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Gaming in Combinatorial Clock Auctions

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Abstract. In recent years, Combinatorial Clock Auctions (CCAs) have been used around the world to allocate frequency spectrum for mobile telecom licenses. CCAs are claimed to significantly reduce the scope for gaming or strategic bidding. In this paper, we show, however, that CCAs significantly enhance the possibilities for strategic bidding. Real bidders in telecom markets are not only interested in the spectrum they win themselves and the price they pay for that, but also in the price competitors pay for that spectrum. Moreover, budget constraints play an important role. When these considerations are taken into account, CCAs provide bidders with significant gaming possibilities, resulting in high auction prices and problems associated with multiple equilibria and bankruptcy (given optimal bidding strategies).

Key Words: Combinatorial auctions, Telecom markets, Raising rivals' cost.

JEL Classification: D440, L960.

[†] We have benefited from comments by Eric van Damme and Emiel Maasland, and audiences in Vienna, Moscow, Rotterdam, and London.

1. Introduction

Combinatorial clock auctions (CCAs) are used around the globe to allocate mobile telecom licenses among interested bidders. The CCA approach has recently been (or will soon be) used in Australia, Canada, and in many European countries (such as Switzerland, Ireland, The Netherlands, Austria, and the United Kingdom). Thus, CCA seems to have superseded the more traditional simultaneous ascending auction (SAA) that until recently was the predominant auction form in telecom auctions. The advantage of the CCA is that it takes package bidding seriously. The CCA is presented to national authorities as a relatively complicated auction model that, if well understood, simplifies bidding as the scope for strategic bidding or "gaming", in the sense of taking risks to manipulate outcomes, is rather limited (see, *e.g.*, Cramton, 2012).[1] Strategic bidding (for example, in the form of demand reduction) is possible in a SSA (see, *e.g.*, Grim*et al.*, 2003).

By means of formal examples, this article shows, however, that the different phases of the CCA (the clock phase and supplementary phase) induce bidders to bid strategically and put them in a position where risky choices have to made, where the risk is endogenous to the auction. This may harm not only bidders, who may achieve undesirable outcomes, but also social welfare if the spectrum is allocated in such a way that unusable packages result. The examples help to sketch a research agenda for further investigating the properties of CCAs.

CCA is developed for markets where bidders only attach a value to their own package and the price they pay (see, *e.g.*, Ausubel, Cramton, and Milgrom, 2006). However, in many auctions (such as those for telecom spectrum) companies are likely to be also interested in

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[1] For example, in the abstract of his paper, Cramton (2012) says, "the pricing rule and information policy [of a CCA auction, MJ and VK] are carefully tailored to mitigate gaming behavior". The Irish regulator Regcom (2012, p. 70) states that their consultancy firm DotEcon notes, "the second price rule is utilized to disincentivize gaming behavior and encourage straightforward bidding".

how much competitors pay for their spectrum.[2,3] After acquiring spectrum, winners have to invest large sums of money in developing or upgrading a network. Given imperfect capital markets, it is likely that the more bidders pay for their licenses, the more expensive it becomes to finance their future investments. This means that firms are interested in raising rivals' costs by increasing the price others pay for their licenses. Another reason why firms are likely interested in raising rival's cost relates to governance issues within a firm. The information released during and at the end of the auction is such that the only way senior management (or shareholders and/or market analysts) can evaluate the success of a firm's bid strategy is to compare the packages different firms obtain at the prices they have to pay. A bid strategy is then easily considered unsuccessful if another firm paid much less for objectively better spectrum. Therefore, it is important that others do not pay less for similar packages.[4]

We show that if firms have a preference for raising rivals' cost,[5] the Vickrey-Clark-Groves (VCG) mechanism underlying the CCA does not have a (weakly) dominant strategy. Nevertheless, the CCA, but not the VCG mechanism, can be solved using iterative elimination of (weakly) dominated strategies. In the resulting equilibrium, bidders express

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[2] That firms are likely interested in other winners paying more for their licenses is confirmed in recent policy discussions (for instance OFCOM, 2012, page 122, point 7.9, for the United Kingdom).

[3] As the value of spectrum also depends on how many winners the auction has, their identity (whether they are incumbents or entrants), and on the quality of the package of licenses the competitors get, bidders valuation may actually be endogenous to the auction outcome. The literature on this issue (see, *e.g.*, Goeree, 2003; Jehiel and Moldovanu, 2000, 2003; Jehiel, Moldovanu, and Stacchetti, 1996; Janssen and Karamychev, 2007, 2010; Klemperer, 2002*a*, 2002*b*) does not consider CCAs, and we do not consider this issue in much detail (apart from a short discussion in the concluding section).

[4] The Swiss auction outcome shows that payments for similar spectrum can be quite unequal. See BAKOM (2010) for the Swiss auction design, and BAKOM (2012) for the outcome.

[5] There is a reasonably large literature on auctions with financial externalities, or auctions with a "spite motive" (see, *e.g.*, Cooper and Fang, 2008; Morgan *et al.*, 2003; Maasland and Onderstal, 2007; and recently Fiat *et al.*, 2012; and Lu, 2012). This literature typically deals with the standard single object auction, and asks the question how bidding behavior is affected if bidders also care about what the winner pays (even if they are not winners themselves). Our paper differs from this literature in that in a multi-object auction, bidders can be winners and still raise rivals' cost of other winners by placing bids on packages they themselves are not winning. This complicated gaming aspect is not present in single object auctions.

bids on packages they do not intrinsically value to raise rivals' costs. This is the first gaming aspect in CCAs. In addition, the clock phase looses much of its purpose as a "price discovery vehicle" as bidders (strategically) prolong the clock phase beyond the duration under sincere bidding. This is the second aspect of gaming.

In addition, it is usually unrealistic to assume that bidders have unlimited budgets. With imperfect capital markets, there are often hard budget constraints (imposed by banks or other creditors). In regular second-price auctions, budget constraints do not impose serious challenges for bidders *if* they do not care about competitors' payments. They can simply bid up to the minimum of their budget and their valuation for each package. In case of multiple objects, such a strategy can lead, however, to extreme inequality in payments, as a budget constraint bidder cannot express higher bids for larger packages (which is an important component in the determination of what other bidders have to pay).

When bidders have a preference for raising rivals' cost, budget constraints impose severe challenges for determining optimal bid strategies. First, it is not obvious what the notion of bidding under a budget constraint means. There are at least three interpretations that are possible: (*i*) do not place any bid above budget, (*ii*) only place bids above budget on packages that – given the development of the clock phase – cannot be winning, and (*iii*) place bids (above budget) such that *equilibrium payments* are not above budget. Second, bidders that stick to the first and second interpretation of a budget constraint run the risk of having to pay more for identical packages than competitors who adhere to the third interpretation of a budget constraint. Bidding under a budget constraint can then lead to multiple equilibria with a Hawk-Dove type flavor: more aggressive bidders perform well if they play against less aggressive bidders, but their bidding can also lead to payments above budget. Bidding under a budget constraint is the third gaming aspect discussed in this paper.

Apart from the papers mentioned above, this paper is also related to a few recent papers that shed some critical light on CCAs. Goeree and Lien (2012) show that the core selection principle introduced in the pricing rule used in CCAs implies that bidding valuation is no longer an optimal bidding strategy. Beck and Ott (2011) show that this principle may imply that bidding both above and below valuations can be optimal in CCA. Knapek and Wambach (2012) show that bidding in a CCA may be strategically complicated. None of these papers analyze, however, the possible equilibrium outcomes when bidders do not only care about own package and own payments, but also about competitors' payments. Moreover, they also do not analyze the implications of budget constraints on bidding behavior.

The rest of the paper is organized as follows. Section 2 provides a more detailed description of the different stages of the CCA. Section 3 discusses how rational bidders will bid in a CCA if the only deviation from the traditional auction set-up is one where bidders have a preference for raising rivals' cost. This preference is introduced in a lexicographic way, *i.e.*, this preference only affects bidders' decisions in case their surplus for the package they win remains the same. Thus, our treatment of raising rivals' cost can be regarded as a robustness check on the standard results regarding CCAs. Section 4 introduces budget constraints, in addition to bidders' preference for raising rivals' cost. Section 5 concludes with an agenda for future research. The appendix contains all the proofs and additional information on CCAs.

2. Combinatorial Clock Auctions

Most recent spectrum auctions allocate spectrum in different frequency bands. In each frequency band, the spectrum is divided in a certain number of blocks. If there are K different frequency bands, with n_k blocks in band k, then the total number of goods (blocks) to be allocated is $\sum_{k=1}^{K} n_k$. Spectrum in different frequency bands has different properties in terms of geographic (or indoor) coverage and capacity. Mobile telecom companies, therefore, want to acquire a mix of spectrum in different frequency bands.

