RISKS OF LEVERAGED PRODUCTS

Antonio Di Cesare
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Risico van leveraged producten

Thesis

to obtain the Degree of Doctor from the
Erasmus University Rotterdam
by command of the
Rector Magnificus

Prof.dr. H.G. Schmidt

and according to the decision of the Doctorate Board

The public defence shall be held on
Thursday 20 December 2012 at 13:30 hrs

by

ANTONIO DI CESARE
born in Pescara, Italy

[Signature]
Doctoral Committee

Promoter: Prof.dr. C.G. de Vries

Other members: Prof.dr. A. Lucas
Prof.dr. P.A. Stork
Prof.dr. M. Verbeek
Dedicated to Ale, 
Annina and Nanni
Acknowledgments

The first time I met Prof.dr. Casper G. de Vries in a small village in the Swiss Alps for a course in extreme value theory, I certainly did not think that I would have owed so much to him in the following years. When he suggested me to come to the Erasmus School of Economics to pursue a PhD, I could not imagine that not only he would have become my advisor for this thesis but also that he would have constantly provided me with such an outstanding academic and human support. Thank you indeed Casper!!! I would have never reached this goal without you.

I am also especially grateful to my coauthors and to my colleagues at the Bank of Italy. The joint work that we did together and the long and fruitful discussions that we had on several topics covered in this thesis taught me a lot. Special thanks are for Giuseppe for the constant support that he has been giving to me since I joined the Bank of Italy several years ago.

I would like to thank all the members of the Erasmus School of Economics that I had the pleasure to meet during the periods I spent in Rotterdam. They proved to me to be excellent and enthusiastic scholars and they also were great in making me feel in a friendly environment. I will always remember with pleasure the nice meals that we had together. In particular, I thank Paul and Oke, my roommates during my stays in Rotterdam, for their friendship and for patiently answering my questions on the Netherlands and the Dutch way of life. Thanks also to Chen for explaining me several things on extreme value theory and to Suzanne for helping me with the translation in Dutch of the summary of this thesis. I thank also the secretaries of the Department of Economics, in particular Sytske and Milky, for providing me with excellent logistic support.

I express gratitude to my parents. They always allowed me to go ahead with my studies when I was younger even when short-term thinking would have probably required me to give a more direct support to my family. I know they love me and are proud of me. They can be sure that I love them and I am proud of them as well.

Dear Anna and Giovanni. Several times during the preparation of this thesis, I was tired of the work, disappointed for some result that I was not able to attain, sad for being so far from my dears. In those cases, I usually took a look at your
smiling faces in the pictures I had with me. Then, immediately a great joy entered
my heart and a large smile appeared on my face. Even without imagining it, you
have been indeed of great support to your dad.

Alessandra, my dear love. I do not think you can imagine how proud I am of
you. You allowed me to go away for long periods and you took care of everything at
home by your own. I know it has not been easy. I also know that you did not care
whether I got a PhD or not but you thought it was important for me and you shared
the effort with me. I believe this is indeed a sign of true love. My most heartfelt
thanks are for you.

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Chapter 1

Introduction

It is usually agreed that financial markets do not only reflect the conditions of the real economy but are also an important determinant of the level of economic activity. The idea that the financial sector can be a powerful amplifier of the business cycle (the concept of pro-cyclicality usually adopted) dates back at least to Fisher (1933). In Fisher’s view, however, financial factors play an asymmetric role: financial frictions limit the availability of external finance to firms and households, worsening downturns, but they do not have a symmetric positive role during upturns.

A vast macroeconomic literature, developed over the last 20 years, has removed this asymmetry with the introduction of the modern theory of the financial accelerator (FA).\textsuperscript{2} The FA works mainly through the value of collateral: a rise in asset prices makes it easier for households and firms to obtain loans, while a decline makes it more difficult. This mechanism is pro-cyclical since asset prices tend to be positively correlated with the business cycle and because credit availability feeds back positively onto investment and consumption, and hence onto economic growth.\textsuperscript{3}

An equally vast literature has ascertained the empirical importance of the FA. Many papers have studied how changes in net worth affect investment by financially constrained firms.\textsuperscript{4} Evidence of financing constraints has also been documented for households and for the functioning of housing markets.\textsuperscript{5} Most analyses have

\textsuperscript{1}Parts of this chapter are based on Panetta et al. (2009).
\textsuperscript{2}The seminal work in this field is Bernanke and Gertler (1989).
\textsuperscript{3}Studies have shown that there are several ways of explaining the FA (Bernanke et al., 1996). One group of models links the FA to fluctuations in the value of collateral assets (e.g., Iacoviello, 2005, and Iacoviello and Neri, 2010, for housing wealth): rising prices allow financially constrained agents to expand borrowing and thus consumption and investment; conversely, collateral devaluations force agents to cut expenditure. A second group of models emphasizes how endogenous changes to firm balance sheets amplify the business cycle (Carlstrom and Fuerst, 1997, and Bernanke et al., 1999). A third line of research directly analyzes banks’ contribution to real fluctuations (Goodfriend and McCallum, 2007, and Gerali et al., 2010).
\textsuperscript{4}See Fazzari et al. (1988) and the review in Hubbard (1998).
\textsuperscript{5}See Campbell and Mankiw (1989), Jappelli and Pagano (1989), Zeldes (1989), and Carroll
focused on the way in which financial market imperfections influence the business cycle indirectly, via their impact on the non-financial sector (firms and households). Much less effort has been devoted to analysis of the direct role of financial firms in amplifying shocks to the real economy.\footnote{A notable exception is the research activity carried out at the Bank for International Settlements on the inherent pro-cyclicality of post-Bretton Woods financial arrangements. See Borio et al. (2001), Borio and White (2004), and White (2006).}

\subsection*{1.1 The new financial accelerator}

A mechanism similar to the FA affects banks’ balance sheets: a negative shock to asset prices depletes capital and increases leverage. Since raising new capital is difficult in a downturn, banks tend to react by reducing credit and selling assets. Disposals feed back onto asset prices, propagating the initial shock. This may have a strong impact on economic activity, especially when shocks hit several banks simultaneously — as is typical of systemic events. In this framework, which is usually referred to as the new financial accelerator (NFA),\footnote{The term “new” is not completely correctly used here as the functioning of this mechanism has been well known at least since Kindleberger (1978). See Caballero and Krishnamurty (2008), Brunnermeier (2009), Brunnermeier and Pedersen (2009), and Adrian and Shin (2010).} the propagating factor is leverage: when banks are highly leveraged, the initial shock and the ensuing reduction in asset prices will induce massive asset liquidations, accentuating the price fall and possibly triggering a vicious circle, especially if banks want to restore a target leverage level. In principle, the mechanism is symmetric: an initial positive shock (e.g., a technological breakthrough, actual or expected) may lead to a broad rise in asset prices and hence to an expansion of intermediaries’ balance sheets, starting a positive circle.

The mechanism, which is complementary to the FA, works as follows.\footnote{The example is adapted from Adrian and Shin (2010).} Suppose a bank has assets and liabilities worth 100 and 90, respectively. Hence, the value of equity — net worth, that is assets less liabilities — is 10. Then its leverage, defined as the ratio of assets to equity, is equal to 10. Suppose an exogenous shock reduces the value of assets by 5\% (to 95) so that equity drops to 5. Then leverage almost doubles to 19. Assume the bank targets some level of leverage — say 10, the pre-shock value. To restore the desired level, the bank could issue equity or sell assets. When losses are large, however, banks tend to liquidate assets, since equity-raising tends to be sluggish and costly, especially in unfavorable market conditions, due

\begin{itemize}
\item and Dunn (1997) for research on households. Almeida et al. (2002) find that house prices are more sensitive to income shocks in countries with higher loan-to-value ratios, so the credit multiplier has greater impact on household spending in those countries.
\end{itemize}
to market frictions. Note that, assuming constant prices, to restore the desired leverage value of 10 the bank will have to liquidate assets and liabilities until, in the new equilibrium, its balance sheet will be 50% smaller than before. This is an enormous effect that is amplified even more if the initial shock hits a sufficiently large number of intermediaries. In this case, the simultaneous wave of asset sales will put further downward pressure on asset prices, generating a vicious circle.

This scheme generates pro-cyclicality as a chain reaction triggered by an exogenous shock (for example, a fall in house prices) and amplified by the interplay between the shock and asset market dynamics. The propagating factor is leverage: when banks are highly leveraged, the initial shock and the ensuing reduction in asset prices induce massive asset liquidation, accentuating the price fall and possibly starting a vicious circle.

Clearly, the empirical strength of the NFA depends on whether banks actually target an optimal leverage value. If banks did not target leverage, letting equity absorb shocks, the vicious circle would be dampened. The actual behavior of leverage over the cycle is therefore crucial for analytical purposes. Adrian and Shin (2010) show that the relationship between increases in leverage and rises in asset prices in the United States differs according to groups of investors. For households it is negative, denoting passive behavior (that is, when the value of their assets falls, households passively accept an increase in leverage, and conversely when it rises). For non-financial firms there is no clear relationship. Commercial banks do appear to target leverage levels. The relationship is also positive for security brokers and dealers (which used to include investment banks). These operators increase leverage when asset prices go up and reduce it when prices go down, making the greatest contribution to the vicious circle described above. Adrian and Shin (2010) argue that for commercial banks the supply curve for assets is downward sloping and the demand curve is upward sloping, generating market instability. For instance, a shock that causes a price drop reduces demand and increases supply, triggering further price declines. The International Monetary Fund (2008b, chap. 4) confirms

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9See Basel Committee on Banking Supervision (2004, para. 757). Kashyap et al. (2008) emphasize two frictions that contribute to this sluggishness: (1) equity issues increase the value of existing debt, thus generating an externality in favor of debt-holders and harming existing shareholders; (2) equity issues may signal forthcoming losses. Kashyap et al. (2008) also note that under Basel II the pressure to liquidate assets is stronger in crisis periods, when risk and hence risk-weighted capital requirements increase. Repullo and Suarez (2004) emphasize that the market for seasoned offerings is plagued by informational frictions, which may entail prohibitive costs of raising new capital.

10Targeting of leverage could be induced by market discipline: investors consider leverage an important gauge of firm behavior and health and so monitor it closely (Gropp and Heider, 2010). For banks, leverage targeting may also be induced by capital regulation or, in some countries, by specific limits on leverage.
this evidence for the top investment banks of 17 advanced economies and for the top commercial banks in the United States and Germany. It also finds that commercial bank leverage tends to be more pro-cyclical in financial systems characterized by a prevalence of arm’s length finance, such the United States and the United Kingdom. However, Panetta et al. (2009) argue that over long periods and broad number of countries the evidence in line with such mechanisms is not so clear-cut.

The above arguments point to the great relevance of the leverage of financial institutions for financial stability — and for economic welfare more generally. It is worth noting that leverage in modern financial markets does not only mean that some investments are financed by borrowed money. More generally, leverage is the capacity of being able to take risky positions without necessarily making an upfront payment that can cover all possible outcomes of those risks. This result usually arises from the usage of particular financial contracts that often have a build-in leverage factor. Due to their characteristics, the risk-return profiles of these instruments are somewhat difficult to understand. The consequences of this lack of knowledge can be severe for investors. This thesis is devoted to the analysis of several aspects of the risks involved in a few leveraged financial instruments, such as collateralized debt obligations (CDOs), credit default swaps (CDSs), and hedge funds.

1.2 Outline

After this introductory section, the thesis analyzes some aspects of the risks related with leveraged instruments. In particular, Chapters 2 and 3 analyze the role of securitization on the risk of incurring large losses for banks. More specifically, Chapter 2 emphasizes the fundamental role of the reinvestment policy of the proceeds of the securitizations, while Chapter 3 focusses on the role of extreme macroeconomic shocks. Chapter 4 studies the determinants of CDS spreads, focussing in particular on the analysis of their changes from before to after the onset of the global financial crisis in 2007. In Chapter 5 we study the impact that the serial correlation of hedge fund returns has on several measures of individual and systemic risk calculated for those financial intermediaries. Finally, Chapter 6 summarizes the main results and concludes.

1.2.1 Chapter 2

Chapter 2 is based on Di Cesare (2009) and studies the impact of securitization on the risk for a bank of incurring large losses. The financial products generated from the securitization processes are interesting instruments that can allow investors to
trade risks (in particular, credit risk) multiple times higher than those of traditional instruments with the same face value. In this sense, securitization products represent leveraged financial instruments.

The first part of the chapter analyzes the consequences for a bank of retaining the most junior tranches of the securitizations, as is often required by market practices and some regulatory frameworks. We rely on a slightly generalized version of the model used by Krahnen and Wilde (2006) to show that there can be several effects on the bank risk — as measured by the value-at-risk (VaR). The risk that a bank faces large losses can either increase or decrease, depending on the individual characteristics (in terms of default probabilities and correlation coefficients) of the loans that are securitized and the new loans that are granted with the proceeds of the securitizations. However, we also show that the final effect on the VaR is usually small as long as the securitizations involve only reasonable shares of the total loan portfolio of the bank.

Because of the importance of the reinvestment process in determining the overall effects on the bank risk, the second part of the chapter provides empirical evidence on how the securitization activity has contributed to changes in the overall composition of the asset side of the bank balance sheets. This part focusses on Italian banks and shows that the banks that securitized their assets have relatively lower shares of both well performing and bad loans to total assets. At the same time, those banks also increased their reliance on investments in the interbank market and in securities other than shares (mainly bonds). Overall, our estimates suggest that the broad changes in the balance sheets involved by the securitization activities have probably reduced the expected credit losses of the Italian banks. We also devote further analysis to individual loan data to compare the default risk of the loans that have been securitized with that of the new loans that have been granted with the proceeds of the securitizations. We show that the Italian banks have usually securitized loans with a better average quality than that of the new loans, thus suggesting that the credit risk embedded in their loan portfolios has increased as a consequence of securitizations (as the securitized loans have been removed from the bank balance sheets).

1.2.2 Chapter 3

Chapter 3 takes the view that painful economic events tend to happen and are inherent to any socio-economic system. The chapter analyzes how common macroeconomic shocks can impact the riskiness of a loan portfolio and the risk of the banks that securitize their assets.
The chapter is based on Di Cesare (2012) and uses a one-factor model to describe the default risk of bank assets that are linked to an underlying macroeconomic factor which can face negative shocks (see Vasicek, 1987, 1991, 2002). The shocks are described by the outcomes of random variables that are either Gaussian or Student’s $t$-distributed (see Lucas et al., 2002, Lucas et al., 2003, and Hull and White, 2004). Using Monte Carlo simulations, we show how the VaR of a bank at high confidence levels is influenced by the presence of common shocks and what happens to the VaR of a bank that securitizes its loan portfolio when the economy is subject to extreme macroeconomic shocks. Given that even borrowers with very low individual default probability are severely affected by extreme macroeconomic shocks, we show that the individual characteristics of the bank assets have only a modest impact on the bank risk. Moreover, the exposure of the bank to credit risk is only slightly influenced by the fact that the bank can securitize its riskiest assets and reinvest the proceeds of the securitizations in assets with lower credit risk.

We thus show that the consequences of extreme negative macroeconomic shocks for the banking system are broadly independent of the quality of the bank assets. For this reason, in order to increase the resilience of individual banks and the whole banking system, banks should focus on holding sufficient capital to absorb the losses resulting from the shocks. Trying to limit the negative consequences of the shocks by securitization practices that are perceived as safer may not be enough in the worst-case events.

As is well known, in the Basel II framework each bank has to satisfy a capital requirement that provides a buffer against unexpected losses at a specific level of statistical confidence, set by regulators at 99.9% (see Basel Committee on Banking Supervision, 2004, 2005). Our results show that reasonable statistical models can easily generate outcomes for which a VaR level that is considered acceptable at that confidence level can suddenly become much larger at slightly higher confidence levels, that is when rare events materialize as a crisis unfolds.

### 1.2.3 Chapter 4

Credit default swaps (CDSs) are the prototype example of credit derivates and represent one of the main instruments for taking leveraged credit exposures. Chapter 4 is based on Di Cesare and Guazzarotti (2010) and analyzes empirically the determinants of CDS spreads for a sample of US non-financial firms. Special attention is devoted to comparing results for the periods before and after the onset of the global financial crisis. In addition to the variables that the literature has found to have a theoretical and empirical impact on CDS spreads, we also include the theoretical
spread implied by the Merton (1974) model as a regressor to deal with the non-linear relationships between the individual characteristics of the firms and CDS spreads.

Our results show that the inclusion of this additional term improves the capacity to explain the CDS spread changes by the changes in the fundamental variables. The extended model is able to explain more than a half of the variations in CDS spreads in both the pre-crisis and the crisis periods, which is higher than previous findings of studies on corporate bond or CDS spread changes. When the theoretical spread calculated using the Merton model is introduced in the regressions, the coefficient of the equity volatility decreases significantly, because of the high sensitivity of the Merton model to this parameter. On the contrary, leverage, which has only second-order effects on the theoretical spreads, maintains its usefulness in explaining CDS spread changes.

The chapter is useful to understand how the global financial crisis has changed the way in which the credit risk is priced in the CDS market for non-financial firms. The contribution of the firm leverage to the explanation of the CDS spread changes is much higher during the crisis than before as investors appear to have become more aware of individual risk factors. At the same time, the impact of the equity volatility substantially decreases possibly because the large swings in implied volatility during the crisis period have made this indicator a poor proxy for the long-term asset volatility — which is the volatility that the theory predicts to be relevant for the pricing of default risk. The overall capacity of the model to explain the CDS spread changes is almost the same before and during the global turmoil, thus highlighting that the underlying risk factors identified by the literature as relevant for the pricing of the credit risk have maintained their explanatory power even in a period of remarkable stress for the CDS market.

Finally, the chapter shows that during the crisis CDS spreads appear to have been moving increasingly together, driven by a common factor that the model was only partly able to explain. Given that the model includes general indicators of economic activity, uncertainty, and risk aversion, our results point to the presence of a market-specific factor that hit the CDS market during the crisis in forms not fully reflected in other markets. The exact identification of this factor is an interesting topic for further research.

1.2.4 Chapter 5

Hedge funds frequently use leverage to optimize their risk-return profiles. Chapter 5, based on Di Cesare et al. (2011), analyzes several risk measures related to the performance of hedge funds.
Due to investments in illiquid assets, and probably also to reporting issues, hedge fund returns frequently exhibit a strong degree of serial correlation. As a consequence, the economic risks of an investment in hedge funds are easily underestimated, and investment decisions may be biased. In the chapter, the seminal work of Getmansky et al. (2004) on the Sharpe ratio (SR) and market beta is extended, by developing a number of smoothing-adjusted downside risk measures and by allowing for heavy tailed return distributions. In particular, the VaR, the expected shortfall, the correlation coefficient, and an extreme linkage measure (ELM) reflecting downside systemic risk are adjusted for the autocorrelation present in reported returns.

We show that unadjusted risk measures tend to understate the true level of risk. An exception is the ELM, for which the direction of the impact of the correction cannot be established a priori. We also show that the adjustment of the downside risk measures for autocorrelation is usually more relevant when returns are fat tailed than when they are normally distributed.

A hedge fund case study reveals that the unadjusted risk measures considerably underestimate the true extent of individual and multivariate risks. It is worth noting that, although the risk-adjustment is applied to hedge funds only, the same framework can also be used to evaluate the risks of other alternative investment strategies. Investments in real estate, art, collectible stamps, and other illiquid or opaque securities are also known to exhibit strong serial correlation in reported returns. Also for these assets, conventional risk measures need adjustments to correctly reflect the true level of investment risk.

\section{Conclusion}

Leveraged investments have become a fundamental feature of modern economies. The new financial products allow people to take greater-than-usual exposures to risk factors.

This thesis analyzes several different aspects of the risks involved by some frequently used leveraged products: CDOs, CDSs, and hedge funds. It is shown that these risks have indeed several facets, and that their misunderstanding can have severe effects, for both individual investors and the global financial stability. However, although leveraged products can be more complex than other traditional instruments, their characteristics in terms of risks and returns can usually be understood rather well by disciplined scholars.
When correctly understood and used, leveraged products can greatly expand the investment and hedging opportunities, and thus the welfare, of economic agents. In this sense, the statement of Buffett (2003) that “derivatives are financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal” is at most partial. Hopefully, this thesis contributes to a better understanding of some of the features of the leveraged products and provides useful insights on how to use these new instruments in the best way.
Chapter 2

Securitization and Bank Stability

2.1 Introduction

Before the onset of the subprime financial crisis in the United States, the use of new financial instruments for credit risk transfer, usually called credit derivatives, was increasing tremendously.\(^1\) The exceptional growth of the market for instruments such as credit-linked notes (CLNs), credit default swaps (CDSs), indices on CDSs (CDXs), and collateralized debt obligations (CDOs) has stimulated an intense debate on the effects of credit derivatives on the entire economic system.

In principle, the use of credit derivatives may have large beneficial effects on both single institutions and the overall financial system as credit derivatives make the dispersion of credit risk among economic sectors easier and more efficient. The risk transfer from the banking sector to less leveraged and more long-term oriented financial institutions, such as insurance companies and pension funds, may contribute to strengthen the whole financial system (Greenspan, 2005). Even sectors that are not net sellers of credit risk may still benefit from a wider dispersion of that risk among their members. However, as Fitch Ratings (2006) pointed out and the recent financial crisis confirmed, the extensive use of credit derivatives may also lead to an extremely high concentration of risks among a few primary dealers, with the consequence that the exit or the failure of one of them has the potential to harm the market liquidity and can result in huge counterpart credit losses for market participants.\(^2\)

---

\(^1\)See Committee on the Global Financial System (2003) for an overview of several techniques used for credit risk transfer. JPMorgan (1999) contains a vast, although admittedly incomplete, taxonomy of credit derivatives.

\(^2\)It has to be emphasized that one of the main drawbacks in most available statistics on the exposure of investors to credit derivatives is that only gross and net positions measured on notional values are available. However, notional values can be misleading since they do not take into account the true economic exposure to the underlying risk factors, that is the sensitivity of the value of the instruments to changes in the underlying risk factors (Cousseran and Rahmouni, 2005). As
During the years before the global financial crisis, it was common practice for banks, especially large banks, to use CDOs to transfer part of the credit risk associated with their loan portfolios to other investors. CDOs are securities issued in tranches with different seniority that are backed by the payoffs of the underlying assets. If some of the underlying loans are not repaid at maturity, the corresponding losses are borne by the tranches with lower seniority up to their notional values. The most senior tranches are affected only in case of very large losses for which the provisions of the other tranches are not sufficient.

Banks that transfer their risks may have less incentives to use sound credit standards and their efforts in that regard cannot be easily verified by the final investors in CDOs. To reduce these moral hazard issues, it is common practice for a bank that securitizes its loans to retain the most junior and riskiest tranche (called the equity tranche). However, Krahnen and Wilde (2006) argue that this practice can represent a source of concern for financial stability. Under very plausible assumptions, the two authors show that selling loans by issuing CDOs and retaining the equity tranche can increase the risk that banks suffer extreme large losses, thus increasing the probability of bank defaults.

In the first part of this chapter, we slightly modify the framework used by Krahnen and Wilde (2006) and show that their results are rather sensitive to the initial hypotheses. Our analysis shows under which conditions the CDO issuance increases or decreases the risk for the issuer to incur large losses and highlights the fundamental role that the reinvestment of the proceeds of the securitization has in this regard. In particular, we show that the level of risk of a bank decreases when the proceeds of the securitization are reinvested in loans with an individual default probability sufficiently smaller than that of the loans that are securitized. Similarly, the securitization can be beneficial for a bank when it is used to increase the diversification of the loan portfolio. This happens, for instance, when the loans that are securitized have a positive correlation with a common risk factor and the proceeds are reinvested in new loans with sufficiently lower correlation with the same risk factor. Hence, our analysis shows that the assessment of the effects of CDO issuance on the risk of incurring large losses for banks is mainly a matter of empirical research.

\[\text{a consequence, apparently balanced positions in terms of notional amounts can hide large risk exposures while apparently unbalanced positions can instead be the result of well-hedged portfolios.}\]

\[\text{3"Collateralized debt obligation" is a generic term for this kind of securities. Depending on whether the assets in the underlying portfolio are loans, bonds or CDSs, one can have collateralized loan obligations (CLOs), collateralized bond obligations (CBOs) or collateralized synthetic obligations (CSOs). CDOs of ABS and CDOs of CDOs (CDO}\textsuperscript{2} \text{were also common. In a CDO of ABS the underlying assets consist of other securities (more precisely, asset backed securities) that are in turn backed by assets such as consumer loans or credit card debts. In a CDO}\textsuperscript{2} \text{the collateral consists of other CDOs. More details about the CDO market can be found in the appendix.} \]
There are three main streams of empirical research on the relationship between banks and securitization. The first one is concerned with the determinants of securitization. In this regard, Bannier and Hänsel (2007) use data on European banks and find that securitizations are more likely to be realized by larger, less liquid, less profitable, and riskier banks. Affinito and Tagliaferri (2010) confirm those results for Italian banks and also find evidence of a role played by regulatory capital relief as a determinant of the securitization activity.

Other papers focus on the empirical effects of CDO issuance on the banking business. Jiangli et al. (2007) and Jiangli and Pritsker (2008), using data that do not include the most recent unfavorable developments for the banking industry, find that securitization was beneficial to US banks in terms of increased profitability and reduced risks. Altunbas et al. (2009) show that the use of securitization shelters loan supply by banks from the effects of monetary policy and strengthen the capacity of banks to supply new loans. The latter capacity depends upon business cycle conditions and banks’ risk positions. Kara et al. (2011) explore the link between securitization and lending standards by examining the pricing behavior of European banks involved in the securitization market when extending credit through syndicated loans. They find that banks more active in originating securitized assets are also more inclined to lower their pricing of credit risk when extending new loans. Pricing standards also change over the business cycle: during an expansionary period, banks more active on funding via securitization are also more likely to relax their pricing standards, probably relying on the possibility of offloading the loans through the financial markets.

A few other papers look at the relationship between securitization and systematic risk for banks. Franke and Krahnen (2006) provide evidence that stock betas rise around the announcement of a CDO issue, thus signaling a perception of increased systematic risk by the market. Also Uhde and Michalak (2010), using a wider sample of securitizations issued by European banks, provide empirical evidence that credit risk securitization has a positive impact on the increase of European banks’ systematic risk.

The second part of this chapter is related to the first two streams of empirical research. First, using data for Italian banks we look for changes in the composition of the asset side of the balance sheets of the banks that have securitized their loans. We provide evidence that the securitization activity has been a relevant factor in explaining those changes. Our results also show that those balance sheet changes have probably contributed to lower the expected credit losses of Italian banks, mainly because of the reduction of the share of bad loans over total assets.
Second, we verify whether Italian banks used, maybe involuntarily, securitizations to modify the overall quality of the most important item of their balance sheets, that is the loan portfolio. To this end, we use loan-by-loan data to compare the default rates of the loans that were securitized with the default rates of the new loans that were granted in the same months by the banks that made the securitizations. Our results show that, on average, the new loans were riskier than the loans that were securitized, thus leading to an increase in the amount of risk borne by the Italian banks as a consequence of the reinvestment of the proceeds of the securitizations. To the best of our knowledge this is the first attempt to measure the effects of securitization on the risks borne by the issuers using loan-by-loan data.

The chapter is organized as follows. Section 2.2 introduces the model used to analyze the effects of CDO issuance on bank stability and discusses the results arising from applying that model. Section 2.3 focuses on the empirical investigation of the impact of securitizations on the risks incurred by Italian banks. Section 2.4 concludes. A short appendix provides some details on the CDO market.

2.2 CDO issuance and bank stability

Due to the growing importance that the CDO market had in the global financial system before the onset of the crisis, it is surprising that, to the best of our knowledge, only the paper by Krahnen and Wilde (2006) has devoted attention to the effects that issuing those instruments can have on the risks of incurring large losses for the originators. Following that paper, we use Monte Carlo simulations to generate the return distribution of a loan portfolio and study how the risk of incurring large losses of the originator changes, depending on the different assumptions regarding the characteristics of the loans that are securitized and the characteristics of the new loans that are granted using the proceeds of the securitization.

2.2.1 The basic model

We assume that a bank owns a portfolio of \( N \) loans granted to \( N \) different borrowers. The capacity of each borrower \( i \) to pay back the loan at maturity \( T \) is described by the variable

\[
V_i = \text{sgn}(\rho_i)\sqrt{|\rho_i|}X + \sqrt{1-|\rho_i|} \epsilon_i,
\]

(2.1)

See Longstaff and Rajan (2008) for an analytical approach to the pricing of CDOs.
with \( i = 1, \ldots, N \), where \( \rho_i \in (-1, 1) \) and \( \text{sgn}(x) \) is the sign function

\[
\text{sgn}(x) = \begin{cases} 
-1, & \text{if } x < 0 \\
0, & \text{if } x = 0 \\
1, & \text{if } x > 0 .
\end{cases}
\] (2.2)

This definition of the model allows \( \rho_i \), the individual correlation coefficient of loan \( i \), to be negative and different for each \( i \), and encompasses the model used by Krahnen and Wilde (2006) that requires \( \rho_i \) to be positive and equal for all borrowers.\(^5\)

The variable \( V_i \) can be interpreted as a normalized measure of the value of the assets of borrower \( i \) and depends on the returns of a common risk factor \( X \) and an idiosyncratic risk factor \( \epsilon_i \) which is borrower specific. Both the common and the idiosyncratic risk factors are assumed to have standard normal distributions. They are also assumed to be pairwise independent. Due to these assumptions, \( V_i \) has a standard normal distribution as well. Moreover, the correlation between \( V_i \) and the common risk factor \( X \) is equal to \( \text{sgn}(\rho_i) \sqrt{|\rho_i|} \) and the correlation between \( V_i \) and \( V_j \), with \( i \neq j \), is equal to \( \text{sgn}(\rho_i \rho_j) \sqrt{|\rho_i \rho_j|} \).

All loans are assumed to have the same face value. The loans pay a coupon yield \( c_i \) at maturity and have an individual default probability \( d_i \). The value of \( d_i \) implicitly defines the default threshold \( D_i = \Phi^{-1}(d_i) \),\(^6\) so that the default of borrower \( i \) takes place when the value of \( V_i \) is lower than \( D_i \). In that case, only the recovery rate \( R \), which is the same for all loans, is paid back to the lender. As in Krahnen and Wilde (2006), we also assume that all loans have a maturity of one year and that defaults can only occur at the maturity of the loans.\(^7\) Finally, a constant interest rate \( r \) for (continuously) discounting the future payoffs is used.

Without loss of generality, the total face value of the \( N \) loans can be normalized to 1, so that the relative weight of each loan is \( 1/N \). The coupon yield of the loans is set equal to

\[
c_i = \frac{\exp(r) - Rd_i}{1 - d_i} - 1,
\] (2.3)

which is the value that makes the discounted expected value of each loan equal to its face value. More generally, we assume that the assets considered in this section (loans and CDOs) are priced in a risk-neutral way. In doing so, we keep outside of our analysis the issues related to the degree of risk aversion of the investors that could influence the valuation of the securities.

---


\(^6\)\( \Phi \) is the standard normal cumulative distribution function.

\(^7\)Although assuming a maturity of one year may appear rather simplistic, we point out that, qualitatively, our results are little influenced by assuming different maturities as long as the other assumptions are unchanged. Actually, allowing for different maturities would only have an impact on the discount factors.
Following Krahnen and Wilde (2006), we assume that the bank decides to sell \( n \) of the \( N \) loans by putting them in the underlying portfolio of a CDO with seven tranches. The sizes of the six more senior tranches are defined by their default probabilities, which are set equal to 1%, 2%, 5%, 10%, 20%, and 30%. That means that some losses are suffered by the first tranche in order of seniority with at most a 1% probability, by the second tranche with at most a 2% probability, and so on. The default probability of the last tranche, that bears all the initial losses and is retained by the bank, is not pre-defined but can be calculated given the previous assumptions on the characteristics of the loans. Immediately after the securitization, the bank reinvests the proceeds of the sale of the six tranches in new loans.

It turns out that the reinvestment strategy is the critical factor determining whether the overall level of risk of the bank increases or decreases when the securitization process has been completed, that is when the tranches are sold and the proceeds are reinvested. Krahnen and Wilde (2006) assume that the new loans have the same characteristics of the loans that are securitized in terms of correlation coefficients and default probabilities. Moreover, they assume that the correlation coefficients and default probabilities are equal for all loans and that the bank securitizes its loan portfolio completely (that is \( n = N \)). Under these hypotheses, reasonable but somewhat simplistic, Krahnen and Wilde (2006) show that the probability that the bank face large losses increases substantially after the securitization.

To study the impact of the reinvestment strategy on the risk of the bank when the assumptions made by Krahnen and Wilde (2006) are relaxed, we estimate the return distribution of the original portfolio (i.e., the loan portfolio of the bank before the starting of the securitization process) and compare it with the return distribution of a new portfolio made of: (1) the loans in the original portfolio that are securitized, (2) the retained equity tranche of the CDO that is issued, (3) the new loans that the bank grants using the proceeds of the sale of the other six tranches of the CDO.

Table 2.1 introduces the notation that is used to describe analytically the payoffs generated by the mechanism just described. The returns associated with those payoffs can be calculated by comparing the payoffs with their initial fair values and are represented, respectively, by \( R_{old} \), \( R_{n} \), \( R_{N-n} \), \( R_{eqt} \), \( R_{rnv} \), and \( R_{new} \).

Using the indicator function

\[
1_{\{x\}} = \begin{cases} 
0, & \text{if } \{x\} \text{ is false} \\
1, & \text{if } \{x\} \text{ is true} 
\end{cases}
\] (2.4)

the probability distributions of the payoffs, and associated returns, are estimated with the following steps:
Table 2.1 – List of symbols
Symbols used in the chapter to refer to the payoffs of loan portfolios

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{old}$</td>
<td>total payoff of the original loan portfolio of the bank</td>
</tr>
<tr>
<td>$P_n$</td>
<td>total payoff of the $n$ loans that are securitized</td>
</tr>
<tr>
<td>$P_{N-n}$</td>
<td>total payoff of the $N-n$ loans that are not securitized</td>
</tr>
<tr>
<td>$P_{eqt}$</td>
<td>payoff of the equity tranche of the CDO</td>
</tr>
<tr>
<td>$P_{rnv}$</td>
<td>total payoff of the loans in which the bank reinvests the proceeds of the securitization</td>
</tr>
<tr>
<td>$P_{new}$</td>
<td>total payoff of the new portfolio</td>
</tr>
</tbody>
</table>

1. Generate a random value for the common risk factor $X$;
2. Generate $N$ random values for the idiosyncratic risk factors $\epsilon_i$, $i = 1, \ldots, N$;
3. For given $\rho_i$, calculate $V_i$, $i = 1, \ldots, N$, as defined in Eq. (2.1);
4. For given default probability $p_i$ for borrower $i$, calculate the default threshold $D_i = \Phi^{-1}(p_i)$;
5. Calculate the total payoff and return of the original portfolio:\(^8\)

\[
P_{old} = \frac{1}{N} \left[ R \sum_{i=1}^{N} \mathbf{1}_{\{V_i \leq D_i\}} + \sum_{i=1}^{N} (1 - \mathbf{1}_{\{V_i \leq D_i\}})(1 + c_i) \right], \quad (2.5)
\]
\[
R_{old} = P_{old} - 1; \quad (2.6)
\]
6. Calculate the total payoff and return of the $n$ loans that are securitized:\(^9\)

\[
P_n = \frac{1}{N} \left[ R \sum_{i=1}^{n} \mathbf{1}_{\{V_i \leq D_i\}} + \sum_{i=1}^{n} (1 - \mathbf{1}_{\{V_i \leq D_i\}})(1 + c_i) \right], \quad (2.7)
\]
\[
R_n = NP_n / n - 1; \quad (2.8)
\]
7. Calculate the total payoff and return of the $N-n$ loans that are not securitized:

\[
P_{N-n} = P_{old} - P_n, \quad (2.9)
\]
\[
R_{N-n} = NP_{N-n} / (N - n) - 1; \quad (2.10)
\]
8. Reiterate the previous steps 100,000 times to calculate the empirical distributions of payoffs and returns;

\(^8\)Remember that the initial total face value of the original portfolio has been normalized to 1, so that the initial face value of each loan is $1/N$. Moreover, given the definition of the coupon yield $c_i$ in Eq. (2.3), also the fair values of the original portfolio and each loan are equal to 1 and $1/N$, respectively.

\(^9\)Without loss of generality, it is assumed that the first $n$ loans are those that are securitized.
9. Using the empirical distribution of $P_n$, calculate the detachment point of the equity tranche:

$$D_{eqt} = \max\{x : \mathbb{P}(P_n < x) \leq 0.3\}; \quad (2.11)$$

10. Calculate the payoff of the equity tranche and its distribution:

$$P_{eqt} = \max(P_n - D_{eqt}, 0); \quad (2.12)$$

11. Using the distribution of $P_{eqt}$, calculate the fair value of the equity tranche and the distribution of its return:

$$V_{eqt} = \exp(-r)\mathbb{E}[P_{eqt}], \quad (2.13)$$

$$R_{eqt} = \frac{P_{eqt}}{V_{eqt}} - 1; \quad (2.14)$$

12. Given that the CDO is priced in a risk-neutral way, its total initial value is equal to that of the underlying loans, which is $n/N$. As a consequence, the proceeds of the sale of the CDO, after retaining the equity tranche, are equal to $n/N - V_{eqt}$. That sum is reinvested by the bank in a portfolio of $n$ new loans with characteristics that are potentially different from those of the loans in the original portfolio. The payoff of the reinvested portfolio ($P_{rnv}$) is calculated in the same way as for $P_{old}$, steps 2 to 5, except for using different values for the parameters of the new loans, and its distribution is determined by reiterating those steps 100,000 times;

13. Finally, calculate the payoff and return, and their distributions, of the portfolio that the bank owns after the securitization and reinvestment processes have taken place:

$$P_{new} = P_{N-n} + P_{eqt} + P_{rnv}, \quad (2.15)$$

$$R_{new} = P_{new} - 1. \quad (2.16)$$

### 2.2.2 The return distribution of a loan portfolio

To analyze the effects of the securitization process on the risk profile of a bank, we study the return distribution of a loan portfolio as a function of three underlying parameters: the number of loans, the individual default probability of the loans, and the individual correlation coefficient of the loans.

Figure 2.1 shows the probability density function (PDF), the cumulative density function (CDF), and the value-at-risk (VaR) for several confidence levels, for the returns of five loan portfolios with different numbers of loans. Admittedly, the
PDF, the CDF and the VaR contain the same information, but each indicator is particularly useful to highlight specific aspects of the return distributions of the portfolios. All the loans in the five portfolios have the same characteristics. As in Krahnen and Wilde (2006), we assume that the loans have an individual annual default probability equal to 20%, an individual correlation coefficient equal to 30%, and a recovery rate equal to 47.5%. We also assume that the risk-free interest rate is equal to 4%. Using Eq. (2.3), the coupon yield of the loans is set to about 18.2%.

Panel A of Figure 2.1 shows that the PDFs of the portfolio returns are rather similar when the number of loans varies from 100 to 10,000.\(^{10}\) Therefore, a fairly high level of diversification can be achieved even with a relatively small number of loans. Panel B of Figure 2.1 shows the CDFs of the portfolio returns, that is the probabilities that returns are lower than given levels.\(^{11}\) For all portfolios, the probability of incurring a loss (i.e., a negative return) is equal to about 30% and the probability of having, for instance, a loss greater than 20% is about 5%. Finally, Panel C of Figure 2.1 reports the VaRs of the portfolios returns for confidence levels from 90.0% to 99.9%.\(^{12}\) This panel shows that, for instance, there is a 1% probability that the losses can be greater than about 31% and that with probability 0.1% the losses can be greater than about 42%.

Other interesting results arise when considering portfolios of loans with different individual annual default probabilities. In this case, the number of loans is set to 1,000 for all portfolios and the assumptions on the individual correlation coefficients, recovery rates, and risk-free interest rate are the same as above. Panel A of Figure 2.2 reports the PDFs of the returns of five portfolios with loans that have individual default probabilities in the 10%–50% range. It is worth noting that the PDFs associated with the portfolios with loans with greater individual default probabilities have both left and right tails much fatter than those of the PDFs of portfolios with loans with smaller individual default probabilities. This result is due to the fact that portfolios with loans with greater individual default probabilities are more likely to incur both large losses (because many defaults can happen) and large positive returns (because loans with greater individual default probabilities earn higher coupon yields when they do not default). From Panel B of Figure 2.2 it

---

\(^{10}\) The PDFs are calculated by simply counting the relative number of outcomes over small intervals of the returns. This numerical approximation is the source of the raggedness of the curve that arises when the number of loans is very small.

\(^{11}\) Formally, Panel B of Figure 2.1 reports the functions \(F_{N_i}(x) = P(R_{N_i} \leq x)\), where \(R_{N_i}\) is the return of the portfolio with \(N_i\) loans.

\(^{12}\) Formally, Panel C of Figure 2.1 reports the function \(\text{VaR} = F_{N_i}^{-1}(1 - x)\), where \(F_{N_i}\) is the cumulative distribution function of the returns of a portfolio with \(N_i\) loans, and \(x\) is the confidence level.
Panel A: Probability density function

Panel B: Cumulative density function

Panel C: Value-at-risk

Figure 2.1 – Characteristics of the returns of a loan portfolio for several values of the number of loans

The legend shows the number of loans in the portfolio. All loans have the same face value, an individual correlation coefficient equal to 30%, an individual annual default probability equal to 20%, and a coupon yield such that the fair value of each loan is equal to its face value (see Eq. 2.3). The recovery rate is set to 47.5% and the risk-free interest rate is equal to 4%.
Figure 2.2 – Characteristics of the returns of a loan portfolio for several values of the individual default probability of the loans

The legend shows the individual annual default probability of the loans. The portfolio is made of 1,000 loans. All loans have the same face value, an individual correlation coefficient equal to 30%, and a coupon yield such that the fair value of each loan is equal to its face value (see Eq. 2.3). The recovery rate is set to 47.5% and the risk-free interest rate is equal to 4%.
Figure 2.3 – Characteristics of the returns of a loan portfolio for several values of the individual correlation coefficient of the loans

The legend shows the individual correlation coefficient of the loans. The portfolio is made of 1,000 loans. All loans have the same face value, an individual annual default probability equal to 20%, and a coupon yield such that the fair value of each loan is equal to its face value (see Eq. 2.3). The recovery rate is set to 47.5% and the risk-free interest rate is equal to 4%.
can be seen that the probability of having losses greater than 10% is in the 5%–30% range or, said in a slightly different way, Panel C of Figure 2.2 shows that when the individual annual default probability increases from 10% to 50% one can expect to have losses greater than about 10% and 35%, respectively, with the same 5% probability.

It is also worth to examine the case in which the individual correlation coefficient varies across portfolios. In this case, the assumptions on the recovery rate and risk-free interest rate are still the same as before, and also the number of loans is kept fixed at 1,000. Moreover, the individual default probability is set to 20%. As shown by Figure 2.3, the effect of changing the individual correlation coefficient from zero (the case of a perfectly diversified portfolio where all loans are independent of each other) to 50% is to transform the return distribution from a normal-like distribution to a distribution with very fat tails. Raising the individual correlation coefficient prompts the return distribution of the portfolio to be more and more concentrated towards the extreme values. This effect is even more noticeable than in the previous case of increasing individual default probabilities. In the limit case in which \( \rho = 1 \), the loans either default all together or none defaults, so that there is exactly a 20% probability that all loans default and a 80% probability that none of the loans defaults. In the first case, the portfolio records a 52.5% loss (as only the recovery rate is received back) while in the second case there is a positive return of 18.2% (which is equal to the coupon yield).

