and Cochrane (1999). The intuition is that although an investor is not that averse to individual consumption risk, she is not inclined to invest in stocks because she is highly averse to per capita consumption risk.

All in all, neither generalizing the standard preference class nor insights from behavioral finance seem capable of offering a resolution. Habit formation (either internal or external) can only resolve the puzzle if individuals are either very averse to their own consumption risk or to per capita consumption risk. However, there is no consensus within the academic profession that either statement is a valid description of an individual's true behavior.

It does not appear easy for individuals to directly insure themselves against all possible fluctuations in their consumption streams. For example, it is difficult to directly insure oneself against fluctuations in labor income. Intuitively, in the absence of complete markets, individual consumption growth will feature risk not present in per capita consumption growth and so it will be more variable. Therefore it might be that individual consumption growth will covary enough with stock returns (remember that in the c-CAPM this covariance is the quantity of risk) to explain the equity premium. Notable contributions are Telmer (1993), Lucas (1994) and Heaton and Lucas (1995, 1996). These papers examine the quantitative predictions of dynamic incomplete market models, calibrated using individual income data, for the equity premium. They find that dynamic self-insurance allows individuals to closely approximate the allocations in a complete markets environment. That is, even though markets might be incomplete individuals will be able to trade away any idiosyncrasies.

Two critical assumptions underlying this numerical work is that (i) income shocks have an autocorrelation less than one and (ii) asset trading must be costless. Regarding the former, Constantinides and Duffie (1996) argue that if labor income shocks are instead permanent (autocorrelation of unity), then it is possible for incomplete market models to explain the large equity premium. However, empirical work is in general not favorable to this proposition. For example, Heaton and Lucas (1996) estimate the autocorrelation of undiversifiable income shocks to be 0.5. Second, the above analysis hinges critically on the assumption of costless trade, is this assumption justifiable? As shown by Aiyagari and Gertler (1991) and Heaton and Lucas (1996), the only way for transaction costs (i.e. informational costs, brokerage fees, bid-ask spread and taxes) to explain the puzzle is to assert that there are significant differences in trading costs across stock and bond markets. There is, however, little evidence to support this proposition. The same goes for borrowing constraints. Heaton and Lucas (1995, 1996) argue that an individual who is constrained in the stock market must in general also be constrained in the bond market and vice versa. As a result, borrowing constraints cannot explain the large equity premium.

Others, e.g. Cecchetti and Mark (1990), Kandel and Stambaugh (1991), Cecchetti et al. (1993) and Burnside (1994), have simply argued that risk aversion is higher than anticipated. Mehra and Prescott (1985), citing others, argue that values of risk aversion exceeding 10 are implausible. Kandel and Stambaugh (1991) show that values of $\gamma$ as high as 30 still imply feasible behavior if agents are faced with a potential wealth loss of just one percent. Despite this evidence, most economists still believe that values in excess of 10 seem to be inconsistent with real-life behavior by individuals.

Another assumption underlying the model is that individuals are sufficiently homogeneous such that their behavior can be described by a representative agent. As shown by Constantinides (1982), even if individuals are heterogeneous in preferences and levels of wealth, it is still possible to construct a representative agent. They key is that asset markets are complete and frictionless so that individuals can trade away any idiosyncratic risk in consumption. In other words, this allows them to become marginally homogeneous.

Despite this result, the validity of a representative agent is challenged. Some claim that only a subset of investors are involved in the stock market and it should be their consumption growth that features in the first-order conditions. It is therefore also known under the heading of limited stock market participation. As these individuals actively trade, the link between their consumption growth and stock returns should be tighter. Thus the covariance between their consumption growth and the excess return should be higher. However, as shown by Mankiw and Zeldes (1991), although this is indeed the case it is not sufficient to explain the puzzle. Subsequent research used household-level data to test this premise; prominent examples include Attanasio et al. (2002), Brav et al. (2002) and Vissing-Jørgenson (2002). All three papers show that this covariance is higher for stockholders than for nonstockholders. Hence this modification clearly helps in resolving the puzzle. These papers tend not to be criticized on theoretical grounds, but rather on their empirical implementation with some economists questioning the reliability and validity of these micro data.

As an example of combining two possible remedies, Constantinides et al. (2002) attempt to resolve the puzzle by combining borrowing constraints with consumer heterogeneity in an overlapping-generations model. During the first period the individual receives a relatively low endowment income and is prohibited from borrowing. This is justifiable given that banks will be reluctant to lend money against future wage income or human capital. In the second period, the individual is employed and receives wage income subject to large uncertainty. During the final period the individual retires and consumes the assets accumulated in the second period. As the correlation of equity income with consumption changes over the life cycle, so does the attractiveness of equity as an asset. Due to borrowing constraints young people, who in an economy with complete markets and without frictions hold equity, are effectively shut out of this market. Thus equity is exclusively priced by middle-aged investors, whose consumption is much more linked to stock returns. Consumption is high when equity income is high, and equity is no longer a hedge against fluctuations in consumption. These individuals must therefore be offered a higher return to hold these assets.

Bansal and Yaron (2004) introduce the concept of long-run risk into the debate. They model both consumption and dividend growth as having a small long-term component and time-varying volatilities where the latter proxies for variation in economic uncertainty. The intuition underlying their results is that individuals require a large equity premium because they are fearful of a reduction in economic growth prospects or rising economic uncertainty which puts downward pressure on asset prices. This paper has been criticized by Hansen et al. (2008), who claim that long-run properties of consumption growth are hard to measure. Moreover, the empirical evidence substantiating these properties is rather weak. Cochrane (2005) argues that movements in long-run consumption growth forecasts are observationally equivalent to unobservable shifts in marginal utility. In sum, the conclusions of Bansal and Yaron (2004) are at a minimum questionable. Nevertheless, it is an important contribution and it has sparked subsequent work along these lines.

Despite this plethora of proposed resolutions, the consensus within the academic community is that the puzzle remains unsolved. Finding a resolution has become even more difficult as any proposed solution also has to provide an answer to the related risk-free rate puzzle of Weil (1989). In addition, it should also be consistent with a number of other asset pricing phenomena such as the procyclical variation in stock prices, the long-horizon predictability of excess stock returns and the countercyclical variation of stock market volatility.

## 1.3 Relevance to Society

The example in Section 1.1 underscores the remarkable wealth building potential of equity compared to bonds. It should come as no surprise therefore, that the equity premium is of central importance in portfolio allocation decisions (i.e. which fraction of funds to invest in the stock market) of pension funds and endowments and in estimating the appropriate cost of capital. More indirectly it also paramount in estimating the effects of business cycle fluctuations and hence has some important policy and welfare implications.

Up until the mid-1980s Dutch pension funds invested mainly in bonds and mortgages. During this decade they diversified into other asset categories, most prominently stock, but also commodities and private equity. Nowadays, equity represents about 50% of pension fund reserves. One of the reasons for this diversification is that the return on bonds was not sufficient to provide employees with a decent and affordable pension. An additional annual return of one percentage points adds about 30% to an employee's pension worth. However, in case the risk attitude of the representative agent is indeed as high as some estimates make believe, pension funds should be very cautious in investing in the stock market.

As examples of the role played by the equity premium in pensions, consider the following. First, Van Ewijk et al. (2009) discuss the welfare gain of intergenerational risk sharing. Their model shows that assuming an equity premium of 2% results in a welfare gain of 4%, while the welfare gain is almost quadrupled if the equity premium is raised to 4%. Second, Bovenberg and van Ewijk (2011) discuss the relation between the degree of risk taking and the required reserves to cover a pension funds' future liabilities. The most appropriate risk premium is deduced from the interest paid on wage-indexed bonds or bonds linked to GDP. However, these assets are not traded. Therefore the Netherlands Bureau of Economic Policy Analysis uses a discount rate based on a long-term equity premium of 4%.

The equity premium also features prominently in the current debate in the U.S. concerning the future of Social Security. When Social Security taxes exceed the program's expenditures, the excess is invested in non-marketable U.S. government bonds, it is prohibited from being invested in any other assets. Critics of this law have proposed to privatize funding, thereby allowing excess funds to be invested in the stock market as well.

Welch (2000) argues, on the basis of several surveys conducted among small and institutional (professional) investors that the latter group often tends to be more conservative with respect to their estimates of the equity premium. In particular, pension fund executives are reported to estimate the expected risk premium at about 3%, which is at the low end of the spectrum of estimates reported in the academic literature. Hence, while the consensus among the pension funds is that its existence cannot be neglected, they do not seem to believe that the difference in return between stocks and bonds is very high. These conflicting views cry for a reconciliation in the interest of public wealth.

Turning to welfare, the discussion in Lucas (1987) serves as the starting point. This paper argues that the welfare costs of fluctuations in consumption are negligible at about 0.0005% of consumption. However, this result is of course critically dependent on the method used for the calculations. The underlying model assumes that markets are complete — this implies that individuals must be able to diversify any idiosyncratic risk in consumption — and preferences are modeled by the power utility function. Given the wealth of evidence against the ability of this model to match key asset pricing phenomena, subsequent research has questioned the conclusion of Lucas (1987). It is again not feasible to do justice to all research in this field, so the following will briefly touch on a small number of contributions; see Lucas (2003) for an excellent survey of this literature.

Atkeson and Phelan (1994) investigate the first assumption and consider the welfare and policy implications when markets are incomplete. The motivation is that the marginal utility of consumption might exhibit much more variation when individuals are confronted with substantial idiosyncratic income risk. Given that agents are not able to trade away these risks themselves, one might wonder whether government policy can be helpful in smoothing consumption streams and hence increasing welfare. The critical point in their analysis is the extent to which countercyclical policy reduces directly the income risk faced by each individual. Based on a model incorporating the wage and unemployment risk faced by individuals over the business cycle, the main effect of countercyclical policy is merely to reduce (or even completely remove) the correlation across individuals in the unemployment risk they face. Hence the only effect on welfare is via its impact on asset prices and so the potential welfare gains from countercyclical policy are, in line with Lucas (1987), negligible.

Krusell et al. (2009) show that accounting for consumer heterogeneity is critically important. Their baseline specification which features idiosyncratic risk and unemployment yields a gain of about 0.1%. However, when they distinguish between short- and long-term unemployment, the gain is approximately 10 times larger. The underlying reasons are simple: if an individual is hit by an unemployment spell that lasts for an extended period of time, her present and future income is diminished significantly. Moreover, the risk of becoming unemployed is highly procyclical. An important result of their analysis is that although the gain is on average equal to 1%, there are substantial differences between wealth groups. For both the poorest and richest individuals the gain of removing cycles is positive and equal to 4% and 2%, respectively. This implies that about 65% of the population, the 'middle class' lose out, their welfare actually decreases.

Others adapted the preferences class to remove the tight link between risk aversion and intertemporal substitution under constant relative risk aversion preferences. For example, Tallarini (2000) uses preferences of the Epstein-Zin type and estimates the welfare cost of aggregate consumption risk to be about 10% of consumption. In sharp contrast, Otrok (2001), while using a different preference class, obtains estimates in the range of 0.002% to 0.006%, which is much closer to the results of Lucas (1987). The concept of habit formation has also been used in this literature and for example Van Wincoop (1994) finds that eliminating consumption fluctuations provides gains ranging from 1 to 25 percent.

Sidestepping the issue of choosing the most appropriate preference class, Alvarez and Jermann (2004) take a nonparametric approach and use asset prices to measure the cost of consumption fluctuations directly. However, their approach does rely on the assumption of complete markets. They emphasize that the frequency at which variation in consumption is removed is paramount. In particular, while removing all variation results in a gain that is in the neighborhood of 30%, focusing on business-cycle frequencies (cycles of at most 8 years) shrinks the gain to a number less than

1%.

If anything, the discussion above reveals that there is no consensus concerning the effect of the equity premium puzzle on welfare and therefore also policy. Estimates vary widely, depending crucially on assumptions concerning the appropriate preference class and idiosyncratic risks (consumer heterogeneity). Until the academic community comes up with an asset pricing model that can account for the equity premium puzzle (and other asset pricing phenomena), it does not seem likely that we can give a meaningful answer to this question.

## 1.4 Outline

All four chapters are self-contained and can be read independently. Although they are all concerned with the equity premium puzzle, the focus of each chapter is different. In other words, every chapter deals with this puzzle from a different point of view. The second chapter develops a method to quantify the uncertainty surrounding estimates of risk aversion and then uses these results to construct a pooled estimate based on data from a number of developed countries. Chapter 3 constitutes the core of this thesis. It introduces a heterogeneous agent model populated by stockholders and nonstockholders. The main innovation is the macroeconomic perspective, using data from national accounts. The fourth chapter tweaks the canonical c-CAPM by focusing on an element that has so far stayed below the radar: the risk-free rate While the first three chapters deal directly with the equity premium puzzle, Chapter 5 is the odd one out. Its central theme is the variability of risk aversion over time, dealing with the equity premium puzzle on the side. In more detail, the outline is as follows.

Chapter 2 is based on Pozzi et al. (2010). The point of this chapter is not to come up with yet another theoretical explanation for the high coefficient of relative risk aversion. Rather, it answers the question to which extent the equity premium puzzle is a statistical phenomenon due to lack of reliability of macroeconomic data. In his authoritative review Campbell (2003) shows that the estimates vary widely across a number of countries, but he has no statistical analysis of this uncertainty. We replicate Campbell's results, extend his data set and use the jackknife resampling method to gauge the uncertainty with which risk aversion is estimated in the c-CAPM. Furthermore, by pooling the country data one obtains a much tighter confidence band on the world coefficient of relative risk aversion than an analysis based on an individual country.

Chapter 3 is based on De Vries and Zenhorst (2012). Limited stock market participation has been proposed as a resolution to the equity premium puzzle. The idea is that stockholders' consumption growth is more tightly linked to excess returns, thereby increasing the covariance and lowering the relative risk aversion coefficient. Existing research in this strand of the literature uses household-level data. In contrast, this chapter use macroeconomic data to distinguish between different sources of income and associated spending. This gives a longer time series than is available for the panel studies and enables an international comparison. The macro approach is not seen as a substitute for, but as complementary to the micro-based approach. The idea is as follows. The stockholders consume only from their capital income, while the non-stockholders consume mainly from their wages. The capital income and wage income are constructed from the categories of net national disposable income. Subsequently aggregate consumption is assigned to the two types of factor income in proportion to their income. These latter series are then used to compute the coefficient of relative risk aversion for both groups. As expected, estimates concerning stockholders are lower than those based on a representative agent. Nevertheless, the level of risk aversion of stockholders exceeds the range of plausible values.

Chapter 4 is based on Zenhorst (2012a). One of the necessary series to estimate the coefficient of relative risk aversion is the risk-free rate. While other parts of the model, such as the functional form of the utility function, have received much attention, the same cannot be said about the risk-free rate. Most studies using U.S. data choose the 3-month T-bill rate to represent the risk-free rate. However, it is questionable whether such an asset is indeed the best proxy of a risk-free asset for the typical consumer. It seems that some kind of return on a savings account (or time deposit) is much closer to the “asset reality” a household is faced with. As the interest rate paid on deposits is in general lower than the rate governments pay on their short-term debt, the excess return when using a deposit rate as the risk-free rate is higher. This implies that, ceteris paribus, the coefficient of relative risk aversion will increase and hence the equity premium puzzle will be even larger.

Chapter 5 is based on Zenhorst (2012b). Most papers concerning this puzzle treat risk aversion as a structural parameter whose value is constant over time. Although convenient, this assumption is hard to justify. Insights from the psychology of decision-making reveal that people are more sensitive to reductions in their levels of well-being than to increases. In other words, a loss of $ 100 hurts more than the joy provided by winning an equal amount. Hence real life observations clearly suggest that there is variation in risk aversion. The central aim of this chapter is to show this time variation in the coefficient of relative risk aversion. It formulates an asset pricing model in which risk aversion varies in response to news in consumption growth, inflation and unemployment growth. Hence besides the standard consumption growth we add two macroeconomic variables that should influence risk aversion. The setup also allows for time-varying volatilities of the innovations. The main yardstick by which to judge the results is to check whether movements in the coefficient of relative risk aversion are in line with official U.S. recession and expansion dates and other important events that took place over the sample period. Risk aversion is estimated to be highly persistent and, as expected, in the run-up to recessions and during these periods it is generally rising.

# Chapter 2

# World Equity Premium based Risk Aversion Estimates

*Joint work with Lorenzo Pozzi en Casper de Vries*

## 2.1 Introduction

The equity premium puzzle is the empirical observation due to Mehra and Prescott (1985) that the coefficient of relative risk aversion estimated from the macro consumption based CAPM with power utility is excessively high on U.S. data. This observation has fascinated many in the economics profession over the past two decades. Progress has been made towards understanding the puzzle by looking into the consequences of limited participation in the stock market (Mankiw and Zeldes, 1991; Attanasio et al., 2002; Vissing-Jørgenson, 2002), habit formation of investors (Campbell and Cochrane, 1999) and decoupling risk aversion from the elasticity of intertemporal substitution (Epstein and Zin, 1989). But the final verdict is not out.

The point of this paper is not to come up with yet another theoretical explanation for the high coefficient of relative risk aversion. Rather, we ask to which extent the equity premium puzzle is a statistical phenomenon due to lack of macro data reliability.

In his authoritative review Campbell (1999) already showed that risk aversion estimates vary widely across a number of countries, but gave no statistical analysis of this uncertainty. We replicate Campbell's results, extend his data set and use the jackknife resampling method to gauge the uncertainty. Furthermore, we show that by pooling the country data one obtains a much tighter confidence band on the world coefficient of relative risk aversion than an analysis based on an individual country.

The remainder of this chapter is organized as follows. In the next section the model is outlined. Our methodology is introduced in the third section. Next, the data used in our empirical investigation are described. Section 2.5 discusses the results and the final section concludes.

## 2.2 Model

In the canonical framework where a representative investor has a time-separable power utility function over aggregate consumption, the Euler equation reads

$$1 = E_t[R_{t+1}M_{t+1}]. \tag{2.1}$$

Here $R$ is the gross return on an asset and $M$ denotes the stochastic discount factor. The discount factor is $M_{t+1} = \delta(C_{t+1}/C_t)^{-\gamma}$, where $\delta$ is the subjective rate of time preference, $C$ is aggregate consumption and $\gamma$ is the coefficient of relative risk aversion. Suppose that asset returns and aggregate consumption are conditionally lognormally distributed with constant variance. Take logs of equation (2.1) and rewrite to get

$$\mathrm{E}_t r_{t+1} = -\log\delta + \gamma \mathrm{E}_t \Delta c_{t+1} - \frac{\sigma_r^2 + (\gamma\sigma_c)^2 - 2\gamma\sigma_{(r,c)}}{2}.$$

The small case letters denote logarithms; $\sigma_r^2$ and $\sigma_c^2$ are, respectively, the unconditional variance of the log return innovations and consumption innovations and $\sigma_{(r,c)}$ is their unconditional covariance. For a riskless asset $rf$, return innovations and the covariance with consumption are both zero, implying that the log risk premium can be conveniently expressed as

$$\mathrm{E}_t[r_{t+1} - rf_{t+1}] + \frac{\sigma^2_{exr}}{2} = \gamma\sigma_{(exr,c)}. \tag{2.2}$$

In words, equation (2.2) says that the expected excess return ($exr$) of equity are equal to the amount of risk aversion times the covariance between consumption growth and excess returns minus a correction factor (i.e. a Jensen inequality term arising as we are using expectations of log returns). The expression permits a calibration of the risk aversion parameter $\gamma$ from estimates of the moments in equation (2.2). In most cases U.S. data on these moments are used. As the U.S. has historically been blessed with high equity returns and rather smooth consumption growth, typical $\gamma$ estimates are high.

Using moment estimates given in equation (2.2) to back out the parameter of relative risk aversion gives a point estimate, but not a confidence interval. For this reason confidence bands are mostly not reported.[1] Given that estimates in the literature vary quite dramatically it is of interest to provide for a confidence interval. The next section uses the jackknife resampling scheme to provide such confidence bands.

## 2.3 Methodology

In order to gauge the uncertainty with which risk aversion is estimated, we use the block-jackknife procedure, see e.g. Shao and Tu (1995), as it is easy to implement and because it can deal with the serial correlation that is present in the consumption series. Let $n$ be the size of the sample, let $m$ denote the number of omitted observations in a resample and let $N$ denote the number of resamples. Note that the total number of resamples is $N = n - m + 1$. To estimate the variance of $\hat{\gamma}_n$, the estimate of $\gamma$ based

[1]Cecchetti et al. (1993) use GMM to construct standard errors of estimated parameters. Their results are difficult to compare with our outcomes, since they employ a Markov switching model.

on all observations, one deletes $m$ subsequent observations at a time and denotes the new estimate of $\gamma$ by $\hat{\gamma}^{(i)}$. The $i$-th pseudovalue of $\hat{\gamma}_n$, denoted by $\tilde{\gamma}^{(i)}$, is defined as $\left[n\hat{\gamma}_n - (n-m)\hat{\gamma}^{(i)}\right]/m$. The resulting vector of pseudovalues across the resamples is used to estimate the variance of $\hat{\gamma}_n$, i.e.

$$S^2_{\hat{\gamma}_n} = \frac{m}{nN} \sum_{i=1}^{N} \left( \tilde{\gamma}^{(i)} - \frac{1}{N} \sum_{i=1}^{N} \tilde{\gamma}^{(i)} \right)^2 . \tag{2.3}$$

These estimates can then be used to form a country-specific confidence interval for $\hat{\gamma}_n$. More specifically, the Quenouille-Tukey mean of the pseudovalues, see Shao and Tu (1995, p. 6), is the bias-corrected version of the estimate of $\gamma_n$ and the required critical value is taken from a $t$-distribution, with $N-1$ degrees of freedom (Miller, 1974).

Our second contribution is to use the cross-sectional dimension of our data set to get a confidence band for the pooled estimator, to which we refer as the world coefficient of relative risk aversion ($\gamma_w$). It is highly likely that the information contained in the estimates of $\gamma_n$ differs between countries, so some kind of weighted average seems to be a natural choice. As the block-jackknife procedure gives us the country-specific sample variances, we can use the optimal weights from Graybill and Deal (1959), defined as

$$w_j = \frac{1/S^2_{\hat{\gamma}_{n,j}}}{\sum_{j=1}^{k} 1/S^2_{\hat{\gamma}_{n,j}}}, \tag{2.4}$$

where $k$ denotes the number of countries. Confidence intervals are formed by weighting the country-specific averages of the pseudovalues to get $\hat{\gamma}_w$. It is straightforward to show that the variance of this estimate is given by

$$S^2_{\hat{\gamma}_w} = \frac{1}{\sum_{j=1}^{k} 1/S^2_{\hat{\gamma}_{n,j}}}. \tag{2.5}$$

The critical values are based on a $t$-distribution with $k-1$ degrees of freedom.

Besides the optimal weighting scheme, we also employ two weighting schemes based on GDP and stock market capitalization. The variance of the world coefficient of risk aversion is then given by

$$S^2_{\hat{\gamma}_w} = \sum_{j=1}^{k} (w_j S_{\hat{\gamma}_{n,j}})^2, \tag{2.6}$$

where $w_j$ is now defined as the GDP (market capitalization) of country $j$ divided by the sum of GDP (market capitalization) of all countries.

## 2.4 Data

The methodology described in the previous section is applied to data for OECD countries. Of the 30 countries, 9 had to be dropped for the following reasons. Iceland, Luxembourg and the Slovak Republic are excluded since we do not have three-month interest rates for these countries. Too few observations has led to the exclusion of Greece, Hungary, Ireland, Mexico, Portugal and Turkey. For all 21 remaining countries, data are taken from three different sources. First, private final consumption expenditure (Quarterly National Accounts, in constant prices)[2] and CPI (Main Economic Indicators, all items) are from the OECD. Second, equity returns (in local currency units) are from MSCI. Finally, the risk-free rate and population figures are from the International Financial Statistics provided by the IMF. The sample period is country-specific, as can be seen in Table 2.1; the panel is thus unbalanced.[3]

For the economic based weighting schemes we need two additional variables. The first is GDP (Annual National Accounts, in U.S. dollars, current prices, current exchange rates; OECD) and the second is market capitalization in U.S. dollars (World Federation of Exchanges).

[2] Note that this implies that our results are independent of the measure of inflation.
[3] Appendix 2.A.1 contains some specific computational details concerning the data.

## 2.5 Results

Estimates of $\gamma_n$ based on equation (2.2) are shown in Table 2.1. The number of omitted observations in each block, $m$, is set to 4 so that we leave out one year of observations at a time. Four of the 21 coefficients of relative risk aversion are negative, implying that agents are risk loving. The remaining estimates are positive and rather high, with the exception of Korea and Poland. Yet the results are in line with those reported by Campbell (1999). In the Appendix we replicate the results of Campbell, but add our estimates for the pseudovalues and the confidence bands.[4]

The averages of the pseudovalues, i.e. $\sum_{i=1}^{N} \tilde{\gamma}_i$, are reported in the next column. Note that these differ considerably from the overall estimates $\hat{\gamma}_n$. Given the small value of $m$, this is indicative for the uncertainty in the estimates of $\gamma$. More than half of these values are negative, perhaps implying that most estimates are severely biased. These average pseudovalues are the basis of the symmetric 95% confidence interval. From this, it is evident that there is a lot of uncertainty in the estimate of the coefficient of relative risk aversion. Only the confidence band for Korea is rather tight, the others are fairly wide. The averages of the pseudovalues may look fluke, but if one considers the confidence interval, or realizes that the confidence band for Campbell (1999) also comprise a very wide range, the point estimates of Table 2.1 look more in line with the generally smaller point estimates from the Campbell subset (see Appendix 2.A.2).

The confidence intervals also show that the null hypothesis of a coefficient of relative risk aversion between one and ten, a range most economists (e.g. Mehra and Prescott, 1985) consider to be a reasonable guess, is never rejected on the basis of the individual country jackknifed confidence bands.

Some preliminary results from pooling can already be deduced from Table 2.1. In particular, consider the means and standard errors of the third and fourth columns. The mean of $\hat{\gamma}_n$ is 97, with a standard error of 50; implying that a confidence interval based

[4]Note that all estimates of $\gamma_n$ are identical, except the one for the U.S. The reason is that the riskfree rate we employ differs slightly from the one used by Campbell.

**Table 2.1: Country-specific results**
Notes: The estimate of the coefficient of relative risk aversion is denoted by $\hat{\gamma}_n$, $\overline{PS}$ is the mean of the pseudovalues, defined in Section 2.3. The confidence interval is based on $\overline{PS}$, with standard error equal to the square root of the variance estimated by equation (2.3) and critical value from a $t$-distribution with $N-1$ degrees of freedom.

| Country | Sample period | $\hat{\gamma}_n$ | $\overline{PS}$ | 95% CI | |
|---|---|---|---|---|---|
| Australia | 1970Q1 - 2003Q3 | 128.14 | -649.74 | -6436.59 | 5137.11 |
| Austria | 1988Q2 - 2003Q3 | 91.79 | 105.83 | -164.52 | 376.18 |
| Belgium | 1995Q2 - 2003Q3 | 194.01 | -890.14 | -3253.38 | 1473.11 |
| Canada | 1970Q1 - 2003Q3 | 45.28 | 31.03 | -63.63 | 125.68 |
| Czech Republic | 1996Q2 - 2003Q3 | -192.06 | -1186.11 | -2549.97 | 177.75 |
| Denmark | 1990Q2 - 2003Q3 | 97.10 | -1148.47 | -3548.99 | 1252.06 |
| Finland | 1990Q2 - 2003Q3 | 205.86 | -0.03 | -824.94 | 824.88 |
| France | 1978Q2 - 2003Q3 | 165.19 | 126.98 | -99.48 | 353.44 |
| Germany | 1975Q4 - 2003Q3 | 165.70 | 408.70 | -1242.74 | 2060.14 |
| Italy | 1981Q2 - 2003Q3 | 109.93 | 57.05 | -831.56 | 945.67 |
| Japan | 1994Q2 - 2003Q3 | -36.89 | -140.97 | -576.13 | 294.19 |
| Korea | 1988Q1 - 2003Q3 | 4.37 | -4.68 | -43.45 | 34.09 |
| Netherlands | 1988Q2 - 2003Q3 | 230.63 | -142.00 | -1208.82 | 924.83 |
| New Zealand | 1988Q1 - 2003Q3 | -108.28 | -6922.59 | -28175.74 | 14330.56 |
| Norway | 1978Q2 - 2003Q3 | -395.14 | 578.19 | -5001.92 | 6158.29 |
| Poland | 1995Q2 - 2003Q3 | 4.88 | -22.44 | -117.17 | 72.28 |
| Spain | 1995Q2 - 2003Q3 | 155.84 | 98.38 | -182.41 | 379.17 |
| Sweden | 1993Q2 - 2003Q3 | 178.29 | -240.76 | -1160.22 | 678.71 |
| Switzerland | 1980Q2 - 2003Q3 | 852.61 | -987.57 | -55476.58 | 53501.45 |
| United Kingdom | 1970Q1 - 2003Q3 | 69.22 | 24.06 | -139.95 | 188.06 |
| United States | 1970Q1 - 2003Q3 | 67.56 | 47.15 | -74.84 | 169.14 |

on these numbers would be rather wide. When using the bias-corrected estimates, the results are even worse. With a mean of -517 and a standard error of 338, the resulting confidence interval would be even wider. However, note that these results are obtained under equal weighting. Hence the estimate of Switzerland would have a weight equal to that of Korea, although the latter is estimated much more precisely. Now we turn to some results in which some alternative weighting schemes are used.

Table 2.2 presents the results based on pooling of country-specific data, which gives an estimate of the world risk aversion parameter. The top panel presents the outcomes obtained while using the optimal variance weights. The optimally weighted mean of the pseudovalues is just over 4.5 (hence within this plausible range) and the 95% confidence interval is rather tight. Note that it is less wide than its counterpart for the U.S., hence pooling of information is clearly beneficial. Moreover, the resultant confidence interval contains the often hypothesized reasonable values of below 10. Again equality to one is not rejected for the world risk aversion parameter. The other two rows hold the results after, respectively, disregarding Korea and Switzerland. The reason for Korea is that its variance of $\gamma$ is estimated much more precisely than any other country. As a result its weight is much larger. In contrast, the estimated variance of Switzerland is the highest. As shown in the top panel of Table 2.2, although Korea has a rather large weight, our results are quite robust to its inclusion. The point estimate is now outside the a priori plausible range, but the confidence interval is still tight. As expected, excluding Switzerland has only a negligible effect.

Looking at the middle panel clearly shows that Switzerland is a bit of an outsider. As it has a large GDP weight, it considerably affects the point estimate. Omitting Switzerland increases $\hat{\gamma}_w$ and the estimate then lies well within the plausible range. However, due to the different weights, the standard error of this estimate is higher than before. Consequently, the confidence interval is wide and uninformative. Comparing the results with and without Korea shows that its large weight has only a minor influence on the results.

Finally, the results based on market capitalization convey more or less the same message as those based on GDP. Korea's influence is minimal, while Switzerland seems to drive the outcomes to a rather large extent. Now the uncertainty in the world coefficient of risk aversion (not excluding Switzerland) is even higher than in the middle panel, as can be inferred from the very wide confidence interval. In contrast, the result excluding Switzerland is more (yet marginally) informative compared to the middle panel. This can be easily explained from the notion that the weight of Switzerland is larger in the bottom panel.

**Table 2.2: Pooled results**

Notes: The world coefficient of relative risk aversion is denoted by $\hat{\gamma}_w$, the standard error for the confidence interval is the square root of equation (2.5) for the top panel and the square root of equation (2.6) for the middle and bottom panels. The critical value is taken from a $t$-distribution with $k-1$ degrees of freedom. The top panel uses the optimal weights ($OPT$) described by equation (2.4), the middle and bottom panels use economic weights based on, respectively, GDP and market capitalization ($MC$ at the end of 2003. In each panel, the top row holds the outcomes using all countries ($All$), the second row disregards Korea ($KO$) while the third row excludes Switzerland ($SW$) from the analysis.

| Description | $\hat{\gamma}_w$ | 95 % CI | |
|---|---|---|---|
| OPT, All | 4.57 | -27.60 | 36.74 |
| OPT, KO | 20.56 | -32.76 | 73.87 |
| OPT, SW | 4.57 | -27.71 | 36.85 |
| GDP, All | -16.90 | -712.01 | 678.21 |
| GDP, KO | -17.17 | -729.90 | 695.57 |
| GDP, SW | -5.65 | -240.95 | 229.65 |
| MC, All | -32.02 | -1737.96 | 1673.91 |
| MC, KO | -32.36 | -1765.13 | 1700.41 |
| MC, SW | -2.85 | -202.37 | 196.67 |

## 2.6 Conclusion

The equity premium puzzle holds that the coefficient of relative risk aversion estimated from the consumption based CAPM with power utility is excessively high. Moreover, estimates in the literature vary considerably. We employ the jackknife resampling method in order to estimate the uncertainty associated with the coefficient of relative risk aversion. Our results show that the country-specific confidence intervals are fairly wide. We never reject equality to one. However, when the data of countries are pooled and a single, optimally weighted point estimate is constructed, the resulting confidence band is tighter and presents less of a puzzle than the individual country estimates.