A combinatorial clock auction allocates the available spectrum using two integrated phases.[6] The first phase is a clock phase, which is divided into several clock rounds. In each round, the auctioneer announces clock prices, one price for each frequency band. The prices in the first round are typically equal to the reserve prices set in advance of the auction. At these given prices, bidders can express how many blocks they would like to acquire in each band. The number of blocks in band k demanded by player i in round t will be denoted by $d_i^t(k)$. To avoid strategic bidding (such as demand reduction), at the end of a round bidders are only informed about the total demand in each band.[7] If, in round t, there is excess demand in band k, the clock prices for that band will increase in clock round $t+1$. The clock phase of the auction stops when there is no excess demand in any band. The clock phase serves to impose restrictions on what bidders can bid in the second phase, the supplementary round. The intention of the clock phase is also to assist bidders in discovering what the prices of the different frequency bands could be (Ausubel, Cramton and Milgrom, 2006). At the end of the clock phase, no spectrum is yet allocated to the bidders.[8]

Each block in a spectrum band is characterized by a certain number of so-called eligibility points. These points form a one-dimensional measurement of a player's demand. If a block in band k requires e_k eligibility points, then the total number of eligibility points of player i's demand in a round is given by $E_i^t = \sum_{k=1}^{K} e_k d_i^t\,(k)$. In any clock round, a bidder cannot express a demand that requires a larger number of eligibility points than the bid

[6] Spectrum auctions typically have a third (assignment) phase where specific location in a spectrum band is allocated. As this is of no concern for our paper, we will not discuss this assignment phase.

[7] In the CCA that was held in Austria in 2010, even that information was not available to bidders. Bidders only knew that there was excess demand in a certain band, as the clock prices in these bands (and only in these bands) increased from this clock round to the next round.

[8] In Romania In 2012, an auction was organized that had some features of a CCA. However, in that auction, frequencies were already allocated to bidders at the end of the clock phase (for the last clock round prices).

expressed in the previous round, *i.e.*, $E_i^t \leq E_i^{t-1}$.[9] If bidder i expresses a demand for a package in round t that requires a strictly smaller number of eligibility points than the bid in the previous round, *i.e.*, if $E_i^t < E_i^{t-1}$, then that round is called an anchor round. Anchor rounds, the band prices in these anchor rounds, and anchor bids play an important role in determining which bids a bidder can express in the supplementary round.

The supplementary round is a simultaneous bid round, where in one round bidders can express a bid for all possible packages, subject to some constraints. The most commonly used constraint is the so-called relative cap,[10] which works as follows.[11] On the package bidder i was bidding for in the final clock round T (and only on this package), this bidder is unconstrained in the supplementary round. Let us denote the bid expressed for this package by b. For all other packages, a bidder is constrained, and the constraints are calculated relative to b as follows. For all packages that require a number of eligibility points E that is smaller than or equal to the number of points E_i^T, the last clock round is the anchor round. Denoting the last clock round prices for band k by $p^T(k)$, the maximum bid B_α in the supplementary round that can be expressed on package α –with demands $d_\alpha(k)$ in band k– is

$$B_\alpha = b + \sum_{k=1}^{K} p^T(k) \cdot \left(d_\alpha(k) - d_i^T(k)\right).$$

In order to compute the cap for a package β that requires a larger number of eligibility points than E_i^T, one first has to track the last round r in the clock phase when bidder i had enough eligibility points to bid for package β. This r is uniquely determined by:

$$E^r < \sum_{k=1}^{K} e_k \cdot d_\beta\,(k) \leq E^{r-1}.$$

Round r is the anchor round for package β. The maximum price bidder i can then bid for package β can be computed iteratively, starting from the last clock round, as follows. Let

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9 In certain proposals, this constraint is relaxed into a Simplified Revealed preference with an Eligibility-Point Safe Harbor, see Ausubel and Cramton (2011).

10 An alternative, the final cap rule is described in the Appendix, part II.

11 This constraint is used in auctions in Switzerland (February 2012), Ireland (October-November 2012), The Netherlands (October-December 2012), and United Kingdom (2013), among others.

$\rho(r)$ be the package bidder i has bid for in clock round r, and let $b_{\rho(r)}$ be the highest actual bid in the clock or supplementary round expressed for package $\rho(r)$, where $b_{\rho(r)} \leq B_{\rho(r)}$. Then, the maximum bid B_β that can be expressed on package β – with demands in each band k of $d_\beta(k)$ – is

$$B_\beta = b_{\rho(r)} + \sum_{k=1}^{K} p^r(k) \cdot \left(d_\beta(k) - d_i^r(k)\right).$$

Thus, the relative cap expresses upper bounds on what bidders can bid for certain packages relative to their bid for the package they were bidding for in the last clock round. The relative cap has a clear economic interpretation in terms of revealed preference (see, *e.g.*, Ausubel, Cramton, and Milgrom, 2006).

Finally, we explain how the winners of the auction and the final auction prices are determined. The winners are determined in the same way as in the VCG mechanism. Of all feasible combinations of bids, one per bidder, a combination is selected that maximizes the total sum of the bids. Bids that are parts of this combination, are winning bids, and bidders who have submitted these bids win their winning bids' packages. The CCA prices are equal to the VCG prices, if the VCG prices are in the core. If the VCG prices are not in the core, CCA typically uses some adjustment such that the CCA prices are in the core (see, *e.g.*, Day and Raghavan, 2007, Day and Milgrom, 2008, and Erdil and Klemperer, 2009).

As we do not want our arguments to depend on the specific core selection principle that is used (as Goeree and Lien, 2012, have already shown that core-selecting pricing rules imply that it is not optimal to bid straightforwardly), we use examples where the CCA prices are in the core and, thus, are equal to the VCG prices. The VCG price a winner has to pay for the package he wins equals the opportunity cost he imposes on others. Thus, a winner pays the maximum price the other bidders are willing to pay additionally for the spectrum he won.

3. Preference for raising rivals' cost

In the Introduction, we have argued that bidders can be interested in raising rivals' cost. In this Section, we present an example showing that with these preferences, the supplementary stage does not have a (weakly) dominant strategy, and bidding on packages that bidders do not intrinsically value is an equilibrium strategy. Symmetric equilibrium strategies in the supplementary round of the CCA can often be found using iterative elimination of weakly dominated strategies. As the elimination procedure uses information concerning maximal bid increments from the clock phase, and this information is not provided in the VCG mechanism, the latter mechanism is not dominance solvable anymore. Using backward induction, bidders find it optimal *not* to bid according to their intrinsic preferences in the clock phase of the CCA, resulting in very high auction prices.

Our lead example has three (*ex ante* identical) bidders, two bands, and three blocks that are available in each of the bands. The reserve prices are 1 for blocks in band 1, and 4 for blocks in band 2. We assume throughout that the auction rules restrict individual bidders to get not more than tree licenses in total. Bidders are intrinsically interested in three packages (x, y), where x ans y stand for the numbers of blocks in bands 1 and 2 correspondingly. Block in band 1 and 2 "costs" 5 and 8 eligibility points correspondingly. Intrinsic values are defined as the values that bidders attach to the different packages independent of their preference to raise rivals' cost. Intrinsic values and eligibility points are given in Table 1.[12]

Apart from these intrinsic values, bidders are also interested in driving up prices their competitors have to pay. We model this preference for raising rivals' cost as a lexicographic

[12] The example is purely hypothetical, but captures important elements that we see in telecom auctions around the world. First, the number of bidders is limited (usually, only the incumbent players, or if there is spectrum reserved for an entrant, there is a limited number of entrants as well). Second, incumbents (especially if they have equal market shares) often have similar values for the spectrum, and they expect others to make similar calculations. Finally, the set of spectrum bands available is also very limited.

Package no.	Package	Package eligibility	Package value
1	(1,2)	21	50
2	(2,1)	18	46.5
3	(1,1)	13	40

Table 1. Values and eligibility points for the three packages.

preference. Bidders first care about their intrinsic values, and only if the intrinsic pay-offs of two strategies are identical, then bidders compare which strategy raises the sum of rivals' cost most. A strategy is preferred to another strategy if, for any bid of the others, it never raises rivals' cost less and sometimes raises rivals' cost more.

For simplicity, the preferences are assumed to be common knowledge. This is, of course, a drastic simplification we have to make for the analysis to become tractable. In real auctions, bidders' valuations (and their budgets) are private information. As the technology and business practices are, however, often common to all firms, firms usually have reasonably accurate beliefs about each other's valuations. Our modeling approach can be considered an extreme case where these beliefs are point beliefs that coincide with the actual realizations.

To start the analysis, we first analyze bidding in the supplementary round in case bidders have bid straightforwardly in the clock phase. The bid behavior of all three bidders in the clock phase is then as represented in Table 2. Later in this section, we show that given the equilibrium behavior in the supplementary round, it is not optimal to bid straightforwardly in the clock phase.

The bidders start bidding on package (1,2) as this is the most profitable package: with a value of 50 and a cost of 9 at the reserve prices, the net surplus is 41. As there is excess demand in band 2 (total demand is 6 as all bidders demand 2 blocks), the band 2 price increases. This is not the case in band 1 as total demand (of 3 as each bidder demands 1 block) is smaller than or equal to supply (3 blocks). In round 2, they switch to package (2,1)

Round no.	Prices	Package costs, values, and surplus				Optimal package	Activity
		Package	Cost	Value	Surplus		
1	(1, 4)	(1, 2)	9	50	41	(1, 2)	21
		(2, 1)	6	46.5	40.5		
		(1, 1)	5	40	35		
2	(1, 5)	(1, 2)	11	50	39	(2, 1)	18
		(2, 1)	7	46.5	39.5		
		(1, 1)	6	40	34		
3	(2, 5)	(2, 1)	9	46.5	37.5	(2, 1)	18
		(1, 1)	7	40	33		
4	(3, 5)	(2, 1)	11	46.5	35.5	(2, 1)	18
		(1, 1)	8	40	32		
5	(4, 5)	(2, 1)	13	46.5	33.5	(2, 1)	18
		(1, 1)	9	40	31		
6	(5, 5)	(2, 1)	15	46.5	31.5	(2, 1)	18
		(1, 1)	10	40	30		
7	(6, 5)	(2, 1)	17	46.5	29.5	(2, 1)	18
		(1, 1)	11	40	29		
8	(7, 5)	(2, 1)	19	46.5	27.5	(1, 1)	13
		(1, 1)	12	40	28		

Table 2. Clock phase development.

and then prices increase for block 1 only. When prices reach $(7,5)$ in round 11, bidders reduce demand to package $(1,1)$ and the clock phase is over.