To conclude this section, we point out that there is another important effect of the individual correlation coefficients on the return distribution of a loan portfolio. This effect is crucial to understand the impact that securitizing loans with a CDO can have on the level of risk of a bank. Remember that in Eq. (2.1) we assumed \( \rho_i \) to be a variable with values in the interval \((-1,1)\). However, until now we only analyzed the effects of \( \rho_i \) on the return distribution of a loan portfolio for positive values of that variable. This was done because we were assuming, as in Krahnen and Wilde (2006), that all loans have the same individual correlation coefficient \( \rho \). In fact, the effect of \( \rho \) on the return distribution of a loan portfolio depends on how it affects the behavior of the variable \( V_i \) in Eq. (2.1). As the common risk factor \( X \) has a symmetric distribution with zero mean, the results described above are actually independent of the sign of \( \rho \).

In the following section we assume that the bank can reinvest the proceeds of the securitization in new loans that have an individual correlation coefficient which is different from that of the loans that are securitized, thus creating a portfolio in which loans with different individual correlation coefficients coexist. In that framework the
Sect. 2.2 – CDO issuance and bank stability

sign of the individual correlation coefficients is relevant as, for instance, the effect of a negative return of $X$ on the value of a loan which is positively correlated with it can be partially offset by the smaller effect on a loan which is less correlated (or possibly negatively correlated) with it.

2.2.3 The effects of securitization on bank stability

We use once again the assumptions made by Krahnen and Wilde (2006) as the benchmark case. Hence, we set $\rho_i = 30\%$ and $d_i = 20\%$ for all loans. Moreover, $R = 47.5\%$ and $r = 4\%$. The continuous line in Panel A of Figure 2.4 shows the PDF of the returns of a portfolio made of 1,000 loans with those characteristics. The other lines in the same panel show the PDFs of the returns of new portfolios that are obtained by securitizing the old loans using a CDO, retaining the equity tranche, and investing the proceeds of the remaining tranches in loans with individual default probabilities as shown in the legend.

It is interesting to notice that the PDFs of the new portfolios are bimodal (see Panel A of Figure 2.2). This result is due to the fact that the new portfolios are a combination of the equity tranche and a portfolio of new loans. The peaks on the left, in particular, reflect the fact that the returns of the equity tranche tend to be negative when the returns of the whole loan portfolio are not large enough. The peaks on the right, on the other hand, reflect the fact that the returns of the equity tranche tend to be extremely large when the returns of the whole portfolio are good, thus compounding the two effects.

As shown by Krahnen and Wilde (2006), the risk for the bank of incurring large losses increases if the bank fully securitizes its loan portfolio by issuing a CDO, retaining the equity tranche, and reinvesting the proceeds in new loans with the same characteristics of the loans that are securitized. This result can be seen clearly from Panels B and C of Figure 2.4, by comparing the lines corresponding to the original portfolio with those of the cases in which the new loans have also an individual default probability of 20%. However, if the bank reinvests the proceeds in loans of better quality, that is with smaller individual default probability, the overall level of risk of the bank can decrease significantly. For instance, if the new loans have an individual default probability of 5%, the VaR at the 99% confidence level goes from $-31\%$ for the original portfolio to $-18\%$ for the new portfolio.

Other interesting results appear if one assumes that the new loans in which the proceeds of the securitization are reinvested have the same individual default probability of the loans that are securitized but a different individual correlation coefficient. Panel C of Figure 2.5 shows that the bank can sharply reduce its VaR
Figure 2.4 – Characteristics of the returns of a loan portfolio, before and after the securitization of 100% of the loans, for several values of the individual default probability of the new loans

All the loans in a portfolio of 1,000 loans, with the same face value and individual annual default probability equal to 20%, are securitized using a CDO of which the bank retains the equity tranche. The remaining proceeds are reinvested in new loans with an individual annual default probability as specified in the legend. All loans have an individual correlation coefficient equal to 30% and a coupon yield such that the value of each loan is equal to its face value (see Eq. 2.3). The recovery rate is set to 47.5% and the risk-free interest rate is equal to 4%.
Figure 2.5 – Characteristics of the returns of a loan portfolio, before and after the securitization of 100% of the loans, for several values of the individual correlation coefficient of the new loans

All the loans in a portfolio of 1,000 loans, with the same face value and individual correlation coefficient equal to 30%, are securitized using a CDO of which the bank retains the equity tranche. The remaining proceeds are reinvested in new loans with an individual correlation coefficient as specified in the legend. All loans have an individual annual default probability equal to 20%, and a coupon yield such that the value of each loan is equal to its face value (see Eq. 2.3). The recovery rate is set to 47.5% and the risk-free interest rate is equal to 4%.
Figure 2.6 – Characteristics of the returns of a loan portfolio, before and after the securitization of 20% of the loans, for several values of the individual default probability of the new loans

The 20% of the loans in a portfolio of 1,000 loans, with the same face value and individual annual default probability equal to 20%, are securitized using a CDO of which the bank retains the equity tranche. The remaining proceeds are reinvested in new loans with an individual annual default probability as specified in the legend. All loans have an individual correlation coefficient equal to 30% and a coupon yield such that the value of each loan is equal to its face value (see Eq. 2.3). The recovery rate is set to 47.5% and the risk-free interest rate is equal to 4%.
Figure 2.7 – Characteristics of the returns of a loan portfolio, before and after the securitization of 20% of the loans, for several values of the individual correlation coefficient of the new loans

The 20% of the loans in a portfolio of 1,000 loans, with the same face value and individual correlation coefficient equal to 30%, are securitized using a CDO of which the bank retains the equity tranche. The remaining proceeds are reinvested in new loans with an individual correlation coefficient as specified in the legend. All loans have an individual annual default probability equal to 20%, and a coupon yield such that the value of each loan is equal to its face value (see Eq. 2.3). The recovery rate is set to 47.5% and the risk-free interest rate is equal to 4%.
for any confidence level if it reinvests in loans with a smaller individual correlation coefficient. For instance, in the case of a reinvestment in loans with $\rho = -10\%$, the VaR at the 99% confidence level drops from $-31\%$ to $-3\%$. By reinvesting in loans that are negatively correlated with the common macro factor, the bank can diversify the residual exposure to the initial portfolio — which is positively correlated with the common risk factor — that it implicitly retains by holding the equity tranche of the CDO.

Although it may appear unusual for a loan to be negatively correlated with a broad macro risk factor, it is worth noting that a bank could obtain the diversification effect described above by buying credit protection in the CDS market or by taking long positions in safe haven assets such as high-rated government bonds or commodities. Finally, it is remarkable that it is not actually necessary to reinvest in instruments with a negative correlation with the common risk factor to reduce the VaR. Sometimes it is sufficient to buy instruments that are just less correlated with the same risk factor than the loans that are securitized (as shown by the blue line in Panel C of Figure 2.5).

Assuming that a bank completely securitizes its portfolio looks like an extreme hypothesis, so we also calculate what happens when only a fraction of the original portfolio is securitized. Figures 2.6 and 2.7 show that the previous results still hold when only 20% of the original portfolio is securitized. However, as one would expect the results are now much less sharp. In this more reasonable case, it is interesting to notice that the overall level of risk of the bank increases by only a negligible amount even if the bank reinvests the proceeds of the securitization in loans with the same characteristics of the loans that it has sold.

According to these results, it is unlikely that CDO issuance may increase the risks for the stability of the financial system through the mechanism highlighted by Krahnen and Wilde (2006). This finding is apparently at odds with the widely accepted opinion that the use of CDOs has been one of the main drivers of the global financial crisis that started in 2007. However, one has to distinguish between the effects coming from the securitization activity by its own — which are analyzed in this paper — and the difficulties in correctly pricing and hedging these complex instruments — which the financial turmoil proved to be rather tough. It is worth to point out once again that one of the main assumptions we have made in this section is that CDOs are correctly priced by both issuers and investors in a risk neutral way.

Finally, Tables 2.2–2.5 report the VaR changes under several hypotheses about the characteristics of the loans that are securitized and the new loans that are granted with the proceeds of the securitization. For instance, Table 2.2 reports that
Table 2.2 – VaR changes for a loan portfolio in case of securitization of 100% of the loans as a function of the individual default probability of the loans

The table shows the changes, in percentages, of the VaR at 99.9%, 99%, and 95% confidence levels, for a portfolio of 1,000 loans (with the same face value and individual correlation coefficient equal to 30%) when the 100% of the loans is securitized using a CDO of which the bank retains the equity tranche. The remaining proceeds are reinvested in new loans with the same characteristics of the loans that are securitized, except for the individual annual default probability. The individual annual default probabilities of the loans in the original portfolio and the new loans are reported in the left column and top row, respectively. All loans have a coupon yield such that the value of each loan is equal to its face value (see Eq. 2.3). The recovery rate is set to 47.5% and the risk-free interest rate is equal to 4%.

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Memo: Pre-Finance VaR old port.

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Memo: Post-Finance VaR old port.
Table 2.3 – VaR changes for a loan portfolio in case of securitization of 100% of the loans as a function of the individual correlation coefficient of the loans

The table shows the changes, in percentages, of the VaR at 99.9%, 99%, and 95% confidence levels, for a portfolio of 1,000 loans (with the same face value and individual annual default probability equal to 20%) when the 100% of the loans is securitized using a CDO of which the bank retains the equity tranche. The remaining proceeds are reinvested in new loans with the same characteristics of the loans that are securitized, except for the individual correlation coefficient. The individual correlation coefficients of the loans in the original portfolio and the new loans are reported in the left column and top row, respectively. All loans have a coupon yield such that the value of each loan is equal to its face value (see Eq. 2.3). The recovery rate is set to 47.5% and the risk-free interest rate is equal to 4%.

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Table 2.4 – VaR changes for a loan portfolio in case of securitization of 20% of the loans as a function of the individual default probability of the loans
The table shows the changes, in percentages, of the VaR at 99.9%, 99%, and 95% confidence levels, for a portfolio of 1,000 loans (with the same face value and individual correlation coefficient equal to 30%) when the 20% of the loans is securitized using a CDO of which the bank retains the equity tranche. The remaining proceeds are reinvested in new loans with the same characteristics of the loans that are securitized, except for the individual annual default probability. The individual annual default probabilities of the loans in the original portfolio and the new loans are reported in the left column and top row, respectively. All loans have a coupon yield such that the value of each loan is equal to its face value (see Eq. 2.3). The recovery rate is set to 47.5% and the risk-free interest rate is equal to 4%.

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Table 2.5 – VaR changes for a loan portfolio in case of securitization of 20% of the loans as a function of the individual correlation coefficient of the loans

The table shows the changes, in percentages, of the VaR at 99.9%, 99%, and 95% confidence levels, for a loan portfolio of 1,000 loans (with the same face value and individual annual default probability equal to 20%) when the 20% of the loans is securitized using a CDO of which the bank retains the equity tranche. The remaining proceeds are reinvested in new loans with the same characteristics of the loans that are securitized, except for the individual correlation coefficient. The individual correlation coefficients of the loans in the original portfolio and the new loans are reported in the left column and top row, respectively. All loans have a coupon yield such that the value of each loan is equal to its face value (see Eq. 2.3). The recovery rate is set to 47.5% and the risk-free interest rate is equal to 4%.

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the VaR at the 99.9%, 99%, and 95% confidence levels would increase by 10%, 15%, and 29%, respectively, if a bank securitized loans with an individual default probability of 20% and reinvests in loans with the same individual default probability. On the other hand, the VaR at the same confidence levels would decrease by 10%, 16%, and 16%, respectively, if the bank reinvested in loans with an individual default probability of 10%.

As a general result, Tables 2.2–2.5 show that a bank can reduce its VaR if it reinvests in loans with sufficiently better credit quality. At the same time, the bank can reduce its VaR also by investing in loans with the same credit quality but with a different correlation with the common risk factor. In particular, the VaR usually decreases when a bank securitizes, in whole or in part, a portfolio of loans that are positively correlated with a common risk factor and reinvests in loans, or other assets, that are less correlated with the same risk factor.

2.3 An empirical investigation on Italian banks

In the previous section, we showed that the overall level of risk of a bank can increase or decrease when it sells part of its loan portfolio by issuing a CDO of which it retains the equity tranche. The final result critically depends on the way in which the proceeds of the securitization are reinvested. The level of risk of a bank tend to decrease (increase) when the bank reinvests in loans with smaller (greater) individual default probability or in loans that are less (more) correlated with the common risk factor than the loans that are securitized. As a consequence of the previous analysis, it is not possible to assess on a general basis the real effects of CDO issuance on the risk of incurring large losses for banks. It is thus necessary to rely on case-by-case studies. In this section we provide some empirical evidence on the effects that securitizations had both on balance sheets and loan portfolios of Italian banks.

Securitization can effect the balance sheet structure of a bank either directly or indirectly. We have a direct effect when some assets are securitized and then replaced with different assets, as assumed in the previous section. For instance, the securitization of bad loans and the investment in new loans automatically tends to reduce the share of the first kind of loans over total assets and to increase the share of performing loans. The indirect effect is instead related to the potential effects that securitization can have on bank balance sheets. As an example, a bank could decide to increase the share of loans granted to particular categories of borrowers because it knows that, if needed, it would be relatively easy to get rid of the new
loans by securitizing them. This is probably what happened to many American
banks that decided to increase their exposure to subprime loans before 2007.

The empirical analysis developed in this section is twofold. First, we analyze
whether the overall structure of Italian bank balance sheets has changed as a con-
sequence of securitizations. To this end, we examine a few balance sheet items to
verify whether their relative shares with respect to total assets have changed during
the years, possibly due to the securitization activities of the banks.

Second, we focus on the characteristics of individual loans and carry out an anal-
ysis which is, in some sense, more in line with the theoretical framework provided
in the previous section. Using supervisory loan-by-loan data, we compare the indi-
vidual default probabilities of the loans that were securitized by Italian banks with
those of the new loans that were granted by the same banks during the same months.
In this way, we can verify whether Italian banks had a tendency to replace the loans
that they securitized with riskier or safer new loans. Unfortunately, for reasons
that will be explained later, the information available in the Italian databases is not
suitable for estimating also whether the correlation with the default probabilities of
the loans that were kept in the bank portfolios was higher or lower for the default
probabilities of the new loans than for the default probabilities of the loans that
were securitized. In this sense, a complete empirical assessment of the theoretical
framework presented in the previous section remains still an interesting subject for
future research.

2.3.1 Analysis of Italian bank balance sheets

We analyze the evolution of Italian bank balance sheets between 1999 (the year in
which the Italian law on bank loan securitization was passed) and 2007 (the last year
before bank balance sheets were hardly hit by the global financial crisis). During
that period, Italian banks securitized assets for € 170 billion. About 75% was
represented by performing loans, that is loans not considered as troublesome by the
banks. Out of the 218 banks that made at least one securitization, 140 securitized
only performing loans, 40 only bad loans, and 38 both types of loans. The market
has been rather concentrated, with only 10 banks accounting for about a half of the
total value of Italian securitizations. That number further declines to 5 if one takes
into account that some banks merged during the period.

Our analysis is based on monthly data on balance sheet items and securitization
proceeds from the Supervisory Reports to the Bank of Italy. Supervisory Reports
permit to distinguish between securitizations concerning performing or bad loans.
According to the Italian regulation, banks are required to classify outstanding loans
to borrowers that are not expected to meet their obligations as bad loans. Clearly, this definition allows banks some discretion in judging whether a loan is bad or not. We will return on this issue later.

We focus on four balance sheet items: performing loans to non-bank customers (i.e., interbank loans and deposits to monetary authorities are excluded), bad loans to non-bank customers, loans to other subjects (i.e., banks), securities other than shares. Overall, these items accounted for about 80% of total assets, both in 1999 and 2007. For each of the items we take into account, we calculate the annual average of their monthly share over total assets.

Before running the econometric analysis, we performed a series of controls on the data. First, we dropped data on branches of foreign banks. There are two reasons for this choice. The first motivation is that, due to the increasing openness of the Italian banking system, there has been a huge increase in the role of foreign banks between 1999 and 2007, both in terms of number of banks and total assets (from 50 to 80 and from €86 to €275 billion, respectively). These developments are likely to be independent of the role of securitization in determining bank capital structures. The second reason for dropping foreign banks is that the Italian law on securitization in 1999 probably did not represent an innovation for most of them as they could already securitize their assets in their home countries.

Then, we take into account mergers and acquisitions by considering all entities involved in those operations as a single entity for all the period under analysis. We do so by summing the values of the corresponding balance sheet items for the banks that were involved in mergers or acquisitions. As we use annual averages of monthly balance sheet data in our estimations, we also dropped the banks for which less than six monthly data are available for either 1999 or 2007. Finally, to get rid of some outliers, we dropped 1% of the observations from the dataset (the best and worst 0.5% of all balance sheet ratios).

The final database consists of 537 banks, about 80% of which are either local mutual or cooperative banks (see Table 2.6). At the end of 2007, the coverage of the sample was about 70% of all Italian banks in terms of number of banks, total assets, loans to non-bank customers and proceeds from securitization (see Table 2.7).

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13Other important items of the asset side of Italian bank balance sheets are repo contracts, equity and other shares, and other assets. We decided not to include those items in the analysis because repo contracts are only used by a minority of banks (although for some of them they represent an important investment), equity and other shares are very volatile due to evaluation effects, and other assets represent a miscellany of residual sub-items.

14Keeping the outliers does not modify the signs or the significance levels of the estimated coefficients but the measures of fit are sometimes lower than those reported here.
Table 2.6 – Number of banks in the sample by type
The numbers take into account mergers and acquisitions. All banks involved in those operations are considered as a single bank. The bank type is that resulting in 2007.

<table>
<thead>
<tr>
<th>Bank type</th>
<th>Number</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mutual banks</td>
<td>401</td>
<td>74.7%</td>
</tr>
<tr>
<td>Limited company banks</td>
<td>106</td>
<td>19.7%</td>
</tr>
<tr>
<td>Cooperative banks</td>
<td>30</td>
<td>5.6%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>537</strong></td>
<td><strong>100.0%</strong></td>
</tr>
</tbody>
</table>

Table 2.7 – Descriptive statistics of the banks included in the sample as a share of all Italian banks
The data on securitizations refer to all operations that took place between 1999 and 2007.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>1999</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of banks</td>
<td>81%</td>
<td>69%</td>
</tr>
<tr>
<td>Total assets</td>
<td>86%</td>
<td>70%</td>
</tr>
<tr>
<td>Loans to non-bank customers</td>
<td>89%</td>
<td>72%</td>
</tr>
<tr>
<td>Value of securitizations</td>
<td>72%</td>
<td></td>
</tr>
</tbody>
</table>

To verify how the main items of the Italian bank balance sheets changed between 1999 and 2007 and whether there were significant differences between banks that securitized their assets and banks that did not, we use a difference-in-difference approach. In particular, we estimate pooled equations of the following type for each of the four asset items we are interested in:

\[ y_{i,t} = \beta_0 + \beta_1 \delta_{t,2007} + \beta_2 \delta_{i,sec} + \beta_3 \delta_{i,sec}\delta_{t,2007} + \beta_4 x_{i,t} + \epsilon_{i,t}, \]

(2.17)

where \( i = 1, \ldots, N \) are the banks included in the sample, \( t \in \{1999, 2007\} \),

\[ \delta_{t,2007} = \begin{cases} 
0, & \text{if } t = 1999 \\
1, & \text{if } t = 2007 
\end{cases} \]

(2.18)

\[ \delta_{i,sec} = \begin{cases} 
0, & \text{if bank } i \text{ did not securitize anything between } 1999 \text{ and } 2007 \\
1, & \text{if bank } i \text{ securitized something between } 1999 \text{ and } 2007 
\end{cases} \]

(2.19)

and \( x_{i,t} \) is a vector of variables that control for size, capitalization, and proprietary structure.\(^{15}\)

Given that the balance sheet shares that we calculated are bounded between zero and one, we use their logistic transformation as dependent variable \( y_{i,t} \).\(^{16}\)

---

\(^{15}\)Results are qualitatively the same when including fixed effects for individual banks.

\(^{16}\)The logistic transformation of \( x \) is \( \log(x/(1 - x)) \).
estimate Eq. (2.17) using the weighted least-squares logistic regression for grouped data described in Greene (2003, pp. 686–689).

As size, capitalization, and proprietary structure have been identified as relevant determinants of the securitization of loans by Italian banks (see Affinito and Tagliaferri, 2010), we include in our regressions the logarithm of the annual average of total assets (in millions of euros), an indicator of capitalization defined as the logistic transformation of the annual average of the ratio of banks’ own capital to total assets, and a dummy variable indicating whether the bank is either a mutual or cooperative bank or not.

As in any difference-in-difference estimation, the most interesting coefficients are those of the dummy variables. In particular, if one excludes the role of the control variables, $\beta_0$ is a measure of the average value of the dependent variable in 1999 for the banks that did not securitize, $\beta_1$ is a measure of the average additional contribution in the value of the dependent variable in 2007 for the banks that did not securitize, $\beta_2$ is a measure of the average additional contribution in the value of the dependent variable in 1999 for the banks that securitized, and $\beta_3$ is a measure of the average additional contribution in value of the dependent variable in 2007 for the banks that securitized some of their loans. Said in other words, $\beta_1$ captures the average structural changes in the Italian bank balance sheets between 1999 and 2007 for the banks that did not securitize, $\beta_2$ measures the average differences in 1999 between banks that securitized and banks that did not, and $\beta_3$ measures the additional average changes in the capital structure of the banks that securitized their loans between 1999 and 2007.

The choice to focus the analysis on just two years deserves some explanation as it may be argued that more data, with the same of higher frequency, should be used. For instance, one could use all yearly or quarterly data available between 1999 and 2007, and not just the first and last yearly data of the period. Although those choices would be fairly reasonable, we prefer to look for structural changes in bank balance sheets only by performing a comparison over a longer period of time. By definition, structural changes are hard to identify in the short-term and, moreover, using data with higher frequency would increase the relevance of the short-term effects of the securitizations. For a bank that securitizes bad loans, for instance, it is not obvious whether, over longer horizons, the bank should have a lower share of bad loans (because it securitizes them) or a higher share of them (because the bank may decide to invest in riskier activities as it knows that it can easily get rid of the bad loans, if needed). By using short-term data, it is very likely to end up with results that give undue relevance to the direct effects of the securitizations only.
Table 2.8 – Shares of some asset items over total assets for the banks included in the sample
Data refer to simple averages across all banks included in the sample.

<table>
<thead>
<tr>
<th>Year</th>
<th>Performing loans</th>
<th>Bad loans</th>
<th>Loans to banks</th>
<th>Securities other than shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>45.9%</td>
<td>3.0%</td>
<td>10.3%</td>
<td>29.6%</td>
</tr>
<tr>
<td>2007</td>
<td>63.1%</td>
<td>2.2%</td>
<td>7.6%</td>
<td>17.8%</td>
</tr>
<tr>
<td>Variation</td>
<td>+37.6%</td>
<td>−24.5%</td>
<td>−26.6%</td>
<td>−40.1%</td>
</tr>
</tbody>
</table>

**All banks**

<table>
<thead>
<tr>
<th>Year</th>
<th>Performing loans</th>
<th>Bad loans</th>
<th>Loans to banks</th>
<th>Securities other than shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>49.8%</td>
<td>3.3%</td>
<td>9.3%</td>
<td>25.3%</td>
</tr>
<tr>
<td>2007</td>
<td>66.7%</td>
<td>1.9%</td>
<td>7.3%</td>
<td>14.1%</td>
</tr>
<tr>
<td>Variation</td>
<td>+34.0%</td>
<td>−42.7%</td>
<td>−22.3%</td>
<td>−44.3%</td>
</tr>
</tbody>
</table>

**Banks that securitized their loans**

<table>
<thead>
<tr>
<th>Year</th>
<th>Performing loans</th>
<th>Bad loans</th>
<th>Loans to banks</th>
<th>Securities other than shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>53.0%</td>
<td>1.9%</td>
<td>7.8%</td>
<td>25.9%</td>
</tr>
<tr>
<td>2007</td>
<td>70.9%</td>
<td>1.6%</td>
<td>5.6%</td>
<td>12.9%</td>
</tr>
<tr>
<td>Variation</td>
<td>+33.8%</td>
<td>−13.5%</td>
<td>−28.3%</td>
<td>−50.2%</td>
</tr>
</tbody>
</table>

**Banks that securitized performing loans only**

<table>
<thead>
<tr>
<th>Year</th>
<th>Performing loans</th>
<th>Bad loans</th>
<th>Loans to banks</th>
<th>Securities other than shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>37.4%</td>
<td>7.6%</td>
<td>15.2%</td>
<td>28.0%</td>
</tr>
<tr>
<td>2007</td>
<td>53.9%</td>
<td>2.5%</td>
<td>11.0%</td>
<td>21.8%</td>
</tr>
<tr>
<td>Variation</td>
<td>+44.2%</td>
<td>−66.9%</td>
<td>−28.0%</td>
<td>−22.1%</td>
</tr>
</tbody>
</table>

**Banks that securitized bad loans only**

<table>
<thead>
<tr>
<th>Year</th>
<th>Performing loans</th>
<th>Bad loans</th>
<th>Loans to banks</th>
<th>Securities other than shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>44.2%</td>
<td>2.8%</td>
<td>10.7%</td>
<td>31.4%</td>
</tr>
<tr>
<td>2007</td>
<td>61.7%</td>
<td>2.4%</td>
<td>7.7%</td>
<td>19.3%</td>
</tr>
<tr>
<td>Variation</td>
<td>+39.4%</td>
<td>−15.8%</td>
<td>−28.1%</td>
<td>−38.7%</td>
</tr>
</tbody>
</table>

Table 2.8 shows how the four balance sheet items that we analyze changed between 1999 and 2007. Italian banks increased considerably the average share of performing loans over total assets and decreased their loans to other banks and their investments in securities other than shares. The increase of the relative size of performing loans was larger for the banks that securitized bad loans only. For those banks, the share of bad loans in their portfolios decreased by about two thirds. Banks that securitized performing loans only were also those with the higher share of that kind of loans in their portfolios, both in 1999 and 2007. The same banks also decreased significantly the relative size of their investments in securities other than share. Finally, it is interesting to notice that the banks that securitized their loans were also more willing to lend to non-bank customers and less inclined to invest in other banks or securities other than shares. This probably reflects the fact that those banks were also technically more advanced and had better credit scoring.
Table 2.9 – Effects of securitization on Italian bank balance sheets: 1

The dependent variable is the logistic transformation of the annual average of the monthly shares of the balance sheet items indicated in the first row over total assets. As for the explanatory variable: \( \delta_{2007} \) is a dummy variable indicating whether the data refer to 2007, \( \delta_{sec} \) is a dummy variable indicating whether the data refer to a bank that securitized some of its loans between 1999 and 2007, \( assets \) is total assets (in millions of euros), \( cap \) is a measure of capitalization defined as the ratio of banks’ own capital to total assets, \( \delta_{mcb} \) is a dummy variable indicating mutual or cooperative banks. The equations are estimated using the weighted least-squares logistic regressions for grouped data described in Greene (2003, pp. 686–689). Significance levels at 1%, 5%, and 10% are denoted by *, **, and *** respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Performing loans</th>
<th>Bad loans</th>
<th>Loans to banks</th>
<th>Securities other than shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_{2007} )</td>
<td>0.809***</td>
<td>0.061</td>
<td>-0.379***</td>
<td>-1.091***</td>
</tr>
<tr>
<td>( \delta_{sec} )</td>
<td>0.686***</td>
<td>0.880***</td>
<td>-0.886***</td>
<td>-0.152**</td>
</tr>
<tr>
<td>( \delta_{2007}\delta_{sec} )</td>
<td>-0.658***</td>
<td>-0.958***</td>
<td>0.806***</td>
<td>0.224***</td>
</tr>
<tr>
<td>( \log(assets) )</td>
<td>-0.152***</td>
<td>-0.007</td>
<td>0.269***</td>
<td>-0.069***</td>
</tr>
<tr>
<td>( \log(cap) )</td>
<td>-0.275***</td>
<td>0.058</td>
<td>-0.223***</td>
<td>0.438***</td>
</tr>
<tr>
<td>( \delta_{mcb} )</td>
<td>-0.069**</td>
<td>-0.456***</td>
<td>0.103**</td>
<td>0.357***</td>
</tr>
<tr>
<td>( constant )</td>
<td>0.326**</td>
<td>-3.694***</td>
<td>-4.665***</td>
<td>0.104</td>
</tr>
</tbody>
</table>

\( R^2 \) (adjusted) 0.405 0.448 0.524 0.480

systems, whereas lending to other banks or investing in securities other than shares usually require less sophisticated risk-management tools.

Table 2.9 reports the first results of the econometric analysis. As expected from the previous review of the data, the coefficients for both \( \delta_{2007} \) and \( \delta_{sec} \) are positive and significant for performing loans. The positive coefficient of \( \delta_{2007} \) reflects the fact that, on average, the share of performing loans over total assets increased markedly for the banks that did not securitize between 1999 and 2007. The positive coefficient of \( \delta_{sec} \) is due to the fact that the banks that securitized their loans had, on average, a higher share of performing loans in 1999 than the banks that did not securitize. The negative sign of the dummy variable that interacts year and securitization \( (\delta_{2007}\delta_{sec}) \) highlights that the banks that securitized their loans had a lower growth of the share of performing loans than the other banks. The negative signs of the control variables related to size and capitalization show that larger and more capitalized banks had lower shares of performing loans.

Overall, the signs of \( \delta_{2007}\delta_{sec} \) show that there was, on average, a negative relationship between the decisions of the banks to securitize some of their loans and the shares of their assets invested in performing or bad loans. At the same time, the relationship was positive for the loans to banks and securities other than shares. As
highlighted by the signs of $\delta_{sec}$, in 1999 the banks that securitized had also significantly higher average levels of bad loans and lower average levels of loans to banks and securities other than share. Given that the dummy variables $\delta_{sec}$ and $\delta_{2007}\delta_{sec}$ have opposite signs for all balance sheet items, it turns out that the banks that securitized their loans reduced the differences in the composition of their balance sheets with respect to the other banks. In the end, the average composition of the balance sheets of the banks that securitized their loans was more similar to that of the other banks in 2007 than in 1999.

To verify the robustness of these results, it is worth to notice that in the first econometric exercise we do not distinguish between banks that relied substantially on securitizations and banks that only securitized a very small part of their assets. Since it seems reasonable that there is some relationship between the size of the securitizations and the effects on the structure of bank balance sheets, we now add the dummy variable $\delta_{sec.high}$ that spots those banks that securitized more than the median (in terms of value of the securitizations over total assets) among the banks that securitized some of their loans between 1999 and 2007.

Table 2.10 shows that the results obtained without taking into account the level of the securitizations are now partially reversed. The dummy variables $\delta_{2007}\delta_{sec}$ and $\delta_{2007}\delta_{sec.high}$ have always opposite signs, meaning that the banks that made more use of securitization had usually smaller changes (in absolute values) in the composition of their balance sheets than the banks that made less use of securitization. While the structure of the balance sheets of the latter banks tended to converge towards that of the banks that did not securitize their loans, the balance sheets of the banks that used securitization widely tended to maintain the differences that they had in 1999 with respect to the banks that did not securitize. These findings are coherent with the results by Affinito and Tagliaferri (2010), who find that one of the main motivations for Italian banks to securitize their loans was to have additional resources for financing their investments, and Altunbas et al. (2009), who find that the use of securitization strengthened the capacity of the banks to supply new loans.

Finally, in Table 2.11 we introduce additional dummy variables to distinguish between possible differences arising from the securitization of performing or bad loans. It turns out that the effects in 2007 for the banks that made little use of securitization (given by the coefficients of the dummy variables $\delta_{2007}\delta_{sec.pl}$ and $\delta_{2007}\delta_{sec.bl}$) were usually very similar to those that we have already seen. For the banks that made greater use of securitization, the additional effects (given by the coefficients of the dummy variables $\delta_{2007}\delta_{sec.pl.high}$ and $\delta_{2007}\delta_{sec.bl.high}$) were significant only for the case in which the banks securitized performing loans and for the impact
Table 2.10 – Effects of securitization on Italian bank balance sheets: 2
The dependent variable is the logistic transformation of the annual average of the monthly shares over total assets of the balance sheet items indicated in the first row. As for the explanatory variable: $\delta_{2007}$ is a dummy variable indicating whether the data refer to 2007, $\delta_{sec}$ is a dummy variable indicating whether the data refer to a bank that securitized some of its loans between 1999 and 2007, $\delta_{sec\text{high}}$ is a dummy variable indicating whether the data refer to a bank that securitized more than the median (as a share over total assets), $assets$ is total assets (in millions of euros), $cap$ is a measure of capitalization defined as the ratio of banks’ own capital to total assets, $\delta_{mcb}$ is a dummy variable indicating mutual or cooperative banks. The equations are estimated using the weighted least-squares logistic regressions for grouped data described in Greene (2003, pp. 686–689). Significance levels at 1%, 5%, and 10% are denoted by *, **, and *** respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Performing loans</th>
<th>Bad loans</th>
<th>Loans to banks</th>
<th>Securities other than shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{2007}$</td>
<td>0.808***</td>
<td>0.053</td>
<td>−0.376***</td>
<td>−1.084***</td>
</tr>
<tr>
<td>$\delta_{sec}$</td>
<td>0.713***</td>
<td>0.775***</td>
<td>−0.795***</td>
<td>−0.071</td>
</tr>
<tr>
<td>$\delta_{sec\text{high}}$</td>
<td>−0.207***</td>
<td>0.109*</td>
<td>−0.100</td>
<td>−0.062</td>
</tr>
<tr>
<td>$\delta_{2007}\delta_{sec}$</td>
<td>−0.837***</td>
<td>−1.096***</td>
<td>0.907***</td>
<td>0.376***</td>
</tr>
<tr>
<td>$\delta_{2007}\delta_{sec\text{high}}$</td>
<td>0.554***</td>
<td>0.316***</td>
<td>−0.226***</td>
<td>−0.414***</td>
</tr>
<tr>
<td>log($assets$)</td>
<td>−0.139***</td>
<td>0.021*</td>
<td>0.244***</td>
<td>−0.096***</td>
</tr>
<tr>
<td>log($cap$)</td>
<td>−0.154***</td>
<td>0.223***</td>
<td>−0.352***</td>
<td>0.325***</td>
</tr>
<tr>
<td>$\delta_{mcb}$</td>
<td>−0.084**</td>
<td>−0.445***</td>
<td>0.098**</td>
<td>0.345***</td>
</tr>
<tr>
<td>constant</td>
<td>0.517***</td>
<td>−3.511***</td>
<td>−4.776***</td>
<td>0.052</td>
</tr>
</tbody>
</table>

We conclude this section with a few back-of-the-envelope calculations to verify whether the changes in the composition of their balance sheets have increased or decreased the expected losses of the Italian banks. These results are driven by the greater relevance of the securitization of performing loans for Italian banks, that has been three times larger than the securitization of bad loans in the period under analysis.
Table 2.11 – Effects of securitization on Italian bank balance sheets: 3

The dependent variable is the logistic transformation of the annual average of the monthly shares over total assets of the balance sheet items indicated in the first row. As for the explanatory variable: $\delta_{2007}$ is a dummy variable indicating whether the data refer to 2007, $\delta_{sec,pl}$ is a dummy variable indicating whether the data refer to a bank that securitized some of its performing loans between 1999 and 2007, $\delta_{sec,pl,high}$ is a dummy variable indicating whether the data refer to a bank that securitized performing loans more than the median (as a share over total assets), $\delta_{sec,bl}$ is a dummy variable indicating whether the data refer to a bank that securitized some of its bad loans between 1999 and 2007, $\delta_{sec,bl,high}$ is a dummy variable indicating whether the data refer to a bank that securitized bad loans more than the median (as a share over total assets), $\delta_{mcb}$ is a dummy variable indicating whether the data refer to a bank that securitized some of its bad loans between 1999 and 2007, and $\delta_{mcb}$ is a dummy variable indicating mutual or cooperative banks. The equations are estimated using the weighted least-squares logistic regressions for grouped data described in Greene (2003, pp. 686–689). Significance levels at 1%, 5%, and 10% are denoted by *, **, and ***, respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Performing loans</th>
<th>Bad loans</th>
<th>Loans to banks</th>
<th>Securities other than shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{2007}$</td>
<td>0.841***</td>
<td>-0.085</td>
<td>-0.459***</td>
<td>-1.067***</td>
</tr>
<tr>
<td>$\delta_{sec,pl}$</td>
<td>0.609***</td>
<td>0.156**</td>
<td>-0.666***</td>
<td>-0.225***</td>
</tr>
<tr>
<td>$\delta_{sec,pl,high}$</td>
<td>-0.160***</td>
<td>-0.266***</td>
<td>-0.050</td>
<td>-0.034</td>
</tr>
<tr>
<td>$\delta_{sec,bl}$</td>
<td>0.048</td>
<td>0.709***</td>
<td>-0.201**</td>
<td>0.259***</td>
</tr>
<tr>
<td>$\delta_{sec,bl,high}$</td>
<td>0.063</td>
<td>0.785***</td>
<td>-0.236***</td>
<td>-0.141</td>
</tr>
<tr>
<td>$\delta_{2007}\delta_{sec,pl}$</td>
<td>-0.590***</td>
<td>-0.232**</td>
<td>0.564***</td>
<td>0.320***</td>
</tr>
<tr>
<td>$\delta_{2007}\delta_{sec,pl,high}$</td>
<td>0.477***</td>
<td>0.253***</td>
<td>-0.201**</td>
<td>-0.380***</td>
</tr>
<tr>
<td>$\delta_{2007}\delta_{sec,bl}$</td>
<td>-0.360***</td>
<td>-0.703***</td>
<td>0.542***</td>
<td>0.063</td>
</tr>
<tr>
<td>$\delta_{2007}\delta_{sec,bl,high}$</td>
<td>0.226***</td>
<td>0.026</td>
<td>-0.063</td>
<td>-0.144</td>
</tr>
<tr>
<td>log(assets)</td>
<td>-0.124***</td>
<td>-0.002</td>
<td>0.246***</td>
<td>-0.112***</td>
</tr>
<tr>
<td>log(cap)</td>
<td>-0.081*</td>
<td>0.186***</td>
<td>-0.462***</td>
<td>0.320***</td>
</tr>
<tr>
<td>$\delta_{mcb}$</td>
<td>-0.133***</td>
<td>-0.306***</td>
<td>0.161***</td>
<td>0.391***</td>
</tr>
<tr>
<td>constant</td>
<td>0.593***</td>
<td>-3.401***</td>
<td>-5.051***</td>
<td>0.147</td>
</tr>
</tbody>
</table>

$R^2$ (adjusted) 0.487 0.634 0.589 0.521

The dependent variables and the linear predictions; (2) calculate the predicted values as the sum of the linear predictions evaluated at the mean values, except for the variable of interest, and the residuals; (3) calculate the inverse of the logistic transformation of the predicted values and take the mean.

Once the predicted values of the balance sheet items are calculated, we can associate to each balance sheet item a given level of expected losses and calculate a measure of the overall level of expected losses as the weighted average of the expected losses for the individual items. Table 2.12 reports the result of this exercise for the average limited company bank for the case in which the expected losses for
Table 2.12 – Impact of the balance sheet changes on the expected losses of Italian banks

The table reports the estimated predicted values for the shares of balance sheet items over total assets obtained using the coefficients reported in Tables 2.9 and 2.10. Data refer to the average limited company bank in terms of assets and capitalization. Given the non-linearities involved by the logistic regressions, predicted values are calculated using the smearing estimate of Duan (1983). Expected losses are calculated using the estimated predicted values for balance sheet shares and assuming the following expected losses for the different asset items: performing loans, 2.27%; bad loans, 60%; loans to banks, 0.04%; securities other than shares, 0.17%.

<table>
<thead>
<tr>
<th>Year</th>
<th>Performing loans</th>
<th>Bad loans</th>
<th>Loans to banks</th>
<th>Securities other than shares</th>
<th>Expected losses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Banks that securitized their loans</td>
<td>59.9%</td>
<td>7.2%</td>
<td>5.4%</td>
<td>24.7%</td>
</tr>
<tr>
<td></td>
<td>1999</td>
<td>63.2%</td>
<td>3.2%</td>
<td>7.9%</td>
<td>12.6%</td>
</tr>
<tr>
<td></td>
<td>2007</td>
<td>60.9%</td>
<td>6.7%</td>
<td>5.8%</td>
<td>26.1%</td>
</tr>
<tr>
<td></td>
<td>Banks that securitized less than the median value</td>
<td>60.3%</td>
<td>2.6%</td>
<td>9.2%</td>
<td>15.3%</td>
</tr>
<tr>
<td></td>
<td>1999</td>
<td>56.3%</td>
<td>7.4%</td>
<td>5.3%</td>
<td>25.0%</td>
</tr>
<tr>
<td></td>
<td>2007</td>
<td>67.7%</td>
<td>3.8%</td>
<td>7.0%</td>
<td>10.3%</td>
</tr>
<tr>
<td></td>
<td>Banks that securitized more than the median value</td>
<td>56.3%</td>
<td>7.4%</td>
<td>5.3%</td>
<td>25.0%</td>
</tr>
<tr>
<td></td>
<td>1999</td>
<td>60.9%</td>
<td>6.7%</td>
<td>5.8%</td>
<td>26.1%</td>
</tr>
<tr>
<td></td>
<td>2007</td>
<td>60.3%</td>
<td>2.6%</td>
<td>9.2%</td>
<td>15.3%</td>
</tr>
<tr>
<td></td>
<td>Banks that did not securitize their loans</td>
<td>44.2%</td>
<td>3.2%</td>
<td>11.6%</td>
<td>27.4%</td>
</tr>
<tr>
<td></td>
<td>1999</td>
<td>44.2%</td>
<td>3.2%</td>
<td>11.6%</td>
<td>27.4%</td>
</tr>
<tr>
<td></td>
<td>2007</td>
<td>62.6%</td>
<td>3.4%</td>
<td>8.4%</td>
<td>11.8%</td>
</tr>
</tbody>
</table>

Performing loans, bad loans, loans to banks, and securities other than shares are equal to 2.27%, 60%, 0.04%, and 0.17%. These numbers correspond, under the usual hypothesis of a 40% recovery rate, to the one-year default probabilities of issuers with ratings B, D, AA, and BBB, respectively (see Ou et al., 2011, Exhibit 26).

Under these assumptions, between 1999 and 2007 the level of expected losses decreased by about 40% for the banks that securitized some of their loans, while increased by more than 15% for the banks that did not securitize. These results are mainly driven by the fact that the share of bad loans, which represents by far the part of the bank balance sheets with the higher expected losses, dropped for the banks that securitized while increased moderately for the banks that did not securitize. It is interesting to notice that the greatest beneficial effects were associated with the banks that made only little use of securitization. Those banks kept the share of performing loans almost unchanged and decreased significantly the share of bad loans. By doing so, they decreased their expected losses much more
than the banks that relied more on securitization. The banks that made greater use of securitization increased substantially their share of performing loans but at the cost of not being able to reduce the share of bad loans to the same extent as the banks that securitized less.

### 2.3.2 Comparing securitized loans with new loans

In this section we compare, for a sample of Italian banks, the individual default probabilities of the loans that were securitized in each semester between 2004 and 2007 with the individual default probabilities of the loans that were granted during the same semesters by the same banks. In particular, for each bank in the sample we calculate the default rate in semester $s$ of all loans that were securitized or granted in semester $t$, where $s$ is any semester between $t$ and the second half of 2007. The default rate is defined as the ratio of the total face value of the loans that were securitized or granted by each bank at time $t$ that run into default at time $s$ to the total face value of the loans that were securitized or granted by the same banks at time $t$. The definition of default that we use is that of bad loan, as defined by the Italian regulation. We also consider a borrower to be in default on all the loans she received when at least one of her loans is classified as bad by one of her lenders. The reason for using this extended definition of default is that the Italian databases identifies the individual borrowers but not the individual loans, thus making extremely difficult to keep track of the transferring of the loans among financial institutions.

