Modeling and Control of Co-generation Power Plants: A Hybrid System Approach

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***Abstract*— In this paper the short term scheduling optimization of a combined cycle power plant is accomplished by exploiting hybrid systems, i.e. systems evolving according to continuous dynamics, discrete dynamics, and logic rules. Discrete features of a power plant are, for instance, the possibility of turning on/off the turbines, operating constraints like minimum up and down times and the different types of start up of the turbines. On the other hand, features with continuous dynamics are power and steam output, the corresponding fuel consumption, etc. The union of these properties characterize the hybrid behavior of a combined cycle power plant. In order to model both the continuous/discrete dynamics and the switching between different operating conditions we use the framework of Mixed Logic Dynamical systems. Then, we recast the economic optimization problem as a Model Predictive Control (MPC) problem, that allows us to optimize the plant operations by taking into account the time variability of both prices and electricity/steam demands. Because of the presence of integer variables, the MPC scheme is formulated as a mixed integer linear program that can be solved in an efficient way via dedicated software.**

***Index Terms*— Hybrid systems; model predictive control; combined cycle power plant; mixed integer linear programming**

I. Introduction

In the last decade, the electric power industry has been subject to deep changes in structure and organization. On the one hand, market liberalization and its associated fierce competition has led to a strong focus on cost reduction and optimal operation strategies. On the other hand, more strict environmental legislation makes operational constraints tighter. In this context, the use of combined cycle power plants (CCPP) has become more and more popular: they are more efficient and flexible than conventional configurations based on boilers and steam turbines, not to speak about nuclear power plants.

A typical CCPP is composed of a gas cycle and a steam cycle. The gas cycle is driven by some fossil fuel (usually natural gas) and produces electric power via expansion of hot gasses in a (gas) turbine. The steam cycle is supplied with the still hot exhaust gases of the gas turbine and generates both electricity and steam for the industrial processes. Clearly, the liberalization of the energy market has promoted the need of operating CCPPs in the most efficient way, that is by maximizing the profits due to the sales of steam and electricity and by minimizing the operating costs.

In this paper we consider the problem of optimizing the short-term operation of a CCPP, i.e. to optimize the plant on an hourly basis over a time horizon that may vary from few hours to one day [31]. A large stream of research in the power systems area focused on this problem. The usual paradigm (also used in this paper) is to recast the economic optimization into the minimization of a cost minus revenues functional and to account for the physical model of the plant through suitably defined constraints. The results available in the literature differ both in the *features* of the CCPP modeled and in the *scope* of optimization.

In [31], [8], [17], [32] the CCPP is assumed in a standard operating condition and optimal scheduling of the resources is performed via non linear programming techniques. The main limitation is that the possibility of turning on/off the turbines is not considered and therefore it is not possible to determine the optimal switching strategy. The discrete features of a CCPP (i.e. the fact that turbines can be turned on/off, the start up dynamics, the minimum up and down time constraints and the priority constraints in start up sequences) can be captured by using binary decision variables along with continuous-valued variables describing physical quantities (e.g. mass, energy and flow rates).

In [27] binary variables are introduced to model the on/off status of the devices and the corresponding optimization problem is solved through the use of genetic algorithms. The same modelling feature is considered in [21] where the automatic computation of the optimal on/off input commands (fulfilling also operational priority constraints) is accomplished through Mixed Integer Linear Programming (MILP). However in both papers, the modelling of the CCPP is done in an ad-hoc fashion and the generalization to plants with different topologies and/or specifications seems difficult. Moreover, other important features such as minimum up and down times or the behavior during start up are neglected. A fairly complete model of a thermal unit, using integer variables for describing minimum up/down time constraints, ramp constraints and different startup procedures, is given in [2]. The behavior of the unit is then optimized by solving MILP problems. Even if this approach could be adapted for modelling a single turbine of a CCPP, no methodological way for describing the coordination between different turbines is provided.

The aim of this paper is to show how both the tasks of modeling and optimization of CCPPs can be efficiently solved by resorting to hybrid system methodologies. Hybrid systems recently have attracted the interest of many researchers, because they can capture in a single model the interaction between continuous and discrete-valued dynamics. Various models for hybrid system have been proposed [24], [26], [10] and the research focused on the investigation of basic

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properties such as stability [9], [20], [22], controllability and observability [4], and the development of control [6] [26], state estimation [15] and verification [7], [1] schemes.

We will use discrete-time hybrid systems in the Mixed Logical Dynamical (MLD) form [6] for two reasons. First, they provide a general framework for modelling many discrete features of CCPPs, including the coordination and prioritization between different devices; second, they are suitable to be used in on-line optimization schemes [6].

In Section II we briefly recall the basic features of MLD systems and in Section III we describe the CCPP plant we consider (the "Island" CCPP). In Section III-B it is shown how to model in the MLD form both the continuous and discrete features of the plant. The operation optimization is then described in Section IV. We show how to recast the economic optimization problem in a Model Predictive Control (MPC) scheme for MLD systems that can be solved via Mixed Integer Liner Programming (MILP). The use of piecewise affine terms in the cost functional allows us to consider various economic factors such as the earnings from selling electric power and steam, the fixed running costs, the start up costs and the cost from the aging of plant components. In Section V the most significant control experiments are illustrated and in Section VI the computational burden of the optimization procedure is discussed. Finally, in Section VII, we discuss how additional features of CCPPs can be incorporated into the resulting MLD model and/or the MPC scheme.

II. Hybrid Systems in the MLD Form

The derivation of the MLD form of a hybrid system involves basically three steps [6]. The first one is to associate with a statement S, that can be either true or false, a binary variable $\delta \in \{0,1\}$ that is 1 if and only if the statement holds true. Then, the combination of elementary statements $S_1, ..., S_q$ into a compound statement via the boolean operators AND $(\wedge)$, OR $(\vee)$, NOT $(\sim)$ can be represented as linear inequalities over the corresponding binary variables $\delta_i,\ i = 1, ..., q$. The inequalities stemming from the compound statements are reported in Table I. As an example consider P3, which says that the statement $S_1 \vee S_2$ holds true if and only if δ_1 and δ_2 sum up at least to 1.

A special statement is given by the condition $a^T x \leq 0$, where $x \in X \subseteq R^n$ is a continuous variable and X is a compact set. If one defines m and M as lower and upper bounds on $a^T x$ respectively, the inequalities in P9 assign the value $\delta = 1$ if and only if the value of $a^T x$ satisfies the threshold condition. Note that in P7 and P9, $\varepsilon > 0$ is a small tolerance (usually close to the machine precision) introduced to replace the strict inequalities by non-strict ones.

The second step is to represent the product between linear functions and logic variables by introducing an auxiliary variable $z = \delta a^T x$. Equivalently, z is uniquely specified through the mixed integer linear inequalities in P10.

The third step is to include binary and auxiliary variables in an LTI discrete-time dynamic system in order to describe in a unified model the evolution of the continuous and logic components of the system. The general MLD form of a hybrid system is [6]

	relation	logic	mixed integer inequalities
P1	AND $(\wedge)$	$S_1 \wedge S_2$	$\delta_1 = 1$ $\delta_2 = 1$
P2		$S_3 \Leftrightarrow (S_1 \wedge S_2)$	$-\delta_1 + \delta_3 \leq 0$ $-\delta_2 + \delta_3 \leq 0$ $\delta_1 + \delta_2 - \delta_3 \leq 1$
P3	OR $(\vee)$	$S_1 \vee S_2$	$\delta_1 + \delta_2 \geq 1$
P4	NOT $(\sim)$	$\sim S_1$	$\delta_1 = 0$
P5	IMPLY $(\Rightarrow)$	$S_1 \Rightarrow S_2$	$\delta_1 - \delta_2 \leq 0$
P6	IFF $(\Leftrightarrow)$	$S_1 \Leftrightarrow S_2$	$\delta_1 - \delta_2 = 0$
P7		$\left[a^T x \leq 0\right] \Rightarrow [\delta = 1]$	$a^T x \geq \varepsilon + (m - \varepsilon)\,\delta$
P8		$[\delta = 1] \Rightarrow \left[a^T x \leq 0\right]$	$a^T x \leq M - M\delta$
P9		$\left[a^T x \leq 0\right] \Leftrightarrow [\delta = 1]$	$a^T x \leq M - M\delta$ $a^T x \geq \varepsilon + (m - \varepsilon)\,\delta$
P10	Mixed product	$z = \delta \cdot a^T x$	$z \leq M\delta$ $z \geq m\delta$ $z \leq a^T x - m\,(1 - \delta)$ $z \geq a^T x - M\,(1 - \delta)$

TABLE I

Basic conversion of logic relations into mixed integer inequalities.