The highest total bids the players made for the three respective packages are 9, 17, and 12. The relative cap, as explained in Section 2, now works as follows. For package $(1,1)$ bidders are unconstrained. Suppose they bid $b_{(1,1)}$ for $(1,1)$. For their supplementary bid on package $(2,1)$, the last round is the anchor round, and at these prices package $(2,1)$ costs $7 \cdot (2-1) + 5 \cdot (1-1) = 7$ more than package $(1,1)$. Therefore, bidders can bid for package $(2,1)$ up to $B_{(2,1)} = b_{(1,1)} + 7$. For package $(1,2)$, round 2 is the anchor round, and at these prices package $(1,2)$ costs $1 \cdot (1-2) + 5 \cdot (2-1) = 4$ more than package $(2,1)$. Therefore, bidders can bid for package $(1,2)$ up to $B_{(1,2)} = b_{(2,1)} + 4$, where $b_{(2,1)} \leq B_{(2,1)}$ is the actual highest bid for package $(2,1)$ in the clock phase or the supplementary round.

In addition, bidders could also make bids on packages $(3,0)$ and $(0,3)$ in the supplementary round. We assume that players do not have an intrinsic value for these packages, but if bidding on these packages increases the prices to be paid by competitors (without obtaining them), then it may be optimal to also express bids for these packages. For package $(3,0)$, the anchor round is the last clock round, and at these prices package $(3,0)$ costs $7 \cdot (3-1) + 5 \cdot (0-1) = 9$ more than package $(1,1)$. Therefore, bidders can bid for package (3,0) up to $B_{(3,0)} = b_{(1,1)} + 9$. For package $(0,3)$, the anchor round is the first clock round, and at these prices package $(0,3)$ costs $1 \cdot (0-1) + 4 \cdot (3-2) = 3$ more than package $(1,2)$. Therefore, bidders can bid for package $(0,3)$ up to $B_{(0,3)} = b_{(1,2)} + 3$, where $b_{(1,2)} \leq B_{(1,2)}$ is the actual bid for package $(1,2)$ in the clock phase or in the supplementary round.

As a reference point on auction prices, we provide two benchmarks. The first is the prices that would result if all bidders bid their intrinsic values. It is easy to see that each bidder imposes an opportunity cost of 16.5 on the two competitors. These opportunity costs imposed upon others could be considered as a kind of natural market price for obtaining the package $(1,1)$. As a second benchmark, if bidders do not make additional bids in the supplementary phase, each of them obtains package $(1,1)$ and pays $(17+12) - (12+12) = 5$. Here, $(12+12)$ is the sum of bidder i's competitors' winning bids, and $(17+12)$ is the total sum of winning bids if bidder i were not present (two remaining bidders would have obtained packages $(2,1)$ and $(1,1)$). It is easy to see that VCG prices in this example are in the core, so that the core selection requirements that CCA imposes on prices are not binding.

The first question we are interested in is what bidders should bid in the supplementary round given truthful bidding in the clock phase. For simplicity, we first assume that the development of the clock phase is public information. Then, we show that the qualitative analysis continues to hold, if bidders are only informed about total demand in every clock round, like in most real-world auctions.

We first show that if all bidders (rationally) believe that no competitor will bid on packages $(3,0)$ and $(0,3)$, for which they do not have intrinsic value, the unique equilibrium outcome in undominated strategies is one where all bidders win the package $(1,1)$ and pay 18 for that package. We subsequently argue that if bidders believe there is a chance that others will bid on packages $(3,0)$ and $(0,3)$, the equilibrium allocation remains the same, but equilibrium payments increase to 23. Both these numbers are higher than the VCG prices.

In the supplementary round, players' bids can be expressed as $\left(b_{(1,1)}, b_{(2,1)}, b_{(1,2)}\right) = (b, x, x+y)$. We look for conditions on b, x, and y such that all requirements of the relative cap are satisfied and such that $(b, x, x+y)$ is a symmetric equilibrium in (weakly) undominated strategies where all bidders win package $(1,1)$.[13]

Statement 1. Let bidders bid truthfully in the clock phase and only bid on packages they intrinsically value in the supplementary round. Then, for any $b < 18$, $x \leq 7$, and $y \leq 4$, a bid $(b, b+x, b+x+y)$ is weakly dominated.

Eliminating all weakly dominated bids with $b < 18$ leaves us with $b \geq 18$ for all bidders. Bidding so high on $(1,1)$ guarantees that each bidder wins this package, which also implies that for any $x \leq 7$ and $y \leq 4$, bidders never obtain packages $(1,2)$ and $(2,1)$. This is exploited in the next statement to argue that given the behavior in the clock round bidders should bid the maximum possible increments on larger packages.

[13] Note there is a continuum of equilibria in weakly dominated strategies. For instance, there exists a symmetric equilibrium where all bidders bid $(16.5, 23, 26.5)$. However, this strategy is weakly dominated by $(17, 23.5, 27)$ as this latter strategy is sometimes winning when the first is not (this happens when, *e.g.*, the rivals bid together more than 16.5 more on (2,1) and (1,2) than they both bid for (1,1)), and if both strategies are winning they yield identical pay-offs.

Statement 2. Let bidders bid truthfully in the clock phase and only bid on packages they intrinsically value in the supplementary round. Then, only strategies of the form $(b, b+7, b+11)$ with $b \geq 18$ survive two rounds of iterative elimination of weakly dominated strategies. In the resulting equilibrium, each bidder gets package $(1,1)$ and pays a price of 18.

It is interesting to note the differences with the VCG mechanism. First, in a CCA bidders are restricted in their bidding in the supplementary round by their bids in the clock phase. This allows to calculate a "knock out" bid, KO-bid hereinafter, a bidder should make on its last clock round package in order to guarantee that he is at least winning this package.[14] In our example, this is 18 (statement 1), much below value. As bidders in a VCG mechanism are not restricted, they cannot calculate such a knock out bid. This implies that any bid below value, with marginal bid increments below marginal values, is dominated. As bidders are unrestricted to bid much more than (marginal) value in a VCG mechanism, no further strategies can be eliminated. Bidding above value may result in a bid that, in combination with high bids of others, is winning in cases where it is better not to win that bid. The restrictions imposed by the CCA give certainty that bidding above marginal value on larger packages than the last clock round package is without risk of winning them *if bidders do not choose weakly dominated strategies*. The information from the clock round can thus be used to bid to raise rivals' cost. This makes the CCA different from the VCG mechanism, where raising rivals' cost is much more risky.

We can now perform a similar analysis for the case where bidders expect their rivals to also consider bidding on packages $(3,0)$ and $(0,3)$. We have argued that given the clock phase bidding, the maximum bid in the supplementary round on package $(3,0)$ equals

[14] The calculations determining the knock-out bid make use of the assumption that bidders know the demand of individual bidders in each clock round. However, even if only total demands are known, a "knock-out" bid can be calculated by considering all possible combinations of individual demands that are consistent with the information concerning total demand and own bidding behavior.

$B_{(3,0)} = b_{(1,1)} + 9$, whereas the maximum bid in the supplementary round on package $(0,3)$ equals $B_{(0,3)} = b_{(1,2)} + 3$, where $b_{(1,2)} \le B_{(1,2)}$ is the actual bid for package $(1,2)$ in the clock phase or the supplementary round. This means that each bidder could express a five-tuple bid $B = (b, b + x, b + x + y, b + z, b + x + y + t)$, where $x \le 7$, $y \le 4$, $z \le 9$, and $t \le 3$, on the packages $(1,1)$, $(2,1)$, $(1,2)$, $(3,0)$, and $(0,3)$ respectively. This implies that the maximal sum of bids of a two-winner combination is $2b + 23$.

In order to guarantee that $(1,1)$ is a winning bid, a bidder should bid at least $b = 23$, which is the KO-bid. If others trust that competitors are bidding $b \ge 23$ on package $(1,1)$, they find it optimal to bid $(23,\ 34,\ 30,\ 32,\ 37)$ on the five possible packages in order to win $(1,1)$ themselves, and to make sure that competitors are paying the maximal amounts for their winning bid.

Again, this outcome follows from iterative elimination of weakly dominated strategies. First, bidders bid $b \ge 23$ on $(1,1)$ to guarantee they are among the winners and bid their marginal valuations in addition to b on packages $(2,1)$ and $(1,2)$. Next, when bidders bid at least 23 on package $(1,1)$, the preference for raising rivals' cost ranks all the undominated strategies with respect to the price that others pay. Thus, the only strategies surviving the second round of elimination are such that bidders bid the largest amounts that are consistent with the relative cap on all packages that have an influence on competitor's prices.