# 2.A Appendices

## 2.A.1 Data issues

Some remarks regarding the data are in order. First, the computation of the real excess return; the difference between the real equity return and the real risk-free rate. In order to compute the return on equity, two series are required: the end-of-month price-index (hereafter denoted by $P$, datatype PIL) and the end-of-month total return index ($R$, datatype RIL). The first step is to use the following definition to back out the monthly dividend yield (DY) at time $t$:

$$\frac{R_t}{R_{t-1}} \equiv \frac{P_t}{P_{t-1}}(1 + DY_t).$$

Next monthly dividends at time $t$ are computed by multiplying $P_t$ by the dividend yield at time $t$ and the quarterly return on equity ($Re$) is computed according to

$$Re_t = \left( \frac{P_t + \sum_{i=t-2}^{t} D(i)}{P_{t-3}} \right) - 1,$$

where $D(i)$ is the monthly dividend series, $P_t$ is the end-of-month price and $P_{t-3}$ is the price at the end of the previous quarter (hence $t - 3$). Finally, log equity returns are computed as

$$r_t = \log(1 + Re_t).$$

The second input series is the risk-free rate. As these are annualized and expressed as percentage points in the database, the log quarterly risk-free rates are computed according to

$$rf_t = \log\left(\left(1 + \frac{Rf_t}{100}\right)^{\frac{1}{4}}\right).$$

These two returns are then made real by subtracting time $t$ inflation, denoted by $\pi_t$.

This is simply the first difference of the logarithmic transformed CPI, i.e.

$$\pi_t = \log(CPI_t) - \log(CPI_{t-1}) = \log\left(\frac{CPI_t}{CPI_{t-1}}\right).$$

It is important to note the timing of the variables is this respect. An investor earns $rf_t$ if he bought the asset at the end of the previous quarter and sells it at the end of the current quarter. This is why the real risk-free rate is computed as

$$rrf_t = rf_{t-1} - \pi_t.$$

The real equity return is simply $rr_t = r_t - \pi_t$. Our series of interest, the real excess return is then $rexr_t = rr_t - rrf_t$.

Second, the population series. As these figures are only available at an annual frequency (mid-year estimate), we need to construct a quarterly series with population estimates ourselves. In line with Campbell (1999), we assume constant growth rates between subsequent annual observations. As data refers to mid-year estimates, the annual observations are taken to represent the second quarter of each year.

Finally, for four countries, the riskfree rate has missing values towards the end of the sample period. Fortunately, in all cases appropriate series have been found that allow us to extrapolate the original series based on these substitutes. Below follows a short description of these four cases.

- Australia: Missing values from July 2002 onwards. We use data on the Australian interbank rate, this series has a correlation of 0.996 for the overlapping part with the original series;

- Austria: Missing values from January 1999 onwards. We use data on the 3 month interbank offered rate, this series has a correlation of 0.995 for the overlapping part with the original series;

- Denmark: Missing values from January 1989 onwards. We use data on T-bills taken from the Central Bank of Denmark, this series has a correlation of 1 for the overlapping part with the original series;

- Netherlands: Missing values from January 1999 onwards. We use on the call money rate, this series has a correlation of 1 for the overlapping part with the original series.

### 2.A.2 Replication

**Table 2.3: Replication of Campbell (1999)**
Notes: The estimate of the coefficient of relative risk aversion is denoted by $\hat{\gamma}_n$, $\overline{PS}$ is the mean of the pseudovalues, defined in Section 2.3. The confidence interval is based on $\overline{PS}$, with standard error equal to the square root of the variance estimated by equation (2.3) and critical value from a $t$-distribution with $N-1$ degrees of freedom. The estimates in the third column, with the exception of the U.S., match those reported in Campbell (1999), the final three columns are our contribution. Mnemonics are as follows: Australia (AU), Canada (CN), France (FR), Germany (BD), Italy (IT), Japan (JP), the Netherlands (NL), Sweden (SD), Switzerland (SW), the United Kingdom (UK) and the United States (US).

| Country | Sample period | $\hat{\gamma}_n$ | $\overline{PS}$ | 95 % CI | |
|---|---|---|---|---|---|
| AU | 1970Q1 - 1996Q2 | 45.70 | 7.11 | -162.63 | 187.91 |
| CN | 1970Q1 - 1996Q2 | 56.43 | 8.97 | -135.34 | 185.90 |
| FR | 1973Q2 - 1996Q2 | -310.32 | 14.63 | -2431.38 | 7700.34 |
| BD | 1978Q4 - 1996Q2 | 343.13 | 13.33 | -45863.66 | 80054.16 |
| IT | 1971Q2 - 1995Q2 | 2465.32 | 4.70 | 35243.95 | 80566.08 |
| JP | 1970Q2 - 1996Q2 | 134.12 | 13.44 | -923.57 | 562.81 |
| NL | 1977Q2 - 1996Q1 | 1050.93 | 23.97 | -33070.35 | 69140.34 |
| SD | 1970Q1 - 1994Q4 | 7215.18 | 20.71 | -2035151.38 | 3726567.77 |
| SW | 1982Q2 - 1996Q2 | -207.29 | 26.79 | -19693.37 | 15176.76 |
| UK | 1970Q1 - 1996Q2 | 156.31 | 14.86 | -5425.25 | 2239.24 |
| US | 1970Q1 - 1996Q3 | 150.82 | 37.45 | -253.48 | 408.73 |

# Chapter 3

# Macro Consumption and Equity Premium based Risk Aversion of Labor and Capitalists

*Joint work with Casper de Vries*

## 3.1 Introduction

The equity premium puzzle holds that estimates of the coefficient of relative risk aversion from consumption data are excessively high. Since Mehra and Prescott (1985) christened the puzzle and despite considerable research efforts, it is still with us today. Yet, our understanding of the puzzle has grown considerably. One strand has investigated the effect of non-standard utility functions, notably habit formation, to explain the puzzle. The other main line of research, introduced by Mankiw and Zeldes (1991), has looked into the effect of limited stock market participation. Their paper uses micro panel data showing that the consumption of stockholders is quite distinct from the consumption of nonstockholders. In particular, the covariance between consumption growth and excess returns is larger for stockholders than for nonstockholders, implying that the resulting estimate of the coefficient of relative risk aversion is lower; thereby reducing the size of the puzzle. This micro panel based research line has been followed up by, amongst others, Vissing-Jørgenson (2002).

In contrast with other papers, we use macro data to distinguish between different sources of income and associated spending. This gives a longer time series than is available for the panel studies and enables an international comparison. We view the macro approach not as a substitute for, but as complementary to the micro-based approach. The idea is as follows. The stockholders consume only from their capital income, while the nonstockholders consume mainly from their wages. The capital income and wage income are constructed from the categories of net national disposable income. Subsequently aggregate consumption is assigned to the two types of factor income in proportion to their income. These latter series are then used to compute the coefficient of relative risk aversion for both groups. The crucial maintained assumption in this approach is the assignment of expenditure in proportion with factor income derived from the national accounts.

As the stock market is associated with risk, one expects equity owners to have a lower relative risk aversion coefficient than wage earners. As a result, equity owners' coefficient of relative risk aversion should also be lower than one that is based on a representative agent model. Applying our model to a selection of OECD countries reveals that this is borne out. In particular, the average coefficient of relative risk aversion for stockholders is 62.3. In contrast, the average estimate of 186.9 for a single representative agent is much higher.

The size of our data set enables us to calculate standard errors by means of the jackknife procedure.[1] The individual country estimates have a wide band of uncertainty. By pooling the information from different countries we can reduce this uncertainty and the *world coefficient of relative risk aversion* of stockholders is shown to be within a range of values that most economists deem plausible. An alternative approach, sug-

[1] There are only few other papers that report standard errors, the most notable examples are Cecchetti et al. (1993) and Vissing-Jørgenson (2002).

gested by Campbell (1999), is to fix the correlation between consumption growth and excess returns at unity while computing coefficients of relative risk aversion. In this case almost all the country-specific values for equity owners take on plausible values. These results suggest that the macro based limited stock market participation by itself cannot completely resolve the equity premium puzzle for individual countries. However, when pooling or fixing the correlation at unity is introduced, our approach yields plausible estimates for the coefficients of risk aversion.

The main motivation underlying our macroeconomic approach is as follows. As in any research program, some of the maintained assumptions in the micro panel data based investigations are stronger than ideal. For example, in Mankiw and Zeldes (1991) consumption is approximated by food consumption, which according to Attanasio and Weber (1995), is a dubious proxy for total consumption. Moreover, the panel data generally have a short time span covering the same households (in Mankiw and Zeldes (1991) there are only 13 observations). Finally, the approach suffers from measurement error as the distinction between stockholders and nonstockholders is static. Due to data limitations, someone is defined as a shareholder over the entire sample period if he owns shares in the final period. Likewise, if a person does not have stock in the final period, he is considered to be a non-stockholder in all periods.

Some of the flaws of the earlier research were overcome in later research, including Attanasio et al. (2002), Brav et al. (2002) and Vissing-Jørgenson (2002). But these more recent studies also suffer from a number drawbacks. As discussed by Vissing-Jørgenson (2002), who makes use of the U.S. Consumer Expenditure Survey (CEX), only 70% of households have data for all interviews so attrition is quite substantial. In addition, households are asked for holdings of 'stocks, bonds, mutual funds and other such securities'. Hence households who do not own stocks, but have some money in a money market fund will nevertheless be classified as a stockholder. Third, it has been documented that many households only hold stock or bonds in their pension plan. However, survey results are unable to show whether households in a defined contribution plan report their holdings. Such households are than mistakenly classified as nonstockholders. This is a likely reason as to why the proportion of stockholders in CEX is lower than in other sources. In addition, Attanasio et al. (2004) note that, for many commodities aggregating CEX data does not conform with the NIPA Personal Consumption Expenditure (PCE) data of the U.S. Bureau of Economic Analysis. In particular, for some items the underestimation is substantial. In terms of aggregate consumption, the CEX aggregate is about 35% lower than its PCE counterpart. Even more detrimental, instead of converging, the difference between both aggregates has been increasing during the second part of the 1990s.

Our macro approach should be seen as complementary to the micro based household level data literature. Our approach, being based on national income data, does not suffer from attrition and sidesteps the issue of classifying households as stockholders or nonstockholders. Furthermore, an advantage is that it yields comparable cross country estimates. Nevertheless, it also has its maintained hypothesis, most notably the assignment of expenditure in proportion with the factor income share. We discuss this issue extensively in Section 3.3.

The remainder of this chapter is organized as follows. Section 3.2 concisely introduces the canonical model and a simple heterogeneous agent model. Our methodology of how to construct quarterly capital and labor income series using macroeconomic data is explained in the third section. Here we also discuss some drawbacks of our method and how standard errors for the estimates of the risk aversion coefficient can be obtained using a simple jackknifing approach. Section 3.4 discusses the data. The results of our approach are given in Section 3.5. In particular, we present estimates of the regular and adjusted risk aversion coefficients, their confidence intervals and the pooled estimates. The final section concludes.

## 3.2 Model

The focus of this paper is on constructing consumption series for stockholders and nonstockholders using only macroeconomic data. We stick to the original model except for disaggregating consumption growth into two parts. The model is explained in terms of a representative investor and aggregate consumption, but due to the aggregation theorem of Grossman and Shiller (1982), it applies just as well to a representative stockholder or nonstockholder and their respective consumption streams.

The canonical framework, (Grossman and Shiller, 1981; Hansen and Jagannathan, 1991; Cochrane and Hansen, 1992), assumes the existence of a representative investor who has a time-separable utility function defined over aggregate consumption. Under these assumptions, the Euler equation reads

$$1 = E_t \left[ R_{t+1} \delta \left( C_{t+1}/C_t \right)^{-\gamma} \right]. \tag{3.1}$$

Here $R$ is the gross return on an asset, $\delta$ is the subjective rate of time preference, $C$ is aggregate consumption and $\gamma$ is the coefficient of relative risk aversion. Next, following Hansen and Singleton (1983), the simplifying assumption is made that asset returns and aggregate consumption are jointly conditionally lognormally distributed with constant variance. Taking logs of equation (3.1) and rewriting leads to

$$\mathrm{E}_t r_{t+1} = -\log \delta + \gamma \mathrm{E}_t \Delta c_{t+1} - \frac{\sigma_r^2 + (\gamma \sigma_c)^2 - 2\gamma \sigma_{(r,c)}}{2}. \tag{3.2}$$

Small case letters denote logarithms; $\sigma_r^2$ and $\sigma_c^2$ are, respectively, the unconditional variance of the log returns and consumption innovations and $\sigma_{(r,c)}$ is their unconditional covariance. Consider the implications of equation (3.2) for a riskless real asset. Obviously, the variance of its returns and its covariance with any random variable are zero, implying that the riskless real interest rate obeys

$$rf_{t+1} = -\log \delta + \gamma \mathrm{E}_t \Delta c_{t+1} - \frac{(\gamma \sigma_c)^2}{2}. \tag{3.3}$$

Combining equations (3.2) and (3.3) gives

$$\mathrm{E}_t [r_{t+1} - rf_{t+1}] + \frac{\sigma_{exr}^2}{2} = \gamma \sigma_{(exr,c)}. \tag{3.4}$$

Equation (3.4) holds that expected excess returns of equity (equity premiums) are equal to the amount of risk aversion times the covariance between consumption growth and excess returns minus a correction factor (due to Jensen's inequality). This expression permits a calibration of the risk aversion parameter $\gamma$ from estimates of the moments in equation (3.4). Most often U.S. data is used to estimate these moments. As the U.S. has historically been blessed with high equity returns and rather smooth (aggregate, per capita) consumption growth, typical values of $\gamma$ are high. For example, Campbell (1999) reports an estimate of 150.[2]

By itself, this does not constitute a puzzle. To understand why these high estimates are disturbing, one needs to resort to other evidence regarding appropriate values for the relative risk aversion coefficient. For example, Mehra and Prescott (1985), after citing a number of studies, argue that values of $\gamma$ exceeding $10$ are not very plausible. An intuitive argument for this bound based on choice under uncertainty is given by Mankiw and Zeldes (1991, p. 105).

As mentioned, the canonical model was initially empirically tested using aggregate, per capita consumption growth data. Given the very high values of $\gamma$ derived from the macro data, one response was to question the paradigm of the representative agent. Mankiw and Zeldes (1991) introduced the notion of limited stock market participation by explicitly separating stockholders from nonstockholders. More precisely, consider two types of agents: one relying on labor income, the other on income derived from share holding. Since the covariance is a linear operator, one can write

$$\sigma_{(exr,c)} = \alpha\sigma_{(exr,cL)} + (1-\alpha)\,\sigma_{(exr,cK)},$$

where $\sigma_{(exr,cL)}$ and $\sigma_{(exr,cK)}$ are the covariances of excess returns and consumption growth of labor and capital owners respectively, while $\alpha$ is the average share of labor in aggregate consumption (assuming that possible fluctuations in $\alpha$ are uncorrelated with

[2]However, the equity premium is not a U.S. phenomenon. Campbell (1999) shows the existence of the puzzle in a number of developed countries, where estimates of relative risk aversion vary between -310 and well over 7000. Hence in some other countries the puzzle seems to be even larger.

the excess returns). Denote the left hand side expectation in equation (3.4) by $A$. Then the risk aversion parameters of the two types of agents simply equal $\gamma_L = A/\sigma_{(exr,cL)}$ and $\gamma_K = A/\sigma_{(exr,cK)}$; where $\gamma_L$ and $\gamma_K$ are the coefficients of relative risk aversion. In the plausible case that $\sigma_{(exr,cK)} > \sigma_{(exr,cL)}$, then $\gamma_K < \gamma_L$. It follows that in per capita terms

$$\gamma_K < \frac{A}{\alpha\sigma_{(exr,cK)} + (1-\alpha)\,\sigma_{(exr,cL)}} < \gamma_L. \tag{3.5}$$

The middle term is the aggregate coefficient of risk aversion. It is biased from the perspective of each of the two types of agents. Hence the rationale for trying to compute these coefficients separately.

Unlike stockholders who have an obvious exposure to equity, nonstockholders have no clear link to this class of assets. Therefore, there is no a priori reason to believe that nonstockholders satisfy the first-order condition underlying the Euler equation as they will not likely optimize consumption taking expected future equity returns into account. As a result, estimates of $\gamma_L$ should be interpreted with care.

Although it appears from equation (3.5) that the representative agent's risk aversion estimate is always in between those of stockholders and nonstockholders, this need not always be the case. If the two covariances (correlations) are both positive, the relation in equation (3.5) obviously holds. If however either $\sigma_{(exr,cK)}$ or $\sigma_{(exr,cL)}$ is negative the relationship may break down. If one covariance is negative while the other is positive, the weighted average covariance of the representative agent specification can be close to zero. As a result, the coefficient of relative risk aversion will then take on a more extreme value than either of those associated with the two subgroups.

### 3.2.1 Heterogeneous agent model

We turn to developing the details of a two-period model populated by two types of consumers both of which supply labor in the first period. Agents in the first group use their wage income for consumption and investment in stocks where the latter is then used to consume in the second period (stockholders). Agents in the second group also earn wage income in the second period. Moreover, they have access to bonds (non-stockholders). The aim of the model is to show that agents will self-select depending on their coefficient of relative risk aversion, i.e. to show the existence of a 'separating' equilibrium.

**Consumption and labor**

The utility function of an agent in the first group is of the standard power utility type, i.e.

$$\mathrm{E}(U)_S = \frac{1}{1-\gamma} C_1^{1-\gamma} + \frac{\delta}{1-\gamma} \mathrm{E}\left[C_2^{1-\gamma}\right] - \psi L_S,$$

where again $\gamma$ is the coefficient of relative risk aversion, $\delta$ is the subjective discount factor and $C_1$ and $C_2$ are, respectively, consumption in the first and second period. Moreover, $\psi$ is the preference for leisure. We focus on the empirically relevant case where $\gamma > 1$. Recall that if $\gamma = 0$ agents are risk neutral, while they are risk averse if $\gamma > 0$; $\gamma = 1$ yields the log utility case.

As mentioned just above, the first type of consumer (stockholders) works the first period and invests this money partly to reap the dividends from stock ownership in the second period. Hence for this type of agent, first and second period consumption are given by

$$C_1 = W L_S - S, \tag{3.6}$$

$$C_2 = R S, \tag{3.7}$$

where $W$ is the real wage rate, $L_S$ is stockholders' labor supply, $S$ denotes stocks and $R$ is the stochastic real equity return. The factor labor works both periods and invests partly in bonds in the first period. Its utility function reads

$$\mathrm{E}(U)_B = \frac{1}{1-\gamma} C_1^{1-\gamma} + \frac{\delta}{1-\gamma} \mathrm{E}\left[C_2^{1-\gamma}\right] - \psi\left(1+\delta\right) L_B.$$

Nonstockholders work both periods and bonds are used to smooth consumption intertemporally

$$C_1 = WL_B - B, \tag{3.8}$$

$$C_2 = WL_B + IB, \tag{3.9}$$

where $B$ denotes bonds and $I$ is the deterministic and exogenously determined return on bonds. The disadvantage of working in both periods is represented by the coefficient $\psi$ times the factor $(1 + \delta)$, as working in the second period is discounted by $\delta$.

**Stockholders**

The first step towards solving for the two endogenous variables of stockholders is to substitute for consumption in the utility function using the expressions in equations (3.6) and (3.7)

$$\mathrm{E}(U)_S = \frac{1}{1-\gamma}(WL_S - S)^{1-\gamma} + \frac{\delta}{1-\gamma}\mathrm{E}\left[(RS)^{1-\gamma}\right] - \psi L_S, \tag{3.10}$$

The first-order conditions with respect to $S$ and $L_S$ are given by

$$\mathrm{E}(U)_S^S : (WL_S - S)^{-\gamma} = \delta \mathrm{E}\left[R^{1-\gamma}\right] S^{-\gamma}, \tag{3.11}$$

$$\mathrm{E}(U)_S^L : W(WL_S - S)^{-\gamma} = \psi. \tag{3.12}$$

Combining equations (3.11) and (3.12) and solving for the first endogenous variable, i.e. $S$, results in

$$S = \left(\frac{\delta W}{\psi}\right)^{\frac{1}{\gamma}} \left(E\left[R^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}, \tag{3.13}$$

In order to solve for stockholders' labor supply, start with equation (3.12) and multiply both sides by $L_S$. Rewriting then leads to

$$\begin{aligned} WL_S &= \left(\frac{W}{\psi}\right)^{\frac{1}{\gamma}} + S, \\ &= \left(\frac{W}{\psi}\right)^{\frac{1}{\gamma}} + \left(\frac{\delta W}{\psi}\right)^{\frac{1}{\gamma}} \left(\mathrm{E}\left[R^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}, \\ &= \left(\frac{W}{\psi}\right)^{\frac{1}{\gamma}} \left(1 + \delta^{\frac{1}{\gamma}} \left(\mathrm{E}\left[R^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}\right), \end{aligned} \tag{3.14}$$

where the second line uses equation (3.13). Dividing the final line of equation (3.14) by $W$ gives an expression for labor supply by stockholders:

$$L_S = W^{\frac{1-\gamma}{\gamma}} \psi^{\frac{-1}{\gamma}} \left(1 + \delta^{\frac{1}{\gamma}} \left(\mathrm{E}\left[R^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}\right). \tag{3.15}$$

The final step is to substitute for both $S$ and $L_S$ in the utility function. Respectively using the first line of equation (3.14), equation (3.13) and equation (3.15) to substitute for the three terms in equation (3.10) gives

$$\mathrm{E}(U)_S = \left(\frac{W}{\psi}\right)^{\frac{1-\gamma}{\gamma}} \frac{\gamma}{1-\gamma} \left(1 + \delta^{\frac{1}{\gamma}} \left(\mathrm{E}\left[R^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}\right). \tag{3.16}$$

**Bondholders**

To solve for labor's two endogenous variables ($B$ and $L_B$), we use the same solution strategy as for the stockholders and obtain

$$\mathrm{E}(U)_B = \left(\frac{W}{\psi}\right)^{\frac{1-\gamma}{\gamma}} \frac{\gamma}{1-\gamma} \left(\frac{1+I}{(1+\delta)\, I}\right)^{\frac{1-\gamma}{\gamma}} \left(1 + \delta^{\frac{1}{\gamma}} I^{\frac{1-\gamma}{\gamma}}\right). \tag{3.17}$$

**Bonds or Stocks?**

We compare the expected utilities of bondholders and stockholders for $\gamma > 1$ and calibrated values for $R$ and $I$.[3] The disutility from work parameter $\psi$ is assumed to be

[3] We investigate $\gamma > 1$ as this is the empirically relevant range.

the same for both agents, as well as the discount factor $\delta$. Equating $\mathrm{E}(U)_S$ to $\mathrm{E}(U)_B$ from equation (3.16) and (3.17) yields the risk aversion coefficient $\bar{\gamma}$ at which bond and equity investors are equally well off

$$\left(\frac{1+1/I}{1+\delta}\right)^{\frac{1-\bar{\gamma}}{\bar{\gamma}}}\left(1+\delta^{\frac{1}{\bar{\gamma}}}I^{\frac{1-\bar{\gamma}}{\bar{\gamma}}}\right)=1+\delta^{\frac{1}{\bar{\gamma}}}\mathrm{E}\left[R^{1-\bar{\gamma}}\right]^{\frac{1}{\bar{\gamma}}}.$$

Note that $\bar{\gamma}$ is solely a non-linear function of $\delta$, $I$, and $R$. As it turns out , for $\gamma > 1$ the solution for $\bar{\gamma}$ is unique.[4]

### Implementation

In order to evaluate the expected utility of stockholders, we first need a specification for $R$. We consider two options: (i) a Bernouilli distribution and (ii) a lognormal distribution. For the former there is an up state and a down state

$$R=\begin{cases}U>1 & \text{with probability } p,\\ D<1 & \text{with probability } 1-p.\end{cases} \tag{3.18}$$

In addition to this, we also assume $U > I > 1$, reflecting that while the net return on bonds is positive, it is smaller than the equity return in the good state of the world.

Moreover, we (roughly) base values of the key parameters on stock market data from January 1871 onwards taken from Robert Shiller's website.[5] In particular, we assign

$$U = 1.16,$$
$$D = 0.90,$$
$$I = 1.02,$$

such that a good state of the world implies an equity return of 16%, while a bad state

[4] This has been evaluated numerically.
[5] http://www.econ.yale.edu/~shiller/data.htm

means a 10% drop in the price of stock. In addition, the return on bonds is $2\%$. The probability of the positive state of the world ($p$) is set to 0.65 such that the equity premium equals $5\%$.

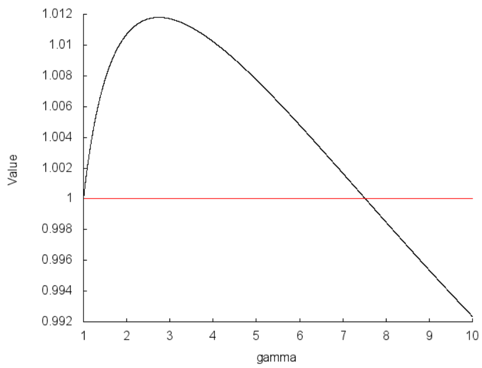
Our second specification assumes that $R$ is lognormally distributed, with $\mu = 0.07$ and $\sigma^2 = 0.03$, where the values are again from Robert Shiller. Although not required, we also assign values to the remaining parameters as follows

$$W = 4,$$
$$\psi = 10.4,$$
$$\delta = 0.99,$$

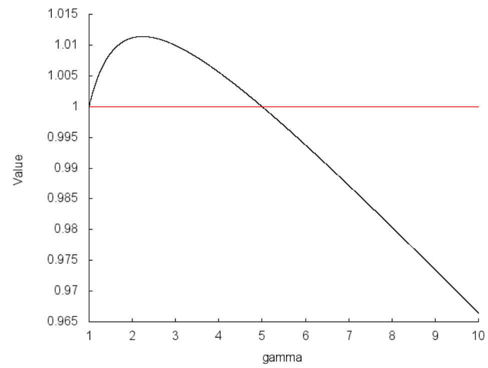
where the value of $\psi$ is taken from Niemann and Pichler (2011) and $\delta$ is the usual choice in calibration exercises, see for example Guvenen (2009, p. 1721). The wage is arbitrarily chosen, but its effect is limited. As mentioned, these three parameters only influence the level of utility, they do not have any impact on the choice between investing in either bonds or stocks. As a result, these parameters do not influence $\bar{\gamma}$, the value of $\gamma$ where an investor would be indifferent between investing in both asset types.

Figure 3.1a (3.1b) shows the ratio of expected utilities based on the Bernouilli (lognormal) distribution specified above. In particular, we compute the expected utility for investing in bonds and stocks for a range of values for $\gamma > 1$ and then divide $\mathrm{E}(U)_B$ by $\mathrm{E}(U)_S$. Since the expected utilities take on negative values for $\gamma > 1$, the relation between $\mathrm{E}(U)_B$ and $\mathrm{E}(U)_S$ is reversed. Hence a value larger than one indicates that investing in stocks is preferred over investing in bonds (the expected utility of stocks is less negative than the expected utility of bonds) and vice versa, while a value equal to one indicates indifference between both types of assets.

Both of the ratios in Figures 3.1a and 3.1b have the shape of a mountain parabola. Although their shape is similar, note that the scale is slightly different. In particular, the lognormal distribution exhibits a steeper decline than the Bernouilli distribution. It




**(a)** Real equity return follows a Bernoulli distribution with $U = 1.16$ and $D = 0.90$. The real bond return equals $I = 1.02$ and the equity premium is set to 5%.




**(b)** Real equity return follows a lognormal distribution with $\mu = 0.07$ and $\sigma^2 = 0.03$. The real bond return equals $I = 1.02$.

**Figure 3.1:** Ratio of expected utilities based on equations (3.16) and (3.17) against $\gamma$ under two alternative specifications for $R$, the real equity return. All values are roughly based on annual stock market data taken from Robert Shiller's website.

is clear that for low values of $\gamma$ an investor has a preference for stocks, while for larger values the preference switches to bonds. Assuming $R$ follows a Bernouilli distribution, the following two regions can be identified:

- Region I (preference for stocks): $\gamma \in (1, 7.52) \Rightarrow \mathrm{E}(U)_B < \mathrm{E}(U)_S$;
- Region II (preference for bonds): $\gamma \in (7.52, \infty) \Rightarrow \mathrm{E}(U)_B > \mathrm{E}(U)_S$.

For $\gamma = 7.52$, an investor is indifferent between investing in bonds or stocks.[6]

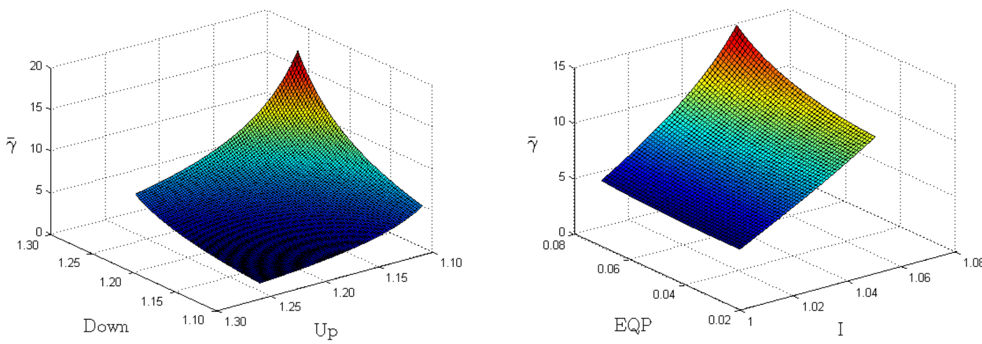
In order to infer to which degree our parameter values influence this value of $\bar{\gamma}$, we conduct two sensitivity tests. As we are merely interested in the effect our parameter values have on value of risk aversion at which investors are indifferent between investing n both asset classes, the resulting effect is not always in line with intuition. The reason is that as one of the parameters varies, the other three are kept fixed. For example, as will be shown below, if we increase $I$, $\bar{\gamma}$ goes up. From a theoretical perspective one would argue that as bonds become more attractive, some agents who

[6]Under the assumption that $R$ is lognormally distributed, $\bar{\gamma}$ equals 4.99.

previously held stocks (those near the former value of indifference) would now prefer to hold bonds such that indifference value of risk aversion should go down. Note that this argument runs through the equity risk premium which of course decreases as the bond return goes up. However, we keep the equity risk premium fixed and hence $\bar{\gamma}$ goes up.

In the first experiment, the bond return and equity premium are kept fixed, while the up and down state of the stock are varied. Figure 3.2a shows the value of $\bar{\gamma}$ where the up and down state vary within the ranges 1.11-1.26 and 0.80-0.95, respectively. The figure clearly shows that as $U$ goes up, $\bar{\gamma}$ goes down, while the break-even value of $\gamma$ moves in the same direction as $D$ is increased. Both relationships can be easily explained by looking at equation (3.16). Regarding the influence of $U$, note that as $U$ is increased, both $p$ and $U^{1-\gamma}$ decrease. Hence $pU^{1-\gamma}$ decreases, but $(1-p)D^{1-\gamma}$ increases. For our range of values of relative risk aversion, the latter effect dominates and hence $\left(E\left[R^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}$ increases. However, remember the negative sign of the fraction $\frac{\gamma}{1-\gamma}$, this implies that $\mathrm{E}(U)_S$ goes down. In other words, $\mathrm{E}(U)_S$ becomes more negative and as $\mathrm{E}(U)_B$ is unaffected, the value of $\bar{\gamma}$ is decreased. Note that this is not a linear effect due to the nonlinear influence of $\gamma$. With respect to $D$ the argument is similar. In this case the effect on $pU^{1-\gamma}$ dominates such that $\left(E\left[R^{1-\gamma}\right]\right)^{\frac{1}{\gamma}}$ decreases. As a result, $\mathrm{E}(U)_S$ is now less negative so the indifference value of risk aversion increases.

In the second robustness check, the up and down state are kept fixed, while the bond return and equity premium are varied between 1.01-1.06 and 0.03-0.08, respectively. Values of risk aversion at which an investor is indifferent between both asset classes for this setup are presented in Figure 3.2b. Both parameters have similar influences on $\bar{\gamma}$. With respect to the bond return we need to consider both $\mathrm{E}(U)_B$ and $\mathrm{E}(U)_S$. Increasing $I$ makes the expected utility of bondholders become less negative, but through $p$, the probability that the stock is in the up state, the stockholder's expected utility 'increases' even more. As a result there is a larger range of values of $\gamma$ for which investing in stocks is more attractive than investing in bonds. Increasing the equity premium, also through $p$, only affects $\mathrm{E}(U)_S$; it becomes less negative and so in similar spirit to the explanation for the bond return, $\bar{\gamma}$ increases.




**(a)** Value of $\bar{\gamma}$ under the assumption that $I = 1.02$ and an equity premium of $5\%$. The up and down state of the stock are varied between 1.11-1.26 and 0.80-0.95, respectively.

**(b)** Value of $\bar{\gamma}$ under the assumption that $U = 1.16$ and $D = 0.90$. The bond return and equity premium are varied between 1.01-1.06 and 0.03-0.08, respectively.

**Figure 3.2:** Impact of $U$, $D$, $I$ and the equity premium on the coefficient of relative risk aversion at which an investor is indifferent between investing in both asset types, based on equations (3.16) and (3.17) with parameter values arbitrarily chosen.

In sum, assuming that the real equity return is Bernouilli distributed, both sensitivity tests indicate that for reasonable values of the parameters the break even coefficient of relative risk aversion has a value in the range between 4 and 9. Values below or above this range only occur when two parameters take on rather extreme values and so, from an empirical point of view, have very limited relevance. Note that under the assumption that $R$ is lognormally distributed we find $\bar{\gamma} = 4.99$, which is within this range.

## 3.3 Methodology

This section describes the construction of separate consumption streams for stockholders and nonstockholders using macroeconomic data. The starting point are the national income accounts. As shown in Table 3.1, net national disposable income is the sum of different components. The first item is labor income and subtracting the third item from the second item yields capital income. What about components (4) through (7)? How should these be assigned to capital and labor income?