$$x(t+1) = Ax(t) + B_1 u(t) + B_2 \delta(t) + B_3 z(t) \qquad (1)$$

$$y(t) = Cx(t) + D_1 u(t) + D_2 \delta(t) + D_3 z(t) \qquad (2)$$

$$E_2 \delta\,(t) + E_3 z\,(t) \leq E_1 u\,(t) + E_4 x\,(t) + E_5 \qquad (3)$$

where $x = \begin{bmatrix} x_c^T & x_l^T \end{bmatrix}^T \in R^{n_c} \times \{0,1\}^{n_l}$ are the continuous and binary states, $u = \begin{bmatrix} u_c^T & u_l^T \end{bmatrix}^T \in R^{m_c} \times \{0,1\}^{m_l}$ are the inputs, $y = \begin{bmatrix} y_c^T & y_l^T \end{bmatrix}^T \in R^{p_c} \times \{0,1\}^{p_l}$ the outputs, and $\delta \in \{0,1\}^{r_l}$, $z \in R^{r_c}$ represent auxiliary binary and continuous variables respectively. All constraints on the states, the inputs, the z and δ variables are summarized in the inequalities (3). Note that, although the description (1)-(2)-(3) seems to be linear, nonlinearity is hidden in the integrality constraints over the binary variables. MLD systems are a versatile framework to model various classes of systems. For a detailed description of such capabilities we defer the reader to [6], [4].

The discrete-time formulation of the MLD system allows developing numerically tractable schemes for solving complex problems, such as stability [12], [29], state estimation and fault detection [15], formal verification of hybrid system [7], and control [6]. In particular, MLD models were proven successful for recasting hybrid dynamic optimization problems into mixed-integer linear and quadratic programs solvable via branch and bound techniques [33]. In this paper, for the optimization of the plant we propose a predictive control scheme (*Model Predictive Control - MPC*) which is able to stabilize MLD systems on desired reference trajectories while fulfilling operating constraints.

From this Section it should be apparent that the procedure for representing a hybrid system in the MLD form (1)-(2)-(3) can be automatized. For this purpose, the compiler HYSDEL (HYbrid System DEscription Language), that generates the matrices of the MLD model starting from a high-level description of the dynamic and logic of the system, was developed at ETH Zürich [35].

III. Hybrid Model of a Combined Cycle Power Plant

The cogeneration combined cycle power plant Island comprises four main components: a gas turbine, a heat recovery steam generator, a steam turbine and a steam supply for a paper mill.

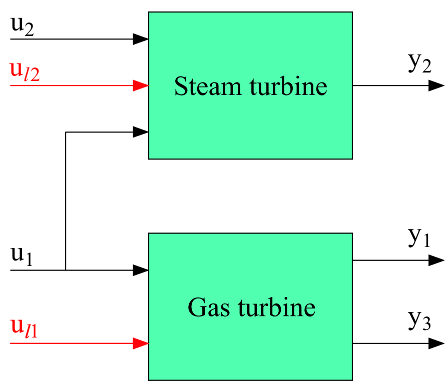

Fig. 1. Block diagram of the Island power plant.

We adopted the simplified input/output description of the plant given in [32] and represented in Figure 1. The plant has two continuous-valued inputs (u_1 and u_2), and two binary inputs (u_{l1} and u_{l2}):

- u_1 is the set point for the gas turbine load (in percent). The permitted operation range for the gas turbine is in the interval $[u_{1,\text{min}}, u_{1,\text{max}}]$;
- u_2 is the desired steam mass flow to the paper mill (in kg/s). The permitted range for the steam flow is in the interval $[u_{2,\text{min}}, u_{2,\text{max}}]$;
- u_{l1} and u_{l2} are, respectively, the on/off commands for the gas and steam turbines; the "on" command is associated with the value one.

We assume that the inputs u_1 and u_2 are independent and all possible combinations within the admissible ranges are permitted. The binary input variables must fulfill the logic condition

$$u_{l2} = 1 \quad \Rightarrow \quad u_{l1} = 1, \tag{4}$$

which defines a priority constraint between the two turbines: The steam turbine can be switched on/off only when the gas turbine is on, otherwise the steam turbine must be kept off.

The output variables of the model are:

- the fuel consumption of the gas turbine, y_1 [kg/s];
- the electric power generated by the steam turbine, y_2 [MW];
- the electric power generated by the gas turbine, y_3 [MW];

Since we aim at optimizing the plant hourly, we choose a sampling time of one hour and we assume that the inputs are constant within each sampling interval. Due to the long sampling time we may also ignore plant dynamics like temperature changes, controller reaction times, etc. As reported in [32], the input/output model of the plant has the form

$$y_1(k+1) = f_1(u_1(k)) \tag{5}$$
$$y_2(k+1) = f_2(u_1(k), u_2(k)) \tag{6}$$
$$y_3(k+1) = f_3(u_1(k)) \tag{7}$$
$$y_4(k+1) = f_4(u_1(k), u_2(k)), \tag{8}$$

where the maps f_1, f_2, f_3 and f_4 can be either affine or piecewise affine and are obtained by interpolating experimental data. The use of piecewise affine input/output relations allows to approximate nonlinear behaviours in an accurate way.

A. Hybrid Features of the Plant

As stated in the introduction the main features which suggest modelling the Island power plant as a hybrid system are the following:

- the presence of the binary inputs u_{l1} and u_{l2};
- the turbines have different start up modes, depending on how long the turbines have been kept off;
- electric power, steam flow and fuel consumption are continuous valued quantities evolving with time.

Furthermore, the following constraints have to be taken into account:

- the operating constraints on the minimum amount of time for which the turbines must be kept on/off (the so-called minimum up/down times);
- the priority constraint (4). This condition, together with the previous one, leads to constraints on the sequences of logic inputs which can be applied to the system;
- the gas turbine load u_1 and the steam mass flow u_2 are bounded.

Finally one would also like to describe the piecewise affine relations (5)-(7) in the model of the CCPP.

B. The MLD Model of the Island Plant

All the features of the Island power plant mentioned in Section III-A can be captured by a hybrid model in the MLD form. For instance, the possibility to incorporate piecewise affine relations in the MLD model is discussed in [6], [4] and the modeling of priority constraints like (4) is detailed in [23]. Moreover the possibility of incorporating bounds on the inputs is apparent from the inequalities (3). In the following we show, as an example, how to derive the MLD description of the different types of start up for the turbines. We focus on the steam turbine. The procedure is exactly the same for the gas turbine.

Typical start up diagrams show that the longer the time for which a turbine is kept off, the longer the time required before producing electric power when it is turned on. This behavior is common to all turbines and is due to the need of heating the materials of the mechanical components in a gradual way, in order to avoid dangerous mechanical stresses. This feature can be modelled, in an approximate way, as a delay between the time instant when the plant is started and the instant when the production of electric power begins.

In our model we consider the four different types of start up procedures for the steam and gas turbines, that are reported

	$time\ spent\ off\quad (h)$	$delay\quad (h)$
normal start up	$[0,8]$	1
hot start up	$]8,60]$	2
warm start up	$]60,120]$	3
cold start up	$]120,+\infty[$	4

TABLE II

TYPICAL TYPES OF START UP PROCEDURES FOR STEAM AND GAS TURBINES

in Table II. Thus, for instance, if a turbine has been kept off for 70 hours, it will produce electric power with a delay of 3 hours from the instant when the start command is given. The shut down procedure is simpler: When a turbine is turned off, at the next time instant (one hour after!) it will produce zero electric power.

In order to take into account in the MLD model the different start up procedures, it is necessary to introduce three clocks with reset (which are state variables), five auxiliary logic variables δ, and three auxiliary real variables z. The clocks are defined as follows:

- ξ_{on} stores the consecutive time during which the turbine produces electric power. If the turbine is producing electric power, ξ_{on} is increased according to the equation
$$\xi_{on}(k+1) = \xi_{on}(k) + 1 \tag{9}$$
otherwise it is kept equal to zero;
- ξ_{off} stores the consecutive time during which the turbine does not produce electric power. So, if the turbine is off or does not produce electric power (as in a start up phase), ξ_{off} is increased according to the equation
$$\xi_{off}(k+1) = \xi_{off}(k) + 1 \tag{10}$$
otherwise it is kept equal to zero;
- ξ_d, when it is positive, stores the delay that must occur between the turning on command and the actual production of electric power. If the turbine is turned on, ξ_d starts to decrease according to the law
$$\xi_d(k+1) = \xi_d(k) - 1 \tag{11}$$
and the energy generation will begin only when the condition $\xi_d < 0$ is fulfilled. Otherwise, if the turbine is off, ξ_d must store the delay corresponding to the current type of start up. In view of Table II, when the turbine is disconnected ($u_{l2} = 0$), the value of ξ_d is given by the following rules:
$$\begin{array}{ll} \xi_{off} \le 8h & \Rightarrow\ \xi_d = 0 \\ 8h < \xi_{off} \le 60h & \Rightarrow\ \xi_d = 1 \\ 60h < \xi_{off} \le 120h & \Rightarrow\ \xi_d = 2 \\ \xi_{off} > 120h & \Rightarrow\ \xi_d = 3 \end{array} \tag{12}$$

For instance, if at the time instant $\bar{k}$ the turbine is off ($u_{l2}(\bar{k}) = 0$) and $\xi_{off}(\bar{k}) = 70$, ξ_d will be set equal to 2. If, at the next time instant $\bar{k}+1$ the turbine is switched on ($u_{l2}(\bar{k}+1) = 1$), ξ_d will evolve according to equation (11) and when, at the time instant $\bar{k}+4$, the condition $\xi_d < 0$ is fulfilled, ξ_{off} is reset to zero ($\xi_{off}(\bar{k}+4) = 0$) and ξ_{on} starts to increase according to equation (9).