Statement 3. Let bidders bid truthfully in the clock phase. Then, only strategies of the form $(b, b + 7, b + 11, b + 9, b + 14)$ with $b \ge 23$ survive iterative elimination of weakly dominated strategies. In the resulting equilibrium, each bidder gets package $(1,1)$ and pays auction price of 23.

One may wonder whether it is risky to bid (high) on packages that do not have intrinsic value, such as $(3,0)$ and $(0,3)$. The answer depends on whether bidders are reasonably certain about the competitors' values and their rationality. If rivals expect others may place a (high) bid on packages $(3,0)$ and $(0,3)$, then they want to protect themselves against not winning, calculate the KO-bid, and bid this amount. In this case, there is no risk for rivals to place (high) bids on $(3,0)$ and $(0,3)$, but this uncertainty is endogenous to the auction in that others' bids depend on their expectations of what others' bid. Of course, if one of the bidders does not understand the iterative dominance logic, there may be two bidders winning a package they do not value.

This extreme potential for gaming (placing relatively high bids on packages that do not have an intrinsic valuation) is not present in the VCG mechanism because a KO-bid does not exist. The risk of winning a package without intrinsic value by bidding on it is, thus, much higher in the VCG mechanism.

The logic of the above example is very strong, and Proposition 1 provides a general condition when the logic applies. The main condition is that given the development of the clock phase, bidders should be able to calculate a KO-bid for their last clock round package, and this KO-bid should be smaller than their valuation. To state and prove this proposition, we need some additional notation. Let v_i^* be the value of bidder i for his last clock round package k_i^*, and let z_i^* be the KO-bid, *i.e.*, the lowest bid on k^* that, given the clock phase behavior, guarantees bidder i winning package k_i^*. Finally, let $B_{(k)i}$ be the maximum (cap) price for package $k \neq k_i^*$ that bidder i is allowed to bid when he bids z_i^* for k_i^*.

Proposition 1. If $v_i^* \geq z_i^*$ for all i, the supplementary round game is dominance solvable by iterative elimination of weakly dominated strategies. In the supplementary round, each bidder who has been active in the last clock round, bids at least z_i^* on package k_i^*, and the maximum (cap) bids $B_{(k)i}$ for (some) other packages k.

The term "some" in the above Proposition requires further explanation. In computing the VCG price that bidder j pays for his package k_j^*, not all bids of bidder i may play a role, *i.e.*, bids $b_{(k)i}$ for some packages k may (but not necessarily) be redundant. In such a case, iterative elimination of weakly dominated strategies does not specify how high such bids should be; these bids are not actively used in the determination of the winning allocation, nor are they used in the VCG pricing.

When clock phase bids of individual bidders are not public information (but the values still are), bidders cannot compute others' relative caps and, therefore, their own KO-bids. However, since the total demands are usually made available, each bidder can calculate for any possible development of the clock phase the corresponding KO-bid. The maximum of all KO-bids, which can be called the "maximum knock out" bid, or MKO-bid, is the bid which guarantees the bidder winning his last clock round package irrespective of the clock phase development.[15]

Given that bidders cannot bid for more than 3 licenses, there are 14 generically different scenarios that generate the same joint demand of bidders 2 and 3. Computing KO-bids for each scenario yields a MKO-bid of 29. This implies that in order to win $(1{,}1)$, bidder 1 has to bid $b \geq 29$.[16] The only remaining ingredient that is necessary to apply Proposition 1 is that the MKO-bids are lower than values. In this case, the gist of proposition 1 continues to hold: we can first eliminate all bids with $b < 29$, and then, all remaining bids for which the relative cap restrictions are not binding. In equilibrium, bidders bid $(b, b+7, b+11, b+9, b+14)$ with $b \geq 29$, win $(1{,}1)$, and pay a price of at most 29 and at least 23.[17]

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[15] The same procedure can also be used in CCA when no information on total demands is made available. The number of scenarios to consider, however, very quickly gets out of hands when the number of clock rounds increases.

[16] When bidders 2 and 3 bid for $(3{,}0)$ and $(1{,}2)$ in rounds 2-6, they can bid up to $b + 20$ for $(0{,}3)$ and up to $b + 9$ for $(3{,}0)$ in the supplementary round. This leads to the KO-bid of 29 in this scenario.

[17] The number 23 is the minimum KO-bid. The details of these derivations are available from the authors.

Clock Phase

We now consider whether it is optimal to bid truthfully in the clock round. To see the consequences of the behavior in the supplementary round for bidding behavior in the clock phase, we mainly consider the last round of the clock phase in our example. Bidders know that if they reduce demand to $(1,1)$ they cannot bid more than $B_{(2,1)} = b_{(1,1)} + 7$ on package $(2,1)$, where $b_{(1,1)}$ is the actual bid for $(1,1)$ in the supplementary round. However, if they continue bidding on $(2,1)$ for one or more rounds, they are able to increase the difference in bids between packages $(2,1)$ and $(1,1)$ in the supplementary round, and can make others pay more in the end. Thus, *individually* bidders have an incentive to overstate their preferences for larger packages in the clock phase.[18]

When all bidders behave this way, however, much higher payments result. To see this, suppose that all bidders continue bidding on $(2,1)$ until the prices reach the amount $(x, 5)$, for some x and then drop demands to $(1,1)$, finishing off the clock phase. This implies that on package $(3,0)$, bidders can pay up to $(2x - 5)$ more than they bid on $(1,1)$ in the supplementary round. The additional amount they can bid on $(0,3)$ stays $(x + 7)$ so that the total amount they can bid more on the combination $(3,0)$ and $(0,3)$ is $(3x + 2)$. The next statement shows which strategies survive the iterative elimination of dominated strategies.

Statement 4. Let bidders have been bidding truthfully in the clock phase until clock round 8. Then:

a) Switching from the package $(2,1)$ to the package $(1,1)$ before round 14 is dominated by keeping bidding for $(2,1)$ up to round 14.

[18] This requires, of course, that bidders have reasonably accurate expectations about the valuations of other bidders.

b) The iteratively undominated strategies in the continuation game are such that in the clock phase, bidders keep bidding for $(2,1)$ until round 14 when prices are $(13,5)$, and then switch to $(1,1)$, and in the supplementary round, bidders bid $B = (40,\ 53,\ 57, b_{(3,0)}, b_{(0,3)})$, where $b_{(3,0)} \in [60,61]$ and $b_{(0,3)} \in [59,60]$. Winners pay a price of at least 39.

Note that the behavior in the clock phase has a Prisoners' Dilemma aspect: it is individually rational to continue the clock phase (even if the others already have reduced demands), but if everyone acts this way the bidders collectively get in a worse outcome. Note also that the supplementary round strategy not only requires bidders to bid 40 on package $(1,1)$, but also to bid in the range 59 – 61 on both packages $(3,0)$ and $(0,3)$, which bidders do not intrinsically value. This implies there are multiple equilibria, and that if bidders do not coordinate on the same equilibrium, some bidders may win packages $(3,0)$ and $(0,3)$ and have to pay 59 or more, which requires to have large budgets, much larger than the valuations.[19] Finally, note that a price of at least 39 is far off from "truthful" opportunity-cost-based market prices. Strategic bidding results in the clock phase not providing useful information concerning the marginal valuations of bidders.

4. Budget Constraint Bidders

In the previous section, we have considered how bidders can raise rivals' cost considerably, by including bids on packages they do not intrinsically value and/or by bidding strategically (much more than marginal value) in the clock phase. This may require, however, that bidders

[19] The multiplicity of equilibria originates from the finite bid increment. The smaller the bid increment is, the smaller the range of multiplicity and, in the limit, when the clock prices increase continuously, there is a unique outcome, which survives iterative elimination of dominated strategies.

have almost unlimited budgets, a condition that is usually not fulfilled. In this Section, we analyze the gaming aspects of budget constraints.

When a budget constraint is soft and *chosen* by senior management or shareholders, it follows immediately from the analysis in the previous section that there is a Prisoners' Dilemma aspect related to the choice of budgets in the clock phase. It is individually optimal in the clock phase to extend the budget constraint to be able to make the largest marginal bids for all bidders in the supplementary round. Collectively, however, bidders are better off if everyone is severely constrained, as this significantly reduces the extent to which bidders can make larger bids on larger packages, thereby reducing prices.

Under hard budget constraints, some fundamental questions arise as to what it means to have a hard budget constraint in a CCA, and how to bid with it. Typically, when bidding under a budget constraint, bidders cannot express a bid that is larger than their budget constraint. This is, however, not the case in a CCA, where a rational bidder will express monetary bids above the budget constraint on certain packages. To understand the issues involved, reconsider the example we analyzed in the previous section, where all bidders have been active in the clock phase until round 14, prices reached (13,5) and then dropped demand from (2,1) to (1,1). Suppose now that bidders have a budget constraint of 35. Note that the highest bid made in the clock phase is 29 (on package (2,1) at prices (12,5)), while the last clock phase bid is 18 on package (1,1) so that the budget constraint was not binding in the clock phase.

As bidders know the others could together bid up to (more than) 40 more on the combination of (3,0) and (0,3), they may actually bid in the supplementary round their entire budget of 35 on package (1,1) to maximizes their chances to get this package. What could they bid on other packages without running the risk of having to pay more than 35? The answer is that on both packages (2,1) and (1,2) they should not bid more than 40, otherwise they will have a chance to pay more than their budget.