**Table 3.1: Composition net national disposable income**
Notes: Modified version of Table 13 of the Annual National Accounts, provided by the OECD. In particular, net property income is the difference between property income received and property income paid. The same applies to net social contributions and benefits other than social transfers in kind and net current transfers. Note that figures are available for the entire economy as well as separately for households and non-profit institutions serving households. See Appendix 3.A.1 for a full description of all items.

| | | |
|---|---|---|
| | (1) | Compensation of employees |
| +/+ | (2) | Gross operating surplus and mixed income |
| −/− | (3) | Consumption of fixed capital |
| +/+ | (4) | Net property income |
| +/+ | (5) | Net social contributions and benefits other than social transfers in kind |
| +/+ | (6) | Net other current transfers |
| −/− | (7) | Current taxes on income, wealth etc. |
| | | Net national disposable income |

Assigning components (4)-(7) involves the following two issues. First, one needs to determine which fraction of each item to assign to equity owners and wage earners. Second, there is only annual information on these items; implying that either one needs to work with annual observations or find a way to accurately translate these observations into quarterly figures. Without further information such an assignment would be rather arbitrary. In the end we decided to disregard net property income, net social contributions and benefits other than social transfers in kind, net current transfers and current taxes on income, wealth etc. in the construction of the income streams. Thus we propose to construct a quarterly proxy for disposable income based merely on the first three items. This is fairly rough, but expecting the alternative to be even harder to justify, we believe it is the best possible solution.

Compensation of employees is available on a stand-alone basis at the quarterly frequency. This is also the case for gross operating surplus and mixed income, and the consumption of fixed capital. But for items (2)-(3) the data is not split between households and non-profit institutions serving households (NPISHs), corporations and the general government. However, this distinction is available at the annual frequency. For this reason we use annual data to assign the percentage that can be attributed to households and then multiply this percentage with the quarterly series for both components. In other words, information is extracted from the annual data in order to assign part of the quarterly series to households.

The final step is to relate income to consumption. There are basically two approaches. We could (i) use these income series as a proxy for consumption or (ii) somehow translate these income figures into consumption streams. The second method has our preference, since it is well-known that consumption is much smoother than income, see e.g. Campbell and Deaton (1989). As a result, the covariance between excess returns and income growth is, ceteris paribus, higher vis-a-vis the situation in which consumption growth is used.

From equation (3.4) it is obvious that a higher covariance translates into a lower estimate of the coefficient of relative risk aversion. Also notice that this choice ensures our estimates of stockholders' risk aversion are more conservative. Finally, using income as a proxy for spending puts the comparison between the estimates for stockholders and nonstockholders on the one hand and the single representative agent specification (in which consumption is used) on unequal footing. In particular, when the estimates for equity owners are indeed lower than for the representative agent model it would not be clear what drives this result. Is it the use of income growth instead of consumption growth or is the consumption stream of stockholders fundamentally different?

Hence we choose to relate the two income streams to consumption streams. We assign aggregate consumption proportional to the share of both production factors in our proxy of net national disposable income. This ensures that in every quarter all consumption is assigned to either of the two groups and implies that current consumption is proportional to current income.

In sum, our approach comprises the following steps.

**Step 1** Use data on the composition of net national disposable income for both the total economy and separately for households to compute annual percentages of gross operating surplus and mixed income, and consumption of fixed capital that flows to households;

**Step 2** Multiply these percentages with quarterly series on gross operating surplus and mixed income, and consumption of fixed capital. This provides quarterly series for gross operating surplus and mixed income, and consumption of fixed capital;

**Step 3** Capital income is then formed by net operating surplus and mixed income;

**Step 4** Labor income is compensation of employees;

**Step 5** Assign consumption proportional to the proxy of net national disposable income (i.e. the sum of capital and labor income described in Steps 3 and 4).

The standard model is derived under the assumption that there is a representative agent whose utility is defined over aggregate consumption. Hence when empirically testing the consumption-based CAPM, one uses real aggregate consumption per capita. To the best of our knowledge, there is no aggregate time series on the number of stockholders and nonstockholders and hence we cannot put our consumption streams for both groups in, respectively, per stockholder and per non-stockholder terms.

### 3.3.1 Drawbacks of the macro approach

There are a number of potential drawbacks of the approach that we follow. First, the proxy implemented to circumvent the issue of how to assign several components of net national disposable income to capital and or labor is likely the most debatable since several important elements of disposable income are disregarded. Nevertheless, the omitted components do not easily fit within either capital or labor income. This implies that if these elements are to be considered, they somehow need to be assigned to either or both of the two categories. As there is no straightforward method to do so, such an approach would be arbitrary. Moreover, we believe it would be much harder to justify. For example, which part of property income belongs to stockholders? Answering such a question requires detailed information on this type of income. In addition, this percentage is likely to vary over time, so we actually require a time series. Similar observations apply to the other components (5)-(7).

Currently we ignore property income, social contributions and benefits other than social transfers in kind, current transfers and current taxes (see Table 3.1). The motive is that these items more or less cancel out. Hence, in general, our proxy is reasonably accurate and the sum of capital income and labor income does not differ much from net national disposable income of households.

Table 3.2 presents some summary statistics regarding our proxy. Note that national disposable income of households is only available at the annual frequency, so we take the sum of our quarterly series of capital and labor income over the year and compare that to the concept we try to proxy. As can be inferred, the average deviation is rather small for most countries.[7] Notable exceptions are Denmark, Italy, the Netherlands and Sweden. Moreover, the proxy does not seem to suffer from a large bias in either direction as we see both negative as well as positive deviations. The variance of these deviations is also not too high.

In order to check whether the average deviation is low because negative and positive deviations cancel over time, the next two columns show the average absolute deviation and its variance. Given the small differences, this does not seem to be the case.

This of course does not rule out the possibility that although the disregarded items more or less cancel out, this does not occur within the two consumption streams but across the streams. However, from the very nature of these four items this does not seem very likely. For example, stockholders will probably receive more property income, but because their income and wealth are also higher, they pay more taxes.

[7] The deviation is computed in nominal terms and relative to net national disposable income for households.

**Table 3.2: Summary statistics proxy**

Notes: $Obs$ denotes the number of yearly observations. $\hat{\mu}(DEV)$ and $\hat{\sigma}^2(DEV)$ denote, respectively, the mean and variance of the deviation. This deviation is defined as the sum of quarterly capital and labor income over the year minus the annual value of net national disposable income for households. $\hat{\mu}(|DEV|)$ and $\hat{\sigma}^2(|DEV|)$ denote, respectively, the mean and variance of the absolute values of the deviation. All columns in percentages.

| Country | $Obs$ | $\hat{\mu}(DEV)$ | $\hat{\sigma}^2(DEV)$ | $\hat{\mu}(\|DEV\|)$ | $\hat{\sigma}^2(\|DEV\|)$ |
|---:|---:|---:|---:|---:|---:|
| Australia | 35 | -4.22 | 4.32 | 4.22 | 4.32 |
| Austria | 10 | 3.20 | 1.88 | 3.20 | 1.88 |
| Belgium | 10 | 3.18 | 6.35 | 3.45 | 4.39 |
| Canada | 35 | -5.21 | 15.29 | 5.45 | 12.64 |
| Czech Republic | 9 | 5.88 | 10.97 | 5.88 | 10.97 |
| Denmark | 10 | 31.66 | 23.94 | 31.66 | 23.94 |
| Finland | 15 | 8.64 | 14.89 | 8.64 | 14.89 |
| France | 27 | 1.66 | 2.50 | 1.77 | 2.10 |
| Germany | 10 | -3.15 | 5.77 | 3.24 | 5.13 |
| Italy | 15 | -13.11 | 1.46 | 13.11 | 1.46 |
| Netherlands | 15 | 15.79 | 6.15 | 15.79 | 6.15 |
| Norway | 10 | 9.05 | 9.58 | 9.05 | 9.58 |
| Sweden | 10 | 25.41 | 15.81 | 25.41 | 15.81 |
| Switzerland | 10 | 10.71 | 2.87 | 10.71 | 2.87 |
| United Kingdom | 10 | -4.90 | 11.96 | 5.00 | 10.84 |
| United States | 35 | 1.01 | 3.88 | 1.73 | 1.83 |

Moreover, note that it is not the level of consumption that determines the value of the coefficient of relative risk aversion, it is the second moment of consumption growth that is paramount.

The same argument also provides an additional justification for our choice to assign consumption proportionally to income, the second issue deserving some more attention. As long as our proxy does well in capturing the dynamics of both income streams, the ultimate consumption growth series of both groups will also have the right dynamics since the proportional assignment causes the consumption streams to inherit their dynamics from the constructed income streams.

The third point deserving attention is the use of percentages derived from annual data to assign part of quarterly series to households. Using annual data on both households and the total economy allows us to compute the percentage that flows to households of gross operating surplus and mixed income, and consumption of fixed capital. This can then be used to assign part of quarterly series to households. This means that we assume that the percentage is constant throughout the year. Although it is probable that this assumption does not hold for every time period and country, quarterly percentage fluctuations are probably not too large. Moreover, the percentages do change every year, so we capture any longer-term trend, only fluctuations within the year are disregarded. Therefore, we believe that the intra year fluctuations have limited influence on our analysis. The U.S. has quarterly data on gross operating surplus and mixed income, and consumption of fixed capital that flows to households. Therefore, the difference between our approach and direct inference can be assessed. Comparing the results shows very small differences and so we are confident that our approach does not lead to a noteworthy bias.

Finally, we are not able to put the consumption streams of equity owners and wage earners in per capita terms. We could use the findings of Poterba and Samwick (1995) about U.S. stock ownership but this seems to be too much of a stretch since their analysis is based on three surveys. This implies that a time-series of stock ownership has to be based on just a few observations. In addition, such a series would be required for each country. Assuming that there is at least some information, there would probably be large differences between the surveys, making it hard to compare the estimates of the coefficient of relative risk aversion cross-country.

How does the per capita issue influence the estimate of $\gamma$ for both groups? Looking at equation (3.4), it is obvious that it affects the covariance between excess returns and consumption growth, i.e. the standard deviation of consumption growth and the correlation between excess returns and consumption growth. In Appendix 3.A.2 we show how these two terms are affected. One can determine the sign of the effect for the standard deviation of consumption growth. In particular, the sign depends on the correlation between consumption growth and population growth. If $\rho(\Delta c_{t+1}, \Delta p_{t+1}) > (<) 0$ then the standard deviation of consumption growth increases (decreases) when it is not considered in per capita terms. Regarding the correlation between excess returns and consumption growth, the sign cannot be determined and so the effect can be in either direction. Given that the influence on estimates of the coefficient of relative risk aversion depends on both terms as well as their relative magnitudes, the ultimate effect is ambiguous.

### 3.3.2 Standard errors

Studies about the equity premium puzzle usually only report point estimates of the coefficient of relative risk aversion.[8] To get an idea of the uncertainty surrounding such an estimate, Pozzi et al. (2010) introduce a simple method to construct standard errors for estimates of the coefficient of relative risk aversion. Moreover, having standard errors implies that we are able to test whether estimates for both groups are statistically significantly different. Again, the following is written in general terms but applies to estimates of $\gamma$ for all three groups.

This method of constructing standard errors uses the block-jackknife procedure, see e.g. Shao and Tu (1995). It is easy to implement and it can deal with the serial correlation that is present in the consumption series of the canonical model, as well as in our constructed consumption series for both stockholders and nonstockholders. Let $n$ be the size of the sample, let $m$ denote the number of omitted observations in a resample and let $N$ denote the number of resamples. Note that the total number of resamples is $N = n - m + 1$. To estimate the variance of $\hat{\gamma}_n$, the estimate of $\gamma$ based on all observations, one first deletes $m$ subsequent observations at a time and denotes the new estimate of relative risk aversion by $\hat{\gamma}^{(i)}$. Then the $i$-th pseudo value of $\hat{\gamma}_n$, denoted by $\tilde{\gamma}^{(i)}$, is defined as $\left[n\hat{\gamma}_n - (n-m)\hat{\gamma}^{(i)}\right]/m$. The resulting vector of pseudo values across the resamples is used to estimate the variance of $\hat{\gamma}_n$, i.e.

$$S^2_{\hat{\gamma}_n} = \frac{m}{nN} \sum_{i=1}^{N} \left( \tilde{\gamma}^{(i)} - \frac{1}{N} \sum_{i=1}^{N} \tilde{\gamma}^{(i)} \right)^2 . \tag{3.19}$$

[8] Cecchetti et al. (1993) and Vissing-Jørgenson (2002) are rare exceptions.

This estimate can then be used to form a country-specific confidence interval (CI) for $\hat{\gamma}_n$. More specifically, the Quenouille-Tukey mean of the pseudo values, see Shao and Tu (1995, p. 6), is the bias-corrected version of the estimate of $\gamma_n$ and the required critical value is taken from a $t$-distribution, with $N-1$ degrees of freedom (Miller, 1974).

### 3.3.3 Pooling

The cross-sectional dimension of our data set allows the computation of pooled estimators for each group, to which we refer as the world coefficient of relative risk aversion ($\gamma_w$). A priori, there is no reason to assume that the country-specific estimates all contain the same amount of information. For example sample periods differ between countries. Therefore using a weighted average seems to be a natural choice. As the block-jackknife procedure supplies the country-specific sample variances, the optimal (i.e. unbiased and variance minimizing) weights of Graybill and Deal (1959) can be used. The optimal weight for country $j$ is defined as

$$w_j = \frac{1/S^2_{\hat{\gamma}_{n,j}}}{\sum_{j=1}^{k} 1/S^2_{\hat{\gamma}_{n,j}}}, \tag{3.20}$$

where $k$ denotes the number of countries. Confidence intervals for the pooled estimators are then formed by weighting the country-specific averages of the pseudo values to get $\hat{\gamma}_w$. It is straightforward to show that the variance of this estimate is given by

$$S^2_{\hat{\gamma}_w} = \frac{1}{\sum_{j=1}^{k} 1/S^2_{\hat{\gamma}_{n,j}}}. \tag{3.21}$$

Critical values are taken from a $t$-distribution with $k-1$ degrees of freedom.

## 3.4 Data

The methodology described in the previous section is applied to data for a selection of OECD countries, i.e. Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Sweden, Switzerland, the United Kingdom and the United States. Various data limitations force us to drop the remaining 18 countries. For Iceland, Luxembourg and the Slovak Republic there is no suitable equity return data; we also miss vital information for the construction of the income series of these countries. The latter is also the reason why Chile, Estonia, Hungary, Ireland, Israel, Japan, Korea, Mexico, New Zealand, Portugal, Slovenia and Turkey are not included in our analysis. Too few observations leads us to disregard Greece (only 13 observations), Poland (14) and Spain (18). The start of the sample period is country-specific, depending on data availability; the end of the timespan is the final quarter of 2004.

Implementation of our approach requires a large number of time series, both annual as well as quarterly series. If applicable, all series are in local currency. The annual series are obtained from the database of the OECD. All can be taken from the simplified non-financial accounts (Table 13) in the Annual National Accounts. We require gross operating surplus and mixed income, consumption of fixed capital and net national disposable income. These three series are in current prices and in two versions: for the total economy, and for households and non profit institutions serving households (NPISHs).

Some quarterly series are also from the OECD. From the Quarterly National Accounts, the following series are needed: net operating surplus and mixed income, consumption of fixed capital and compensation of employees; all in current prices, the latter also at quarterly levels. Private final consumption expenditure in constant prices is also from this source. The Consumer Price Index (all items) is from the Main Economic Indicators. For Australia this series is quarterly, for the other countries there is a monthly index; for these we use the value of the final month of the quarter. All these series are taken directly from the OECD database.

Finally, three series are from other sources and obtained through Datastream. Data to construct the equity return are from Morgan Stanley Capital International. For the riskfree rate, we use the best proxy available in the International Financial Statistics, provided by the IMF. When available, the 3-month T-bill rate (line 60C) is used; otherwise a money market rate (line 60B) is selected. Annual mid-year population figures are also taken from the IFS. Again, more details about the construction of the real excess return and the transformation of the annual population figures into quarterly estimates are given in Appendix 2.A.1.

## 3.5 Results

We first compute the coefficient of relative risk aversion denoted by $\gamma$ for all three specifications (i.e. stockholders, nonstockholders and the representative agent) and discuss the driving force behind the differences in point estimates. In addition, confidence intervals are constructed and we present the pooled estimates. In the second subsection we follow Campbell (1999) and provide results based on an adjusted relative risk aversion coefficient (denoted by $\theta$) by setting the correlation between consumption growth and equity returns equal to unity. The argument is that the correlation term on the RHS of equation (3.4) is difficult to estimate accurately. Confidence intervals and pooled estimates are also presented for this alternative estimator.

To be able to judge the performance of our heterogeneous agent approach, the standard model with a single representative agent is also estimated. In order to keep the comparison fair, consumption growth is used instead of consumption growth per capita (see the discussion in Section 3.1) The influence on the results for this specification are quantitatively small and none of the qualitative conclusions are affected.[9] This can also be seen as an indication that not computing the consumption streams of stockholders

[9]The results of the representative agent specification are shown throughout this entire section; these can be compared to the results in per capita terms which are given in Appendix 3.A.3.

and nonstockholders in per recipient terms — one of the drawbacks of our approach mentioned earlier — does not substantially influence the results.

### 3.5.1 Coefficient of relative risk aversion

Estimates of the coefficient of relative risk aversion based on equation (3.4) are presented in Table 3.3. Consistent with previous research, e.g. Campbell (1999), the standard single representative agent model (subscript $R$) provides a wide spectrum of risk aversion estimates, running from Norway's estimate of -677 to Austria's $\gamma$ of 1084. None of the estimates comes close to the values that are considered plausible. Coefficient values for stockholders (subscript $K$) shown in the next column, are in general closer to zero. With the exception of the Netherlands and the United Kingdom, all values of $\gamma$ are lower for stockholders than for the representative agent. The final column reveals that estimates for nonstockholders (subscript $L$) are both negative as well as positive. However, remember that there is no a priori reason to believe that nonstockholders satisfy the first-order condition underlying the Euler equation, so estimates of $\gamma$ have no clear interpretation for this group.[10]

Note that the estimate for the representative agent is in between those for the two subgroups for 8 out of the 16 countries, recall the discussion of equation (3.5) in Section 2. It is worth mentioning that for all countries the average excess return is positive over the sample period. Hence any negative value for $\gamma$ is the result of a negative correlation between excess returns and the respective consumption growth (see Panel B of Table 3.4 and the discussion later on). This correlation is also the reason why stockholders for the Netherlands and the United Kingdom have such a high estimate; their correlation is near zero.

[10] All estimates of $\gamma$ are obtained using equation (3.4). Hence for nonstockholders we use excess returns and the proportion of consumption that is assigned to labor. As there does not seem to be a direct channel through which equity returns impact nonstockholders' consumption, the first-order condition is not likely satisfied. Our two agent model developed in Section 2.1 provides another expression for labor's coefficient of relative risk aversion. In this case it has a clear interpretation, but this model is not empirically implemented.

**Table 3.3: Estimates of risk-aversion coefficient**
Notes: Obs denotes the number of observations; $\hat{\gamma}$ is the estimate of the coefficient of relative risk aversion based on equation (3.4). Subscripts refer to the three groups: the representative agent ($R$), stockholders ($K$) and nonstockholders ($L$). Sample periods are country-specific, depending on data availability; the end of the timespan is the final quarter of 2004.

| Country | Sample period | Obs | $\hat{\gamma}_R$ | $\hat{\gamma}_K$ | $\hat{\gamma}_L$ |
|---|---|---|---|---|---|
| Australia | 1970Q1 - 2004Q3 | 139 | 137.66 | 23.13 | -1135.49 |
| Austria | 1995Q2 - 2004Q3 | 38 | 1083.51 | 63.02 | -440.54 |
| Belgium | 1995Q2 - 2004Q3 | 38 | 355.03 | 152.86 | 493.34 |
| Canada | 1970Q2 - 2004Q3 | 138 | 47.78 | 30.95 | 50.18 |
| Czech Republic | 1996Q2 - 2004Q3 | 34 | -318.12 | 36.81 | -74.79 |
| Denmark | 1995Q2 - 2004Q3 | 38 | 129.43 | 108.99 | 130.38 |
| Finland | 1990Q2 - 2004Q3 | 58 | 195.38 | 27.56 | 2312.13 |
| France | 1978Q2 - 2004Q3 | 106 | 187.21 | 54.96 | 426.42 |
| Germany | 1995Q2 - 2004Q3 | 38 | 532.32 | 54.40 | -777.71 |
| Italy | 1990Q2 - 2004Q3 | 58 | 96.22 | 18.96 | -126.26 |
| Netherlands | 1990Q2 - 2004Q3 | 58 | 156.55 | 580.14 | 141.49 |
| Norway | 1995Q2 - 2004Q3 | 38 | -676.77 | -17.13 | 127.10 |
| Sweden | 1995Q2 - 2004Q3 | 38 | 182.62 | -3.03 | 18.16 |
| Switzerland | 1995Q2 - 2004Q3 | 38 | 734.60 | 34.40 | -821.24 |
| United Kingdom | 1995Q2 - 2004Q3 | 38 | 74.06 | -210.20 | 61.13 |
| United States | 1970Q2 - 2004Q3 | 138 | 72.59 | 40.25 | 89.04 |

Our results clearly indicate that estimates for stockholders constitute less of puzzle in comparison with the results for the representative agent model. What drives this result? A glance at equation (3.4) reveals that the LHS is equal for all three groups, so any difference has to come from the only other unknown: the covariance between excess returns and consumption growth. In particular, it has to be due to (i) higher standard deviations of consumption growth for stockholders, (ii) higher correlations between excess returns and consumption growth for stockholders or (iii) a combination of these two factors.

Table 3.4, Panel A gives the standard deviations of consumption growth for all three groups. As shown, the consumption growth of stockholders (column 3) is much more variable than for nonstockholders (column 4). On average, it is approximately a factor 3 higher. Comparing the standard deviations for the representative agent (column 2) on one hand and those for the two subgroups on the other hand reveals that, with the exception of the United States, the estimate is lowest for the representative agent model. This suggests that the two consumption streama are negatively correlated. The United States is the only country for which consumption growth of both groups has gone hand in hand. In all other countries, consumption growth seems to be better characterized as a zero-sum game: positive consumption growth for stockholders implies negative consumption growth for nonstockholders and vice versa.

The other possible explanation comes from the correlation between excess returns and consumption growth as presented in Panel B of Table 3.4. Just like before, there is no a priori reason to expect the correlation between excess returns and the consumption growth of nonstockholders to satisfy certain conditions. We do expect the correlation for stockholders to be higher than for the representative agent because they are the group whose consumption stream is most likely affected by changes in excess returns.

The results are somewhat mixed. For seven of the countries, the correlation is indeed higher for stockholders than for the single agent specification. However, this implies that for the majority of countries (i.e. nine), the correlation is closer to zero, turns negative or becomes more negative. The negative correlations for Norway, Sweden and the United Kingdom are difficult to explain from a theoretical perspective. Moreover, the presented correlations for nonstockholders are also occasionally puzzling; estimates for Sweden and the United Kingdom are even higher than for the representative agent specification so that for these countries nonstockholders' consumption is most linked to asset returns. The most likely explanation for these abstruse results lies in the suggestion of Campbell (1999), who claims short-term measurement errors in consumption make the correlation difficult to measure accurately. Therefore, he also presents results for which this correlation has been set equal to one; an assumption that is also widely used in calibration exercises (e.g. Guvenen, 2009). In the next subsection we presents results based on this maintained hypothesis.

In sum, for a small majority of the countries the correlation is higher for the representative agent model than for stockholders, but in light of the discussion above, one should be careful in emphasizing this too much. More importantly, from a quantitative point of view, it seems to be the higher standard deviation of consumption growth that drives the lower estimates of the coefficient of relative risk aversion for stockholders compared to the representative agent model.

**Table 3.4: Standard deviation and correlation**
Notes: $\hat{\sigma}_c$ is the estimated standard deviation of consumption growth and $\rho_{(exr,c)}$ is the estimated correlation between the excess return and consumption growth. Subscripts refer to the three groups: the representative agent ($R$), stockholders ($K$) and nonstockholders ($L$). Country-specific sample periods are given in Table 3.3.

| | Panel A | | | Panel B | | |
|---|---|---|---|---|---|---|
| Country | $\hat{\sigma}_{cR}$ | $\hat{\sigma}_{cK}$ | $\hat{\sigma}_{cL}$ | $\rho_{(exr,cR)}$ | $\rho_{(exr,cK)}$ | $\rho_{(exr,cL)}$ |
| Australia | 1.57 | 7.88 | 2.36 | 0.08 | 0.10 | -0.01 |
| Austria | 0.83 | 2.25 | 0.85 | 0.04 | 0.25 | -0.10 |
| Belgium | 1.07 | 3.48 | 1.25 | 0.12 | 0.08 | 0.07 |
| Canada | 1.72 | 7.43 | 1.91 | 0.28 | 0.10 | 0.24 |
| Czech Republic | 1.53 | 6.01 | 2.34 | -0.07 | 0.15 | -0.19 |
| Denmark | 2.35 | 10.37 | 2.52 | 0.17 | 0.05 | 0.16 |
| Finland | 1.67 | 8.51 | 2.27 | 0.12 | 0.17 | 0.01 |
| France | 1.13 | 4.36 | 1.35 | 0.17 | 0.15 | 0.06 |
| Germany | 1.26 | 6.47 | 1.73 | 0.05 | 0.09 | -0.02 |
| Italy | 1.46 | 3.44 | 1.73 | 0.10 | 0.21 | -0.06 |
| Netherlands | 1.70 | 6.34 | 1.90 | 0.12 | 0.01 | 0.11 |
| Norway | 2.33 | 23.64 | 3.61 | -0.02 | -0.07 | 0.06 |
| Sweden | 1.25 | 38.02 | 5.65 | 0.19 | -0.37 | 0.42 |
| Switzerland | 0.94 | 7.64 | 0.99 | 0.07 | 0.19 | -0.06 |
| United Kingdom | 0.95 | 4.80 | 1.06 | 0.26 | -0.02 | 0.29 |
| United States | 1.35 | 3.81 | 1.29 | 0.34 | 0.22 | 0.29 |

What is the uncertainty surrounding these point estimates? And are the estimates of both groups statistically significantly different from each other? To answer these questions, we apply the block-jackknife method of Pozzi et al. (2010) to construct 95% confidence intervals for all estimates of $\gamma$, where $m$ is set to 4 so that we leave out one year of information when computing the variances of $\hat{\gamma}$. Outcomes are presented in Table 3.5. As mentioned in Section 3.2, the mean of the pseudo values can be seen as the bias-corrected relative risk aversion coefficients. For an easier comparison, the values of $\gamma$ are also shown. Comparing both sets of estimates for each of the three groups reveals that the bias is in general (relatively) small. In addition, the bias is not in a single direction. For all three groups, the division is almost equal between positive and negative bias. Interestingly, the bias is always in the same direction for each country. So when the bias in the estimate for the representative agent model is positive, it is also positive for the estimates of stockholders and nonstockholders. All in all, the bias in estimates of $\gamma$ is not too alarming.

The width of the confidence intervals varies considerably from country to country. It seems that the standard error is higher if the mean pseudo value (or point estimate) takes on a more extreme negative or positive value. For example, the mean pseudo values of nonstockholders for Australia and Finland are the most extreme; they also have the largest width of all reported confidence intervals. Interestingly, the confidence interval for the United States is the only one that does not encompass negative values. Confidence intervals for the estimate of $\gamma$ for stockholders are the smallest, so besides the result that the point estimates for this group are lowest, they are also estimated with the highest precision. Although, as can be inferred from the confidence intervals, the mean pseudo values (or point estimates) differ considerably between stockholders and nonstockholders, this difference is not significant at statistically plausible levels. The confidence intervals are just too wide.

Finally, the standard errors for the country-specific risk aversion coefficients can be used to construct a pooled estimate, denoted by $\gamma_w$, by weighing individual estimates with the inverse of their relative uncertainty. The results in Table 3.6 are encouraging in the sense that the pooled estimate for stockholders is 0.3, hence just below the plausible range of values between one and ten (Mehra and Prescott, 1985). Moreover, the accompanying 95% confidence interval is rather tight. For nonstockholders the estimate is outside of the plausible range and its confidence interval is less informative. Recall that if labor has no investment at all the approach does not deliver an estimate of $\gamma$ that can be related to the risk aversion of labor. The representative agent model has the highest estimate as well as the most uncertainty. Combined, these results in-

**Table 3.5: Confidence interval for risk-aversion coefficient**

Notes: $\hat{\gamma}$ is the estimate of the coefficient of relative risk aversion based on equation (3.4). The mean pseudo value is denoted by $\overline{PS}$. The confidence intervals are centered around the mean pseudo value, the standard error is based on the square root of equation (3.19) and the critical value is taken from a $t$-distribution with degrees of freedom equal to $N-1$; see Section 3.2 for more details. Subscripts refer to the three groups: the representative agent ($R$), stockholders ($K$) and nonstockholders ($L$). Country-specific sample periods are given in Table 3.3. Mnemonics are as follows: Australia (AU), Austria (OE), Belgium (BG), Canada (CN), Czech Republic (CZ), Denmark (DK), Finland (FN), France (FR), Germany (BD), Italy (IT), the Netherlands (NL), Norway (NW), Sweden (SD), Switzerland (SW), the United Kingdom (UK) and the United States (US).

| Country | $\hat{\gamma}_R$ | $\overline{PS}_R$ | 95% CI | | $\hat{\gamma}_K$ | $\overline{PS}_K$ | 95% CI | | $\hat{\gamma}_L$ | $\overline{PS}_L$ | 95% CI | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AU | 137.66 | 143.45 | -91.25 | 378.16 | 23.13 | 24.11 | -15.33 | 63.55 | -1135.49 | -1183.29 | -3119.31 | 752.72 |
| OE | 1083.51 | 957.03 | -682.35 | 2596.41 | 63.02 | 55.66 | -39.69 | 151.02 | -440.54 | -389.11 | -1055.66 | 277.43 |
| BG | 355.03 | 307.49 | -213.78 | 828.76 | 152.86 | 132.39 | -92.04 | 356.82 | 493.34 | 427.28 | -297.07 | 1151.63 |
| CN | 47.78 | 50.46 | -23.87 | 124.79 | 30.95 | 32.68 | -15.46 | 80.82 | 50.18 | 52.99 | -25.06 | 131.04 |
| CZ | -318.12 | -262.31 | -888.19 | 363.56 | 36.81 | 30.36 | -42.07 | 102.78 | -74.79 | -61.67 | -208.81 | 85.47 |
| DK | 129.43 | 122.78 | -65.74 | 311.31 | 108.99 | 103.40 | -55.37 | 262.16 | 130.38 | 123.69 | -66.23 | 313.61 |
| FN | 195.38 | 234.20 | -48.51 | 516.91 | 27.56 | 33.04 | -6.84 | 72.92 | 2312.13 | 2771.50 | -574.09 | 6117.09 |
| FR | 187.21 | 180.20 | -10.23 | 370.63 | 54.96 | 52.90 | -3.00 | 108.81 | 426.42 | 410.45 | -23.30 | 844.20 |
| BD | 532.32 | 534.12 | -560.63 | 1628.87 | 54.40 | 54.58 | -57.29 | 166.45 | -777.71 | -780.34 | -2379.74 | 819.07 |
| IT | 96.23 | 119.36 | -253.50 | 492.22 | 18.96 | 23.52 | -49.95 | 96.99 | -126.26 | -156.63 | -645.92 | 332.66 |
| NL | 156.55 | 183.40 | -74.80 | 441.60 | 580.14 | 679.62 | -277.18 | 1636.43 | 141.49 | 165.76 | -67.60 | 399.11 |
| NW | -676.77 | -516.72 | -2084.22 | 1050.78 | -17.13 | -13.08 | -52.74 | 26.59 | 127.10 | 97.04 | -197.34 | 391.43 |
| SD | 182.62 | 173.12 | -133.51 | 479.75 | -3.03 | -2.87 | -7.95 | 2.21 | 18.16 | 17.22 | -13.28 | 47.71 |
| SW | 734.60 | 691.97 | -301.46 | 1685.40 | 34.40 | 32.40 | -14.12 | 78.91 | -821.24 | -773.58 | -1884.19 | 337.02 |
| UK | 74.06 | 55.71 | -201.39 | 312.81 | -210.20 | -158.12 | -887.79 | 571.55 | 61.13 | 45.98 | -166.22 | 258.19 |
| US | 72.59 | 76.13 | 5.10 | 147.15 | 40.25 | 42.21 | 2.83 | 81.59 | 89.04 | 93.38 | 6.25 | 180.51 |

dicate that pooling is a very effective way to reduce the uncertainty in the estimate of the coefficient of relative risk aversion, but not to such a degree that estimates are statistically different for stockholders and nonstockholders. However, we do find that the confidence intervals for the representative agent and stockholders do not overlap. Hence the difference between estimates of relative risk aversion between these two groups is statistically significant.