Since the energy production depends on the condition $\xi_d < 0$, we introduce the logic variable δ_d defined by the threshold condition
$$\delta_d = 1 \quad \Leftrightarrow \quad \xi_d < 0 \tag{13}$$
which, written as
$$\delta_d = 1 \quad \Leftrightarrow \quad \xi_d + 0.5 \le 0$$
can be translated into mixed integer linear inequalities using the rule P9 in Table I.

Then we introduce the logic variable δ_{on}, which represent the condition ”*the turbine is on and produces electric power*” through the logic statement
$$\delta_{on} = 1 \quad \Leftrightarrow \quad (u_{l2} = 1) \wedge (\delta_d = 1) \tag{14}$$
In order to find the mixed-integer linear inequalities representing (14) one has two possibilities. The first one is to re-write (14) in *Conjunctive Normal Form* (CNF) [11] and then use the rules of Table I. Alternatively, one can use the algorithm described in [5] that allows computing the inequalities representing the proposition (14) in an automated way starting from the truth-table of the proposition (14).

The dynamics of the clocks ξ_{on}, ξ_{off} can be written as
$$\xi_{on}(k+1) = [\xi_{on}(k) + 1]\delta_{on}(k) \tag{15}$$
$$\xi_{off}(k+1) = [\xi_{off}(k) + 1](1 - \delta_{on}(k)) \tag{16}$$
As explained in Section II, the product between logic variables (as δ_{on}) and continuous variables (as ξ_{on} and ξ_{off}) can be translated in the MLD form by introducing the auxiliary real variables z_{on} and z_{off} defined as
$$z_{on}(k) = (\xi_{on}(k) + 1)\delta_{on}(k) \tag{17}$$
$$z_{off}(k) = \xi_{off}(k)\delta_{on}(k) \tag{18}$$
These relations can be represented through linear inequalities by using the rule P10 of Table I. Finally the dynamics of the counters ξ_{on} and ξ_{off} in the MLD form is given by the equations:
$$\begin{array}{rcl} \xi_{on}(k+1) & = & z_{on}(k) \\ \xi_{off}(k+1) & = & z_{off}(k) \end{array}$$

In order to represent the dynamics of the counter ξ_d, three more auxiliary binary variables and one auxiliary real variable are needed. The binary variables δ_h, δ_w, δ_c are necessary to distinguish the different types of start up and so their definition depends on the value of ξ_{off}. According to the Table II, we have
$$\delta_h = 1 \quad \Leftrightarrow \quad \xi_{off} \ge 8h \tag{19}$$
$$\delta_w = 1 \quad \Leftrightarrow \quad \xi_{off} \ge 60h \tag{20}$$
$$\delta_c = 1 \quad \Leftrightarrow \quad \xi_{off} \ge 120h \tag{21}$$
Let z_d be defined as
$$z_d = \begin{cases} \xi_d(k) - 1 & if\ u_{l2} = 1 \\ \delta_h(k) + \delta_w(k) + \delta_c(k) & if\ u_{l2} = 0 \end{cases} \tag{22}$$

Again, (22) can be translated into mixed-integer linear inequalities by using the rules P8 and P10 of Table I. It is now possible to write the dynamics of the state ξ_d as

$$\xi_d(k+1) = z_d(k)$$

that is compatible with the MLD form (1) and must be complemented with the inequalities representing (19), (20), (21) and (22).

Remark 1: From the equations (9), (10), (11), it follows that the clocks ξ_{on}, ξ_{off} and ξ_d are unbounded, but it is easy to make them bounded by a value $\overline{\xi}_{on}$ by introducing further auxiliary variables modeling rules like *"if* $\xi_{on}>\overline{\xi}_{on}$ *then* $\xi_{on}=\overline{\xi}_{on}$".

By using the methodology outlined in this Section, it is possible to derive an MLD model capturing every hybrid feature of the Island power plant. The complete model is described in [34] and involves 12 state variables, 25 δ-variables and 9 z-variables.

The 103 inequalities stemming from the representation of the δ and z variables are collected in the matrices E_i, $i = 1, ..., 5$ of (3) and are not reported here due to the lack of space. Some significative simulations which test the correctness of the MLD model of the Island power plant are also available in [34].

IV. Plant Optimization

The control technique we use to optimize the operation of the Island power plant is Model Predictive Control (MPC) [30], [6] [28]. The main idea of MPC is to use a model of the plant (the MLD model in our case) to *predict* the future evolution of the system within a fixed prediction horizon. Based on this prediction, at each time step k the controller selects a sequence of future command inputs through an optimization procedure, which aims at minimizing a suitable cost function and enforces fulfillment of the constraints. Then, only the first sample of the optimal sequence is applied to the plant at time k and at time $k+1$, the whole optimization procedure is repeated. This on-line "re-planning" provides the desired feedback control action.

Economic optimization is achieved by designing the inputs of the plant that minimize a cost functional representing the operating costs minus the obtained revenues. The terms composing the cost functional we consider are described in Section IV-A. In particular, some terms appearing in the cost functional are naturally non linear and in Section IV-B we will show how to recast them in a linear form using suitably defined auxiliary optimization variables.

This allows reformulating the MPC problem as a Mixed Integer Linear Programming (MILP) problem, for which efficient solvers exist [16].

A. Cost Functional

The following cost functional is minimized:

$$J = C_{dem} + C_{change} + C_{fuel} + C_{start\ up} + \\ + C_{fixed} - E + C_{start\ up\ gas} + C_{fixed\ gas} \tag{23}$$

Let k and M be the current time instant and the length of the control horizon, respectively. We use the notation $f(t|k)$ for indicating a time function, defined for $t \geq k$, that depends also on the current instant k. Then, the terms appearing in (23) have the following meaning:

- C_{dem} is the penalty function for not meeting the electric and steam demands over the prediction horizon:

$$C_{dem} = \sum_{t=k}^{k+M-1} p_{dem\ el}(t|k)\,|y_{pow}(t|k) - d_{el}(t|k)| \\ + \sum_{t=k}^{k+M-1} p_{dem\ st}(t|k)\,|u_2(t|k) - d_{st}(t|k)|$$

  where $y_{pow}(t|k) = y_2(t|k) + y_3(t|k)$, $p_{dem\ el}(t|k)$ and $p_{dem\ st}(t|k)$ are suitable positive weight coefficients. Further, $d_{el}(t|k)$ and $d_{st}(t|k)$, $t = k, ..., k+M-1$ represent the profile of the electric and steam demands within the given prediction horizon. Both the coefficients and the demands are supposed to be known over the prediction horizon. In actual implementation they are usually obtained by economic forecasting. The values of $p_{dem\ el}(t|k)$ and $p_{dem\ st}(t|k)$ weigh the fulfillment of the electric power demand and the fulfillment of the steam demand, respectively. Finally, we note that this cost functional penalizes production surpluses. Technological reasons are behind this design. For instance, electrical network stability related issues make overproduction undesirable.
- C_{change} is the cost for changing the operation point between two consecutive time instants:

$$C_{change} = \sum_{t=k}^{k+M-2} p_{\Delta u_1}(t|k)\,|u_1(t+1|k) - u_1(t|k)| + \\ \sum_{t=k}^{k+M-2} p_{\Delta u_2}(t|k)\,|u_2(t+1|k) - u_2(t|k)|$$

  where $p_{\Delta u_1}(t|k)$, $p_{\Delta u_2}(t|k)$ are positive weights. Note that this term is not a rate-of-change constraint because big changes in u_1 and u_2 are allowed, even if penalized. Clearly, rate-of-change constraints can also be added.
- C_{fuel} takes into account the cost for fuel consumption (represented in our model by the output y_1).

$$C_{fuel} = \sum_{t=k}^{k+M-1} p_{fuel}(t|k) y_1(t|k)$$

  where $p_{fuel}(t|k)$ is the price of the fuel.
- $C_{start\ up}$ is the cost for the start up of the steam turbine. In fact, during the start up phase, no energy is produced and an additional cost related to fuel consumption is paid. $C_{start\ up}$ is then given by

$$C_{start\ up} = \sum_{t=k}^{k+M-2} p_{start\ up}(t|k)\ \Delta u_{l1}(t|k)$$

  where $\Delta u_{l1}(t|k) = \max\{u_{l1}(t+1|k) - u_{l1}(t|k), 0\}$ and $p_{start\ up}(t|k)$ represents a positive weight coefficient. Note that $\max\{[u_{l1}(t+1|k) - u_{l1}(t|k)], 0\}$ is equal to one only if the start up of the steam turbine occurs, otherwise it is equal to zero. Since, as discussed in Section

III-B, different start up modes are allowed, in principle $p_{start\ up}(t|k)$ should increase as the delay between the "on" command and the production of electric power increases (see Table II).