To see this, consider bids $B = \left(b_{(1,1)}, b_{(2,1)}, b_{(1,2)}\right)$ with $b_{(1,1)} = 35$. The relative cap restrictions are $b_{(2,1)} \leq b_{(1,1)} + 13$ and $b_{(1,2)} \leq b_{(1,1)} + 17$. This implies bidders 2 and 3 can together maximally bid 30 more for the total spectrum than they can together bid for (2,2). Thus, if bidder 1 does not bid more than 5 more for a package (2,1) or (1,2) he does not risk paying above 35. To see that a bid larger than 40 is risky, suppose that bidder 1 bids $B_1 = \left(35,\ 40,\ 40 + x\right)$, $x > 0$. It is easy to see that if bidder 2 bids $B_2 = \left(18 + x,\ 29,\ 35 + \frac{1}{2}x\right)$ while bidder 3 bids $B_3 = \left(22,\ 35,\ 35\right)$, bidder 1 wins (1,2) and pays $p_{(1,2)} = 35 + \frac{1}{2}x$, *i.e.*, above the budget. A similar argument holds true for bidder 1 bidding$(35, 40 + x, 40)$, $x > 0$.

Two things are worth noting about this conclusion. First, if all bidders behave in this way and bid $B = \left(35,\ 40,\ 40\right)$, the actual payments will be 10 (!) for the package (1,1), quite a bit lower than the reasonable market prices of 16.5, and much less than 40 (when bidders are not constrained by budgets). Second, a budget constraint of 35 does not mean that a bidder will not express a bid that is larger than 35. A bidder can calculate that bidding 40 on (2,1) and/or (1,2) will never be winning bids, if combined with a bid of 35 on (1,1). Thus, even a hard budget constraint does not imply that you cannot (and should not) make a bid above the budget constraint!

The problem with the behavior assumed above is that it is not an equilibrium. In particular, assuming its rival bidders to be avoiding any risk (of not winning (1,1)) by bidding $B = \left(35,\ 40,\ 40\right)$, a bidder may actually *increase* its rivals' prices by *reducing* its bid on the packages (1,1) and (2,1). By doing so to the maximum extent possible, the bidder, in fact, increases the marginal bids on (2,1) and (1,2), which increases the payments others have to make for their winning package of (1,1) to 22, whereas the bidder itself continues paying 10.

Such a bid of $B' = (23,\ 36,\ 40)$ is, however, also more risky as, depending on the strategies chosen by the competitors, the bidder is more likely to win nothing. However, if the others avoid all risks and bid their entire budget on their last clock phase package and only marginally more on larger packages, then it is optimal to incur this risk. If everyone does so, however, then the outcome is that two bidders win packages $(2,1)$ and $(1,2)$ and pay 36, respectively 40. This outcome does not respect the budget constraints of the bidders.

Thus, bidding under a budget constraint is fairly complicatedin a CCA where bidders are interested in raising rivals' cost. It is not optimal to have all bids satisfy the budget constraint, and one can "safely"[20] make some bids above this constraint. Generally, it is not optimal, however, in a CCA to make only these "safe" bids; it can be optimal to take risks. How far one should go, depends on the expectations of how far others are willing to go and in a one-off round, this is very difficult to predict.

The next statement shows that there are usually many asymmetric equilibria that have a Hawk-Dove nature: the more aggressive, more risk-taking bidder does better than the less aggressive bidder. It presents one such an equilibrium.

Statement 5. Let all bidders have dropped demand in clock round 14 at prices $(13,\ 5)$ from $(2,1)$ to $(1,1)$, and let all bidders have a budget constraint of 35. The following is an equilibrium: one "hawk-like" bidder bids $B^H = (35,\ 48,\ 52,\ 56,\ 55)$, while the other two "dove-like" bidders bid $B^D = (35,\ 48,\ 52,\ 50,\ 49)$. All bidders win package $(1,1)$, the "hawk-like" bidder pays 30, while the other two pay 35.

The equilibrium in Statement 5 satisfies the budget constraint "in equilibrium", but all bidders run the risk of having to pay more than their budget if a non-equilibrium allocation results.

[20] Of course, what is important here is our assumption that players know or have correct beliefs about the bidding behavior of individual bidders. In real CCAs, bidders are only informed about the total demand per band in each clock round. In that case, bidders can still make inferences about which bids they can safely make, but this requires more complicated combinatorial calculations.

This “equilibrium” interpretation of a budget constraint is the third interpretation of a budget constraint (apart from the “never make a bid above the budget constraint” – which is not optimal in a CCA, and “make only bids above the budget constraint that are never winning”). It is not clear which of these interpretations are most natural in this context.

The equilibrium in Statement 5 is only one of the equilibria of the budget constrained game. Multiple equilibria create endogenous uncertainty for bidders, as they have no way to coordinate on one of them. As a result, if they do not coordinate, they may have to pay above their budget constraint and obtain (and pay for) a package they do not value. In the real world, this would imply the firm goes bankrupt. Bidding under a budget constraint is, therefore, a risky endeavor, especially as the most careful bidder is likely to pay the most at the end of the auction.

5. Discussion and Conclusion

This paper has considered combinatorial clock auctions (CCAs) where bidders are not only interested in the package they acquire and the price they have to pay, but also care about the price competitors have to pay. We have also considered bidders that are budget constrained. We argued that these preferences and constraints are likely to be important in markets where CCAs are nowadays used.

We have shown that given these preferences, CCAs give rise to many strategic gaming possibilities. First, by placing bids on non-winning packages (in the extreme, on packages bidders do not intrinsically value) a CCA gives players the possibility to raise rivals’ cost. Information provided in the clock phase, together with the rules capping the bids that rivals can make in the supplementary round, allows bidders to calculate a “knock-out” price, which is the bid bidders minimally have to make to guarantee themselves winning at least the last clock round bid.[21] We have shown that in many circumstances, all undominated strategies for the supplementary round have bidders bid at least this amount on the last clock round package. Given the conclusion that rational bidders are able to calculate this "knock-out" price and bid at least this amount, it is without risk to bid above value on other packages.

Second, gaming in the clock phase is possible to soften the constraints imposed by the relative cap for bidding in the supplementary phase. In this way, the clock phase looses much of its appeal as a "price discovery vehicle". There is an important danger both for the bidders and for auctioneer interested in allocating packages efficiently that if one of the players does not fully understand the scope for gaming, the auction outcome can be very inefficient.

We have also pointed out that it is not clear how to interpret "bidding under a budget constraint" in a CCA. We have shown it is not optimal to place only bids that satisfy the budget constraint. A bidder can calculate that certain bids above budget do not have a chance of winning if properly combined with other bids. These bids may nevertheless be essential in raising rivals' cost. If other bidders are also constrained, a bidder may even make higher bids (much above budget) on certain packages he hopes not to win as he knows that if others are constrained, he will not win these bids. These bids are, however, risky (and can be winning bids) if others will also make these risky bids. Not making these bids may result, however, in having to pay much more than competitors for similar packages. With budget constraints, CCAs place bidders in a situation where risky choices are unavoidable, similar to the choices players have to make in a Hawk-Dove game with multiple, asymmetric equilibria.

These issues lead to several questions on which more research is required. First, there is the issue of how much information bidders should get in the clock phase and whether or not there should be a clock phase. Without the clock phase, a CCA reduces to a VCG

[21] In this paper, we have considered the relative cap rule that is often adopted in CCAs. In Appendix II, we argue that similar considerations apply to the final cap rule.

mechanism, with a different, core-selecting pricing rule.[22] *If* there is a clock phase, bidders could, in principle at the end of each round of the clock phase get information on anonymous individual demands, on total demand only, or only on the fact whether or not in a band demand is larger than supply. In this paper, we have pointed at gaming possibilities that arise if players get too much information during the clock phase (either info about individual demand or total demand). In the Austrian 2010 CCA auction, bidders did not get any information in the clock phase apart from the fact that in certain rounds the prices on certain bands increased (implying that demand was higher than supply in these bands). The Austrian model has, however, an important drawback in case bidders have a preference ordering over the full auction outcome (as in the case where the auction outcome partially determines the market structure after the auction and thereby the profits firms make; see footnote 4). It is important to understand the pros and cons of different information structures better. This is an area where future research is necessary.

A second issue relates to the pricing rule. With auction prices being determined by competitors' bids, and the possibility of making bids on many different packages, the CCA lends itself to bidders raising rivals' cost. In 2012, Romania had a package auction, but the second-price principle was replaced by a first price principle (see Ancom, 2012), and goods being allocated at the end of the clock phase (and only unsold goods being offered in the second round). In a first-price auction, one can only raise rivals' cost by also raising the cost for oneself. To better understand the advantages and disadvantages of the second-price principle in package auctions is another area for more research.

Finally, we have made our points by mainly considering a specific example. It is not difficult to construct other examples that deliver the same messages. We have a proposition showing the main features that are relevant for the properties of our example to hold. It is

[22] We did not consider core selecting pricing rules, and focused on cases where the CCA allocation and prices are in the core, as Goeree and Lien (2012) already point at the fact that bidding valuation is not optimal under alternative pricing rules.

important to know to what extent our examples generalize, and how to modify the auction design such that these unfavorable outcomes do not arise. We see this paper as a first attempt to raise interesting issues in market design that need to be understood in more detail.