**Table 3.6: Pooled results**

Notes: The pooled estimate of the relative risk aversion coefficient is denoted by $\gamma_w$, using weights specified according to equation (3.20). The confidence interval uses the square root of equation (3.21) as standard error and critical value from a $t$-distribution with $k-1$ degrees of freedom, where $k$ denotes the number of countries; see Section 3.3 for more details. Subscripts refer to the three groups: the representative agent ($R$), stockholders ($K$) and nonstockholders ($L$).

| Specification | Estimate | 95% CI | |
|---|---|---|---|
| $\hat{\gamma}_{wR}$ | 88.93 | 41.99 | 135.87 |
| $\hat{\gamma}_{wK}$ | 0.31 | -4.75 | 5.37 |
| $\hat{\gamma}_{wL}$ | 29.70 | 2.59 | 56.82 |

### 3.5.2 Adjusted coefficients of relative risk aversion

As mentioned, Campbell (1999) argues that short-term measurement error in consumption is the reason why the correlation between excess returns and consumption growth is difficult to measure accurately in the canonical model. To alleviate this problem, Campbell (1999) sets this correlation at a value of unity and presents estimates of this adjusted risk-aversion coefficient, which we denote by $\theta$. As this specification allows us to determine the extent to which the equity premium puzzle is due to the smoothness of consumption, it offers a method to figure out whether the more variable consumption growth of stockholders is enough to solve the puzzle.

As shown in Table 3.7, this seems to be the case for the majority of the countries under investigation. In particular, the estimates for Austria (15.7) and Belgium (12.6) are the only two that do not fall within the plausible range of below 10. The remaining estimates of $\theta$ for stockholders take on plausible values. This is in sharp contrast to the adjusted estimates of relative risk aversion for the representative agent specification. Here the estimate for Italy (9.4) is just below the upper boundary, all other values are (well) above this threshold. Hence, in line with Campbell (1999), the standard deviation of consumption growth for the representative agent specification is not high enough to explain the equity premium puzzle. However, if one separates the consumption stream of stockholders and nonstockholders, it seems that the variability of stockholders' consumption growth is enough to explain the equity premium puzzle for the large majority of the countries under investigation.

**Table 3.7: Estimates of the adjusted risk-aversion coefficient**
Notes: $\hat{\theta}$ is the estimate of the adjusted coefficient of relative risk aversion based on equation (3.4), but with the implicit correlation term fixed at a value of unity. Subscripts refer to the three groups: the representative agent ($R$), stockholders ($K$) and nonstockholders ($L$). Country-specific sample periods are given in Table 3.3.

| Country | $\hat{\theta}_R$ | $\hat{\theta}_K$ | $\hat{\theta}_L$ |
|---:|---:|---:|---:|
| Australia | 11.56 | 2.30 | 7.71 |
| Austria | 42.66 | 15.73 | 41.80 |
| Belgium | 40.99 | 12.56 | 34.84 |
| Canada | 13.49 | 3.12 | 12.16 |
| Czech Republic | 21.70 | 5.53 | 14.24 |
| Denmark | 22.46 | 5.10 | 20.99 |
| Finland | 23.18 | 4.54 | 17.02 |
| France | 31.80 | 8.24 | 26.68 |
| Germany | 23.73 | 4.61 | 17.26 |
| Italy | 9.37 | 3.97 | 7.91 |
| Netherlands | 18.03 | 4.84 | 16.16 |
| Norway | 12.58 | 1.24 | 8.12 |
| Sweden | 34.03 | 1.12 | 7.53 |
| Switzerland | 52.81 | 6.46 | 50.12 |
| United Kingdom | 19.52 | 3.85 | 17.50 |
| United States | 24.76 | 8.78 | 26.07 |

Our interest is not solely in point estimates. The standard deviation of such estimates is of equal importance. Applying the block-jackknife method to the adjusted estimates of the coefficient of relative risk aversion provides more informative confidence intervals, for two reasons. First, recall the apparent relation between the value of the estimate and its standard deviation in the sense that more extreme estimates seem to be accompanied by a higher level of uncertainty. Second, since the correlation between excess returns and consumption growth is fixed, this term no longer contributes to the uncertainty associated with the estimate of the adjusted coefficient of relative risk aversion.

Before judging the uncertainty surrounding these adjusted estimates of the coefficient of relative risk aversion, we first focus our attention towards the possible bias in these point estimates, see Table 3.8. These results are qualitatively similar to those presented in Table 3.5. More specifically, (i) the bias is almost as likely positive as it is negative, (ii) the bias is always in the same direction for all three specifications and (iii) the direction of the bias corresponds directly to that in Table 3.5, i.e. for 7 countries the bias is negative for all 6 specifications, for the remaining 9 countries, this bias is always positive. Similarly to the situation with estimates of $\gamma$, the confidence intervals are smaller for stockholders than for the other two specifications and the U.S. is the only country for which the confidence interval does not contain negative values. As expected, all confidence intervals are smaller than their counterparts in Table 3.5; but, again, they do not give rise to significant differences between estimates.

Although estimates of $\theta$ are already estimated with higher precision than estimates of $\gamma$, we can reduce this uncertainty even more by pooling all country-specific information. Table 3.9 shows that the outcome for the stockholders is very appealing, the pooled point estimate is 2.3 and its 95% confidence interval is extremely small. The representative agent specification has an estimate that is just under 20, with a confidence interval that is not too large. In sum, these results show that if the consumption streams of stockholders and nonstockholders are separated, the correlation between excess returns and consumption growth is fixed and country-specific estimates are pooled in an optimal sense, there seems to be a solution to the equity premium puzzle. Notice that the confidence intervals for stockholders on one hand, and those for the represen-

**Table 3.8: Confidence interval for the adjusted risk-aversion coefficient**

Notes: $\hat{\theta}$ is the estimate of the adjusted coefficient of relative risk aversion based on equation (3.4), but with the correlation term fixed at a value of unity. The mean pseudo value is denoted by $\overline{PS^{\theta}}$. The confidence intervals are centered around the mean pseudo value, the standard error is based on the square root of equation (3.19) applied to the adjusted risk-aversion coefficients and the critical value is taken from a $t$-distribution with degrees of freedom equal to $N-1$; see Section 3.2 for more details. Subscripts refer to the three groups: the representative agent ($R$), stockholders ($K$) and nonstockholders ($L$). Country-specific sample periods are given in Table 3.3.

| Country | $\hat{\theta}_R$ | $\overline{PS^{\theta}}_R$ | 95% CI | | $\hat{\theta}_K$ | $\overline{PS^{\theta}}_K$ | 95% CI | | $\hat{\theta}_L$ | $\overline{PS^{\theta}}_L$ | 95% CI | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Australia | 11.56 | 12.05 | -7.67 | 31.77 | 2.30 | 2.40 | -1.53 | 6.33 | 7.71 | 8.03 | -5.11 | 21.17 |
| Austria | 42.66 | 37.68 | -26.87 | 102.23 | 15.73 | 13.89 | -9.91 | 37.69 | 41.80 | 36.92 | -26.32 | 100.16 |
| Belgium | 40.99 | 35.50 | -24.68 | 95.68 | 12.56 | 10.88 | -7.56 | 29.32 | 34.84 | 30.17 | -20.98 | 81.33 |
| Canada | 13.49 | 14.25 | -6.74 | 35.23 | 3.12 | 3.30 | -1.56 | 8.16 | 12.16 | 12.84 | -6.07 | 31.75 |
| Czech Republic | 21.70 | 17.89 | -24.80 | 60.59 | 5.53 | 4.56 | -6.32 | 15.45 | 14.24 | 11.74 | -16.28 | 39.76 |
| Denmark | 22.46 | 21.31 | -11.41 | 54.03 | 5.10 | 4.83 | -2.59 | 12.26 | 20.99 | 19.91 | -10.66 | 50.49 |
| Finland | 23.18 | 27.78 | -5.76 | 61.32 | 4.54 | 5.45 | -1.13 | 12.02 | 17.02 | 20.40 | -4.23 | 45.02 |
| France | 31.80 | 30.61 | -1.74 | 62.95 | 8.24 | 7.93 | -0.45 | 16.30 | 26.68 | 25.68 | -1.46 | 52.81 |
| Germany | 23.73 | 23.81 | -25.00 | 72.62 | 4.61 | 4.62 | -4.85 | 14.09 | 17.26 | 17.31 | -18.17 | 52.80 |
| Italy | 9.37 | 11.63 | -24.69 | 47.94 | 3.97 | 4.93 | -10.46 | 20.32 | 7.91 | 9.81 | -20.83 | 40.45 |
| Netherlands | 18.03 | 21.13 | -8.62 | 50.87 | 4.84 | 5.67 | -2.31 | 13.66 | 16.16 | 18.93 | -7.72 | 45.58 |
| Norway | 12.58 | 9.60 | -19.53 | 38.74 | 1.24 | 0.95 | -1.93 | 3.82 | 8.12 | 6.20 | -12.61 | 25.02 |
| Sweden | 34.03 | 32.26 | -24.88 | 89.40 | 1.12 | 1.06 | -0.82 | 2.94 | 7.53 | 7.14 | -5.51 | 19.79 |
| Switzerland | 52.81 | 49.75 | -21.67 | 121.17 | 6.46 | 6.09 | -2.65 | 14.83 | 50.12 | 47.21 | -20.57 | 114.99 |
| United Kingdom | 19.52 | 14.69 | -53.09 | 82.46 | 3.85 | 2.90 | -10.47 | 16.27 | 17.50 | 13.17 | -47.59 | 73.93 |
| United States | 24.76 | 25.97 | 1.74 | 50.20 | 8.78 | 9.21 | 0.62 | 17.80 | 26.07 | 27.34 | 1.83 | 52.84 |

tative agent and nonstockholders on the other hand do not overlap, implying that the pooled estimate of stockholders differ significantly from the other two estimates at the conventional significance level of 5%.

**Table 3.9: Pooled results for the adjusted coefficients**
Notes: The pooled estimate of the adjusted risk aversion coefficient is denoted by $\hat{\theta}_w$, using weights specified according to equation (3.20). The confidence interval uses the square root of equation (3.21) as standard error and critical value from a $t$-distribution with $k-1$ degrees of freedom; see Section 3.3 for more details. Subscripts refer to the three groups: the representative agent ($R$), stockholders ($K$) and nonstockholders ($L$).

| Situation | Estimate | 95% CI | |
|---|---|---|---|
| $\hat{\theta}_{wR}$ | 19.670 | 10.88 | 28.52 |
| $\hat{\theta}_{wK}$ | 2.33 | 1.02 | 3.65 |
| $\hat{\theta}_{wL}$ | 12.99 | 6.75 | 19.23 |

## 3.6 Conclusion

Unlike previous research that attempts to explain the equity premium puzzle using the notion of limited stock market participation based on household-level data, our approach is based exclusively on using macroeconomic data. The greatest advantages of our approach compared to previous studies is that we have longer timeseries (i.e. more observations) and given that these data are gathered by the OECD, the results can be compared more easily across countries.

Despite these benefits, limited stock market participation does not entirely explain the puzzle. Estimates for stockholders are much more in line with plausible values of the coefficient of relative risk aversion, so the macro approach does go a long way. This result is mainly due to the higher standard deviation of consumption growth for the stockholders group compared to that of nonstockholders and the combination of these two groups into a single representative agent; the correlation between excess returns and consumption growth plays at best a modest role.

By using a simple jackknifing method we have been able to construct standard errors for these estimates. Although estimates of the coefficient of relative risk aversion are lower for stockholders, the difference with risk aversion estimates of the single representative agent is not statistically significant. Nevertheless, using these country-specific estimates to compute pooled estimators reduces this uncertainty and provides encouraging results, i.e. the pooled estimate of stockholders is within the range of plausible values and differs significantly from the estimate of the representative agent specification.

Short-term measurement errors in consumption growth imply that the correlation between this variable and excess returns is easily distorted. This plays a large role in the conclusion that our approach does not offer a full solution to the puzzle. Following the suggestion of Campbell (1999) and setting this correlation equal to a value of unity results in estimates that do not only vary less from country to country, for almost all countries they are within the range of plausible values. Hence the higher variability of stockholders' consumption growth, combined with a fixed value of the correlation between excess returns and consumption growth, is able to provide a solution to the equity premium puzzle. Despite their more precise estimation, there is still too much uncertainty to obtain statistically significant differences between relative risk aversion estimates for stockholders and nonstockholders. However, when the country-specific estimates are combined in an optimal way, we are able to draw the conclusion that the pooled estimate for stockholders is significantly lower than those for the single representative agent and nonstockholders.

## 3.A Appendices

### 3.A.1 Definitions Table 3.1

**Compensation of employees** Compensation of employees is the total remuneration, in cash or in kind, payable by an enterprise to an employee in return for work done by the latter during the accounting period.

Compensation of employees has two main components:

1. Wages and salaries payable in cash or in kind;
2. The value of the social contributions payable by employers: these may be actual social contributions payable by employers to Social Security schemes or to private funded social insurance schemes to secure social benefits for their employees; or imputed social contributions by employers providing unfunded social benefits.

**Gross operating surplus and mixed income** The operating surplus measures the surplus or deficit accruing from production before taking account of any interest, rent or similar charges payable on financial or tangible non-produced assets borrowed or rented by the enterprise, or any interest, rent or similar receipts receivable on financial or tangible non-produced assets owned by the enterprise.

Note: for unincorporated enterprises owned by households, this component is called “mixed income”.

**Consumption of fixed capital** Consumption of fixed capital represents the reduction in the value of the fixed assets used in production during the accounting period resulting from physical deterioration, normal obsolescence or normal accidental damage.

**Net property income** Property income is the income receivable by the owner of a financial asset or a tangible non-produced asset in return for providing funds to or putting the tangible non-produced asset at the disposal of, another institutional unit; it consists of interest, the distributed income of corporations (i.e. dividends and withdrawals from income of quasi-corporations), reinvested earnings on direct foreign investment, property income attributed to insurance policy holders, and rent.

**Net social contributions & benefits other than social transfers in kind** Social contributions are actual or imputed payments to social insurance schemes to make provision for social insurance benefits to be paid. Social benefits other than social transfers in kind consist of all social benefits except social transfers in kind; in other words, they consist of: (a) all social benefits in cash — both social insurance and social assistance benefits — provided by government units, including social security funds, and NPISHs; and (b) all social insurance benefits provided under private funded and unfunded social insurance schemes, whether in cash or in kind.

**Net current transfers** Other current transfers consist of net premiums and claims for non-life insurance, current transfers between different kinds of government units, usually at different levels of government and also between general government and foreign governments, and current transfers (i.e. non-financial transfers) such as those between different households.

**Current taxes on income, wealth etc.** Most current taxes on income, wealth etc. consist of taxes on the incomes of households or profits of corporations and taxes on wealth that are payable regularly every tax period (as distinct from capital taxes levied infrequently).

**Net national disposable income** National disposable income may be derived from national income by adding all current transfers in cash or in kind receivable by resident institutional units from non-resident units and subtracting all current transfers in cash or in kind payable by resident institutional units to non-resident units.

### 3.A.2 Influence 'per capita' streams

The coefficient of relative risk aversion is calibrated according to

$$\hat{\gamma} = \frac{\mathrm{E}_t[r_{i,t+1} - r_{f,t+1}] + \frac{\sigma_{exr}^2}{2}}{\sigma_{(exr,c)}}.$$

Obviously, consumption only enters the denominator of this expression, so we do not have to consider how it will affect the numerator. The covariance can be decomposed as

$$\sigma_{(exr,c)} = \sigma_{exr} \cdot \sigma_c \cdot \rho_{(exr,c)},$$

where $\rho_{(exr,c)}$ denotes the correlation between excess returns and real consumption growth per capita. This shows that we need to determine how not using per capita terms affects (i) the standard deviation of consumption and (ii) the correlation between excess returns and consumption.

#### Standard deviation consumption

For convenience, the derivation will be in terms of the variance instead of standard deviation. Moreover, we assume that consumption is already in real terms. Using consumption in nominal terms just adds to the algebra, it does not affect our final result. The variance of real consumption growth is given by

$$\begin{aligned}\mathrm{Var}(\Delta c_{t+1}) &= \mathrm{Var}(c_{t+1} - c_t), \\ &= \mathrm{Var}(c_{t+1}) + \mathrm{Var}(c_t) - 2 \cdot \mathrm{Cov}(c_{t+1}, c_t).\end{aligned}$$

Using the fact that $c_t = \log[C_t/P_t]$, where $C_t$ is real consumption and $P_t$ denotes population size, this can be written as

$$\begin{aligned}
\text{Var}(\Delta c_{t+1}) &= \text{Var}(\log\left[C_{t+1}/P_{t+1}\right]) + \text{Var}(\log\left[C_t/P_t\right]) \\
&\quad - 2\cdot\text{Cov}(\log\left[C_{t+1}/P_{t+1}\right], \log\left[C_t/P_t\right]), \\
&= \text{Var}(c_{t+1} - p_{t+1}) + \text{Var}(c_t - p_t) \\
&\quad - 2\cdot\text{Cov}(c_{t+1} - p_{t+1}, c_t - p_t), \\
&= \text{Var}(c_{t+1}) + \text{Var}(p_{t+1}) - 2\cdot\text{Cov}(c_{t+1}, p_{t+1}) + \text{Var}(c_t) \\
&\quad + \text{Var}(p_t) - 2\cdot\text{Cov}(c_t, p_t) - 2\cdot\text{Cov}(c_{t+1}, c_t) \qquad (3.22) \\
&\quad + 2\cdot\text{Cov}(c_{t+1}, p_t) + 2\cdot\text{Cov}(p_{t+1}, c_t) - 2\cdot\text{Cov}(p_{t+1}, p_t).
\end{aligned}$$

When consumption growth is not denoted in per capita terms (labeled $\Delta c^*_{t+1}$), this expression boils down to

$$\text{Var}(\Delta c^*_{t+1}) = \text{Var}(c_{t+1}) + \text{Var}(c_t) - 2\cdot\text{Cov}(c_{t+1}, c_t). \qquad (3.23)$$

The difference between equations (3.22) and (3.23) is given by

$$\begin{aligned}
\text{Var}(\Delta c^*_{t+1}) - \text{Var}(\Delta c_{t+1}) &= -\text{Var}(p_{t+1}) - \text{Var}(p_t) + 2\cdot\text{Cov}(p_{t+1}, p_t) \\
&\quad + 2\cdot\text{Cov}(\Delta c_{t+1}, \Delta p_{t+1}).
\end{aligned}$$

Using that $\text{Var}(p_{t+1}) \approx \text{Var}(p_t)$ and $\text{Cov}(p_{t+1}, p_t) \approx \text{Var}(p_t)$ allows us to approximate this expression by

$$\text{Var}(\Delta c^*_{t+1}) - \text{Var}(\Delta c_{t+1}) \approx 2\cdot\text{Cov}(\Delta c_{t+1}, \Delta p_{t+1}).$$

Thus the difference is approximately equal to two times the covariance between real consumption growth and population growth; implying that the sign is determined by their correlation. In particular, if $\rho(\Delta c_{t+1}, \Delta p_{t+1}) > (<)\ 0$ then the variance of consumption growth increases (decreases) when it is not considered in per capita terms.

**Correlation excess returns and consumption**

Unfortunately there is no simple expression that shows the difference between the correlation terms. In per capita terms, the correlation equals

$$\begin{aligned}\rho(exr_t, \Delta c_{t+1}) &= \rho(exr_t, c_{t+1} - c_t)\\ &= \rho(exr_t, c_{t+1} - p_{t+1} - c_t + p_t)\\ &= \frac{\text{Cov}(exr_t, c_{t+1} - p_{t+1} - c_t + p_t)}{\sqrt{\text{Var}(exr_t)}\sqrt{\text{Var}(c_{t+1} - p_{t+1} - c_t + p_t)}}. \end{aligned} \tag{3.24}$$

In case population growth is disregarded, it is given by

$$\rho(exr_t, \Delta c^*_{t+1}) = \frac{\text{Cov}(exr_t, c_{t+1} - c_t)}{\sqrt{\text{Var}(exr_t)}\sqrt{\text{Var}(c_{t+1} - c_t)}}.$$

The difference, after decomposing the covariance in equation (3.24) and rewriting, is given by

$$\begin{aligned}\rho(exr_t, \Delta c^*_{t+1}) - \rho(exr_t, \Delta c_{t+1}) = \rho(exr_t, \Delta c_{t+1}) &- \frac{\rho(exr_t, \Delta c_{t+1})\sqrt{\text{Var}(\Delta c_{t+1})}}{\sqrt{\text{Var}(\Delta c_{t+1} - \Delta p_{t+1})}}\\ &+ \frac{\rho(exr_t, \Delta p_{t+1})\sqrt{\text{Var}(\Delta p_{t+1})}}{\sqrt{\text{Var}(\Delta c_{t+1} - \Delta p_{t+1})}}.\end{aligned}$$

As there is no way to split the denominator of the second and third term, the expression for the difference cannot be simplified any further. It is also not possible to determine the sign of the expression a priori. The reason is that the sign depends on the sign of the two correlation terms as well as the relative magnitudes of all terms in this expression. Hence, the effect on the correlation can not be determined analytically.

### 3.A.3 Results representative agent in per capita terms

**Table 3.10: Country-specific results**

Notes: $\hat{\gamma}^{\dagger}_R$ is the estimate of the coefficient of relative risk aversion based on equation (3.4) and $\hat{\sigma}_{cR}$ is the estimated standard deviation of consumption growth. The mean pseudo value is denoted by $\overline{PS}^{\dagger}_R$. The confidence intervals are centered around the mean pseudo value, the standard error is based on the square root of equation (3.19) and the critical value is taken from a $t$-distribution with degrees of freedom equal to $N-1$; see Section 3.2 for more details. $\hat{\theta}^{\dagger}_R$ is the estimate of the adjusted coefficient of relative risk aversion based on equation (3.4), but with the correlation term fixed at a value of unity. The subscript $(R)$ refers to the representative agent and the (†) indicates that these results are based on consumption growth per capita. Mnemonics are as follows: Australia (AU), Austria (OE), Belgium (BG), Canada (CN), Czech Republic (CZ), Denmark (DK), Finland (FN), France (FR), Germany (BD), Italy (IT), the Netherlands (NL), Norway (NW), Sweden (SD), Switzerland (SW), the United Kingdom (UK) and the United States (US).

| Country | Sample period | Obs | $\hat{\gamma}^{\dagger}_R$ | $\hat{\sigma}^{\dagger}_{cR}$ | $\overline{PS}^{\dagger}_R$ | 95% CI | | $\hat{\theta}^{\dagger}_R$ | $\overline{PS^{\theta}}^{\dagger}_R$ | 95% CI | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AU | 1970Q1 - 2004Q3 | 139 | 136.11 | 1.58 | 141.84 | -90.23 | 373.92 | 11.47 | 11.47 | -7.61 | 31.52 |
| OE | 1995Q2 - 2004Q3 | 38 | 1466.76 | 0.84 | 1295.54 | -923.71 | 3514.80 | 41.96 | 41.96 | -26.43 | 100.56 |
| BG | 1995Q2 - 2004Q3 | 38 | 317.25 | 1.08 | 274.76 | -191.03 | 740.55 | 40.52 | 40.52 | -24.40 | 94.59 |
| CN | 1970Q2 - 2004Q3 | 138 | 47.29 | 1.71 | 49.94 | -23.62 | 123.50 | 13.57 | 13.57 | -6.78 | 35.45 |
| CZ | 1996Q2 - 2004Q3 | 34 | -294.12 | 1.52 | -242.53 | -821.19 | 336.14 | 21.89 | 21.89 | -25.02 | 61.13 |
| DK | 1995Q2 - 2004Q3 | 38 | 124.93 | 2.36 | 118.51 | -63.46 | 300.48 | 22.39 | 22.39 | -11.37 | 53.86 |
| FN | 1990Q2 - 2004Q3 | 58 | 190.07 | 1.70 | 227.83 | -47.19 | 502.85 | 22.68 | 22.68 | -5.63 | 60.00 |
| FR | 1978Q2 - 2004Q3 | 106 | 179.22 | 1.13 | 172.51 | -9.79 | 354.81 | 31.70 | 31.70 | -1.73 | 62.75 |
| BD | 1995Q2 - 2004Q3 | 38 | 570.99 | 1.26 | 572.92 | -601.36 | 1747.20 | 23.70 | 23.70 | -24.96 | 72.51 |
| IT | 1990Q2 - 2004Q3 | 58 | 93.57 | 1.49 | 116.08 | -246.53 | 478.69 | 9.18 | 9.18 | -24.18 | 46.95 |
| NL | 1990Q2 - 2004Q3 | 58 | 154.57 | 1.70 | 181.08 | -73.85 | 436.00 | 18.08 | 18.08 | -8.64 | 51.01 |
| NW | 1995Q2 - 2004Q3 | 38 | -517.74 | 2.33 | -395.30 | -1594.45 | 803.86 | 12.61 | 12.61 | -19.58 | 38.84 |
| SD | 1995Q2 - 2004Q3 | 38 | 170.63 | 1.27 | 161.76 | -124.75 | 448.27 | 33.44 | 33.44 | -24.45 | 87.85 |
| SW | 1995Q2 - 2004Q3 | 38 | 706.41 | 0.95 | 665.42 | -289.90 | 1620.73 | 52.16 | 52.16 | -21.41 | 119.68 |
| UK | 1995Q2 - 2004Q3 | 38 | 70.39 | 0.95 | 52.95 | -191.39 | 297.29 | 19.35 | 19.35 | -52.62 | 81.74 |
| US | 1970Q2 - 2004Q3 | 138 | 73.11 | 1.35 | 76.67 | 5.13 | 148.21 | 24.76 | 24.76 | 1.74 | 50.19 |

**Table 3.11: Pooled results**

Notes: The pooled estimate of the coefficient of relative risk aversion is denoted by $\gamma_{wR^\dagger}$, and the pooled estimate of the adjusted risk aversion coefficient is denoted by $\hat{\theta}_{wR^\dagger}$. Both use weights specified according to equation (3.20). The confidence interval uses the square root of equation (3.21) as standard error and critical value from a $t$-distribution with $k-1$ degrees of freedom; see Section 3.3 for more details. The subscript ($R$) refers to the representative agent and the (†) indicates that these results are based on consumption growth per capita.

| Situation | Estimate | 95% CI | |
|---|---|---|---|
| $\hat{\gamma}_{wR^\dagger}$ | 88.01 | 41.61 | 134.40 |
| $\hat{\theta}_{wR^\dagger}$ | 19.64 | 10.85 | 28.43 |

# Chapter 4

# A Better Risk-free Rate Proxy, a Larger Equity Premium Puzzle

## 4.1 Introduction

In the realm of macroeconomics and financial economics, the equity premium puzzle remains one of the most intriguing empirical observations. The notion that one needs a (very) high coefficient of relative risk aversion in order to explain the difference between stock returns and bond returns in a consumption based asset pricing model has been at the cradle of a huge literature about the subject. Where previous studies have predominantly focused on trying to offer a resolution to this puzzle, this paper shows that the puzzle may be even larger than previously anticipated.

The novelty in our approach is that it uses data which have never been applied in this context before. One of the necessary series to estimate the coefficient of relative risk aversion is the risk-free rate. While other parts of the model, such as the functional form of the utility function, have received much attention, the same cannot be said about the risk-free rate. Most studies using U.S. data choose the 3-month T-bill rate to represent the risk-free rate. However, it is questionable whether such an asset is indeed the best proxy of a risk-free asset for the typical consumer. For the average investor, holding T-bills directly was too costly. According to the U.S. Bureau of Public Debt, the minimum purchase of Treasury bills, notes and bonds has been \$ 10,000 until August 9, 1998. From August 10, 1998 until April 6, 2008 it was \$ 1000. From April 7, 2008 until now it is \$ 100. Hence although this latter threshold will not deter retail investors too much from buying U.S. debt, the \$ 10,000 threshold that has been in effect for much of the previous century has meant that average investors have not been able to directly purchase Treasury bills to hold in their portfolio.

The 1970s saw the birth of money market mutual funds as a way for U.S. retail investors to indirectly hold U.S. government debt. Due to the large amounts of money these funds invest on behalf of their clients, this might be seen as evidence that savings deposits are losing importance. Recent evidence by the International Monetary Fund (International Monetary Fund, 2010) shows that this is not the case. As recent as 2008, the total amount of outstanding bank deposits was still twice as high as total net assets of money market mutual funds. For other developed nations, this ratio is (much) higher. In Europe money market mutual funds are predominantly used by institutional investors. Moreover, money market funds' holdings for the area as a whole are small relative to banks retail deposits (around 8 percent). Part of the reason is that some of the incentives to move from deposits to money market funds investments that are present in the U.S. (such as higher reserve requirements and no interest payable on demand deposits) are absent in the European financial system. Hence although such funds may have a central role in the U.S. financial system, this is not the case in other developed countries such as the U.K., Japan and Australia as well as those located in Europe. As a result, for inhabitants of these countries, it is still more of a challenge to directly hold short-term government debt.

It seems that some kind of return on a savings account (or time deposit) is much closer to the 'asset reality' a household is faced with. A recent study by the Investment Company Institute and the Securities and Financial Markets Association (Investment Company Institute, 2008) provides evidence supporting this view. In 2008, nearly half (47%) of U.S. households held bonds as well as equities. Nearly two-thirds of these households hold bonds, so this evidence suggests that the proportion of bond ownership in the U.S. is approximately 30%. The survey results also seem to suggest that 88% of those who own equity or bonds also hold a bank deposit account. More importantly, of the non-owners, 60% has a bank deposit account. Finally, less than one out of five U.S. households only hold equities or bonds outside of employer-sponsored retirement plans. We are most interested in those holding equities or bonds outside of such a plan since they are most likely to change consumption in response to asset price changes. Therefore the percentage of 30% mentioned just above should probably be seen as the upper threshold. In sum, at least in the U.S., the proportion of households holding government bonds is likely to be less than 30%, while the ownership of a bank deposit account is at least twice as likely.

Such survey information does not seem to be readily available for other countries. As the proportion of the population that invests in bonds does not likely differ substantially between advanced countries, we feel the U.S. evidence is representative for other developed nations as well. With respect to the percentage of the people having access to a savings account, we feel it is justified to take the survey results of Investment Company Institute (2008) as a lower bound. It is well-known that the U.S. has a low savings rate compared to other developed nations, and so one would expect a higher proportion of inhabitants of other developed countries to have a bank deposit account. Thus for countries other than the U.S., a savings rate would be even closer to the 'asset universe' a typical household is faced with.

The interest rate paid on deposits is in general lower than the rate governments pay on their short-term debt. Consequently, the excess return when using a deposit rate as the risk-free rate is higher. This implies that, ceteris paribus, the coefficient of relative risk aversion will increase and hence the equity premium puzzle will be even larger.

This hypothesis is generally supported by data from a selection of OECD countries. On average, estimates of relative risk aversion when using a deposit rate instead of a T-bill rate are approximately 40% higher. The only developed countries for which the opposite is true are Australia and the U.S.; for these two nations the deposit rate seems to be higher than the short-term government debt rate, so coefficients of relative risk aversion are lower. While puzzling at first sight, the explanations are straightforward. Regarding Australia, the solution lies in one of the characteristics of the deposit rate. Unlike other countries, deposits at Australian financial institutions are not risk-free and so the deposit rates carry an implicit default premium. The explanation for the U.S. lies in the definition of the deposit rate: instead of a proper deposit rate, there is only information on a certificates of deposit rate. As these are deposits with a fixed term (up to five years) that are intended to be held to maturity, they pay a higher interest rate than savings accounts from which the money may be withdrawn on demand. Subsequent analysis using alternative data on Australian deposit rates points in the same direction, but as these rates also carry a default premium, all results pertaining to Australian data should be interpreted with care. For the U.S., using an alternative deposit rate results in an higher estimate of relative risk aversion. In sum, for the overwhelming majority of developed nations, the equity premium puzzle is larger under deposit rates than under T-bill rates.

The remainder of this chapter is organized as follows. Section 4.2 introduces the canonical model. The third section explains our choice of the risk-free rate and compares it with the standard proxy. In addition, we briefly explain a jackknife approach for the purpose of obtaining standard errors and discuss how these estimates can be used to efficiently pool information. Data are discussed in Section 4.4. In particular, the impact of changing the risk-free rate is tested on three sets of data: a selection of OECD countries, other proxies for individual countries using national sources and monthly U.S. data. Section 4.5 presents the results, i.e. we show how estimates of the coefficient of relative risk aversion differ for the two risk-free rates in the three situations identified just above. The final section concludes and offers suggestions for future research.

## 4.2 Model

The canonical framework, (Grossman and Shiller, 1981; Hansen and Jagannathan, 1991; Cochrane and Hansen, 1992), assumes the existence of a representative investor who has a time-separable utility function defined over aggregate consumption. Under these assumptions, the Euler equation reads

$$1 = E_t[R_{t+1}\delta(C_{t+1}/C_t)^{-\gamma}]. \tag{4.1}$$

Here $R$ is the gross return on an asset, $\delta$ is the subjective discount factor, $C$ is aggregate consumption and $\gamma$ is the coefficient of relative risk aversion. Next, following Hansen and Singleton (1983), the simplifying assumption is made that asset returns and aggregate consumption are jointly conditionally lognormally distributed with constant variance. Taking logs of equation (4.1) and rewriting leads to

$$\mathrm{E}_t r_{t+1} = -\log\delta + \gamma \mathrm{E}_t \Delta c_{t+1} - \frac{\sigma_r^2 + (\gamma\sigma_c)^2 - 2\gamma\sigma_{(r,c)}}{2}. \tag{4.2}$$

Small case letters denote logarithms; $\sigma_r^2$ and $\sigma_c^2$ are, respectively, the unconditional variance of log returns and consumption innovations and $\sigma_{(r,c)}$ is their unconditional covariance. Consider the implications of equation (4.2) for a riskless real asset. Obviously, the variance of its returns and its covariance with any random variable are zero, implying that the riskless real interest rate obeys

$$rf_{t+1} = -\log\delta + \gamma \mathrm{E}_t \Delta c_{t+1} - \frac{(\gamma\sigma_c)^2}{2}. \tag{4.3}$$

Combining equations (4.2) and (4.3) gives

$$\mathrm{E}_t[r_{i,t+1} - rf_{t+1}] + \frac{\sigma_{exr}^2}{2} = \gamma\sigma_{(exr,c)}. \tag{4.4}$$

Equation (4.4) shows that expected excess returns of equity are equal to the amount of risk aversion times the covariance between consumption growth and excess returns minus a correction factor (i.e. a Jensen inequality term arising as we are using expectations of log returns). This expression permits a calibration of the risk aversion parameter $\gamma$ from estimates of all three moment conditions.

