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A Simple Approximation to the Normal Distribution Function

with an application to the Black & Scholes Option Pricing Model

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A SIMPLE APPROXIMATION TO THE NORMAL DISTRIBUTION FUNCTION

WITH AN APPLICATION TO THE BLACK & SCHOLES OPTION PRICING MODEL

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CONTENTS:

ABSTRACT:

The standard normal (cumulative) distribution $\Phi(x)$ plays a central role in statistics. As there exists no closed form solution to this integral, practical applications call for adequate approximations. There exist well-known accurate approximations, but their application may be cumbersome. This raises the question whether more simple approximations to $\Phi(x)$ can be derived, which in a large number of cases provide a sufficient degree of accuracy.

In this note, we present simple approximations to the standard normal distribution function, based on rational functions. The expansions of these rational approximating functions match the first terms in the series expansions of $\Phi(x)$ exactly and provide a fair approximation to the higher order terms.

Aside from an insight into the approximation accuracy of the proposed expressions, we provide an application to the Black & Scholes option pricing model. We show how a rational approximation can be employed, either to estimate option prices or to estimate implied volatilities of at-the-money options.

A Simple Approximation to the Normal Distribution Function

With an Application to the Black & Scholes Option Pricing Model

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1. Introduction

The standard normal (cumulative) distribution plays a central role in statistics and is defined by:

$$(1) \quad \Phi(x) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{x} e^{-\frac{1}{2}u^2} \, du$$

As there exists no closed form solution to this integral, practical applications call for adequate approximations. The approximations provided by Hastings [1955], in particular, are widely used. For example, the highly accurate five point approximation for x > 0 has the form:

$$(2) \quad \Phi(x) \approx 1 - (2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2}x^2} \, z(z(z(z(a_5 z + a_4) + a_3) + a_2) + a_1)$$

where: $z = (1+0.2316419 \cdot x)^{-1}$
$a_1 = 0.319381530$
$a_2 = -0.356563782$
$a_3 = 1.781477937$
$a_4 = -1.821255978$
$a_5 = 1.330274429$

For $x < 0$, the identity $\Phi(-x) = 1 - \Phi(x)$ is used.

Although this approximation is extremely good, its application may be cumbersome. This raises the question whether more simple approximations to $\Phi(x)$ can be derived, which in a large number of cases provide a sufficient degree of accuracy.

In this note, we present simple approximations for the standard normal distribution function. Section 2 discusses a simple approximation for moderate values of the argument x, whereas section 3 focusses on the case where x is large. In section 4, one approximation will be applied to the Black & Scholes [1973] option pricing model. Section 5 summarizes the paper.

2. Approximations for moderate argument values

In some applications, the cumulative probability for moderate values of $|x|$ is needed. A Maclaurin series expansion of the exponential function yields:

$$(3) \qquad f(x) \equiv e^{-\frac{1}{2}x^2} = 1 + \sum_{k=1}^{\infty} (-\tfrac{1}{2})^k \frac{x^{2k}}{k!}$$

Finding the primitive function F(x) term by term gives:

$$(4) \qquad F(x) = x + \sum_{k=1}^{\infty} \frac{1}{2k+1} (-\tfrac{1}{2})^k \frac{x^{2k+1}}{k!}$$

$$= x - x^3/6 + x^5/40 - x^7/336 + \ldots$$

As this is a converging series, f(x) can be integrated term by term. Combining eq. (4) and (1) and rewriting, we get:

$$(5) \qquad \Phi(x) = \tfrac{1}{2} + (2\pi)^{-\frac{1}{2}} \int_0^x e^{-\frac{1}{2}u^2}\, du = \tfrac{1}{2} + (2\pi)^{-\frac{1}{2}} F(u) \Big|_0^x$$

$$= \tfrac{1}{2} + (2\pi)^{-\frac{1}{2}} x \{ 1 - x^2/6 + x^4/40 - x^6/336 + \ldots\ldots \}$$

For very small values of $|x|$, the series can be truncated after some term.

