Risk-averse and Risk-seeking Investor Preferences
for Oil Spot and Futures*

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Abstract

This paper examines risk-averse and risk-seeking investor preferences for oil spot and futures prices by using the mean-variance (MV) criterion and stochastic dominance (SD) approach. The MV findings cannot distinguish between the preferences of spot and futures markets. However, the SD tests show that spot dominates futures in the downside risk, while futures dominate spot in the upside profit. On the other hand, the SD findings suggest that spot dominates futures in downside risk, while futures dominate spot in upside profit. Risk-averse investors prefer investing in the spot index. Risk seekers are attracted to the futures index to maximize their expected utility but not expected wealth in the entire period, as well as for both the OPEC and Iraq War sub-periods. The SD findings show that there is no arbitrage opportunity between the spot and futures markets, and these markets are not rejected as being efficient.

Keywords: Stochastic dominance, mean-variance, risk averter, risk seeker, futures market, spot market.

JEL: C14, G12, G15.
1. Introduction

There is a large literature examining the relationships between spot and futures prices of petroleum products. Among many, Bopp and Sitzer (1987) and Crowder and Hamid (1993) test the market efficiency hypothesis. The long-run and lead-lag relationships between oil spot and futures prices have been analyzed by Schwartz and Szakmary (1994), Gulen (1999), and Bekiros and Diks (2008), among others.

Besides the relationships between oil spot and futures prices, empirical studies have compared different crude oil markets. NYMEX, among other crude oil markets, has been examined in, for example, Lin and Tamvakis (2001) and Hammoudeh et al. (2003). The volatility of oil prices is another important area in the energy literature. Wilson et al. (1996) note that exogenous shocks will cause sudden changes in the variance of oil prices. Fong and See (2003) show that a regime switching model is superior for short-term volatility forecasting. Recently, Kang et al. (2009) and Arouri et al. (2012) use GARCH conditional volatility models to analyze and forecast the volatility of oil spot and futures prices.

Alternative approaches that could be used to address the issue include the mean-variance (MV) criterion and the CAPM statistics. However, these approaches rely on the normality assumption and the first two moments. However, the presence of non-normality in portfolio stock distributions is well documented (Beedles, 1979). The stochastic dominance (SD) approach differs from conventional parametric approaches in that comparing portfolios by using the SD approach is equivalent to the choice of assets by utility maximization. It endorses the minimum assumptions of investors’ utility functions and studies the entire distributions of returns directly.

The advantage of SD analysis over parametric tests becomes apparent when the assets return distributions are non-normal as the SD approach does not require any assumption about the
nature of the distribution, and hence can be used for any type of distribution. In addition, SD rules offer superior criteria on prospects investment decisions as SD incorporates information on the entire returns distribution, rather than the first two moments, as in the MV and CAPM, or higher moments in the extended MV. The SD approach has been regarded as one of the most useful tools to rank investment prospects as the ranking of financial assets has been shown to be equivalent to utility maximization for the preferences of risk averters and risk seekers (Tesfatsion, 1976; Li and Wong, 1999).

Consider an expected utility maximizing investor who holds a portfolio of two assets, namely oil spot and oil futures. The objective of investors is to rank the preference of these two assets to maximize expected utility. Lean et al. (2010) apply the SD test developed by Linton et al. (LMW, 2005) to show that investors are indifferent to investing spot or futures based on West Texas Intermediate crude oil data. The advantage of the LMW test is that the observations need not be independently and identically distributed, but its disadvantage is that the power is relatively low.

In order to extend the work of Lean et al. (2010), we apply both the MV criterion and the SD test proposed by Davidson and Duclos (2000) (hereafter DD test), and modified by Bai et al. (2011), to examine the behaviour of both risk averters and risk seekers with regard to oil spot and futures prices. In order to complement the results from the SD test, we also apply the MV and CAPM approaches to address the issue. The advantage of the DD test is that it investigates the characteristics of the entire distributions for oil futures and spot returns.