These moments also take center stage in explaining why there is an equity premium puzzle. On the left-hand side is (i) the expected excess return which for almost all countries is rather high over an extended period of time and (ii) a half times the variance of this excess return. On the right-hand side is the covariance between excess returns and consumption growth. It is empirically well-known that consumption growth is very smooth. As a result, this covariance is rather small. Hence the only way for this equation to hold is through a (very) high value of $\gamma$, i.e. the equity premium puzzle.

Estimates of the relative risk aversion coefficient vary substantially across countries and sample periods — sometimes estimates even have a negative sign, implying risk-loving behavior — but values above 50 are not uncommon. Such a value is puzzling as it is irreconcilable with micro-based evidence derived from individual choice under uncertainty. Mehra and Prescott (1985) already argue than values above 10 are implausible; for an illuminating example using a simple lottery, see Mankiw and Zeldes (1991).

## 4.3 Methodology

Existing research proposing resolutions to the equity premium puzzle have looked at for example limited stock market participation (e.g. Mankiw and Zeldes, 1991; Vissing-Jørgenson, 2002), incomplete markets and transaction costs (Aiyagari, 1993), and adjusting the utility function (e.g. Epstein and Zin, 1989; Campbell and Cochrane, 1999). Our modification to the model is of a different nature. To the best of our knowledge, the 3-month T-bill rate is almost always used as a proxy for the risk-free rate. The reason for this choice is likely twofold. First, the T-bill rate is widely used in applied work as (a proxy for) the risk-free rate. Second, the maturity of this asset matches the quarterly frequency at which detailed information regarding consumption is available in countries' national accounts.

We propose to use a savings rate as a proxy for the risk-free rate instead. In particular, a deposit rate.[1] The primary reason is that this rate is more closely related to the assets households have access to. As outlined in the introduction, there do not seem to be many households that directly hold government debt, while most of us have some kind of savings account at a financial institution. So, from the average household's perspective, this is a more natural proxy for the risk-free rate. The majority of savings account holders do not have a balance exceeding the deposit guarantee, so for them the savings account rate is not just a proxy, it is the actual risk-free rate.[2]

### 4.3.1 Standard errors

Studies about the equity premium puzzle usually only report the point estimates of the coefficient of relative risk aversion. To get an idea of the uncertainty surrounding such an estimate, Pozzi et al. (2010) introduce a simple method to construct standard errors for estimates of the coefficient of relative risk aversion.

This method uses the block-jackknife procedure, see e.g. Shao and Tu (1995). The main reasons are that it is easy to implement and it can deal with the serial correlation that seems to be present in the consumption series. Let $n$ be the size of the sample, let $m$ denote the number of omitted observations in a resample and let $N$ denote the number of resamples. Note that the total number of resamples is $N = n - m + 1$. To estimate the variance of $\hat{\gamma}_n$, the estimate of $\gamma$ based on all observations, one first deletes $m$ subsequent observations at a time and denotes the new estimate of the risk aversion coefficient by $\hat{\gamma}^{(i)}$. Then the $i$-th pseudovalue of $\hat{\gamma}_n$, denoted by $\tilde{\gamma}^{(i)}$, is defined as $\left[n\hat{\gamma}_n - (n-m)\hat{\gamma}^{(i)}\right]/m$. The resulting vector of pseudovalues across the resamples

[1] Definitions vary from country to country, but it predominantly refers to a 3-month deposit.

[2] As mentioned in the introduction, this is not the case for Australia.

is used to estimate the variance of $\hat{\gamma}_n$, i.e.

$$S^2_{\hat{\gamma}_n} = \frac{m}{nN}\sum_{i=1}^{N}\left(\tilde{\gamma}^{(i)} - \frac{1}{N}\sum_{i=1}^{N}\tilde{\gamma}^{(i)}\right)^2. \tag{4.5}$$

This estimate can then be used to form a country-specific confidence interval for $\hat{\gamma}_n$. More specifically, the Quenouille-Tukey mean of the pseudovalues, see Shao and Tu (1995, p. 6), is the bias-corrected version of the estimate of $\gamma_n$ and the required critical value is taken from a $t$-distribution, with $N-1$ degrees of freedom (Miller, 1974).

### 4.3.2 Pooling

The cross-sectional dimension of our data set allows the computation of pooled estimators for both proxies, to which we refer as the world coefficient of relative risk aversion ($\gamma_w$). A priori, there is no reason to assume that the information contained in the country-specific estimates is equal, so using a weighted average seems to be a natural choice. As the block-jackknife procedure supplies the country-specific sample variances, the optimal (i.e. unbiased and variance minimizing) weights of Graybill and Deal (1959) can be used. These are defined as

$$w_j = \frac{1/S^2_{\hat{\gamma}_{n,j}}}{\sum_{j=1}^{k} 1/S^2_{\hat{\gamma}_{n,j}}}, \tag{4.6}$$

where $k$ denotes the number of countries. Confidence intervals for the pooled estimators are then formed by weighting the country-specific averages of the pseudo values to get $\hat{\gamma}_w$. It is straightforward to show that the variance of this estimate is given by

$$S^2_{\hat{\gamma}_w} = \frac{1}{\sum_{j=1}^{k} 1/S^2_{\hat{\gamma}_{n,j}}}. \tag{4.7}$$

Critical values are taken from a $t$-distribution with $k-1$ degrees of freedom.

## 4.4 Data

The above model and methodology is applied to a selection of OECD countries. Most studies concerning the equity premium puzzle use U.S. data but it is well-known that the U.S. savings rate is quite low. In contrast, European citizens tend to save a larger proportion of their income. Using data from a number of developed countries can determine whether the difference in risk aversion estimates between the two proxies is related to a country's saving behavior. In addition, Australia and New Zealand have data on alternative deposit rates, we investigate their impact as well. The main drawback is that the number of observations in these analyses is limited since consumption is only available at the quarterly frequency. In contrast, the U.S. has monthly consumption data. Therefore, we also estimate the model using monthly U.S. data, taken from national sources.

### 4.4.1 Selection of OECD countries

The selection of OECD countries consists of Australia, Austria, Belgium, Canada, Czech Republic, Finland, France, Germany, Italy, Japan, Korea, the Netherlands, New Zealand, Norway, Poland, Spain, Sweden, Switzerland, the United Kingdom and the United States. Various data limitations force us to drop the remaining 14 countries. For Iceland, Luxembourg and the Slovak Republic there is no suitable equity return data; Denmark misses data on the deposit rate. The four countries that joined the OECD in 2010 (Chile, Estonia, Israel and Slovenia) do not have data on consumption and inflation. Too few observations leads us to disregard Greece, Hungary, Ireland, Mexico, Portugal and Turkey. The start of the sample period is country-specific, depending on data availability; the end of the timespan is restricted to the final quarter of 2008. Since consumption and population series are often revised until a few years after initial publication, the sample period is restricted to ensure only final values enter the empirical investigation.

Data for this part are taken from three sources, but all are obtained through Datastream. First, private final consumption expenditure (in constant prices) is taken from the Quarterly National Accounts of the OECD. The Consumer Price Index (all items) is from the OECD's Main Economic Indicators.[3] For Australia and New Zealand this series is quarterly, for the other countries there is a monthly index; for these we use the value of the final month of the quarter. Second, data to construct the equity return are from Morgan Stanley Capital International. In particular, we use the monthly total return and price index for the 'Standard Country' type in local currency units. Finally, the risk-free rates are taken from the International Financial Statistics, provided by the IMF. The deposit rate (line 60L) is readily available, for the standard risk-free rate the 3-month T-bill rate (60C) is selected; if this series is not available we use the money market rate (line 60B) instead. Annual mid-year population figures (line 99Z) are also taken from the IFS. Again, more details about the construction of the real excess return and the transformation of the annual population figures into quarterly estimates are given in Appendix 2.A.1.

### 4.4.2 Australia and New Zealand

Australia and New Zealand have data on alternative deposit rates as well. The former has the most extensive data on deposit rates of any developed nation. In total, the Royal Bank of Australia has information on 12 types of deposit accounts (acronyms in parentheses)[4]: (i) Banks's bonus savings accounts (BSA), (ii) Online savings accounts (OSA), (iii) Cash management accounts at banks (CM1), (iv) Cash management accounts at banks (minimum balance of AUD 50,000; CM5), (v) Cash management trusts (CMT), (vi) Banks' term deposits for 1 month (T1M), (vii) Banks' term deposits for 3 months (T3M), (viii) Banks' term deposits for 6 months (T6M), (ix) Banks' term deposits for 1 year (T1Y), (x) Banks' term deposits for 3 years (T3Y), (xi) Banks' term deposits, average rate (all terms; TAN) and (xii) Banks' term deposits, average

[3] Note that this implies that our results are independent of the measure of inflation used to make the nominal interest rates real. Of course, they still depend on the implicit deflator used in the computation of consumption denoted in constant prices.

[4] All have minimum balance of AUD 10,000; unless indicated otherwise.

'special' rate (all terms; TAS).

The Royal Bank of New Zealand keeps track of only one series, i.e. the interest paid for a new six month term deposit of $ 10,000 (TDR). For both countries, the other series are are identical to those used in the first part of the empirical investigation.

### 4.4.3 Monthly U.S. data

The U.S. is the only country that has data on consumption at a monthly frequency, which allows us to benefit from the increase in the number of observations. The first category of data is macroeconomic, i.e. real consumption per capita and inflation. The Bureau of Economic Analysis (BEA) has monthly data on total consumption in chained dollars, but we are interested in real nondurables and services consumption. Hence we compute this series ourselves using the 'divisia approximation' as explained in Whelan (2002, p. 225). This method requires data on total consumption and durable goods consumption in threefold: nominal terms, the price indices and in chained dollars. The resulting real nondurables and services consumption is divided by monthly population estimates, also provided by the BEA. An implicit price deflator is then constructed by using data on nominal nondurables and services consumption and its newly created chained dollars counterpart.

Financial data comprise the second category of data. In particular, we require an equity return and a risk-free rate. For the former, the monthly log return on the S&P Composite Index is used; this series is provided by CRSP. As mentioned, we use and contrast two different proxies for the risk-free rate. The first is the standard 3-month T-bill rate, taken from FRED. For the interest rate on savings deposits, we use the average interest rate on savings deposits, provided by BankRate Monitor. As an additional check, we also use the deposit rate (i.e. certificate of deposit rate) supplied by the IMF. A more detailed description of the data is given in Appendix 4.A.1.

## 4.5 Results

This section discusses the results of using the two proxies for the risk-free rate. First, the focus is on the group of OECD countries. Here we compute standard errors for the coefficient of relative risk aversion and then use these statistics to compute the optimally weighted pooled estimators. Hereafter we discuss the outcomes of using alternative deposit rates of Australia and New Zealand. Finally, we contrast three proxies using monthly U.S. data.

### 4.5.1 Selection of OECD countries

Before discussing the effect that our different risk-free rate has on estimates of the coefficient of relative risk aversion, it might be insightful to first show how the two proxies relate to each other. As shown in Table 4.1, the average deposit rate is lower than the average T-bill rate for most countries. Given that monetary financial institutions $pay$ the deposit rate and $earn$ the T-bill rate on some of their investments, this is what one expects to see. There are only two exceptions: Australia and the United States. Although puzzling at first sight, there is a straightforward explanation for both countries. First, deposits at an Australian Authorized Deposit-taking Institution are not risk-free, i.e. unlike other countries, there is no explicit guarantee on deposits. This has only been introduced in late 2008. Hence for the sample period in question, the deposit rate cannot be seen as a risk-free rate for households as it contains an implicit default premium. With respect to the U.S., the explanation also lies in the characterization of the deposit rate. In contrast to the other countries included in this analysis, there is no U.S. savings deposit rate available. The closest substitute is an average certificate of deposit rate of term deposits up to five years. This is much longer than other countries' rates which are based on three-month fixed term deposits or deposits than can be withdrawn at notice. Therefore, the U.S. deposit rate carries a term premium. In sum, results for both countries should be interpreted with care.

In addition, besides the lower average, the deposit rates also show the least amount of variability; i.e. there seems to be a correspondence between lower averages and lower standard deviations. Taken together, these results already suggest that using the deposit rate as the risk-free rate instead of the T-bill rate will increase the coefficients of relative risk aversion and hence lead to a larger equity premium puzzle. The reason is simple. Notice that the left-hand side of equation (4.4) consists of two terms. With respect to the first term it is easy to see that a lower risk-free rate implies a higher excess return. There is no clear distinction between the variance of the excess returns under the two competing proxies, they are almost as often higher under the T-bill rate as vice versa. However, the differences are very small (the largest value is 0.015 for Korea) so this second term is clearly dominated by the first term. Hence the left-hand side of equation (4.4) is higher under the deposit rate. Since it is well-known that consumption growth is smooth, the covariance will not be affected much and so the right-hand sides will be more or less equal for both proxies. It then follows that relative risk aversion is expected to be higher under the deposit rate than under the T-bill rate.

Estimates of the coefficient of relative risk aversion based on equation (4.4) are shown in Table 4.2. Note that the sample period is country-specific, depending on data availability. The second and third column present the excess returns (plus half of the variance) for both proxies of the risk-free rate, where the subscripts refer to the deposit rate ($D$) and the T-bill (or money market) rate ($T$). Comparing the numbers in both columns reveals that the choice of risk-free rate generally has a limited influence (the average difference is 124 basis points), but for some countries the difference is substantial. Another striking feature is that for Australia and the United States, the excess return under the T-bill rate is higher than under the deposit rate; a result in line with the outcomes presented in Table 4.1.

Given that the excess return is the numerator in the fraction that determines the coefficient of relative risk aversion, one expects a higher excess return to be associated with a larger value of $\gamma$ and this is indeed the case. For Australia and the United States the coefficient of relative risk aversion is higher under the T-bill rate (column 5) than under the deposit rate (column 4), and vice versa for the other countries. On average,

**Table 4.1: Descriptive statistics risk-free rate proxies**

Notes: $Obs$ is the number of observations. $\mu$ and $\sigma$ denote, respectively, the average and standard deviation. Subscripts refer to the applied risk-free rate: $D$ for the deposit rate, $T$ for the T-bill (or money market) rate. All statistics in annualized percentages. The standard deviation is the square root of the sample variance. Sample periods are country-specific, depending on data availability. Due to revisions in consumption and population figures, the sample period is restricted to 2008Q4. Average is the simple cross-country arithmetic average.

| Country | Sample period | Obs | $\mu_D$ | $\sigma_D$ | $\mu_T$ | $\sigma_T$ |
|---:|---:|---:|---:|---:|---:|---:|
| Australia | 1972Q4 - 2008Q3 | 144 | 9.11 | 4.61 | 8.54 | 3.77 |
| Austria | 1988Q2 - 2000Q3 | 50 | 2.69 | 0.70 | 5.68 | 2.45 |
| Belgium | 1995Q2 - 2003Q4 | 35 | 2.82 | 0.67 | 3.35 | 0.72 |
| Canada | 1971Q2 - 2008Q3 | 150 | 5.78 | 3.80 | 7.13 | 3.82 |
| Czech Republic | 1996Q2 - 2008Q2 | 49 | 3.29 | 2.53 | 5.49 | 3.96 |
| Finland | 1990Q2 - 1997Q4 | 31 | 4.61 | 2.31 | 7.92 | 4.28 |
| France | 1978Q2 - 2008Q3 | 122 | 4.52 | 1.83 | 7.13 | 4.05 |
| Germany | 1977Q3 - 2003Q2 | 104 | 4.76 | 2.14 | 5.25 | 2.20 |
| Italy | 1983Q2 - 2003Q4 | 83 | 6.00 | 3.28 | 9.32 | 4.44 |
| Japan | 1980Q2 - 2008Q3 | 114 | 1.74 | 1.64 | 2.16 | 2.15 |
| Korea | 1988Q1 - 2008Q3 | 83 | 7.72 | 2.75 | 9.07 | 5.09 |
| Netherlands | 1988Q2 - 2008Q3 | 82 | 3.29 | 0.68 | 4.53 | 2.30 |
| New Zealand | 1990Q2 - 2008Q3 | 74 | 6.90 | 1.69 | 7.12 | 2.02 |
| Norway | 1986Q3 - 2008Q3 | 89 | 6.33 | 3.12 | 7.51 | 4.17 |
| Poland | 1995Q2 - 2006Q4 | 47 | 11.06 | 7.37 | 12.95 | 7.06 |
| Spain | 1995Q2 - 2003Q1 | 32 | 3.65 | 1.81 | 4.78 | 2.07 |
| Sweden | 1980Q2 - 2006Q2 | 105 | 6.25 | 3.97 | 8.11 | 4.31 |
| Switzerland | 1981Q2 - 2008Q3 | 110 | 3.15 | 2.52 | 3.28 | 2.44 |
| United Kingdom | 1970Q1 - 1998Q4 | 116 | 7.83 | 3.46 | 9.36 | 3.02 |
| United States | 1970Q1 - 2008Q3 | 155 | 6.59 | 3.35 | 5.82 | 2.95 |
| Average | | | 5.40 | 2.71 | 6.72 | 3.36 |

the difference in estimates is close to 40%. Also notice the large variability in estimates across countries, a feature that is in line with previous research (e.g. Campbell, 1999). Looking at the values of $\gamma$ under the T-bill rate, the equity premium puzzle is prominent. The sole exception is Korea, where the value of 9.5 is still within the plausible range. New Zealand's estimate is even lower but has a negative sign, which would imply risk-loving behavior. Note that these low values are the result of the extremely low average excess return. The closer this statistic is to zero, the smaller the resulting relative risk aversion coefficient, see equation (4.4).

**Table 4.2: Estimates of relative risk aversion coefficient**
Notes: $aeRe$ is the is the average excess log return on the MSCI index over the risk-free rate plus one half of the variance of this excess return. The estimated coefficient of relative risk aversion is denoted by $\hat{\gamma}$. Subscripts refer to the applied risk-free rate: $D$ for the deposit rate, $T$ for the T-bill (or money market) rate. See Table 4.1 for country-specific sample periods. Average is the simple cross-country arithmetic average.

| Country | Obs | $aeRe_D$ | $aeRe_T$ | $\hat{\gamma}_D$ | $\hat{\gamma}_T$ |
|---:|---:|---:|---:|---:|---:|
| Australia | 144 | 4.60 | 5.13 | 189.53 | 221.05 |
| Austria | 50 | 8.40 | 5.58 | 300.44 | 199.35 |
| Belgium | 35 | 8.90 | 8.37 | 285.54 | 267.69 |
| Canada | 150 | 5.84 | 4.57 | 63.47 | 48.67 |
| Czech Republic | 49 | 16.46 | 14.45 | -1496.98 | -1574.62 |
| Finland | 31 | 16.43 | 13.25 | 277.38 | 174.02 |
| France | 122 | 10.27 | 7.81 | 190.52 | 139.97 |
| Germany | 104 | 6.61 | 6.13 | 209.28 | 198.07 |
| Italy | 83 | 8.63 | 5.57 | 268.61 | 162.52 |
| Japan | 114 | 5.15 | 4.74 | 92.74 | 87.77 |
| Korea | 83 | 7.01 | 5.89 | 11.40 | 9.59 |
| Netherlands | 82 | 8.24 | 7.02 | 266.05 | 216.74 |
| New Zealand | 74 | 0.16 | -0.09 | 2.97 | -1.67 |
| Norway | 89 | 7.12 | 5.98 | 773.85 | 477.52 |
| Poland | 47 | 8.59 | 7.00 | 101.23 | 77.09 |
| Spain | 32 | 13.69 | 12.55 | 186.66 | 170.98 |
| Sweden | 105 | 17.00 | 15.27 | 973.18 | 660.32 |
| Switzerland | 110 | 9.11 | 8.95 | 595.30 | 589.31 |
| United Kingdom | 116 | 9.01 | 7.60 | 104.41 | 87.18 |
| United States | 155 | 4.39 | 5.09 | 55.77 | 65.49 |
| Average | | 8.78 | 7.54 | 172.57 | 113.85 |

Given the one-to-one correspondence between the average excess returns and the coefficients of relative risk aversion, the standard deviation of the excess return and the correlation between consumption growth and the excess return do not seem to influence the results much. This is verified by Table 4.3. There is no clear distinction between the standard deviations of the two excess returns across countries, they are almost as often higher under the deposit rate than under the T-bill rate and vice versa. Moreover, the differences are relatively small and hence do not affect the estimate of

$\gamma$ to any considerable extent. The same is true for the correlations. They are predominantly more or less equal and when there is a difference it does not exceed 0.02. Therefore, the covariances between consumption growth and excess returns are close to each other, the average difference is less than 0.24, which is small compared to the difference between the average excess returns.

These numbers also show what drives the low values of relative risk aversion for Korea. Compared to the other countries, Korea's excess return under both specifications displays the highest degree of variability. In addition, the correlations between consumption growth and the excess returns are also (much) higher than for other OECD countries. As a result, the covariances between consumption growth and the excess returns are much larger, as indicated by the final two columns of Table 4.3.

Hence for all countries the difference in estimates of $\gamma$ between the two proxies for the risk-free rate is predominantly driven by differences in average excess returns (i.e. the numerator). Sometimes the covariance in the denominator has the opposing effect, but due to the small relative differences, this does not exert much influence on the value of relative risk aversion.

In order to infer the degree of precision with which coefficients of relative risk aversion are estimated, Table 4.4 presents the mean of the pseudovalues and the 95% confidence intervals. In line with the results in Pozzi et al. (2010) and De Vries and Zenhorst (2012), we obtain evidence that the mean pseudovalues are occasionally far removed from the value of the coefficient of relative risk aversion, but at the same time some country-specific $\gamma$'s only seem to suffer from a marginal bias. The standard deviations are substantial and hence the confidence intervals are in general wide.

As discussed in Section 3.2, the sample variances of the country-specific estimates of the coefficient of relative risk aversion can be used to construct an optimally weighted pooled estimator. As Panel A of Table 4.5 shows, the pooled estimators differ substantially. When one focuses solely on these point estimates, it appears that risk aversion estimates are indeed larger under the deposit rates than under the T-bill rates. The country-specific results in Table 4.2 also support this point of view.

**Table 4.3: Standard deviations, correlations and covariances**

Notes: The standard deviation of the excess return is denoted by $\sigma_{exr}$, the correlation between the excess return and consumption growth is denoted by $\rho$ and $cov$ is the covariance between these two series. Subscripts refer to the applied risk-free rate: $D$ for the deposit rate, $T$ for the T-bill (or money market) rate. See Table 4.1 for country-specific sample periods. Average is the simple cross-country arithmetic average.

| Country | $\sigma_{(exr,D)}$ | $\sigma_{(exr,T)}$ | $\rho_D$ | $\rho_T$ | $cov_D$ | $cov_T$ |
|---:|---:|---:|---:|---:|---:|---:|
| Australia | 20.40 | 20.41 | 0.08 | 0.07 | 2.43 | 2.32 |
| Austria | 26.18 | 26.15 | 0.24 | 0.24 | 2.80 | 2.80 |
| Belgium | 23.43 | 23.43 | 0.16 | 0.16 | 3.12 | 3.13 |
| Canada | 17.14 | 17.16 | 0.34 | 0.34 | 9.20 | 9.38 |
| Czech Republic | 24.86 | 25.13 | -0.03 | -0.02 | -1.10 | -0.92 |
| Finland | 34.10 | 34.30 | 0.08 | 0.10 | 5.92 | 7.62 |
| France | 22.48 | 22.53 | 0.22 | 0.23 | 5.39 | 5.58 |
| Germany | 24.50 | 24.50 | 0.06 | 0.06 | 3.16 | 3.09 |
| Italy | 25.25 | 25.39 | 0.09 | 0.09 | 3.21 | 3.43 |
| Japan | 21.50 | 21.49 | 0.14 | 0.13 | 5.55 | 5.40 |
| South Korea | 35.03 | 35.29 | 0.41 | 0.41 | 61.48 | 61.38 |
| Netherlands | 19.62 | 19.65 | 0.10 | 0.10 | 3.10 | 3.24 |
| New Zealand | 17.00 | 17.03 | 0.18 | 0.18 | 5.32 | 5.35 |
| Norway | 28.07 | 28.03 | 0.02 | 0.02 | 0.92 | 1.25 |
| Poland | 29.63 | 29.77 | 0.24 | 0.26 | 8.49 | 9.09 |
| Spain | 28.29 | 28.28 | 0.27 | 0.27 | 7.34 | 7.34 |
| Sweden | 28.82 | 28.75 | 0.03 | 0.04 | 1.75 | 2.31 |
| Switzerland | 20.04 | 20.02 | 0.09 | 0.09 | 1.53 | 1.52 |
| United Kingdom | 21.31 | 21.32 | 0.17 | 0.17 | 8.63 | 8.71 |
| United States | 16.57 | 16.54 | 0.35 | 0.34 | 7.87 | 7.77 |
| Average | 24.21 | 24.26 | 0.16 | 0.16 | 7.30 | 7.49 |

The standard deviations for the pooled estimators are much smaller than those for the country-specific estimates and so the confidence intervals are much tighter. However, they span more or less the same range and so the difference between the pooled estimators is not statistically significant at any conventional significance level. In light of the discussion regarding the Australian and American deposit rates, Panel B presents the pooled estimators while excluding the risk aversion estimates of these two countries. The results are qualitatively similar and so the pooled estimators are still not statistically significant from each other.

**Table 4.4: Confidence intervals**

Notes: The estimate of the coefficient of relative risk aversion based on equation (4.4) is denoted by $\hat{\gamma}$. The mean pseudovalue is denoted by $\overline{PS}$. The confidence intervals are centered around the mean pseudovalue, the standard error is based on the square root of equation (4.5) and the critical value is taken from a $t$-distribution with degrees of freedom equal to $N-1$; see Section 4.2 for more details. Subscripts refer to the applied risk-free rate: $D$ for the deposit rate, $T$ for the T-bill (or money market) rate. See Table 4.1 for country-specific sample periods.

| Country | $\hat{\gamma}_D$ | $\overline{PS}_D$ | 95% CI | | $\hat{\gamma}_T$ | $\overline{PS}_T$ | 95% CI | |
|---|---|---|---|---|---|---|---|---|
| Australia | 189.53 | 1449.67 | -5972.42 | 8871.76 | 221.05 | -137.20 | -4651.87 | 4377.46 |
| Austria | 300.44 | 225.05 | -175.69 | 625.80 | 199.35 | 361.44 | -356.17 | 1079.06 |
| Belgium | 285.54 | 28.85 | -524.33 | 582.04 | 267.69 | 32.33 | -504.67 | 569.33 |
| Canada | 63.47 | 60.46 | -29.46 | 150.39 | 48.67 | 46.53 | -35.39 | 128.45 |
| Czech Republic | -1496.98 | -16544.72 | -28651.79 | -4437.64 | -1574.62 | -14220.52 | -24224.26 | -4216.78 |
| Finland | 277.38 | -2313.52 | -20788.48 | 16161.44 | 174.02 | 982.56 | -4105.16 | 6070.27 |
| France | 190.52 | 176.96 | -37.28 | 391.21 | 139.97 | 132.31 | -46.73 | 311.36 |
| Germany | 209.28 | -6634.97 | -35043.30 | 21773.35 | 198.07 | 4844.28 | -11667.71 | 21356.26 |
| Italy | 268.61 | -2021.21 | -7753.81 | 3711.39 | 162.52 | -403.45 | -2189.11 | 1382.22 |
| Japan | 92.74 | 50.12 | -194.86 | 295.09 | 87.77 | 43.48 | -204.44 | 291.41 |
| Korea | 11.40 | 3.96 | -40.65 | 48.58 | 9.59 | 2.80 | -40.61 | 46.20 |
| Netherlands | 266.05 | 85.20 | -590.05 | 760.44 | 216.74 | 93.69 | -446.16 | 633.53 |
| New Zealand | 2.97 | -1.60 | -183.71 | 180.52 | -1.67 | -3.79 | -178.35 | 170.78 |
| Norway | 773.85 | 7363.51 | -40873.70 | 55600.72 | 477.52 | 968.12 | -5104.47 | 7040.71 |
| Poland | 101.23 | 35.47 | -264.81 | 335.74 | 77.09 | 22.24 | -236.15 | 280.63 |
| Spain | 186.66 | 90.69 | -262.28 | 443.66 | 170.98 | 89.33 | -248.59 | 427.26 |
| Sweden | 973.18 | 2416.34 | -28547.44 | 33380.12 | 660.32 | 980.67 | -10688.96 | 12650.30 |
| Switzerland | 595.30 | 349.76 | -649.87 | 1349.38 | 589.31 | 342.49 | -652.05 | 1337.02 |
| United Kingdom | 104.41 | 29.73 | -210.49 | 269.94 | 87.18 | 26.25 | -180.27 | 232.77 |
| United States | 55.77 | 50.86 | -48.91 | 150.64 | 65.49 | 59.30 | -46.85 | 165.45 |

**Table 4.5: Pooling**
Notes: The pooled estimate of the coefficient of relative risk aversion is denoted by $\hat{\gamma}_w$, using weights specified according to equation (4.6). The confidence interval uses the square root of equation (4.7) as standard error and critical value from a $t$-distribution with $k-1$ degrees of freedom; see Section 4.3 for more details. Panel A: Subscripts refer to the applied risk-free rate: $D$ for the deposit rate, $T$ for the T-bill (or money market) rate. Panel B: Same as Panel A but Australia and the U.S. are excluded. Panel C: Pooled estimator of 12 Australian deposit rates.

| Specification | Estimate | 95% CI | |
|---|---|---|---|
| Panel A | | | |
| $\hat{\gamma}_{wD}$ | 26.83 | -10.05 | 63.71 |
| $\hat{\gamma}_{wT}$ | 23.53 | -12.07 | 59.13 |
| Panel B | | | |
| $\hat{\gamma}_{wD}$ | 23.59 | -15.99 | 63.17 |
| $\hat{\gamma}_{wT}$ | 19.71 | -18.06 | 57.47 |
| Panel C | | | |
| $\hat{\gamma}_{AU}$ | 119.98 | 12.88 | 227.08 |

### 4.5.2 Australia and New Zealand

As mentioned before, Australia and New Zealand are the only two countries for which there are long time series on alternative deposit rates. Results of using these series in our model are presented in Table 4.6. Estimates of $\gamma$ for Australia range between 57.5 when using 'online savings accounts' and 228.8 when using 'one-month term deposits'. The estimate of using the deposit rate from the IMF is 189.5; this is within the range of values shown here. An outcome worth noticing is that there seems to be a relation between the number of observations and the value of the correlation coefficient between consumption growth and excess returns. The shorter the sample period, the larger the correlation coefficient seems to be. This is also the main driving force between differences in values of relative risk aversion. As these estimates all apply to the same country, it is insightful to investigate what value the pooled estimate would take. Using the jackknife approach to estimate standard deviations and then weighting optimally gives a value of 120 (see Table 4.5). This is almost twice as low as the value for the T-bill rate, but as mentioned before, all Australian deposit rates carry an implicit default premium and hence these results should be interpreted with care.

The final row of Table 4.6 shows that the low value of $\gamma$ for New Zealand in Table 4.2 is quite robust, given that it is 2.9 for this six-month term deposit rate. This is no surprise as the standard deviation of the excess return and the correlation between excess returns and consumption growth are practically similar. However, just like before, the main reason for this low value of relative risk aversion is the small average excess return.

**Table 4.6: In-depth analysis of Australia and New Zealand**
Notes: $aeRe$ is the is the average excess log return on the MSCI index over the risk-free rate plus one half of the variance of this excess return. The standard deviation of the excess return is denoted by $\sigma_{exr}$, $\rho$ denotes the correlation between the excess return and consumption growth and $cov$ is the covariance between these two series. The estimated coefficient of relative risk aversion is denoted by $\hat{\gamma}$. See Section 4.2 for an explanation of mnemonics.

| Country | Sample period | Obs | $aeRe$ | $\sigma_{exr}$ | $\rho$ | $cov$ | $\hat{\gamma}$ |
|---|---|---|---|---|---|---|---|
| Australia | | | | | | | |
| BSA | 2002Q3 - 2008Q3 | 25 | 7.23 | 13.05 | 0.55 | 10.12 | 71.43 |
| OSA | 2004Q2 - 2008Q3 | 18 | 6.60 | 14.26 | 0.69 | 11.47 | 57.53 |
| CM1 | 1989Q4 - 2008Q3 | 76 | 6.17 | 12.88 | 0.30 | 4.56 | 135.13 |
| CM5 | 1989Q4 - 2008Q3 | 76 | 4.90 | 12.83 | 0.30 | 4.53 | 108.16 |
| CMT | 1983Q2 - 2008Q3 | 102 | 6.98 | 17.57 | 0.16 | 3.90 | 179.11 |
| T1M | 1982Q3 - 2008Q3 | 105 | 8.70 | 17.41 | 0.16 | 3.81 | 228.77 |
| T3M | 1982Q2 - 2008Q3 | 106 | 7.64 | 17.37 | 0.16 | 3.73 | 204.98 |
| T6M | 1982Q2 - 2008Q3 | 106 | 7.37 | 17.34 | 0.15 | 3.70 | 199.25 |
| T1Y | 1982Q2 - 2008Q3 | 106 | 7.01 | 17.38 | 0.15 | 3.71 | 189.01 |
| T3Y | 1982Q3 - 2008Q3 | 105 | 6.99 | 17.46 | 0.15 | 3.68 | 189.63 |
| TAN | 2002Q2 - 2008Q3 | 26 | 5.73 | 13.07 | 0.52 | 9.36 | 61.17 |
| TAS | 2002Q2 - 2008Q3 | 26 | 4.42 | 13.19 | 0.52 | 9.48 | 46.60 |
| New Zealand | | | | | | | |
| TDR | 1990Q2 - 2008Q3 | 74 | 0.16 | 16.99 | 0.18 | 5.32 | 2.92 |

### 4.5.3 Monthly U.S. data

Again we start by showing how the three proxies relate to each other. As shown in Table 4.7, the certificate of deposit rate has the highest average, followed by the T-bill rate while the savings rate has the lowest average. But the higher the average, the higher the variability of the specific risk-free rate. Just like before, the averages of the risk-free rates are important as they will largely determine the value of relative risk aversion. Since the savings rate has the lowest average, it is expected to have the highest coefficient of relative risk aversion. In contrast, the certificate of deposit rate has the highest average, so it is likely to produce the lowest value of $\gamma$, while relative risk aversion under the short-term government debt rate is expected lie between those of the other two proxies.