However, instead of ignoring all higher order terms, we can specify a rational function, whose expansion matches the series between braces to some degree.[1] A rational function $R(\cdot)$ takes the form:

$$(6) \qquad R(x^2) = R_{m,k}(x^2) = \frac{P_m(x^2)}{Q_k(x^2)}$$

where $P_m(x^2)$ and $Q_k(x^2)$ are polynomials in x^2 of degree m and k, respectively:

$$(7) \qquad \begin{aligned} P_m(x^2) &= a_0 + a_1 x^2 + \ldots + a_m x^{2m} \\ Q_k(x^2) &= b_0 + b_1 x^2 + \ldots + b_k x^{2k} \end{aligned}$$

We assume that $P_m(x^2)$ and $Q_k(x^2)$ have no common zero and are reduced to their lowest degree by the cancellation of common factors. Clearly, we require that $Q_k(x^2) \neq 0$. More in particular, we require that $b_0 \neq 0$ in order to prevent singularities for $x = 0$. Without loss of generality, we normalize by setting $b_0 = 1$.

As we are looking for a simple approximation, we choose m = 0 and k = 1, which gives:

$$(8) \quad R_{0,1}(x^2) = \frac{a_0}{1 + b_1 x^2} = a_0(1 - b_1 x^2 + b_1^2 x^4 - b_1^3 x^6 + \ldots.)$$

For $a_0 = 1$ and $b_1 = 1/6$, we have:

$$(9) \quad R_{0,1}(x^2) = 1 - x^2/6 + x^4/36 - x^6/216 + \ldots..$$

which matches the first two terms in the braced expression in eq. (5) exactly and provides a fair approximation to the higher order terms. Incorporating this rational function in eq. (5) finally gives:

$$(10) \quad \Phi(x) \approx \tfrac{1}{2} + (2\pi)^{-\frac{1}{2}} \frac{x}{1 + x^2/6}$$

In contrast to eq. (2), this approximation can directly be used for positive as well as negative values of x. Neither does the approximation involve an exponential function. Note, however, that eq. (10) has a maximum for $x = \sqrt{6} \approx 2.45$ (and a minimum for $x = -\sqrt{6}$), so the use of this formula should in any case be restricted to $|x| < \sqrt{6}$. For larger values of x, we refer to the next section.

To provide insight into the approximation accuracy of the proposed formula, Table 1 contains cumulative probabilities $\Phi(x)$, generated by eq. (2) and by eq. (10), as well as the absolute value of the difference. It follows that the approximation is very good for smaller values of x and acceptable for larger values, although the approximation should not be used for $|x| \geq 2.30$.[2)]

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Insert Table 1 here

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3. Approximations for large argument values

In some applications, information about the tails of the distribution ($|x| \geq 2$) is needed. Rewriting eq. (1) as:

$$(11) \quad \Phi(x) = 1 - (2\pi)^{-\frac{1}{2}} \int_x^{\infty} e^{-\frac{1}{2}u^2} \, du$$

and partially integrating, we get:

$$(12)\quad \Phi(x) = 1 - (2\pi)^{-½}\left[\frac{1}{x} e^{-½x^2} - \int_x^\infty \frac{1}{u^2} e^{-½u^2}\, du\right]$$

Continuing this process, we arrive at (cf. Feller [1968, p.193], e.g.):

$$(13)\quad \Phi(x) = 1 - (2\pi)^{-½} e^{-½x^2}\left[\frac{1}{x} - \frac{1}{x^3} + \frac{1\cdot3}{x^5} - \frac{1\cdot3\cdot5}{x^7} + \frac{1\cdot3\cdot5\cdot7}{x^9} - \ldots\right]$$

$$= 1 - (2\pi)^{-½} e^{-½x^2}\left[\sum_{k=0}^{\infty} (-1)^k \frac{(2k-1)!!}{x^{2k+1}}\right]$$

where $(2k-1)!! \equiv 1\cdot3\cdots(2k-1)$.

For large x, 1/x will be small. Truncating the bracketed series after two terms yields an error of:

$$(14)\quad \xi(x) = -(2\pi)^{-½}\int_x^\infty \frac{3}{u^4} e^{-½u^2}\, du$$

As:

$$(15)\quad |\xi(x)| \le (2\pi)^{-½}\frac{3}{x^5}\int_x^\infty u\cdot e^{-½u^2}\, du = (2\pi)^{-½}\frac{3}{x^5} e^{-½x^2}$$

it follows that the absolute error for $|x| \ge 2$ is bounded by $5\cdot10^{-3}$. For the tails of the distribution, however, $\Phi(-|x|)$ is close to zero, so the relative error will be still considerable. So, instead of ignoring all higher order terms, we can again look for some rational function, whose expansion matches the bracketed series to some degree. As:

$$(16)\quad \frac{-1/x^3}{1 + 3/x^2} = -\frac{1}{x^3} + \frac{3}{x^5} - \frac{9}{x^7} + \frac{27}{x^9} - \ldots..$$

we find the approximation:

$$(17)\quad \Phi(x) \approx 1 - (2\pi)^{-½} e^{-½x^2}\left[\frac{1}{x} - \frac{1/x^3}{1 + 3/x^2}\right]$$

$$= 1 - (2\pi)^{-1/2} e^{-1/2x^2} \frac{1}{x} \left[\frac{1 + 2/x^2}{1 + 3/x^2} \right]$$