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Appendix

Proof of Statement 1.

The proof consists of four parts. Depending on whether $x \leq 6.5$ or $x \in (6.5,\ 7]$ and whether $(x+y) \leq 10$ or $(x+y) \in (10,\ 11]$, we construct for bidder 1 a strategy $\tilde{B}$ that weakly dominates bid $B = (b, b+x, b+x+y)$. The main reason for distinguishing these four cases is that depending on whether a bidder's marginal bids on larger packages are larger than marginal valuation, a bidder may win a different package. In all cases, a feasible bid in the supplementary round of bidder $i \in \{2,3\}$ is $B_i = (b_i, b_i + x_i, b_i + x_i + y_i)$, where $x_i \leq 7$ and $y_i \leq 4$.

Since the clock phase includes bids $b_{(1,1)} = 12$, $b_{(2,1)} = 17$, and $b_{(1,2)} = 9$, we need to consider bids that satisfy $b_i \geq 12$, $b_i + x_i \geq 17$, and $b_i + x_i + y_i \geq 9$. By v_{-1} we denote the value of the optimal allocation if bidder 1 were absent:

$$v_{-1} = b_2 + b_3 + Z, \text{ where } Z \equiv \max\begin{Bmatrix} 0, x_2, x_3, (x_2+y_2), (x_3+y_3), \\ (x_2+y_2+x_3), (x_2+x_3+y_3) \end{Bmatrix} \leq 18.$$

Part 1. Let $x \leq 6.5$ and $(x+y) \leq 10$. We will show that bid $\tilde{B} = (18, 18+x, 18+x+y)$ weakly dominates any bid $B = (b, b+x, b+x+y)$ with $b < 18$.

Similar to the analysis of VCG mechanisms, $\tilde{B}$ either yields the same allocation as B, in which case bidder 1 pays the same price and gets the same surplus, or $\tilde{B}$ is winning, whereas B is not: as the marginal bids are identical, it cannot be that a different package is winning under the two bids. In the latter case, $\tilde{B}$ wins either $(1,1)$, or $(2,1)$, or $(1,2)$, which we consider separately. It suffices to show that if $\tilde{B}$ is winning, the surplus is nonnegative.

a) $\tilde{B}$ wins $(1,1)$. Then, it is necessarily the case that bids of players i and j satisfy $b + b_2 + b_3 \geq v_{-1}$. Bidder 1 pays VCG price $p_{(1,1)} = v_{-1} - (b_2 + b_3) = Z \leq b < 18$, and gets surplus $s_{(1,1)} = v_{(1,1)} - p_{(1,1)} \geq 40 - 18 = 22 > 0$. Thus, $\tilde{B}$ is profitable.

b) $\tilde{B}$ wins $(2,1)$. Suppose w.l.o.g. that the second winner is bidder 2.

If bidder 2 wins $(1,2)$, it must be that $(b+x) + (b_2 + x_2 + y_2) \geq b + b_2 + b_3$, which implies $b_3 \leq x + x_2 + y_2$. Bidder 1 pays VCG price

$$p_{(2,1)} = v_{-1} - (b_2 + x_2 + y_2) = b_3 + Z - (x_2 + y_2) \leq x + Z \leq x + 18,$$

and gets the surplus $s_{(2,1)} = v_{(2,1)} - p_{(2,1)} \geq 46.5 - (x+18) = 28.5 - x \geq 22 > 0$.

If, in the other hand, bidder 2 wins $(1,1)$, it must be that $(b+x) + b_2 \geq b + b_2 + b_3$, which implies $b_3 \leq x$. Bidder 1 pays VCG price

$$p_{(2,1)} = v_{-1} - b_2 = b_3 + \mathrm{Z} \leq x + Z \leq x + 18,$$

and the same result $s_{(2,1)} \geq 22$ holds. Thus, $\tilde{B}$ is profitable when it is winning.

c) $\tilde{B}$ wins $(1,2)$. Suppose w.l.o.g. that the second winner is bidder 2.

If bidder 2 wins $(2,1)$, it must be that $(b+x+y) + (b_2 + x_2) \geq b + b_2 + b_3$, which implies $b_3 \leq x + y + x_2$. Bidder 1 pays VCG price

$$p_{(1,2)} = v_{-1} - (b_2 + x_2) = b_3 + Z - x_2 \leq (x+y) + Z \leq (x+y) + 18,$$

and gets the surplus $s_{(1,2)} = v_{(1,2)} - p_{(1,2)} \geq 50 - (x+y+18) \geq 22 > 0$.

If, in the other hand, bidder 2 wins $(1,1)$, it must be that $(b+x+y)+b_2 \geq b+b_2+b_3$, which implies $b_3 \leq (x+y)$. Bidder 1 pays VCG price

$$p_{(2,1)} = v_{-1} - b_2 = b_3 + \mathrm{Z} \leq (x+y) + Z \leq x + 18,$$

and the same result $s_{(1,2)} \geq 22$ holds. Thus, $\tilde{B}$ is profitable when it is winning.

Part 2. Let $x \in (6.5,\ 7]$ and $(x+y) \leq 10$. We will show that bid $\tilde{B} = (b+\Delta, b+x, b+\Delta+x+y)$ weakly dominates any bid $B = (b, b+x, b+x+y)$ with $b < 18$, where $\Delta = (x-6.5)$. Here again, as in Part 1, if $\tilde{B}$ yields the same allocation as B does, it yields equal pay-off as well. If B is not winning but $\tilde{B}$ is, the very same arguments as in Part 1 apply to show that $\tilde{B}$ yields strictly positive pay-off. The only difference here is that $\tilde{B}$ can win a different package than B: it can win either $(1,1)$ or $(1,2)$ whereas B wins $(2,1)$. We consider these cases separately.

a) If $\tilde{B}$ wins $(1,1)$, the restrictions $b_i \in [12,18)$, $x_i \leq 7$, and $y_i \leq 4$ imply that bidders $i \in \{2,3\}$ win $(1,1)$ as well, and the price for bidder 1 is $\tilde{p}_{(1,1)} = v_{-1} - b_2 - b_3 = Z$, so that the surplus is $\tilde{s}_{(1,1)} = v_{(1,1)} - \tilde{p}_{(1,1)} = 40 - Z$.

Suppose that when B wins $(2,1)$, the second winner is bidder 2 who wins either $(1,2)$ or $(1,1)$.

Suppose that 2 wins $(1,2)$. When $\tilde{B}$ wins $(1,1)$, it must be that $(b+\Delta)+b_2+b_3 \geq (b+x)+(b_2+x_2+y_2)$, which implies $b_3 \geq x + x_2 + y_2 - \Delta$. Thus, when B wins $(2,1)$ the price is

$$p_{(2,1)} = v_{-1} - (b_2 + x_2 + y_2) = b_3 - x_2 - y_2 + Z \geq x - \Delta + Z,$$

and surplus is $s_{(2,1)} = v_{(2,1)} - p_{(2,1)} = 46.5 - \ x + \Delta - Z$. Then, the difference between $\tilde{s}_{(1,1)}$ and $s_{(2,1)}$ is:

$$\tilde{s}_{(1,1)} - s_{(2,1)} = (40 - Z) - (46.5 - \ x + \Delta - Z) = x - \Delta - 6.5 \geq 0.$$

Thus, $\tilde{B}$ yields a higher surplus than B does.

Suppose, to the contrary, that 2 wins (1,1). When $\tilde{B}$ wins (1,1), it must be that $(b+\Delta)+b_2+b_3 \geq (b+x)+b_2$, which implies $b_3 \geq x-\Delta$. Thus, when B wins (1,1) the price is

$$p_{(2,1)} = v_{-1} - b_2 = b_3 + Z \geq x - \Delta + Z,$$

and the same result $\tilde{s}_{(1,1)} - s_{(2,1)} \geq 0$ holds. Thus, $\tilde{B}$ yields a higher surplus than B does.

b) If $\tilde{B}$ wins(1,2), the second winner, let it be bidder 2, can win either (2,1) or (1,1).