**Table 4.7: Descriptive statistics risk-free rates**
Notes: The sample period runs from January 1990 until December 2008. The certificate of deposit rate is the average of dealer offering rates on nationally traded certificates of deposit. The savings rate is the average interest rate on savings deposits, provided by BankRate Monitor. The T-bill rate is the interest rate on 3-month T-bills. All statistics are annualized and in percentage points. The standard deviation is the square root of the sample variance.

| Proxy | Average | Standard deviation |
|---:|---:|---:|
| Certificate of deposit rate | 4.46 | 1.84 |
| Savings rate | 2.17 | 1.48 |
| T-bill rate | 3.96 | 1.76 |

The results in Table 4.8 show that the average excess return (plus half of its variance) is indeed much higher for the savings rate than for the T-bill rate, with the certificate of deposit rate having the lowest value. But the correlations between the excess return and consumption growth (indicated by $\rho$) is identical for all three proxies. Given that the standard deviations of the excess returns are very close and the standard deviation of consumption growth is obviously identical, the differences in covariances between the excess returns and consumption growth are almost negligible. In terms of equation (4.4), the left-hand side when using the savings rate is much higher than when using the T-bill rate, but the right-hand side only differs marginally. So it comes as no surprise that the estimate of the coefficient of relative risk aversion is higher for our proxy than for the canonical choice. With respect to the certificate of deposit rate the resulting value of relative risk aversion is much lower than that under the 3-month T-bill rate; a finding similar to the quarterly empirical investigation shown in Table 4.2. However, we feel that the savings rate is a more appropriate risk-free rate proxy than the certificate of deposit rate, so for U.S. we also feel it is justified to claim that the equity premium puzzle is larger under a deposit rate than under a short-term government debt rate.

**Table 4.8: Estimates of the coefficient of relative risk aversion**
Notes: The sample period runs from January 1990 until December 2008. Due to the computation of consumption growth rates the first observation is lost, so the results are based on 227 observations. $aeRe$ is the average excess log return on the S&P index over the risk-free rate plus one half of the variance of this excess return. $\sigma_{exr}$ and $\sigma_c$ are, respectively, the standard deviations of the log excess return and consumption growth; $\rho_{(exr,c)}$ is their correlation and $\sigma_{(exr,c)}$ is their covariance. The estimated coefficient of relative risk aversion is denoted by $\hat{\gamma}$. See Table 4.7 for an explanation of the three series.

| | CD | Savings | T-bill |
|---|---|---|---|
| aeRe | 0.55 | 2.76 | 1.00 |
| $\sigma_{exr}$ | 14.99 | 14.99 | 14.98 |
| $\sigma_c$ | 0.89 | 0.89 | 0.89 |
| $\rho_{(exr,c)}$ | 0.14 | 0.14 | 0.14 |
| $\sigma_{(exr,c)}$ | 1.86 | 1.86 | 1.84 |
| $\hat{\gamma}$ | 29.41 | 148.45 | 54.54 |

## 4.6 Conclusion

Although other assumptions and empirical implementations of the consumption CAPM model have been modified in order to provide a resolution to the equity premium puzzle, the choice of the appropriate risk-free rate has been taken for granted. Particularly, the 3-month T-bill rate has been the canonical choice in this respect. However, we conjecture that an interest rate on a savings account is, from the average household's perspective, closer to their concept of a risk-free rate. Therefore it is more plausible to serve as a benchmark to which equity returns are compared.

This idea is first applied to data from a selection of OECD countries. Here we find that for the overwhelming majority of countries, the resulting relative risk aversion estimate is higher when the deposit rate instead of the T-bill rate is used. The two exceptions are Australia and the U.S., but their deposit rates are not comparable to those of the other countries: Australian deposits are not risk-free and the U.S. only has data on certificates of deposit, not savings deposits. On average, the relative difference between the country-specific estimates is close to 40%. The pooled estimators, constructed using standard deviations computed via a jackknife approach, also support this result. However, although these pooled estimators are less plagued by estimation uncertainty, the difference in estimates is not statistically significant at any conventional significance level.

Two countries have data on alternative proxies for the risk-free rate, i.e. Australia and New Zealand. For Australia, the pooled estimator constructed using the 12 alternative deposit rates is also lower than the value of relative risk aversion under the T-bill rate, but these results should be interpreted with care. New Zealand's alternative deposit rate produces an estimate of $\gamma$ almost identical to that obtained in the first part of the empirical investigation.

Finally, we also investigate the impact of alternative risk-free rates on monthly U.S. data. Here we show that the average interest rate on savings accounts has been lower than the T-bill rate, while the interest paid on certificates of deposit has exceeded the short-term government debt rate. This implies that the resulting equity premium is higher for the savings rate than for the T-bill rate. As a result, the estimate of the coefficient of relative risk aversion is higher under the former and hence constitutes more of a puzzle. The certificate of deposit rate, in line with the quarterly analysis, produces a lower coefficient of relative risk aversion, but we are inclined to attach more weight to the results under the savings rate and hence for the U.S., the equity premium puzzle also seems to be larger.

After all modifications to the model and methodology that have produced lower and lower values of relative risk aversion in the last decades, this paper has shown that the equity premium puzzle is larger. In a way, this is a setback since it will be more difficult to get a resolution to this famous puzzle. However, when the time comes that the puzzle has been solved, we need to make sure that the model is as close to reality as possible. The notion of limited stock market participation is already a large step into the right direction, but a short-term government debt rate is not likely part of an average household's asset reality, especially in developed countries other than the U.S. where money market funds are not very popular. A savings rate is more plausible to be the benchmark against which asset returns are compared and should therefore be used in future empirical analyzes trying to offer a solution to the equity premium puzzle.

In 2003 the European Central Bank started gathering more detailed data with respect to deposit rates. For now, the number of available observations is not large enough to use in an empirical investigation, but as time goes by these can be used to infer an even more detailed picture regarding the influence of the choice of risk-free rate on estimates of relative risk aversion.

## 4.A Appendices

### 4.A.1 Monthly U.S. data: Data definitions and important information

**Consumption** Two input series: Personal Consumption Expenditures (i.e. total consumption) and durable goods consumption. Both series from BEA and in three specifications. First, in nominal terms (Table 2.8.5; billions of dollars and seasonally adjusted at annual rates). Second, price indices (Table 2.8.4; index numbers, 2005 = 100, seasonally adjusted). Third, chained dollar estimates (Table 2.8.6; Billions of chained (2005) dollars; seasonally adjusted at annual rates). The final series is only available from 1995 onwards. For the pre-1995 period, the real series are constructed by using nominal and price index data. The average deviation of this method for the 1995-2010 period is -0.00438% for total consumption and 0.00879% for durables consumption.

**Population** Monthly population figures from BEA (Table 2.6.3). Population is the total population of the United States, including the Armed Forces overseas and the institutionalized population. The monthly estimate is the average of the estimates for the first of the month and the first of the following month; the annual estimate is the average of the monthly estimates. Official source: Census.

**Equity return** Log return on the monthly S&P Composite Index, provided by CRSP.

**T-bill rate** Secondary market rates on 3-month Treasury Bills from FRED. Monthly data in percentages, averages of business days on a discount basis.

**Savings rate** Average interest rate on savings deposits. Gathered by BankRate Monitor, obtained from the Investment Company Institute.

**CPI** Implicit price deflator from constructed nominal series and price index of non-durables and services consumption. As possible robustness checks, CPI series from FRED (officially from Bureau of Labor Statistics) can be used. All series are consumption price indices for all urban consumers, monthly data, 1982-84 = 100. Four alternatives: either all items or excluding food and energy; and seasonally adjusted or not.

# Chapter 5

# A State Space Approach to Time-varying Risk Aversion

## 5.1 Introduction

Ever since Mehra and Prescott (1985) christened the equity premium puzzle, a vast amount of research time has been devoted to this topic. Existing research has contemplated a large number of possible solutions among which heterogeneous agents (Mankiw and Zeldes, 1991; Attanasio et al., 2002; Brav et al., 2002; Vissing-Jørgenson, 2002), and habit formation (Abel, 1990; Campbell and Cochrane, 1999) are most prominent; others include disaster risk (Rietz, 1988; Barro, 2006) and long run risk (Bansal and Yaron, 2004).

This paper's purpose is not to come up with yet another solution to this puzzle. Most papers concerning this empirical observation treat risk aversion as a structural parameter whose value is constant over time. Although convenient, this assumption is hard to justify. Insights from the psychology of decision-making reveal that people are more sensitive to reductions in their levels of well-being than to increases. In other words, a loss of $100 hurts more than the joy provided by winning an equal amount. This is better known as loss aversion and this concept plays a central role in the prospect theory of Kahneman and Tversky (1979). Hence real life observations clearly suggest that there is variation in risk aversion.

The central aim of this paper is to show this time variation in the coefficient of relative risk aversion. We formulate a consumption-based asset pricing model in which risk aversion varies in response to news in consumption growth, inflation and unemployment growth. Hence besides the standard consumption growth we add two macroeconomic variables that, from our point of view, should influence risk aversion. In addition we also allow for time-varying innovation volatilities for a total of four different specifications. We check whether movements in the coefficient of relative risk aversion are in line with official U.S. recession and expansion dates and other important events that took place over the sample period. During expansions we expect this coefficient to be small while it should rise around recessions. The main novelty is the use of state space techniques. As risk aversion is a latent variable, this methodology is ideally suited for this purpose.

Brandt and Wang (2003) is the paper closest to ours in the sense that they formulate a model in which news concerning consumption growth and inflation induce variation in risk aversion. However, their focus is on the pricing implications of this framework for the term structure of interest rates and the cross-section of stock returns.

Another motivation for this paper is Cochrane (2011) who, in a nutshell, argues that current asset-pricing research is centered around variation in discount rates. Time-varying risk aversion can be seen as a manifestation of this variation. This is most obvious when he reasons that the 'representative investor' did not ignore the high premiums offered by stocks in December 2008 because he had wrong expectations or intermediaries were making asset-pricing decisions for him. No, these premiums were ignored because he could be losing his job or his business might go under and he feels that he is currently not in a position to take any risks. Hence during a recession his risk aversion went up. Cochrane (2011) also writes that such a statement is vacuous unless one restricts discount rates by ideally tying them to other data. This is exactly what we do in this paper.

Based on parameter estimates news concerning consumption growth is most important. Of the two additional variables unemployment growth has the largest impact, while inflation plays a more modest role. Moreover, the path of risk aversion generated by our specifications is generally in line with the official U.S. recession and expansion dates. This implies that during expansions risk aversion either goes down or is already at a subdued level. In contrast, in the run up to recessions risk aversion starts to rise, remains at elevated levels throughout these periods and on several occasions it does not decrease for a few years after the recession officially ended. Three of our four specifications indicate that the lowest levels of risk aversion over the last half a decade were attained at the end of the 1980s / start of the 1990s. With respect to the highest level our four specifications are very clear, this occurred at the end of the 'Great Recession' (June 2009) or in one of the following six months.

The remainder of this chapter is organized as follows. The second section introduces our starting model and extends it along two dimensions for a total of four different specifications. This section also covers the state space technique we use for estimation. Section 5.3 addresses the data. Results for our four models are discussed in Section 5.4. The final section concludes.

## 5.2 Model

Our starting point is the consumption-based capital asset pricing model (henceforth c-CAPM) by Breeden (1979) and extended by Grossman and Shiller (1981), Hansen and Jagannathan (1991) and Cochrane and Hansen (1992). Studies concerned with the equity premium puzzle usually assume that the coefficient of relative risk aversion and the covariance between per capita consumption growth and the excess return are constant, i.e.

$$\mathrm{E}_{t-1}[exr] = \gamma\sigma_{(exr,c)},$$

where $exr$ is the excess return calculated as the difference between the equity return and the risk-free rate, $\gamma$ is the log coefficient of relative risk aversion and $\sigma_{exr,c}$ is the unconditional covariance between the excess return and per capita consumption growth ($c$). Note that all lower case letters denote logarithms and that we leave out a small correction factor for simplicity.[1]

As our purpose is to study time-varying risk aversion we have to use a more general version of the c-CAPM in which $\gamma$ is no longer constant. In addition, in the following we sometimes assume that the covariance term varies as well. Hence, our most general model for the excess return is given by

$$exr_t = \gamma_t cov_{t-1}[\epsilon_t^{exr}, \epsilon_t^{c}] + \epsilon_t^{exr},$$

where $exr$ and $\gamma$ are as before. Note that in contrast to the previous specification in which the covariance refers to the excess return and consumption growth, the conditional covariance is calculated between the innovations in these two variables. The final term is a residual that has expectation zero.

We propose to generalize the model even further by focusing on

$$exr_t = \gamma_t cov_{t-1}[\epsilon_t^{exr}, \epsilon_t^{\text{Macro}}] + \epsilon_t^{exr},$$

where $exr$, $\gamma$ and $\epsilon_t^{exr}$ are as before. The conditional covariance is again calculated between innovations but instead of the shock to per capita consumption growth, we now have a more general 'macroeconomic shock'. This shock is given by

$$\epsilon_t^{\text{Macro}} = \theta^c \epsilon_t^c + \theta^\pi \epsilon_t^\pi + \theta^u \epsilon_t^u,$$

[1]The expression for the equity premium in the c-CAPM is given by

$$\mathrm{E}_{t-1}[exr] + \frac{\sigma_{exr}^2}{2} = \gamma\sigma_{(exr,c)}.$$

The second term on the left-hand side is a correction factor due to Jensen's inequality, see De Vries and Zenhorst (2012) for more details. As it is small we disregard it.

where $c$ still refers to per capita consumption growth, $\pi$ denotes inflation and $u$ is unemployment growth. Hence the general macroeconomic shock is modeled as a linear combination of three innovations. The first shock comes from the c-CAPM.

The second innovation originates from the postulation by Brandt and Wang (2003) that news concerning inflation also influences $\gamma$. As documented in for example Campbell and Shiller (1996) and Barr and Campbell (1997), inflation is correlated with real asset prices. Third is our suggestion that in addition to news about consumption growth and inflation, news concerning unemployment growth affects risk aversion as well. The reason that we include unemployment growth is that it is countercyclical and lagging (in relation to the business cycle), while both consumption growth and inflation are procyclical and coincident. In the recent recession, the plunge in U.S. stock prices was swiftly followed by a large increase in the unemployment rate. The work of Phelps (1999), Farmer (2011a) and Farmer (2011b) documents this link between the stock market and unemployment more formally.

Note that this setup allows us to gauge the relative importance of these three variables, i.e. the variables with the highest parameter value ($\theta$) is most important. In addition, if $\theta^{\pi}$ and/or $\theta^{u}$ differs significantly from zero then we have evidence that the variability in excess returns is better described by multiple macroeconomic variables than by consumption growth alone.

The final building block is that we allow all four innovations (i.e. $\epsilon_t^{exr}$, $\epsilon_t^{c}$, $\epsilon_t^{\pi}$ and $\epsilon_t^{u}$) to have time-varying volatilities. Financial variables are known to have non-constant variances and existing research has shown that consumption growth (e.g. Kandel and Stambaugh, 1991; Bansal et al., 2005; Beeler and Campbell, 2009) and inflation (e.g. Friedman, 1977; Engle, 1982) exhibit this as well. In contrast, unemployment growth does not seem to have this property. A simple LM test developed by Lee (1991) for GARCH effects supports this point. As can be inferred from Table 5.1, there is overwhelming evidence against the null hypothesis of constant variance for the real excess return, consumption growth and inflation: all three p-values are even smaller than 0.1%. In contrast, the p-value associated with the test statistic for unemployment growth is 0.51. Hence for this variable there is not enough support in favor of a varying volatility.

**Table 5.1: LM GARCH Test**
Test for first-order GARCH effects in the variance of the innovations as in Lee (1991). Series abbreviations are as follows: real excess return ($exr$), real per capita consumption growth ($c$), inflation ($\pi$) and unemployment growth ($u$).

| | exr | c | $\pi$ | u |
|---|---|---|---|---|
| Test value | 155.027 | 23.254 | 263.099 | 0.431 |
| p-val | 0.000 | 0.000 | 0.000 | 0.512 |

All four volatilities are modeled according to the QGARCH(1,1) model by Sentana (1995). This specification allows for a different impact of positive and negative innovations on volatility. This is imperative as it is well documented in the literature that the conditional variance of stock returns often increases when returns are negative. This is better known as the 'leverage' effect as a negative return results in a higher leverage for a firm. In addition, inflation volatility tends to be high if the level of inflation is high as well. Such dynamics are easily captured through the use of the asymmetric GARCH model.[2]

Combining all of the above, our model is given by[3]

$$exr_t = \gamma_t[\theta^c k_t^c + \theta^\pi k_t^\pi + \theta^u k_t^u] + \epsilon_t^{exr}, \tag{5.1}$$

$$c_t = \bar{c} + \phi^c c_{t-1} + \epsilon_t^c, \tag{5.2}$$

$$\pi_t = \bar{\pi} + \phi^\pi \pi_{t-1} + \epsilon_t^\pi, \tag{5.3}$$

$$u_t = \bar{u} + \phi^u u_{t-1} + \epsilon_t^u, \tag{5.4}$$

$$\gamma_t = \bar{\gamma} + \phi^\gamma \gamma_{t-1} + \epsilon_t^\gamma, \tag{5.5}$$

$$(\sigma_t^i)^2 = \beta_1^i + \beta_2^i \epsilon_{t-1}^i + \beta_3^i (\epsilon_{t-1}^i)^2 + \beta_4^i (\sigma_{t-1}^i)^2, \quad \text{for i} = exr, c, \pi \text{ and } u, \tag{5.6}$$

$$k_t^i = \rho_{(exr,i)} \sigma_t^{exr} \sigma_t^i, \quad \text{for i} = c, \pi \text{ and } u. \tag{5.7}$$

[2]To guarantee that the conditional variance is positive, the following restrictions are imposed: (i) $\beta_1 > 0$, (ii) $\beta_3, \beta_4 \geq 0$ and (iii) $\beta_2 < 4\beta_3(1 - \beta_3 - \beta 4)$.

[3]See Appendix 5.A.1 for an explicit derivation of equation (5.1).

The covariance between the excess return and our three macroeconomic variables, from now on denoted by $k^i$ for $i = c, \pi$ and $u$, is simply the product of $\rho_{exr,i}$ — the correlation between $\epsilon_t^{exr}$ and $\epsilon_t^i$ — and their standard deviations, denoted by $\sigma_t^{exr}$ and $\sigma_t^i$, respectively. Real per capita consumption growth, inflation, unemployment growth and time-varying risk aversion all follow AR(1) specifications. Note that as a consequence of these time-varying volatilities, the covariances between the excess return and respectively consumption growth, inflation and unemployment growth will also exhibit variation.

### 5.2.1 Methodology

Our model is estimated using the state space framework. Due to the time-varying innovation variances it does not fit into a standard Gaussian linear state space system. To accommodate this non-standard feature, we augment the state vector with the innovations $\epsilon_{t+1}^{exr}$, $\epsilon_{t+1}^{c}$, $\epsilon_{t+1}^{\pi}$ and $\epsilon_{t+1}^{u}$ and hence follow the approach suggested by Harvey et al. (1992). Using the Kalman filter and smoother we obtain estimates of all parameters, including those related to the conditional variances. Appendix 5.A.2 has a more detailed exposition of all system matrices and provides more computational details concerning our approach.

The covariances vary over time due to the time-varying nature of the variances. An alternative approach would be to model these covariances directly. However, there is no study in the line of Harvey et al. (1992) that focuses on covariances instead of variances. Although these two concepts are of course closely linked, the presence of the correlation complicates matters. This extension is left for future research.

## 5.3 Data

The model is estimated using U.S. data. The required series are available at varying frequencies, i.e. the highest frequency at which consumption is available is monthly, while equity returns can be computed on a daily basis. As a result, our analysis is conducted at the monthly frequency. We require both macroeconomic as well as financial variables.

The first macroeconomic series is real consumption of nondurables and services per capita. This series is not directly available, so we have to construct it ourselves. The Bureau of Economic Analysis has data on monthly 'real' (i.e. chained dollars) personal consumption expenditure as well as on durable consumption. Combining this with price indexes for both series allows us to apply the 'chain-subtraction' method to get a so-called Fisher index; see Whelan (2002) for more details on this procedure. This index is then divided by a monthly population series, originally from the Bureau of Census. The chained series are only available from 1995 onwards. By combining personal consumption expenditure and durable consumption in nominal terms with the price indexes, these series can be easily constructed for the pre-1995 period.

The consumer price index is the implicit price deflator derived from nondurables and services consumption in nominal terms and chained values. Finally, unemployment is from the Bureau of Labor Statistics. In particular, we use the seasonally adjusted unemployment rate for people aged 16 or older (series LNS14000000).

The two financial variables are the equity return and the risk-free rate. For the former we chose the log return of the S&P 500 index from CRSP. As a proxy for the risk-free rate we use monthly data on the secondary market rate of the 3-month T-bill (series TB3MS, FRED).

The sample period starts in January 1960 and runs until June 2011, for a total of 618 observations. Table 5.2 presents some descriptive statistics of the data. On average, real nondurables and services consumption growth per capita is about 0.15% with a standard deviation approximately two and a half times this value. The monthly real equity return on the S&P 500 is approximately 0.25%, but the standard deviation of 4.35% is fairly large, especially when compared with that of consumption growth. The worst month for equity investors is October 1987 with a return of -22.09%, while the market showed its best performance, a return of 15.67%, thirteen years earlier in

October 1974. The real risk-free rate is about one promille per month. Of all the variables considered, it shows the lowest degree of variability. The mean of the real excess return is equal to the difference between the real equity return and the real risk-free rate and its standard deviation is just slightly below that of the former. The worst month in terms of consumption growth is May 1960 (-1.35%), the highest growth (i.e. 1.64%) occurred in October 1965. Inflation is on average 0.33% per month, with a standard deviation that is comparable with that of the risk-free rate. Inflation peaked in August 1973, while the price level fell the most in November 2008. Unemployment fell by almost 9% in April 1998 and soared by approximately 12% in March 1960; on average the change in unemployment is a tenth of a percent, but note the rather large standard deviation.

**Table 5.2: Descriptive statistics**
All statistics in percentages and not annualized. The sample period is from January 1960 until June 2011, for a total of 618 observations. *StDev* denotes the sample standard deviation. Series abbreviations are as follows: real excess return ($exr$), real per capita consumption growth ($c$), inflation ($\pi$) and unemployment growth ($u$).

| | Average | StDev | Min | Max |
|---|---|---|---|---|
| Real equity return | 0.25 | 4.35 | -22.09 | 15.67 |
| Real risk-free rate | 0.10 | 0.25 | -0.92 | 1.35 |
| $exr$ | 0.15 | 4.32 | -22.28 | 15.66 |
| $c$ | 0.15 | 0.38 | -1.35 | 1.64 |
| $\pi$ | 0.33 | 0.26 | -1.29 | 1.32 |
| $u$ | 0.10 | 3.06 | -8.89 | 11.78 |

## 5.4 Results

This section discusses the results of estimating our model using the state space approach. But before we discuss these results, we first consider three simpler specifications. In the first subsection we examine a model which only includes consumption growth ($c$) and hence disregards inflation ($\pi$) and unemployment growth ($u$). In addition, the covariance between the innovations in the excess return and consumption growth is kept fixed. Note that this implies that both $\epsilon^{exr}$ and $\epsilon^{c}$ need to have constant variances. The reason is that this model resembles the standard c-CAPM but with time-varying risk aversion. The second specification extends this setup by allowing the shocks in the real excess return and consumption growth to have a time-varying volatility. This then induces a time-varying covariance; this specification is discussed in Section 5.4.2. Our expectation is that this second model captures part of the variation in $\gamma$ through $k_t^c$.

While the first two specifications only involve consumption growth, the third and fourth model no longer disregard inflation and unemployment growth as additional explanatory variables. The relation between these two specifications is the same as before: the third model assumes constant variances (and covariances) while the fourth specification models the volatility of the real excess return, consumption growth and inflation to be time-varying, keeping $(\sigma^u)^2$ fixed.[4] Remember from Table 5.1 that there is not enough evidence to reject the null hypothesis that the variance of unemployment growth is constant. The outcomes of the model in which this volatility is nevertheless assumed to vary are qualitatively similar and so this does not influence our results to a significant extent.

### 5.4.1 First Specification

We first discuss the results from the simplest model which only includes consumption growth. Moreover, we assume that the variance of the innovations and therefore the covariance between the innovations (i.e. between the excess return and consumption growth) are constant over time. Table 5.3 presents the parameter estimates; sandwich based standard errors are given in parentheses. Note that $\theta^c$ has been normalized to unity.

The persistence parameter in the estimated autoregressive equation for consumption growth is 0.48 which indicates that this series is not very persistent. In sharp

[4]The equations describing the first three specifications are shown in Appendix 5.A.3.

contrast, the AR(1) parameter for relative risk aversion — conveniently labeled $\gamma_1$ to distinguish it from the series generated by the other specifications — is 0.97, close to upper bound of unity for non-explosive series. The bottom panel shows the correlation between the innovations in consumption growth and (1) the innovations in the excess return and (2) the innovations in relative risk aversion. The first is positive and hence indicates that these innovations are positively associated. This implies that when there is a positive shock to consumption growth it is likely that real excess returns are higher as well. The correlation between (the innovations in) consumption growth and relative risk aversion is negative which is in line with our a priori expectation: a positive surprise in consumption growth should be associated with a decline in risk aversion. A positive shock to consumption growth is considered to be good news and hence risk aversion should decline. Note that all parameters estimates are statistically significant from zero at the 5% significance level.

Our main variable of interest, the time-varying estimate of log relative risk aversion, is shown in Figure 5.1. As indicated by the parameter estimate in Table 5.3, this series is highly persistent. Although the value of 0.97 is not significantly different from the value of unity which would make the variable of an explosive nature, the figure clearly indicates that over the period from January 1960 until June 2011 it exhibits mean reversion. The shaded regions indicate the NBER recession dates and are the main tool to judge our model. Our sample period contains the following eight recession periods:[5]

I. April 1960 until February 1961

II. December 1969 until November 1970

III. November 1973 until March 1975

IV. January 1980 until July 1980

V. July 1981 until November 1982

[5] We refer to Labonte and Makinen (2002) and Bordo and Landon-Lane (2010) for a more detailed exposition of these eight periods including likely causes, severity and policy responses.

**Table 5.3: Parameter estimates of the first specification**

Notes: Series abbreviations are as follows: real excess return ($exr$), real per capita consumption growth ($c$) and the log coefficient of relative risk aversion (labeled $\gamma_1$). Estimation results refer to equations (5.13)-(5.16), see Appendix 5.A.3. This model only features consumption growth and keeps the innovation variances fixed. As a result the covariance between excess returns and consumption growth is also constant. In addition, $\theta^c$ is normalized to unity. Correlations (denoted by $\rho$) are calculated between the innovations of the series in question. Every column pertains to a single series and shows its relevant parameter estimates, with superscript $i$ referring to these series. The sample period runs from January 1960 until June 2011; sandwich based standard errors in parentheses.

| | exr | c | $\gamma_1$ |
|---|---|---|---|
| $\theta^i$ | - | 1.00 | - |
| | - | (0.36) | - |
| $\bar{i}$ | - | 0.02 | 1.32 |
| | - | (5.3E-3) | (0.54) |
| $\phi^i$ | - | 0.48 | 0.97 |
| | - | (0.11) | (0.22) |
| $(\sigma^i)^2$ | 1.9E-3 | 1.6E-5 | 489.45 |
| | (4.9E-4) | (5.3E-6) | (236.76) |
| $\rho_{(exr,i)}$ | - | 0.76 | - |
| | - | (0.21) | - |
| $\rho_{(i,\gamma_1)}$ | - | -0.49 | - |
| | - | (0.14) | - |

VI. July 1990 until March 1991

VII. March 2001 until November 2001

VIII. December 2007 until June 2009

Before we discuss the path of log relative risk aversion in more detail, it is important to realize the following. As we keep the covariance between the innovations in the real excess return and consumption growth ($k^c$) constant, one might be tempted to conclude that all variation in $\gamma_1$ comes from variation in the variance of the real excess return. This is not the case. The reason is the presence of $\epsilon^{exr}$ and $\epsilon^{\gamma}$ which are both unobserved. In other words, not all variation in the real excess return is reflected in the path of risk aversion. Moreover, we present the smoothed series, i.e. we also apply the Kalman smoothing technique. This implies that all information is used to estimate the most likely path of risk aversion: historical, current but also future realizations of excess returns and consumption growth. This is standard practice when using the state space technique, but one should be aware of its influence.

Although risk aversion rises during the first recession period, it keeps increasing for an extended period of time after the downturn officially ended. The leveling of log risk aversion between 3 and 4 during 1962 until 1964 is caused by a spell of mostly negative real excess returns in this period, but the innovations in $\gamma_1$ also absorbed part of the variation in the dependent variable. During periods II and III risk aversion rises, but while it keeps on rising after the first recession period in the 1970s, it approximately peaks at the end of the trough for the recession related to the oil crises. The declining path of $\gamma_1$ during recession period IV (also known as the first leg of the 'Double Dip' recession) is due to the predominantly positive returns in this period. During the second leg of the 'Double Dip' recession (period V), risk aversion rises and again peaks near the end of the trough.

Risk aversion rises in late 1987 due to the stock market crash but the effect of this event is rather limited. This is partly due to negative innovations in the excess return and some positive innovations in $\gamma_1$. A likely reason for the somewhat muted impact on risk aversion is that this crash was unexpected and although its impact was huge, its impact on equity returns was short-lived: the excess return of November is negative but returns are positive for the period thereafter. The path slopes downward in period VI but here negative and positive returns are more balanced so the decline is less steep than during the fourth recession period. During the longest expansion in the history of the U.S. (i.e. in between period VI and VII), $\gamma_1$ is predominantly rising, especially during the late 1990s. This can be explained by the events that took place during the summer of 1998, i.e. the Russian ruble crises and the subsequent default of Long-Term Capital Management which contributed to a spell of negative excess returns. The fact that these events span a longer time period and have such an impact only supports our view concerning the relatively small response of risk aversion to the 1987 stock market crash.

The decade of the 1990s also exemplifies the impact of the Kalman smoother. As mentioned, through the use of the smoother future realizations are used to get the best estimate of risk aversion at time $t$. This implies that the level of $\gamma$ just after the 1990 recession is to some degree influences by the events in 1998 and the recession starting in the first quarter of 2001. In other words, in 1990 it is already known that risk aversion should increase later in the decade as a result of these two events. In combination with the high level of persistence, this produces the upward sloping path of risk aversion during this decade.[6]

Surprisingly, risk aversion peaks just before 2001 recession and is already declining during the recession period. Returns are mostly negative in this timespan so this can only be the result of the unobserved innovations and this is indeed the case. Note that this period also comprises the September 2001 terrorist attacks, for which one would expect to observe rising levels of risk aversion. However, in light of the discussion concerning the 1987 stock market crash, this results is less surprising. The path slopes

[6]As our interest is to show the time variation in risk aversion, applying the smoother is validated. If one is interested in forecasting future levels of risk aversion one should of course use the output generated by the Kalman filter.

upward again in the run-up the ‘Great Recession’, rises during this period and peaks in November 2009, five months after the recession officially ended. The decline in $\gamma_1$ at the end of the sample period is the result of the good performance of the stock market after the most recent recession and the very low yields on government debt which, combined, produce a large excess return.

With respect to the level of relative risk aversion, it has its minimum at the late 1960s (0.67) and in the period after the 1990s recession (0.74). Based on this specification, the final recession of the sample period has been accompanied by the highest level of risk aversion (6.81), but the 1975 (6.27) and 2001 (6.23) recessions are a close second and third. The average log relative risk aversion is equal to 3.51 such that the coefficient of relative risk aversion is approximately 98. This value corresponds well to the literature on the equity premium puzzle in which such high values are not uncommon.