- C_{fixed} represents the fixed running cost of the steam turbine. It is non-zero only when the device is on and it does not depend on the level of the steam flow u_2 and/or on the level of power output y_3. C_{fixed} is then given by

$$C_{fixed} = \sum_{t=k}^{k+M-1} p_{fixed}(t|k)u_{l1}(t|k)$$

where p_{fixed} represents the increase of the maintenance cost (per hour) due to the use of the turbine. Note that C_{fixed} causes the steam turbine to be turned on only if the earnings by having it running are greater than the fixed costs.

- E represents earnings from sale of steam and electricity; this term has to take into account that the surplus production can not be sold:

$$E = \sum_{t=k}^{k+M-1} p_{el}(t|k)(\min[y_{pow}(t|k), d_{el}(t|k)]) + \sum_{t=k}^{k+M-1} p_{st}(t|k)(\min[u_2(t|k), d_{st}(t|k)])$$

where $p_{el}(t|k)$ and $p_{st}(t|k)$ represent the prices for electricity and steam, respectively. Note that the cost functional (23) has to be minimized and so the term E appears with a minus sign.

- $C_{start\ up\ gas}$ is the start up cost for the gas turbine. It plays the same role as the term $C_{start\ up}$ and it is defined via the logic input u_{l2}. $C_{start\ up\ gas}$ is given by

$$C_{start\ up\ gas} = \sum_{t=k}^{k+M-2} p_{start\ up\ gas}(t|k)\ \Delta u_{l2}(t|k)$$

where $\Delta u_{l2}(t|k) = \max\{[u_{l2}(t+1|k) - u_{l2}(t|k)], 0\}$ and $p_{start\ up\ gas}(t|k)$ is a positive weight.

- $C_{fixed\ gas}$ represents the running fixed cost of the gas turbine and is analogous to the term C_{fixed}:

$$C_{fixed\ gas} = \sum_{t=k}^{k+M-1} p_{fixed\ gas}(t|k)u_{l2}(t|k) \qquad (24)$$

where $p_{fixed\ gas}(t|k)$ is a positive weight.

B. Constraints and Derivation of the MILP

The optimization problem can be written as one of minimizing (23) subject to $x(t|k) = x_k$, and for $t = k,...,k+M-1$, to equations of the form (1)-(3). The optimization variables are $\{u(t|k)\}_{t=k}^{k+M-1}$, $\{\delta(t|k)\}_{t=k}^{k+M-1}$, $\{z(t|k)\}_{t=k}^{k+M-1}$.

In order to state the problem in a more explicit form let us introduce the following notation. In the sequel, for any signal $p(t|k)$ we denote by $\underline{p}_k$ the vector

$$\underline{p}_k = \left[\ p(k|k) \quad \cdots \quad p(k+M-1|k)\ \right]'. \qquad (25)$$

Then, the optimization problem can be written as follows: minimize

$$\begin{aligned} J[\underline{u}, \underline{\delta}, \underline{z}] \quad = \quad & C_{dem} + C_{change} + C_{fuel} + C_{start\ up} + \\ & C_{fixed} - E + C_{start\ up\ gas} + C_{fixed\ gas} \end{aligned}$$

subject to

$$x(k|k) = x_k$$

and for $t = k$,...,$k+M-1$ to

$$\begin{aligned} &\underline{x}_{k+1} = T_x\underline{x}_k + T_u\underline{u}_k + T_\delta\underline{\delta}_k + T_z\underline{z}_k \\ &\underline{y}_k = C_C\underline{x}_{k+1} + D_{D_1}\underline{u}_k + D_{D_2}\underline{\delta}_k + D_{D_3}\underline{z}_k + \widetilde{C}x_k \\ &E_{E_2}\underline{\delta}_k + E_{E_3}\underline{z}_k \leq E_{E_1}\underline{u}_k + E_{E_4}\underline{x}_{k+1} + E_{E_5}, \end{aligned}$$

where the entries of matrices T_x, T_u, T_δ and T_z can be computed by successive substitutions involving the equation

$$x(t) = A^{t-k}x(k) + \sum_{i=0}^{t-1-k} A^i[B_1u_{it} + B_2\delta_{it} + B_3z_{it}]. \quad (26)$$

Here we used the notation $a_{it} \equiv a(t-1-i)$, for $a = \{u, \delta, z\}$. Equation (26) is also used to generate the matrices $\widetilde{C}$, D_{D_i}, $i = 1, ..., 3$, E_{E_j}, $j = 1, ..., 5$.

Due to the nonlinearities appearing in the terms C_{dem}, C_{change}, $C_{start\ up}$, E and $C_{start\ up\ gas}$ this optimization problem is a mixed integer *non linear* program. However, note that the nonlinearities in the cost functional are of a special type. Indeed, both the absolute value appearing in C_{dem} and C_{change}, and the $\min/\max$ functions appearing in E, $C_{start\ up}$ and $C_{start\ up\ gas}$ are piecewise affine maps and the optimization of a piecewise affine cost functional subject to linear inequalities can be formulated as an MILP via introduction of certain binary and continuous optimization variables [14].

The case of the cost functional J is even simpler because, using the fact that all the weight coefficients are positive, it is possible to write J as a linear function of the unknowns without increasing the number of binary optimization variables. To illustrate this point, we consider the term

$$C_{dem\ el} = \sum_{t=k}^{k+M-1} p_{dem\ el}(t|k)\,|y_{pow}(t|k) - d_{el}(t|k)|$$

.

For the absolute value, we exploit the fact that if $p > 0$, then

$$\begin{array}{ccc} \min\limits_{\substack{x \epsilon R \\ Ax \leq b}} \; p\,|x| & \equiv & \min\limits_{\substack{\eta, x\epsilon R \\ Ax \leq b \\ x \leq \eta \\ -x \leq \eta}} \; p\eta \end{array} \qquad (27)$$

Then, using (27) it is possible to express $C_{dem\ el}$ as follows

$$C_{dem\ el} = \underline{p}'_{dem\ el,k}\underline{\eta}_{dem\ el,k},$$

where $\underline{p}_{dem\ el,k}$ is a column vector of positive elements and $\underline{\eta}_{dem\ el,k}$ is a vector of auxiliary variables defined according to the notation (25) and subject to the following constraints

$$\underline{\eta}_{dem\ el,k} \geq$$

$$\begin{bmatrix} y_2(k) + y_3(k) - d_{el}(k) \\ \vdots \\ y_2(k+M-1) + y_3(k+M-1) - d_{el}(k+M-1) \end{bmatrix}$$

$$\geq$$

$$\underline{\eta}_{dem\ el,k}$$

$$- \begin{bmatrix} y_2(k) + y_3(k) - d_{el}(k) \\ \vdots \\ y_2(k+M-1) + y_3(k+M-1) - d_{el}(k+M-1) \end{bmatrix}$$

By using similar procedures (see [25] for details) it is easy to obtain the following linear expressions for the other nonlinear terms:

- $C_{dem\ st} = \underline{p}'_{dem\ st,k} \underline{\eta}_{dem\ st,k}$
- $C_{change} = \underline{p}'_{\Delta u_1,k} \underline{\eta}_{\Delta u_1,k} + \underline{p}'_{\Delta u_2,k} \underline{\eta}_{\Delta u_2,k}$
- $C_{start\ up} = \underline{p}'_{start\ up,k} \underline{\eta}_{start\ up,k}$
- $E = -\underline{p}'_{el,k} \underline{\eta}_{el,k} - \underline{p}'_{st,k} \underline{\eta}_{st,k}$
- $C_{start\ up\ gas} = \underline{p}'_{start\ up\ gas,k} \underline{\eta}_{start\ up\ gas,k}$

where $\underline{\eta}_{dem\ st,k}$, $\underline{\eta}_{\Delta u_1,k}$, $\underline{\eta}_{\Delta u_2,k}$, $\underline{\eta}_{start\ up,k}$, $\underline{\eta}_{el,k}$, $\underline{\eta}_{st,k}$, $\underline{\eta}_{start\ up\ gas,k}$ are suitably defined vectors of auxiliary continuous-valued optimization variables.