If 2 wins (2,1) under $\tilde{B}$, bidder 1 pays price $\tilde{p}_{(1,2)} = v_{-1} - (b_2 + x_2) = b_3 - x_2 + Z$ and gets surplus $\tilde{s}_{(1,2)} = v_{(1,2)} - \tilde{p}_{(1,2)} = 50 - b_3 + x_2 - Z$. When B wins (2,1), the second winner can be:

- Bidder 2, who wins (1,2). Bid B yields price $p_{(2,1)} = v_{-1} - (b_2 + x_2 + y_2) = b_3 + Z - x_2 - y_2$ and surplus $s_{(2,1)} = v_{(2,1)} - p_{(2,1)} = 46.5 - b_3 - Z + x_2 + y_2$. Hence, $\tilde{s}_{(1,2)} - s_{(2,1)} = 3.5 - y_2$. $\tilde{B}$ wins (1,2) only when $(b+\Delta+x+y)+(b_2+x_2) \geq (b+x)+(b_2+x_2+y_2)$, which implies $\Delta + y \geq y_2$. Thus, using $(x+y) \leq 10$:

  $\tilde{s}_{(1,2)} - s_{(2,1)} = 3.5 - y_2 \geq 3.5 - \Delta - y \geq x - 6.5 - \Delta \geq 0.$

- Bidder 2, who wins (1,1). Bid B yields price $p_{(2,1)} = v_{-1} - b_2 = b_3 + Z$ and surplus $s_{(2,1)} = v_{(2,1)} - p_{(2,1)} = 46.5 - b_3 - Z$. Hence, $\tilde{s}_{(1,2)} - s_{(2,1)} = 3.5 + x_2$. $\tilde{B}$ wins(1,2) only when $(b+\Delta+x+y)+(b_2+x_2) \geq (b+x)+b_2$, which implies $x_2 \geq -\Delta - y$. Thus, again:

  $\tilde{s}_{(1,2)} - s_{(2,1)} = 3.5 + x_2 \geq 3.5 - \Delta - y \geq x - 6.5 - \Delta \geq 0.$

- Bidder 3, who wins (1,2). Bid B yields price $p_{(2,1)} = v_{-1} - (b_3 + x_3 + y_3) = b_2 + Z - x_3 - y_3$ and surplus $s_{(2,1)} = v_{(2,1)} - p_{(2,1)} = 46.5 - b_2 - Z + x_3 + y_3$. Hence, $\tilde{s}_{(1,2)} - s_{(2,1)} = 3.5 + (b_2 + x_2) - (b_3 + x_3 + y_3)$. $\tilde{B}$ wins(1,2) only when

$(b + \Delta + x + y) + (b_2 + x_2) \geq (b + x) + (b_3 + x_3 + y_3)$, which implies $(\Delta + y) + (b_2 + x_2) \geq (b_3 + x_3 + y_3)$. Thus, using $(x + y) \leq 10$:

$\tilde{s}_{(1,2)} - s_{(2,1)} \geq 3.5 - \Delta - y \geq x - 6.5 - \Delta \geq 0$.

- Bidder 3, who wins (1,1). Bid B yields price $p_{(2,1)} = v_{-1} - b_3 = b_2 + Z$ and surplus $s_{(2,1)} = v_{(2,1)} - p_{(2,1)} = 46.5 - b_2 - Z$. Hence, $\tilde{s}_{(1,2)} - s_{(2,1)} = 3.5 - b_3 + (b_2 + x_2)$. $\tilde{B}$ wins(1,2) only when $(b + \Delta + x + y) + (b_2 + x_2) \geq (b + x) + b_3$, which implies $(\Delta + y) + (b_2 + x_2) \geq b_3$. Thus, again:

  $\tilde{s}_{(1,2)} - s_{(2,1)} \geq 3.5 - \Delta - y \geq x - 6.5 - \Delta \geq 0$.

Summarizing, when 2 wins (2,1) under $\tilde{B}$, $\tilde{B}$ yields a higher surplus than B does.

If 2 wins (1,1) under $\tilde{B}$, similar argument hold so that $\tilde{s}_{(1,2)} - s_{(2,1)} \geq 0$. Thus, when $\tilde{B}$ wins (1,2), $\tilde{B}$ yields a higher surplus than B does.

Part 3. Let $x \leq 6.5$ and $(x + y) \in (10,\ 11]$. We take $\tilde{B} = (b + \Delta, b + \Delta + x, b + x + y)$ where $\Delta = (x + y - 10)$. As in Part 2, we only need to consider cases where B wins (1,2) whereas $\tilde{B}$ wins either (1,1) or (2,1). Since the arguments are almost identical to the arguments in part 2, they are omitted.

Part 4. Let $x \in (6.5,\ 7]$ and $(x + y) \in (10,\ 11]$. We take $\tilde{B} = (b + \Delta, b + x, b + x + y)$ where $\Delta \equiv \min(x + y - 10, x - 6.5)$. As in Parts 2 and 3, we only need to consider cases where B wins either (1,2) or (2,1) whereas $\tilde{B}$ wins (1,1). Again, the detailed arguments are omitted as they are almost identical to the ones in part 2. ∎

Proof of Statement 2.

According to Statement 1, we only need to consider bids $(b, b + x, b + x + y)$ with $b \geq 18$, $x \leq 7$, and $y \leq 4$, which always result in allocating (1,1) to every bidder. We consider bidders $i \in \{2,3\}$ bidding $B_i = (b_i, b_i + x_i, b_i + x_i + y_i)$ with $b_i \geq 18$, $x_i \leq 7$, and $y_i \leq 4$. Prices that bidder j pays for package (1,1) is:

$$p_j = \max\{0, x, x_i, (x+y), (x_i+y_i), (x+y+x_i), (x+x_i+y_i)\}.$$

It is easy to see that p_j weakly increases in x and y. Moreover, at $x_i = 7$ and $y_i = 0$, p_j strictly increases in x and y at $x = 7$ and $y = 4$. Thus, choosing $x = 7$ and $y = 4$, *i.e.*, their maximal values, maximizes p_i and, hence, dominates all other remaining $x \le 7$ and $y \le 4$. ■

Proof of Statement 3.

The proof combines the proofs of statements 1 and 2 above. In the first round of elimination of weakly dominated strategies, we eliminate all bids $B = (b, b+x, b+x+y, b+z, b+x+y+t)$, where $b < 23$, $x \le 7$, $y \le 4$, $z \le 9$, and $t \le 3$. As in the proof of Statement 1, we have four parts where different bids $\tilde{B}$ dominate bid B:

Part 1. If $x \le 6.5$ and $(x+y) \le 10$, we take $\tilde{B} = \left(23,\ 23+x, 23+x+y, b+z, b+x+y+t\right)$ and show it weakly dominates B with $b < 23$. The arguments are very similar to that of Part 2 of the proof of Statement 1. The only difference is that now it may happen that B wins (0,3) or (3,0) resulting in a negative surplus whereas $\tilde{B}$ wins (1,1), (2,1), or (1,2), resulting in a positive surplus.

Part 2. If $x \in \left(6.5,\ 7\right]$ and $(x+y) \le 10$, bid $\tilde{B} = (b+\Delta, b+x, b+\Delta+x+y, b+z, b+x+y+t$ weakly dominates B with $b<23$, where $\Delta = x - 6.5$.

Part 3. If $x \le 6.5$ and $(x+y) \in \left(10,\ 11\right]$, bid $\tilde{B} = (b+\Delta, b+\Delta+x, b+x+y, b+z, b+x+y+t$ weakly dominates B with $b<23$, where $\Delta = x+y-10$.

Part 4. If $x \in \left(6.5,\ 7\right]$ and $(x+y) \in \left(10,\ 11\right]$, bid $\tilde{B} = (b+\Delta, b+x, b+x+y, b+z, b+x+y+t$ weakly dominates B with $b<23$, where $\Delta = \min x+y-10, x-6.5$.

After the first elimination round, only bids $B = (b, b+x, b+x+y, b+z, b+x+y+t)$ remain, where $b \ge 23$, $x \le 7$, $y \le 4$, $z \le 9$, and $t \le 3$. All such bids win (1,1). In the second round of elimination, we will show that bids $\tilde{B} = (b, b+7, b+11, b+9, b+14)$ with $b \ge 23$ dominate all the other bids.

Let bidders $i \in \{2,3\}$ bid $B_i = (b_i, b_i + x_i, b_i + x_i + y_i, b_i + z_i, b_i + x_i + y_i + t_i)$ with $b_i \geq 23$, $x_i \leq 7$, $y_i \leq 4$, $z_i \leq 9$, and $t_i \leq 3$, and bidder 1 bids B. Bidder j wins $(1,1)$ and pays the price:

$$p_j = \max\left\{\begin{matrix} 0, x, x_j, (x+y), (x_j+y_j), (x+y+x_j), (x+x_j+y_j), \\ (z+x_j+y_j+t_j), (x+y+t+z_j) \end{matrix}\right\}.$$

It is easy to see that p_j weakly increases in x, y, z, and t. Moreover, at $x_j = y_j = t_j = 0$ and $z_j = 9$, p_j strictly increases in x, y, and t at $x = 7$, $y = 4$, and $t = 3$; at $x_j = 7$, $y_j = 4$, $t_j = 3$, and $z_j = 0$, p_j strictly increases in z at $z = 9$. Thus, all bids of the form $B = (b, b+7, b+11, b+9, b+14)$, with $b \geq 23$, maximize prices that bidders $i \in \{2,3\}$ pay and, therefore, only those bids survive the second elimination round. ■

Proof of Proposition 1.

In the first round of elimination of weakly dominated strategies, we concentrate on bids of the following forms:

a) Bid b_i^* for the last clock round package is lower than z_i^*, and bids for all other packages b_i^k do not exceed b_i^* plus bidder i's marginal value for packages k. All such bids are dominated by bidding $b_i^* = z_i^*$. The reason is exactly the same as in VCG mechanism: for bids that are below own valuations, higher bids dominate the lower ones.

b) Bid b_i^* for the last clock round package is lower than z_i^*, and some bid b_i^n for another package n strictly exceeds b_i^* plus bidder i's marginal value for this packages n. All such bids can be improved (*i.e.*, dominated) by raising b_i^* and all b_i^k for packages for which b_i^k does not exceed b_i^* plus bidder i's marginal value. The reason is again the same.