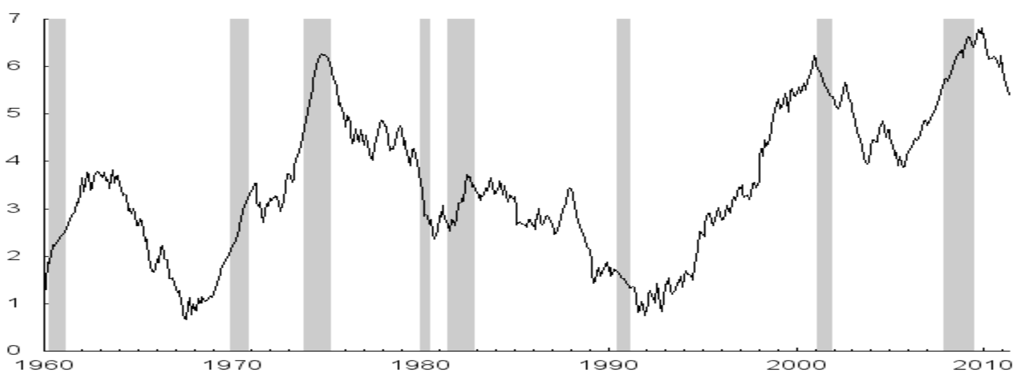



**Figure 5.1:** Smoothed log relative risk aversion coefficient generated by the first specification ($\gamma_1$) over the sample period running from January 1960 until June 2011. Shaded bars indicate official recession dates. Risk aversion is estimated from equations (5.13)-(5.16), see Appendix 5.A.3. This specification only involves consumption growth and innovations in this series and the excess return are assumed to have a constant variance. As a result the covariance between shocks in the excess return and consumption growth innovations is also constant.

### 5.4.2 Second Specification

The second model still only involves consumption growth, but the shocks to both the excess return and consumption growth now have a time-varying volatility. Moreover, this time-varying volatility implies that the covariance between the innovations in the excess return and consumption growth, denoted by $k_t^c$, varies as well. We again normalize $\theta^c$ to unity. The results of this setup are presented in Table 5.4.

In line with our a priori expectation, remember our discussion of the 'leverage' effect in Section 5.2, the asymmetry coefficient ($\beta_2^{exr}$) of the conditional volatility process for excess return innovations is negative. The AR(1) parameter in the equation for consumption growth is about 0.12 larger than before, hence increasing its persistence. The GARCH parameters regarding consumption growth reveal that the response of volatility to a negative innovation does not differ significantly from that of a positive innovation, i.e. the asymmetry parameter is not significantly different from zero at a conventional significance level. Estimates for relative risk aversion are comparable to those for the first model, i.e. $\gamma_2$ is estimated to be highly persistent. The correlations between the real excess return and, respectively, consumption growth and risk aversion again have the expected sign (positive and negative) and are, in an absolute sense, larger than in the first model. With the exception of $\beta_2^c$, all parameters are statistically significant from zero at the conventional significance level.

Figure 5.2 shows the estimated path of log relative risk aversion for the second model. Compared to $\gamma_1$ there are a couple of differences.[7] The first is that $\gamma_2$ is higher in the period between recession periods I and II. The second difference is that risk aversion does not increase as much in the 'oil crises' period. Third, log risk aversion declines sharper between periods V and VI and there is also no clear movement during October 1987. Fourth, the level of $\gamma_2$ is lower in the run-up to recession VII. In other words, the upward sloping pattern is less steep than before. Relatedly, while $\gamma_1$ is clearly impacted by the events that took place in August 1998, the same cannot be said about the path of risk aversion generated by this specification. Just like 1987, there is also no clear response to the events that took place in September 2001. Finally, while

[7] One might be tempted to think that any differences between $\gamma_1$ and $\gamma_2$ can be ascribed to the time-varying covariance $k_t^c$. However, this is not the case. The reason lies in the presence of $\epsilon_{t+1}^{exr}$ and $\epsilon_{t+1}^c$ which are of course not necessarily equal between these two specifications.

**Table 5.4: Parameter estimates of the second specification**

Notes: Series abbreviations are as follows: real excess return ($exr$), real per capita consumption growth ($c$) and the coefficient of relative risk aversion (labeled $\gamma_2$). Estimation results refer to equations (5.17)-(5.21), see Appendix 5.A.3. This model only features consumption growth and allows for variation in the innovation variances. As a result the covariance between excess returns and consumption growth is also time-varying. In addition, $\theta^c$ is normalized to unity. Correlations (denoted by $\rho$) are calculated between the innovations of the series in question. Every column pertains to a single series and shows its relevant parameter estimates, with superscript $i$ referring to these series. The sample period runs from January 1960 until June 2011; sandwich based standard errors in parentheses.

| | exr | g | $\gamma_2$ |
|---|---|---|---|
| $\theta^i$ | - | 1.00 | - |
| | - | (0.27) | - |
| $\bar{i}$ | - | 0.03 | 0.65 |
| | - | (7.7E-3) | (0.24) |
| $\phi^i$ | - | 0.61 | 0.98 |
| | - | (0.19) | (0.33) |
| $(\sigma^i)^2$ | - | - | 53.09 |
| | - | - | (17.20) |
| $\beta_1^i$ | 9.6E-5 | 3.8E-5 | - |
| | (4.1E-5) | (5.9E-6) | - |
| $\beta_2^i$ | -0.32 | 0.08 | - |
| | (0.09) | (0.07) | - |
| $\beta_3^i$ | 0.19 | 0.24 | - |
| | (0.07) | (0.09) | - |
| $\beta_4^i$ | 0.68 | 0.66 | - |
| | (0.24) | (0.27) | - |
| $\rho_{(exr,i)}$ | - | 0.81 | - |
| | - | (0.32) | - |
| $\rho_{(i,\gamma_2)}$ | - | -0.65 | - |
| | - | (0.29) | - |

the path of risk aversion generated by the first model rises sharply during the 'Great Recession', the increase in $\gamma_2$ is less pronounced. Later on we will come back to some of these differences.

The minimum value is 0.23 and it is attained at the start of the 1990s while the maximum of 5.52 is observed just after the end of recession VIII. The average log value of risk aversion equals 3.18 which translates to a value of 47.49 for the coefficient of relative risk aversion over the whole sample period.

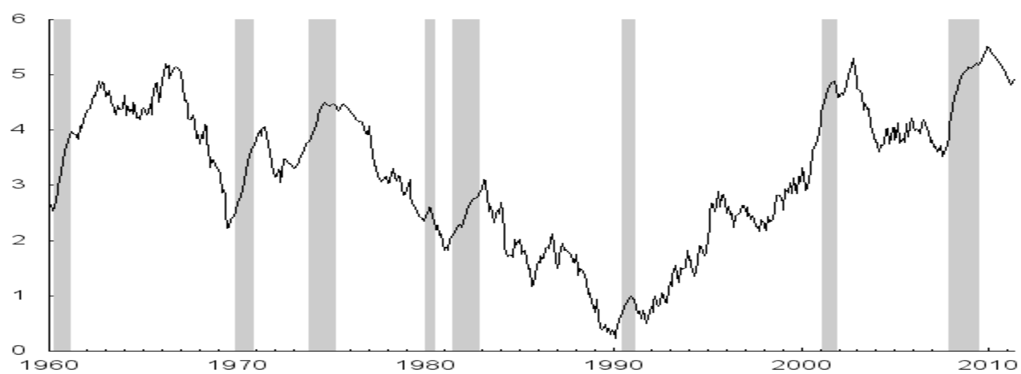



**Figure 5.2:** Smoothed log relative risk aversion coefficient generated by the first specification ($\gamma_2$) over the sample period running from January 1960 until June 2011. Shaded bars indicate official recession dates. Risk aversion is estimated from equations (5.17)-(5.21), see Appendix 5.A.3. This specification only involves consumption growth and innovations in this series and excess returns are assumed to have a time-varying variance. As a result the covariance between shocks in the excess return and consumption growth innovations also varies.

The conditional volatility of excess returns innovations is shown in Figure 5.3. There are clear spikes in the conditional volatility for all eight recession periods, although there are major differences in the levels of volatility attained during these periods. For example, while the average level of the volatility is 0.002 it rises to 0.009 and 0.007 during the 'oil crises' and the 'Great Recession', respectively. In addition, there are also spikes at four other instances. First, during the final months of 1962 which is caused by three large and negative returns during the second quarter of that year. Second, at the final quarter of 1987 which is the result of the stock market crash in October 1987; rising to its second highest level over this sample period. Third, during the final months of 1998 which is related to the Russian crises in the summer of 1998 and the resulting default of Long-Term Capital Management. Finally, at the final quarter of 2002 which is the result of a large negative excess return in September 2002. Note that these are all periods in which large negative returns result in higher conditional variances, hence substantiating the asymmetric GARCH effect.

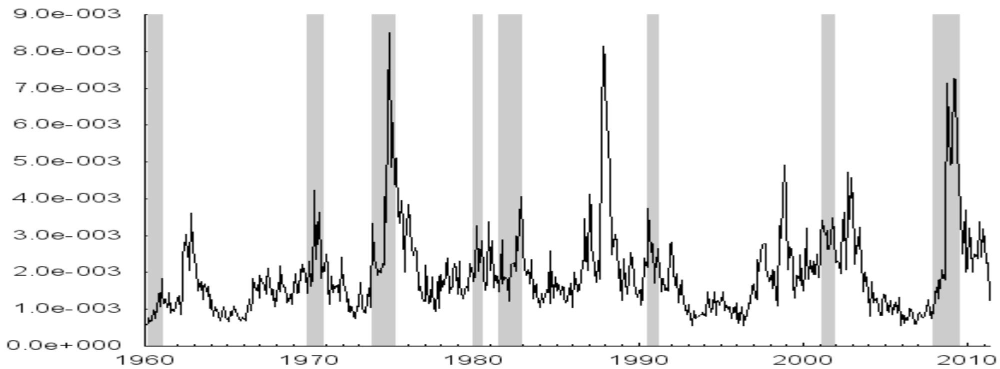



**Figure 5.3:** Conditional variance of innovations in the real excess return ($exr$) innovations over the sample period running from January 1960 until June 2011, given by equation (5.20). Shaded bars indicate official recession dates.

Figure 5.4 presents the GARCH series for consumption growth. During all recessions we observe spikes in the conditional volatility, except for recessions II and V. The highest level of volatility is attained during the first recession which is due to the fact that the largest monthly drop in consumption growth took place in May 1960. The spike in volatility around 1965 comes from the highest gain in October 1965. Given that we observe spikes during these two occasions provides additional evidence that there should not be an asymmetry effect for consumption growth; this is exactly what we find from Table 5.2. Note that while the conditional volatility series of the real excess return innovations is quite erratic, the series for innovations in consumption growth is much smoother.

The conditional covariance series between the innovations in the excess return and the innovations in consumption growth is shown in Figure 5.5. Note that this series is the product of 0.81 — the correlation between these innovations — and the square root of the conditional volatilities shown in Figures 5.3 and 5.4. As it inherits its dynamics from the latter two series, $k_t^g$ shows spikes during all eight recession periods, the four

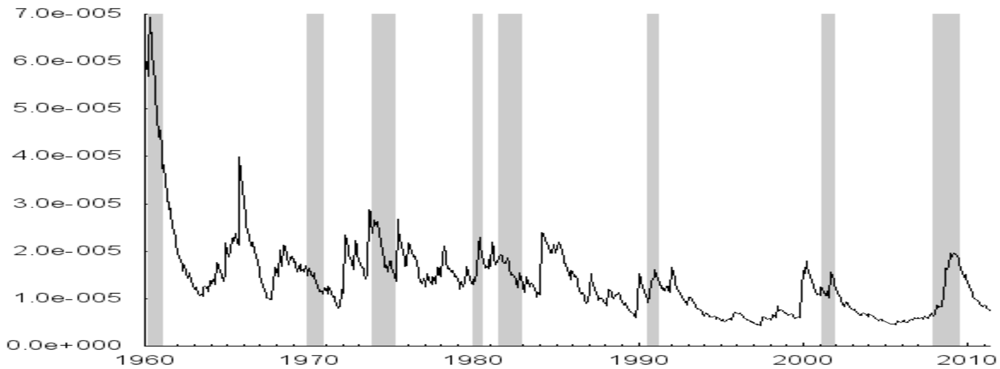



**Figure 5.4:** Conditional variance of consumption growth ($c$) innovations over the sample period running from January 1960 until June 2011, given by equation (5.20). Shaded bars indicate official recession dates.

additional periods identified for excess return volatility and, to a lesser extent, the two instances where consumption growth shocks are characterized by a high level of volatility. Note that it is more closely related to the conditional volatility of the real excess return than to the GARCH process of consumption growth. The reason is that the former is much larger than the latter, hence dominating their product.

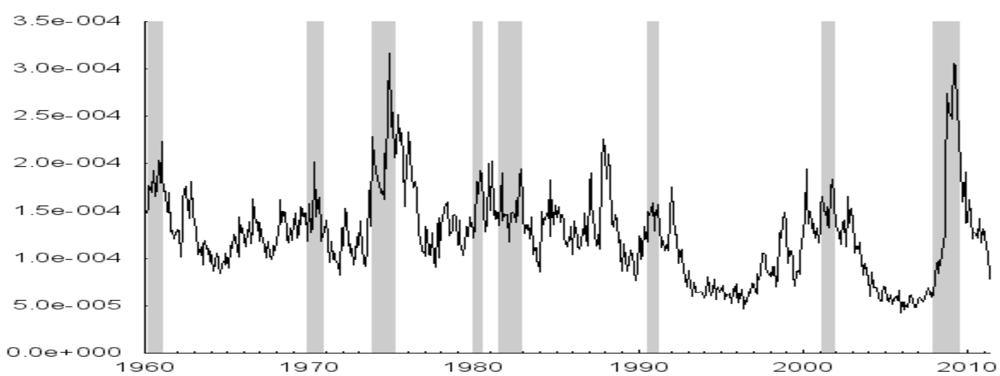



**Figure 5.5:** Conditional covariance of innovations in the real excess return ($exr$) and consumption growth ($c$) over the sample period running from January 1960 until June 2011, given by equation (5.21). Shaded bars indicate official recession dates.

As mentioned before, differences between $\gamma_1$ and $\gamma_2$ are not necessarily the consequence of having variation in the covariance between the excess return and consumption growth. Nevertheless, there are at least two differences which are likely the result of introducing a varying covariance: recession periods III and VIII. Comparing $\gamma_1$ and

$\gamma_2$ reveals the latter increases less during these two periods. At the same time, $k_t^c$ is much higher than its average during these times. Hence some of the variation in the excess return that had to be ascribed to risk aversion in the first specification is assigned to the conditional covariance in the second model. This is, to a lesser degree, also the case for the events that took place in 1987, 1998 and 2001.

### 5.4.3 Third specification

The third model uses information on inflation and unemployment growth along with consumption growth, normalizing the absolute sum of $\theta^c$, $\theta^\pi$ and $\theta^u$ to unity. The first row of Table 5.5 reveals that the parameter of consumption growth is (in absolute sense) largest (0.46), followed by unemployment growth (-0.32) and inflation (-0.21). Hence consumption growth is, based on this specification, most important. Note that the estimate of $\theta^\pi$ is not statistically significant at the 5% level. A likely reason is that the shock to inflation contains more or less the same information as the innovation in consumption growth as both are procyclical and coincident. In contrast, unemployment growth is countercyclical and lagging so it makes sense that it contains information not present in the consumption growth shock.

While the persistence parameter of consumption growth is 0.48 in the first specification, it has a value of 0.64 in this more elaborate model. The parameter values of the equation for inflation are comparable to those of consumption growth, but unemployment is estimated to be considerably less persistent. As before, relative risk aversion remains very persistent.

The two correlations involving consumption growth keep their correct sign. Moreover, they are in an absolute sense larger than those of the first model. The correlation between the real excess return and inflation is negative but not statistically significant. This is surprising as we expected to find a clear negative relationship between these two variables. The finding that the correlation with innovations in unemployment growth is not significant is less puzzling: the stock and bond market adjust immediately to the current state of the economy (or perhaps even reflect this), while unemployment adjusts very slowly and is usually lagging. The negative sign is in line with our expectation as one would expect that these two variables move in opposite direction over longer time horizons.

The final line shows the correlation between the innovations in relative risk aversion and news in our three explanatory variables. With regards to inflation we find a positive and significant relationship. If there is positive shock to inflation (i.e. inflation increases) then risk aversion rises as well. Finally, while unemployment news does not appear to have a direct impact on excess returns, it does have a positive and statistically significant impact on $\gamma_3$. In others words, if unemployment goes up, risk aversion tends to increase as well.

**Table 5.5: Parameter estimates of the third specification**

Notes: Series abbreviations are as follows: real excess return ($exr$), real per capita consumption growth ($c$), inflation ($\pi$), unemployment growth ($u$) and the coefficient of relative risk aversion (labeled $\gamma_3$). Estimation results refer to equations (5.22)-(5.27), see Appendix 5.A.3. This model keeps the innovation variances fixed. As a result the covariances between excess returns and, respectively, consumption growth, inflation and unemployment growth are also constant. In addition, the absolute sum of $\theta^c$, $\theta^\pi$ and $\theta^u$ is normalized to unity. Correlations (denoted by $\rho$) are calculated between the innovations of the series in question. Every column pertains to a single series and shows its relevant parameter estimates, with superscript $i$ referring to these series. The sample period runs from January 1960 until June 2011; sandwich based standard errors in parentheses.

| | exr | g | $\pi$ | u | $\gamma_3$ |
|---|---|---|---|---|---|
| $\theta^i$ | - | 0.46 | -0.21 | -0.32 | - |
| | - | (0.18) | (0.11) | (0.13) | - |
| $\bar{i}$ | - | 0.07 | 0.11 | 4.0E-3 | 1.02 |
| | - | (0.02) | (0.04) | (6.9E-4) | (0.47) |
| $\phi^i$ | - | 0.64 | 0.63 | 0.39 | 0.95 |
| | - | (0.27) | (0.21) | (0.13) | (0.35) |
| $(\sigma^i)^2$ | 9.5E-3 | 4.5E-4 | 8.7E-4 | 3.3E-3 | 261.68 |
| | (3.6E-3) | (9.3E-5) | (3.3E-4) | (4.9E-4) | (121.43) |
| $\rho_{(exr,i)}$ | - | 0.89 | -0.33 | -0.18 | - |
| | - | (0.37) | (0.21) | (0.16) | - |
| $\rho_{(i,\gamma_3)}$ | - | -0.56 | 0.35 | 0.72 | - |
| | - | (0.19) | (0.13) | (0.32) | - |

Figure 5.6 shows the log relative risk aversion generated by the third model. Risk aversion rises sharply during the first two recessions. Although inflation is low, unemployment growth is skyrocketing during both periods so this result is likely due to the inclusion of $u$ as an additional source of information regarding risk aversion. The sharp increase in $\gamma_3$ during the third recession and the results that risk aversion remains at an elevated level for quite some time afterwards is not surprising given that this recession is also known as the 'Stagflation recession'. In general the 1970s can be characterized as a period in which economic growth was low, unemployment therefore remained elevated, but inflation was high as well. Given the two additional innovations in this specification, the path of $\gamma_3$ during this decade is not surprising.

Although risk aversion increases in late 1987, this rise is less pronounced compared to the first model. A likely reason is that both inflation is relatively low and unemployment growth is decreasing, so these two variables are not hit by shocks that suggest that risk aversion should increase. Concerning the events that took place during the second half of 1998 and in September 2001 a similar reasoning applies; inflation is low and unemployment growth is more or less constant so the response of $\gamma_3$ is small(ler). During the latest recession we again observe a sharp increase in log relative risk aversion. As inflation is decreasing, this effect is again due to increasing levels of unemployment. In sum, $\gamma_3$ differs from $\gamma_1$ and this is primarily due to the inclusion of unemployment growth.

The minimum is attained in May 1966 and has a value of 1.16. The overall maximum value is 6.49 and it is again during the latest recession that this value is observed but the value of 6.30 just after recession III is a close second. Finally, the average is 3.35 such that the coefficient of relative risk aversion takes on a value of about 64. Although still too high to resolve the equity premium puzzle, it does appear that adding inflation and unemployment growth can substantially reduce the coefficient of relative risk aversion.

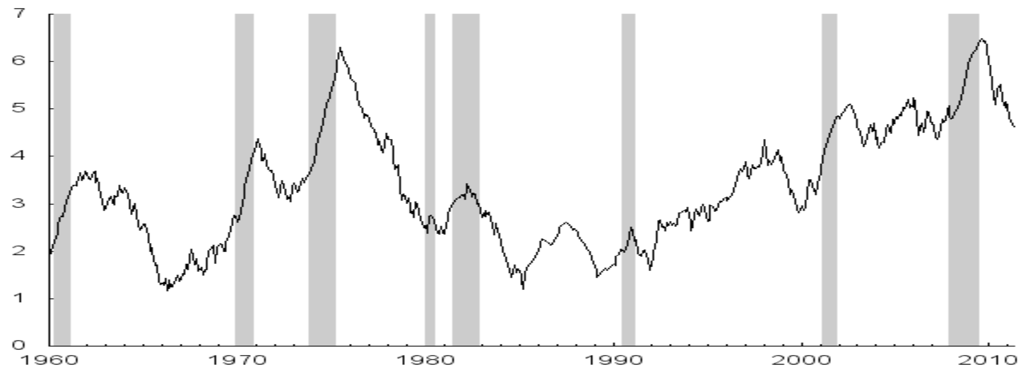



**Figure 5.6:** Smoothed log relative risk aversion coefficient generated by the first specification ($\gamma_3$) over the sample period running from January 1960 until June 2011. Shaded bars indicate official recession dates. Risk aversion is estimated from equations (5.22)-(5.27), see Appendix 5.A.3. This specification involves consumption growth, inflation and unemployment. Moreover, innovations in these three series and the excess return are assumed to have a constant variance. As a result the covariance between shocks in the excess return and, respectively, consumption growth, inflation and unemployment innovations is also constant.

### 5.4.4 Fourth specification

Parameter estimates of our fourth and final model are presented in Table 5.6. With respect to the $\theta$'s — the absolute sum of which has again been normalized to unity — we observe the same pattern as in the previous specification: consumption growth is most important, followed by unemployment growth and inflation. Both $\theta^c$ and $\theta^u$ are larger than before and hence $\theta^\pi$ is smaller, but in contrast to the third specification it is now also statistically significantly different from zero at the 5% level.

The constants, $\phi$ and $\sigma^2$ parameters are more or less in line with those of the previous model with only slight changes in persistence. The GARCH parameters of consumption growth are also similar to the estimates reported in Table 5.4 with $\beta_4$ being the largest and the parameter of asymmetry $\beta_2$ insignificant. In contrast, and in line with our expectation, this parameter is positive and significantly different from zero for inflation. If there is a positive shock to inflation, its volatility tends to go up. The parameter on the GARCH lag is very high (0.92). Remember that there is not enough evidence for a varying volatility in unemployment growth innovations hence this is variance is constant.

The first three correlations (i.e. those involving the real excess return) are comparable to those in the previous model. The only exception is $\rho_{(exr,c)}$ which is now much larger, positive but still not significant. All three correlations involving $\gamma$ have kept their correct sign, but inflation no longer has (in an absolute sense) the largest correlation. It is now consumption growth that seems to be most associated with risk aversion.

**Table 5.6: Parameter estimates of the fourth specification**

Notes: Series abbreviations are as follows: real excess return ($exr$), real per capita consumption growth ($c$), inflation ($\pi$), unemployment growth ($u$) and the coefficient of relative risk aversion (labeled $\gamma_4$). Estimation results refer to equations (5.1)-(5.6), see Section 2. This model allows for variation in the innovation variances. As a result the covariances between excess returns and, respectively, consumption growth, inflation and unemployment growth are also time-varying. In addition, the absolute sum of $\theta^c$, $\theta^\pi$ and $\theta^u$ is normalized to unity. Correlations (denoted by $\rho$) are calculated between the innovations of the series in question. Every column pertains to a single series and shows its relevant parameter estimates, with superscript $i$ referring to these series. The sample period runs from January 1960 until June 2011; sandwich based standard errors in parentheses.

| | exr | g | $\pi$ | u | $\gamma_4$ |
|---|---|---|---|---|---|
| $\theta^i$ | - | 0.51 | -0.14 | -0.35 | - |
| | - | (0.15) | (0.06) | (0.16) | - |
| $\bar{i}$ | - | 0.01 | 2.0E-3 | 3.8E-3 | 0.30 |
| | - | (4.9E-3) | (9.7E-4) | (5.2E-4) | (0.11) |
| $\phi^i$ | - | 0.58 | 0.69 | 0.32 | 0.96 |
| | - | (0.11) | (0.24) | (0.15) | (0.25) |
| $(\sigma^i)^2$ | - | - | - | 4.3E-3 | 39.53 |
| | - | - | - | (6.9E-4) | (17.05) |
| $\beta_1^i$ | 7.2E-5 | 7.3E-3 | 5.8E-3 | - | - |
| | (3.1E-5) | (2.1E-3) | (2.7E-3) | - | - |
| $\beta_2^i$ | -0.37 | 0.05 | 0.12 | - | - |
| | (0.16) | (0.03) | (0.02) | - | - |
| $\beta_3^i$ | 0.21 | 0.20 | 0.05 | - | - |
| | (0.10) | (0.07) | (0.01) | - | - |
| $\beta_4^i$ | 0.72 | 0.72 | 0.92 | - | - |
| | (0.25) | (0.14) | (0.20) | - | - |
| $\rho_{(exr,i)}$ | - | 0.73 | -0.52 | -0.47 | - |
| | - | (0.28) | (0.21) | (0.27) | - |
| $\rho_{(i,\gamma_4)}$ | - | -0.59 | 0.31 | 0.46 | - |
| | - | (0.26) | (0.12) | (0.15) | - |

Figure 5.7 shows the estimated log relative risk aversion coefficient of this specification. While risk aversion rises during the first two recessions, its behavior during period III is a little puzzling. Although it first goes up, it falls afterwards. However, note that its movement during this period is quite limited. In other words, $\gamma_4$ does not move a lot within this time span. In contrast to the third specification we do not observe an economically significant rise in risk aversion while it is known that both inflation and unemployment growth were high. The only possibility is that the time-varying covariances pick up this effect; we will come back to this later. Risk aversion rises in periods IV and V and there is also an upward movement in late 1987. It rises sharply in the sixth recession and keeps on increasing for most of the 1990s. Recession VII is the only downturn in which $\gamma_4$ falls, but note that it goes up just afterwards. During the 'Great Recession' we again observe a sharp rise and risk aversion attains its highest value.

For this final specification the maximum value of risk aversion also occurs at the latest recession (5.47). Again, the minimum of 0.68 is attained in the late 1980s. Not surprisingly, this specification yields the lowest average of 2.97 which translates into an average risk aversion coefficient of 51 over the sample period.

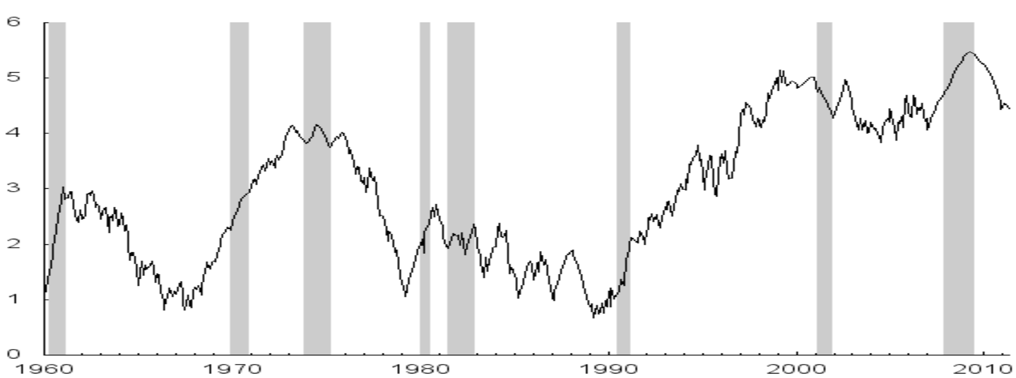



**Figure 5.7:** Smoothed log relative risk aversion coefficient generated by the first specification ($\gamma_4$) over the sample period running from January 1960 until June 2011. Shaded bars indicate official recession dates. Risk aversion is estimated from equations (5.1)-(5.7), see Section 2. This specification involves consumption growth, inflation and unemployment. Moreover, innovations in these three series and the excess return are assumed to have a time-varying variance. As a result the covariance between shocks in the excess return and, respectively, consumption growth, inflation and unemployment innovations also varies.

The conditional variance series for real excess return innovations for this fourth specification is not shown. The reason is that it closely resembles the QGARCH series from the second model. This is not true for consumption growth, Figure 5.8 presents this conditional variance series. There is still a large spike in volatility (in response to a very negative consumption growth realization) during the first recession but note that the level of this series is smaller compared to the one depicted in Figure 5.4. In contrast, there is no clear response to the highest growth rate in late 1965. There are, however, some additional spikes in 1972Q2, 1983Q2, 1984Q2 and during the first six months of 2000. A look at the unobserved error term of the equation for unemployment growth, i.e. $\epsilon^c_{t+1}$, reveals that all these periods are characterized by either very negative or highly positive innovations. Despite some notable differences between the conditional volatility series of consumption growth generated by the second and fourth specification, the conditional covariances series look fairly similar. As mentioned in the discussion of Figure 5.5, the real excess return is the dominant driving force in $k^c_t$ and as they are quite similar, the conditional variances series is fairly similar too. For sake of brevity we again decide not to show it.

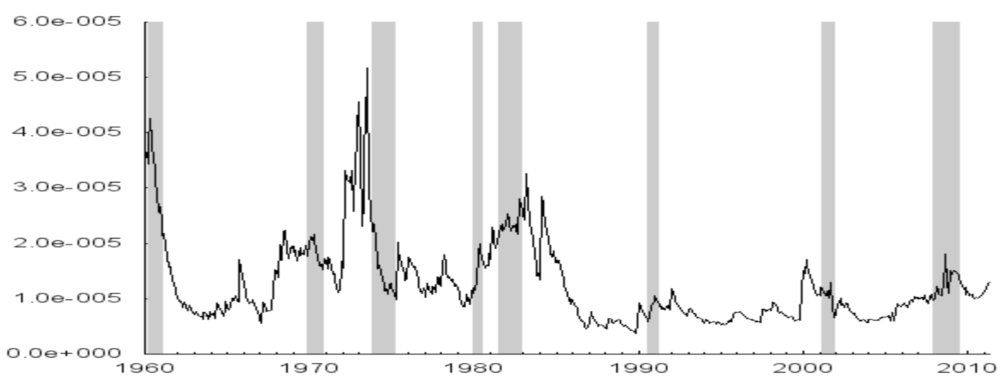



**Figure 5.8:** Conditional variance of consumption growth ($c$) innovations over the sample period running from January 1960 until June 2011, given by equation (5.6). Shaded bars indicate official recession dates.

The final two models also use information in inflation and unemployment growth to model risk aversion. While there is no evidence that the variance of the unemployment growth shocks is time-varying, such evidence is abundant when it comes to inflation.

In particular, it is well documented that during periods of high inflation, the volatility is high as well. The path of the estimated QGARCH(1,1) process of inflation depicted in Figure 5.9 supports this. As mentioned above, most of the 1970s can be characterized as a period of high inflation and high unemployment. Hence the peak of inflation during the third recession is no surprise. The finding that the volatility of inflation does not really respond to the events of 1987, 1998 and 2001 is also probable as inflation is not at an elevated level during any of these three periods. The spike in 2005 is the result of a monthly inflation rate of 1.2% in September; the spike just the latest recession is the result of the deflation rate of -1.2% in November 2008.

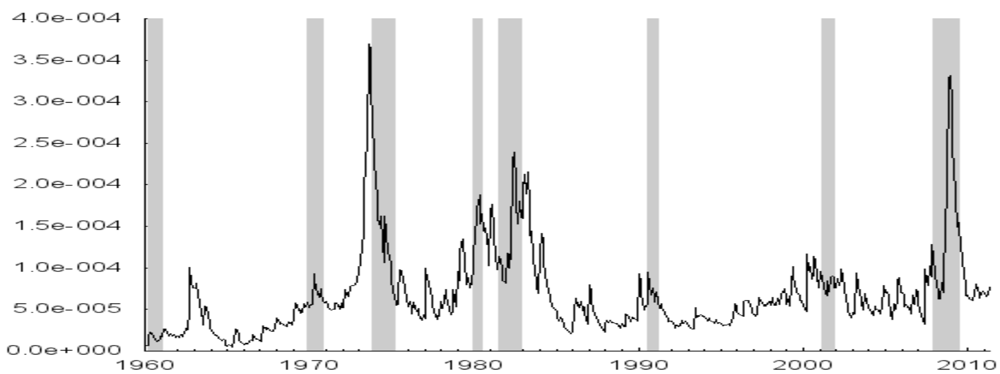



**Figure 5.9:** Conditional variance of inflation ($\pi$) innovations over the sample period running from January 1960 until June 2011, given by equation (5.6). Shaded bars indicate official recession dates.

Note that due to the negative correlation between the innovations in the excess return and inflation the conditional covariance series, i.e. $k_t^\pi$ in Figure 5.10 is negative over the whole sample period. As the average conditional volatility of inflation shocks is about 24 times smaller than the average conditional volatility of real excess return innovations, $k_t^\pi$ predominantly follows the path of the latter. Nevertheless, as inflation volatility peaks at recessions III and VIII, it does contribute to the large values in these two downturns.