It is now possible to recast the optimization problem as the MILP

$$\begin{array}{ll} \min\limits_{V} & K'V \\ \text{subject to} & FV \leq G \end{array} \tag{28}$$

where the vector K collects all the weight coefficients, the optimization vector V is defined as

$$\begin{aligned} V = & [\underline{\eta}'_{dem\ el,k} \ \ \underline{\eta}'_{dem\ st,k} \\ & \underline{\eta}'_{\Delta u_1,k} \ \ \underline{\eta}'_{\Delta u_2,k} \\ & \underline{\eta}'_{start\ up,k} \ \ \underline{\eta}'_{el,k} \\ & \underline{\eta}'_{st,k} \ \ \underline{\eta}'_{start\ up\ gas,k} \\ & \underline{u}'_k \ \ \underline{\delta}'_k \ \ \underline{z}'_k]' \end{aligned}$$

and F and G are suitable matrices that collect the inequalities of the original optimization problem and the ones related to the definition of the η-variables.

In the whole MILP problem, the number of optimization variables is $46 \cdot M - 4$ ($27 \cdot M$ of which are integer and comprise the logic inputs and the δ-variables) and the number of mixed integer linear constraints is $119 \cdot M - 8$.

V. Control Experiments

In this section we demonstrate the effectiveness of the proposed optimization procedure through some simulations. The input-output equations describing the plant are given by (5)-(7) where

$$f_1(u_1) = 0.0748 \cdot u_1 + 2.0563 \tag{29}$$

$$f_2(u_1, u_2) = 0.62 \cdot u_1 - 0.857 \cdot u_2 + 29.714 \tag{30}$$

$$f_3(u_1) = 1.83 \cdot u_1 \tag{31}$$

Input	Minimum	Maximum
u_1	50%	100%
u_2	2 kg/s	37 kg/s

TABLE III

Upper and lower bounds on the inputs

The permitted ranges for u_1 and u_2 are summarized in Table III. For the Island plant, the affine models (29) and (31) are sufficiently accurate [32], whereas equation (30) is just an approximation of the true nonlinear behavior. Again, we stress the fact that a more precise MLD model could be obtained by using more accurate (and complex) piecewise affine approximations for the function f_2.

In Section V-A and V-B we consider a profile of the electric and steam demands over 48 hours reported in [32]. In Section V-C a control experiment over four days is illustrated.

A. Minimum Prediction Horizon M=2

In the first control experiment we use the shortest possible horizon $M = 2$. At the initial instant ($k = 0$) the state of the system has been chosen in a way such that both turbines are on. In particular, the counters of the steam turbine have the following values: $\xi_{on}(0) = 10$, $\xi_{off}(0) = 0$, $\xi_d(0) = -10$. The other components of the state were initialized randomly.

The following constant values for the weights were used in the cost functional :

$p_{dem\ el} = 10$ [MW]	$p_{fuel} = 0.02$ [kg/s]
$p_{dem\ st} = 1$ [kg/s]	$p_{fixed} = 1$
$p_{\Delta u_1} = 0.01$ [%]	$p_{fixed\ gas} = 1$
$p_{\Delta u_2} = 0.01$ [kg/s]	$p_{el} = 0.2$ [MW]
$p_{start\ up} = 50$	$p_{st} = 0.2$ [kg/s]
$p_{start\ up\ gas} = 50$	

The results are shown in Figure 2. Note that the steam turbine is always kept off. This is desirable until $k = 14$ because the gas turbine can produce enough power to satisfy the electric demand. But when, at $k = 15$, the electric demand exceeds the maximum electric power that can be supplied by the gas turbine ($\simeq 183$ MW), the contribution of the steam turbine would be required.

Note that, at time $k = 15$, the steam turbine is in the condition of a hot start up. In fact, the turbine has been kept off for more than eight hours ($\xi_{off}(15) > 8$), and then $\xi_d(15) = 1$ (see Table II). If the command on were given, it would imply that ξ_d begins to decrease and the turbine would start to produce electric power with a delay of two hours (i.e. when the condition $\xi_d < 0$ is fulfilled). However, the short prediction horizon ($M = 2$) is equal to the delay and the earnings from power production (that may compensate for the start up costs) cannot be taken into account in the cost functional. This explains why, as represented in Figure 2, the optimizer always decides to keep the steam turbine off. The natural remedy to this undesirable behavior is to increase the prediction horizon.

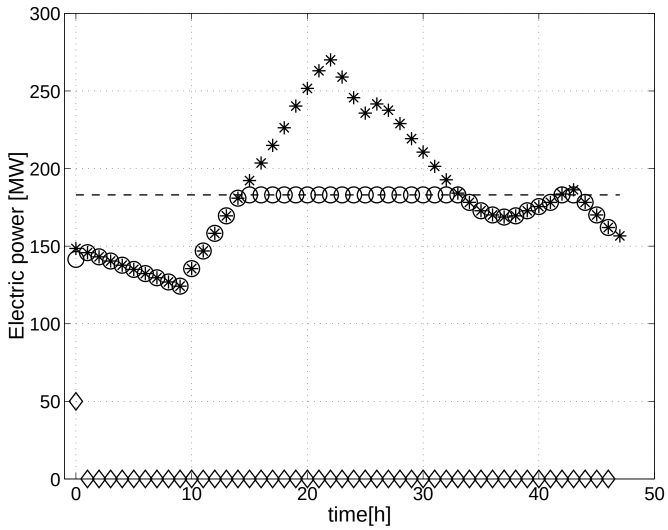

Fig. 2. Control experiment over 48 hours with $M = 2$: Electric demand (stars), electric power produced by both turbines (circles) and by the steam turbine only (diamonds). The maximum level of electric power producible by the gas turbine is depicted with a dashed line

In Figure 3 the control results obtained by starting from the same initial state and using a horizon $M = 3$ are shown. As expected now the steam turbine is turned on when needed.

The previous results highlight that a prediction horizon of $M \geq 3$ is required to enable a hot start up. Similar considerations suggest a horizon $M \geq 5$ to activate all the other start up procedures reported in Table II. In the same spirit, even larger horizons might be needed if the underlying electricity and fuel prices are such that it takes long time to recover the costs of an startup.

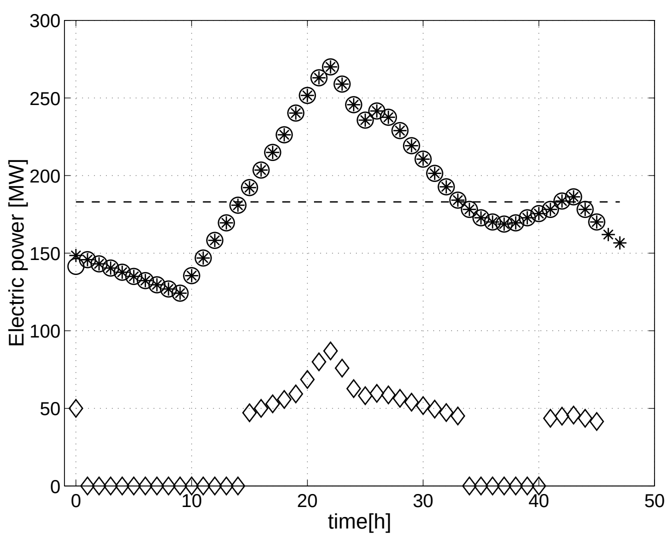

Fig. 3. Control experiment over 48 hours with $M = 3$: electric demand (stars), electric power produced by both the turbines (circles) and by the steam turbine (diamonds). The maximum electric power producible by the gas turbine is depicted with the dashed line

The results of the latter experiment concerning the control of the steam supplied to the paper mill are shown in Figure 4. Note that the steam demand is not always satisfied. In fact, due to the typical running of a combined cycle power plant, in order to increase the electric power output, some steam has to be deviated for this purpose. Since the steam demand is close to the maximum amount of steam that can be produced by the plant and $p_{dem\ el}$ was chosen higher than $p_{dem\ st}$ (i.e. $p_{dem\ el} = 10 \cdot p_{dem\ st}$), the fulfillment of the electric demand is forced at the cost of the nonfulfillment of the steam demand. In order to obtain the opposite behavior, it is necessary to decrease the value of the ratio $p_{dem\ el}/p_{dem\ st}$ sufficiently.