As already mentioned, the Italian regulation requires banks to classify the outstanding loans to borrowers that are not expected to meet their obligations as bad loans. Undoubtedly, this definition allows banks to have a certain degree of arbitrariness as they have to judge when the loans are likely to be repaid to not. It turns out that monthly default rates show a strong seasonality, probably reflecting the policies applied by the main Italian banks to review the quality of their loans. To mitigate the seasonal effects, we use semi-annual data. Moreover, we believe that six months is a period of time long enough to allow the banks to reinvest the proceeds of their securitizations but not excessively long to permit banks to make their investments using predominantly other sources of financing.

To assess the level of risk of a bank, in Section 2.2 we showed that the characterizing features of the loans are the individual default probability and the correlation with the other loans in the bank portfolio. Unfortunately, as mentioned before, we are actually only able to provide empirical evidence on the first characteristic of the loans granted or securitized by Italian banks. In fact, it is extremely difficult to say
anything on the correlation between the loans that remained in the bank balance sheets and the new loans, or the loans that were securitized, because of the arbitrariness with which Italian financial institutions can classify a loan as performing or bad. What happens in practice is that a loan that is securitized is defined as performing or bad by the special purpose vehicle (SPV) that buy it, whereas the loans that remain in the portfolio of the bank are classified as performing or bad by the bank itself. The fact that loans are classified by different institutions, which potentially use different criteria, makes the calculation of any correlation coefficient for defaults strongly unreliable. In principle, this problem could be circumvented by using a sufficiently large period of time, say six months, that is deemed to capture most of the possible differences in the timing with which loans are classified as performing or bad by different institutions. However, using semi-annual windows with only four years of data available would lead to having only eight triples of default rates (eight default rates for each group of new loans, securitized loans, and loans that remained in the portfolio of the bank), thus leading to completely unreliable estimations of the correlation coefficients.

For our analysis, we rely on two sources of data with loan-by-loan details. We gather the individual data on defaults and loans that were securitized from the Italian Central Credit Register (Centrale dei Rischi). The second source of data is the Sample Survey of Active and Passive Rates (Rilevazione Campionaria dei Tassi Attivi e Passivi) which contains individual information on the new loans, and the corresponding borrowers, granted from a sample of Italian banks since 2004.

Given that our primary interest is to know when a loan goes into default, we only take into account the loans securitized through institutions reporting to the Italian Central Credit Register, thus excluding foreign SPVs. We also use data on securitizations of performing loans only as bad loans are already in default by definition when they are securitized. Given that particularly small or large loans can sometimes be treated in special ways by banks, we dropped from the dataset the loans with an outstanding amount smaller than €75,000 or greater than €1,000,000. Finally, for each semester we consider only the new loans that were granted by the banks that made at least one securitization during the same period. Overall, the final dataset includes about 493,000 new loans and about 346,000 securitized loans.

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17 The Central Credit Register is a department of the Bank of Italy that collects data on borrowers from their lending banks. Reporting banks file detailed information for each borrower with total loans or credit lines of more than €75,000. Banks are required to report smaller exposures only if the borrower goes into default. Bad loans are defined on a customer basis and therefore include all the outstanding credit extended by a bank to a borrower considered insolvent.

18 The survey covers about 70% of the total amount of the new loans granted during the sample period.
Figure 2.8 – Average default rates of the old loans that remained in the Italian bank balance sheets after a securitization and of the new loans that were granted with the proceeds of the securitization

The panels show the simple averages of the semi-annual default rates across all banks included in the sample. The legends report the semesters in which the securitizations took place and the new loans were granted. The horizontal axes show the number of semesters following the securitization and the granting of the new loans.

Figure 2.8 shows the evolution over time of the default rates of the loans that were securitized or granted in each semester for the overall banking system. Data refer to simple averages across all banks included in the sample. The first interesting aspect highlighted by the figure is that the overall quality of securitized and new loans did not change significantly as a function of the semester in which they were securitized or granted. This result appears from the fact that the lines corresponding to the different semesters tend to cross each other repeatedly and do not stand on clearly defined different levels. This finding suggests the absence of a credit cycle during the sample period, at least when all kinds of loans are taken into
account and the analysis is limited to banks that used securitization to finance their investments.\textsuperscript{19}

The second relevant aspect of the figure is that the new loans were, on average, riskier than the securitized loans. Overall, the new loans had an average default rate of 0.41\%, against 0.28\% for securitized loans. The significance of the positive difference between the default rates of the new loans and the loans that were securitized, for the whole sample period, is confirmed by the $t$-test reported in Table 2.13. The table also reports the averages of the differences across all banks included in the sample for all semesters of inception and default. Positive values appear for almost all possible combinations of semesters of inception and default and several of them are statistically significant. Overall, these findings show that the banks that securitized their loans were also increasing their risks.

These results deserve a few last comments. It may be argued that one should control for other variables when comparing the level of risk of the loans that were securitized with that of the new loans. In principle, it could be possible that the different levels of risk of the two categories of loans were due to the fact that the loans were granted to different sets of borrowers (with possible different credit worthiness) or had different characteristics in terms of yield (fixed or variable), maturity (shorter or longer), or anything else that could influence the probability to run into default. It may also be argued that one should control for the time in which new and securitized loans were initially granted as Ioannidou et al. (2009) and Jiménez et al. (2010) show that the loans that are granted when the interest rates are low (high) have higher (lower) probability to run into default when interest rates increase (decrease). Hence, the previous results could be explained with the argument that, say, securitized loans are less risky only because they were granted when interest rates were higher. While explaining the determinants of the differences among default rates is a very interesting issue, we feel that it is outside of the scope of the present analysis. As shown in Section 2.2, what matters for the risk of a bank of incurring large losses is the difference between the level of risk of the loans that are securitized and the level of risk of the new loans, no matter where that difference comes from. Whatever the reason, securitizing good loans and substituting them with loans with lower quality is not a good deal for the stability of a bank.

\textsuperscript{19}Bonaccorsi di Patti and Felici (2008) use a larger sample of banks, that includes banks that did not use securitizations, to analyze mortgages to Italian households and find that default rates tended to rise during the same sample period as a function of the semester of inception (the semester in which the loans were granted or securitized).
Table 2.13 – Comparison of the default probabilities of the new loans with the default probabilities of the loans that were securitized by Italian banks

The table reports, in percentage points, the differences between the default rates of new and securitized loans by semester of inception (left column) and semester of default (top row). Simple averages across all banks included in the sample. Significance levels, calculated using t-tests, at 1%, 5%, and 10% are denoted by *, **, and ***, respectively.

<table>
<thead>
<tr>
<th>Semester of inception</th>
<th>Semester + 0</th>
<th>Semester + 1</th>
<th>Semester + 2</th>
<th>Semester + 3</th>
<th>Semester + 4</th>
<th>Semester + 5</th>
<th>Semester + 6</th>
<th>Semester + 7</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004 S1</td>
<td>0.090</td>
<td>0.137</td>
<td>−0.121</td>
<td>0.384*</td>
<td>0.103</td>
<td>0.155</td>
<td>0.332*</td>
<td>0.438*</td>
<td>0.190***</td>
</tr>
<tr>
<td>2004 S2</td>
<td>0.136**</td>
<td>0.151*</td>
<td>0.240</td>
<td>0.044</td>
<td>0.191</td>
<td>0.522***</td>
<td>−0.054</td>
<td>—</td>
<td>0.176***</td>
</tr>
<tr>
<td>2005 S1</td>
<td>−0.094</td>
<td>0.348**</td>
<td>−0.090</td>
<td>0.296***</td>
<td>0.299*</td>
<td>0.181*</td>
<td>—</td>
<td>—</td>
<td>0.157***</td>
</tr>
<tr>
<td>2005 S2</td>
<td>0.040</td>
<td>0.086</td>
<td>0.033</td>
<td>0.003</td>
<td>0.321*</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.097</td>
</tr>
<tr>
<td>2006 S1</td>
<td>0.187</td>
<td>0.002</td>
<td>0.068</td>
<td>0.126</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.096</td>
</tr>
<tr>
<td>2006 S2</td>
<td>0.061</td>
<td>−0.246</td>
<td>0.181</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>−0.002</td>
</tr>
<tr>
<td>2007 S1</td>
<td>0.059</td>
<td>0.255**</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.157**</td>
</tr>
<tr>
<td>2007 S2</td>
<td>−0.233</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>−0.233</td>
</tr>
<tr>
<td>Total</td>
<td>0.035</td>
<td>0.105**</td>
<td>0.041</td>
<td>0.176**</td>
<td>0.235***</td>
<td>0.257***</td>
<td>0.172</td>
<td>0.438*</td>
<td>0.126***</td>
</tr>
</tbody>
</table>
2.4 Conclusion

This chapter analyzes the effects of CDO issuance on the risk of incurring large losses for a bank. Using Monte Carlo simulations, we show that the practice for a bank to securitize part of its loans using CDOs, of which it retains the equity tranches, and reinvest the proceeds of the securitization in other loans can increase as well as decrease the risk that the bank faces large losses. The final result depends on the different characteristics of the loans that are securitized and the new loans that are granted in terms of individual default probabilities and correlations with the loans that the bank do not securitize. We also document that the final effect on the VaR of a bank is usually almost negligible as long as the securitization involves only a reasonable share of the total loan portfolio.

We then focus on Italian banks and provide evidence that the use of securitization has contributed to change the overall composition of the asset side of the bank balance sheets. The Italian banks that securitized their assets had lower increases in their shares of performing and bad loans over total assets and increased their reliance on investments in loans to other banks and securities other than shares. However, the Italian banks that made greater use of securitization recorded greater increases in their shares of performing and bad loans. Overall, the changes in the balance sheets involved by the use of securitization have probably reduced the expected credit losses of Italian banks, mainly because of the reduction in the share of bad loans. Finally, we examine loan-by-loan data to compare the default risk of the loans that were securitized with that of the new loans that were granted by the same banks during the same months. We show that, on average, Italian banks securitized loans with less default risk than the new loans, thus suggesting that the risks embedded in the loan portfolios of the banks that securitized their loans have increased during the years before the onset of the global financial crisis.
2.A Appendix: The CDO market

Collateralized debt obligations are securities backed by the cash flows of portfolios of different financial instruments (called underlying assets). The main characteristic of CDOs is tranching. A CDO is actually made by several different securities with given seniorities in terms of rights on the cash flows generated by the underlying assets. In this respect, senior, mezzanine, and junior tranches rank in a decreasing order. Risks and returns offered by those tranches vary accordingly. The splitting of CDOs into different tranches dictates a sequential allocation of the losses that the underlying portfolio can incur. The structure of a CDO guarantees that the holders of each tranche, with the exception of the equity tranche, are protected from the risk of incurring losses by one or more of the other tranches. The most junior tranche, called equity tranche, is the first to absorb the losses deriving from one or more defaults of the assets in the underlying portfolio. If the losses exceed the notional value of the equity tranche, they are absorbed by the other junior and mezzanine tranches. The senior tranches are affected only if the losses in the underlying portfolio are very large. In that case, they sustain the remaining part of the losses that cannot be absorbed by the other tranches. Usually, senior and mezzanine tranches are also protected by other specific credit enhancement techniques, such as overcollateralization, reserve accounts, and the trapping of excess spreads.\(^{20}\)

Collateralized debt obligations are usually classified according to the final aim of the transaction and the way in which the credit risk of the underlying portfolio is transferred. With respect to the first dimension, one can have balance sheet CDOs and arbitrage CDOs depending on whether the main purpose of the transaction is to modify the composition of the balance sheet of the seller (also named originator) or to carry out an arbitrage transaction by exploiting the potential differences between the returns required from the investors in the tranches of the CDOs and the returns of the assets included in underlying portfolio.\(^{21}\) In a balance sheet CDO the originator is usually a financial institution (most of the times, a bank) that wants to get rid of some of its assets in order to have additional resources for other investments, increase its profitability, or reduce the regulatory capital.\(^{22}\) Balance sheet CDOs determine a transferring of the risks traditionally taken by the banking system to other investors.

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\(^{20}\)See Melennec (2000) for a description of several credit enhancement techniques.

\(^{21}\)Carlstrom and Samolyk (1995), Gorton and Pennacchi (1995), and DeMarzo and Duffie (1999) provide alternative explanations for the growth in securitization activity based on the fact that certain institutions have a natural comparative advantage in originating, but not holding, illiquid assets.

\(^{22}\)Jones (2000) describes several securitization techniques used by banks to reduce their regulatory capital requirements.
The aim of arbitrage CDOs is instead to carry out a market arbitrage by putting together a portfolio of underlying assets and a CDO structure such that the overall return of the underlying portfolio (the arbitrageurs cash-in-flows) is greater than the overall return of the CDO tranches (the arbitrageurs cash-out-flows).\textsuperscript{23}

The credit risk can be transferred with CDOs either through a true sale of the assets (\textit{cash flow} CDOs) or using credit derivatives (\textit{synthetic} CDOs). In a cash flow CDO the property rights on the assets in the underlying portfolio are actually transferred from the originator to a \textit{special purpose vehicle} (SPV) which in turn finances the purchase using the proceeds of the issuance of the CDO. A synthetic CDO has a more complex structure as the transferring of risk is not achieved with the true sale of risky assets but by using CDSs.\textsuperscript{24} In a synthetic CDO the originator gets rid of the credit risk by buying protection from the SPV using CDSs and the SPV in turn buys protection from the holders of the CDO tranches. If some of the underlying assets default, the originator asks the SPV to be compensated for the losses. Then, the SPV transfers the losses to the final investors in the CDO according to the tranching structure. Given that there is not any initial sale of assets from the originator to the SPV, there is not even the necessity for the SPV to raise cash when issuing the CDO. That means that the buyers of the tranches of the CDO can be required to pay nothing at the inception of the contract (\textit{unfunded synthetic} CDO). The buyers of the tranches eventually pay what is due according to the tranching structure only when there is some default among the underlying assets. In an unfunded CDO the ability of the SPV to compensate the originator if a credit event occurs depends only on the creditworthiness of the buyers of the CDOs. Therefore, the risks for the originator are higher. However, the fact that the buyers of the CDO pay nothing at inception makes this product more attractive for investors.\textsuperscript{25} On the other side, in a \textit{funded synthetic} CDO the final investors are required to pay the notional amounts of their tranches to the SPV, which then invests the proceeds in high-rated bonds (usually, AAA-rated government bonds) and eventually use them to compensate the originator for the losses recorded in the underlying portfolio.

Usually, the overall compensation paid to CDO investors is significantly smaller than the returns of the underlying assets as the difference goes to pay the profession-

\textsuperscript{23}Amato and Remolona (2005) use evidence from the market of arbitrage CDOs to argue that the high level of the spreads on corporate bonds relative to expected losses from default is due to the difficulty of fully diversify a credit portfolio.

\textsuperscript{24}In a CDS the protection seller agrees to pay to the protection buyer some amount of money in case of default of the reference entity, in exchange of a periodic fee. See JPMorgan (1999) and Chapter 4 for more details.

\textsuperscript{25}Tranches of CDS indices such as ITraxx or IBoxx are examples of unfunded synthetic CDOs.
als involved in the transaction (originators, security firms, asset managers, trustees, rating agencies, attorneys, and accountants). Hence, one might ask why investors are interested in products with returns smaller than those of the underlying assets. The answer is that CDOs can create customized exposures that investors desire and cannot achieve in any other way. Using CDOs it is possible to fit into investors’ various risk appetites and capital constraints. For instance, less risk-averse investors, such as hedge-funds and investment banks, usually prefer to be exposed to the riskier tranches, whereas pension funds and insurance companies prefer to invest in more senior tranches. CDOs slice the overall credit risk of the underlying assets into different tranches and sell each of them to the investors that feel more confident with the corresponding risk-return profile.

Due to the characteristics of the CDOs, it should not be surprising if the rate of growth of CDO issuance has been exceptionally high in the years before the onset of the global financial crisis. Worldwide CDO issuance was about $500 billion in 2007, in spite of a strong decrease in the second half of the year, or more than three times greater than in 2004 (see Panel A of Figure 2.9). As in previous years, the bulk of the issues was represented by cash flow CDOs (about 70% of the total). A breakdown by purpose shows that arbitrage CDOs accounted for about 85% of the total issues (see Panel B of Figure 2.9). About 50% of the total issues were backed by other structured products — such as residential mortgage backed securities (RMBS), commercial mortgage backed securities (CMBS), other CDOs, CDSs, and other securitized/structured products — and 30% were backed by high-yield loans. New issues were mainly denominated in US dollars (70%) and euros (25%).

\[26\text{Data are from the Securities Industry and Financial Markets Association (SIFMA), an organization born of the merger between The Securities Industry Association and The Bond Market Association. SIFMA represents more than 650 member firms of all sizes, in all financial markets in the United States and around the world. Data do not include unfunded synthetic tranches.}

\[27\text{High-yield loans are defined as transactions of borrowers with senior unsecured debt ratings below Baa3 from Moody’s or BBB- from S&P.}\]
Figure 2.9 – Global CDO market issuance in 2004–07
Chapter 3

Securitization and Extreme Events

Just as on the Titanic,

not even first class passengers can save themselves

Mr. Giulio Tremonti
(Former Italian Treasury Minister)

3.1 Introduction

Is there a best way to protect against a tsunami? Common sense reasonings can offer some guidance. First, the tsunamis cannot be avoided and cannot be foreseen, as the earthquakes that generate them cannot be avoided and foreseen either. Second, a complete ex ante physical protection against tsunamis is unfeasible, either technically or economically, because it would involve suspending human activities along coasts. However, the destructive consequences of the tsunamis can certainly be reduced by using swift information systems and building refuge places. The bottom line is: you may not be able to avoid the disaster, but you can manage it in order to limit its negative consequences.

Not only natural disasters occur, but also economic history is full of unpredicted extreme events that had significant negative impact on human lives. For instance, one may think of the unprecedented reduction in residential property prices that occurred in the last few years in several industrial countries. Because of the importance of housing as a collateral for bank financing, the reduction of property prices led close to the collapse of the worldwide banking system. Global output decreased sharply and unemployment surged in many countries.\footnote{See Panetta et al. (2009) for a review of the analyses of the causes that led to the overvaluation of residential and commercial properties in several countries around the world, to the burst of the housing bubble, and to the quick spread of the negative consequences of those events all around the world.}
This chapter takes the view that painful economic events tend to happen and are inherent to any socio-economic system. In this framework, we study the consequences that such events may have on the banking system, especially when banks securitize their assets. To this end, bank assets are described by a standard one-factor model in which the underlying macroeconomic factor faces negative shocks, possible quite severe, as described by the realization of a random variable with a Student’s $t$-distribution. Using Monte Carlo simulations, this chapter documents several interesting results obtained from that model. First, at very high confidence levels, the value-at-risk (VaR) of a bank is little influenced by the quality of its assets, especially when macroeconomic shocks can be very severe (i.e., when the Student’s $t$-distribution has a lower number of degrees of freedom). Second, in most of the cases the securitization of loans with poor credit rating and the reinvestment in loans of much better quality do not improve the tail risk exposure of the bank to macroeconomic shocks. Third, the most extreme consequences of negative macroeconomic shocks have to be studied at confidence levels that are higher than those usually used for management and regulatory purposes.

This chapter shows that even the best borrowers are negatively affected and defaults tend to cluster when severe macroeconomic shocks occur. To increase the resilience of the banking system to extreme macroeconomic shocks banks should therefore strengthen their capital base to survive when a shock occurs. Steering banking activities towards investments and management practices deemed to be safer is usually not useful to protect banks against extreme events.

The chapter is organized as follows. Section 3.2 describes the model that is used to simulate numerically the behaviour of the economy and the impact of macroeconomic shocks on the level of risk of the bank. Section 3.3 discusses the results and Section 3.4 concludes.

### 3.2 The model

To study the impact that extreme macroeconomic shocks may have on the risk of a bank to incur large losses, especially when the bank securitizes its assets, we use a simple one-factor model. Li (2000) suggests to model the default correlation among $N$ borrowers, or issuers, using a Gaussian copula approach.\(^2\) A common way to implement this framework is to assume the existence of $N + 1$ independent standard

\(^2\)See Appendix 3.A for a brief review of the main properties of copulas and numerical methods to simulate copulas.
normal random variables $X$ and $\varepsilon_i$, $i = 1, \ldots, N$, and define

$$V_i = \sqrt{\rho} X + \sqrt{1 - \rho} \varepsilon_i,$$

for some $\rho \in [0, 1]$ (see Vasicek, 1987, 1991, 2002). Given these assumptions, the random variable $V_i$ has a standard normal distribution as well. Then, issuer $i$ is assumed to default when $\Phi(V_i) \leq p_i$, where $\Phi$ is the cumulative distribution function for the standard normal distribution and $p_i$ is the individual default probability of issuer $i$. It is easy to verify that $\rho$ measures the linear correlation between two issuers.

Equation (3.1) can be interpreted as a measure of the asset returns of a borrower. A default occurs when returns are particularly negative. The factors that determine the performance of the borrower are of two kinds: the random variable $X$ describes the common shocks that hit all issuers contemporaneously, and can be interpreted as a macroeconomic factor; the variables $\varepsilon_i$ are issuer specific, and can be interpreted as idiosyncratic risk factors. The parameter $\rho$ determines the exposure of the borrowers to macroeconomic shocks.

Building on this interpretation, we modify Eq. (3.1) in two directions to better meet the needs of this chapter. First, the standard normal random variables are replaced with $t$-distributed random variables that allow for extreme shocks that are not possible in the normal framework (this model is usually called double $t$-distribution copula). Second, to fully exploit the diversification properties of a portfolio of assets, we assume that the parameter $\rho$ can be issuer-specific and non-positive. The asset returns of the borrowers are thus described by

$$\tilde{V}_i = \text{sgn}(\rho_i) \sqrt{|\rho_i|} \tilde{X} + \sqrt{1 - |\rho_i|} \tilde{\varepsilon}_i,$$

where $\text{sgn}(x)$ is the sign function

$$\text{sgn}(x) = \begin{cases} 
-1, & \text{if } x < 0 \\
0, & \text{if } x = 0 \\
1, & \text{if } x > 0,
\end{cases}$$

the random variables $\tilde{X}$ and $\tilde{\varepsilon}_i$ are independent and $t$-distributed with unit variance, and $\rho_i \in [-1, 1]$, $i = 1, \ldots, N$. In this framework, the correlation between two borrowers, $i$ and $j$, is given by $\text{sgn}(\rho_i \rho_j) \sqrt{|\rho_i \rho_j|}$. Notice that the correlation between pairs of borrowers can also be negative, but it is always positive and equal to $|\rho|$ if

---

3Early studies on the double $t$-distribution copula include Lucas et al. (2002), Lucas et al. (2003) and Hull and White (2004).

4For an example of negative correlation, one may think of the relationship that takes place between stock and risk-free government bonds during periods of distress.
\( \rho_i = \rho_j = \rho. \)

Given that the distribution of \( \hat{V}_i \) in Eq. (3.2) is unknown, we use Monte Carlo simulations to calculate \( \Psi \), the cumulative distribution function of \( \hat{V}_i \). Issuer \( i \) is assumed to default when \( \Psi(\hat{V}_i) \) is below the individual default probability \( p_i \).

We also assume that the loans granted by the bank have the same notional value, interest rate, and recovery ratio \( R \). This means that, for a given individual default probability \( p_i \), that implicitly defines the default threshold \( d_i = \Phi^{-1}(p_i) \) for the normal case and \( d_i = \Psi^{-1}(p_i) \) for the Student’s \( t \) case, only the share \( R \) of the notional value of the loan is paid back when the value of \( \hat{V}_i \) drops below \( d_i \) and borrower \( i \) is considered to default. We discount future payoffs at a constant interest rate \( r \). Finally, all loans are assumed to have a maturity of one year and defaults can only occur at the end of the year.\(^6\)

Without loss of generality, the total notional value of the \( N \) loans are normalized to 1, so that the size of each loan is \( 1/N \). The interest amount (coupon) that has to be paid by borrower \( i \) on its loan at the end of the year is set equal to

\[
C_i = \frac{1}{N} \frac{r + p_i(1 - R)}{1 - p_i},
\]

which is the value that makes the discounted expected value of the loan equal to its face value. More generally, to get rid of the issues related to investors’ degree of risk aversion, we assume that all assets are priced in a risk-neutral way.

Following Krahnen and Wilde (2006) and Di Cesare (2009), we assume that the bank sell \( n \) of the \( N \) loans by putting them in the underlying pool of a CDO. The CDO has 7 tranches with attachment points defined by their default probabilities. The attachment points of the tranches are set at the 1st, 2nd, 5th, 10th, 20th, and 30th percentiles of the distribution of the losses of the pool of loans, which means that the first tranche (the most senior) only suffers losses with a 1% probability, the second tranche has a 2% probability of not being repaid in full, and so on. The default probability of the last tranche (the most junior) is not pre-defined but can be calculated given the previous assumptions on the characteristics of the loans and taking into account that it bears all initial losses.\(^7\) The last tranche is retained by the bank and immediately after the securitization the bank reinvests the proceeds of the sale of the other six tranches in new loans. It turns out that the reinvestment

\(^5\)An alternative version of Eq. (3.2) is \( \hat{V}_i = \rho_i \hat{X} + \sqrt{1 - \rho_i^2} \hat{\epsilon}_i \), which implies a correlation of \( \rho^2 \) between borrowers with the same parameter \( \rho \). Equation (3.2) has been preferred to the alternative version to maintain the interpretation of \( \rho \) as the correlation between borrowers with the same positive parameter.

\(^6\)Allowing for loans with longer maturities and for defaults that can occur before the maturity of the loans would make the notation and empirical implementation more cumbersome, without changing the overall quality of the results.

\(^7\)Because of its high level of risk, the last tranche is also called equity tranche.
Table 3.1 – List of symbols
Symbols used in the chapter to refer to the payoffs of several assets, or pool of assets

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{old}$</td>
<td>total payoff of the original loan portfolio of the bank</td>
</tr>
<tr>
<td>$P_n$</td>
<td>total payoff of the $n$ loans that are securitized</td>
</tr>
<tr>
<td>$P_{N-n}$</td>
<td>total payoff of the $N-n$ loans that are not securitized</td>
</tr>
<tr>
<td>$P_{eqt}$</td>
<td>payoff of the equity tranche of the CDO</td>
</tr>
<tr>
<td>$P_{rnv}$</td>
<td>total payoff of the loans in which the bank reinvests the proceeds of the securitization</td>
</tr>
<tr>
<td>$P_{new}$</td>
<td>total payoff of the new portfolio, obtained after the securitization and the reinvestment process</td>
</tr>
</tbody>
</table>

Strategy is the critical factor determining whether the overall level of risk of the bank (i.e., the risk of incurring large losses) increases or decreases after the securitization process has been completed.

Krahnen and Wilde (2006) show that, due to a leverage effect of the risk, the securitization of the assets and the retaining of the most junior tranche tend to increase the level of risk of the bank under the assumption that the new loans have the same characteristics as the loans that have been securitized in terms of $\rho$, $p$, and $R$. Following the lead of Krahnen and Wilde (2006), Di Cesare (2009) shows that the risk of the bank of incurring large losses can also decrease when the proceeds of the securitization are reinvested in safer assets or in assets that are less correlated with the assets retained in the bank portfolio. In the end, what happens with the risk level of a bank after a securitization depends on the reinvestment strategy.

The return distribution of the original portfolio (i.e., the loan portfolio held by the bank before the securitization process starts) can be calculated using Monte Carlo simulations, and then compared with the return distribution of a new portfolio made of: (1) the $N-n$ loans in the original portfolio that are not securitized; (2) the equity tranche of the CDO that has been used for the securitization; (3) the new loans that the bank has granted using the proceeds of the sale of the other tranches of the CDO.

To describe in detail how the simulations work, it is useful to refer to the symbols described in Table 3.1. The returns associated with the payoffs listed in the table are indicated by substituting $R$ to the corresponding $P$ (i.e., one has $R_{old}$, $R_n$, $R_{N-n}$, $R_{eqt}$, $R_{rnv}$, and $R_{new}$) and are calculated by comparing the payoffs with their initial fair values (i.e., the discounted expected values of the payoffs). The probability distributions of the returns are estimated using the following steps:
1. Generate a random value for the common risk factor $X$;\(^8\)
2. Generate $N$ random values for the idiosyncratic risk factors $\varepsilon_i$, for $i = 1, \ldots, N$;
3. For given $\rho_i$, compute $\tilde{V}_i$ as defined in Eq. (3.2), for $i = 1, \ldots, N$;
4. For given default probability $p_i$ for borrower $i$, calculate the default threshold $d_i = \Phi^{-1}(p_i)$ for the normal case or $d_i = \Psi^{-1}(p_i)$ for the Student’s $t$ case;
5. Calculate the total payoff and return of the original portfolio:\(^9\)
   \[
P_{old} = \frac{1}{N} \left[ R \sum_{i=1}^{N} \mathbf{1}_{\{\tilde{V}_i \leq d_i\}} + (1 + C) \left( N - \sum_{i=1}^{N} \mathbf{1}_{\{\tilde{V}_i \leq d_i\}} \right) \right], \tag{3.5}\]
   \[R_{old} = P_{old} - 1; \tag{3.6}\]
6. Calculate the total payoff and return of the $n$ loans that are securitized (assuming that the first $n$ in the portfolio are those that are securitized):
   \[
P_n = \frac{1}{N} \left[ R \sum_{i=1}^{n} \mathbf{1}_{\{\tilde{V}_i \leq d_i\}} + (1 + C) \left( n - \sum_{i=1}^{n} \mathbf{1}_{\{\tilde{V}_i \leq d_i\}} \right) \right], \tag{3.7}\]
   \[R_n = NP_n/n - 1; \tag{3.8}\]
7. Calculate total payoff and return of the $N - n$ loans that are not securitized:
   \[
P_{N-n} = P_{old} - P_n, \tag{3.9}\]
   \[R_{N-n} = NP_{N-n}/(N - n) - 1; \tag{3.10}\]
8. Repeat the previous steps 1 million times to calculate the distributions of $P_{old}$, $R_{old}$, $P_n$, $R_n$, $P_{N-n}$, and $R_{N-n}$;
9. Using the estimated distribution for $P_n$, calculate the detachment point of the equity tranche in terms of total payoff of the underlying portfolio:
   \[
   D_{eqt} = \max(x : \mathbb{P}(P_n < x) \leq 0.3); \tag{3.11}\]
10. Using the estimated distribution for $P_n$, calculate the distribution of the payoff of the equity tranche:
    \[
    P_{eqt} = \max(P_n - D_{eqt}, 0); \tag{3.12}\]
---
\(^8\)Here and in step 2 we use the term “random value” to refer to the outcome of independent normal or $t$-distributed random variables, depending on the case we want to analyze.

\(^9\)Remember that the total notional value of the $N$ loans has been normalized to 1, so that the size of each loan is $1/N$. Moreover, because of the risk-neutral assumption and Eq. (3.4), the fair value of the loan portfolio is equal to its notional value.
11. Using the estimated distribution of \( P_{eqt} \), calculate the fair value of the equity tranche and the distribution of its return:

\[
V_{eqt} = \mathbb{E}[P_{eqt}]/(1 + r),
\]

(3.13)

\[
R_{eqt} = P_{eqt}/V_{eqt} - 1;
\]

(3.14)

12. Given that the CDO is priced in a risk-neutral framework, its total initial value is equal to that of the underlying loans, which is \( n/N \). This implies that the proceeds of the sale of the CDO, after retaining the equity tranche, are equal to \( n/N - V_{eqt} \). That amount of money is reinvested by the bank in a portfolio of \( n \) new loans with characteristics potentially different from those of the loans in the original portfolio. The distribution of the payoff of the reinvested portfolio \( P_{rnv} \) is calculated by repeating 1 million times the steps 2 to 5 used to calculate \( P_{old} \) (setting \( N = n \));

13. Using the estimated distributions for \( P_{N-n} \), \( P_{eqt} \), and \( P_{rnv} \), calculate the distributions of the payoff and return of the new portfolio that the bank owns after the securitization and reinvestment process have taken place:

\[
P_{new} = P_{N-n} + P_{eqt} + P_{rnv},
\]

(3.15)

\[
R_{new} = P_{new} - 1.
\]

(3.16)

Once the previous steps have been done and the distributions of the returns of both the original and new portfolios have been estimated, one can calculate and compare synthetic indicators of tail risk for the two portfolios, such as the value-at-risk (VaR) and the expected shortfall (ES). The rest of the chapter focuses on the VaR, which is the most common tail risk measure, but results using the ES are qualitative the same.\textsuperscript{10}

\section*{3.3 Analysis of the numerical results}

To shed light on the potential effects of extreme macroeconomic shocks on a loan portfolio, before and after a securitization process takes place, the numerical simulations described in the previous section have been performed using standard normal and \( t \)-distributed random variables (with 5 and 3 degrees of freedom).\textsuperscript{11} The choice

\textsuperscript{10}Detailed results for the ES are available from the author on request.

\textsuperscript{11}Lucas et al. (2002) use the same degrees of freedom for the \( t \)-distributed random variables. Lucas et al. (2001) claim that the return series of the S&P index can best be described by a Student’s \( t \)-distribution with 5 degrees of freedom. For comparability reasons, the \( t \)-distributed random variables have been normalized to have unit variance. To this end, the stochastic realizations of the random variables have been divided by \( \sqrt{\nu/(\nu - 2)} \), which is the standard deviation of a \( t \)-distributed random variable with \( \nu \) degrees of freedom.
of three types of random variables permits to compare the results obtained using a distribution with thin tails, for which all moments exists, with those arising from two distributions with fat tails, for which only the first 4 and 2 moments exist, respectively. The three types on random variables permit to simulate economies in which extreme random events are progressively more likely to occur.

For instance, the first percentile of a standard normal random variable is $-2.326$, while the same percentile for $t$-distributed random variables with 5 and 3 degrees of freedom and unit variance is equal to $-2.606$ and $-2.622$, respectively. The differences are even larger for lower quantiles. However, the fact that low quantiles are smaller for the $t$-distributed random variables than for the normal random variable is of minor importance in our framework, because also the default thresholds of the loans that are used in the simulations are modified accordingly.\textsuperscript{12}

To understand the role that common shocks have on the default risk of individual loans, the magnitude of the expected values of the three random variables conditional on the variables being smaller than their respective first percentiles is of fundamental importance. This conditional expected value is equal to $-2.665$ for a standard normal random variable and to $-3.449$ and $-4.043$ for $t$-distributed random variables with 5 and 3 degrees of freedom and unit variance, respectively. So, while the conditional expected value is about 15% smaller than the first percentile for the normal case, it is about 32% and 54% per cent smaller for the other two distributions. This means that, while for the normal case one can be rather confident that when the outcomes are lower than a given low threshold they are actually not much smaller than that threshold, this is not the case for the $t$-distributed random variables.\textsuperscript{13} The latter distributions can thus be used to mimic economies in which really bad things can happen once the outlook is ugly enough.

In the following numerical analysis there are some characteristics that we keep fixed. First, we assume that the bank owns a portfolio of 1,000 loans,\textsuperscript{14} with the same notional value and recovery rate (set equal to 40%).\textsuperscript{15} Second, the risk-free rate, which is used to discount the expected payoffs, is set equal to 4%.

\textsuperscript{12}See step 4 on page 60.

\textsuperscript{13}Appendix 3.B shows that, when the threshold goes to minus infinity, the ratio of the conditional expected value to the threshold goes to 1 for the normal case and to $\nu/(\nu - 1)$ for the Student’s $t$ case, where $\nu$ is the number of degrees of freedom.

\textsuperscript{14}Further increasing the number of loans does not lead to any appreciable difference, except for taking more computational time.

\textsuperscript{15}According to Ou et al. (2011, Exhibit 7), the average corporate debt recovery rate measured by post-default trading prices for senior unsecured bank loans has been equal to 39.9% over the period 1982–2010.
<table>
<thead>
<tr>
<th>Rating (Moody’s scale)</th>
<th>Rating (S&amp;P’s and Fitch’s scales)</th>
<th>Default rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>AAA</td>
<td>0.000%</td>
</tr>
<tr>
<td>Aa</td>
<td>AA</td>
<td>0.068%</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>0.092%</td>
</tr>
<tr>
<td>Baa</td>
<td>BBB</td>
<td>0.280%</td>
</tr>
<tr>
<td>Ba</td>
<td>BB</td>
<td>1.292%</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>3.781%</td>
</tr>
<tr>
<td>Caa</td>
<td>CCC</td>
<td>12.358%</td>
</tr>
<tr>
<td>Ca–C</td>
<td>CC–C</td>
<td>23.350%</td>
</tr>
</tbody>
</table>

### 3.3.1 The riskiness of the original loan portfolio

To analyze the risk properties of a loan portfolio, we use Eq. (3.2) to model the correlation between loan defaults. The risk of the portfolio is described through the VaR, which is the function

\[
\text{VaR}(x) = -F^{-1}(1 - x),
\]

where F is the cumulative distribution function of the portfolio returns, and x is the confidence level.

In a first exercise, we calculate the VaR of a loan portfolio for several levels of the individual default probability of the loans. For the moment, the individual default probability is assumed to be the same for all the loans, as in Krahnen and Wilde (2006). The values of the individual default probabilities that are used in our analyses correspond approximately to the average annual default probabilities of borrowers to whom the rating agency Moody’s assigned its credit ratings over the period 1920–2010 (see Table 3.2, taken from Exhibit 26 of Ou et al., 2011). We also assume that the loans have the same individual correlation coefficient, set equal to 15%. The individual correlation coefficient is a key parameter, as it determines the exposure to the common macroeconomic factor. The higher the individual correlation coefficient the higher the impact on the portfolio of a macroeconomic shock. A correlation coefficient equal to 15% is a conservative choice that nonetheless highlights several interesting results.\(^{16}\)

\(^{16}\)Under the assumption of normality for the risk factors, Krahnen and Wilde (2008) and Mann and Metz (2011) also use a correlation coefficient of 15%, while Krahnen and Wilde (2006) and Di Cesare (2009) use a value of 30%. Hull and White (2004) use correlations of 0, 0.3 and 0.6, for normal and Student’s t risk factors. For Student’s t copulas, Wanitjirattikal and Kiatsupaibul (2007) use values of 0.1, 0.5 and 0.9.
Figure 3.1 – Value-at-risk of a loan portfolio for several levels of the individual default probability of the loans

The figure reports the VaR of a loan portfolio as a function of the confidence level. The dependence among loans is modeled using Eq. (3.2), where the common and idiosyncratic factors have the distribution mentioned in the title of the panels. The VaR is measured as a percentage of the initial value of the portfolio. The legend shows the individual annual default probability of the loans and, between parenthesis, the corresponding credit rating according to Ou et al. (2011). Number of loans: 1,000; individual correlation coefficient: 15%; recovery rate: 40%; risk-free rate: 4%. Each loan pays an interest amount given by Eq. (3.4) if it does not default.
Panel A of Figure 3.1 reports the VaR, from 99.0% to 99.999% confidence levels, for the case in which both the macroeconomic and idiosyncratic shocks follow standard normal distributions. As one intuitively would expect, the VaR levels for the different default probabilities differ from each other at any confidence level. For instance, the VaR for Caa-rated loans is about 2 to 3 times larger than the VaR for B-rated loans between 99% and 99.95% confidence levels. It is worth noting that, in this framework, the VaR is always smaller than 40% at the highest confidence level, even in the worst scenario in which the portfolio is made of loans with the lowest rating and the highest level of individual default probability. The fact that the VaR is always significantly lower than the maximum attainable VaR (which is equal to 60% as the recovery rate is 40%) means that even in the worst case there is a significant share of loans that do not default.

The picture changes radically when distributions with fat tails are used. In this case, the VaR is much higher than in the case of normal returns at high confidence levels. Moreover, the VaR tends to converge to the highest admissible value of 60% at the highest confidence levels, independently on the rating of the individual loans (see Panels B and C of Figure 3.1). For instance, in Panel B the VaR at the 99.99% confidence level for A-rated loans is just 10%, but it jumps to 56.5% at the 99.999% confidence level. This result is due to the fact that, as mentioned above, for high confidence levels the expected outcomes are much larger when the distributions are fat-tailed. At high confidence levels, there is always a chance that the outcome of the common macroeconomic shock is so negative that even very good idiosyncratic returns are not enough to avoid the default of almost all of the borrowers.

It is also worth noting that the differences between thin- and fat-tailed distributions are only clearly visible at very high confidence levels. For instance, with the exception of the loans with the lowest rating, the VaR at the 99% confidence level is broadly the same for all of the three distributions that we are examining. The severe consequences of extreme macroeconomic shocks can thus be fully evaluated only when looking deeply in the tails of the loss distributions.

As is well known, in the Basel II framework each bank has to satisfy a capital requirement that provides a buffer against unexpected losses at a specific level of statistical confidence, set by regulators at 99.9% (see Basel Committee on Banking Supervision, 2004, 2005). The previous results show that reasonable statistical models can easily generate outcomes for which a VaR level that is considered acceptable at that confidence level can become much larger at slightly higher confidence levels.
Sect. 3.3 – Analysis of the numerical results

Panel A: Normal distribution

Panel B: Student’s t-distribution with 5 degrees of freedom

Panel C: Student’s t-distribution with 3 degrees of freedom

Figure 3.2 – Value-at-risk of a loan portfolio for several levels of the individual correlation coefficient of the loans

The figure reports the VaR of a loan portfolio as a function of the confidence level. The dependence among loans is modeled using Eq. (3.2), where the common and idiosyncratic factors have the distribution mentioned in the title of the panels. The VaR is measured as a percentage of the initial value of the portfolio. The legend shows the individual correlation coefficient of the loans. Number of loans: 1,000; individual annual default probability: 1.3%; recovery rate: 40%; risk-free rate: 4%. Each loan pays an interest amount given by Eq. (3.4) if it does not default.
In Figure 3.2, the individual correlation coefficient of the borrowers varies while the individual default probability is kept fixed at 1.3% (which is the average annual default rate for BB-rated borrowers). The BB rating is the median rating for which Moody’s provides the information on default rates, after excluding the AAA rating for which the average annual default probability is essentially null (see Table 3.2). The results of our simulations under this new framework are somewhat similar to those obtained when only the individual default probability is allowed to change. In that case, even a small default probability was enough to cause severe portfolio losses in the worst scenarios when $t$-distributed random variables were used. In this case, when the same fat-tailed distributions are used, even a very small (but not null) individual correlation coefficient is enough to determine very high default rates in the most extreme cases. Continuing with our interpretation of the model described by Eq. (3.2), macroeconomic shocks can be so large that any non-null exposure to them is enough to determine extremely high default rates in the worst scenarios. Once again, the differences between normal and $t$-distributed random variables are only apparent for confidence levels higher than 99%.

Notice that the VaR is always negative when the individual correlation coefficient is null, that is the portfolio always record a positive return for both normal and $t$-distributed random variables. This result is due to the fact that, in this case, the returns of the loans are independent of the return of the macro factor, and are thus independent of each other. Using the Gaussian approximation for binomial random variables, it can be calculated that the probability of having more than 30 defaults in a portfolio of 1,000 loans with individual default probability equal to 1.3% is almost null, and even in that case the return of the portfolio is positive (about 2.9%).

The model described by Eq. (3.2) with $t$-distributed random variables can be interpreted as a stylized version of an economy in which macroeconomic shocks happen rarely, but when they occur they can be extremely severe. Under these conditions, many borrowers tend to default together in spite of their low individual default probabilities and their low exposures to macroeconomic shocks.