Although there is not enough evidence to conclude that unemployment shocks have a time-varying variance, $k_t^u$ still varies as a result of the variation in the variance of excess return innovations; see Figure 5.11. This series therefore derives its time-varying

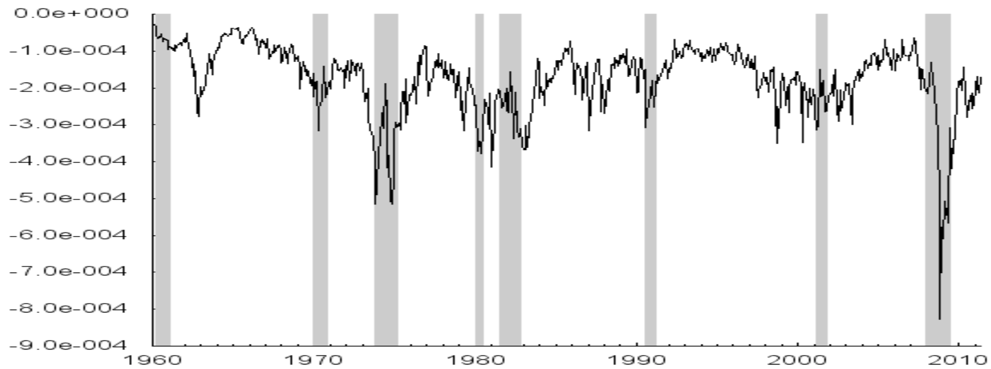



**Figure 5.10:** Conditional covariance of innovations in the real excess return ($exr$) and inflation ($\pi$) over the sample period running from January 1960 until June 2011, given by equation (5.7). Shaded bars indicate official recession dates.

nature solely from the variation in the variance of shocks to the real excess return. But note that it is not a scaled version of this series due to the non-linear effect of the square root.

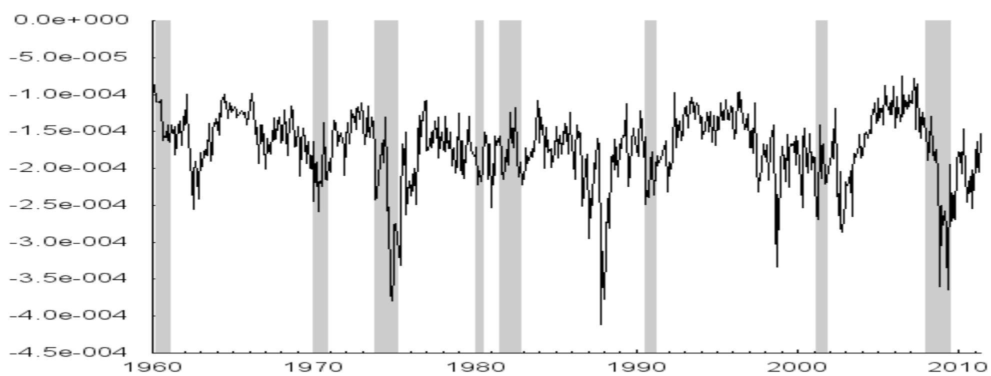



**Figure 5.11:** Conditional covariance of innovations in the real excess return ($exr$) and unemployment growth ($u$) over the sample period running from January 1960 until June 2011, given by equation (5.7). Shaded bars indicate official recession dates.

### 5.4.5 Comparison

Comparing the four estimated paths of the log coefficient of relative risk aversion (labeled $\Gamma$) reveals quite some differences. Table 5.7 provides a concise overview of some descriptive statistics. The top panel reveals that risk aversion in the first and third specification (those with non-varying covariances) is lowest in the years 1966 and 1967, while this occurs in in the years 1989 and 1990 for the two models in which there is variation in the covariances as well. Regarding the maximum, all four models are very clear: over the last fifty years log risk aversion has never been higher than during or just after the latest recession. Note that while the dispersion between the minimum and maximum values is, respectively, 0.93 and 1.34, average values of $\Gamma$ range from 2.97 (model 4) to 3.51 (model 1). Standard deviations of the the four series are comparable.

Average values of risk aversion over the sample period reveal that all four models suffer from the equity premium puzzle, i.e. coefficients of risk aversion exceeding 10. Nevertheless, the value attained by the second and fourth specification is much closer to this upper bound than the first and third model. Of course this is not surprising as the former two specifications encompass time-varying covariances, hence part of the variation is captured through these terms.

**Table 5.7: Descriptive statistics risk aversion**

Notes: Models 1-3 are respectively based on equations (5.13)-(5.16), (5.17)-(5.21) and (5.22)-(5.27); see Appendix 5.A.3. The fourth model is based on equations (5.1)-(5.7), see Section 2. The first specification excludes inflation and unemployment growth and keeps the variance of consumption growth innovations fixed. The second specification is similar to the first but allows this variance to be time-varying. The third specification includes consumption growth, inflation and unemployment growth but keeps their innovations variances fixed. The fourth specification is similar to the third but allows these variances to be time-varying. Log risk aversion is denoted by $\Gamma$, while risk aversion is denoted by $\gamma$ and so $\Gamma \equiv \log \gamma$. The sample standard deviation is denoted by *StDev*. Sample period runs from January 1960 until June 2011.

| | Model 1 | Model 2 | Model 3 | Model 4 |
|---|---|---|---|---|
| Minimum $\Gamma$ | 0.67 | 0.23 | 1.16 | 0.68 |
| Minimum $\gamma$ | 1.95 | 1.26 | 3.19 | 1.96 |
| Date | Sep-1967 | Mar-1990 | May-1966 | May-1989 |
| | | | | |
| Maximum $\Gamma$ | 6.81 | 5.52 | 6.49 | 5.47 |
| Maximum $\gamma$ | 908.29 | 249.64 | 658.52 | 237.46 |
| Date | Dec-2009 | Jan-2010 | Sep-2009 | May-2009 |
| | | | | |
| Average $\Gamma$ | 3.51 | 3.18 | 3.35 | 2.97 |
| StDev | 1.52 | 1.30 | 1.24 | 1.31 |
| Average $\gamma$ | 98.33 | 47.49 | 64.42 | 42.37 |
| StDev | 156.13 | 50.89 | 102.09 | 51.02 |

## 5.5 Conclusion

Instead of treating risk aversion as a structural parameter that remains constant over time, existing research has recognized that this assumption can no longer be maintained given the abundance of real life evidence against it. Another motivation for this paper is the recent supposition that discount-rate variation is at the heart of current asset-pricing research and time-varying risk aversion can be seen as a manifestation of this.

This paper formulates four asset pricing models. The first is the standard c-CAPM with the single difference that risk aversion is time-varying. We then extend this model by allowing for time-varying volatility of consumption growth innovations. While this increases the validity of the model it also has one important side-effect: it induces variation in the covariance between the excess return and consumption growth. Our third specification adds inflation and unemployment growth to the identity such that risk aversion now also varies in response to news concerning these two variables. Finally we also allow for variability in the volatility of inflation innovations, keeping the volatility of unemployment growth constant. The main novelty is that our models are estimated using state space techniques which are ideally suited for this purpose as risk aversion is a latent variable.

In general the paths of risk aversion generated by our four specifications are in line with the business cycle implying that risk aversion is low during expansion periods and rises around recession dates. Moreover, we also observe that aversion increases in late 1987 (stock market crash), during the summer of 1998 (Russian ruble crises) and in late 2001 (September terrorist attacks). Over the last 50 years the lowest level of risk aversion occurred at the late 1980s / early 1990s while the highest level is seen at the end or just after the latest recession, also known as the ‘Great Recession’. The level of risk aversion decreases as the models are extended but our most elaborate specification still generates an average risk aversion of about 42 which is outside of the range of values deemed plausible. However, given the plethora of resolutions offered by existing research, this should not come as a surprise.

## 5.A Appendices

### 5.A.1 Derivation

Assume there is a representative agent who maximizes a time-separable power utility function defined over aggregate consumption $C_t$, price level $P_t$ and unemployment $U_t$, i.e.

$$U(C_t, P_t, U_t) = \frac{\left[C_t^{\theta^c} P_t^{\theta^\pi} U_t^{\theta^u}\right]^{1-\gamma} - 1}{1-\gamma},$$

where $\Gamma$ is the coefficient of relative risk aversion and the $\theta' s$ are parameters constrained to lie between -1 and 1. Their purpose becomes apparent later. Moreover, both the price level and unemployment are exogenous to the agent's choice set.

Marginal utility is given by

$$U_c(C_t, P_t, U_t) = \left[C_t^{\theta^c} P_t^{\theta^\pi} U_t^{\theta^u}\right]^{-\gamma},$$

where we made use of the fact that $P_t$ and $U_t$ are exogenous and that $\frac{\partial P}{\partial C} = \frac{\partial U}{\partial C} = 0$. This implies that the stochastic discount factor, denoted by $M_{t+1}$, reads

$$\begin{aligned} M_{t+1} &= \frac{\delta U_c\left(C_{t+1}^{\theta^c}, P_{t+1}^{\theta^\pi}, U_{t+1}^{\theta^u}\right)}{U_c\left(C_t^{\theta^c}, P_t^{\theta^\pi}, U_t^{\theta^u}\right)} \\ &= \delta \frac{\left[C_{t+1}^{\theta^c} P_{t+1}^{\theta^\pi} U_{t+1}^{\theta^u}\right]^{-\gamma}}{\left[C_t^{\theta^c} P_t^{\theta^\pi} U_t^{\theta^u}\right]^{-\gamma}} \\ &= \delta \left[\left(\frac{C_{t+1}}{C_t}\right)^{\theta^c} \left(\frac{P_{t+1}}{P_t}\right)^{\theta^\pi} \left(\frac{U_{t+1}}{U_t}\right)^{\theta^u}\right]^{-\gamma}. \end{aligned} \tag{5.8}$$

The conditional pricing relation is

$$1 = \mathrm{E}_t\left[(1 + R_{t+1}) M_{t+1}\right], \tag{5.9}$$

where $R_{t+1}$ is the net return on an asset held from time $t$ to time $t+1$. Next we follow Hansen and Singleton (1983) and assume that joint conditional distribution of asset returns and the stochastic discount factor is lognormal. Taking logs of equation (5.9) leads to

$$\mathrm{E}_t r_{t+1} = -\mathrm{E}_t m_{t+1} - \frac{1}{2}\left[\left(\sigma_r^2\right)_t + \left(\sigma_m^2\right)_t + 2\left(\sigma_{r,m}\right)_t\right].$$

Lower case denote logarithms, $(\sigma_r^2)_t = \mathrm{E}_t\left[(\log r - \mathrm{E}_t \log r)^2\right]$ is the conditional variance of the log equity return innovations, $(\sigma_m^2)_t = \mathrm{E}_t\left[(\log m - \mathrm{E}_t \log m)^2\right]$ is the conditional variance of the stochastic discount factor innovations and $\left(\sigma_{r,m}\right)_t = \mathrm{E}_t[\log r - \mathrm{E}_t \log r, \log m - \mathrm{E}_t \log m]$ is their conditional covariance. Using equation (5.8) to substitute for the stochastic discount factor yields

$$\begin{aligned}
\mathrm{E}_t r_{t+1} = &- \log(\delta) + \Gamma \mathrm{E}_t\left[\theta^c \Delta c_{t+1} + \theta^\pi \Delta \pi_{t+1} + \theta^u \Delta u_{t+1}\right] \\
&- \frac{1}{2}\left\{\left(\sigma_r^2\right)_t + \gamma^2\left[\left(\theta^c\right)^2\left(\sigma_c^2\right)_t + \left(\theta^\pi\right)^2\left(\sigma_\pi^2\right)_t + \left(\theta^u\right)^2\left(\sigma_u^2\right)_t\right]\right\} \\
&+ \gamma^2\left[\theta^c\theta^\pi\left(\sigma_{c,\pi}\right)_t + \theta^c\theta^u\left(\sigma_{c,u}\right)_t + \theta^\pi\theta^u\left(\sigma_{\pi,u}\right)_t\right] \\
&- \gamma\left[\theta^c\left(\sigma_{r,c}\right)_t - \theta^\pi\left(\sigma_{r,\pi}\right)_t - \theta^u\left(\sigma_{r,u}\right)_t\right]. \qquad (5.10)
\end{aligned}$$

Lower care letters denote logarithms and $\Delta c_{t+1}, \Delta \pi_{t+1}$ and $\Delta u_{t+1}$ refer to consumption growth, inflation and unemployment growth, respectively. The conditional variances and covariances are similar to those defined just above.

To get an expression for the expected excess return, we repeat the above exercise for the riskfree rate. Its innovation variance is obviously zero and hence so is its covariance with any other series. This implies that

$$\begin{aligned}
\mathrm{E}_t rf_{t+1} = &- \log(\delta) + \gamma \mathrm{E}_t\left[\theta^c \Delta c_{t+1} + \theta^\pi \Delta \pi_{t+1} + \theta^u \Delta u_{t+1}\right] \\
&- \frac{\gamma^2}{2}\left[\left(\theta^c\right)^2\left(\sigma_c^2\right)_t + \left(\theta^\pi\right)^2\left(\sigma_\pi^2\right)_t + \left(\theta^u\right)^2\left(\sigma_u^2\right)_t\right] \\
&+ \gamma^2\left[\theta^c\theta^\pi\left(\sigma_{c,\pi}\right)_t + \theta^c\theta^u\left(\sigma_{c,u}\right)_t + \theta^\pi\theta^u\left(\sigma_{\pi,u}\right)_t\right], \qquad (5.11)
\end{aligned}$$

where all variables are as defined before.

Subtracting equation (5.11) from equation (5.10) yields an expression for the expected equity premium

$$\mathrm{E}_t[r_{t+1} - rf_{t+1}] + \frac{(\sigma_r^2)_t}{2} = \gamma[\theta^c(\sigma_{r,c})_t + \theta^\pi(\sigma_{r,\pi})_t + \theta^u(\sigma_{r,u})_t]. \tag{5.12}$$

Compared to the canonical c-CAPM there are two differences. First, besides consumption growth, the excess return is now also a function of inflation and unemployment growth. The second difference is the presence of the $\theta's$. As mentioned, these parameters are constrained to lie between -1 and 1 and are used to test whether inflation and unemployment growth contain information not present in consumption growth that helps explain the equity premium. In other words, if either $\theta^\pi$ or $\theta^u$ is statistically different from zero it helps explain the excess return.

### 5.A.2 State Space Representation

The general state space representation is given by a measurement equation

$$y_{t+1} = Z_{t+1}\alpha_{t+1} + H_{t+1}\epsilon_{t+1}, \quad \text{where } \epsilon_{t+1} \sim N(0, G_{t+1}),$$

a transition equation

$$\alpha_{t+1} = T_{t+1}\alpha_t + R_{t+1}\eta_{t+1} \quad \text{where } \eta_{t+1} \sim N(0, Q_{t+1}),$$

and an initialization of the state vector

$$\alpha_1 \sim N(a_1, P_1).$$

For our model, we have the following state space representation

$$y_{t+1} = \begin{bmatrix} exr_{t+1} & c_{t+1} & \pi_{t+1} & u_{t+1} \end{bmatrix}',$$

$$Z_{t+1} = \begin{bmatrix} 0 & 0 & 0 & 0 & k^c_{t+1} + k^{\pi}_{t+1} + k^u & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\alpha_{t+1} = \begin{bmatrix} 1 & c_{t+1} & \pi_{t+1} & u_{t+1} & \gamma_{t+1} & \epsilon^{exr}_{t+1} & \epsilon^c_{t+1} & \epsilon^{\pi}_{t+1} & \epsilon^u_{t+1} \end{bmatrix}'.$$

$$T_{t+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{c} & \phi^c & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{\pi} & 0 & \phi^{\pi} & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{u} & 0 & 0 & \phi^u & 0 & 0 & 0 & 0 & 0 \\ \bar{\gamma} & 0 & 0 & 0 & \phi^{\gamma} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$R_{t+1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\eta_{t+1} = \begin{bmatrix} \epsilon^{exr}_{t+1} & \epsilon^{c}_{t+1} & \epsilon^{\pi}_{t+1} & \epsilon^{u}_{t+1} & \epsilon^{\gamma}_{t+1} \end{bmatrix}',$$

$$Q_{t+1} = \begin{bmatrix} (\sigma^{exr}_{t+1})^2 & k^c_{t+1} & k^{\pi}_{t+1} & k^u & 0 \\ k^c_{t+1} & (\sigma^c_{t+1})^2 & 0 & 0 & m^c_{t+1} \\ k^{\pi}_{t+1} & 0 & (\sigma^{\pi}_{t+1})^2 & 0 & m^{\pi}_{t+1} \\ k^u & 0 & 0 & (\sigma^u)^2 & m^u \\ 0 & m^c_{t+1} & m^{\pi}_{t+1} & m^u & (\sigma^{\gamma})^2 \end{bmatrix},$$

where

$$\begin{aligned} k^c_{t+1} &= \rho_{(exr,c)}(\sigma^{exr}_{t+1})^2(\sigma^c_{t+1})^2, \\ k^{\pi}_{t+1} &= \rho_{(exr,\pi)}(\sigma^{exr}_{t+1})^2(\sigma^{\pi}_{t+1})^2, \\ k^u &= \rho_{(exr,u)}(\sigma^{exr}_{t+1})^2(\sigma^u)^2, \\ m^c_{t+1} &= \rho_{(c,\gamma)}(\sigma^c_{t+1})^2(\sigma^{\gamma})^2, \\ m^{\pi}_{t+1} &= \rho_{(\pi,\gamma)}(\sigma^{\pi}_{t+1})^2(\sigma^{\gamma})^2, \\ m^u &= \rho_{(u,\gamma)}(\sigma^u)^2(\sigma^{\gamma})^2 \quad \text{and} \\ (\sigma^i_{t+1})^2 &= \beta^i_1 + \beta^i_2\epsilon^i_t + \beta^i_3(\epsilon^i_t)^2 + \beta^i_4(\sigma^i_t)^2 \quad \text{for} \quad i = exr, c \text{ and } \pi. \end{aligned}$$

and

$$\begin{aligned} a_1 &= \begin{bmatrix} 1 & \frac{\bar{c}}{1-\phi^c} & \frac{\bar{\pi}}{1-\phi^{\pi}} & \frac{\bar{u}}{1-\phi^u} & \frac{\bar{\gamma}}{1-\phi^{\gamma}} & 0 & 0 & 0 & 0 \end{bmatrix}', \\ diag(P_1) &= \begin{bmatrix} 0 & \frac{\sigma^2_{c1}}{1-(\phi^c)^2} & \frac{\sigma^2_{\pi 1}}{1-(\phi^{\pi})^2} & \frac{\sigma^2_u}{1-(\phi^u)^2} & \frac{\sigma^2_{\gamma}}{1-(\phi^{\gamma})^2} & \sigma^2_{exr1} & \sigma^2_{c1} & \sigma^2_{\pi 1} & \sigma^2_u \end{bmatrix}', \end{aligned}$$

where

$$\sigma^2_{exr1} = \frac{\beta_1^{exr}}{1-\beta_3^{exr}-\beta_4^{exr}},$$
$$\sigma^2_{g1} = \frac{\beta_1^{c}}{1-\beta_3^{c}-\beta_4^{c}},$$
$$\sigma^2_{\pi 1} = \frac{\beta_1^{\pi}}{1-\beta_3^{\pi}-\beta_4^{\pi}}.$$

**Technical Notes**

1. The method proposed by Harvey et al. (1992) requires that the conditional distribution of the error $\eta_{t+1}$ is Gaussian. Note that the unconditional distribution is non-Gaussian, see Hamilton (1994, p. 662).

2. The system matrices $Z_{t+1}, T_{t+1}, R_{t+1}$ and $Q_{t+1}$ need to be constant, exogenous or predetermined (see Hamilton, 1994, Chapter 13)]. Our model satisfies this condition.

3. The filter is initialized with the matrices $A_1$ and $P_1$ which, given the assumption of stationary states, contain the unconditional means and variances of the states.

4. The time-varying conditional variances $(\sigma^i_{t+1})^2$, for $i = exr, c$ and $\pi$, complicate the linear Gaussian state space framework. To deal with this we follow the approach by Harvey et al. (1992) and include the stocks $\epsilon^{exr}_{t+1}$, $\epsilon^{c}_{t+1}$, $\epsilon^{\pi}_{t+1}$ and $\epsilon^{u}_{t+1}$ in the state vector. We then note that the conditional variances and therefore $Q_{t+1}$ are functions of the unobserved states $\epsilon^{exr}_{t+1}$, $\epsilon^{c}_{t+1}$, $\epsilon^{\pi}_{t+1}$ and $\epsilon^{u}_{t+1}$. Harvey et al. (1992) replace $(\sigma^i_{t+1})^2$ in the system by $(\sigma^{i*}_{t+1})^2 = \beta_1 + \beta_2\epsilon^{i*}_t + \beta_3(\epsilon^{i*}_t)^2 + \beta_4(\sigma^{i*}_t)^2$ where the unobserved $(\epsilon^i_t)^2$ are replaced by their conditional expectations $(\epsilon^{i*}_t)^2 = \mathrm{E}_t(\epsilon^i_t)^2$. Note that $\mathrm{E}_t(\epsilon^i_t)^2 = [\mathrm{E}_t\epsilon^i_t]^2 + [\mathrm{E}_t(\epsilon^i_t - \mathrm{E}_t\epsilon^i_t)^2]$ where the quantities between square brackets are period $t$ Kalman filter output (conditional means and variances of the states $\epsilon^i_t$. Thus, given $(\sigma^i_t)^2$ (which is initialized by the unconditional variances of $\epsilon^i_t$, i.e. $\sigma^2_{i1}$ and given the Kalman filter output from

period $t$, namely $\mathrm{E}_t(\alpha_t)$ and $\mathrm{V}_t(\alpha_t)$, we can calculate $(\sigma^i_{t+1})^2$ and the system matrix $Q_{t+1}$ which makes it possible to calculate $\mathrm{E}_t(\alpha_{t+1})$, $\mathrm{V}_t(\alpha_{t+1})$, $\mathrm{E}_{t+1}(\alpha_{t+1})$, $\mathrm{V}_{t+1}(\alpha_{t+1})$ and so on.

### 5.A.3 Specifications

This appendix provides details of the first three specifications, the fourth model is described in Section 2.

#### First model

The first specification only includes consumption growth ($c$). In addition, the variance of shocks in this variable $(\sigma^c)^2$ and the real excess return $(\sigma^{exr})^2$ is kept constant. As a result, the covariance between the innovations in the excess return and consumption growth ($k^c$) is also fixed. More formally,

$$exr_t = \gamma_t k^c + \epsilon_t^{exr}, \tag{5.13}$$

$$c_t = \bar{c} + \phi^c c_{t-1} + \epsilon_t^c, \tag{5.14}$$

$$\gamma_t = \bar{\gamma} + \phi^\gamma \gamma_{t-1} + \epsilon_t^\gamma, \tag{5.15}$$

$$k^c = \rho_{(exr,c)} \sigma^{exr} \sigma^c. \tag{5.16}$$

#### Second model

This specification extends the setup of the first model by allowing the shocks in the real excess return and consumption growth to have time-varying volatilities. This variation is modeled through the use of the QGARCH(1,1) model. It is given by

$$exr_t = \gamma_t k^c_{t-1} + \epsilon^{exr}_t, \tag{5.17}$$

$$c_t = \bar{c} + \phi^c c_{t-1} + \epsilon^c_t, \tag{5.18}$$

$$\gamma_t = \bar{\gamma} + \phi^\gamma \gamma_{t-1} + \epsilon^\gamma_t, \tag{5.19}$$

$$(\sigma^i_t)^2 = \beta^i_1 + \beta^i_2 \epsilon^i_{t-1} + \beta^i_3 (\epsilon^i_{t-1})^2 + \beta^i_4 (\sigma^i_{t-1})^2, \quad \text{for i} = exr \text{ and } c \tag{5.20}$$

$$k^c_{t-1} = \rho_{(exr,c)} \sigma^{exr}_t \sigma^c_t. \tag{5.21}$$

**Third model**

The third model adds inflation ($\pi$) and unemployment growth ($u$) to the list of explanatory variables. Just like the first specification, all innovation variances (and covariances) are kept fixed. Hence we estimate

$$exr_t = \gamma_t [k^c + k^\pi + k^u] + \epsilon^{exr}_t, \tag{5.22}$$

$$c_t = \bar{c} + \phi^c c_{t-1} + \epsilon^c_t, \tag{5.23}$$

$$\pi_t = \bar{\pi} + \phi^\pi \pi_{t-1} + \epsilon^\pi_t, \tag{5.24}$$

$$u_t = \bar{u} + \phi^u u_{t-1} + \epsilon^u_t, \tag{5.25}$$

$$\gamma_t = \bar{\gamma} + \phi^\gamma \gamma_{t-1} + \epsilon^\gamma_t, \tag{5.26}$$

$$k^i = \rho_{(exr,i)} \sigma^{exr} \sigma^i, \quad \text{for i} = c, \pi \text{ and } u. \tag{5.27}$$

# Chapter 6

# Nederlandse samenvatting (Summary in Dutch)

Het is algemeen bekend dat op lange termijn het rendement op aandelen veelal hoger is dan op staatsobligaties. Dit verschil in rendement staat in de economische wetenschap bekend als de aandelenpremie. Hoe langer de looptijd van de investering, des te beter renderen aandelen ten opzichte van obligaties. Dit houdt ook in dat op de korte termijn het tegenovergestelde vaak waar is. Zo verloor de AEX-index gedurende het jaar 2008 meer dan de helft van haar waarde, terwijl obligaties dat jaar met een positief rendement afsloten. Aangezien aandelenkoersen heftiger fluctueren dan obligaties, worden ze over het algemeen als risicovoller beschouwd. Gelet hierop kan de aandelenpremie worden beschouwd als compensatie voor dit hogere risico.

Het consumptie *Capital Asset Pricing Model* (hierna c-CAPM) wordt veelvuldig gebruikt om de prijs van financiële activa vast te stellen. Het genoemde model gaat uit van een belegger die zijn nut — de bevrediging die een goed bij consumptie verschaft — maximaliseert. Met andere woorden, een investeerder is niet primair geïnteresseerd in de eigenlijke rendementen, maar meer in de consumptie die hij zich kan veroorloven door in aandelen en obligaties te handelen. Uit data-onderzoek blijkt dat werkelijke aandelenpremies niet consistent zijn met consumptiegegevens, tenzij we ervan uitgaan dat huishoudens extreem risico-avers zijn. Dit is in het kort de aandelenpremiepuzzel. De puzzel is niet dat aandelen beter renderen dan obligaties, maar dat de hoogte van de premie niet verenigbaar is met consumptiegegevens als we uitgaan van beschikbare gegevens ten aanzien van risico-aversie.

De kern van de aandelenpremiepuzzel is de relatief lage variatie in de waargenomen consumptiegroeicijfers. Een risicovol activum heeft een laag rendement indien het niet met consumptiefluctuaties meebeweegt door het goed (slecht) te doen in periodes van lage (hoge) consumptiegroei. De achterliggende gedachte is dat een activum dat hieraan voldoet een investeerder verzekert tegen moeilijke tijden. Hierdoor zal er veel belangstelling voor zijn waardoor het rendement lager uitvalt. Andersom is een activum dat weinig betaalt in moeilijke tijden en veel in goede tijden minder wenselijk. Als gevolg zal het beleggers een hoger rendement moeten bieden om hen over te halen het in hun portefeuille op te nemen. Aangezien consumptiegroei maar weinig varieert, kan een hoog rendement op risicovolle activa alleen worden ondersteund als men er van uitgaat dat zelfs zeer geringe consumptie-schommelingen zeer pijnlijk zijn voor de consument. Met andere woorden, de enige verklaring is dat consumenten een buitengewoon hoge mate van risico-aversie moeten hebben.

In dit proefschrift wordt de aandelenpremiepuzzel vanuit vier verschillende invalshoeken benaderd. Het eerste hoofdstuk introduceert deze puzzel, bespreekt relevante literatuur en gaat in op de maatschappelijke relevantie. Het tweede hoofdstuk ontwikkelt een methode om de onzekerheid rondom de schatting van de coëfficiënt van relatieve risico-aversie te kwantificeren. Deze uitkomsten worden vervolgens gebruikt om de gegevens van een aantal landen op een optimale manier samen te voegen om zodoende een meer precieze schatting te krijgen. Hoofdstuk 3 vormt de kern van dit proefschrift. Het introduceert een heterogeen-agent model voor het gedrag van aandeelhouders en niet-aandeelhouders. De belangrijkste innovatie is het macroeconomische perspectief, geoperationaliseerd met behulp van gegevens uit nationale rekeningen. Hoofdstuk 4 past het c-CAPM aan door de focus te leggen op een element dat tot nu toe in de literatuur onderbelicht is gebleven: de risicovrije rentevoet.

In tegenstelling tot de andere hoofdstukken houdt hoofdstuk 5 zich niet rechtstreeks met de aandelenpremiepuzzel bezig. Het centrale thema hierin is de variabiliteit van risico-aversie door de tijd heen; de puzzel wordt in de kantlijn besproken. Meer in detail is de indeling als volgt.

Hoofdstuk 2 is gebaseerd op Pozzi et al. (2010). Het zoekt een antwoord op de vraag in welke mate de aandelenpremiepuzzel een statistisch fenomeen is, te wijten aan de gebrekkige betrouwbaarheid van de macro-economische gegevens. Uit het literatuuroverzicht van Campbell (2003) blijkt dat schattingen van risico-aversie behorend bij verschillende landen sterk uiteenlopen, maar dit onderzoek onderwerpt deze onzekerheid niet aan een statistische analyse. Dit hoofdstuk repliceert de resultaten van Campbell, voegt recente gegevens toe en gebruikt de "jackknife methode om de onzekerheid waarmee risico-aversie wordt geschat in het c-CAPM te kwantificeren. Bovendien, door het bundelen van land-specifieke gegevens ontstaat een veel kleiner betrouwbaarheids-interval rondom de coëfficiënt van relatieve risico-aversie vergeleken met een analyse op basis van een individueel land.

De Vries and Zenhorst (2012) vormt de basis voor Hoofdstuk 3. Èèn van de voorgestelde oplossingen voor de aandelenpremiepuzzel is de beperkte toegankelijkheid van aandelenmarkten. Oftewel, niet iedereen heeft de mogelijkheid om in aandelen te beleggen. Het idee is dat de consumptiegroei van aandeelhouders nauwer verbonden is met de aandelenpremie dan die van niet-aandeelhouders. Dit betekent dat de covariantie groter is en de coëfficiënt van relatieve risico-aversie lager. Bestaand onderzoek op dit gebied maakt over het algemeen gebruik van gegevens op het niveau van huishoudens. Daarentegen gebruikt dit hoofdstuk macro-economische gegevens om onderscheid te maken tussen de verschillende bronnen van inkomsten en bijbehorende consumptie. Naast het gegeven dat dit langere tijdreeksen oplevert, heeft deze aanpak ook als voordeel dat het een internationale vergelijking vergemakkelijkt. De gedachte achter deze macro-economische benadering is als volgt. Aandeelhouders consumeren uitsluitend uit hun kapitaalinkomen, terwijl niet-aandeelhouders alleen de beschikking hebben over arbeidsinkomen. Het inkomen uit kapitaal en de loon-inkomsten zijn opgebouwd uit de categorieën van het beschikbaar nationaal inkomen. Aansluitend wordt de geaggregeerde consumptie verhoudingsgewijs toegewezen aan deze twee vormen van factorinkomen. Deze reeksen worden vervolgens gebruikt om de coëfficiënt van relatieve risico-aversie voor beide groepen te berekenen. Zoals verwacht zijn de schattingen behorende bij aandeelhouders lager dan die van de representatieve agent. Toch is de risico-aversie van aandeelhouders nog steeds te hoog om plausibel te zijn.

Hoofdstuk 4 heeft zijn oorsprong in Zenhorst (2012a). Èèn van de noodzakelijke variabelen om de relatieve risico-aversie coëfficiënt te schatten is de risicovrije rentevoet. Terwijl andere aspecten van het model, zoals de functionele vorm van de nutsfunctie, veel aandacht hebben gekregen, gaat dit niet op voor deze variabele. De meeste studies maken gebruik van de rente op een staatsobligatie met een looptijd van drie maanden om als risicovrije rente dienst te doen. Het is echter de vraag of dit activum voor de gemiddelde consument inderdaad de beste maatstaf is. Het lijkt erop dat de geboden rente op een spaarrekening of termijndeposito veel dichter bij de investeringsmogelijkheden van een gemiddeld huishouden ligt. Aangezien deze rente over het algemeen lager is dan de rente die overheden betalen op hun korte-termijn schulden, zal de aandelenpremie toenemen. Dit houdt in dat de coëfficiënt van relatieve risico-aversie hoger uitvalt en daarmee wordt de aandelenpremiepuzzel dus groter.

Hoofdstuk 5 komt voort uit Zenhorst (2012b). Bestaand onderzoek met betrekking tot de aandelenpremiepuzzel behandelt risico-aversie als een structurele parameter waarvan de waarde constant is door de tijd heen. Hoewel dit het model aanzienlijk versimpelt, is deze aanname moeilijk te rechtvaardigen. Inzichten uit de psychologie van besluitvorming laten zien dat mensen verliezen en winsten asymmetrisch waarderen. Met andere woorden, een verlies van € 100 komt harder aan dan de vreugde waarmee het winnen van ditzelfde bedrag gepaard gaat. Uit waarnemingen blijkt dus duidelijk dat risico-aversie varieert. Het centrale doel van dit hoofdstuk is om deze tijdsvariatie in de coëfficiënt van relatieve risico-aversie zichtbaar te maken. Hiervoor formuleert het een model waarin variatie in risico-aversie wordt gedreven door nieuws in consumptiegroei, inflatie en de groei in werkloosheid. Het belangrijkste criterium om het model te beoordelen is de mate waarin de variatie in de coëfficiënt van relatieve risico-aversie in lijn is met recessieperiodes en andere belangrijke gebeurtenissen die gedurende de onderzoeksperiode plaatsvinden. De resultaten laten zien dat risico-aversie zeer persistent is en, zoals verwacht, stijgt in de aanloop naar en gedurende recessies.

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