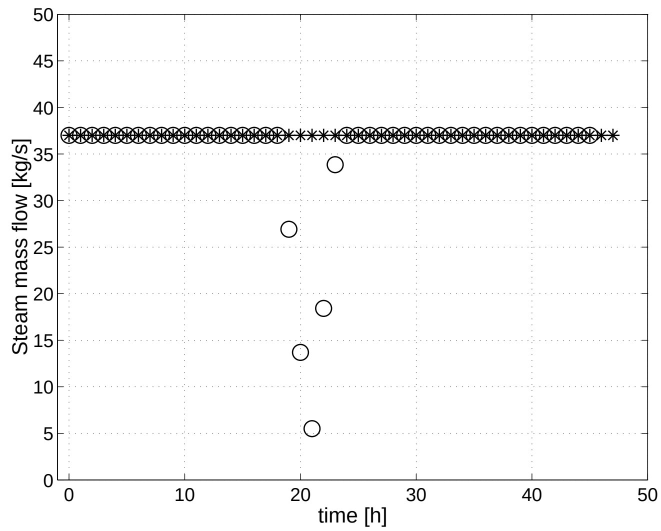

Fig. 4. Control experiment over 48 hours with $M = 3$: steam demand (stars) and steam supplied(circles)

B. Effect of Penalizing the Input Changes

In the second experiment we used a prediction horizon $M = 3$ hours and the same initial state and weight coefficients reported in Section V-A. The only difference is that we scaled the profile of the electric demand in order to make it zero at time $k = 9$. The results are shown in Figure 5. Note that, until $k = 11$ the gas turbine is off (and therefore also the steam turbine is off in view of the constraint (4)). Indeed, the electric power that can be supplied by the gas turbine is bounded from below by the value $y_{3\,\min} = 91.5$. Therefore, it is not economical to run the gas turbine for low values of the electric demand, because the surplus production cannot be sold and the earnings can not compensate for the fixed cost (24). The gas turbine is turned on at $k = 10$ and starts producing power at $k = 12$. As in the previous experiment, the steam turbine is turned on exactly when required, i.e. at $k = 18$ when the power demand exceeds the maximum capabilities of the gas turbine.

If we conduct an experiment changing only the weight coefficients in the term C_{change} and setting them to $p_{\Delta u_1} = 1$, $p_{\Delta u_2} = 1$, we obtain a different control action as shown in Figure 6. The start up of the steam turbine occurs three hours earlier compared to the previous experiment. This is because the term C_{change} in (23) becomes very big if abrupt changes of the inputs u_1 and u_2 occur. Therefore, the joint use of the gas and steam turbines is anticipated in order to allow for smoother input profiles.

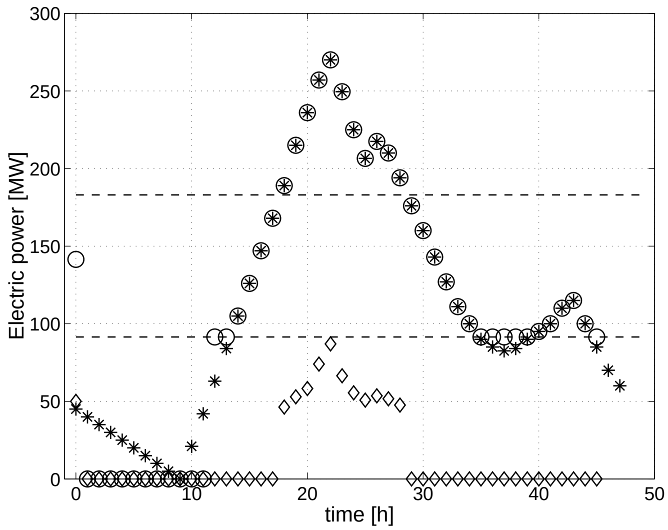

Fig. 5. Control experiment over 48 hours with $M = 3$ and low values for the weights $p_{\Delta u_1}$ and $p_{\Delta u2}$. Profile of the electric demand (stars), electric power produced by both the turbines (circles) and electric power produced by the steam turbine alone (diamonds). Dashed line: maximum and minimum electric power producible by the gas turbine

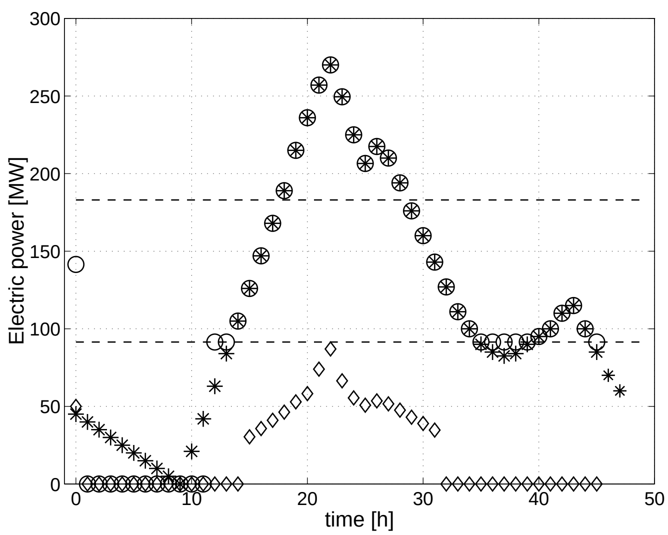

Fig. 6. Control experiment over 48 hours with $M = 3$ and high values for the weights $p_{\Delta u_1}$ and $p_{\Delta u2}$: Electric demand (stars) and electric power produced by both turbines (circles) and by the steam turbine alone (diamonds).

C. Four Days Experiment

The last control experiment we present lasts four days and a prediction horizon $M = 24$ hours was adopted. The profile of the electric demand is a scaled version of the one reported in the "IEEE reliability test" [19]. As is apparent from Figure 7, the demand during the weekend differs from the one during the working days. Moreover, the electricity prices are chosen proportional to the profile of the electricity demand. The steam demand is constant and assumed to be near to the maximum level that can be generated by the plant (see Figure 10).

Different start up costs for different start up procedures have been used. As remarked in Section IV-A, this can be done by properly choosing the weight coefficients $p_{start\ up}$ and $p_{start\ up\ gas}$. The admissible values for $p_{start\ up}$ and $p_{start\ up\ gas}$ are summarized in Table IV. In order to illustrate how the startup coefficients are assigned, we focus on the gas turbine, the procedure for the steam turbine is analogous. If at time k the gas turbine is off, the type of startup is determined by the value of the counter ξ_d (see formula (12)). Then, the numerical values of $p_{start\ up}(t|k)$, $t = k, \ldots, k + M - 1$, are determined according to Table IV and by assuming that only one startup will occur in the control horizon. For instance, if $\xi_{off}(k) = 60$ a "hot startup" should occur if the turbine is turned on in the next hour and a "warm startup" should occur if the turbine is tuned on in the next 60 hours. Then, if $M < 60$, the values $p_{start\ up}(k|k) = 58$ and $p_{start\ up}(t|k) = 115$, $t = k+1, \ldots, k+M-1$ are used for determining the optimal inputs at time k. This guarantees that at least the first startup of the turbine within the control horizon is correctly penalized.

NORMAL start up	$p_{start\ up} = 30$	$p_{start\ up\ gas} = 30$
HOT start up	$p_{start\ up} = 58$	$p_{start\ up\ gas} = 58$
WARM start up	$p_{start\ up} = 115$	$p_{start\ up\ gas} = 115$
COLD start up	$p_{start\ up} = 152$	$p_{start\ up\ gas} = 152$

TABLE IV

WEIGHTS FOR THE STARTUP OF THE GAS AND STEAM TURBINES

The other weight coefficients have the constant values

$$\begin{array}{ll} p_{dem\ el} = 20 \quad [\text{MW}] & p_{fuel} = 0.02 \quad [\text{kg/s}] \\ p_{dem\ st} = 1 \quad [\text{kg/s}] & p_{fixed} = 1 \\ p_{\Delta u_1} = 0.001 & p_{fixed\ gas} = 1 \\ p_{\Delta u_2} = 0.001 & p_{st} = 0.2 \quad [\text{kg/s}] \end{array}$$

Note, in particular, that the fulfillment of the electric demand has a higher priority than the fulfillment of the steam demand because $p_{dem\ el} \gg p_{dem\ st}$.