Although we do not present a table with a comparison between the approximations, we found that eq. (17) adds considerable accuracy to the approximation on the basis of eq. (13), truncated after two terms.

4. An application to the Black & Scholes option pricing model

One field in finance in which we inevitably encounter the normal distribution function, is option pricing theory. Undoubtly the most popular option pricing model is derived by Black & Scholes [1973] (henceforth B&S). The B&S-model applies to European options (i.e. options that can only be exercised on the maturity date) and the pricing formula is:

$$\text{(18)} \quad C = S\Phi(d_1) - Ke^{-rt}\Phi(d_2)$$

with: $d_1 = \dfrac{\ln(S/Ke^{-rt})}{\sigma\sqrt{t}} + \tfrac{1}{2}\sigma\sqrt{t}$ and $d_2 = d_1 - \sigma\sqrt{t}$

where: C = the value of the European call option
S = the value of the underlying asset (a stock, e.g.)
K = the exercise price
r = the continuously compounded annual risk free rate of interest
σ = the annual standard deviation of the logarithmic stock return
t = the time to expiration (in years).

Dividing both sides of eq. (18) by Kexp(-rt) yields the call price in terms of the present value of the exercise price, as a function of only two variables: S/Ke^{-rt} and $\sigma\sqrt{t}$.

Using eq. (10) to approximate $\Phi(x)$ and collecting terms, we get:

$$\text{(19)} \quad \frac{C}{Ke^{-rt}} \approx \tfrac{1}{2} \left[\frac{S}{Ke^{-rt}} - 1 \right] + \frac{1}{\sqrt{(2\pi)}} \left[\frac{S}{Ke^{-rt}} \cdot \frac{d_1}{1 + d_1^2/6} - \frac{d_2}{1 + d_2^2/6} \right]$$

Table 2 presents exact and approximated option values for various choices of S/Ke^{-rt} and $\sigma\sqrt{t}$. For options that are no more than about 5% in or out of the money, the approximation is virtually exact (even for $\sigma\sqrt{t} << .15$). However, when the option is deep in- or out-of-the-money and when $\sigma\sqrt{t}$ is small, $|d_1|$ and $|d_2|$ become very large. As indicated in section 2, the approximation to $\Phi(x)$ then deteriorates quickly, resulting in large (theoretical) pricing errors. In that case, the approximation eq. (17) can be used.

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Insert Table 2 here
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For an at-the-money call option (defined as $S = Ke^{-rt}$), we have $d_2 = -d_1$. As $1/\sqrt{(2\pi)} \approx .399$, eq. (19) can be simplified to:

$$(20) \quad \frac{C}{S} \approx .399 \left[\frac{d_1}{1 + d_1{}^2/6} - \frac{d_2}{1 + d_2{}^2/6} \right]$$

$$= .399 \frac{\sigma\sqrt{t}}{1 + \sigma^2 t/24}$$

Brenner & Subrahmanyam [1988] ignore all terms of order x^2 and higher in eq. (5) to derive a simple way to estimate the implied volatility of at the money options. Their approximation is:

$$(21) \quad C/S \approx .399\sigma\sqrt{t}$$

which is quite accurate for small values of $\sigma\sqrt{t}$. However, as eq. (10) provides much more accuracy than their first order approximation, eq. (20) is more accurate to estimate either the option price for at-the-money options or to estimate their implied volatility.

5. Summary

This note presents two rational approximations to the standard normal (cumulative) distribution function $\Phi(x)$. One approximation applies to large values of the argument ($|x| \geq 2$; see eq.(17)), and the other applies to moderate values of the argument ($|x| \leq 2.3$; see eq.(10)). The latter approximation is applied in the context of Black & Scholes' [1973] option pricing model. The accuracy of this approximation is shown in Table 2. Eq. (20) shows an application of the approximation in estimating the implied volatility of at-the-money options.