These findings seem to mimic reasonably well what happened in the housing market in the United States in 2007–08. When house prices stopped to increase, the so-called subprime borrowers started to default because they had the lowest capacity to repay their loans (that is, they had the highest individual default probabilities). However, when house prices started to decrease, that is when a mild shock turned into a severe macroeconomic shock, higher-rated borrowers started to default as well, with dreadful consequences for the world economy (see Gorton, 2008, and Mishkin, 2010).
Sect. 3.3 – Analysis of the numerical results

Figure 3.3 – Value-at-risk of a loan portfolio, before and after the securitization, for several levels of the individual default probability of the new loans granted with the proceeds of the securitization.

The figure reports the VaR of a loan portfolio as a function of the confidence level, both before and after the securitization. The dependence among loans is modeled using Eq. (3.2), where the common and idiosyncratic factors have the distribution mentioned in the title of the panels. The VaR is measured as a percentage of the initial value of the portfolio. The legend reports between parentheses the individual annual default probability of the original loans (for the original portfolio) and new loans (for the new portfolios) granted with the proceeds of the securitization, after retaining the equity tranche. The legend also reports the credit rating corresponding to the quality of the loans according to Ou et al. (2011). Number of loans: 1,000; individual correlation coefficient: 30%; recovery rate: 40%; risk-free rate: 4%. Each loan pays an interest amount given by Eq. (3.4) if it does not default.
3.3.2 The riskiness of the loan portfolio after a securitization process

The same framework used before can be applied to study how the risk of incurring large losses for a bank changes when the bank decides to securitize part of its assets and reinvest the proceeds in new loans. According to market practices and to recent regulatory requirements, the bank is assumed to retain the equity tranche of the CDO that is used to securitize its loans. In a similar framework, Krahnen and Wilde (2006) show that the risk of the bank tends to increase when the proceeds of the securitization are reinvested in loans with the same characteristics of the loans the have been securitized. Di Cesare (2009) shows that the level of risk of the bank may decrease when the proceeds of the securitization are reinvested in loans with lower individual default probabilities or lower individual correlation coefficients. Both Krahnen and Wilde (2006) and Di Cesare (2009) use standard normal random variables for their analysis.

To sharpen the results of this section, we now assume that the individual correlation coefficient of the loans is higher than before (30%, instead of 15%). We also assume that the quality of the original loans is rather poor, with an expected annual default rate of 12.4% (corresponding to CCC-rated borrowers), while the new loans are granted to borrowers with better creditworthiness. Finally, we assume that the bank securitizes 50% of its loans. These assumptions are clearly rather extreme but they are useful to highlight that the following results hold even when a very large share of extremely bad loans is replaced with loans of much better quality.

Under the previous assumptions, Figure 3.3 reports the VaR as a function of the confidence level, for both the original portfolio and the new portfolios made of the loans that are not securitized, the equity tranche of the resulting CDO, and the new loans that are granted with the proceeds of the securitization.

Because the new loans are of much better quality than the original loans, Panel A of Figure 3.3 shows that, under the normality assumption, the level of risk of the bank is substantially reduced after the securitization, for any confidence level. However, based on what we said before, it should not be surprising now to see that the tail risk of the bank, at the highest confidence levels, remains broadly unchanged from before to after the securitization when extreme macroeconomic events are allowed to happen (see Panels B and C of Figure 3.3). When fat-tailed random variables are used, the securitization of bad loans and the reinvestment in high-quality loans can reduce the level of risk of the bank at high confidence levels more than for the normal case. However, this is not the case for the highest confidence levels. Indeed, extreme tail risk is unaffected by the securitization process.
Panel A: Normal distribution

Panel B: Student’s \( t \)-distribution with 5 degrees of freedom

Panel C: Student’s \( t \)-distribution with 3 degrees of freedom

Figure 3.4 – Value-at-risk of a loan portfolio, before and after the securitization, for several levels of the individual correlation coefficient of the new loans granted with the proceeds of the securitization

The figure reports the VaR of a loan portfolio as a function of the confidence level, both before and after the securitization. The dependence among loans is modeled using Eq. (3.2), where the common and idiosyncratic factors have the distribution mentioned in the title of the panels. The VaR is measured as a percentage of the initial value of the portfolio. The legend reports between parenthesis the individual correlation coefficient of the original loans (for the original portfolio) and new loans (for the new portfolios) granted with the proceeds of the securitization, after retaining the equity tranche. Number of loans: 1,000; individual annual default probability: 12.4%; recovery rate: 40%; risk-free rate: 4%. Each loan pays an interest amount given by Eq. (3.4) if it does not default.
Similar results also hold when the new loans are assumed to have the same individual default probability of the original loans (which is set equal to 12.4%) but have a different individual correlation coefficient. As long as the individual correlation coefficient of the new loans is positive, even if much lower than that of the original loans, the tail risk of the bank at the highest confidence levels is unaffected by the securitization process when the underlying risk factors have fat tails (see Panels B and C of Figure 3.4).

The only way to reduce the level of risk of the bank in this case is to reinvest in assets that are uncorrelated with the macroeconomic factor, or are negatively correlated with it. Both strategies allow for a greater diversification of the risk in the portfolio, so that a significant share of the loans does not default even at the highest confidence levels. Whether or not one or both of the two strategies is feasible for a bank is an interesting topic for empirical research.\footnote{Actually, the desired assets may not exist.}

### 3.3.3 Further analyses of the riskiness of the loan portfolio

One may wonder how much of the previous results is due to the fact that normal and \( t \)-distributed random variables are used or to the specific model that is used (in which the firm value is represented as a linear combination of common and idiosyncratic factors). In other words, our previous results were dependent on the fact that we used either Gaussian or double \( t \)-distribution copulas. In this section we use alternative copulas to model the dependence among loans and check whether the previous results still hold.

In particular, one may wonder what happens when one uses Gaussian and Student’s \( t \) copulas with different marginal distributions (for instance, a Gaussian copula with Student’s \( t \) marginals or a Student’s \( t \) copula with Gaussian marginals). It turns out that, in our framework, the choice of the marginal distributions is completely irrelevant, as only the functional form of the copula has an impact on the final results. This feature of the approach that we are using is due to the fact that the only thing that matters to establish whether a loan is in default or not is whether the asset return of the borrower is below the value \( \Psi^{-1}(p) \) or not (where \( \Psi \) is the marginal cumulative distribution function of the asset returns and \( p \) is the individual default probability). Hence, using a different \( \Psi \) is actually unimportant, as the individual probability of default is kept fixed and changing \( \Psi \) would only change the threshold under which the loan is deemed to default.
Figures 3.5-3.8 are equivalent to Figures 3.1-3.4 except for the fact that now Gaussian and Student’s $t$ copulas are used. The Gaussian cases in the new exercises are obtained using a different numerical approach (described in Appendix 3.A), but not surprisingly the results are the same as before (as shown by the comparison of the Panels A of Figures 3.5-3.8 with the Panels A of Figures 3.1-3.4, respectively).

The comparison of Panels B and C of Figures 3.5-3.6 with the corresponding panels of Figures 3.1-3.2 highlights several interesting differences:

- In Figures 3.5-3.6 the tail behavior of the VaR is much smoother than in Figures 3.1-3.2, where it tends to increase sharply at the highest confidence levels;

- The VaR is often lower in Figures 3.1-3.2 for several values of the confidence level, but it is always higher at the highest confidence levels. In Figures 3.5-3.6 the VaR often does not reach the highest admissible value (60%);

- In Figure 3.6 the VaR is positive also when the individual correlation coefficient is zero. This result is due to the fact that, as shown by Embrechts et al. (2002), the tail dependence of jointly $t$-distributed random variables is positive even when the variables are uncorrelated or negatively correlated (provided that $\rho > -1$). In the double $t$-distribution copula instead, zero correlation implies independence.

These results broadly hold true also when comparing Panels B and C of Figures 3.7-3.8 with the corresponding panels of Figures 3.3-3.4. It is worth noticing that, because of the tail dependence of jointly $t$-distributed random variables, in case of a securitization in which the proceeds are reinvested in loans which are uncorrelated or negatively correlated with the original loans, the diversification effect is much lower when Student’s $t$ copulas are used than when double $t$-distribution copulas are used. In the former cases, the VaR of the new portfolio is usually higher at the highest confidence levels.

Using different copulas to model the dependence among loans in a portfolio is thus particularly relevant at the highest confidence levels, which are probably those where the financial stability of an economic system has to be evaluated. Modeling correctly the dependence among loans is thus important not only for pricing credit derivatives (as shown by Wanitjirattikal and Kiatsupaibul, 2007, and Burtschell et al., 2009) but also for policy and risk management purposes.
Figure 3.5 – Value-at-risk of a loan portfolio for several levels of the individual annual default probability of the loans

The figure reports the VaR of a loan portfolio as a function of the confidence level. The dependence among loans is modeled using the copula mentioned in the title of the panels. The VaR is measured as a percentage of the initial value of the portfolio. The legend shows the individual annual default probability of the loans and, between parenthesis, the corresponding credit rating according to Ou et al. (2011). Number of loans: 1,000; individual correlation coefficient: 15%; recovery rate: 40%; risk-free rate: 4%. Each loan pays an interest amount given by Eq. (3.4) if it does not default.
Figure 3.6 – Value-at-risk of a loan portfolio for several levels of the individual correlation coefficient of the loans

The figure reports the VaR of a loan portfolio as a function of the confidence level. The dependence among loans is modeled using the copula mentioned in the title of the panels. The VaR is measured as a percentage of the initial value of the portfolio. The legend shows the individual correlation coefficient of the loans. Number of loans: 1,000; individual annual default probability: 1.3%; recovery rate: 40%; risk-free rate: 4%. Each loan pays an interest amount given by Eq. (3.4) if it does not default.
The figure reports the VaR of a loan portfolio as a function of the confidence level, both before and after the securitization. The dependence among loans is modeled using the copula mentioned in the title of the panels. The VaR is measured as a percentage of the initial value of the portfolio. The legend reports between parenthesis the individual annual default probability of the original loans (for the original portfolio) and new loans (for the new portfolios) granted with the proceeds of the securitization, after retaining the equity tranche. The legend also reports the credit rating corresponding to the quality of the loans according to Ou et al. (2011). Number of loans: 1,000; individual correlation coefficient: 30%; recovery rate: 40%; risk-free rate: 4%. Each loan pays an interest amount given by Eq. (3.4) if it does not default.
Figure 3.8 – Value-at-risk of a loan portfolio, before and after the securitization, for several levels of the individual correlation coefficient of the new loans granted with the proceeds of the securitization

The figure reports the VaR of a loan portfolio as a function of the confidence level, both before and after the securitization. The dependence among loans is modeled using the copula mentioned in the title of the panels. The VaR is measured as a percentage of the initial value of the portfolio. The legend reports between parenthesis the individual correlation coefficient of the original loans (for the original portfolio) and new loans (for the new portfolios) granted with the proceeds of the securitization, after retaining the equity tranche. Number of loans: 1,000; individual annual default probability: 12.4%; recovery rate: 40%; risk-free rate: 4%. Each loan pays an interest amount given by Eq. (3.4) if it does not default.
3.4 Conclusion

This chapter takes the view that painful economic events tend naturally to happen and studies the consequences that those events may have on the banking system. In doing so, bank assets are described by a standard one-factor model in which the underlying macroeconomic factor can face negative shocks, as described by the realization of normal or $t$-distributed random variables.

Using Monte Carlo simulations, we analyze what happens to the VaR of a bank that securitizes part of its loan portfolio when the economy is subject to extreme macroeconomic shocks. Since usually there is no way for individuals to be protected against those macroeconomic shocks, this chapter shows that the individual characteristics of the loans of the bank have only a very limited impact on the risk of the bank of incurring large losses. Moreover, the level of risk of the bank is only slightly influenced by the fact that the bank securitizes its loans and reinvest the proceeds of the securitization in new loans with different characteristics. We also show that the behaviour of the VaR of a bank at the highest confidence levels can change significantly when different kinds of copulas are used to model the dependence among loans.

Overall, the results of this chapter suggest that, in order to increase the resilience of the banking system to extreme macroeconomic shocks, banks should focus on how to increase their capital base to survive when a shock occurs. Looking for investment and risk management practices that are perceived as safer is usually of little help when the worst is coming.
3.A Appendix: A brief review of copulas

In this appendix we recall a few definitions and properties regarding copulas. For the sake of clarity, we limit our review to the bivariate case, although the extension to the multivariate case is usually straightforward. We also assume that the random variables have invertible distribution functions. For more details and technicalities about copulas and their applications in finance, see Jäckel (2002) and Cherubini et al. (2004).

A copula is the joint distribution function of two standard uniform random variables:

\[ C(u_1, u_2) = \mathbb{P}(U_1 \leq u_1, U_2 \leq u_2). \]  

(3.18)

Denoting with \( F \) the distribution function of the random variable \( X \), it is well known that \( F(X) = U \), or \( X = F^{-1}(U) \), where \( U \) is a standard uniform random variable:

\[ \mathbb{P}(F(X) \leq x) = \mathbb{P}(X \leq F^{-1}(x)) = F(F^{-1}(x)) = x. \]  

(3.19)

Using this result and the definition of a copula, it is easy to see that a copula computed at \( (F_1(x_1), F_2(x_2)) \) gives a joint distribution function at \( (x_1, x_2) \):

\[ C(F_1(x_1), F_2(x_2)) = \mathbb{P}(U_1 \leq F_1(x_1), U_2 \leq F_2(x_2)) \]  

(3.20)

\[ = \mathbb{P}(F_1^{-1}(U_1) \leq x_1, F_2^{-1}(U_2) \leq x_2) \]  

(3.21)

\[ = \mathbb{P}(X_1 \leq x_1, X_2 \leq x_2) \]  

(3.22)

\[ = F(x_1, x_2) \]  

(3.23)

Sklar (1959) shows that the converse also holds true. For every joint distribution function \( F \) with marginal distribution functions \( F_1 \) and \( F_2 \), there exists a copula \( C \) such that \( F(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \). Moreover, if the marginal distribution functions are continuous, then \( C \) is unique.

Elliptical copulas are commonly used in finance, and among them we have:

- The Gaussian copula

\[ C^{Ga}(x_1, x_2; \rho) = \Phi_\rho(\Phi^{-1}(x_1), \Phi^{-1}(x_2)), \]  

(3.24)

where \( \Phi_\rho(x_1, x_2) \) is the distribution function of a bi-dimensional standard normal random variable, with linear correlation coefficient \( \rho \), and \( \Phi \) is a standard normal distribution function;

- The Student’s \( t \) copula

\[ C^t(x_1, x_2; \rho, \nu) = t_\rho(t^{-1}(x_1; \nu), t^{-1}(x_2; \nu); \nu), \]  

(3.25)
where \( t_p(x_1, x_2; \rho, \nu) \) is the distribution function of a bi-dimensional Student’s \( t \) random variable, with linear correlation coefficient \( \rho \) and \( \nu \) degrees of freedom, and \( t \) is a Student’s \( t \) distribution function.

It is possible to simulate numerically a Gaussian copula\(^{18} \) by generating random variables \( \tilde{X}_1 \) and \( \tilde{X}_2 \) with a joint standard normal distribution with correlation \( \rho \) and then setting \( \tilde{U}_1 = \Phi(\tilde{X}_1) \) and \( \tilde{U}_2 = \Phi(\tilde{X}_2) \), where \( \Phi \) is a standard normal distribution function (see Jäckel, 2002, p. 46, for details). Actually, the joint distribution of \((\tilde{U}_1, \tilde{U}_2)\) is given by

\[
F(\tilde{u}_1, \tilde{u}_2) = \mathbb{P}(\tilde{U}_1 \leq \tilde{u}_1, \tilde{U}_2 \leq \tilde{u}_2) = \mathbb{P}(\Phi(\tilde{X}_1) \leq \tilde{u}_1, \Phi(\tilde{X}_2) \leq \tilde{u}_2) = \mathbb{P}(\tilde{X}_1 \leq \Phi^{-1}(\tilde{u}_1), \tilde{X}_2 \leq \Phi^{-1}(\tilde{u}_2)) = \Phi_\rho(\Phi^{-1}(\tilde{u}_1), \Phi^{-1}(\tilde{u}_2)) = C^{Ga}(\tilde{u}_1, \tilde{u}_2; \rho). \tag{3.26}
\]

It is also possible to simulate a Gaussian copula with arbitrary marginal distribution functions \( F_1 \) and \( F_2 \).\(^{19} \) In this case, one has to follow the same procedure as before and then set \( \tilde{Y}_1 = F_1^{-1}(\tilde{U}_1) \) and \( \tilde{Y}_2 = F_2^{-1}(\tilde{U}_2) \). In fact, the joint distribution of \((\tilde{Y}_1, \tilde{Y}_2)\) is given by

\[
F(\tilde{y}_1, \tilde{y}_2) = \mathbb{P}(\tilde{Y}_1 \leq \tilde{y}_1, \tilde{Y}_2 \leq \tilde{y}_2) = \mathbb{P}(F_1^{-1}(\tilde{U}_1) \leq \tilde{y}_1, F_2^{-1}(\tilde{U}_2) \leq \tilde{y}_2) = \mathbb{P}(F_1^{-1}(\Phi(\tilde{X}_1)) \leq \tilde{y}_1, F_2^{-1}(\Phi(\tilde{X}_2)) \leq \tilde{y}_2) = \mathbb{P}(\tilde{X}_1 \leq \Phi^{-1}(F_1(\tilde{y}_1)), \tilde{X}_2 \leq \Phi^{-1}(F_2(\tilde{y}_2))) = \Phi_\rho(\Phi^{-1}(F_1(\tilde{y}_1)), \Phi^{-1}(F_2(\tilde{y}_2))) = C^{Ga}(F_1(\tilde{y}_1), F_2(\tilde{y}_2); \rho). \tag{3.29}
\]

3.B Appendix: Some tail properties of normal and \( t \)-distributed random variables

It is possible to show that the limit of the ratio of the expected value of a standard normal random variable conditional on the random variable being greater than a given threshold \( x \) to the same threshold is equal to 1, for \( x \) going to infinity. The same ratio for a \( t \)-distributed random variable with \( \nu \) degrees of freedom is equal to \( \nu/(\nu - 1) \), as long as \( \nu > 1 \). Because of the symmetry of the normal and Student’s \( t \) distributions, of course the same results holds when the threshold goes to minus

\(^{18}\) A similar procedure holds for simulating a Student’s \( t \) copula.

\(^{19}\) Analogously, it is possible to simulate a Student’s \( t \) copula with arbitrary marginal distribution functions.
infinity and the expected values of the random variables are calculated conditional on the variables being smaller than the threshold.

In fact, the expected value of a standard normal random variable $X$, conditional on the variable being greater than $x$, is equal to $E[X|X \geq x] = \phi(x)/(1 - \Phi(x))$. As $x$ goes to infinity, it is possible to use the Laplace approximation $1 - \Phi(x) \simeq \phi(x)/x$, so that one has

$$E[X|X \geq x] = \phi(x)/(1 - \Phi(x)) \rightarrow \phi(x)/(\phi(x)/x) = x. \quad (3.35)$$

For the case of a $t$-distributed random variable $T$ with $\nu$ degrees of freedom, it is possible to use the approximation $P(T > t) \simeq At^{-\nu}$, for some constant $A$, when $t$ goes to infinity. Then,

$$E[T|T \geq t] \rightarrow -\int_t^\infty s dA s^{-\nu} At^{-\nu} = At^{1-\nu} \frac{\nu}{\nu-1} = t \frac{\nu}{\nu-1}. \quad (3.36)$$
Chapter 4

The Determinants of CDS Spread Changes

4.1 Introduction

Empirical work on the determinants of credit risk – the risk of default of borrowers – has traditionally looked at corporate spreads, that is spreads between corporate and government bonds. The common finding has been that only a small share of observed spreads can be actually considered as a compensation for the risk of default of borrowers, whereas the greatest share can be traced back to taxes and liquidity issues, and to the presence of other systematic risk factors (e.g., Elton et al., 2001; Amato and Remolona, 2005; Driessen, 2005).

Although the bond market has traditionally been regarded as the best place in which the creditworthiness of a borrower can be evaluated, in recent years there has been a huge development of financial instruments, called credit derivatives, that are specifically designed to make credit risk easily tradable. Among these innovative instruments, credit default swaps (CDSs) have proved particularly successful. Essentially, a CDS works like an insurance contract: the policy holder (i.e., the buyer of protection) pays a premium to the insurer (i.e., the seller of protection) in order to receive compensation if a particular event, called credit event, occurs. In the case of CDSs, this event usually includes bankruptcy, failure to pay, and restructuring. For sovereign issuers, repudiation and moratoria are considered as well. Generally speaking all these events are subsumed in the term “default”. The main difference between a CDS and an insurance contract is that, while in the case of insurance

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1 We use the terms “credit spreads” and “credit premiums” as synonyms of “corporate spreads” and “CDS spreads”. By “corporate spreads” or “bond spreads” we mean the differential between the yields of bonds issued by private companies and the yields of government bonds with similar characteristics. We also use the term “risk-free yields” as a synonym of “government bond yields”.

the policy holder gets a reimbursement only for the damages that he has actually suffered, with a CDS it is possible to buy or sell protection against credit events independently from the real exposure to the risk of default of the reference entity. A CDS can thus be used not only for hedging credit risk but also for taking pure speculative positions, as with forward and futures contracts. This aspect clearly distinguishes the CDS market from the bond market. In the bond market the short selling of credit risk requires the short selling of bonds, which is usually limited by the low level of liquidity of the repo market, especially for high-yield bonds, and by the short maturity of repo contracts. On the contrary, CDSs allow investors to short sell credit risk easily.

The CDS market has been growing dramatically during the last few years. According to the International Swap and Derivatives Association (ISDA), the notional value of outstanding CDSs has increased from 1 to 62 trillion USD between 2001 and 2007. The notional amount of outstanding CDSs was so large that it turned out to be a serious potential source of instability during the financial turmoil triggered by the subprime crisis in the United States because of the corresponding counterparty risks. In order to reduce these risks, CDS market dealers started entering multilateral netting of CDS contracts, thus reducing the notional amount of outstanding CDS to 39 trillion by the end of 2008 (European Central Bank, 2009b). According to Fitch Ratings (2006), CDSs represent more than a half of the whole credit derivatives market.\footnote{More detailed information about the CDS market can be found in European Central Bank, 2009a.}

Because of its particular features, the CDS market is potentially much more efficient than the bond market in signaling the creditworthiness of borrowers. The relationship between bond and CDS markets has been the subject of several academic papers. Blanco et al. (2005) find that CDS and bond markets reflect firm-specific characteristics equally in the long run, while in the short run CDS prices appear to be more efficient than bond spreads in the price discovery process. Zhu (2004) confirms that credit risk tends to be priced equally in the two markets in the long run and that the derivatives market seems to lead the cash market in anticipating rating events and in price adjustments. Other literature which analyzes the relationship between CDS spreads and credit ratings show that the CDS market is usually very effective in anticipating rating changes (Hull et al., 2004; Norden and Weber, 2004; Di Cesare, 2005).

The development of the CDS market has also attracted the interest of researchers to analyze whether factors that determine corporate spreads are also relevant for CDS spreads. As said before, there are differences between corporate spreads and
CDSs that make this question not obvious. The aim of this chapter is to contribute to this literature by analyzing how the global financial turmoil has changed the way that credit risk is priced in the CDS market. Most of existing literature use data from the early stage of the CDS market only, thus not taking into account more recent developments. To the best of our knowledge, only Annaert et al. (2009) and Raunig and Scheicher (2009) analyze the period following the onset of the current financial turmoil in July 2007, but they focus on the banking sector. Moreover, with the exception of Alexander and Kaeck (2008) and Pires et al. (2009), the issue of the non-linear relationship between CDS spreads and default factors is usually not taken into account.

In this chapter, we use a large data set of CDS quotes on US non-financial companies from January 2002 to March 2009. This data set allows us to analyze the effects of the global financial turmoil on the determinants of CDS spread changes. We also take care explicitly of the issue of non-linearity in our regressions by using a theoretical CDS spread calculated using the Merton model. In this respect, this chapter is similar in spirit to Byström (2006) which uses the CreditGrades model to calculate theoretical CDS spreads and compare these with empirically observed spreads for CDS indices (iTraxx) covering Europe. Byström (2006) finds that theoretical and empirical spread changes are significantly correlated. Given that the specific functional form provided by a theoretical model could misspecify the true relationship between CDS spreads and default factors, we also add in our regressions the default factors separately. We also include other factors that theory and empirical evidence found to be significant in explaining credit spreads.

The main results of this chapter are: (i) the inclusion of a theoretical CDS spread in the regressions improves the explanatory power of the fundamental variables. The extended model is able to explain 54% of the variations in CDS spreads in the pre-crisis period (from January 2002 to June 2007) and 51% in the crisis period (from July 2007 to March 2009), which is higher than most previous findings of studies on corporate bond and CDS spread changes; (ii) when the theoretical spread calculated from the Merton model is introduced in the regressions, the coefficient of equity volatility decreases significantly; that of leverage, on the contrary, maintains its usefulness in explaining CDS spread changes; (iii) the contribution of leverage to the explanation of CDS spread changes is much higher during the crisis, as investors appear to have become more aware of individual risk factors; at the same time, the impact of equity volatility substantially decreases, possibly because the large swings in implied volatility that have characterized the crisis period have made this indicator a poor proxy for long-term asset volatility; (iv) the overall capacity of the
model to explain CDS changes is almost the same before and during the turmoil, thus highlighting that the underlying risk factors identified by the literature as relevant for the pricing of the credit risk have maintained their explanatory power also in a period of remarkable stress for the CDS market; (v) during the crisis CDS spreads appear to have been moving increasingly together, driven by a common factor that our model was able to explain only in part.

This chapter is organized as follows. The following section surveys the theoretical and empirical literature on the determinants of credit spreads. Section 4.3 describes the model introduced by Merton (1974) and the way in which it can be used to estimate model-based CDS spreads. Section 4.4 summarizes the characteristics of the data set and describes the statistical methodology that we use to analyze the determinants of CDS spread movements. The results are reported in Section 4.5. Section 4.6 concludes.

4.2 A review of the literature on credit spreads

In this section we briefly review the main contributions to the literature on the determinants of credit spreads. Early research on this topic has focused on assessing how much of observed bond spreads can be explained by structural default factors, that is by factors that are suggested to be linked to credit risk by theoretical models of default. The results usually showed that bond spreads predicted by theoretical models were much lower than observed spreads. Thus, the academic research has turned its attention to testing for alternative hypotheses to investigate the origins of these unexplained extra yields.

4.2.1 Main determinants of credit spreads

Credit risk models are usually divided into structural and reduced-form models. Under the structural approach, the liabilities of a firm are seen as a contingent claim on the assets of the firm itself. Default occurs when the market value of the assets, which is modeled as a stochastic process, reaches some limit. The reduced-form approach, instead, postulates that default occurs randomly, due to one or more exogenous factors. These factors can occur with a probability, dubbed the “intensity”, which is modeled and calibrated using market data. Models belonging to this latter class are also called intensity-based models. Reduced-form models have been praised by practitioners because of their capacity to fit market data by construction. On the contrary, structural models have been usually seen by academics as better

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suited to analyzing the determinants of credit risk. The capacity of individual structural models to describe the actual behavior of credit risk is still a matter of debate (e.g., Anderson and Sundaresan, 2000; Eom et al., 2004; Tarashev, 2005; Ericsson et al., 2007). During the last few years there has been a new strand of literature that, departing from any specific model, aimed at measuring the explanatory power for observed credit spreads of the variables that structural models predict to be theoretically linked to credit risk.

The first structural model on credit risk was introduced by Merton (1974). Because of its importance, the Merton model is described in detail in the next section. For the moment it will suffice to say that in the Merton model a default occurs when the market value of a firm, which is assumed to be described by a random process, turns out to be below the face value of the outstanding debt at the maturity of the debt. In case of default, the shareholders give the assets of the firm up to the bondholders. Merton’s intuition was that a bond subject to credit risk can be seen as a combination of a long position in a risk-free bond and short position in a European put option that the bondholders sell to the shareholders. The strike price of this option is equal to the face value of the risky bond. In this setting, the price of the risky bond can be determined through standard option pricing methods. These methods link the value of the bond to the parameters of the stochastic process driving the firm value and to the level of outstanding debt. The prediction of the Merton model is that credit spreads are a function of the following variables:

- **risk-free interest rate**: the risk-free interest rate represents the drift of the process describing the value of the assets of the firm under the risk-neutral measure. Higher interest rates increase the future expected value of the assets, thus reducing credit spreads;

- **nominal outstanding amount of debt**: the nominal value of the debt represents the threshold at which default is triggered. A higher amount of debt makes default more likely so that higher credit spreads are expected;

- **firm value**: higher values for the assets of the firm make the regular payment of the debt more likely and credit spreads are expected to be lower;

- **asset volatility**: higher asset volatility increases the value of the put option granted to the shareholders, thus increasing the compensation required by the bondholders through higher credit spreads.

Merton model and other structural models aim at explaining bond spreads using a small number of factors such as those just described. However, there are other
factors, not linked to credit risk, that have been shown to have a non-negligible impact on bond spreads. These include:

- **taxes**, to the extent that credit instruments and government bonds are subject to different tax rates;
- **liquidity**, as risky credit instruments usually have lower volumes and higher transaction costs than government bonds;
- **supply and demand shocks**, which can affect both corporate and government bond markets;
- **systematic risk factors**, such as those prevailing in the equity and in the corporate bond markets (e.g., Elton et al., 2001).

During the last few years, a growing empirical literature on credit spreads has used the CDS market as the object of its research. Actually, working on CDS quotes has at least three advantages with respect to using bond spreads. First, the CDS market already quotes in terms of a “spread”, so that it is not necessary to introduce another market in the analysis, like that of government bonds, which can be subject to its own specific factors. Second, CDS contracts are rather standardized, while bonds are characterized by a high level of non-homogeneity in terms of coupons, maturities, outstanding amounts, embedded options, and so on. All these features make comparisons among bonds difficult. Finally, because of the nature of CDSs as derivatives contracts, CDS quotes should be less prone to supply and demand effects than the bond market. In fact, transactions in the bond market are often affected by the physical nature of the instruments.

### 4.2.2 Empirical studies

Given the relatively recent development of the CDS market, empirical work on the determinants of credit risk has traditionally looked at corporate spreads. A first group of empirical studies identifies factors that can explain why corporate spreads are much larger than what would be predicted by historical rates of default and recovery rates (the so-called “credit spread puzzle”). Elton et al. (2001) identify the main components of credit spreads in the expected default loss, a tax premium, and a risk premium for the systematic, and thus not diversifiable, part of the risk on corporate bonds. Following the same line of research, Driessen (2005) shows that risk premiums for liquidity and jump-to-default risks are also important components of bond spreads.
Amato and Remolona (2005) propose an alternative explanation for the “credit spread puzzle” arguing that, while previous studies mostly focused on the expected loss component of credit risk, extra spreads may actually compensate for non-diversifiable credit risk. Their argument is that the high degree of skewness in bond returns makes diversification more difficult. The size of the portfolio required to reduce the probability of extreme losses by diversification is actually very large, and thus difficult to attain in practice. Due to this difficulty, investors in corporate bonds would require additional premiums in the form of higher spreads.

A second group of studies on corporate spreads departs from trying to reconcile historical default losses with observed credit premiums and instead aims at explaining credit spreads in a purely statistical way by regressing observed spreads on factors that theoretical models suggest are relevant in determining both default and non-default components of credit spreads. The main advantage of this approach is that it allows the impact of any single fundamental factor on credit spreads to be estimated directly. Collin-Dufresne et al. (2001) initiate this strand of research. They show that factors that should represent the main explanatory variables for credit spreads, according to structural default models, actually explain only 25% of observed bond spread changes. They also show that the missing component is represented by a common risk factor which is independent of equity, swap, and Treasury returns. Collin-Dufresne et al. (2001) conclude that bond spread changes are mostly driven by supply and demand shocks which are specific to the corporate bond market and independent of both default and liquidity factors. Campbell and Taksler (2003) focus on the effect of equity volatility on corporate bond yields. They show that idiosyncratic firm-level volatility can explain as much cross-sectional variation in yields as can credit ratings.

In the same vein, Guazzarotti (2004) investigates the determinants of changes in individual credit spreads of non-financial European corporate bonds by looking at the relevance of both structural default factors (like leverage, asset volatility, and the level and slope of the risk-free yield curve) and of some non credit-risk factors (e.g., market liquidity and market risk). He finds that: (i) default risk factors account for less than 20% of total variation of credit spreads; (ii) liquidity risk and aggregate market risk factors, although significant, explain only an additional 10%; (iii) the remaining part of credit spread changes remains unexplained. More recently, Avramov et al. (2007) analyze the capacity of structural models to explain changes in corporate credit risk using a set of common factors and company-level fundamentals. They are able to explain more than 54% and 67% of the variation in credit spread changes for medium-grade and low-grade bonds, respectively, with no clearly
dominant latent factor left in the unexplained variation. Cremers et al. (2008) introduce measures of volatility and jump risks that are based on individual stock options. They show that implied volatilities of individual options contain useful information for credit spreads and improve on historical volatilities when explaining the cross-sectional and time-series variations in a panel of corporate bond spreads.

Turning to the empirical analysis on CDS spreads, one of the first papers on the topic is that by Aunon-Nerin et al. (2002). As the credit derivatives market was still in its infancy when that paper was written, the authors use a sample of CDSs which is rather small and with a predominance of financial companies. Moreover, the analysis is conducted on the levels of the variables thus leaving the door open to econometric problems related to the non-stationary nature of the data. Abid and Naifar (2006) perform an analysis on CDS spread levels as well. Greatrex (2008) digs deeper into the relevance of the econometric issues for the study of the determinants of CDS spreads and shows that variables commonly used in this analysis are usually non-stationary in levels but stationary in first differences. Running regressions on non-stationary variables could result in high values for the $R^2$ statistics but also in inefficient coefficient estimates, sub-optimal forecasts, and invalid significance tests (Granger and Newbold, 1974). Ericsson et al. (2009) use data for the period 1999–2002 and show that estimated coefficients of a limited number of theoretical determinants of default risk are consistent with the theory. Among these factors, volatility and leverage have substantial explanatory power. Moreover, a principal component analysis of spreads and residuals indicates limited evidence for a residual common factor, confirming that the variables can explain a significant amount of the variation in the data.

Zhang et al. (2005) focus on information arising from the equity market. They show that volatility and jump risk measures derived from the equity market using high frequency data, together with credit ratings, macroeconomic conditions, and firms’ balance sheet information, can explain up to 77% of the total variation of CDS spreads. However, their results are obtained using variables in levels and are subject to the same econometric problems mentioned above. More recently, a few papers have focussed on CDSs on banks and on the differences between the periods before and after the global financial crisis. Annaert et al. (2009) show that: (i) the determinants of changes in bank CDS spreads exhibit significant time variation; (ii) variables suggested by structural credit risk models are not significant in explaining bank CDS spread changes, either in the period prior to the crisis or in the crisis period itself; (iii) some of the variables used as proxies of the general economic conditions are significant, but the magnitude of the coefficient estimates
and their sign have changed over time. Raunig and Scheicher (2009) also find the crisis had an impact on the pricing of CDSs on banks. In particular, they show that the perception of bank credit risk by market participants approaches the level of the most risky non-bank companies.

4.3 Merton model

As already mentioned, Merton (1974) introduces the first model of credit risk based on the structural approach. Merton assumes that the whole debt of a firm is represented by a zero-coupon bond maturing at time $T$, with face value $D$. Moreover, the market value at time $t$ of the assets of a firm, denoted by $V_t$, follows a geometric Brownian motion given by

$$dV_t = rV_t dt + \sigma V_t dW_t,$$

with $t \in [0,T]$, where $r$ and $\sigma$ are constants representing, respectively, the risk-free interest rate and the volatility of the process. Merton assumes that a default occurs if the value of the firm at time $T$ is lower than $D$. In that case the ownership of the firm is transferred from the shareholders to the bondholders. In the framework introduced by Merton (1974), the bondholders of the firm can be seen as holding a long position in a risk-free zero-coupon bond and a short position in a European put option that they granted to the shareholders. The underlying asset of the option is represented by the assets of the firm, the strike price is equal to $D$ and the maturity is $T$. The spread between corporate and government yields is the compensation required by the bondholders for granting the put option to the shareholders.

Similarly, the equity value of the firm can be seen as the value of a European call on the assets of the firm, with strike price and maturity equal to $D$ and $T$, respectively. The equity value at time $T$ is thus given by $E_T = \max(0, V_T - D)$. Using standard option valuation tools (Black and Scholes, 1973), one obtains

$$E_t = V_t \Phi(d_1) - e^{-r(T-t)} D \Phi(d_2),$$

at time $t < T$, where

$$d_1 = \frac{\log(V_t/D) + (r + \sigma^2/2)(T-t)}{\sigma \sqrt{T-t}},$$
$$d_2 = d_1 - \sigma \sqrt{T-t},$$

and $\Phi(x)$ is the standard normal cumulative distribution function evaluated at $x$.

The value of the debt at time $t$ is thus equal to $D_t = V_t - E_t$. It is also possible to
recover the expected costs of default $C_t$ as the difference between the face value of the debt, discounted at the risk-free rate, and the actual value of the debt, that is

$$C_t = e^{-r(T-t)}D - D_t.$$  \hfill (4.4)

In this framework, in which the default can occur only at the maturity of the debt, the annual spread $s$ of a CDS with quarterly payments and $T$ years to maturity is such that it makes $C_0$ equal to the present value of the premiums paid by the CDS,

$$C_0 = D^2 \sum_{n=1}^{4T} \exp \left(-r \frac{n}{4} \right),$$

that is

$$s_t = \frac{4C_0 / D}{\sum_{n=1}^{4T} \exp \left(-r \frac{n}{4} \right)}.$$ \hfill (4.5)

In order to use the Merton model to estimate the CDS spread it is necessary to have four ingredients: the level of the risk-free interest rate, the face value of the total debt of the firm, the market value and the volatility of the assets of the firm. Although the last two variables are not directly observable, for listed firms it is nonetheless possible to observe the market value of equity and to estimate the historical volatility. When there are options written on the shares of the firm, it is also possible to use the volatility implied by stock option prices, which is a forward-looking measure of the equity volatility. The equity value and equity volatility can be used as proxies for the asset value and asset volatility. However, they can also be used to obtain more direct estimates of the asset value and asset volatility. In fact, using Ito’s lemma it can be shown that the dynamics of $E_t$ is given by a geometric Brownian motion with diffusion coefficient equal to $\sigma V_t \frac{dE_t}{dV_t}$. The term $\frac{dE_t}{dV_t}$ is the derivative of the equity with respect to the value of the assets (a quantity commonly called “the delta”), and is equal to $\Phi(d_1)$. Under the further assumption that the dynamics of the equity can be described by a geometric Brownian motion with diffusion coefficient $\sigma E \frac{dE_t}{dV_t}$, one has that

$$E_t = V_t \Phi(d_1) \frac{\sigma}{\sigma_E}.$$ \hfill (4.7)

Given the observable variables $r$, $D$, $E_t$, and $\sigma_E$, it is thus possible to use numerical methods to solve the system of non-linear equations (4.2) and (4.7) for the values of the two unknown variables $V_t$ and $\sigma$.

Figure 4.1 shows the theoretical spread $s$ on a 5-year CDS calculated using the Merton model as a function of the equity volatility (Panel A) for three different levels of the leverage (the ratio of total assets $A_t = D + E_t$ to equity $E_t$) and as a
function of the leverage (Panel B) for three different levels of equity volatility. The risk-free interest rate is assumed to be fixed at 4%. The figure shows clearly that the main driver of the CDS spread in the Merton model is the level of equity volatility. When the equity volatility falls below 30%, the CDS spread is almost negligible for any level of leverage. For instance, when $\sigma_E = 30\%$ the theoretical CDS spread is still below 10 basis points even when the debt is twice as large as the equity value (so that the leverage is equal to 300%). As the level of equity volatility increases, the theoretical CDS spread surges sharply and the differences among firms with different levels of leverage tend to decrease in relative terms (although the differences increase in absolute terms). Panel A shows that the relationship between the theoretical CDS

Figure 4.1 – Theoretical CDS spreads generated by the Merton model
Theoretical CDS spreads on the debt of firms with different leverage (Panel A) or equity volatility (Panel B) as a function of equity volatility or leverage. The spreads are calculated using the Merton model assuming that they are paid with a quarterly frequency, that the risk-free interest rate is constant and equal to 4%, and that the maturity of the debt is 5 years. Leverage is defined as the ratio of total assets (total liabilities plus market capitalization) to market capitalization. Equity volatility and leverage are in percentages; CDS spreads are in basis points.
spread and the equity volatility is non-linear. An even stronger non-linearity arises when the theoretical CDS spread is expressed as a function of the leverage. In fact, Panel B not only confirms the relevance of the equity volatility for the level of the theoretical spread but also shows a non-monotone relationship between the CDS spread and the leverage of the firm. This pattern arises from the fact that Eqs. (4.2) and (4.7) imply that higher levels of debt are associated with lower levels of asset volatility. This is because the higher the leverage the lower the weight that the equity volatility receives as an estimator of the volatility of the whole value of the firm. Hence, at some point the negative impact that the higher debt has on the credit risk of the firm starts to be more than compensated by the positive impact of a decreasing asset volatility.

The Merton model provides a nice tool for understanding how fundamental variables, such as leverage and asset volatility, affect credit spreads. However, there are several features of real financial markets that are not taken into account by the model and that have led to many other extensions.\(^5\) The three main drawbacks of the model are:

- **There is only one kind of debt securities.** In the Merton model only the case of a single zero-coupon bond is considered. However, most corporate bonds pay coupons and firms usually issue several kinds of bonds with different characteristics;

- **Default occurs only at the maturity of the debt and if and only if the value of the assets is below the face value of the debt.** In the real world default can occur at any time before the maturity of the debt if the firm fails to meet any kind of obligation, such as the payment of coupons. Moreover, bankruptcy is a process that usually involves several forms of cost which can result in models with default barriers which are different from the face value of the debt (e.g., Leland and Toft, 1996);

- **Volatility is assumed to be constant for all maturities.** Empirical work on option pricing shows that implied volatility is not constant, neither for different strikes nor for different maturities. This means that volatility implied by equity options, which have maturity usually up to one year, could be a biased estimator of the equity volatility over longer horizons.

All these caveats have to be borne in mind when evaluating the results of any empirical application of the Merton model like the one described in the next section.

\(^5\)See Cossin and Pirotte (2001) for several extensions of the Merton model.
4.4 Methodology

In this section we describe the data, the model, and the statistical methodology used in our empirical analysis.

4.4.1 Data description

We built the data set by selecting all CDS contracts in US dollars available from Bloomberg written on the senior debt of US non-financial firms and with maturity equal to 5 years (the maturity which is usually associated with the most liquid contracts). We used end-of-day mid quotes for CDS spreads. In order to have data on market capitalization, the sample is restricted to listed firms. For each company, we gathered from Bloomberg quarterly data on current liabilities, non-current liabilities, and cash and equivalents, for the period between January 2002 and March 2009. For each company, we collected daily data on market capitalization, stock returns, and implied volatility derived from equity options. We then selected the companies for which we were able to find all data for at least 3 years. The final sample is made of 167 companies. We used the zero-coupon curve calculated by Datastream on US government bonds as our risk-free interest rate curve. We also collected daily data on the average levels of credit spreads for AA and BBB-rated US industrial companies (the option-adjusted spreads for the Merrill Lynch corporate industrial indices for US companies with AA and BBB ratings), a broad US equity market index (the S&P Composite index), and a broad index of US market implied volatility (the VIX index).