At time $k = 0$ the two turbines are assumed to have been off for one hour. By looking at the electric power produced by each turbine (depicted in Figure 8) and the sequence of logic inputs (Figure 9), one notes that early morning Friday and Monday the gas turbine is kept off because the demand is significantly below the minimum level that can be produced by the plant (the dashed line in Figure 7). On the other hand, Friday and Saturday night, the gas turbine is kept on because the drop in demand is not big enough. From Figures 8 and 9 it is also apparent that the steam turbine is turned on when required. For the steam control, note that, as expected, the demand is not fulfilled when high levels of power and steam are needed simultaneously (see Figure 10).

VI. COMPUTATIONAL COMPLEXITY

It is well known that MILP problems are NP-complete and their computational complexity strongly depends on the number of integer variables [33]. Therefore, the computational burden must be analyzed in order to decide about the possibility of optimizing the CCPP on-line. In the case studies reported in Section V, at every time instant, an MILP problem with $(46 \cdot M - 4)$ optimization variables ($(27 \cdot M)$ of which are

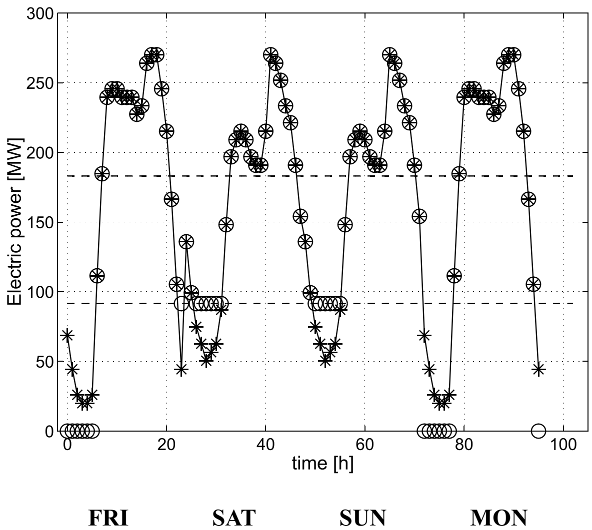

Fig. 7. Control experiment over 4 days with $M = 24$ hours: Electric power demanded (stars and solid line) and produced by both the turbines (circles). The dashed lines represent the maximum and minimum electric power that can be produced by the gas turbine

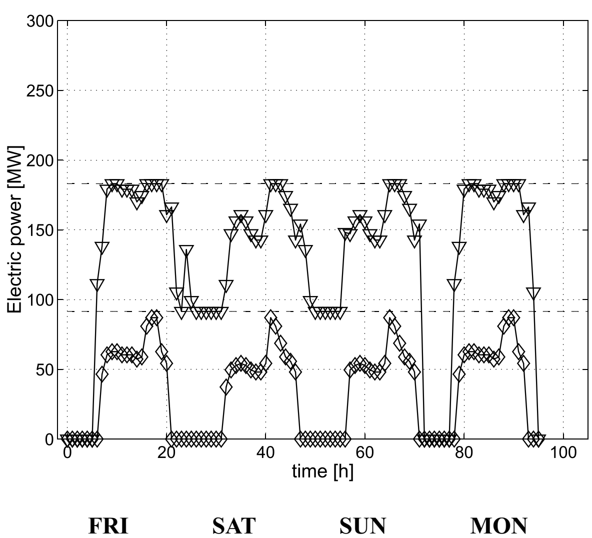

Fig. 8. Control experiment over 4 days with $M = 24$ hours: Electric power produced by the steam turbine (diamonds and solid line) and by the gas turbine (triangles and solid line). The dashed lines represent the maximum and minimum electric power that can be produced by the gas turbine

integer), and $(119 \cdot M - 8)$ mixed integer linear constraints was solved.

The computational times (in the average and worst cases) needed for solving the MILPs on a Pentium II$\cdot 400$ (running Matlab 5.3 for building the matrices K, F, and G appearing in (28) and running CPLEX for solving the MILP (28)) are reported in Table V.

Note that the computation times increase as the prediction horizon M becomes longer. However, the solution to the optimization problem took at most 102 s, a time much shorter than the sampling time of one hour. We conclude that the proposed optimization technique is suitable for online implementation.

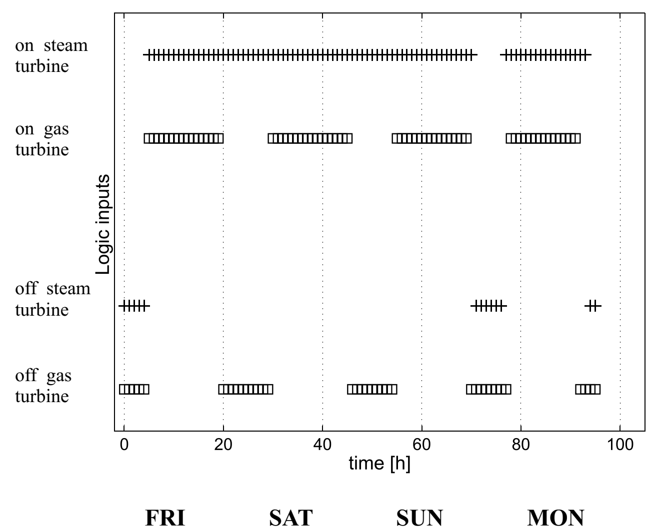

Fig. 9. Control experiment over 4 days with $M = 24$ hours: Logic input of the gas turbine (squares) and of the steam turbine (plus)

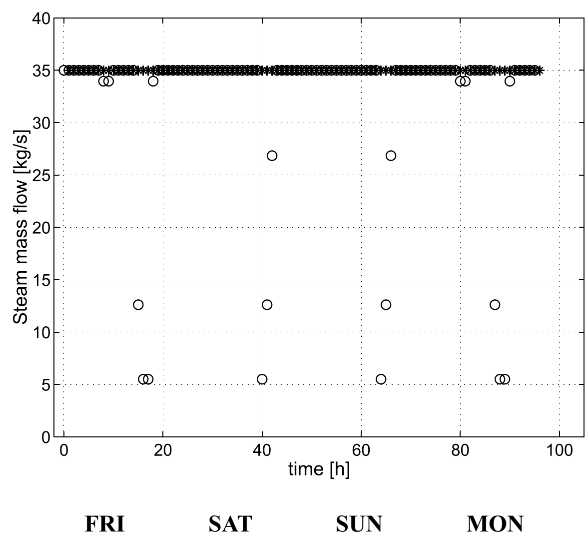

Fig. 10. Control experiment over 4 days with $M = 24$ hours: Steam demanded (stars) and supplied (circles)

M	Average times [s]	Worst case times [s]
2	0.7705	0.8110
3	1.1335	1.2720
5	2.0996	4.4860
9	4.7323	9.7040
24	33.6142	101.7370

TABLE V

COMPUTATIONAL TIMES FOR SOLVING THE MILP (29)

VII. ADDITIONAL FEATURES IN A REALISTIC IMPLEMENTATION

In the preceding sections we have illustrated how to model the basic features of a CCPP in the MLD form and how plant operation optimization can be recast as MPC problems for MLD systems. However, for a realistic implementation, several other CCPP characteristics and electricity markets features must be added either to the model or to the optimization problem. In this section we discuss some of these issues and indicate briefly how they can be included in our ”MLD systems + MPC” framework.

The first example is the inclusion of slope constraints on the power output. Physically, these constraints represent bounds on the rate at which the power plant output may be changed. In other words, at each time instant t they define a cone to which the power output at time $t+1$ must belong. More formally, slope constraints can be expressed as

$$\begin{aligned} y_3(k+1) &= \delta_{max}(k)\min\{y_3(k)+Slope, u_{Max}(k)\} + \\ &+ (1-\delta_{max}(k))\max\{y_3(k)-Slope, u_{Max}(k)\} \end{aligned}$$

where

$$u_{Max}(k) = u_1(k)MaxLoad,$$

$u_1(t)$ and $y_3(t)$ have been defined in (7), $Slope$ and $MaxLoad$ are suitable constants and the auxiliary variable $\delta_{max}(k)$ is given by

$$\delta_{max}(k) = 1 \Leftrightarrow u_{Max}(k) > y_3(k).$$

Clearly, these expressions can be embedded in the MLD system representing the plant by using the modelling procedures presented in Sections II and III.

Another important effect to be taken into account is the dynamics of the heat recovery steam generators and/or additional boilers. Indeed, the characteristics of these devices may have a strong influence on the CCPP behavior and efficiency. In this case, the corresponding nonlinear input/output relationships should be approximated by piecewise affine functions and incorporated into the MLD description of the power plant.