FOOTNOTES:

1) This procedure is related to the Padé rational approximation (cf. for example Ralston [1965, Ch. 7.3] or Young & Gregory [1973, Ch. 6.12]), in which an analytical function Z(y) is approximated by a rational function $R_{m,k}(y)$ around y=0. This rational approximation can be seen as a kind of generalization of a Taylor series approximation. Given m, k and y, the (m+k+1) free coefficients of $P_m(y)$ and $Q_k(y)$ are chosen such that at the spanning point:

1. their values are equal: R(0) = Z(0); and
2. the first (m+k) derivatives of R(y) are equal to the corresponding derivatives of Z(y): $R^{(j)}(0) = Z^{(j)}(0)$ for j=1,...,m+k.

Phillips [1982] uses the technique to approximate probability density functions. For a more detailed exposition and other applications, we refer to Hallerbach [1994, pp. 203-208].

2) The derivative of eq. (10), $(2\pi)^{-\frac{1}{2}}(1-x^2/6)/(1+x^2/6)^2$, in turn, forms a close approximation to the standard normal density function $\phi(x) \equiv \Phi'(x) = (2\pi)^{-\frac{1}{2}}\exp(-\frac{1}{2}x^2)$.

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Table 1: The cumulative probabilities Φ(x), generated by the 5-point approximation eq. (2) and by eq. (10), and the absolute value of the difference (computed before rounding to 4 digits)

	Φ(x)				Φ(x)		
x	eq. (2)	eq. (10)	|diff.|	x	eq. (2)	eq. (10)	|diff.|
.00	.5000	.5000	.0000	1.30	.9032	.9046	.0015
.05	.5199	.5199	.0000	1.35	.9115	.9131	.0016
.10	.5398	.5398	.0000	1.40	.9192	.9210	.0018
.15	.5596	.5596	.0000	1.45	.9265	.9284	.0019
.20	.5793	.5793	.0000	1.50	.9332	.9352	.0020
.25	.5987	.5987	.0000	1.55	.9394	.9416	.0021
.30	.6179	.6179	.0000	1.60	.9452	.9474	.0022
.35	.6368	.6368	.0000	1.65	.9505	.9528	.0023
.40	.6554	.6554	.0000	1.70	.9554	.9577	.0023
.45	.6736	.6737	.0000	1.75	.9599	.9622	.0023
.50	.6915	.6915	.0000	1.80	.9641	.9663	.0022
.55	.7088	.7089	.0000	1.85	.9678	.9700	.0021
.60	.7257	.7258	.0001	1.90	.9713	.9733	.0020
.65	.7422	.7423	.0001	1.95	.9744	.9762	.0018
.70	.7580	.7582	.0001	2.00	.9773	.9787	.0014
.75	.7734	.7736	.0002	2.05	.9798	.9810	.0011
.80	.7881	.7884	.0002	2.10	.9821	.9829	.0007
.85	.8023	.8027	.0003	2.15	.9842	.9845	.0003
.90	.8159	.8163	.0004	2.20	.9861	.9858	.0003
.95	.8289	.8294	.0005	2.25	.9878	.9868	.0009
1.00	.8413	.8420	.0006	2.30	.9893	.9876	.0016
1.05	.8531	.8539	.0007				
1.10	.8643	.8652	.0009	*2.35*	*.9906*	*.9882*	*.0024*
1.15	.8749	.8759	.0010	*2.40*	*.9918*	*.9885*	*.0033*
1.20	.8849	.8861	.0011	*2.45*	*.9929*	*.9886*	*.0043*
1.25	.8944	.8956	.0013	*2.50*	*.9938*	*.9885*	*.0053*

Table 2: The Black & Scholes value of a call option, as a percentage of the present value of the exercise price, as well as the difference between the exact and approximated value (computed before rounding to 4 digits).