In the regressions, we used monthly averages for CDS spreads and for all other daily financial indicators. Quarterly balance sheet data were linearly interpolated. In order to control for the quality of CDS data, we dropped daily observations whenever equal to the value of the previous day. We also dropped the monthly observations based on less than 5 daily data. The final data set, after dropping any firm-month observation with one or more missing values, includes 11,084 firm-month observations: 8,140 in the pre-crisis period (from January 2002 to June 2007) and 2,944 in the crisis period (from July 2007 to March 2009).

The historical behavior of a few of the variables used in the analysis is shown in Figure 4.2. The plot shows the huge increase of the average CDS spread after the onset of the financial crisis. It also makes apparent the negative relationships of CDS spreads with stock returns and government bond yields as well as the positive relationship with the level of volatility. All these facts are coherent with the theory underlying the Merton model. Some descriptive statistics of the data set are reported in Tables 4.1–4.3. As highlighted earlier, average CDS spreads increase
**Panel A: Average CDS spread and S&P Composite**

![Graph of CDS spread and S&P Composite over years 2002 to 2009]

**Panel B: Equity market volatility and risk-free interest rates**

![Graph of VIX and risk-free interest rate over years 2002 to 2009]

*Figure 4.2 – Time series of the average CDS spread and of other financial variables*

Monthly averages. The CDS spread is the average value of all CDS spreads included in the sample. The risk-free interest rate is the zero-coupon 5-year rate on US government bonds. The CDS spread is in basis points; the risk-free interest rate is in percentages.

... sharply in the crisis period, from 87 to 199 basis points (Table 4.1). They are much higher for firms in the Consumer cyclical and the Communication/Technology sectors (respectively, 137 and 93 basis points before the crisis and 392 and 200 basis points in the crisis period; Table 4.2). As expected, average CDS spreads increase with leverage (Table 4.3). The theoretical spreads derived from the Merton model are consistently much lower than the observed ones (the average values in the whole sample period are 71 and 116, respectively). However, it is noteworthy that theoretical spreads appear to replicate rather well variations in observed spreads through sectors and leverage classes. Leverage and implied volatility also rise drastically during the crisis. Leverage is highest in the Utilities and Consumer cyclical sectors, lowest in Communication/Technology. Volatility is highest in the Consumer cyclical sector and lowest in the Consumer non-cyclical and Utilities sectors.
Table 4.1 – Descriptive statistics of the data set
CDS spreads are end-of-day mid quotes. The theoretical spread is the spread calculated using the Merton model. Leverage is defined as the ratio of total assets (total liabilities plus market capitalization) to market capitalization. Volatility is the mean of implied volatilities on call and put stock options. CDS spreads and theoretical spreads are in basis points; leverage and volatility are in percentages; market capitalization is in millions of US dollars.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Standard deviation</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Whole period</strong> (January 2002 – March 2009)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDS spreads</td>
<td>116</td>
<td>54</td>
<td>221</td>
<td>6</td>
<td>7,995</td>
</tr>
<tr>
<td>Theoretical spreads</td>
<td>71</td>
<td>3</td>
<td>213</td>
<td>0</td>
<td>1,862</td>
</tr>
<tr>
<td>Leverage</td>
<td>219</td>
<td>175</td>
<td>190</td>
<td>94</td>
<td>6,044</td>
</tr>
<tr>
<td>Volatility</td>
<td>35</td>
<td>30</td>
<td>19</td>
<td>7</td>
<td>369</td>
</tr>
<tr>
<td>Market capitalization</td>
<td>25,939</td>
<td>13,616</td>
<td>39,549</td>
<td>58</td>
<td>420,623</td>
</tr>
<tr>
<td><strong>Pre-crisis period</strong> (January 2002 – June 2007)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDS spreads</td>
<td>87</td>
<td>47</td>
<td>122</td>
<td>6</td>
<td>2,811</td>
</tr>
<tr>
<td>Theoretical spreads</td>
<td>31</td>
<td>1</td>
<td>108</td>
<td>0</td>
<td>1,793</td>
</tr>
<tr>
<td>Leverage</td>
<td>214</td>
<td>174</td>
<td>162</td>
<td>94</td>
<td>4,420</td>
</tr>
<tr>
<td>Volatility</td>
<td>31</td>
<td>28</td>
<td>13</td>
<td>7</td>
<td>369</td>
</tr>
<tr>
<td>Market capitalization</td>
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<td>13,555</td>
<td>39,810</td>
<td>481</td>
<td>386,416</td>
</tr>
<tr>
<td><strong>Crisis period</strong> (July 2007 – March 2009)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDS spreads</td>
<td>199</td>
<td>89</td>
<td>366</td>
<td>10</td>
<td>7,995</td>
</tr>
<tr>
<td>Theoretical spreads</td>
<td>184</td>
<td>23</td>
<td>348</td>
<td>0</td>
<td>1,862</td>
</tr>
<tr>
<td>Leverage</td>
<td>234</td>
<td>179</td>
<td>252</td>
<td>104</td>
<td>6,044</td>
</tr>
<tr>
<td>Volatility</td>
<td>47</td>
<td>39</td>
<td>25</td>
<td>7</td>
<td>223</td>
</tr>
<tr>
<td>Market capitalization</td>
<td>25,906</td>
<td>14,086</td>
<td>38,824</td>
<td>58</td>
<td>420,623</td>
</tr>
</tbody>
</table>

4.4.2 Empirical models and testing methodology
As described in Section 4.3, the Merton model posits that credit spreads are a function of the asset value, asset volatility, face value and maturity of the debt and risk-free interest rate. As discussed in that section, the Merton model also implies that the relationship between CDS spreads and default factors is highly non-linear.

In order to test for the capacity of the Merton model to explain observed CDS spreads, we run the following univariate regression:

\[
s(i, t) = \beta_0 + \beta_1 \bar{s}(i, t) + \epsilon(i, t), \quad \text{(Model 1)}
\]

where \( i = 1, \ldots, N \) denotes a specific firm and \( t = 1, \ldots, T \) a specific time period. The variable \( \bar{s} \) is the theoretical spread on a 5-year CDS calculated using the Merton
Table 4.2 – Descriptive statistics of the firms: breakdown by sector
The table reports the mean values of the indicated variables, expect for the number of observations. CDS spreads are end-of-day mid quotes. The theoretical spread is the spread calculated using the Merton model. Leverage is defined as the ratio of total assets (total liabilities plus market capitalization) to market capitalization. Volatility is the mean of implied volatilities on call and put stock options. CDS spreads and theoretical spreads are in basis points; leverage and volatility are in percentages; market capitalization is in millions of US dollars.

<table>
<thead>
<tr>
<th>Leverage quartiles</th>
<th>Number of observations</th>
<th>CDS spreads</th>
<th>Theoretical spreads</th>
<th>Leverage</th>
<th>Volatility</th>
<th>Market capitalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry</td>
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<td>83</td>
<td>69</td>
<td>206</td>
<td>34</td>
<td>27,991</td>
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<tr>
<td>Communications/Technology</td>
<td>1,738</td>
<td>118</td>
<td>66</td>
<td>178</td>
<td>35</td>
<td>41,476</td>
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<tr>
<td>Basic materials/Energy</td>
<td>1,973</td>
<td>76</td>
<td>57</td>
<td>186</td>
<td>35</td>
<td>20,086</td>
</tr>
<tr>
<td>Consumer cyclical</td>
<td>2,752</td>
<td>204</td>
<td>127</td>
<td>279</td>
<td>42</td>
<td>18,746</td>
</tr>
<tr>
<td>Consumer non-cyclical</td>
<td>1,752</td>
<td>74</td>
<td>31</td>
<td>185</td>
<td>29</td>
<td>31,790</td>
</tr>
<tr>
<td>Utilities</td>
<td>885</td>
<td>88</td>
<td>24</td>
<td>285</td>
<td>27</td>
<td>14,656</td>
</tr>
<tr>
<td>Industry</td>
<td>1,416</td>
<td>65</td>
<td>36</td>
<td>202</td>
<td>30</td>
<td>28,025</td>
</tr>
<tr>
<td>Communications/Technology</td>
<td>1,328</td>
<td>93</td>
<td>42</td>
<td>170</td>
<td>32</td>
<td>41,661</td>
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<tr>
<td>Basic materials/Energy</td>
<td>1,432</td>
<td>57</td>
<td>14</td>
<td>186</td>
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<td>18,475</td>
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<tr>
<td>Consumer cyclical</td>
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<td>137</td>
<td>48</td>
<td>268</td>
<td>35</td>
<td>19,098</td>
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<tr>
<td>Consumer non-cyclical</td>
<td>1,285</td>
<td>61</td>
<td>11</td>
<td>177</td>
<td>26</td>
<td>32,529</td>
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<tr>
<td>Utilities</td>
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<td>80</td>
<td>18</td>
<td>292</td>
<td>25</td>
<td>14,212</td>
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<tr>
<td><strong>Crisis period</strong> (Jul. 2007 – Mar. 2009)</td>
<td>2,944</td>
<td>199</td>
<td>184</td>
<td>234</td>
<td>47</td>
<td>25,906</td>
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<td>Industry</td>
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<td>127</td>
<td>152</td>
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<tr>
<td>Communications/Technology</td>
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<td>200</td>
<td>144</td>
<td>205</td>
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<tr>
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<td>127</td>
<td>171</td>
<td>184</td>
<td>48</td>
<td>24,352</td>
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<td>Consumer cyclical</td>
<td>726</td>
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<td>309</td>
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<td>17,764</td>
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<tr>
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<td>467</td>
<td>110</td>
<td>87</td>
<td>208</td>
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<td>29,756</td>
</tr>
<tr>
<td>Utilities</td>
<td>232</td>
<td>111</td>
<td>41</td>
<td>266</td>
<td>32</td>
<td>15,904</td>
</tr>
</tbody>
</table>
Table 4.3 – Descriptive statistics of the firms: breakdown by leverage

The breakdown refers to the average leverage of the firms in the whole period. The table reports the mean values of the indicated variables, except for the number of observations. CDS spreads are end-of-day mid quotes. The theoretical spread is the spread calculated using the Merton model. Leverage is defined as the ratio of total assets (total liabilities plus market capitalization) to market capitalization. Volatility is the mean of implied volatilities on call and put stock options. CDS spreads and theoretical spreads are in basis points; leverage and volatility are in percentages; market capitalization is in millions of US dollars.

<table>
<thead>
<tr>
<th>Leverage quartiles</th>
<th>Number of observations</th>
<th>CDS spreads</th>
<th>Theoretical spreads</th>
<th>Leverage</th>
<th>Volatility</th>
<th>Market capitalization</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Whole period</strong> (Jan. 2002 – Mar. 2009)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>108–148</td>
<td>11,084</td>
<td>116</td>
<td>71</td>
<td>219</td>
<td>35</td>
<td>25,939</td>
</tr>
<tr>
<td>148–186</td>
<td>2,789</td>
<td>59</td>
<td>27</td>
<td>132</td>
<td>32</td>
<td>43,972</td>
</tr>
<tr>
<td>186–242</td>
<td>2,809</td>
<td>82</td>
<td>62</td>
<td>166</td>
<td>33</td>
<td>24,719</td>
</tr>
<tr>
<td>242–1,527</td>
<td>2,894</td>
<td>101</td>
<td>71</td>
<td>210</td>
<td>35</td>
<td>18,069</td>
</tr>
<tr>
<td>242–1,527</td>
<td>2,592</td>
<td>233</td>
<td>128</td>
<td>380</td>
<td>40</td>
<td>16,643</td>
</tr>
<tr>
<td>108–148</td>
<td>8,140</td>
<td>87</td>
<td>31</td>
<td>214</td>
<td>31</td>
<td>25,950</td>
</tr>
<tr>
<td>148–186</td>
<td>2,009</td>
<td>45</td>
<td>7</td>
<td>131</td>
<td>28</td>
<td>44,197</td>
</tr>
<tr>
<td>186–242</td>
<td>2,023</td>
<td>59</td>
<td>26</td>
<td>161</td>
<td>29</td>
<td>24,752</td>
</tr>
<tr>
<td>242–1,527</td>
<td>2,174</td>
<td>77</td>
<td>25</td>
<td>206</td>
<td>31</td>
<td>17,764</td>
</tr>
<tr>
<td>242–1,527</td>
<td>1,934</td>
<td>170</td>
<td>66</td>
<td>364</td>
<td>35</td>
<td>17,451</td>
</tr>
<tr>
<td><strong>Crisis period</strong> (Jul. 2007 – Mar. 2009)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>108–148</td>
<td>2,944</td>
<td>199</td>
<td>184</td>
<td>234</td>
<td>47</td>
<td>25,906</td>
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<tr>
<td>148–186</td>
<td>780</td>
<td>96</td>
<td>79</td>
<td>135</td>
<td>41</td>
<td>43,391</td>
</tr>
<tr>
<td>186–242</td>
<td>786</td>
<td>142</td>
<td>156</td>
<td>180</td>
<td>44</td>
<td>24,635</td>
</tr>
<tr>
<td>242–1,527</td>
<td>720</td>
<td>172</td>
<td>213</td>
<td>223</td>
<td>49</td>
<td>18,990</td>
</tr>
<tr>
<td>242–1,527</td>
<td>658</td>
<td>416</td>
<td>311</td>
<td>428</td>
<td>55</td>
<td>14,265</td>
</tr>
</tbody>
</table>
model with the following assumptions: the face value of the debt, whose maturity is assumed to be equal to 5 years, is equal to the total of balance sheet liabilities net of cash; the value of the equity is equal to the market capitalization of the firm; the equity volatility is equal to the mean of implied volatilities calculated from call and put options; the risk-free interest rate is given by the 5-year zero-coupon rate on US government bonds.

Model 1 may not be able to fully describe the relationship between observed CDS spreads and default factors because of its simplifying assumptions. So, we also estimate the following three-factor linear model which has been used in other empirical works on the determinants of CDS spreads (e.g., Ericsson et al., 2009):

\[ s(i, t) = \beta_0 + \beta_1 \sigma_E(i, t) + \beta_2 L(i, t) + \beta_3 r(t) + \epsilon(i, t). \]  

(Model 2)

As for the explanatory variables, $\sigma_E$ is the implied volatility calculated as the mean of implied volatilities derived from call and put options, $L$ is the leverage calculated as the ratio of total assets (current and non-current liabilities, net of cash and equivalents, plus market capitalization) to market capitalization, and $r$ is the 5-year zero-coupon rate on US government bonds. We opted to use implied volatility as a proxy of equity volatility to avoid the backward-looking nature of the historical equity volatility.

If the non-linearities implied by the Merton model had some additional explanatory power, one would expect that the portion of total variation explained by the model increases and the linear terms lose significance when the theoretical spread is added to the linear model. So, we also estimate a third model which is a combination of the previous two models:

\[ s(i, t) = \beta_0 + \beta_1 \bar{s}(i, t) + \beta_2 \sigma_E(i, t) + \beta_3 L(i, t) + \beta_4 r(i, t) + \epsilon(i, t). \]  

(Model 3)

Finally, we estimate an extended model in which we include other variables usually used in the literature on the determinants of credit spreads. These variables are the (log) stock values of the firms, the slope of the yield curve (the difference between the 10-year and the 1-year zero-coupon rates on US government bonds), an index of the premium required by investors to hold riskier assets (the difference between the average OASs of the Merrill Lynch indices for US industrial companies with BBB and AA ratings), the (log) value of a broad equity index (the S&P Composite index), and an index of market uncertainty (the VIX index). Thus, the specification of the fourth model is:

\[ s(i, t) = \beta_0 + \beta_1 \bar{s}(i, t) + \beta_2 \sigma_E(i, t) + \beta_3 L(i, t) + \beta_4 r(i, t) + \beta_5 \text{STOCK}(t) + \beta_6 \text{SLOPE}(t) + \beta_7 \text{OAS}(t) \]

\[ + \beta_8 \text{SPCI}(t) + \beta_9 \text{VIX}(t) + \epsilon(i, t). \]  

(Model 4)
The additional variables have the following expected signs:

- **STOCK**: a negative sign is expected, as higher stock values should signal higher future profitability and higher capacity of the firms to meet their obligations;

- **SLOPE**: the sign is uncertain. On the one hand, higher values for the slope should predict higher future risk-free rates, which should have a negative impact on CDS spreads. On the other hand, the increase in expected future interest rates may reduce the number of profitable projects available to the company and, in turn, increase credit spreads. Moreover, it has to be pointed out that we are already including a 5-year interest rate in the regressions. In this setting, a higher level for the slope is more likely associated, *ceteris paribus*, with a lower level for the short-term interest rate which is usually associated with worsening economic conditions and higher credit spreads;

- **OAS**: a positive sign is expected, as the higher the premium required by investors to hold riskier securities the higher should be the compensation required to hold credit risk;

- **SPCI**: the sign is uncertain. On the one hand a broad market increase of equity values should signal better economic conditions and a lower probability of default for the companies, and have a negative impact on CDS spreads. On the other hand, in our regressions we have already included individual stock returns that should take into account firm-specific expectations about profitability much better than broad market measures. In our setting, a broad market increase of equity values signals, *ceteris paribus*, a relatively bad performance of individual firms, so that a positive effect on CDS spreads can be expected;

- **VIX**: a positive sign is expected, as the higher the uncertainty in the market the higher the value of the put option that the bondholders implicitly sell to the shareholders when buying credit risk.

The four models are estimated in first-differences by running pooled OLS regressions with standard errors that allow for time correlation at firm level. Models in first-differences are preferred to those in levels in order to isolate unobserved individual factors which do not vary over time and account for the possible problem of non-stationarity of the processes for CDS spreads.\(^6\) As a robustness check, Model 4 is also estimated with a panel regression with fixed effects.\(^7\)

\(^6\)We found that the augmented Dickey-Fuller test, run separately for each CDS time series, does not reject the null hypothesis of the presence of a unit-root in almost all cases.

\(^7\)The test by Hausman (1978) implies that the model with random effects cannot be rejected; a panel data analysis with random effects did not result in appreciable differences.
We compare the results across groups of firms defined according to the level of leverage or the economic sector. Generally, we abstract from factors not linked to credit risk which the literature has documented are significant in the corporate bond market (such as taxes, liquidity and supply and demand shocks) on the assumption that for the CDS market they are less relevant. However, we recognize that the lack of liquidity in the CDS market following the onset of the crisis could have an impact on our estimates and so we try to control for that by using the CDS bid-ask spread change as an instrument to distinguish between contracts that were hit by different liquidity shocks during the crisis.\(^8\) We were not able to estimate directly the effect of liquidity on spreads as we found that changes in both CDS spreads (defined as the average between bid and ask prices) and bid-ask spreads were mainly determined by changes in ask quotes. So, changes in CDS spreads and bid-ask spreads were often extremely correlated and resulted in a misspecified model. For this reason, we prefer to run a set of regressions for subgroups of contracts defined according to the size of the change in the average bid-ask spread on CDS contracts from before to after the onset of the crisis.

Finally, we perform a principal component analysis on CDS spread changes and regression residuals from Model 4 in order to assess to what extent the model reduces the weight of the main principal components of spread variations. This analysis is conducted on a balanced sub-sample of 34 firms with complete monthly data from January 2003 to December 2008.

\section*{4.5 Results}

In this section we report the results of our estimates. To have a comparison with the previous literature, which was mainly concentrated in the pre-crisis period, we first discuss the results for the four models presented in Section 4.4 in the pre-crisis period (January 2002 to June 2007) and then we assess how those results have changed after the onset of the crisis (July 2007 to March 2009).

\subsection*{4.5.1 The pre-crisis period}

As shown in Table 4.4, by regressing the observed CDS spreads on the theoretical spreads obtained using the Merton model for the period before the onset of the crisis we find that, as expected, the coefficient of the theoretical spread is positive (0.32) and highly significant. Moreover, the explanatory power of the model, measured by the adjusted \(R^2\) statistics, is comparable to previous studies on the determinants

\(^8\)For a study on the determinants of CDS bid-ask spreads, see Meng and ap Gwilym (2008).
Table 4.4 – The determinants of CDS spread changes
The values in the table are obtained by running a pooled OLS regression for all observations in the selected period, with standard errors that allow for time correlation at firm level. Monthly data from January 2002 to March 2009; the pre-crisis period goes from January 2002 to June 2007; the post-crisis period from July 2007 to March 2009. The explanatory variables are changes in the monthly averages of: the estimated theoretical spread for firm $i$ at time $t$; the implied volatility of options written on the stocks of firm $i$ at time $t$; the leverage ratio of firm $i$ at time $t$; the 5-year zero-coupon interest rate on the US government bond at time $t$; the log of the stock value of firm $i$ at time $t$; the slope of the zero-coupon curve on US government bonds (10-1 yrs) at time $t$; the Merrill Lynch industrial bond average spread (BBB-AA) at time $t$; the log of the S&P Composite stock index at time $t$; the VIX volatility index at time $t$; a constant term. Observed and theoretical spreads and the corporate spread are in basis points, all other variables are in percentages. Significance levels: $\texttt{***} = 1\%; \texttt{**} = 5\%; \texttt{*} = 10\%$.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th></th>
<th>Model 2</th>
<th></th>
<th>Model 3</th>
<th></th>
<th>Model 4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical spread</td>
<td>0.32***</td>
<td>0.20***</td>
<td>-0.12***</td>
<td>0.18***</td>
<td>0.00</td>
<td>-0.18</td>
<td>0.16***</td>
<td>-0.03</td>
</tr>
<tr>
<td>Volatility</td>
<td>2.72***</td>
<td>1.29***</td>
<td>-1.43***</td>
<td>1.22</td>
<td>1.25</td>
<td>0.03</td>
<td>0.99</td>
<td>1.67</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.35***</td>
<td>0.92***</td>
<td>0.57***</td>
<td>0.37***</td>
<td>0.92***</td>
<td>0.55***</td>
<td>0.35***</td>
<td>0.91***</td>
</tr>
<tr>
<td>Interest rate</td>
<td>-3.56***</td>
<td>-41.21***</td>
<td>-37.65***</td>
<td>-2.49***</td>
<td>-41.26***</td>
<td>-38.78***</td>
<td>-3.19***</td>
<td>-15.61</td>
</tr>
<tr>
<td>Stock return</td>
<td>-0.42***</td>
<td>-0.17</td>
<td>0.26</td>
<td>3.29*</td>
<td>9.44**</td>
<td>6.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope yield curve</td>
<td>0.56***</td>
<td>0.65***</td>
<td>0.09</td>
<td>1.01***</td>
<td>1.01</td>
<td>0.00</td>
<td>0.27</td>
<td>-0.53</td>
</tr>
<tr>
<td>Corporate spread</td>
<td>0.27</td>
<td>-0.53</td>
<td>-0.79</td>
<td>0.44*</td>
<td>-3.10**</td>
<td>-3.54***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VIX</td>
<td>-0.64**</td>
<td>14.26***</td>
<td>14.9***</td>
<td>-0.13</td>
<td>-1.17</td>
<td>-1.04</td>
<td>-0.35</td>
<td>-1.16</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.25</td>
<td>0.07</td>
<td>0.48</td>
<td>0.50</td>
<td>0.52</td>
<td>0.50</td>
<td>0.54</td>
<td>0.51</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.25</td>
<td>0.07</td>
<td>0.48</td>
<td>0.50</td>
<td>0.52</td>
<td>0.50</td>
<td>0.54</td>
<td>0.51</td>
</tr>
<tr>
<td>No. obs.</td>
<td>8,140</td>
<td>2,944</td>
<td>8,140</td>
<td>2,944</td>
<td>8,140</td>
<td>2,944</td>
<td>8,140</td>
<td>2,944</td>
</tr>
</tbody>
</table>
of CDS spread changes (e.g., Ericsson et al., 2009). However, the coefficient is significantly different from the value of one that is predicted by theory.

Turning to Model 2, all the estimated coefficients have the expected signs and are highly significant. In particular, a one percentage point increase in the levels of volatility and leverage raises the average CDS spread by 2.72 and 0.35 basis points, respectively. A similar increase in the interest rate level determines a drop of 3.56 basis points for the CDS spread. The adjusted $R^2$ statistics is significantly higher than in Model 1; this aspect suggests that the Merton model constrains the effects of the single factors on CDS spreads in ways that are not fully consistent with market data.

In Model 3, where the theoretical spread is added to the more traditional three-factors model, the coefficients of the theoretical spread, volatility and interest rate are all lower in absolute terms when compared with the previous models. Moreover, the coefficient of volatility shows a large loss of significance, which is probably due to the high level of correlation with the theoretical spread (70 per cent). The linear contribution of leverage, on the contrary, remains basically unchanged and highly significant; this could be due to the fact that the Merton model is not very sensitive to changes in leverage (see Figure 4.1). The adjusted $R^2$ statistics increases further from 0.48 to 0.52, highlighting the positive contribution to the model of the theoretical spread, which should take into account the non-linearities embedded in the pricing of CDSs.

In Model 4, the other explanatory variables that the literature has indicated as important determinants of credit spreads (the individual stock returns, the slope of the yield curve, an indicator of risk aversion in the bond market, and a broad market stock index return) have the expected signs and all but the indicator of broad market uncertainty (the VIX index) have significant effects on CDS spreads. 9 However, their overall contribution to the explanatory power of the model is marginal, as the adjusted $R^2$ statistic only rises from 0.52 to 0.54.

Table 4.5 shows that the results obtained running a panel regression with fixed effects confirm those obtained with the pooled OLS regression, both in terms of absolute values and the significance of the coefficients. We also checked that the results are robust to the filtering procedures applied to the original data such as the dropping of stale quotations and of observations referring to months with few CDS quotes. We also verified that the results remain basically unchanged when the analysis is limited to a balanced data set including only a subset of companies with complete data in the period from 2003 to 2008.

9The coefficients of SLOPE and SPCI, whose signs are theoretically uncertain, are positive.
Table 4.5 – The determinants of CDS spread changes: pooled and panel regressions
The table compares the results from pooled OLS and panel fixed-effect regressions run on the same sample. Standard errors allow for time correlation at firm level. Monthly data from January 2002 to March 2009; the pre-crisis period goes from January 2002 to June 2007; the post-crisis period from July 2007 to March 2009. The explanatory variables are changes in the monthly averages of: the estimated theoretical spread for firm i at time t; the implied volatility of options written on the stocks of firm i at time t; the leverage ratio of firm i at time t; the 5-year zero-coupon interest rate on the US government bond at time t; the log of the stock value of firm i at time t; the slope of the zero-coupon curve on US government bonds (10-1 yrs) at time t; the Merrill Lynch industrial bond average spread (BBB-AA) at time t; the log of the S&P Composite stock index at time t; a constant term. Observed and theoretical spreads and the corporate spread are in basis points, all other variables are in percentages. Significance levels: *** = 1%; ** = 5%; * = 10%.

<table>
<thead>
<tr>
<th></th>
<th>Pre-crisis</th>
<th>Crisis</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coeff.</td>
<td>t-stat.</td>
<td>Coeff.</td>
</tr>
<tr>
<td>Theoretical spread</td>
<td>0.16***</td>
<td>2.63</td>
<td>-0.03</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.99</td>
<td>1.25</td>
<td>1.67</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.35***</td>
<td>9.65</td>
<td>0.91***</td>
</tr>
<tr>
<td>Interest rate</td>
<td>-3.19**</td>
<td>-2.52</td>
<td>-15.61</td>
</tr>
<tr>
<td>Stock return</td>
<td>-0.42***</td>
<td>-3.57</td>
<td>-0.17</td>
</tr>
<tr>
<td>Slope yield curve</td>
<td>3.29*</td>
<td>1.84</td>
<td>9.44***</td>
</tr>
<tr>
<td>Corporate spreads</td>
<td>0.56***</td>
<td>8.22</td>
<td>0.65***</td>
</tr>
<tr>
<td>S&amp;P Composite</td>
<td>1.01***</td>
<td>4.27</td>
<td>1.01</td>
</tr>
<tr>
<td>VIX</td>
<td>0.27</td>
<td>0.65</td>
<td>-0.53</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.44*</td>
<td>1.76</td>
<td>-3.10**</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.54</td>
<td></td>
<td>0.51</td>
</tr>
</tbody>
</table>

Pooled OLS regression

Panel fixed effect regression
Overall, our results suggest that factors predicted by the theory as being relevant for credit spreads have indeed a good explanatory power before the onset of the crisis. Moreover, the theoretical spread calculated using the Merton model does convey specific information on credit risk that cannot be captured by the linear models, supporting the hypothesis that non-linearities are important in explaining credit spreads.

4.5.2 The crisis period

The financial crisis has disrupted credit markets, causing spreads and volatility to surge and market liquidity to evaporate (e.g., International Monetary Fund, 2008a). The impact of these events on the pricing of credit risk is a matter of debate. On the one hand, one could expect that the crisis, by amplifying the importance of systemic risk factors, has caused a generalized increase of spreads (almost) independently from the fundamental characteristics of firms. On the other hand, the crisis could have exacerbated the differences between firms belonging to different classes of risk, as investors have become more aware of idiosyncratic risk factors.

In order to assess the impact of the financial crisis on the pricing of risk, we compare the results in the pre-crisis period with those obtained during the crisis period. Table 4.4 reports the results of the regressions for the four models in both periods, as well as the tests on the differences between the two periods.

As for Model 1, the striking fact is the drop in both the coefficient of the theoretical spread and the explanatory power of the model, from 0.32 and 25% before the start of the crisis, to 0.20 and just 7%, respectively. This indicates that during the crisis the empirical relationship between CDS spreads and default factors is no longer described by the specific functional form of the Merton model.

This interpretation is confirmed by looking at the results from Model 2, where the default factors are included in the regression separately and the functional form between the default factors and the CDS spreads is not determined by the theoretical model. In this case, we actually find a moderate increase in the explanatory power of the model from before to during the crisis. Moreover, all the coefficients maintain the expected signs and are highly significant. However, the impact of equity volatility on CDS spreads more than halves, suggesting that the wide swings in implied volatility, which have characterized the crisis period, have probably made this indicator a poor proxy for long-term asset volatility. This fact could also explain the loss of importance in Model 1 of the theoretical spread, which is mostly driven by volatility changes (see Figure 4.1). At the same time, during the crisis spreads have become much more sensitive to changes in leverage (from 0.35 to 0.92) and interest rates.
(from $-3.56$ to $-41.21$). On the one hand, the increased importance of leverage may reflect a greater awareness by investors of firm-specific characteristics. On the other hand, the increased relevance of interest rates probably reflects the fact that during the crisis risk-free interest rates have been interpreted as better proxies of economic activity: lower interest rates should signal worse economic conditions and higher credit risks.

Changes in the results of Model 3 from the pre-crisis period to the crisis period are rather similar to those of Model 2, given the negligible impact of the theoretical spread during the crisis.

In Model 4, the interest rate coefficient also loses its significance during the crisis, possibly in favor of the slope of the yield curve. Given the negative relationship between the slope of the yield curve and short-term interest rates, for a given level of longer term yields – for which our regressions already control for – the positive coefficient on the slope of the yield curve seems to indicate that the CDS market has been looking at short-term interest rates as a better indicator of economic activity than longer-term interest rates. Lower short-term rates (and a higher slope) are associated with worsening economic conditions and greater CDS spreads.

Overall, during the crisis the proportion of explained variations decreases only slightly, from 54% to 51% in Model 4, confirming that the model works rather well also in this case. This result highlights that the underlying risk factors identified by the literature as relevant for the pricing of credit risk have maintained their explanatory power also in a period of remarkable stress for the CDS market. In this regard, the CDS market appears to have continued to price credit risk in much the same fashion as it did before the crisis.

Table 4.5 shows that the estimates for the crisis period are confirmed by a panel regression with fixed effects.

Finally, we perform a principal component analysis on CDS spread changes and regression residuals from Model 4. The analysis shows that during the crisis CDS spreads appear to have been moving increasingly together, as reported in Figure 4.3. The fraction of CDS variations explained by the first component increases from 45% to 62% during the crisis period. When the analysis is repeated using the residuals of Model 4, the two values drop to 25% and 41%, respectively, thus confirming the ability of the model to capture a substantial part of the common factors underlying the spread changes. This result suggests that spread changes during the crisis are increasingly driven by common or systematic factors and less by firm-specific factors. In order to take into account time-varying factors, we repeated the analysis including monthly dummy variables, but the findings were mostly unchanged. Given that

Panel B: Crisis period: July 2007 – December 2008

Figure 4.3 – Principal component analysis

The principal component analysis has been conducted on a balanced sub-sample of 34 firms with complete data from January 2003 to December 2008 for the CDS changes and the regression residuals of Model 4. The explained variations by the first 5 components are in percentages.

general indicators of economic activity, uncertainty, and risk aversion are already included in our model, these results seem to point to the presence of a market-specific factor that hit CDSs during the crisis in forms that are not fully reflected in other markets. A large part of CDS spread variations during the crisis is thus still to be explained.

4.5.3 Further analyses

In order to dig deeper into previous results, we repeat the analysis based on Model 4 on sub-groups of firms defined according to the level of leverage, the economic sector, and the size of the change in the average bid-ask spreads for CDSs from before to during the crisis.
Analysis by leverage quartiles

Table 4.6 reports the results for an analysis based on the average level of leverage of the firms. First of all, we note that the explanatory power of the model is highest for the companies with leverage in the highest quartile (0.63 in the pre-crisis period) and lowest for the companies in the lowest quartile (0.25). As the two groups are also associated with the highest and lowest levels of CDS spreads, these results confirm previous findings that structural models can better explain the credit spreads for firms with relatively lower credit quality (e.g., Collin-Dufresne et al., 2001, for an analysis on corporate spreads, and Greatrex, 2008, for comparable results on CDSs).

We also note that the explanatory power of the model increases substantially during the crisis for firms with low leverage, to 0.35. The increased explanatory power of the model for firms with low levels of leverage does not derive from factors related to firm-specific characteristics (such as leverage and volatility), but from market-wide factors such as the interest rate and the market price of risk (as captured by the significant increase of the coefficient of the corporate bond spread). For firms with the highest levels of leverage, the leverage itself is the variable whose coefficient increased more significantly during the crisis, pointing to the fact the investors became more concerned about the particular weaknesses of the balance sheets of those firms.

Analysis by economic sector

Table 4.7 reports the results for the sectoral analysis. We notice that, across sectors, the model explains the highest proportion of variation for companies in the Utilities and Consumer Cyclical sectors, which are also the ones with the highest levels of leverage. During the crisis model performance sharply increases for firms in the Basic materials/Energy and Consumer non cyclical sectors, which are characterized by relatively low levels of leverage, volatility, and CDS spreads, probably reflecting the fact that these firms have been perceived as relatively riskier than before the crisis. Actually, for these sectors the theoretical spreads calculated using the Merton model show the highest levels of relative increase from before to after the onset of the crisis (see Table 4.2). Overall, this is further evidence of the capacity of the model to price CDSs of riskier companies better.

Analysis by liquidity change

In order to check whether the change of liquidity in the CDS market had any impact on the capacity of the model to explain CDS spread variations, we repeat our analysis on the basis of the bid-ask spread on CDS contracts. In particular, we group
Table 4.6 – The determinants of CDS spread changes by leverage

The values in the table are obtained by running a pooled OLS regression for all observations in the selected period and quartile of the firms’ average leverage, with standard errors that allow for time correlation at firm level. Monthly data from January 2002 to March 2009; the pre-crisis period goes from January 2002 to June 2007; the post-crisis period from July 2007 to March 2009. The explanatory variables are changes in the monthly averages of: the estimated theoretical spread for firm $i$ at time $t$; the implied volatility of options written on the stocks of firm $i$ at time $t$; the leverage ratio of firm $i$ at time $t$; the 5-year zero-coupon interest rate on the US government bond at time $t$; the log of the stock value of firm $i$ at time $t$; the slope of the zero-coupon curve on the US government bonds (10-1 yrs) at time $t$; the Merrill Lynch industrial bond average spread (BBB-AA) at time $t$; the log of the S&P Composite stock index at time $t$; the VIX volatility index at time $t$; a constant term. Observed and theoretical spreads and the corporate spread are in basis points, all other variables are in percentages. Significance levels: *** = 1%; ** = 5%; * = 10%.

<table>
<thead>
<tr>
<th></th>
<th>1st quartile (≤148)</th>
<th>2nd quartile (148–186)</th>
<th>3rd quartile (186–242)</th>
<th>4th quartile (&gt;242)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Theoretical spread</strong></td>
<td>0.17*** 0.04</td>
<td>0.21*** 0.09** −0.12*</td>
<td>0.18** 0.03 −0.15</td>
<td>0.09 −0.18 −0.27</td>
</tr>
<tr>
<td><strong>Volatility</strong></td>
<td>0.09 −0.06 −0.15</td>
<td>−0.20 −0.67 −0.48</td>
<td>0.08 0.46 0.37</td>
<td>2.19** 4.04 1.84</td>
</tr>
<tr>
<td><strong>Leverage</strong></td>
<td>0.91** 0.95** 0.04</td>
<td>0.21 0.66*** 0.45*</td>
<td>0.46** 0.75*** 0.29</td>
<td>0.28*** 0.86*** 0.58***</td>
</tr>
<tr>
<td><strong>Interest rate</strong></td>
<td>1.19 −20.61***−21.80***−4.43***−28.24***−23.81***−3.64***−28.66***−25.01***−5.83 8.56 14.38</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Stock return</strong></td>
<td>−0.05 −0.26 −0.22</td>
<td>−0.59***−0.47 0.12</td>
<td>−0.05 −0.16 −0.11</td>
<td>−0.96*** −0.49 0.47</td>
</tr>
<tr>
<td><strong>Slope yield curve</strong></td>
<td>0.36 8.67** 8.31*</td>
<td>3.95* 9.95** 6.00</td>
<td>2.43 2.17 −0.26</td>
<td>5.04 8.71 3.68</td>
</tr>
<tr>
<td><strong>Corporate spreads</strong></td>
<td>0.24*** 0.44*** 0.20**</td>
<td>0.39*** 0.40*** 0.01</td>
<td>0.52*** 0.57*** 0.05</td>
<td>1.15*** 1.15** 0.00</td>
</tr>
<tr>
<td><strong>S&amp;P Composite</strong></td>
<td>0.35 0.76*** 0.41</td>
<td>0.82*** 0.57 −0.25</td>
<td>0.46 0.78 0.33</td>
<td>2.70*** 1.62 −1.08</td>
</tr>
<tr>
<td><strong>VIX</strong></td>
<td>0.74 0.13 −0.61</td>
<td>0.83** 0.11 −0.72</td>
<td>0.63 −0.44 −1.08</td>
<td>0.35 −0.49 −0.84</td>
</tr>
<tr>
<td><strong>Intercept</strong></td>
<td>−0.47 −3.45***−2.98***</td>
<td>0.05 −2.95* −2.99*</td>
<td>0.24 −3.76***−3.99***</td>
<td>1.24* −0.64 −1.87</td>
</tr>
<tr>
<td><strong>Adjusted $R^2$</strong></td>
<td>0.25 0.35</td>
<td>0.43 0.33</td>
<td>0.30 0.39</td>
<td>0.63 0.52</td>
</tr>
<tr>
<td><strong>No. obs.</strong></td>
<td>2,009 780</td>
<td>2,023 786</td>
<td>2,174 720</td>
<td>1,934 658</td>
</tr>
</tbody>
</table>
Table 4.7 – The determinants of CDS spread changes by sector

The values in the table are obtained by running a pooled OLS regression for all observations in the selected sector and period, with standard errors that allow for time correlation at firm level. Monthly data from January 2002 to March 2009; the pre-crisis period goes from January 2002 to June 2007; the post-crisis period from July 2007 to March 2009. The explanatory variables are changes in the monthly averages of: the estimated theoretical spread for firm \( i \) at time \( t \); the implied volatility of options written on the stocks of firm \( i \) at time \( t \); the leverage ratio of firm \( i \) at time \( t \); the 5-year zero-coupon interest rate on the US government bond at time \( t \); the log of the stock value of firm \( i \) at time \( t \); the slope of the zero-coupon curve on US government bonds (10-1 yrs) at time \( t \); the Merrill Lynch industrial bond average spread (BBB-AA) at time \( t \); the log of the S&P Composite stock index at time \( t \); the VIX volatility index at time \( t \); a constant term. Observed and theoretical spreads and the corporate spread are in basis points, all other variables are in percentages. Significance levels: *** = 1%; ** = 5%; * = 10%.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Industrial</th>
<th>Communications &amp; Technology</th>
<th>Basic materials &amp; Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical spread</td>
<td>0.27**</td>
<td>0.04</td>
<td>−0.22*</td>
</tr>
<tr>
<td>Volatility</td>
<td>−0.71**</td>
<td>1.33</td>
<td>2.04*</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.14</td>
<td>0.51</td>
<td>0.37*</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.12</td>
<td>−21.34***</td>
<td>−21.46***</td>
</tr>
<tr>
<td>Stock return</td>
<td>−0.49*</td>
<td>−0.11</td>
<td>0.37</td>
</tr>
<tr>
<td>Slope yield curve</td>
<td>1.08</td>
<td>−0.14</td>
<td>−1.22</td>
</tr>
<tr>
<td>Corporate spreads</td>
<td>0.41***</td>
<td>0.36*</td>
<td>−0.05</td>
</tr>
<tr>
<td>S&amp;P Composite</td>
<td>0.40</td>
<td>−0.35</td>
<td>−0.75</td>
</tr>
<tr>
<td>VIX</td>
<td>0.41</td>
<td>−1.81**</td>
<td>−2.21*</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.11</td>
<td>−2.87</td>
<td>−2.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Adjusted ( R^2 )</th>
<th>No. obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial</td>
<td>0.33</td>
<td>1,416</td>
</tr>
<tr>
<td>Communications &amp; Technology</td>
<td>0.46</td>
<td>568</td>
</tr>
<tr>
<td>Basic materials &amp; Energy</td>
<td>0.25</td>
<td>1,328</td>
</tr>
</tbody>
</table>

(cont.)
Table 4.7 – The determinants of CDS spread changes by sector (cont.)
The values in the table are obtained by running a pooled OLS regression for all observations in the selected sector and period, with standard errors that allow for time correlation at firm level. Monthly data from January 2002 to March 2009; the pre-crisis period goes from January 2002 to June 2007; the post-crisis period from July 2007 to March 2009. The explanatory variables are changes in the monthly averages of: the estimated theoretical spread for firm $i$ at time $t$; the implied volatility of options written on the stocks of firm $i$ at time $t$; the leverage ratio of firm $i$ at time $t$; the 5-year zero-coupon interest rate on the US government bond at time $t$; the log of the stock value of firm $i$ at time $t$; the slope of the zero-coupon curve on US government bonds (10-1 yrs) at time $t$; the Merrill Lynch industrial bond average spread (BBB-AA) at time $t$; the log of the S&P Composite stock index at time $t$; the VIX volatility index at time $t$; a constant term. Observed and theoretical spreads and the corporate spread are in basis points, all other variables are in percentages. Significance levels: **∗∗∗ = 1%; ** = 5%; * = 10%.