More complicated is the treatment of intrinsically time varying effects like the dependency of plant efficiency and maximal power output on the ambient conditions. There are at least two ways to address this problem. The first one is to model ambient conditions as an additional uncontrollable but otherwise known and predictable input and define plant efficiency and power output as piecewise linear functions of this variable. This approach fits perfectly into the MLD framework and can be used in a straightforward manner. An interesting alternative is to model these phenomena through additional time varying constraints that are updated at every time step. This approach (that requires the use of time-varying MLD models and that is not detailed here for space reasons) has proven to be flexible and efficient.

Finally, we mention that the economic factors we considered in the definition of the cost functional J are not the only possible choices. In fact, different piecewise affine terms, reflecting other performance criteria could be added without changing the structure of the resulting optimization problem [14]. For instance, asset depreciation due to plant aging can be incorporated by exploiting piecewise linear lifetime consumption models, see [18] and [13] for details.

Using the same ideas, one can also tackle the so called ”generalized unit commitment problem” [3], i.e. the case where the optimal scheduling of several dozens of power generation units has to be found. It will be a matter of future investigations to prove the scalability of the approach presented in this paper to that case.

VIII. Conclusions

The main goal of this paper has been to show that hybrid systems in the MLD form provide a powerful framework for modelling and computation of optimal schedules for combined cycle power plants. In particular, we have shown that many features like the possibility of switching on/off the turbines, the presence of minimum up and down times, priority constraints between turbines and different startup procedures can be captured by an MLD model in a natural manner. We have also shown that the optimization of the operation can be recast as an MPC problem and that this problem can be solved efficiently by resorting to MILP solvers.

References

[1] R. Alur, T. A. Henzinger, and P. H. Ho. Automatic symbolic verification of embedded systems. *IEEE Trans. on Software Engineering*, 22(3):181–201, March 1996.

[2] J.M. Arroyo and A.J. Conejo. Optimal response of a thermal unit to an electricity spot market. *IEEE Trans. on Power Systems*, 15(3):1098–1104, 2000.

[3] R. Baldick. The generalized Unit Commitment Problem. *IEEE Transactions on Power Systems*, 10(1):465–475, 1995.

[4] A. Bemporad, G. Ferrari-Trecate, and M. Morari. Observability and Controllability of Piecewise Affine and Hybrid Systems. *IEEE Trans. on Automatic Control*, 45(10):1864–1876, 2000.

[5] A. Bemporad, D. Mignone, and M. Morari. A Framework for Control, Fault Detection, State Estimation and Verification of Hybrid Systems. *Proc. American Control Conference*, pages 134–138, 1999.

[6] A. Bemporad and M. Morari. Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3):407–427, 1999.

[7] A. Bemporad and M. Morari. Verification of Hybrid Systems via Mathematical Programming. In F.W. Vaandrager and J.H. van Schuppen, editors, *Hybrid Systems: Computation and Control; Second International Workshop - Lecture Notes in Computer Science*, pages 31–45. Springer Verlag, 1999.

[8] G. Bonzani, M. Mennucci, M. Scala, and G. Sormani. Technical and economical optimization of a 450 mw combined cycle plant. *ASME COGEN-TURBO IGTI*, 6:131–143, 1991.

[9] M.S. Branicky. Multiple Lyapunov functions and other analysis tools for switched and hybrid systems. *IEEE Trans. on Automatic Control*, 43(4):475–482, 1998.

[10] M.S. Branicky, W.S. Borkar, and S.K. Mitter. A unified framework for hybrid control: model and optimal control theory. *IEEE Trans. on Automatic Control*, 43(1):31–45, 1998.

[11] T. M. Cavalier, P. M. Pardalos, and A. L. Soyster. Modeling and integer programming techniques applied to propositional calculus. *Computers Opns Res.*, 17(6):561–570, 1990.

[12] G. Ferrari-Trecate, F.A. Cuzzola, D. Mignone, and M. Morari. Analysis of discrete-time piecewise affine and hybrid systems. *Automatica*, 38(12):2139–2146, 2002.

[13] G. Ferrari-Trecate, E. Gallestey, A. Stothert, G. Hovland, P. Letizia, M. Spedicato, and M. Antoine M. Morari. Modelling and control of co-generation power plants under consideration of lifetime consumption: A hybrid system approach. *Proc. 15th IFAC World Congress on Automatic Control, Barcelona, Spain*, 2002.

[14] G. Ferrari-Trecate, P. Letizia, and M. Spedicato. Optimization with piecewise-affine cost functions. Technical report, AUT00-13, Automatic Control Laboratory, ETH Zurich, 2001.

[15] G. Ferrari-Trecate, D. Mignone, and M. Morari. Moving horizon estimation for hybrid systems. *IEEE Trans. on Automatic Control*, 47(10):1663–1676, 2002.

[16] R. Fletcher and S. Leyffer. A mixed integer quadratic programming package. Technical report, Department of Mathematics, University of Dundee, Scotland, U.K., 1994.

[17] C. A. Frangopoulos, A. I. Lygeros, C. T. Markou, and P. Kaloritis. Thermoeconomic operation optimization of the Hellenic Aspropyrgos Refinery combined-cycle cogeneration system. *Applied Thermal Engineering*, 16(12):949–958, 1996.

[18] E. Gallestey, A. Stothert, M. Antoine, and S. Morton. Model predictive control and the optimisation of power plant load while considering lifetime consumption. *IEEE Trans. on Power Systems*, 2001. To appear.

[19] C. Grigg and P. Wong. The IEEE Reliability Test System-1996. *IEEE Trans. on Power Systems*, 14(3):1010–1020, August 1999.

[20] A. Hassibi and S. Boyd. Quadratic stabilization and control of piecewise-linear systems. *Proc. American Control Conference,* pages 3659–3664, 1998.

[21] K. Ito and R. Yokoyama. Operational strategy for an industrial gas turbine cogeneration plant. *International Journal of Global Energy Issues*, 7(3/4):162–170, 1995.

[22] M. Johannson and A. Rantzer. Computation of piecewise quadratic Lyapunov functions for hybrid systems. *IEEE Trans. on Automatic Control*, 43(4):555–559, 1998.

[23] E.C. Kerrigan, A. Bemporad, D. Mignone, M. Morari, and J.M. Maciejowski. Multi-objective prioritisation and reconfiguration of hybrid systems using model predictive control. *Proceedings of the 2000 American Control Conference (ACC)*, 3:1694–1698, June 2000.

[24] G. Labinaz, M.M. Bayoumi, and K. Rudie. A Survey of Modeling and Control of Hybrid Systems. *Annual Reviews of Control*, 21:79–92, 1997.

[25] P. Letizia. Controllo di impianti cogenerativi mediante sistemi ibridi, 2001. M. Sc. thesis, Universita' degli Studi di Pavia.

[26] J. Lygeros, C. Tomlin, and S. Sastry. Controllers for reachability specifications for hybrid systems. *Automatica*, 35(3):349–370, 1999.

[27] D. A. Manolas, C. A. Frangopoulos, T. P. Gialamas, and D. T. Tsahalis. Operation optimization of an industrial cogeneration system by a genetic algorithm. *Energy Conversion Management*, 38(15-17):1625–1636, 1997.

[28] D.Q. Mayne, J.B. Rawlings, C.V. Rao, and P.O.M. Scokaert. Constrained model predictive control: Stability and optimality. *Automatica*, 36(6):789–814, 1999.

[29] D. Mignone, G. Ferrari-Trecate, and M. Morari. Stability and stabilization of piecewise affine and hybrid systems: A LMI approach. *Proc. 39th Conference on Decision and Control*, 2000.

[30] M. Morari, J. Lee, and C. Garcia. *Model Predictive Control*. Prentice Hall, Draft Manuscript, 2001.

[31] K. Moslehi, M. Khadem, R. Bernal, and G. Hernandez. Optimization of multiplant cogeneration system operation including electric and steam networks. *IEEE Trans. on Power Systems*, 6(2):484–490, 1991.

[32] K. Mossig. Load optimization. Technical report, ABB Corporate Research, Baden (Zurich), 2000.

[33] G.L. Nemhauser and L.A. Wolsey. *Integer and Combinatorial Optimization*. Wiley, 1988.

[34] M. Spedicato. Modellizzazione di impianti cogenerativi mediante sistemi ibridi, 2001. M. Sc. thesis, Universita' degli Studi di Pavia.

[35] F. D. Torrisi, A. Bemporad, and D. Mignone. HYSDEL - A Tool for Generating Hybrid Models. Technical report, AUT00-03, Automatic Control Laboratory, ETH Zurich, 2000.

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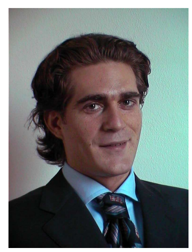

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