	$\sigma\sqrt{t}$ = .15			$\sigma\sqrt{t}$ = .20		
	$100 \cdot C/Ke^{-rt}$			$100 \cdot C/Ke^{-rt}$		
S/Ke^{-rt}	exact	approx	diff	exact	approx	diff
.80	.404	.476	.0720	1.186	1.254	.0677
.82	.588	.656	.0688	1.526	1.578	.0517
.84	.831	.888	.0566	1.932	1.969	.0365
.86	1.145	1.186	.0409	2.409	2.432	.0237
.88	1.539	1.565	.0260	2.960	2.974	.0139
.90	2.022	2.036	.0142	3.589	3.596	.0072
.91	2.299	2.309	.0098	3.934	3.939	.0049
.92	2.602	2.608	.0064	4.299	4.303	.0032
.93	2.930	2.934	.0039	4.685	4.687	.0019
.94	3.285	3.287	.0022	5.092	5.093	.0011
.95	3.666	3.667	.0011	5.520	5.520	.0006
.96	4.074	4.075	.0005	5.968	5.968	.0003
.97	4.510	4.510	.0002	6.437	6.437	.0001
.98	4.972	4.972	.0001	6.926	6.926	.0000
.99	5.462	5.462	.0000	7.436	7.436	.0000
1.00	5.979	5.979	.0000	7.966	7.966	.0000
1.01	6.522	6.522	.0000	8.515	8.515	.0000
1.02	7.091	7.091	.0001	9.084	9.085	.0000
1.03	7.685	7.686	.0002	9.673	9.673	.0001
1.04	8.305	8.305	.0004	10.280	10.280	.0003
1.05	8.949	8.950	.0010	10.906	10.906	.0005
1.06	9.616	9.618	.0019	11.549	11.550	.0010
1.07	10.305	10.309	.0033	12.210	12.212	.0016
1.08	11.017	11.022	.0052	12.888	12.890	.0026
1.09	11.749	11.756	.0079	13.582	13.586	.0039
1.10	12.500	12.511	.0112	14.292	14.298	.0056
1.12	14.059	14.079	.0202	15.758	15.768	.0104
1.14	15.685	15.717	.0319	17.281	17.298	.0172
1.16	17.371	17.416	.0456	18.856	18.882	.0261
1.18	19.108	19.169	.0600	20.480	20.517	.0370
1.20	20.891	20.965	.0736	22.147	22.197	.0496

(**Table 2** continued)

	$\sigma\sqrt{t}$ = .30			$\sigma\sqrt{t}$ = .40		
	$100 \cdot C/Ke^{-rt}$			$100 \cdot C/Ke^{-rt}$		
S/Ke^{-rt}	exact	approx	diff	exact	approx	diff
0.80	3.534	3.568	.0338	6.391	6.410	.0191
0.82	4.115	4.139	.0235	7.135	7.148	.0133
0.84	4.753	4.768	.0156	7.925	7.934	.0090
0.86	5.448	5.458	.0097	8.762	8.768	.0058
0.88	6.201	6.207	.0056	9.644	9.648	.0036
0.90	7.013	7.016	.0030	10.571	10.573	.0021
0.91	7.440	7.442	.0021	11.052	11.053	.0016
0.92	7.882	7.884	.0014	11.543	11.544	.0011
0.93	8.339	8.339	.0009	12.045	12.046	.0008
0.94	8.809	8.810	.0006	12.557	12.558	.0006
0.95	9.294	9.294	.0004	13.081	13.081	.0004
0.96	9.792	9.792	.0002	13.615	13.615	.0003
0.97	10.305	10.305	.0001	14.159	14.159	.0002
0.98	10.831	10.831	.0000	14.713	14.713	.0001
0.99	11.371	11.371	.0000	15.278	15.278	.0001
1.00	11.924	11.924	.0000	15.852	15.852	.0001
1.01	12.490	12.490	.0000	16.436	16.436	.0001
1.02	13.069	13.069	.0000	17.030	17.030	.0001
1.03	13.661	13.661	.0001	17.633	17.633	.0002
1.04	14.265	14.265	.0002	18.246	18.246	.0003
1.05	14.882	14.882	.0003	18.867	18.868	.0004
1.06	15.510	15.511	.0005	19.498	19.499	.0005
1.07	16.151	16.152	.0008	20.138	20.138	.0007
1.08	16.803	16.804	.0012	20.786	20.787	.0010
1.09	17.466	17.468	.0017	21.443	21.444	.0014
1.10	18.141	18.143	.0024	22.108	22.110	.0018
1.12	19.522	19.527	.0042	23.463	23.466	.0029
1.14	20.945	20.952	.0069	24.850	24.854	.0044
1.16	22.407	22.418	.0107	26.267	26.274	.0064
1.18	23.906	23.922	.0155	27.714	27.723	.0090
1.20	25.441	25.462	.0216	29.188	29.200	.0123

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