<table>
<thead>
<tr>
<th></th>
<th>Consumer cyclical</th>
<th></th>
<th>Consumer non-cyclical</th>
<th></th>
<th>Utilities</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical spread</td>
<td>0.06</td>
<td>-0.14</td>
<td>-0.19</td>
<td>0.11</td>
<td>0.07**</td>
<td>-0.04</td>
</tr>
<tr>
<td>Volatility</td>
<td>2.11**</td>
<td>2.57</td>
<td>0.46</td>
<td>1.01***</td>
<td>0.03</td>
<td>-0.97**</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.30***</td>
<td>0.94***</td>
<td>0.63***</td>
<td>-0.16***</td>
<td>0.23***</td>
<td>0.39***</td>
</tr>
<tr>
<td>Interest rate</td>
<td>0.00</td>
<td>19.49</td>
<td>19.48</td>
<td>-2.42</td>
<td>-25.43***</td>
<td>-23.01***</td>
</tr>
<tr>
<td>Stock return</td>
<td>-0.75***</td>
<td>-1.86</td>
<td>-1.11</td>
<td>-0.60*</td>
<td>-0.64***</td>
<td>-0.04</td>
</tr>
<tr>
<td>Slope yield curve</td>
<td>-2.27</td>
<td>16.89</td>
<td>19.16</td>
<td>3.98</td>
<td>19.41***</td>
<td>15.44**</td>
</tr>
<tr>
<td>Corporate spreads</td>
<td>0.90***</td>
<td>1.45***</td>
<td>0.55</td>
<td>0.16***</td>
<td>0.19**</td>
<td>0.03</td>
</tr>
<tr>
<td>S&amp;P Composite</td>
<td>1.67**</td>
<td>2.11</td>
<td>0.44</td>
<td>-0.10</td>
<td>0.89***</td>
<td>0.99**</td>
</tr>
<tr>
<td>VIX</td>
<td>0.05</td>
<td>-0.25</td>
<td>-0.30</td>
<td>-0.66</td>
<td>0.42</td>
<td>1.08</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.70</td>
<td>-4.04</td>
<td>-4.74</td>
<td>0.57</td>
<td>-4.19***</td>
<td>-4.75***</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.64</td>
<td>0.62</td>
<td></td>
<td>0.20</td>
<td>0.57</td>
<td></td>
</tr>
<tr>
<td>No. obs.</td>
<td>2,026</td>
<td>726</td>
<td></td>
<td>1,285</td>
<td>467</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.8 – The determinants of CDS spread changes by liquidity change

The values in the table are obtained by running a pooled OLS regression for all observations in the selected period and quartile of the firms’ liquidity change, with standard errors that allow for time correlation at firm level. The liquidity change is defined as the change in the average bid-ask spread of the CDSs, at the firm level, from before to after the onset of the crisis. Monthly data from January 2002 to March 2009; the pre-crisis period goes from January 2002 to June 2007; the post-crisis period from July 2007 to March 2009. The explanatory variables are changes in the monthly averages of: the estimated theoretical spread for firm i at time t; the implied volatility of options written on the stocks of firm i at time t; the leverage ratio of firm i at time t; the 5-year zero-coupon interest rate on the US government bond at time t; the log of the stock value of firm i at time t; the slope of the zero-coupon curve on US government bonds (10-1 yrs) at time t; the Merrill Lynch industrial bond average spread (BBB-AA) at time t; the log of the S&P Composite stock index at time t; the VIX volatility index at time t; a constant term. Observed and theoretical spreads and the corporate spread are in basis points, all other variables are in percentages. Significance levels: ** = 1%; *** = 5%; * = 10%.

<table>
<thead>
<tr>
<th></th>
<th>1st quartile</th>
<th></th>
<th>2nd quartile</th>
<th></th>
<th>3rd quartile</th>
<th></th>
<th>4th quartile</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-&lt;0.9)</td>
<td></td>
<td>(-0.9-1.1)</td>
<td></td>
<td>(1.1-5.5)</td>
<td></td>
<td>(&gt;5.5)</td>
<td></td>
</tr>
<tr>
<td>Theoretical spread</td>
<td>0.25***</td>
<td>0.02</td>
<td>-0.23**</td>
<td>0.20***</td>
<td>0.04*</td>
<td>-0.16**</td>
<td>0.21***</td>
<td>0.01</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.07</td>
<td>0.36</td>
<td>0.29</td>
<td>0.44**</td>
<td>0.07</td>
<td>-0.37</td>
<td>-0.06</td>
<td>0.72*</td>
</tr>
<tr>
<td>Leverage</td>
<td>0.26**</td>
<td>1.46***</td>
<td>1.21***</td>
<td>0.11</td>
<td>0.65**</td>
<td>0.55**</td>
<td>0.09</td>
<td>0.32***</td>
</tr>
<tr>
<td>Interest rate</td>
<td>-6.65**</td>
<td>-18.20***</td>
<td>-11.55***</td>
<td>-0.87</td>
<td>-17.03***</td>
<td>-16.16***</td>
<td>-2.01**</td>
<td>-24.84***</td>
</tr>
<tr>
<td>Stock return</td>
<td>-0.66*</td>
<td>0.86**</td>
<td>1.52**</td>
<td>-0.13</td>
<td>0.16</td>
<td>0.28</td>
<td>-0.03</td>
<td>-0.33</td>
</tr>
<tr>
<td>Slope yield curve</td>
<td>10.84***</td>
<td>7.93***</td>
<td>-2.91</td>
<td>-0.52</td>
<td>6.73***</td>
<td>7.25***</td>
<td>2.01</td>
<td>4.92</td>
</tr>
<tr>
<td>Corporate spreads</td>
<td>0.65***</td>
<td>0.2***</td>
<td>-0.45***</td>
<td>0.15***</td>
<td>0.24***</td>
<td>0.09*</td>
<td>0.38***</td>
<td>0.74***</td>
</tr>
<tr>
<td>S&amp;P Composite</td>
<td>0.91**</td>
<td>0.57*</td>
<td>-0.33</td>
<td>0.11</td>
<td>0.03</td>
<td>-0.08</td>
<td>0.41***</td>
<td>0.41</td>
</tr>
<tr>
<td>VIX</td>
<td>0.50</td>
<td>0.46</td>
<td>-0.04</td>
<td>0.09</td>
<td>-0.07</td>
<td>-0.16</td>
<td>0.92**</td>
<td>-0.81*</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.28</td>
<td>-3.37***</td>
<td>-3.65***</td>
<td>-0.26</td>
<td>-3.17***</td>
<td>-2.91***</td>
<td>-0.03</td>
<td>-4.01***</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.51</td>
<td>0.48</td>
<td>-0.21</td>
<td>0.43</td>
<td>0.33</td>
<td>0.41</td>
<td>-0.03</td>
<td>-0.40</td>
</tr>
<tr>
<td>No. obs.</td>
<td>2,155</td>
<td>709</td>
<td>1,953</td>
<td>750</td>
<td>1,757</td>
<td>733</td>
<td>1,811</td>
<td>752</td>
</tr>
</tbody>
</table>
firms into four quartiles according to the change in the average bid-ask spread they experienced from before to after the onset of the crisis. The results are reported in Table 4.8. The interesting result is that the overall explanatory power of the model remains broadly the same during the crisis period for both the contracts in the lowest and highest quartiles of bid-ask spread changes. For firms in the highest quartile, moreover, all coefficients of the regression, except for that related to leverage, do not show any significant change during the crisis. These results suggest that the lack of liquidity experienced in the CDS market during the crisis, as proxied by larger bid-ask spreads, did not modify the basic relationships between CDS spread changes and their explanatory factors.

4.6 Conclusion

In this chapter we analyze the determinants of CDS spread changes (based on a sample of 167 US non-financial firms over the period between January 2002 and March 2009), using the variables that the literature has found to have a theoretical and empirical impact on CDS spreads. We include in our regressions the theoretical spread implied by the Merton model in order to deal with the non-linear relationships between the individual characteristics of the firms and CDS spreads. We find that the inclusion of this additional term improves the capacity of changes in the fundamental variables to explain changes in CDS spreads. When the theoretical spread calculated using the Merton model is introduced in the regressions, the coefficient of the equity volatility decreases significantly, because of the high sensitivity of the Merton model to this parameter. On the contrary, leverage, which has only second-order effects on the theoretical spreads, maintains its usefulness in explaining CDS spread changes. The extended model is able to explain 54% of the variations in CDS spreads in the pre-crisis period and 51% in the crisis period, which is higher than previous findings of studies on corporate bond and CDS spread changes.

We also analyze how the financial crisis has changed the way in which credit risk is priced in the CDS market. We find that the contribution of the leverage of the firms to the explanation of CDS spread changes is much higher during the crisis, as investors appear to have become more aware of individual risk factors. At the same time, the impact of equity volatility substantially decreases, possibly because the large swings in implied volatility during the crisis period have made this indicator a poor proxy for long-term asset volatility. We also find that the overall capacity of the model to explain CDS changes is almost the same before and during the turmoil, thus highlighting that the underlying risk factors identified
by the literature as relevant for the pricing of the credit risk have maintained their explanatory power even in a period of remarkable stress for the CDS market.

Finally, we show that during the crisis CDS spreads appear to have been moving increasingly together, driven by a common factor that our model was only partly able to explain. Given that the model includes general indicators of economic activity, uncertainty, and risk aversion, our results point to the presence of a market-specific factor that hit the CDS market during the crisis in forms not fully reflected in other markets. The exact identification of this factor is an interesting topic for further research.
Chapter 5

Risk Measures of Autocorrelated Hedge Fund Returns

5.1 Introduction

After two decades of strong growth, hedge funds have developed into a mature and widely accepted asset class. On the back of relatively high historical returns, the hedge fund industry has enjoyed a near-continuous inflow of new money, with the credit crisis period being the exception. Moreover, hedge fund risk levels are frequently reported to be lower than those of the more traditional investments in equities. These performance characteristics of hedge funds have also attracted considerable academic attention.\(^1\)

One particular feature of hedge fund returns is the strong autocorrelation. Fung and Hsieh (2001), Brooks and Kat (2002), and Agarwal and Naik (2004) demonstrate that this feature invalidates standard mean-variance analysis for hedge funds. Getmansky et al. (2004) argue that the autocorrelation stems from the illiquidity of the assets held by hedge funds and the smoothing of the returns because of reporting practices. Based on a moving average representation of reported returns, they show how this process affects the Sharpe ratio (SR) and beta in a standard single index market model. As the smoothing lowers the variance of the returns and the covariance (with the market index), but leaves the mean unaffected, the standard risk measures tend to underestimate the actual risk (SR is overstated). In Chan et al. (2006), this framework is used to evaluate the systemic risk posed by hedge funds for the banking sector. Bollen and Pool (2009) use this autocorrelation

\(^{1}\)For research on the risk and return characteristics of hedge funds, see Fung and Hsieh (1997), Ackermann et al. (1999), Agarwal and Naik (2000, 2004), Amin and Kat (2003), Morton et al. (2006), Bali et al. (2007), Eling and Schuhmacher (2007), Kosowski et al. (2007), Fung et al. (2008), Bollen and Whaley (2009), Ding et al. (2009), Sadka (2010), and Dichev and Yu (2011).
structure to detect misreporting. Recently, Avramov et al. (2011) and Teo (2011) use the algorithm of Getmansky et al. (2004) to unsmooth hedge fund returns.

This chapter extends the lead taken by Getmansky et al. (2004) along three dimensions. First, whereas Getmansky et al. (2004) consider two global measures of risk (SR and market beta), here the scope is broadened by evaluating also popular downside measures of risk. There is considerable evidence from behavioral finance that individuals do not symmetrically treat the upside potential and the downside risk. Moreover, regulatory frameworks such as Solvency II and Basel II focus on downside risk measures.

The second dimension is the distinction between univariate (or individual) and multivariate (or systemic) measures. SR is a univariate measure of risk, while beta measures the interdependency with the market. As variance and covariance are global concepts, covering the upside and the downside, SR and beta are global risk measures. This chapter provides for two individual and one systemic risk measures that are focused on the downside.

Finally, the third dimension adds the distinction between light tails and heavy tails. The measures considered by Getmansky et al. (2004) fully characterize the risk aspects in the case that the noise is multivariate normally distributed, that is, in the case of light tails. In practice, it is known that return distributions of most assets are heavy tailed. An example of a heavy tailed distribution is the Student’s t-distribution. Such distributions exhibit hyperbolic or power-like decline in the tails, whereas light tailed distributions have exponential declining tails. While the SR and beta measures also apply in case of heavy tails (as long as second moments are finite), the downside risk measures do respond quite differently to smoothing depending on whether the returns are light or heavy.

More specific, what are the risk measures that are studied in this chapter? Apart from the univariate global SR measure considered by Getmansky et al. (2004), we also investigate the value-at-risk (VaR) and expected shortfall (ES) measures. The VaR and ES downside measures play a central role in risk management practices of the financial sector and are also sensitive to the type of tail behavior (light or fat) of the returns under consideration. As for the multivariate risk measures, we examine the correlation coefficient $\rho$, which is a global measure of risk, and a multivariate measure that focuses on the downside systemic risk. The latter measure reflects the amount of interdependence among two or more returns deep into the joint tail loss area. It exclusively picks up the extreme linkages in crisis situations and is termed the Extreme Linkage Measure (ELM). Most of these measures are well known, except perhaps ELM, which is explained in Section 5.2.2. ELM is one of the measures used
to capture the systemic risk (see, e.g., International Monetary Fund, 2009, chap. 2). One of the lessons of the credit crisis is that supervisory frameworks should pay more attention to the risk exposures of the entire financial sector. Table 5.1 summarizes the investigation of this chapter for both light and heavy tails.

After investigating how these risk measures are affected by the kind of smoothing proposed by Getmansky et al. (2004), the smoothing-adjusted risk measures are applied on four broad-based hedge fund indices between 1990 and 2009. The results show that the smoothing-adjusted hedge fund investment returns indicate levels of risk that can be considerably higher than the risk measures based on reported returns would suggest. This finding holds in particular for the downside risk measures. The size of these distortions corresponds to the findings by Getmansky et al. (2004, p. 551) for the global SR measure.

Using the smoothing-adjusted economic risk measures is important both for investors trying to determine the proportion to invest in hedge funds and for investors constructing a hedge fund portfolio based on the relative risks of those funds. Correct risk measures are instrumental to prevent overpaying for an investment in hedge funds caused by overestimating the attractiveness of hedge funds. Finally, the ELM results can be of interest for policy makers and regulators who are concerned about the effects that hedge funds could have on financial stability.

The chapter proceeds as follows. Section 5.2 models the impact of smoothing and derives the adjusted risk measures. Section 5.3 presents the empirical methodology. In Section 5.4, the adjusted risk measures are applied to a sample of hedge fund indices. Section 5.5 concludes.

### 5.2 Modeling the impact of smoothing

In this section it is derived how smoothing affects the risk measures introduced before. Following Getmansky et al. (2004), the reported or observed returns are considered to be a weighted average of the fund’s actual returns over a number of the most recent periods, including the current period. This assumption turns the observed returns into a moving average of the actual returns. Consider two hedge

<table>
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Table 5.1 – Risk measures analyzed in the chapter
funds with actual returns $X_{1,t}$ and $X_{2,t}$ in period $t$, and assume that these returns adhere to a single factor market model. Thus, if $R_t$ is the market return in period $t$ and if $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are idiosyncratic risk factors in period $t$, then

$$X_{i,t} = \beta_i R_t + \varepsilon_{i,t},$$

(5.1)

for $i = 1, 2$. For the sake of the presentation, it is assumed that both $\beta_1$ and $\beta_2$ are strictly positive constants, except when indicated otherwise, and that $R_t$, $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are independent and identically distributed (i.i.d.) with distributions that are defined in Section 5.2.1. The following results are subsequently generalized to the case in which there are multiple market factors. The corresponding equations are reported in Appendix 5.A. Further, the results can be easily generalized to allow for negative beta, that can be important for certain hedge fund strategies, such as Dedicated Short Bias.

As in Getmansky et al. (2004, eqs. 21–23), it is assumed that the actual returns $X_{i,t}$ cannot be observed directly and that reported returns $S_{i,t}$ are governed by the following $MA(K)$ process

$$S_{i,t} = \sum_{k=0}^{K} \theta_{i,k} X_{i,t-k},$$

(5.2)

$$\theta_{i,k} \in [0, 1], \quad k = 0, \ldots, K,$$

(5.3)

$$\sum_{k=0}^{K} \theta_{i,k} = 1,$$

(5.4)

for $i = 1, 2$. We refer to the $MA$ coefficients $\theta_{i,k}$ as the “smoothing coefficients”.

### 5.2.1 Smoothing effects on univariate risk measures

In this subsection, the smoothing-adjusted formulae for the univariate risk measures SR, VaR, and ES are derived. For each measure, the thin tail case is first treated, followed by the fat tail case. What is meant by the thin tail case is focused on the normal distribution, which is the standard fare in finance. It provides a benchmark against which the case of heavy tails is judged. Considering the normal distribution as representative for the thin tail case is sometimes overly restrictive. For example, in the case of the SR analysis, this it is only required that the first two moments exist, so that the results for the normal case also apply to other distributions such as the uniform, the exponential, and heavy tailed distributions with the first two moments bounded.
A distribution is said to be (symmetrically) heavy tailed if it is regularly varying at infinity, that is to say, the tails of the distribution satisfy

$$\lim_{t \to \infty} \frac{F(-tx)}{F(-t)} = \lim_{t \to \infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha},$$

(5.5)

for all $x > 0$ and some $\alpha > 0$. Somewhat loosely formulated, this definition means that to a first order at infinity the distribution follows the Pareto distribution $P(|X| > x) = x^{-\alpha}$. The tail index $\alpha$ determines how heavy the tails are, as only the moments up to $\alpha$ are bounded. For example, it is readily verified that the Student’s $t$-distribution with $v$ degrees of freedom has regularly varying tails with $\alpha = v$. Moreover, it will be assumed that the following first order condition holds

$$P(|X| > x) = 2Ax^{-\alpha} + o(x^{-\alpha}),$$

(5.6)

where $A > 0$, although Eq. (5.5) also permits $A$ not to be constant (but requires slow variation, that is $\lim_{t \to \infty} A(tx)/A(t) = 1$ for any $x > 0$).

To derive the implications of the $MA(K)$ process for the mentioned risk measures, one needs to know the distribution of the convolution of the random variables in Eq. (5.2). How to do this exercise for the normal distribution is well known as the square root rule. For example, assuming that the market factor $R \sim N(\mu_R, \sigma_R)$ and the idiosyncratic factor $\varepsilon \sim N(\mu_\varepsilon, \sigma_\varepsilon)$ in Eq. (5.1), it follows (omitting subindices $i$ whenever no confusion is possible)

$$X \sim N(\beta\mu_R + \mu_\varepsilon, (\beta^2\sigma_R^2 + \sigma_\varepsilon^2)^{1/2}).$$

(5.7)

For the case of heavy tailed distributions there does not generally exist such a simple rule. For our purposes, however, it suffices to know what happens to the tail probabilities under summation, which is a considerably simpler problem. To derive the results, we use the celebrated convolution theorem by Feller (1971, chap. VIII.8, see Appendix 5.B). The flavor of the convolution theorem is demonstrated for the case of the single index model from Eq. (5.1). Suppose, for example, that both the market factor $R$ and the idiosyncratic risk $\varepsilon$ have a Student’s $t$-distribution with $v$ degrees of freedom, and hence satisfy Eq. (5.6). Then, the Feller convolution result holds that

$$\lim_{t \to \infty} P(X > t) = \frac{1}{(\beta^v + 1)A^{t-v}} = 1.$$  

(5.8)

It transpires that one can just add the tails if these are of equal order, provided that one scales the weights with the tail index. If the tail indices are unequal, then the tail with the lowest index dominates the sum. Note that the convolution changes the scale factor, but leaves the power $v$ unaffected. In other words, if one studies
the convolution of two independent heavy tail distributed random variables with the same tail index at large quantiles, then it suffices to take the sum of the scales divided by the quantile to the power of tail index. To a first order, all mass in the plane concentrates along the axes and this property determines the sum. With these preparations at hand, we can now turn to the specific measures and investigate the effects of the moving average feature of reported returns.

**Sharpe ratio**

Given the single index model in Eq. (5.1) for the MA($K$) process in Eq. (5.2), and assuming that the market factor $R \sim N(\mu_R, \sigma_R)$ and the idiosyncratic factor $\varepsilon \sim N(\mu_\varepsilon, \sigma_\varepsilon)$, it readily follows that

$$
\sigma_S = \left( \sum_{k=0}^{K} \theta_k^2 \right)^{1/2} \left( \beta^2 \sigma_R^2 + \sigma_\varepsilon^2 \right)^{1/2} = \left( \sum_{k=0}^{K} \theta_k^2 \right)^{1/2} \sigma_X. \tag{5.9}
$$

Thus, $\sigma_S \leq \sigma_X$, as Eqs. (5.3) and (5.4) imply that $\sum_{k=0}^{K} \theta_k^2 \leq 1$. On the other hand, the mean is unaffected

$$
\mu_S = \left( \sum_{k=0}^{K} \theta_k \right) (\beta \mu_R + \mu_\varepsilon) = \beta \mu_R + \mu_\varepsilon = \mu_X. \tag{5.10}
$$

These results are used to determine how the SR measure is affected by smoothing

$$
\text{SR}_X = \frac{\mu_X - R_f}{\sigma_X} \leq \frac{\mu_X - R_f}{\sigma_S} = \text{SR}_S, \tag{5.11}
$$

where $R_f$ is the risk-free rate. In other words, as $\text{SR}_S = (\sum \theta_k^2)^{1/2} \text{SR}_X$, smoothing understates the (global) risk and hence SR is overstated. This finding is the same as Proposition 1 in Getmansky et al. (2004).

Next, consider the case that $R$ and $\varepsilon$ do not follow the normal distribution, but rather have distributions with heavy tails as defined above. Then Eq. (5.11) nevertheless applies as long as $\alpha > 2$, meaning that the second moments are bounded. Hence, under the mild restriction that the variance is bounded, this result merely corroborate Getmansky et al. (2004) for the SR measure. But things will be different for the downside risk measures.

**Value-at-risk**

VaR is probably the most widely used univariate measure in risk management. It has numerous applications in banking and insurance. Jorion (2007) provides an extensive overview of the VaR measure. VaR is sometimes criticized for its lack
of subadditivity. Such criticism is why we also treat the globally subadditive ES measure below. Nevertheless, for the normal distribution VaR is subadditive below the mean. This result also holds, for example, for the Student’s t-distribution. In fact, Danielsson et al. (2010) show that for all fat tailed distributions, the VaR measure is subadditive in the tail area. So both for practical and theoretical reasons it is of interest to analyze how VaR is affected by the presence of autocorrelation in hedge fund returns.

For the random variable $Y_t$ with a continuous distribution, VaR with confidence level $1 - p$ is defined as the unique real number $\text{VaR}(Y_t, p)$ for which

$$\mathbb{P}(Y_t > \text{VaR}(Y_t, p)) = p,$$

(5.12)

or

$$\text{VaR}(Y_t, p) = \varphi_{Y_t}^{-1}(1 - p),$$

(5.13)

where $\varphi_{Y_t}^{-1}(x)$ is the inverse of the cumulative density function of $Y_t$ evaluated at $x$. Note that VaR usually is a loss return and hence a positive number. Therefore, the focus is on the right hand tail of the loss distribution.

— Normal case

Let $\text{VaR}(S_t, p; N)$ denote VaR at the confidence level $1 - p$ for the reported return $S_t$ under the assumption that the i.i.d. random variables $R_t$ and $\varepsilon_t$ have normal distributions $R_t \sim N(\mu_R, \sigma_R)$ and $\varepsilon_t \sim N(\mu_\varepsilon, \sigma_\varepsilon)$. Under these assumptions, one has from the above that $S_t \sim N(\mu_S, \sigma_S)$. So that

$$p = \mathbb{P}(S_t > \text{VaR}(S_t, p; N)) = 1 - \Phi\left(\frac{\text{VaR}(S_t, p; N) - \mu_S}{\sigma_S}\right),$$

(5.14)

where $\Phi(x)$ is the standard normal cumulative distribution function evaluated at $x$. VaR is given by

$$\text{VaR}(S_t, p; N) = \sigma_S \Phi^{-1}(1 - p) + \mu_S.$$

(5.15)

Since $\mu_S = \mu_X$ and $\sigma_S \leq \sigma_X$ (see Eqs. 5.9 and 5.10), VaR calculated on the smoothed returns, $\text{VaR}(S_t, p; N)$, is always smaller than or equal to VaR calculated on the actual returns, $\text{VaR}(X_t, p; N)$. In particular, from Eqs. (5.9) and (5.15) we have:

**Proposition 5.1** If the market factor $R$ and idiosyncratic risk are independently normally distributed, then VaR of the reported returns is related to VaR of the actual

\[\text{See Appendix 5.C for the case in which returns behave according to a Pareto law.}\]
returns in the following way

\[
\frac{\text{VaR}(S_t, p; N) - \mu_S}{\text{VaR}(X_t, p; N) - \mu_S} = \frac{\sigma_S}{\sigma_X} = \left(\frac{1}{\sum_{k=0}^{K} \theta_k^2}\right)^{1/2}.
\] (5.16)

Under the assumption of a normal distribution, the presence of autocorrelation in the actual hedge fund returns reduces the reported VaR by the reduction in the volatility of the returns. The square root rule applies again (cf. Eq. 5.11).

--- Heavy tails case

Suppose now that the distributions of \(R_t\) and \(\varepsilon_t\) are heavy tailed as in Eq. (5.6). As a slight generalization, allow the scale \(A\) in Eq. (5.6) to be different for \(R\) and \(\varepsilon\), as \(\gamma_R\) and \(\gamma_{\varepsilon}\), respectively. Let \(\text{VaR}(S_t, p; H)\) denote VaR at confidence level \(1 - p\) for the case of heavy tailed distributions. Invoking Feller’s convolution theorem gives

\[
\lim_{p \downarrow 0} \frac{\mathbb{P}(S_t > \text{VaR}(S_t, p; H))}{(\gamma_R \beta^\alpha + \gamma_{\varepsilon}) \left(\sum_{k=0}^{K} \theta_k^a\right) \text{VaR}(S_t, p; H)^{-\alpha}} = 1.
\] (5.17)

On first order inversion, for small \(p\) approximately

\[
\text{VaR}(S_t, p; H) \simeq \left(\frac{\gamma_R \beta^\alpha + \gamma_{\varepsilon} \left(\sum_{k=0}^{K} \theta_k^a\right)}{p}\right)^{1/\alpha}.
\] (5.18)

Similarly, it is shown that for the actual returns

\[
\text{VaR}(X_t, p; H) \simeq \left(\frac{\gamma_R \beta^\alpha + \gamma_{\varepsilon}}{p}\right)^{1/\alpha}.
\] (5.19)

we thus obtain the following:

**Proposition 5.2** If the market factor and idiosyncratic factor exhibit heavy tails as in Eq. (5.6), then

\[
\lim_{p \downarrow 0} \frac{\text{VaR}(S_t, p; H)}{\text{VaR}(X_t, p; H)} = \left(\sum_{k=0}^{K} \theta_k^a\right)^{1/\alpha}.
\] (5.20)

Proposition 5.2 is readily generalized to allow for negative betas, by taking the absolute value of beta.

Given that Eqs. (5.3) and (5.4) imply \(\sum_{k=0}^{K} \theta_k^a \leq 1\) for any \(\alpha \geq 1\), VaR calculated on smoothed returns deep into the tail area is always smaller than or equal to VaR calculated on actual returns. The latter condition just requires that the mean is
bounded. In case of the Cauchy distribution, which has \( \alpha = 1 \), VaR calculated on smoothed returns is equal to VaR calculated on actual returns.

As the first derivative of \( (\sum \theta_k^\alpha)^{1/\alpha} \) in Eq. (5.20) with respect to the tail index \( \alpha \) is negative, this result implies that the presence of autocorrelation affects the smoothed returns’ VaR relatively less when the reported return distribution has fatter tails. In general, for any given value of the tail index \( \alpha > 1 \), VaR of the smoothed returns VaR(\( S_t, p; H \)) is minimized when the smoothing coefficients \( \theta_k \) equal \( 1/(K + 1) \) for all \( k = 0, \ldots, K \). In that case, the current and past true economic returns are equally weighted and together make up the reported returns. As a result, the ratio of VaR(\( S_t, p; H \)) to VaR(\( X_t, p; H \)) equals \( (K + 1)^{1-\alpha}/\alpha \). For \( K = 2 \) and \( \alpha = 5 \) this finding implies, for instance, that the reported VaR could be equal to just about 40% the true VaR.

Finally, note that the term on the right-hand side of Eq. (5.20) for the heavy tail case looks exactly like the correction term in Eq. (5.16) for the normal case, but with 2 replaced by \( \alpha \). This result implies that the correction term for the heavy tail case is always smaller than the correction term for the normal case as long as \( \alpha > 2 \). In other words, the impact of smoothing is usually larger in the heavy tail case than in the normal case.

**Expected shortfall**

Artzner et al. (1999) argue that monotonicity, subadditivity, homogeneity and translation invariance are desirable properties for a risk measure. VaR satisfies three of the four criteria, but can fail subadditivity. For this reason Artzner et al. (1999) advance the ES measure as it satisfies all four criteria (provided that \( \alpha > 1 \) in the heavy tail case). For a given loss return threshold \( y \), the ES measure is the conditional expectation

\[
\text{ES}(Y_t, y) = \mathbb{E}[Y_t | Y_t > y].
\]

In general, the ES measure is difficult to compute for the convolution induced by the \( MA(K) \) process. Fortunately, for the normal case and the heavy tail case at sufficiently large \( y \), we can obtain explicit results.

--- **Normal case**

Using the previous assumptions, Eqs. (5.9) and (5.10) give ES as

\[
\text{ES}(S_t, y; N) = \frac{\int_y^\infty \frac{x - \mu_S}{\sigma_S} \phi \left( \frac{x - \mu_S}{\sigma_S} \right) dx}{\int_y^\infty \phi \left( \frac{x - \mu_S}{\sigma_S} \right) dx} = \frac{\sigma_S \phi \left( \frac{y - \mu_S}{\sigma_S} \right)}{\Phi \left( \frac{y - \mu_S}{\sigma_S} \right)} + \mu_S,
\]

(5.22)
where $\phi(x)$ is the standard normal probability density function evaluated at $x$. It can be shown that the first derivative of ES in Eq. (5.22) with respect to $\sigma_S$ is positive\(^3\) so that, given that $\sigma_S \leq \sigma_X$, ES calculated on smoothed returns is always lower than or equal to ES calculated on actual returns. Furthermore, when $y$ equals the VaR that corresponds to the confidence level $1 - p$, one has

$$
\text{ES}(S_t, \text{VaR}(S_t, p; N); N) = \frac{\sigma_S \phi\left(\frac{\text{VaR}(S_t, p; N) - \mu_S}{\sigma_S}\right)}{p} + \mu_S
$$

(5.23)

The last equation shows that ES is proportional to VaR. Given this proportionality result, it is not so surprising that the following is obtained:

**Proposition 5.3** If the market factor $R$ and the idiosyncratic risk are independently normally distributed, then ES of the reported returns is related to ES of the actual returns as follows

$$
\frac{\text{ES}(S_t, \text{VaR}(S_t, p; N); N) - \mu_S}{\text{ES}(X_t, \text{VaR}(X_t, p; N); N) - \mu_S} = \frac{\sigma_S}{\sigma_X} = \left(\sum_{k=0}^{K} \theta_k^2\right)^{1/2}.
$$

(5.24)

As in the case of VaR, we find that after the correction for the mean, the smoothed ES is proportional to the actual ES. The presence of autocorrelation

\(^3\)To show that the first derivative of

$$
f(\sigma) = \frac{\sigma \phi\left(\frac{y - \mu}{\sigma}\right)}{1 - \Phi\left(\frac{y - \mu}{\sigma}\right)}
$$

is positive, notice that

$$
\frac{d}{d\sigma} \sigma \phi\left(\frac{y - \mu}{\sigma}\right) = \frac{(\sigma^2 + (y - \mu)^2)}{\sigma^2} \phi\left(\frac{y - \mu}{\sigma}\right),
$$

so that the sign of the derivative of $f$ with respect to $\sigma$ is equal to the sign of

$$
(\sigma^2 + (y - \mu)^2) \left(1 - \Phi\left(\frac{y - \mu}{\sigma}\right)\right) + (y - \mu)\sigma \phi\left(\frac{y - \mu}{\sigma}\right),
$$

or, setting $\theta = (y - \mu)/\sigma$, to the sign of $g(\theta) = (1 + \theta^2) (1 - \Phi(\theta)) + \theta \phi(\theta)$. When $\theta \geq 0$ the last equation is clearly positive, so that only the sign for $\theta < 0$ has to be checked. Notice that for $\theta = 0$ the last equation is equal to 0.5, so that it is sufficient to show that its derivative is negative when $\theta < 0$. This is indeed the case, as

$$
\frac{d}{d\theta} g(\theta) = 2\theta (1 - \Phi(\theta)) - 2\theta^2 \phi(\theta).
$$
in the reported hedge fund returns reduces the estimated ES, and the reduction is proportional to the ratio of the two volatility estimates.

— Heavy tails case

Under the same assumptions as before, we have that

\[
\lim_{x \to \infty} \frac{\mathbb{P}(S_t > x)}{(\gamma R \beta^\alpha + \gamma \epsilon)(\sum_{k=0}^K \theta_k^\alpha) x^{-\alpha}} = 1.
\]

(5.25)

If the distribution function is monotonic in the tail area, it holds furthermore that the density satisfies the following asymptotic expansion (see Bingham et al., 1987)

\[
\lim_{x \to \infty} \frac{f_{S_t}(x)}{\alpha(\gamma R \beta^\alpha + \gamma \epsilon)(\sum_{k=0}^K \theta_k^\alpha) x^{-\alpha-1}} = 1.
\]

(5.26)

Hence, for a sufficiently large threshold \(y\) and if \(\alpha > 1\), the ES measure is approximately equal to

\[
\text{ES}(S_t, y; H) \approx \frac{\alpha(\gamma R \beta^\alpha + \gamma \epsilon)(\sum_{k=0}^K \theta_k^\alpha) \int_y^\infty xx^{-\alpha-1} \, dx}{(\gamma R \beta^\alpha + \gamma \epsilon)(\sum_{k=0}^K \theta_k^\alpha) y^{-\alpha}} = \frac{\alpha}{\alpha - 1} y.
\]

(5.27)

This result shows that ES is independent of the smoothing coefficients \(\theta_k\) and of the sensitivity to the market risk \(\beta\), as well as of the scale parameters \(\gamma R\) and \(\gamma \epsilon\). The reason for this independence is that the scale parameters \(\gamma R\) and \(\gamma \epsilon\) affect the expected value of the exceedances and the probability of exceeding the threshold in the same proportion. As a result, both effects cancel each other out. For heavy tails, ES is therefore invariant to smoothing of the returns as well as to the extent to which the returns are sensitive to movements of the stock market.

In the event that \(y = \text{VaR}(S_t, p; H)\), it can be obtained from Eq. (5.27) that

\[
\text{ES}(S_t, \text{VaR}(S_t, p; H); H) \approx \frac{\alpha}{\alpha - 1} \text{VaR}(S_t, p; H).
\]

(5.28)

Danielsson et al. (2006) already obtained the analogous result for the unsmoothed returns. In Eq. (5.28), ES does depend on the smoothing coefficients, the market exposure, and the scale parameters. The reason for this dependence is that the VaR depends on these parameters as well (cf. Eq. 5.18). Consequently, the properties of ES exactly match those of the VaR. In particular, one has:

**Proposition 5.4** If the market factor and idiosyncratic factor exhibit heavy tails as in Eq. (5.6), then

\[
\lim_{p \to 0} \text{ES}(S_t, \text{VaR}(S_t, p; H); H) = \left(\sum_{k=0}^K \theta_k^\alpha\right)^{1/\alpha}.
\]

(5.29)
This last result shows that ES calculated on smoothed returns is always smaller than or equal to ES calculated on actual returns. This result mimics the one found for the VaR metric in Eq. (5.20).

To conclude, both in the case of normally distributed returns and in case of heavy tails, the VaR and ES measures are proportional to each other and are similarly affected by the smoothing because of reporting.

5.2.2 Smoothing effects on multivariate risk measures

In this subsection we investigate two systemic risk measures. Getmansky et al. (2004) already consider how the estimate of the market beta for the single index model is reduced because of smoothing. As their results apply for the normal case and the heavy tail case as long as \( \alpha > 2 \), those results are not reproduced here. Instead, we focus on the correlation coefficient \( \rho \) and the downside systemic risk measure ELM.

**Pairwise correlation**

We investigate the effects of smoothing on the correlation between the reported returns \((S_{1,t}, S_{2,t})\) of two hedge funds. The case of a hedge fund and an equity index would be similar to the case of beta studied by Getmansky et al. (2004) and is therefore not analyzed here. The correlation is defined as

\[
\rho(S_{1,t}, S_{2,t}) = \frac{\text{Cov}(S_{1,t}, S_{2,t})}{(\text{Var}(S_{1,t})\text{Var}(S_{2,t}))^{1/2}}. \tag{5.30}
\]

Assume, as before, that the market return \( R_t \) and the idiosyncratic risk factors \( \varepsilon_{1,t}, \varepsilon_{2,t} \) are i.i.d. and independent at any point in time, with variances given by, respectively, \( \sigma^2_R, \sigma^2_{\varepsilon_1}, \) and \( \sigma^2_{\varepsilon_2} \). In this rather general framework only the second moments of the relevant random variables are required to exist. Standard results imply

\[
\rho(X_{1,t}, X_{2,t}) = \frac{\beta_1 \beta_2 \sigma^2_R}{((\beta_1^2 \sigma^2_R + \sigma^2_{\varepsilon_1})(\beta_2^2 \sigma^2_R + \sigma^2_{\varepsilon_2}))^{1/2}}. \tag{5.31}
\]

For the reported hedge fund returns one has

\[
\text{Cov}(S_{1,t}, S_{2,t}) = \beta_1 \beta_2 \sigma^2_R \sum_{k=0}^{K} \theta_{1,k} \theta_{2,k} = \text{Cov}(X_{1,t}, X_{2,t}) \sum_{k=0}^{K} \theta_{1,k} \theta_{2,k}, \tag{5.32}
\]

and it can be easily shown that

\[
\rho(S_{1,t}, S_{2,t}) = \frac{\sum_{k=0}^{K} \theta_{1,k} \theta_{2,k}}{\left(\left(\sum_{k=0}^{K} \theta_{1,k}^2 \right) \left(\sum_{k=0}^{K} \theta_{2,k}^2 \right)\right)^{1/2}} \rho(X_{1,t}, X_{2,t}). \tag{5.33}
\]
To compare $\rho(X_{1,t}, X_{2,t})$ from Eq. (5.31) with $\rho(S_{1,t}, S_{2,t})$ from Eq. (5.33), recall the Cauchy-Schwarz inequality which holds that for real numbers $\theta_{1,k}$ and $\theta_{2,k}$, $k = 0, \ldots, K$,

\[
\left( \sum_{k=0}^{K} \theta_{1,k} \theta_{2,k} \right)^2 \leq \left( \sum_{k=0}^{K} \theta_{1,k}^2 \right) \left( \sum_{k=0}^{K} \theta_{2,k}^2 \right).
\]  
(5.34)

This result is used to show:

**Proposition 5.5** Suppose that the second moments of the factors of the market model are bounded. Then for the correlation coefficients

\[
\rho(S_{1,t}, S_{2,t}) = \frac{\sum_{k=0}^{K} \theta_{1,k} \theta_{2,k}}{\left( \sum_{k=0}^{K} \theta_{1,k}^2 \right)^{1/2} \left( \sum_{k=0}^{K} \theta_{2,k}^2 \right)^{1/2}} \rho(X_{1,t}, X_{2,t}) \leq \rho(X_{1,t}, X_{2,t}).
\]  
(5.35)

Note that the correlation calculated on the reported returns $\rho(S_{1,t}, S_{2,t})$ equals the correlation calculated on the actual returns $\rho(X_{1,t}, X_{2,t})$ if the actual returns of the two hedge funds show exactly the same pattern of autocorrelation, thus when $\theta_{1,k} = \theta_{2,k}$ for all $k$. Except for this rather exceptional case, the Cauchy-Schwarz inequality implies that the correlation calculated on the smoothed returns is always strictly smaller than the correlation calculated on the actual returns. Therefore, the correlation calculated on reported returns is likely to underestimate the true correlation calculated on actual returns.4

**The extreme linkage measure**

For policy makers and risk managers, it is of utmost interest to understand the dependence of financial institutions when extreme events occur. Although the behavior under normal market circumstances is also of interest, most insight is gained when the greatest shocks occur. Contingency planning and hedging are most relevant when the markets and institutions are in extreme turmoil.

The correlation measure is often criticized as an inappropriate measure of dependence as it is well-known that it performs well for normally distributed variables but much less so when returns are heavy tailed. The concept of correlation is very much tied to the specifics of the multivariate normal distribution but, from the perspective of systemic risk, it tends to give too much weight to observations in the

4If the smoothing coefficients are allowed to be negative, there could also be the extreme case in which $\sum_{k=0}^{K} \theta_{1,k} \theta_{2,k} = 0$, so that $\rho(S_{1,t}, S_{2,t}) = 0$ irrespective of the value of $\rho(X_{1,t}, X_{2,t})$. Hence, the reported returns would always appear to be uncorrelated even if, for example, the correlation between the actual returns equals one.
middle of the distribution. For example, suppose that $Q$ and $W$ are two i.i.d. asset classes and that a hedge fund is long in both exposures while a second hedge fund is long in $Q$ and short in $W$ (such as in a market neutral fund). Then the portfolios $Q+W$ and $Q-W$ have zero correlation and would be independent under normality. However, in the case of the Student’s $t$-distribution the two portfolios are dependent (albeit uncorrelated) because of the outliers of the two portfolios that line up along the two diagonals. Bae et al. (2003) give a clear explanation of the shortcomings of the pairwise correlation measure. Therefore, to study the systemic dependence of hedge fund returns requires turning to another measure.

ELM is a nonparametric measure of dependence based on Extreme Value Theory (EVT). It was introduced by Huang (1992) and has been applied in several empirical studies of systemic risk (see, e.g., Hartmann et al., 2004; Straetmans et al., 2008). ELM is defined as the probability that two hedge funds face losses above a threshold $s$, given that at least one of the funds faces a loss in excess of that same threshold $s$,

$$
ELM(S_{1,t}, S_{2,t}; s) = \frac{\mathbb{P}(S_{1,t} > s, S_{2,t} > s)}{1 - \mathbb{P}(S_{1,t} \leq s, S_{2,t} \leq s)}.
$$

(5.36)

For theoretical purposes, ELM is evaluated in the limit as $s$ tends to infinity,

$$
ELM(S_{1,t}, S_{2,t}) = \lim_{s \to \infty} ELM(S_{1,t}, S_{2,t}; s).
$$

(5.37)

EVT then shows that the value obtained has (empirical) relevance at finite levels, as long as $s$ is very large. If desired, the threshold levels can be easily scaled differently to account for differences in capital or size of the institution.

It can be shown that this measure is equal to the expected number of hedge funds that are stressed, $n$, given that at least one of the hedge funds is stressed, minus one:

$$
\mathbb{E}[n|n \geq 1] = 1 - \frac{\mathbb{P}(S_{1,t} > s, S_{2,t} \leq s)}{1 - \mathbb{P}(S_{1,t} \leq s, S_{2,t} \leq s)} + \frac{\mathbb{P}(S_{1,t} > s, S_{2,t} > s)}{1 - \mathbb{P}(S_{1,t} \leq s, S_{2,t} \leq s)}
$$

$$
= \frac{\mathbb{P}(S_{1,t} > s) + \mathbb{P}(S_{2,t} > s)}{1 - \mathbb{P}(S_{1,t} \leq s, S_{2,t} \leq s)}
$$

$$
= 1 + ELM(S_{1,t}, S_{2,t}; s).
$$

(5.38)

In fact, $\mathbb{E}[n|n \geq 1]$ readily applies to higher dimensions, by extending Eq. (5.38). In the bivariate case, moreover, ELM can also be expressed as

$$
ELM(S_{1,t}, S_{2,t}; s) = \frac{\mathbb{P}(\min(S_{1,t}, S_{2,t}) > s)}{\mathbb{P}(\max(S_{1,t}, S_{2,t}) > s)}.
$$

(5.39)
where the variables \( \min(S_{1,t}, S_{2,t}) \) and \( \max(S_{1,t}, S_{2,t}) \) denote the minimum and the maximum values of the variables \( S_{1,t} \) and \( S_{2,t} \). By counting the number of excesses in the numerator and denominator of Eq. (5.39) at high levels of \( s \), a simple non-parametric count estimator of ELM is obtained.