After all bids with $b_i^* \leq z_i^*$ are eliminated, bidders win their last clock round package k^* with certainty. The prices that competitors pay will necessarily depend on the bids that a bidder puts on other packages. In the second round of elimination, we eliminate all bids $b_{(k)i}$ that are strictly below the caps $B_{(k)i}$ and that determine the prices that competitors pay. ■

Proof of Statement 4.

Suppose that bidders 2 and 3 have reduced their demands to package $(1,1)$ in rounds τ_2 and τ_3 correspondingly, where $8 \leq \tau_2 \leq \tau_3 \leq 12$. Let us consider bidder 1 who decides whether to continue bidding on $(2,1)$ or to drop to $(1,1)$ at round $\tau \geq \tau_3$. In round τ, prices are $(\tau - 1,5)$. Following the arguments of the proof of Statement 3, after switching to $(1,1)$, undominated bids of all bidders in the supplementary round are:

$$B_1 = (b, b + \tau - 1, b + \tau + 3, b + 2\tau - 7, b + \tau + 6),$$

$$B_2 = (b_2, b_2 + \tau_2 - 1, b_2 + \tau_2 + 3, b_2 + 2\tau_2 - 7, b_2 + \tau_2 + 6),$$

$$B_3 = (b_3, b_3 + \tau_3 - 1, b_3 + \tau_3 + 3, b_3 + 2\tau_3 - 7, b_3 + \tau_3 + 6),$$

where $b \geq \tau_2 + 2\tau_3 - 1$, $b_2 \geq \tau_3 + 2\tau - 1$, $b_3 \geq \tau_2 + 2\tau - 1$, provided $\tau_3 + 2\tau - 1 \leq 40$, *i.e.*, $\tau \leq \frac{1}{2}(41 - \tau_3)$. Each bidder wins $(1,1)$ and pays:

$p_1 = \tau_2 + 2\tau_3 - 1$, $p_2 = \tau_3 + 2\tau - 1$, and $p_3 = \tau_2 + 2\tau - 1$.

Since prices p_2 and p_3 are increasing in τ, dropping to $(1,1)$ at round $\tau < \frac{1}{2}(41 - \tau_3)$ for bidder 1 is dominated by continuing bidding on $(2,1)$. Consequently, dropping to $(1,1)$ at any round τ satisfying $\tau < \frac{1}{2}(41 - \tau)$, *i.e.*, $\tau \leq 13$, is dominated. Thus, the last (remaining) bidder in the clock phase never drops to $(1,1)$ before round $\tau = 14$.

Similar argument holds when bidder 1 decides whether to continue bidding on $(2,1)$ or to drop to $(1,1)$ at round $\tau < \tau_3 \leq 14$. Indeed, the prices that bidders 2 and 3 pay are given by the same above expressions, and continuing bidding on $(2,1)$ dominates dropping to $(1,1)$ in round τ. Therefore, whether other bidders have dropped to $(1,1)$ or not, bidding on $(2,1)$ dominates dropping to $(1,1)$ at any round $\tau \leq 13$.

Part (b) is proven by direct verification. Let bidders keep bidding on $(2,1)$ up to (and including) round 13 and drop to $(1,1)$ in round 14. The KO-bid is then 41, which is above the value. Thus, bidding $b = 40$ is dominant. In the second elimination round, all bids are eliminated which are (1) below caps, and (2) are never winning when $b = 40$. Since the caps allow bids with $b_{(3,0)} = b + 21$, and $b_{(0,3)} = b + 20$, all bids with $b_{(3,0)} \in [60,61]$ or $b_{(0,3)} \in [59,60]$ remain undominated. ■

Proof of Statement 5.

If the three bidders choose these strategies, B^H, B^D, and B^D, the total sum of winning bids is $35 + 35 + 35 = 105$. The two "dove-like" bidders are willing to pay 100 together for the full spectrum, while one "dove-like" bidder and one "hawk-like" bidder have together also expressed a willingness to pay of 105. Each of the "dove-like" bidders pay 35 (as for each of them, the other two bidders are together willing to pay 105 for the total spectrum). The "hawk-like" bidder pays 5 less (as others are together maximally willing to pay 100 for the full spectrum – the combination $(2,1)$ and $(1,2)$).

It is not difficult to see that no bidder individually has an incentive to deviate. If by deviating, they change the final allocation, at least one bidder does not get any spectrum. If this is the deviating bidder, he strictly gets a smaller pay-off than in the proposed equilibrium. If it is another bidder, who does not win spectrum after the deviation, then the deviating player has to pay the opportunity cost – which is what the losing bidder expressed to be willing to pay for a larger package – which is more than the budget constraint (and valuation). Finally, as bidders do not affect their own payment, if the final allocation is not affected and in the equilibrium they already bid so as to maximally increase others' payments without affecting the allocation, deviating can only result in payments of other bidders that are not larger. Thus, no deviation is profitable.

Appendix II: Final Cap Rule

Ausubel and Cramton (2011) have proposed a new rule that imposes more constraints on bids in the supplementary rounds. They require that all bids on packages that require more eligibility points than the last bid in the clock phase satisfy a revealed preference constraint with respect to *all* rounds after the relevant anchor round where the bidders reduced eligibility points, and the last round as well. They term this rule the Simplified Revealed Preference Cap. In several recent CCA designs, for example in Australia and Canada, this Simplified Revealed Preference Cap, or final cap rule, has been adopted in practice. Under this rule, all bids b_α for packages α in the supplementary round should satisfy the following condition:

$$b_\alpha \leq B_\alpha = b + \sum_{k=1}^{K} p^T(k) \cdot \left(d_\alpha(k) - d^f(k)\right).$$

Ausubel and Cramton (2011) show that this rule implies that bidders win the package they were bidding for in the final clock round if the clock phase ends without excess supply (like in our example). The rule implies that *if bidders bid truthfully in the clock phase*, bidders cannot bid more than respectively $b + 7$, $b + 5$, $b + 9$, and $b + 3$ on the packages $(2,1)$, $(1,2)$, $(3,0)$, and $(0,3)$. As a result, no two bidders can outbid the other remaining bidder. For the total supply of $(3,3)$, they together cannot bid more than 12 more than they jointly bid on the supply of $(2,2)$. Thus, the final clock bid of $b = 12$ is sufficient to win $(1,1)$, and the total payment is not more than 12, substantially reducing bidders' possible payments.

Knowing that bids $b + 7$, $b + 5$, $b + 9$, and $b + 3$ on the packages $(2,1)$, $(1,2)$, $(3,0)$, and $(0,3)$ cannot be winning, players can submit these higher bids without any risk and make sure that rivals pay 12 for their winning package $(1,1)$. Thus, under the final cap rule as well, bidders can game and bid without any risk on packages they do not value intrinsically. This consideration has led the UK regulator OFCOM to withdraw the final cap rule from its auction design and to revert back to the relative cap rule.[23]

Our analysis shows few important differences between the relative and final cap rules. First, the equilibrium payments under the relative cap rule are, in our example, almost twice as high as under the final cap rule. Second, it is true that by bidding on $(3{,}0)$ and $(0{,}3)$, the bidder runs a risk, under the relative cap rule, to obtain these packages that do not have an intrinsic value. This risk is absent under the final cap rule. However, the above iterative dominance argument shows that also under the relative cap rule, this risk is minimal knowing that rational rivals will try to avoid any risk of not winning any package and, therefore, will bid higher on $(1{,}1)$ than what others together can add to their total combined bid for $(2{,}2)$.

If bidders bid truthfully in the clock phase, the trade-off is then clear: under the final cap rule, the potential for gaming in the supplementary round is severely restricted, but gaming is without any risk. Under the relative cap rule, the potential for gaming is very large, but carries some risk (that depends on whether other players protect themselves against this gaming by rivals). There is, thus, no clear-cut case to be made for either one of these rules. In particular, it is not the case that under the relative cap rule, gaming will not arise, and if it arises, it can have a severe impact on auction prices.

Finally, and related, the relative cap rule has the disadvantage that the outcome of the auction may be inefficient. If two out of the three players follow the logic of iterative dominance and bid $B = (23,\ 30,\ 34,\ 32,\ 37)$ on the five possible packages, whereas the other bidder bids strictly less than 23 on $(1{,}1)$, the outcome is that there are two winners who win packages $(3{,}0)$ and $(0{,}3)$ which they do not have any intrinsic value for.

If bidders want to raise rivals' cost in a CCA under the final cap rule further, they can behave strategically in the clock phase. The reasons are the same as before (and potentially

[23] See, OFCOM (2012), Assessment of future mobile competition and award of800 MHz and 2.6 GHz, 24 July 2012.

stronger) as to get around the severe restrictions of the supplementary round bids, bidders demand larger packages in the clock phase than they would normally do in truthful bidding.

To see this, suppose that all bidders start the clock phase in the same way as before, but continue bidding on $(2,1)$ until the prices reach the amount $(x, 5)$, for some x and then drop demand to $(1,1)$ finishing off the clock phase. Knowing that the maximum bids on the packages $(2,1)$, $(1,2)$, $(3,0)$, and $(0,3)$ are then given by $b + x$, $b + 5$, $b + 2x - 5$, and $b + 10 - x$, competitors can together bid maximally $2b + x + 5$ on the total supply, if they together bid $2b$ on two blocks in each band. Thus, by choosing a large x, bidders can also raise rivals' cost under the final cap rule.