— Normal case

Given the previous assumptions for the single index model, the \( X_{1,t} \) and \( X_{2,t} \) are multivariate normally distributed with correlation \( \rho \) and standard deviations \( \sigma_1 \) and \( \sigma_2 \), respectively. To derive ELM, we adopt the proof of Sibuya (1960). Note that by elementary manipulations

\[
E[n | n \geq 1] = \frac{\mathbb{P}(S_{1,t} > s) + \mathbb{P}(S_{2,t} > s)}{1 - \mathbb{P}(S_{1,t} \leq s, S_{2,t} \leq s)}
\]

\[= \frac{1}{1 - \frac{\mathbb{P}(S_{1,t}>s,S_{2,t}>s)}{\mathbb{P}(S_{1,t}>s)+\mathbb{P}(S_{2,t}>s)}} \]

\[\leq \frac{1}{1 - \frac{\mathbb{P}(S_{1,t}+S_{2,t}>2s)}{\mathbb{P}(S_{1,t}>s)+\mathbb{P}(S_{2,t}>s)}}, \quad (5.40)\]

as the line \( S_{1,t} + S_{2,t} = 2s \) cuts the slab \( (S_{1,t} > s, S_{2,t} > s) \) from below. Note that \( (S_{1,t} + S_{2,t})/2 \) has variance equal to \( (\sigma_1^2 + \sigma_2^2 + 2\sigma_1\sigma_2\rho)/4 \), which is strictly smaller than \( \max(\sigma_1^2, \sigma_2^2) \) as long as \( \rho < 1 \). The classical Laplace’s tail expansion of a standard normal distribution \( \Phi(s) \) with density \( \phi(s) \) holds that, for large \( s \), \( 1 - \Phi(s) \approx \phi(s)/s \). It then follows that

\[
\lim_{s \to \infty} \frac{\mathbb{P}((S_{1,t} + S_{2,t})/2 > s)}{\mathbb{P}(S_{1,t} > s) + \mathbb{P}(S_{2,t} > s)} = 0, \quad (5.41)
\]

as the rate of the exponential decay of the density of the sum (divided by two), dictated by the inverse of its variance, is greater than the rate of the exponential decay of at least one of the individual probabilities. Hence, \( E[n | n \geq 1] = 1 \) for large \( s \) and \( ELM(S_{1,t}, S_{2,t}) = 0 \). As the proof does not depend on the particular values of the variances and correlation, it immediately follows that \( ELM(X_{1,t}, X_{2,t}) = 0 \) as well. In summary:

**Proposition 5.6** In the case of normally distributed returns, the correlation measure summarizes the interdependency and is reduced because of smoothing. ELM tends to be nondiscriminatory and is not affected by smoothing.

Thus in the case of light tails, ELM is uninformative, as it does not depend on the particular values of the variances and the correlation, nor does it hinge on the
moving average parameters.\(^5\) The global correlation measure is more informative in the case of light tails. ELM simply pierces too deeply into tails. This outcome is in sharp contrast with the case of fat tails.

— Heavy tails case

For the heavy tail case, assume again that
\[
\lim_{s \to \infty} \frac{\mathbb{P}(R_t > s)}{\gamma_R s^{-\alpha}} = \lim_{s \to \infty} \frac{\mathbb{P}(\varepsilon_{1,t} > s)}{\gamma_{\varepsilon_1} s^{-\alpha}} = \lim_{s \to \infty} \frac{\mathbb{P}(\varepsilon_{2,t} > s)}{\gamma_{\varepsilon_2} s^{-\alpha}} = 1, \tag{5.42}
\]
where the scale parameters \(\gamma_R, \gamma_{\varepsilon_1}, \gamma_{\varepsilon_2}\) are strictly positive constants. For values of the threshold \(s\) high enough such that Feller’s theorem provides for a good approximation for the convolution of the random variables, one has
\[
\mathbb{P}(X_{1,t} > s) \simeq (\beta_1^a \gamma_R + \gamma_{\varepsilon_1}) s^{-\alpha}, \tag{5.43}
\]
\[
1 - \mathbb{P}(X_{1,t} \leq s, X_{2,t} \leq s) \simeq (\gamma_{\varepsilon_1} + \gamma_{\varepsilon_2} + \max(\beta_1, \beta_2)^a \gamma_R) s^{-\alpha}. \tag{5.44}
\]
The first expression in Eq. (5.43) is a straightforward application of Feller’s theorem as used in Eq. (5.17) and explained above in Eq. (5.8). With equal tail indices, Feller’s theorem basically states that sufficiently far from the origin the probability mass above a hyperplane or a multidimensional figure of any shape that separates the space into two parts, is determined by the mass along the axes above this hyperplane or multidimensional figure. So one can just add the univariate probability mass loaded on these axes above the points where the figure cuts the axes.

For the second expression in Eq. (5.44), the three axes with univariate probability mass are the two idiosyncratic risk factors and the one with market risk. The boundary of \(1 - \mathbb{P}(X_{1,t} \leq s, X_{2,t} \leq s)\) is a pyramid shaped figure with respective boundaries of \(\gamma_{\varepsilon_1} s^{-\alpha}, \gamma_{\varepsilon_2} s^{-\alpha}\) and \(\max(\beta_1, \beta_2)^a \gamma_R s^{-\alpha}\). Summation then yields the right-hand side of Eq. (5.44).\(^6\)

From Eq. (5.38) we then have
\[
\text{ELM}(X_{1,t}, X_{2,t}) = \frac{\beta_1^a \gamma_R + \gamma_{\varepsilon_1} + \beta_2^a \gamma_R + \gamma_{\varepsilon_2}}{\gamma_{\varepsilon_1} + \gamma_{\varepsilon_2} + (\max(\beta_1, \beta_2))^a \gamma_R} - 1
\]
\[
= \frac{(\min(\beta_1, \beta_2))^a \gamma_R}{\gamma_{\varepsilon_1} + \gamma_{\varepsilon_2} + (\max(\beta_1, \beta_2))^a \gamma_R}, \tag{5.45}
\]
\(^5\)This conclusion is true also in the case in which \(\rho = 1\). It is easy to verify that even in that special case, the previous results still hold as long as \(\sigma_1 \neq \sigma_2\). Moreover, if \(\sigma_1 = \sigma_2\) then \(\mathbb{E}[|n| n \geq 1] = 2\) for large \(s\) and \(\text{ELM}(S_{1,t}, S_{2,t}) = 1\). These results imply that ELM is uninformative anyway.
\(^6\)See Proposition 5.10 in Appendix 5.B for a formal proof of Eq. (5.44).
For the actual hedge fund returns $S_{1,t}$ and $S_{2,t}$ the following equations apply

$$\mathbb{P}(S_{i,t} > s) = \mathbb{P}(X_{i,t} > s) \sum_{k=0}^{K} \theta_{i,k}^\alpha = (\beta_i^\alpha \gamma_R + \gamma_{\varepsilon_i}) s^{-\alpha} \sum_{k=0}^{K} \theta_{i,k}^\alpha, \quad (5.46)$$

$$1 - \mathbb{P}(S_{1,t} \leq x, S_{2,t} \leq x) = \sum_{k=0}^{K} \left( \theta_{1,k}^\alpha \gamma_{\varepsilon_1} + \theta_{2,k}^\alpha \gamma_{\varepsilon_2} + \left( \max(\beta_1 \theta_{1,k}, \beta_2 \theta_{2,k}) \right)^\alpha \gamma_R \right) s^{-\alpha}, \quad (5.47)$$

so that

$$\text{ELM}(S_{1,t}, S_{2,t}) = \frac{(\beta_1^\alpha \gamma_R + \gamma_{\varepsilon_1}) \sum_{k=0}^{K} \theta_{1,k}^\alpha + (\beta_2^\alpha \gamma_R + \gamma_{\varepsilon_1}) \sum_{k=0}^{K} \theta_{2,k}^\alpha}{\sum_{k=0}^{K} \left( \theta_{1,k}^\alpha \gamma_{\varepsilon_1} + \theta_{2,k}^\alpha \gamma_{\varepsilon_2} + \left( \max(\beta_1 \theta_{1,k}, \beta_2 \theta_{2,k}) \right)^\alpha \gamma_R \right)} - 1$$

$$= \frac{\sum_{k=0}^{K} \left( \theta_{1,k}^\alpha \gamma_{\varepsilon_1} + \theta_{2,k}^\alpha \gamma_{\varepsilon_2} + \left( \max(\beta_1 \theta_{1,k}, \beta_2 \theta_{2,k}) \right)^\alpha \gamma_R \right)}{\sum_{k=0}^{K} \left( \theta_{1,k}^\alpha \gamma_{\varepsilon_1} + \theta_{2,k}^\alpha \gamma_{\varepsilon_2} + \left( \max(\beta_1 \theta_{1,k}, \beta_2 \theta_{2,k}) \right)^\alpha \gamma_R \right)} \cdot \left( 1 + (1 + \beta_2) \sum_{k=0}^{K} \theta_{2,k}^\alpha / \sum_{k=0}^{K} \theta_{1,k}^\alpha \right). \quad (5.48)$$

Note that Eq. (5.48) simplifies to Eq. (5.45) when both of the hedge funds’ actual returns are smoothed in exactly the same way, that is, when $\theta_{1,k} = \theta_{2,k}$ for all $k$. In this case, as in the pairwise correlation, the measure of linkage calculated on reported returns is equal to that calculated on actual returns.

We have already shown that the presence of autocorrelation reduces SR, VaR, ES, and pairwise correlation, both in the case of normally distributed returns and in case of heavy tails. In the case of heavy tails, we have the following corresponding result for ELM (proven in Appendix 5.D):

**Proposition 5.7** Suppose that the hedge fund returns $X_{1,t}$ and $X_{2,t}$ have the same market exposure as $\beta_1 = \beta_2 = \beta$. Moreover, the smoothed $S_{1,t}$ and $S_{2,t}$ both follow MA($K$) processes, possibly with different coefficients. Then, ELM based on the reported returns is lower than the true ELM if not all smoothing coefficients are equal.

However, if the market betas of the hedge funds are sufficiently different, then ELM of the smoothed returns can be larger than ELM of the true underlying returns. To show this outcome, consider the case in which the scale parameters $\gamma_R$, $\gamma_{\varepsilon_1}$, $\gamma_{\varepsilon_2}$ all equal one and $\beta_1 \theta_{1,k} < \beta_2 \theta_{2,k}$ for all $k$. In this case, Eq. (5.48) implies

$$\text{ELM}(S_{1,t}, S_{2,t}) = \frac{\beta_1^\alpha \sum_{k=0}^{K} \theta_{1,k}^\alpha}{\sum_{k=0}^{K} \theta_{1,k}^\alpha + (1 + \beta_2) \sum_{k=0}^{K} \theta_{2,k}^\alpha} \cdot \frac{\beta_1^\alpha \sum_{k=0}^{K} \theta_{2,k}^\alpha}{\sum_{k=0}^{K} \theta_{2,k}^\alpha} \cdot \left( 1 + (1 + \beta_2) \sum_{k=0}^{K} \theta_{2,k}^\alpha / \sum_{k=0}^{K} \theta_{1,k}^\alpha \right). \quad (5.49)$$
Equation (5.49) shows that ELM depends on the ratio $\sum \theta_{2,k}/\sum \theta_{1,k}$. The presence of autocorrelation can either increase or decrease the estimated ELM as compared with the no-smoothing case when $\theta_{1,0} = \theta_{2,0} = 1$ and $\theta_{1,k} = \theta_{2,k} = 0$ for $k = 1, \ldots, K$. Therefore, we have:

**Proposition 5.8** If the factors of the market model for two series of returns exhibit heavy tails as in Eq. (5.6), then it is not possible to establish ex-ante the impact of smoothing on ELM of the two series.

In summary, if the market betas of the hedge funds are very similar, then it can be expected that the reported returns induce a lower measure of systemic risk than is factual. However, in general, the sign of the impact of smoothing cannot be established a priori for ELM and the smoothing coefficients must be estimated to determine which direction the reported returns bias the systemic risk measure.

### 5.3 Empirical methodology

The theory developed in Section 5.2 is applied using data from Hedge Fund Research (HFR). HFR identifies four primary strategy classes of hedge funds: Equity Hedge, Event-Driven, Macro, and Relative Value. We use HFR’s equally weighted total return indices denominated in US dollars for each of these four classes, which together include around 2,000 hedge funds. The HFR’s web site contains additional information on these indices. For the empirical analysis, monthly returns across the period January 1990 to August 2009 are used, which amounts to 236 months in total.

Figure 5.1 reports the cumulative returns of the four indices and the Standard and Poor’s 500 (S&P 500) total return index. It illustrates the main reason for the success and growth of the hedge fund industry. Over the last two decades, each of the four indices has outperformed the S&P 500 index by showing higher returns as well as lower volatility. The most striking period is the last decade where the cumulative return of the S&P 500 index since 1999 has been negligible, while the four hedge fund indices have delivered large positive cumulative returns. It can also be gleaned from Figure 5.1 that the recent financial crisis has affected the main hedge fund indices markedly less than the S&P 500 index. The key statistics are reported in Table 5.2.

Table 5.2 shows that the hedge fund indices have SRs that are four to six times larger than that of the S&P 500 index. Table 5.2 also illustrates that volatility is

\[7\text{See www.hedgefundresearch.com.}\]
Figure 5.1 – Cumulative monthly log-returns of four hedge fund indices for the period January 1990 to August 2009
Source: Hedge Funds Research and Thomson Reuters Datastream.

Table 5.2 – Descriptive statistics of hedge fund index returns
All statistics are based on monthly log-returns for the period January 1990 to August 2009. Mean, standard deviation and Sharpe ratio are annualized. The Sharpe ratio is calculated using the USD 3-month Libor as the risk-free rate. The Jarque-Bera statistic tests the hypothesis of normality of the returns, which is the joint hypothesis that skewness = 0 and kurtosis = 3; significance levels are denoted by ** for 1% (critical value = 11.69) and * for 5% (critical value = 5.72).

<table>
<thead>
<tr>
<th>Investment strategy</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>Sharpe ratio</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Jarque-Bera statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity hedge</td>
<td>13.19</td>
<td>9.22</td>
<td>0.96</td>
<td>−0.38</td>
<td>5.08</td>
<td>48.43**</td>
</tr>
<tr>
<td>Event-driven</td>
<td>11.56</td>
<td>7.02</td>
<td>1.03</td>
<td>−1.50</td>
<td>7.89</td>
<td>323.64**</td>
</tr>
<tr>
<td>Macro</td>
<td>13.14</td>
<td>7.70</td>
<td>1.15</td>
<td>0.35</td>
<td>3.79</td>
<td>11.02*</td>
</tr>
<tr>
<td>Relative value</td>
<td>10.00</td>
<td>4.55</td>
<td>1.25</td>
<td>−2.46</td>
<td>18.45</td>
<td>2,586.47**</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>7.54</td>
<td>15.23</td>
<td>0.21</td>
<td>−0.84</td>
<td>4.74</td>
<td>57.79**</td>
</tr>
</tbody>
</table>

not an appropriate risk indicator for hedge fund indices. For instance, the Relative Value index has the lowest standard deviation and the highest SR, but it has also an extremely negative skewness and a very large kurtosis. Next, Figure 5.2 shows the monthly returns of the four indices.

The four plots in Figure 5.2 highlight two other features of hedge fund returns that are particularly important when studying systemic risk. First, although returns are stable and mostly positive, outliers occasionally do occur. In most cases these outliers are negative. Second, the negative outliers tend to occur simultaneously. An example is found in August 1998, caused by the Russian crisis and the LTCM
collapse. Another example of strong negative co-movement of hedge funds occurs in the last quarter of 2008, after the failure of Lehman Brothers.

This kind of extreme dependence is seen more clearly in Figure 5.3 which reports a scatter plot of the returns of the Equity Hedge and Event-Driven indices. In Figure 5.3, the hedge fund total return series have been transformed to have uniform marginal distributions. The elliptical shape of the plot indicates that the returns of the two indices tend to move together. More important, the greater density of points in the lower-left part of the plot highlights that the most negative extreme returns of the two indices tend to occur simultaneously.

5.3.1 Estimation of the smoothing coefficients

In this subsection, the smoothing coefficients $\theta_k$ are estimated. Before doing this, we first inspect the raw returns data and their squares for their autocorrelation properties. All four series exhibit significant $MA$ behavior. The Macro index only displays $MA(1)$, while the other three strategy indices display significant $MA(2)$ behavior, as in Getmansky et al. (2004). Some ARCH effects are also present in the raw series. Following Getmansky et al. (2004), we therefore applied both maximum likelihood and linear regression estimations to an $MA(2)$ smoothing model. More precise, in the maximum likelihood estimation we assume the process of observed returns to be $S_t = \theta_0 X_t + \theta_1 X_{t-1} + \theta_2 X_{t-2}$, with $X_t$ i.i.d., $\theta_0 + \theta_1 + \theta_2 = 1$ and $\theta_k \in [0, 1]$, for $k = 0, 1, 2$. In our numerical procedures, we do not explicitly impose the additional restrictions $\theta_1 < 1/2$ and $\theta_1 < 1 - 2\theta_2$, which ensure that the $MA(2)$ process is invertible (cf. Proposition 3 in Getmansky et al., 2004). However, our estimates satisfy these restrictions in all cases. Furthermore, $X_t$ is not required to be normally distributed, as in Getmansky et al. (2004). Nevertheless, we are still allowed to exploit the asymptotic normality of the maximum likelihood estimator (cf. Paragraph 8.8 in Brockwell and Davis, 1991).

Getmansky et al. (2004) also argue that consistent, although not efficient, estimates of the smoothing coefficients $\theta_k$, $k = 0, 1, 2$, can be obtained by running ordinary least squares regressions of the equation

$$S_t = \mu + \beta(\theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}) + u_t,$$

where $u_t = \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$, with $\varepsilon_t$ i.i.d., $\theta_0 + \theta_1 + \theta_2 = 1$ and $\theta_k \in [0, 1]$, for $k = 0, 1, 2$. In this case, $\hat{\beta} = \gamma_0 + \gamma_1 + \gamma_2$ and $\hat{\theta}_k = \gamma_k/\hat{\beta}$.

To have efficient estimates, Getmansky et al. (2004) suggest using the maximum likelihood estimator. However, in our case the results are almost identical for both the OLS and the maximum likelihood estimators.
Figure 5.2 – Monthly log-returns of four hedge fund indices for the period January 1990 to August 2009
Source: Hedge Fund Research.
Sect. 5.3 – Empirical methodology

Figure 5.3 – Normalized monthly returns of two hedge fund indices
Scatter plot of the pairs \((y_{1,t}, y_{2,t})_{t=1,...,236}\), where \(y_{1,t} = \hat{F}_{X_1}(x_{1,t})\), \(y_{2,t} = \hat{F}_{X_2}(x_{2,t})\), and \(\hat{F}_{X_1}\) and \(\hat{F}_{X_2}\) are the empirical cumulative distribution functions of the monthly log-returns of the Equity Hedge and Event-Driven indices for the period January 1990 to August 2009. The plot shows the empirical joint distribution of the returns of the two indices that have been normalized to have uniform marginal distributions.

factors to the hedge fund model could increase the model’s explanatory power (see Hasanhodzic and Lo, 2007). However, our focus is to investigate the proposed methodological improvements to hedge fund risk measures, for which a direct comparison with Getmansky et al. (2004) gives the most clarity. Therefore, we use the S&P 500 index as the only underlying factor in the regressions.

After applying the \(MA(2)\) correction, we find that using the maximum likelihood estimation eliminates the linear correlation structure. But the OLS estimates in case of the Event-Driven strategy, for example, still display some \(MA(1)\) behavior, while this is not the case for the Equity Hedge strategy. The reason is that the S&P500 index is not necessarily the most appropriate factor for all strategies. For the Equity Hedge index, some non-linear dependence in the squared unsmoothed returns was present as well. But since our downside risk measures are unconditional and the estimation procedure is consistent under ARCH effects, this should not thwart our results.

Table 5.3 reports the estimated smoothing coefficients for the four indices and the S&P 500 index using both the maximum likelihood and the linear regression estimators.

The estimates of both methods are similar. Moreover, the results are comparable with those in Getmansky et al. (2004), even though they use different indices. It can be gleaned from Table 5.3 that the estimated smoothing coefficients \(\hat{\theta}_1\) and \(\hat{\theta}_2\) are
Table 5.3 – Estimates of the smoothing coefficients
All estimates are based on monthly log-returns for the period January 1990 to August 2009. The table shows maximum likelihood and linear regression estimates of an MA(2) smoothing process $S_t = \theta_0 X_t + \theta_1 X_{t-1} + \theta_2 X_{t-2}$. In the linear regression case, the underlying hypothesis is that $X_t$ is defined by a linear single-factor model in which the factor is the Standard and Poor’s 500 (S&P 500) index. Significance levels for the maximum likelihood estimates are calculated using the asymptotic result of Theorem 3 in Getmansky et al. (2004) (significance levels are denoted by ** for 1% and * for 5%). The null hypothesis for $\hat{\theta}_0$ is that it is equal to one, those for $\hat{\theta}_1$, $\hat{\theta}_2$ are that they are equal to zero.

<table>
<thead>
<tr>
<th>Investment strategy</th>
<th>Maximum likelihood</th>
<th>Linear regression</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$\hat{\theta}_0$</td>
<td>$\hat{\theta}_1$</td>
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<tr>
<td>Equity hedge</td>
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<td>Event-driven</td>
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</tr>
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<td>Macro</td>
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</tr>
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<td>Relative value</td>
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<td>0.262**</td>
</tr>
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<td>S&amp;P 500</td>
<td>0.923</td>
<td>0.089</td>
</tr>
</tbody>
</table>

Statistically different from zero for all four hedge fund indices. The only exception is parameter estimate $\hat{\theta}_2$ for the Macro index. Conversely, parameter estimate $\hat{\theta}_0$ is always statistically different from one. The linear regression estimation approach allow also to estimate the parameter $\beta$, which measures the sensitivity of the hedge fund index returns to the market index returns.

These estimates are now used to determine how smoothing affects SR. Table 5.2 reports that the uncorrected SRs for the Equity Hedge and Event-Driven indices equal 0.96 and 1.03, respectively. The correction for smoothing effects takes the same approach as Proposition 1 in Getmansky et al. (2004). We use the parameter values obtained through the maximum likelihood estimation method. After unsmoothing the returns, SRs drop approximately 20% and 30%, respectively, to a value of 0.74 for both indices. Although results are not directly comparable, it is worth noting that this large change in SR exceeds the impact of unsmoothing found by Getmansky et al. (2004, p. 588).\footnote{One difference between this empirical exercise and Getmansky et al. (2004) is that we use data from HFR for the period 1990–2009 while Getmansky et al. (2004) use data from TASS for the period 1977–2001. Moreover, we run the estimators on indices while the other authors use data on individual funds. Finally, our SR is calculated with respect to the USD 3-month Libor rate while Getmansky et al. (2004) use a zero interest rate benchmark.}

Next, two hedge fund indices are selected to illustrate the empirical relevance of the risk measurement correction methodology presented in this chapter. Table 5.3 shows that the adjusted $R^2$ is highest for the Equity Hedge and Event-Driven indices at 0.54 and 0.58, respectively. The $R^2$ is much lower for the Macro and the Relative...
Value indices at 0.11 and 0.34, respectively. For reasons of brevity, the empirical applications in the remainder of the chapter are limited to the Equity Hedge and Event-Driven indices only.

Because hedge funds change their investment exposures frequently, it is likely that the smoothing coefficients are not constant over time. As a result the calculation of the risk measures could be affected as well. For this reason we also estimate the smoothing coefficients using rolling windows of 60 months. Figures 5.10 and 5.11 in Appendix 5.E report the parameter estimates for both the Equity Hedge and Event-Driven indices, using the maximum likelihood as well as the linear regression approach.

We find that the coefficients tend to remain fairly stable across the sample period 1995–2009. Furthermore, the resulting parameter levels are quite similar across the two estimation methods. These results provide support to the choice of the S&P 500 index as underlying risk factor in the estimates.

### 5.3.2 Estimation of the tail index and scale parameters

How does the presence of autocorrelation affect the various downside risk measures when the market return and the idiosyncratic risk factors follow a heavy tail distribution? To answer this question, we first estimate the tail index ($\alpha$) and the scale parameters ($\gamma_R, \gamma_{\varepsilon_1}, \gamma_{\varepsilon_2}$). Thereafter, we calculate VaR using Eqs. (5.18) and (5.19) and ELM from Eqs. (5.45) and (5.48).

To estimate the tail index, we use the standard Hill (1975) estimator

$$\frac{1}{\hat{\alpha}} = \frac{1}{m} \sum_{i=1}^{m} \log \left( \frac{X_{(i)}}{X_{(m+1)}} \right),$$

(5.51)

where the $X_{(i)}$ are the largest descending order statistics $X_{(1)} \geq X_{(2)} \geq \cdots \geq X_{(m)} \geq X_{(m+1)} \geq X_{(m+2)} \geq \cdots \geq X_{(n)}$ of the sample of $n$ return observations $X_1, \ldots, X_n$.\(^{10}\)

Parameter $m$ equals the number of extreme returns exceeding $X_{(m+1)}$, which is the threshold return level above which the Pareto approximation applies (see Jansen and de Vries, 1991; Embrechts et al., 1997).

The scale parameter $\gamma$ is estimated as

$$\hat{\gamma} = \frac{m}{n} (X_{(m+1)})^{\hat{\alpha}}.$$  

(5.52)

The idea behind Eq. (5.52) is to approximate the probability $\mathbb{P}(S > X_{m+1}) \simeq \gamma (X_{m+1})^{-\alpha}$ by its empirical value $m/n$.

\(^{10}\)The Hill estimator is consistent in the presence of autocorrelation in the returns or their squares; see, e.g., Drees (2008).
For the market factor $R$, the estimates of its tail index and its scale parameter are obtained by inserting the order statistics of the S&P 500 index into Eqs. (5.51) and (5.52). Before the tail index and the scale estimates for the residuals $\varepsilon_t$ can be obtained, the residuals themselves need to be estimated first. Accordingly, we first calculate the values

$$
\hat{u}_t = S_t - \left( \hat{\mu} + \hat{\beta} \left( \hat{\theta}_0 R_t + \hat{\theta}_1 R_{t-1} + \hat{\theta}_2 R_{t-2} \right) \right),
$$

(5.53)

where $\hat{\mu}, \hat{\beta}, \hat{\theta}_0, \hat{\theta}_1, \hat{\theta}_2$ are estimates obtained by the linear regression method described in Section 5.3.1. Given the assumption that $u_t = \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$, the residuals are obtained recursively using

$$
\hat{\varepsilon}_t = \left( \hat{u}_t - \hat{\theta}_1 \hat{\varepsilon}_{t-1} - \hat{\theta}_2 \hat{\varepsilon}_{t-2} \right) / \hat{\theta}_0.
$$

(5.54)

Hence, tail indices and the scale parameters for the residuals are estimated by inserting the order statistics of the estimated residuals from Eq. (5.54) into the Hill estimator from Eq. (5.51) and the scale estimator from Eq. (5.52).

Given the relatively small sample size of only 236 monthly observations, Hill plots are used to determine the number of higher left tail order statistics to be used in the estimators from Eqs. (5.51) and (5.52). A Hill plot depicts the value of the Hill estimator of $\alpha$ as a function of the number $m$ of extreme returns above the threshold $X_{m+1}$. It is used to select the number of higher order statistics such that the variance and bias squared are balanced in order to minimize the mean squared error (see Embrechts et al., 1997, for a description). The Hill plots for the hedge fund and stock indices returns and hedge fund indices residuals are shown if Figure 5.4. In all cases the tail index seems to hover around three for reasonably high levels of the threshold. This value is similar to the estimates provided by Jansen and de Vries (1991) and Hyung and de Vries (2005) for individual US stocks. So, $\alpha = 3$ is set for both the S&P 500 index and the residuals of the hedge-fund indices in the calculations that follow.

To estimate the scale parameters, we use a method analogous to the Hill plot, but with the $\gamma$ estimates on the vertical axis (fixing $\alpha$ at 3; see Figure 5.5). As the plots for the tail indices turn out to be relatively stable for $m \leq 24$ (corresponding to 10% of the observations) the scale parameters are set equal to the average of their estimates obtained using Eq. (5.52) across all $m \leq 24$ extreme returns

$$
\hat{\gamma} = \frac{1}{24} \sum_{k=1}^{24} \frac{k}{n} \left( X_{(k+1)} \right)^3.
$$

(5.55)
Figure 5.4 – Hill plot of the tail index of hedge fund and stock indices returns and of hedge fund indices residuals

Estimates of the tail index, obtained using the Hill estimator, as a function of the number of extreme returns used in the estimation. The vertical lines correspond to the 97.5, 95.0 and 90.0 percentile of the whole dataset, respectively. Estimates are based on data from the left tail of the distribution of the returns (i.e., only negative returns are taken into account).

For the scale parameters $\gamma_R$ (for the stock index), $\gamma_{\epsilon_1}$ (for the idiosyncratic term of the Equity Hedge index), and $\gamma_{\epsilon_2}$ (for the idiosyncratic term of the Event-Driven index), these mean estimates equal 48.0, 5.0, and 2.6, respectively.

This section can be concluded with a few remarks regarding the reliability of the methodology and the results that will be presented in Section 5.4. Following the previous discussion, it could be argued that the approach used in this chapter suffers from a potential small sample bias. Usually EVT estimates are based on relatively large amounts of data. Per contrast, our sample contains only 236 observations which can leave the results vulnerable to biases. Moreover, the statistical significance of the results could be weak because of the limited number of observations available. Therefore in Appendix 5.F, we investigate how the adjusted EVT estimators are
Panel A: Hedge fund and stock indices returns

Panel B: Hedge fund indices residuals

Figure 5.5 – Estimates of the scale index of hedge fund and stock indices returns and of hedge fund indices residuals

Estimates of the scale index as a function of the number of extreme returns used in the estimation. The reported scale parameters are calculated under the assumption that the tail index is always equal to 3. The vertical lines correspond to the 97.5, 95.0 and 90.0 percentile of the whole dataset, respectively. The straight lines represent the average values of the estimates across the worst 10% of the observations. Estimates are based on data from the left tail of the distribution of the returns (i.e., only negative returns are taken into account).

affected by the small sample size. In short, two main conclusions are drawn from the Monte Carlo simulation analysis. First, the simulations show that the estimates for VaR and ELM indeed tend to be biased in small samples. The study shows that the bias is very small for VaR and somewhat larger for ELM. In both cases, the bias tends to disappear as the sample size increases. Second, the confidence intervals tend to be fairly wide in small samples but they significantly decrease in larger samples. Hence, we conclude that our empirical estimates could be somewhat affected by the small size of the sample but any potential bias will likely be fairly small and will not affect the main findings.
5.4 Effects of autocorrelation on risk measures

This section analyzes how autocorrelation affects various risk measures. As hedge funds frequently modify their investment exposures, it is likely that the smoothing coefficients change over time as well. To capture these time-varying changes, rolling windows of 60 months are used in most of the following calculations.

5.4.1 Univariate measures of risk

We start with calculating VaR. Figure 5.6 reports the time series estimates of the reported and uncorrected VaR($S, p$) as well as the true and unobservable VaR($X, p$) for the Equity Hedge and Event-Driven indices, calculated under the hypothesis that returns are normally distributed (cf. Eq. 5.15). Figure 5.7 depicts similar estimates, but instead of assuming normal returns, fat tailed returns are assumed (cf. Eq. 5.19).

A comparison of the Figures 5.6 and 5.7 yields a number of interesting conclusions. First, the uncorrected and corrected VaRs often increase at the same points in time. Apparently the dynamics of the two VaR series are fairly similar, although the corrected VaR shows larger jumps. It is intuitive that the estimates of VaR for unsmoothed returns exhibit larger jumps.

Second, the levels of the two measures of VaR are substantially different. On several occasions the corrected VaR exceeds the uncorrected VaR by 50% or more. For both indices the differences between the uncorrected and corrected VaR levels increase substantially in 2009. The reason for these increases is that the presence of smoothing becomes stronger at the end of the sample period. This effect is also evident in Figure 5.10, which shows that the value of $\theta_0$ decreases in 2008 and 2009.

Third, VaRs are markedly higher when hedge fund returns are assumed to be fat tailed in comparison with the case of a normal distribution. The levels of VaRs in Figure 5.7 clearly exceed the corresponding VaRs under normality in Figure 5.6. Both the corrected and uncorrected series show such an effect. The normal distribution underrepresents the mass in the tails of the returns distributions.

Finally, our fourth conclusion is that the difference between the corrected and uncorrected VaR series is larger when the distribution is fat tailed (see, e.g., Figure 5.6, Panel A, and Figure 5.7, Panel A), although the adjustment factors in Eqs. (5.24) and (5.29) at the parameters values that we estimated are not so much different. For example, using the values for the mean and the standard deviation over the entire sample period from Table 5.2 and the corresponding regression-based point estimates for the smoothing coefficients $\theta_k$ from Table 5.3, for the Equity Hedge index the difference between the smoothed and unsmoothed VaR is 2.09 percentage
Figure 5.6 – Value-at-risk for the Equity Hedge and Event-Driven indices at the 99% confidence level under the assumption of normally distributed monthly returns

The uncorrected VaR, which is based on reported returns, and the corrected VaR, which is based on the true unobservable returns, are equal, respectively, to \( \sigma_S \Phi^{-1}(1-p) + \mu_S \) and \( \sigma_X \Phi^{-1}(1-p) + \mu_S \), where \( 1-p \) is the confidence level, \( \sigma_S \) is the volatility of the reported returns, \( \mu_S \) is the mean of the reported returns, and \( \sigma_X = \sigma_S \left( \sum_{k=0}^{2} \theta_k^2 \right)^{1/2} \). The maximum likelihood estimate (MLE) of the smoothing coefficients (\( \theta_k \)) is based on the model \( S_t = \sum_{k=0}^{2} \theta_k X_{t-k} \) in which returns reported by hedge funds (\( S_t \)) are weighted averages of current and past unobservable returns (\( X_t \)) that are assumed to be i.i.d. The linear regression estimates (ordinary least squares [OLS]) make the additional assumption that \( X_t = \beta R_t + \varepsilon_t \), where \( R_t \) is the market return and \( \varepsilon_t \) is an i.i.d. idiosyncratic risk factor. In both cases, the smoothing coefficients are assumed to satisfy the constraints \( \theta_k \in [0,1] \), for \( k = 0, 1, 2 \), and \( \sum_{k=0}^{2} \theta_k = 1 \). In the OLS estimation, the Standard and Poor’s 500 total return index is used as market factor. The estimations are based on rolling windows of 60 months ending in the reference month.
Fig. 5.7 – Value-at-risk for the Equity Hedge and Event-Driven indices at the 99% confidence level under the assumption of fat tailed distributed monthly returns

The uncorrected VaR, which is based on reported returns, and the corrected VaR, which is based on the true unobservable returns, are equal, respectively, to \((\gamma_R^\alpha + \gamma_\varepsilon)(\sum_{k=0}^{\alpha} \theta_k)/p\)\(^{\frac{1}{\alpha}}\) and \((\gamma_R^\alpha + \gamma_\varepsilon)/p\)\(^{\frac{1}{\alpha}}\), where \(1 - p\) is the confidence level. The smoothing coefficients \((\theta_k)\) and the market exposure \((\beta)\) are estimated by linear regressions (ordinary least squares \([\text{OLS}]\)) of the model \(S_t = \sum_{k=0}^{2} \theta_k X_{t-k}\) in which returns reported by hedge funds \((S_t)\) are weighted averages of current and past unobservable returns \((X_t)\). Unobservable returns are assumed to be of the form \(X_t = \beta R_t + \varepsilon_t\), where \(R_t\) is the market return and \(\varepsilon_t\) is an i.i.d. idiosyncratic risk factor. The smoothing coefficients are assumed to satisfy the constraints \(\theta_k \in [0, 1]\), for \(k = 0, 1, 2\), and \(\sum_{k=0}^{2} \theta_k = 1\). The estimations use the Standard and Poor’s 500 total return index as market factor and are based on rolling windows of 60 months ending in the reference month. The tail index \((\alpha)\) and the scale parameter of the market factor \((\gamma_R)\), estimated using Hill plots, are kept fixed at 3 and 48.0, respectively. The scale parameters \((\gamma_\varepsilon)\) of the idiosyncratic terms of the Equity Hedge and Event-Driven indices are estimated using Hill plots and kept fixed at 5.0 and 2.6, respectively.
Figure 5.8 – Correlation between the Equity Hedge and Event-Driven indices

The uncorrected correlation is the usual Pearson’s correlation between reported returns whereas the corrected correlation, which is based on the true unobservable returns, is adjusted by the factor \((\sum_{k=0}^{2} \theta_{1,k} \theta_{2,k})/((\sum_{k=0}^{2} \theta_{1,k}^2)(\sum_{k=0}^{2} \theta_{2,k}^2))^{1/2}\). The smoothing coefficients \((\theta_{1,k})\) and \((\theta_{2,k})\) are estimated by a maximum likelihood estimation of the model \(S_t = \sum_{k=0}^{2} \theta_k X_{t-k}\) in which returns reported by hedge funds \((S_t)\) are weighted averages of current and past unobservable returns \((X_t)\). Unobservable returns are assumed to be i.i.d. and the smoothing coefficients are assumed to satisfy the constraints \(\theta_k \in [0, 1]\), for \(k = 0, 1, 2\), and \(\sum_{k=0}^{2} \theta_k = 1\). The estimations are based on rolling windows of 60 months ending in the reference month.

points in the case of normal returns (from 5.09 to 7.18 percent), and 3.12 percentage points for the fat tail case (from 8.25 to 11.37 percent). However, the normal correction term is 0.75 and the fat tail correction term is 0.73.

In this subsection, the focus is on the VaR metric. However, for the other univariate risk metric, the ES, similar conclusions can be drawn. The difference between the true unobservable ES and its unreported counterpart mimics that of VaR (see Eqs. 5.23 and 5.28). Thus, also for ES it is highly relevant to adjust the fat tailed hedge fund returns for smoothing effects.

5.4.2 Bivariate measures of risk

Next, we discuss how autocorrelation impacts on the bivariate measures of risk. First, consider the correlation measure. Figure 5.8 depicts the development over time of the uncorrected and corrected correlations between the two hedge fund indices. Figure 5.8 shows that the impact of smoothing on correlation is almost negligible. The reason for the limited difference is that the correction term in Eq. (5.35) has a value close to one for our sample. Note that this correction term can be interpreted as the raw correlation between the smoothing coefficients. Thus, the similarity in
Sect. 5.4 – Effects of autocorrelation on risk measures

The uncorrected ELM, which is based on reported returns, and the corrected ELM, which is based on the true unobservable returns, are equal, respectively, to

\[
\left( \frac{\min(\beta_1, \beta_2)^\gamma_R}{\sum_{k=0}^2 \theta_k \gamma_k} \right) / \left( \gamma_{e_1} + \gamma_{e_2} + \max(\beta_1, \beta_2)^\gamma_R \right)
\]

and

\[
\left( \frac{\sum_k (\min(\beta_1 \theta_{1,k}, \beta_2 \theta_{2,k}))^\gamma_R}{\sum_{k=0}^2 \theta_k \gamma_k} \right) / \left( \gamma_{e_1} + \gamma_{e_2} + \max(\beta_1 \theta_{1,k}, \beta_2 \theta_{2,k})^\gamma_R \right).
\]

The smoothing coefficients \( \theta_k \) and the market exposure \( \beta \) are estimated by linear regressions of the model

\[
S_t = \sum_{k=0}^2 \theta_k X_{t-k}
\]

in which returns reported by hedge funds \( S_t \) are weighted averages of current and past unobservable returns \( X_t \). Unobservable returns are assumed to be of the form

\[
X_t = \beta R_t + \varepsilon_t,
\]

where \( R_t \) is the market return and \( \varepsilon_t \) is an i.i.d. idiosyncratic risk factor. The smoothing coefficients are assumed to satisfy the constraints \( \theta_k \in [0, 1] \), for \( k = 0, 1, 2 \), and \( \sum_{k=0}^2 \theta_k = 1 \). The estimations use the Standard and Poor’s 500 total return index as market factor and are based on rolling windows of 60 months ending in the reference month. The tail index \( \alpha \) and the scale parameter of the market factor \( \gamma_R \), estimated using Hill plots, are kept fixed at 3 and 48.0, respectively. The scale parameters \( \gamma_{e_1} \) and \( \gamma_{e_2} \) of the idiosyncratic terms of the Equity Hedge and Event-Driven indices, estimated using Hill plots, are kept fixed at 5.0 and 2.6, respectively.

The autocorrelation structure of the Equity Hedge and Event-Driven indices (see Table 5.3) shows that the estimated correlation is hardly affected by the smoothing effects of the reported data.

Given the presence of fat tails, a better measure for the tail dependence is ELM. Equation (5.48) shows that the parameter values determine how the presence of autocorrelation affects ELM, and in particular whether the smoothing-adjusted measure is greater or smaller than its unadjusted counterpart (recall Propositions 5.7 and 5.8). Using the regression-based point estimates over the entire sample period for the smoothing coefficients \( \theta \) and the exposure to the market factor \( \beta \) from Table 5.3 and the scale parameters \( \gamma \) reported above, we calculate the two ELM measures as \( ELM(S_{1,t}, S_{2,t}) = 0.21 \) and \( ELM(X_{1,t}, X_{2,t}) = 0.31 \). Thus, the true
measure of systemic risk is almost 50% higher than ELM based on smoothed returns. Figure 5.9 depicts ELM estimates for the Equity Hedge and Event-Driven indices based on rolling windows of 60 months. It shows that, on average, the smoothing-adjusted ELM significantly exceeds its unadjusted counterpart, confirming the estimates based on the whole sample. The relative size of the adjustment can, at times, even exceed the 50% estimation error that was found above. For instance, at the beginning of 2005 the corrected probability of one hedge fund index being under stress given that the other index is under stress, is more than 80% higher than the uncorrected probability. Also, at the end of the sample period the corrected measure exceeds the uncorrected measure by about 55%. Per contrast, the corrected and uncorrected correlation estimates in Figure 5.8 are of a very comparable level and nearly indistinguishable.

The large difference between the correlation and ELM estimates underscores the relevance of our proposed adjustments to the tail dependence metric ELM. Evidently, unsmoothing the observed hedge fund returns is especially important when studying the extreme tail dependence. This part of the distribution is of most interest to policy makers and risk managers because of its relevance for financial stability issues and loss prevention. Our empirical analysis illustrates the economic importance of adjusting the metrics of extreme co-movement risks to prevent autocorrelation-induced distortions. Otherwise, a potentially serious underestimation error is a likely consequence.

5.5 Conclusion

Hedge fund returns frequently exhibit a strong degree of autocorrelation. As a result, the economic risks of an investment in hedge funds are easily underestimated and investment decisions can become biased. In this chapter we extend the seminal work of Getmansky et al. (2004) on SR and market beta, by developing a number of smoothing-adjusted downside risk measures and by allowing for nonnormal fat tailed return distributions. In particular, VaR, ES for individual risk exposures, correlation coefficient, and ELM reflecting downside systemic risk, are adjusted for the autocorrelation present in reported returns. We show that the adjustment of the downside risk measures for autocorrelation is more relevant when returns are fat tailed than when they are normally distributed. A hedge fund case study reveals that the unadjusted risk measures can considerably underestimate the true extent of individual and multivariate risks. Finally, note that, although the risk-adjustment
introduced in this chapter is applied to hedge funds only, the framework can also be used to evaluate the risks of other alternative investment strategies. Investments in real estate, art, collectible stamps, and other illiquid or opaque securities are also known to exhibit strong autocorrelation in the reported returns.\textsuperscript{11} Also for these assets, conventional risk measures need adjustments to correctly reflect the true level of risk.

\textsuperscript{11}See, for example, Ross and Zisler (1991), Campbell (2008), and Dimson and Spaenjers (2011).
5.A Appendix: A more general framework

In the main text it is assumed that the hedge funds’ actual returns are a function of only one market factor. In this appendix, it is shown that the results can also be generalized to multiple factor models. See Fung and Hsieh (2004), Kosowski et al. (2007), and Teo (2011) for examples of the use of such multi-factor models. Assume that the actual returns of the two hedge funds have the following structure

\[ X_{1,t} = \sum_{n=1}^{N} \beta_{1,n} R_{n,t} + \varepsilon_{1,t} \quad \text{and} \quad X_{2,t} = \sum_{n=1}^{N} \beta_{2,n} R_{n,t} + \varepsilon_{2,t}, \]  

(5.56)

where the variables \( R_{n,t}, n = 1, \ldots, N \), denote different factors. Furthermore, the idiosyncratic risk factors \( \varepsilon_{1,t}, \varepsilon_{2,t} \) and the factors \( R_{n,t} \) are i.i.d. and the reported returns are smoothed according to Eqs. (5.2)–(5.4). It can be shown that the results of the previous cases are generalized into the following equations

- **Value-at-risk**
  - Normal case
    \[ \text{VaR}(S_t, p) = \sigma_S \Phi^{-1}(1 - p), \]  
    (5.57)
    where \( \sigma_S^2 = \left( \sigma_R^2 \sum_{n=1}^{N} \beta_n^2 + \sigma_\varepsilon^2 \right) \sum_{k=0}^{K} \theta_k^2. \)
  - Heavy tail case
    \[ \text{VaR}(S_t, p) = \left( \frac{\gamma_R \sum_{n=1}^{N} \beta_n^\alpha + \gamma_\varepsilon \sum_{k=0}^{K} \theta_k^\alpha}{p} \right)^{\frac{1}{\alpha}}. \]  
    (5.58)

- **Expected shortfall**
  - Normal case
    \[ \text{ES}(S_t, y) = \frac{\sigma_S \phi \left( \frac{y}{\sigma_S} \right)}{1 - \Phi \left( \frac{y}{\sigma_S} \right)}, \]  
    (5.59)
    where \( \sigma_S^2 = \left( \sigma_R^2 \sum_{n=1}^{N} \beta_n^2 + \sigma_\varepsilon^2 \right) \sum_{k=0}^{K} \theta_k^2. \)
  - Heavy tail case
    \[ \text{ES}(S_t, y) = \frac{\alpha}{\alpha - 1} y. \]  
    (5.60)

- **Correlation**
  \[ \rho(S_{1,t}, S_{2,t}) = \frac{\sum_{k=0}^{K} \theta_{1,k} \theta_{2,k}}{\left( \left( \sum_{k=0}^{K} \theta_{1,k}^2 \right) \left( \sum_{k=0}^{K} \theta_{2,k}^2 \right) \right)^{1/2}} \left\{ \frac{\sum_{n=1}^{N} \beta_{1,n} \beta_{2,n} \sigma_{R_n}^2}{\text{Var}(X_1) \text{Var}(X_2)} \right\}^{1/2}, \]  
    (5.61)

with \( \text{Var}(X_i) = \sum_{n=1}^{N} \beta_{i,n}^2 \sigma_{R_n}^2 + \sigma_\varepsilon^2, \quad i = 1, 2. \)
Sect. 5.B – Appendix: Feller convolution theorem

- Extreme linkage measure

\[
\text{ELM}(S_{1,t}, S_{2,t}) = 1 + \frac{\sum_{k=0}^{K} \sum_{n=1}^{N} (\min(\beta_{1,n}\theta_{1,k}, \beta_{2,n}\theta_{2,k}))^\alpha \gamma_R}{\sum_{k=0}^{K} (\theta_{1,k}^\alpha \gamma_{\epsilon_1} + \theta_{2,k}^\alpha \gamma_{\epsilon_2} + \sum_{n=1}^{N} (\max(\beta_{1,n}\theta_{1,k}, \beta_{2,n}\theta_{2,k}))^\alpha \gamma_R)}.
\]

These results facilitate the estimation of the various risk measures when the hedge fund returns are best modeled by more than one (market) factor alone.

5.B Appendix: Feller convolution theorem

This appendix recall a simplified version of the convolution theorem by Feller (1971) and prove a related result.

**Theorem 5.9** For \( n = 1, \ldots, N \), let \((X_n)\) be independent random variables for which

\[
\lim_{s \to \infty} \frac{\mathbb{P}(X_n > s)}{\gamma_n s^{-\alpha}} = 1,
\]

for some positive scale parameters \((\gamma_n)\) and exponent \(\alpha\), and let \((\lambda_n)\) be non-negative constants (with at least one positive \(\lambda_n\)), then

\[
\lim_{s \to \infty} \frac{\mathbb{P} \left( \sum_{n=1}^{N} \lambda_n X_n > s \right)}{\sum_{n=1}^{N} \lambda_n^\alpha \gamma_n s^{-\alpha}} = 1.
\]

The proof of Theorem 5.9 can be found in Feller (1971, chap. VIII.8). According to this theorem, the probability that the convolution of independent random variables with Pareto tails and the same exponent is greater than some high threshold can be approximated with the corresponding probability for a random variable which has Pareto tails and the same exponent as well, but a different scale parameter.

The following proposition defines formally the result used in Eq. (5.44).

**Proposition 5.10** For \( n = 1, \ldots, N \), let \((X_n)\) be independent random variables for which

\[
\lim_{s \to \infty} \frac{\mathbb{P}(X_n > s)}{\gamma_n s^{-\alpha}} = 1,
\]

for some positive scale parameters \((\gamma_n)\) and exponent \(\alpha\), and let \((\lambda_{1,n})\) and \((\lambda_{2,n})\) be non-negative constants (with at least one positive \(\lambda_{1,n}\) and one positive \(\lambda_{2,n}\)), then

\[
\lim_{s \to \infty} \frac{1 - \mathbb{P} \left( \sum_{n=1}^{N} \lambda_{1,n} X_n \leq s, \sum_{n=1}^{N} \lambda_{2,n} X_n \leq s \right)}{\sum_{n=1}^{N} \tilde{\lambda}_n^\alpha \gamma_n s^{-\alpha}} = 1,
\]

where \(\tilde{\lambda}_n = \max(\lambda_{1,n}, \lambda_{2,n})\).
**Proof** First of all notice that
\[
1 - \mathbb{P}\left( \sum_{n=1}^{N} \lambda_{1,n} X_n \leq s, \sum_{n=1}^{N} \lambda_{2,n} X_n \leq s \right)
\]
\[
\leq 1 - \mathbb{P}\left( \sum_{n=1}^{N} \tilde{\lambda}_n X_n \leq s, \sum_{n=1}^{N} \tilde{\lambda}_n X_n \leq s \right)
\]
\[
= 1 - \mathbb{P}\left( \sum_{n=1}^{N} \tilde{\lambda}_n X_n \leq s \right)
\]
\[
= \mathbb{P}\left( \sum_{n=1}^{N} \tilde{\lambda}_n X_n > s \right)
\]
so that, using Theorem 5.9,
\[
\lim_{s \to \infty} \frac{1 - \mathbb{P}\left( \sum_{n=1}^{N} \lambda_{1,n} X_n \leq s, \sum_{n=1}^{N} \lambda_{2,n} X_n \leq s \right)}{\sum_{n=1}^{N} \tilde{\lambda}_n^{\alpha} \gamma_{n} s^{-\alpha}} \leq 1.
\]
(5.68)

Only to simplify the notation in the remaining part of the proof, let us assume that \( N = 3 \). In this case,
\[
1 - \mathbb{P}\left( \sum_{n=1}^{3} \lambda_{1,n} X_n \leq s, \sum_{n=1}^{3} \lambda_{2,n} X_n \leq s \right)
\]
\[
\geq \mathbb{P}(\tilde{\lambda}_1 X_1 > s) + \mathbb{P}(\tilde{\lambda}_2 X_2 > s) + \mathbb{P}(\tilde{\lambda}_3 X_3 > s)
\]
\[
- \mathbb{P}(\tilde{\lambda}_1 X_1 > s, \tilde{\lambda}_2 X_2 > s)
\]
\[
- \mathbb{P}(\tilde{\lambda}_1 X_1 > s, \tilde{\lambda}_3 X_3 > s)
\]
\[
- \mathbb{P}(\tilde{\lambda}_2 X_2 > s, \tilde{\lambda}_3 X_3 > s)
\]
\[
+ \mathbb{P}(\tilde{\lambda}_1 X_1 > s, \tilde{\lambda}_2 X_2 > s, \tilde{\lambda}_3 X_3 > s)
\]
\[
= \mathbb{P}(\tilde{\lambda}_1 X_1 > s) + \mathbb{P}(\tilde{\lambda}_2 X_2 > s) + \mathbb{P}(\tilde{\lambda}_3 X_3 > s)
\]
\[
- \mathbb{P}(\tilde{\lambda}_1 X_1 > s)\mathbb{P}(\tilde{\lambda}_2 X_2 > s)
\]
\[
- \mathbb{P}(\tilde{\lambda}_1 X_1 > s)\mathbb{P}(\tilde{\lambda}_3 X_3 > s)
\]
\[
- \mathbb{P}(\tilde{\lambda}_2 X_2 > s)\mathbb{P}(\tilde{\lambda}_3 X_3 > s)
\]
\[
+ \mathbb{P}(\tilde{\lambda}_1 X_1 > s)\mathbb{P}(\tilde{\lambda}_2 X_2 > s)\mathbb{P}(\tilde{\lambda}_3 X_3 > s)
\]
(5.69)

so that
\[
\lim_{s \to \infty} \frac{1 - \mathbb{P}\left( \sum_{n=1}^{3} \lambda_{1,n} X_n \leq s, \sum_{n=1}^{3} \lambda_{2,n} X_n \leq s \right)}{\sum_{n=1}^{3} \tilde{\lambda}_n^{\alpha} \gamma_{n} s^{-\alpha}}
\]
\[
\geq \lim_{s \to \infty} \left( \sum_{n=1}^{3} \frac{\mathbb{P}(\tilde{\lambda}_n X_n > s)}{s^{-\alpha}} + o(s^{-\alpha}) \right)
\]
\[
= \sum_{n=1}^{3} \tilde{\lambda}_n^{\alpha} \gamma_{n},
\]
(5.70)
or
\[
\lim_{s \to \infty} \frac{1 - \mathbb{P}\left(\sum_{n=1}^{N} \lambda_1 n X_n \leq s, \sum_{n=1}^{N} \lambda_2 n X_n \leq s\right)}{\sum_{n=1}^{N} \lambda_0 n s^{-\alpha}} \geq 1, \tag{5.71}
\]
which completes the proof. \qed

Note that equation (5.66) implies
\[
1 - \mathbb{P}\left(\beta X_1 + \lambda_1 X_3 \leq s, \theta X_2 + \lambda_2 X_3 \leq s\right) \simeq \left(\beta^\alpha \gamma_1 + \theta^\alpha \gamma_2 + (\max(\lambda_1, \lambda_2))^{\alpha} \gamma_3\right)s^{-\alpha}. \tag{5.72}
\]
for \(\beta, \theta, \lambda_1, \lambda_2\) strictly positive and \(s\) sufficiently large. This result has been used in Eq. (5.44).

5.C Appendix: Subadditivity of VaR

This appendix shows that the VaR is subadditive when the tails of the distribution of the returns of the assets behave according to a Pareto law.\(^{12}\)

Proposition 5.11 For \(n = 1, 2\), let \((X_n)\) be independent random variables for which, for any \(s\) sufficiently large,
\[
\mathbb{P}(X_n > s) = \gamma_n s^{-\alpha}, \tag{5.73}
\]
for some positive scale parameters \((\gamma_n)\) and \(\alpha \geq 1\), then the VaR measure is subadditive, that is
\[
\text{VaR}(\lambda_1 X_1, p) + \text{VaR}(\lambda_2 X_2, p) \geq \text{VaR}(\lambda_1 X_1 + \lambda_2 X_2, p), \tag{5.74}
\]
for any \(\lambda_1, \lambda_2 \geq 0\) and any \(p\) sufficiently small.

Proof: For any \(p\) sufficiently small for the VaR to fall in the tail of the distribution, one has
\[
p = \mathbb{P}\left(\lambda_n X_n > \text{VaR}(\lambda_n X_n, p)\right) = \lambda_0^\alpha \gamma_n \text{VaR}(\lambda_n X_n, p)^{-\alpha}, \tag{5.75}
\]
or,
\[
\text{VaR}(\lambda_n X_n, p) = \theta_n p^{-1/\alpha}, \tag{5.76}
\]
where \(\theta_n = \lambda_n^{1/\alpha}\). Hence,
\[
\text{VaR}(\lambda_1 X_1, p) + \text{VaR}(\lambda_2 X_2, p) = (\theta_1 + \theta_2)p^{-1/\alpha}. \tag{5.77}
\]
\(^{12}\)See Danielsson et al. (2010) for a more general framework.
On the other hand, denoting $Y = \lambda_1 X_1 + \lambda_2 X_2$ and using Theorem 5.9, it follows that
\[
p = \mathbb{P}(Y > \text{VaR}(Y, p)) = (\theta_1^\alpha + \theta_2^\alpha)\text{VaR}(Y, p)^{-\alpha},
\] (5.78)

or
\[
\text{VaR}(Y, p) = (\theta_1^\alpha + \theta_2^\alpha)^{1/\alpha} p^{-1/\alpha}.
\] (5.79)

The final result follows directly from the fact that \((\sum_{i=1}^n x_i)^\theta \geq \sum_{i=1}^n x_i^\theta\) when all \(x_n \geq 0\) and \(\theta \geq 1\).

The previous arguments can be easily generalized to the case of \(N\) independent random variables \((X_n)\) with Pareto tails, with scale parameters \((\gamma_n)\) and the same exponent \(\alpha \geq 1\),
\[
\sum_{n=1}^N \text{VaR}(\lambda_n X_n, p) \geq \text{VaR}\left(\sum_{n=1}^N \lambda_n X_n, p\right).
\] (5.80)

Notice that when the exponent \(\alpha\) is not the same for all the random variables \((X_n)\), then the previous calculations are still valid, but one has to take into account only the random variables with the heaviest tails, that is the ones with the smallest \(\alpha\).

5.5 Appendix: Proof of Proposition 5.7

Given the equal betas, the true \(\text{ELM}(X_{1,t}, X_{2,t})\) from Eq. (5.45) reduces to
\[
\text{ELM}(X_{1,t}, X_{2,t}) = \frac{\beta^\alpha \gamma_R}{\gamma_{\epsilon_1} + \gamma_{\epsilon_2} + \beta^\alpha \gamma_R} = \frac{1}{\frac{\gamma_{\epsilon_1} + \gamma_{\epsilon_2}}{\beta^\alpha \gamma_R} + 1}.
\] (5.81)

The corresponding measure for the smoothed returns from Eq. (5.48) becomes
\[
\text{ELM}(S_{1,t}, S_{2,t}) = \sum_{k=0}^K (\min(\theta_{1,k}, \theta_{2,k}))^\alpha \geq \sum_{k=0}^K (\max(\theta_{1,k}, \theta_{2,k}))^\alpha.
\] (5.82)

Comparing the two measures \(\text{ELM}(X_{1,t}, X_{2,t}) \geq \text{ELM}(S_{1,t}, S_{2,t})\) shows
\[
\frac{\gamma_{\epsilon_1}}{\beta^\alpha \gamma_R} \sum_{k=0}^K \theta_{1,k}^\alpha + \frac{\gamma_{\epsilon_2}}{\beta^\alpha \gamma_R} \sum_{k=0}^K \theta_{2,k}^\alpha + \sum_{k=0}^K (\max(\theta_{1,k}, \theta_{2,k}))^\alpha \geq \frac{\gamma_{\epsilon_1} + \gamma_{\epsilon_2}}{\beta^\alpha \gamma_R} \sum_{k=0}^K (\min(\theta_{1,k}, \theta_{2,k}))^\alpha + \sum_{k=0}^K (\min(\theta_{1,k}, \theta_{2,k}))^\alpha,
\] (5.83)
or

\[
\sum_{k=0}^{K} \left( \left( \max(\theta_{1,k}, \theta_{2,k}) \right)^\alpha - \left( \min(\theta_{1,k}, \theta_{2,k}) \right)^\alpha \right) \gtrless
\]

\[
\frac{\gamma_1}{\beta^\alpha \gamma_R} \sum_{k=0}^{K} \left( \left( \min(\theta_{1,k}, \theta_{2,k}) \right)^\alpha - \theta_{1,k}^\alpha \right) + \frac{\gamma_2}{\beta^\alpha \gamma_R} \sum_{k=0}^{K} \left( \left( \min(\theta_{1,k}, \theta_{2,k}) \right)^\alpha - \theta_{2,k}^\alpha \right).
\]

(5.84)

The elements on the left-hand side are all nonnegative (and some are strictly positive if not all smoothing coefficients are equal), while the terms on the right-hand side are all nonpositive (and some are strictly negative if not all smoothing coefficients are equal). Hence, the left-hand side is always at least as large as the right-hand side, or \( \text{ELM}(X_{1,t}, X_{2,t}) \geq \text{ELM}(S_{1,t}, S_{2,t}) \).

\[ \square \]
5.E Appendix: Estimates of the smoothing coefficients

![Panel A: Equity Hedge index](image)

![Panel B: Event-Driven index](image)

Figure 5.10 – Maximum likelihood estimates for the Equity Hedge and Event-Driven indices of the smoothing coefficients ($\theta_k$)

Returns reported by hedge funds ($S_t$) are weighted averages of current and past unobservable returns ($X_t$), according to the model $S_t = \sum_{k=0}^{2} \theta_k X_{t-k}$. Unobservable returns are assumed to be i.i.d. and the smoothing coefficients are assumed to satisfy the constraints $\theta_k \in [0, 1]$, for $k = 0, 1, 2$, and $\sum_{k=0}^{2} \theta_k = 1$. The estimations are based on rolling windows of 60 months ending in the reference month.
Returns reported by hedge funds \((S_t)\) are weighted averages of current and past unobservable returns \((X_t)\), according to the model
\[
S_t = \sum_{k=0}^{2} \theta_k X_{t-k}.
\]
Unobservable returns are assumed to be of the form \(X_t = \beta R_t + \varepsilon_t\), where \(R_t\) is the market return and \(\varepsilon_t\) is an i.i.d. idiosyncratic risk factor. The smoothing coefficients are assumed to satisfy the constraints \(\theta_k \in [0, 1]\), for \(k = 0, 1, 2\), and \(\sum_{k=0}^{2} \theta_k = 1\). As suggested by Getmansky et al. (2004), consistent estimates of the parameters are obtained by running ordinary least squares (OLS) regressions of the equation
\[
S_t = \mu + \beta (\theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}) + u_t = \mu + \gamma_0 R_t + \gamma_1 R_{t-1} + \gamma_2 R_{t-2} + u_t,
\]
where \(u_t = \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}\) and \(\varepsilon_t\) is i.i.d. In this case, \(\hat{\beta} = \hat{\gamma}_0 + \hat{\gamma}_1 + \hat{\gamma}_2\) and \(\hat{\theta}_k = \hat{\gamma}_k / \hat{\beta}\). The estimations use the Standard and Poor’s 500 total return index as market factor and are based on rolling windows of 60 months ending in the reference month.
5.F Appendix: Small sample effects

This appendix analyzes how the EVT estimators used in this chapter are affected by the small size of the sample. The reason for conducting this analysis is that EVT estimators are usually applied to large data samples. Per contrast, our data set is relatively small as it contains only 236 return observations. Therefore, it is uncertain how reliable the parameter estimates presented in this chapter are.

To analyze this issue, Monte Carlo simulations of the market and idiosyncratic components of hedge funds returns are used. First, a random sample from a standard normal distribution is generated. Next, the outcomes in both the first and last deciles of the sorted return observations are replaced by the corresponding random numbers drawn from a Pareto distribution with tail index $\alpha = 3$ and scale parameter $\gamma = 48.0$. The result is a random sample normally distributed in the middle of the distribution but with Pareto tails. A similar approach is used by Danéiasson and de Vries (2000). The parameter values are chosen to mimic the estimated S&P 500 index tail behavior.

Next, the same method is applied to generate two other samples. The scale parameters are set equal to 5.0 and 2.6, respectively, which mimic the estimated series of the idiosyncratic factors. With these random samples, Eqs. (5.1) and (5.2) are used to simulate the behavior of the unobservable true returns of two hedge fund indices as well as those of the corresponding reported smoothed returns. The betas are set equal to 0.61 and 0.53, respectively. The thetas are set equal to 0.79, 0.10, and 0.11, respectively, for the Equity Hedge index and to 0.67, 0.18, and 0.15 for the Event-Driven index. These values equal the estimates for the two hedge fund indices at the end of the sample (cf. Figure 5.11). This procedure is repeated 5,000 times to estimate the mean, as well as confidence bands, for the parameters.

Then, linear regressions are used to estimate the market sensitivity parameter ($\beta$) and the smoothing coefficients ($\theta$). We use moving windows of 60 observations of random samples of 261 observations. Actually, the length of the moving windows equals 62, but two observations are lost in the estimation process because of the $MA(2)$ nature of the model. Moreover, the random samples are chosen to slightly exceed the length of our data set, which has 236 observations. As a result, we have exactly 200 parameters estimates obtained from 200 moving windows of 62 observations. The results, reported in Table 5.4, show that all parameter estimates are unbiased. However, because of the small size of the sample (only 60 observations), the confidence intervals and the mean absolute errors are rather large.

We also performed Monte Carlo simulations using Student’s $t$-random variables with three degrees of freedom. The results are qualitatively similar.
Table 5.4 – Monte Carlo simulations: Descriptive statistics of the estimates of the market exposure and the smoothing coefficients

The table reports the true values and some descriptive statistics of the distributions of their estimates of the market exposure ($\beta$) and the smoothing coefficients ($\theta$) of the Equity Hedge and Event-Driven indices. The calculations are based on ordinary least squares regressions on 5,000 random samples designed to mimic the behavior of the negative tails of the returns of the Standard and Poor’s 500 index and the idiosyncratic components of the returns of the hedge fund indices. The true values of the parameters for the two hedge fund indices are set equal to the estimates at the end of the sample (cf. Figure 5.11).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True Value</th>
<th>Mean</th>
<th>$5^{th}$ percentile</th>
<th>$10^{th}$ percentile</th>
<th>$90^{th}$ percentile</th>
<th>$95^{th}$ percentile</th>
<th>Mean absolute error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>0.61</td>
<td>0.619</td>
<td>0.464</td>
<td>0.498</td>
<td>0.743</td>
<td>0.790</td>
<td>0.078</td>
</tr>
<tr>
<td>$\theta_{1,1}$</td>
<td>0.79</td>
<td>0.790</td>
<td>0.644</td>
<td>0.678</td>
<td>0.910</td>
<td>0.949</td>
<td>0.072</td>
</tr>
<tr>
<td>$\theta_{1,2}$</td>
<td>0.10</td>
<td>0.102</td>
<td>0.000</td>
<td>0.000</td>
<td>0.187</td>
<td>0.216</td>
<td>0.054</td>
</tr>
<tr>
<td>$\theta_{1,3}$</td>
<td>0.11</td>
<td>0.109</td>
<td>0.000</td>
<td>0.003</td>
<td>0.194</td>
<td>0.223</td>
<td>0.054</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.53</td>
<td>0.532</td>
<td>0.394</td>
<td>0.429</td>
<td>0.635</td>
<td>0.672</td>
<td>0.065</td>
</tr>
<tr>
<td>$\theta_{2,1}$</td>
<td>0.67</td>
<td>0.677</td>
<td>0.562</td>
<td>0.588</td>
<td>0.775</td>
<td>0.818</td>
<td>0.059</td>
</tr>
<tr>
<td>$\theta_{2,2}$</td>
<td>0.18</td>
<td>0.176</td>
<td>0.072</td>
<td>0.104</td>
<td>0.242</td>
<td>0.263</td>
<td>0.043</td>
</tr>
<tr>
<td>$\theta_{2,3}$</td>
<td>0.15</td>
<td>0.147</td>
<td>0.035</td>
<td>0.070</td>
<td>0.217</td>
<td>0.240</td>
<td>0.046</td>
</tr>
</tbody>
</table>

We find that the Hill estimates are quite stable. The simulations show that the Hill estimates are very close to the true values (i.e., the values used to generate the random samples). The average absolute difference found for the range between the tenth and 25th observation equals 0.64. Furthermore, the confidence intervals considerably reduce in size when the number of observations is increased. The scale parameter estimates are also unbiased and show confidence intervals that decrease in size with the number of observations used.

Subsequently, we calculate Hill estimates of the idiosyncratic term of the hedge fund returns, and find that this Hill plot strongly resembles the one generated for the S&P 500 index. The reason for this similarity is that the tail index of the market index equals that of the idiosyncratic terms. Additionally, it is noted that the width of the confidence intervals found for the idiosyncratic term estimates exceeds that found for the market index. The reason is that the idiosyncratic terms are not directly observable but need to be estimated. This estimation procedure increases the statistical uncertainty. Next, the scale parameter of the idiosyncratic terms is estimated. The results are very similar to those found for the Hill estimates of the idiosyncratic terms.

Given that the tails of the systematic and idiosyncratic risk factors are Pareto distributed, we use Eqs. (5.19) and (5.45) to calculate the exact values of VaR and ELM and compare them with the Monte Carlo outcomes. Figure 5.12 depicts the
Table 5.5 – Monte Carlo simulations: Bias and mean absolute error of the estimates of the value-at-risk and extreme linkage measure

The table reports the bias and the mean absolute error of the estimates of VaR of the Equity Hedge index and ELM between the Equity Hedge and Event-Driven indices, for a few values of the size of the sample. The calculations are based on 5,000 random samples designed to mimic the behavior of the negative tails of the returns of the Standard and Poor’s 500 index and the idiosyncratic components of the returns of the hedge fund indices. In the simulations, we set the values of the market exposure and the smoothing coefficients for the two indices equal to the estimates at the end of the sample (cf. Figure 5.11). Based on the estimates for the market exposures and the smoothing coefficients, the true values of the risk measures are VaR = 11.7 and ELM = 0.39 (cf. Figure 5.7, Panel A, and Figure 5.9).

<table>
<thead>
<tr>
<th>Number of observations</th>
<th>VaR</th>
<th>ELM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bias</td>
<td>Mean absolute error</td>
</tr>
<tr>
<td>60</td>
<td>0.181</td>
<td>1.019</td>
</tr>
<tr>
<td>150</td>
<td>0.055</td>
<td>0.658</td>
</tr>
<tr>
<td>300</td>
<td>0.019</td>
<td>0.468</td>
</tr>
</tbody>
</table>

estimated and true values of both VaR and ELM of the Equity Hedge index. Panel A of Figure 5.12 shows that the procedure generates a minor positive bias of around 1.5% in the estimate of the true unobservable VaR. The bias is caused by the functional form of Eq. (5.19) that can be shown to be convex in \( \beta \). Given that \( \beta \) is imprecisely estimated around its true value in small samples, the estimate for VaR tends to be larger than the true value of VaR because of Jensen’s inequality. Of course, the larger the sample used in the estimation, the more precise the estimate of \( \beta \) and the smaller the bias. Our results show that the bias is about 0.5% when estimations are performed on samples of 150 observations and it is smaller than 0.2% on samples of 300 observations (see Table 5.5). Although the bias is almost negligible even in our small samples of 60 observations, the mean absolute errors are somewhat larger and equal to about 10% of the true VaR. Similar to the case of the bias, we find that mean absolute errors are much smaller in greater samples.

Compared with the above VaR results, our findings are less favorable for ELM (see Figure 5.12, Panel B). In the ELM’s case, the bias is negative and approximately 10% of the true ELM. Moreover, the mean absolute error is much larger than that.

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14We also tried an alternative method to estimate the true VaR by running a linear regression of a log-transformed version of Eq. (5.18). For this procedure the mean estimate is close to the true value although it also appears to be biased. Moreover, the confidence intervals are wider than those found through the method described above.

15Eq. (5.19) can be written as \( f(\beta) = (a \beta^\alpha + c)^{1/\alpha} \), where \( a = \gamma_R/p \) and \( c = \gamma_e/p \) are both positive. It can be shown that the second derivative of \( f(\beta) \) with respect to \( \beta \) is positive when \( \beta > 0 \) and \( \alpha > 1 \), so that Jensen’s inequality implies \( E[f(\hat{\beta})] \geq f(E[\hat{\beta}]) = f(\beta) \).
for VaR, and equals around 30% of the true ELM. However, note that results of the Monte Carlo simulations further reinforce the conclusions in Section 5.4 for ELM. Unreported simulations show that the estimated ELM for smoothed returns is unbiased. Hence, the true difference between ELM calculated on smoothed observable returns and that calculated on true unobservable returns even exceeds the difference reported in Figure 5.9. The simulation analysis further underscores the importance of correcting the risk measures when returns are smoothed.
Figure 5.12 – Monte Carlo simulations: Bias and confidence intervals of the estimates of the value-at-risk of the Equity Hedge index and of the extreme linkage measure between the Equity Hedge and Event-Driven indices

The figure shows the mean values, and the real values as well, of the estimates of VaR (Panel A) and ELM (Panel B) based on 200 moving windows of 60 observations of 5,000 random samples of 261 datapoints. The random samples are designed to mimic the behavior of the negative tails of the returns of the Standard and Poor’s 500 index and the idiosyncratic components of the returns of the Equity Hedge and Event-Driven hedge fund indices. In the simulations we set the values of the market exposure and the smoothing coefficients for the two indices equal to the estimates at the end of the sample (cf. Figure 5.11). Based on the estimates for the market exposures and the smoothing coefficients, the true values of the risk measures are VaR = 11.7 and ELM = 0.39 (cf. Figures 5.7 and 5.9). The dotted lines represent 80% confidence intervals.
Chapter 6
Summary and General Conclusions

Leverage is an important element of financial activities. It is present in essentially any field of the economy, from house financing to innovative financial instruments for hedge funds. A large literature has analyzed the positive and negative effects that leverage can have on economic growth and financial stability. This thesis studies the characteristics of some of the financial instruments that are designed specifically to exploit a leverage effect. The analysis aims at improving the understanding of leveraged products and their role in modern financial markets. Proper insights into the risks and benefits of leverage for the whole society may help to design a balanced institutional framework in which these novel instruments can operate properly.

Securitization represented an important source of financing for banks before the onset of the global financial crisis in 2007 and will likely return to be so in a few years. The main driver of securitization was initially the diversification of risk, but at some stage it also enabled leverage. Thus, a major question is whether the securitization process is a factor that tends to increase or decrease the riskiness of the banks that securitize. Nowadays, the tendency for regulators is to ask banks that securitize their assets to retain a share of the issued bonds to mitigate moral hazard problems. This requirement was often already satisfied in the past due to market practices. However, the practice to retain the most junior tranches of the securitizations could actually increase the risks of banks to incur large losses, because of the high level of credit risk that is retained in those highly leveraged instruments. The first part of Chapter 2 analyzes this issue and shows the relevance of the reinvestment process on the final impact of securitization on bank risk. Because of the importance of the reinvestment process, the second part of Chapter 2 looks at the impact that securitizations had on the composition of the asset side of the balance sheets of Italian banks. The analysis is also extended to data on individual loans, to study
whether the loans that were securitized by Italian banks were more or less risky than the new loans that were granted with the proceeds of the securitizations. The results show that, on average, securitizations helped Italian banks to reshape their balance sheets towards investments that were usually safer from the point of view of credit risk (such as, interbank deposits and securities other than shares). At the same time, the new loans that were granted with the proceeds of the securitizations had higher average default rates than the loans that were securitized, thus signaling that the loan portfolios of Italian banks were made riskier by securitizations.

Following the global financial crisis, many regulatory reforms have been adopted worldwide. Broadly speaking, as a result of these overhauls banks are now required to hold more capital and are incentivized to invest in safer and more liquid assets. Holding more capital should help banks to face economic downturns, while holding safer assets should reduce the likelihood that large losses materialize in bank balance sheets. Chapter 3 analyzes the impact that extreme macroeconomic shocks can have on bank losses and studies whether securitization can help to reduce the effects of these common shocks. Because extreme macroeconomic shocks have significant consequences even for the borrowers with the lowest default risk, it is shown that the tail risk for banks is broadly independent of the quality of the assets in which the banks invest. In addition, the securitization of riskier assets and the reinvestment in safer instruments has only a mild impact on the tail risk of banks. The main drivers are the macro shocks, not so much the portfolio composition. Given that having less risky portfolios cannot mitigate the impact of extreme macroeconomic shocks, this chapter highlights the importance of bank capital for financial stability.

Credit default swaps (CDSs) are the prototype example of credit derivates and they represent one of the main instruments for taking leveraged credit exposures. The determinants of CDS prices have been the subject of broad analyses, both theoretical and empirical. Chapter 4 analyzes empirically whether the factors that have been identified in the literature as important determinants of CDS spreads have changed their role from before to after the onset of the global financial crisis. The results show that CDS spreads have recently become much more sensitive to the amount of leverage while volatility has lost its importance. Since the beginning of the crisis, CDS spread changes have also been increasingly driven by a common factor that our model was able to explain only in part. The exact identification of this factor is an interesting topic for future research.

This thesis also studies the characteristics of downside and global measures of individual and systemic risks for hedge funds. Since hedge funds invest in illiquid assets — and possibly also due to reporting issues — the returns of this asset class
usually exhibit serial dependence. Given that hedge fund returns are autocorrelated, standard measures of risk are biased. Moreover, the distribution of hedge fund returns tend often to show fat tails, thus making some common measures of risk not appropriate. Chapter 5 provides for a methodology to correct risk measures in order to deal with both problems of serial correlation and heavy-tailness. This methodology is applied to global measures of risk, such as the Sharpe ratio and pairwise correlation, and also to downside measures of risk, such as the value-at-risk, the expected shortfall, and the extreme linkage measure. The latter measure reflects the amount of interdependence among two or more returns deep into the joint tail loss area. Corrected risk measures are found to be usually larger than uncorrected measures. Hence, uncorrected measures tend to understate the true risks for hedge funds. An empirical analysis shows that the correction can actually be rather sizable. Moreover, correcting for autocorrelation is more relevant when returns also exhibit heavy tails than in the usual setup based on normally distributed returns.

Overall, this thesis sheds light on several aspects related to the riskiness of leveraged products, such as CDOs, CDSs, and hedge funds. In doing so, this thesis possibly contributes to achieving better investment and regulatory practices.
Bibliography


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Financiële instellingen maken bij de exploitatie van hun activiteiten in grote mate gebruik van de financiering door middel van vreemd vermogen (ook wel “leverage” of het hefboomeffect genoemd). Financiële instellingen ontwikkelen in rap tempo nieuwe producten om dit hefboomeffect optimaal uit te buiten en op deze wijze ogenschijnlijk goedkopere financiering te bewerkstelligen. In de literatuur is reeds veel onderzoek gedaan naar de positieve en negatieve effecten van deze vorm van financiering door financiële instellingen op de economische groei en financiële stabilitéit in het vestigingsgebied van deze instelling. Dit proefschrift levert een bijdrage aan deze literatuur door de kenmerken van een gedeelte van deze nieuwe producten te analyseren. De analyse is gericht op het vergroten van de kennis ten aanzien van deze producten en de invloed van deze producten op de economische omgeving vast te stellen. Inzicht in de risico’s, kosten en baten van deze nieuwe financiële producten is essentieel voor het bewerkstelligen van een evenwichtig economisch klimaat binnen financiële instellingen en hun economische omgeving.

Een specifieke vorm van financiering welke een sterke opkomst heeft gekend in de afgelopen jaren binnen financiële instellingen betreft de securisatie. Securisatie is ontstaan uit de behoefte van financiële instellingen om de risico’s te diversifiëren. Echter securisatie is ook verantwoordelijk voor het in grote mate verschuiven van financiële producten naar afzonderlijke instituties om het hefboomeffect verder te kunnen exploiteren. Een belangrijke vraag is of dat securisatie het aanwezige risico binnen een financiële instelling heeft verkleind of juist vergroot. Toezichthouders zijn geneigd financiële instellingen te verplichten een gedeelte van de gesecuriseerde producten zelf aan te houden op haar balans, om aangepast gedrag (“moral hazard”) te voorkomen. Financiële instellingen komen in de praktijk tegemoet aan deze eis door de laagste tranches van de securisatie producten zelf aan te houden. Echter, dit heeft tot gevolg dat banken de grootste kredietrisico’s, ondanks de securisatie, nog steeds op hun balans houden. Deze werkwijze kan tijdens een crisis mogelijk grote verliezen tot gevolg hebben. In het eerste gedeelte van Hoofdstuk 2 wordt
dit probleem geanalyseerd en wordt aangetoond dat het achterhouden van de gese-
curiseerde producten op de balans van een bank van grote invloed is op het risico
profiel van de betreffende bank. Het tweede gedeelte van Hoofdstuk 2 bestudeert
de impact van securisatie op de samenstelling van de activazijde van de balans van
Italiaanse banken. Tevens worden data ten aanzien van individuele leningen geanaly-
seerd om vast te stellen of de gesecuriseerde leningen een hoger risicoprofiel
hebben dan de leningen welke tot stand zijn gekomen door de herinvestering van de
vrijgekomen gelden uit de securisatie. De resultaten tonen aan dat de securisatie
Italiaanse banken in staat heeft gesteld de vrijgekomen gelden uit de securisatie te
besteden aan veiligere activa uit het oogpunt van kredietrisico, zoals bijv. interban-
caire deposito’s en andere niet-risicodragende effecten. Maar omdat juist de betere
leningen werden securiseerd, is het afbetalingsrisico in de bestaande portefeuille wel
toegenomen.

Naar aanleiding van de huidige financiële crisis zijn veel hervormingen op het
gebied van wet- en regelgeving doorgevoerd over de gehele wereld. Eén van deze
maatregelen betreft de verplichting van banken om meer kapitaal aan te houden,
om op deze wijze banken weerbaarder maken tegen negatieve economische schokken.
Een andere maatregel betreft het stimuleren van banken om te investeren in veiligere
een meer liquide producten, om op deze wijze zorg te dragen voor een lagere kans op
grote verliezen. Hoofdstuk 3 analyseert de invloed van extreme macro-economische
schokken op de resultaten van banken en onderzocht of dat de toepassing van se-
curisatie deze effecten van macro-economische schokken kan verminderen. Wij to-
nen aan dat de extreme resultaten van banken (een groot verlies) grotendeels on-
afhankelijk zijn van de kwaliteit van de activa waarin the banken investeren. Dit
wordt veroorzaakt door het feit dat macro-economische schokken niet alleen de
activa met een lage kredietstatus treffen, maar ook de meer kredietwaardige ac-
tiva. Investeren in veiligere en meer liquide producten door banken, heeft dan ook
een zeer beperkt effect op de extreme resultaten van banken als gevolg van macro-
economische schokken. Extreme resultaten zijn een gevolg van macro-economische
schokken en niet zozeer van de samenstelling van de activa-portefeuille van een bank.
Dit onderbouwt het belang van het aanhouden van meer kapitaal.

Credit Default Swaps (CDS’s) zijn derivaten, welke veelvuldig door financiële in-
stellingen worden gebruikt voor het afdekken van kredietrisico. In de literatuur zijn
zowel empirische als ook theoretische onderzoeken beschikbaar die de determinanten
voor een CDS prijs analyseren. In Hoofdstuk 4 wordt geanalyseerd of de in de liter-
atuur aangeduide determinanten van de prijs van een CDS, tijdens de financiële crisis
dezelfde waarde behouden. Deze analyse toont aan dat de CDS prijzen gevoeliger
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zijn gebleken voor de financiering met vreemd vermogen (hefboomeffect), terwijl de volatiliteit van minder invloed lijkt te zijn. Tevens lijkt de CDS prijs sinds het begin van de crisis ook te worden bepaald door een gemeenschappelijke factor, welke door het model slechts gedeeltelijk is te verklaren. Het is interessant om vervolg onderzoek te doen naar deze gemeenschappelijke factoren.

Hoofdstuk 5 analyseert de karakteristieken van hedge fondsen en de invloed van deze hedge fondsen en geïmplementeerde wet- en regelgeving op diversificatie en systeemrisico. Hedge fondsen investeren voornamelijk in illiquide activa, welke meestal in hoge mate serie correlatie vertonen. Deze hoge mate van serie correlatie tussen de rendementen van hedge fondsen en het feit dat de rendementen grote uitschieters (extreme waarden) vertonen, heeft tot gevolg dat het toepassen van de standaard risico management praktijk die is gebaseerd op de normale verdeling niet voldoende is om deze risico’s te beheersen. Hoofdstuk 5 voorziet in een nieuwe vorm van risico management om aan deze karakteristieken, grote uitschieters en een hoge mate van correlatie tussen de extreme waarden, tegemoet te komen. Naast de meetinstrumenten van het standaard risico management, zoals bijv. de Sharpe ratio en de paargewijze correlatie, worden ook specifieke neerwaarts risico maatstaven, zoals de Value-at-Risk maatstaf, expected shortfall en de extreme samenhang maatstaven geanalyseerd. De extreme samenhang maatstaf maakt de mate van wederzijdse afhankelijkheid van twee of meer financiële producten voor de grootste uitschieters zichtbaar. Het gebruik van specifieke risico meetinstrumenten geeft een beter inzicht dan het aanpassen van de standaardmeetinstrumenten, daar deze vaak een onder- schatting van het daadwerkelijke risico binnen de activaportefeuille van hedge fondsen weergeeft. Het empirische onderzoek wijst uit dat de invloed van correlatie en de extreme waarden in de opbrengstenverdeling van grote invloed zijn op de gebruikte meetinstrumenten.

Dit proefschrift levert een bijdrage aan meer inzicht in het gebruik van innovatieve producten, zoals CDO’s en CDS’s, door financiële instellingen op het risiconiveau van een financiële instelling. Tevens wordt de invloed van hedge funds op de economische omgeving geanalyseerd. Met deze analyses wordt beoogd een bijdrage te leveren aan de creatie van een transparanter investeringsklimaat en effectievere wet- en regelgeving.
Leveraged investments have become a fundamental feature of modern economies. The new financial products allow people to take greater-than-usual exposures to risk factors. This thesis analyzes several different aspects of the risks involved by some frequently used leveraged products: CDOs, CDSs, and hedge funds. It is shown that these risks have indeed several facets and that misjudging them can have severe effects for both individual investors and the global financial stability. However, although leveraged products can be more complex than other financial instruments, their characteristics in terms of risks and returns can usually be understood rather well by careful scholars. The aim of this thesis is to contribute to a better understanding of some of the features of leveraged products and provide useful insights on how to best use these new instruments.

Antonio Di Cesare (1971) graduated cum laude in Economics at D’Annunzio University in Pescara (Italy). He was granted scholarships by the Bank of Italy and the Scuola Normale Superiore in Pisa to take advanced international courses in finance. In 2000 he was hired by the Bank of Italy as an economist in the Economic Research Department. In December 2009 he asked to join the doctorate programme at Erasmus University Rotterdam, where he was admitted in June 2010. During his PhD he continued to work at the Bank of Italy in the Economic Outlook and Monetary Policy Department and became head of the Financial Markets Unit of the Financial Analysis Division in July 2010. His research interests are the analysis of innovative financial instruments and their impact on financial stability. He is the author of academic and policy papers on financial markets and financial stability. He is occasional referee for international journals.