The contributions of the paper include the following: (i) the issue is revisited by using a new dataset, namely Brent crude oil data; (ii) we apply the SD test developed by Davidson and Duclos (2000) and modified by Bai et al. (2011), which is a more powerful procedure; and (iii) we examine the preferences for risk seekers, which is novel.
The empirical findings from applying the MV criterion and CAPM statistics could not indicate any preference between these two assets. As the data are found to be non-normal, the inferences drawn from the MV criterion and CAPM statistics may be misleading. Therefore, we recommend using the SD analysis to address the issue. The findings from the SD test imply that the hypotheses that futures stochastically dominate spot, or vice-versa, at first order are rejected, implying no arbitrage opportunity. We also find that oil spot dominates futures in downside returns, while oil futures dominate spot in upside profit. In addition, the SD results imply that risk-averse investors prefer the spot index, whereas risk seekers are attracted to the futures index to maximize expected utility, though not their expected wealth, for the entire period, as well as for both the OPEC and Iraq War sub-periods. The SD findings also suggest that the oil spot and futures markets are not rejected as being efficient.

2. Data and Methodology

We examine the performance of Brent Crude oil spot and futures for the period January 1, 1989 to June 30, 2008. For the purpose of comparison, we use the same sample period as in Lean et al. (2010). The daily closing prices for Brent Crude oil spot and futures are obtained from Datastream. Daily log returns, $R_{i,t}$, for the oil spot and futures prices are defined to be $R_{i,t} = \ln \left( \frac{P_{i,t}}{P_{i,t-1}} \right)$, where $P_{i,t}$ is the daily price at day $t$ for asset $i$, with $i = S$ (Spot) and $F$ (Futures), respectively. For computing the CAPM statistics, we use the 3-month U.S. T-bill rate and the Morgan Stanley Capital International index returns (MSCI) to proxy the risk-free rate and the global market index, respectively.

During the sample period, there are two major oil crises, namely the reduction in oil production by OPEC on October 29, 1999 and the Iraq War, which began on March 20, 2003. We divide the full sample period into two sub-periods on the crisis date. For the first crisis, we have
the pre-OPEC sub-period (pre-OPEC) and the sub-period thereafter (OPEC), using October 29, 1999 as the cut-off point. For the second crisis, we have the pre-Iraq War sub-period (pre-Iraq War) and the sub-period thereafter (Iraq War), using March 20, 2003 as the cut-off point.

2.1. Mean-Variance criterion and CAPM statistics

For any two investments of returns, $X$ and $Y$, with means, $\mu_X$ and $\mu_Y$, and standard deviations, $\sigma_X$ and $\sigma_Y$, respectively, $Y$ is said to dominate $X$ by the MV criterion for risk averters if $\mu_Y \geq \mu_X$ and $\sigma_Y \leq \sigma_X$, with at least one inequality holding (Markowitz, 1952a). Thus, the MV rule for risk averters is to check whether $\mu_Y \geq \mu_X$ and $\sigma_Y \leq \sigma_X$. If both are not rejected, with at least one strict inequality relationship, then we conclude that $Y$ dominates $X$ significantly by the MV rule.

On the other hand, Wong (2007) defines the MV rule for risk seekers such that, if $\mu_Y \geq \mu_X$ and $\sigma_Y \geq \sigma_X$ with at least one strict inequality relationship, then $Y$ dominates $X$ by the MV rule of risk seekers. Wong (2007) has shown that if both $X$ and $Y$ belong to the same location-scale family or the same linear combination of location-scale families, and if $Y$ dominates $X$ by the MV criterion for risk averters (seekers), then risk averters (seekers) will attain higher expected utility by holding $Y$ than $X$. Bai et al. (2012) have developed the mean-variance ratio statistic to test the performance among assets for small samples. CAPM statistics include the beta, Sharpe ratio, Treynor’s index and Jensen (alpha) index to measure performance.1

2.2. Stochastic Dominance Theory

1 Readers may refer to Sharpe (1964), Treynor (1965) and Jensen (1969) for details on the definitions of these indices and statistics. Readers may refer to Leung and Wong (2008) and the references therein for the test statistic of the Sharpe ratios, Morey
As developed by Hadar and Russell (1969), Hanoch and Levy (1969) and Rothschild and Stiglitz (1970), SD theory is one of the most useful tools in investment decision-making under uncertainty to rank investment prospects. Let $F$ and $G$ be the cumulative distribution functions (CDFs), and $f$ and $g$ be the corresponding probability density functions (PDFs) of two investments, $X$ and $Y$, respectively, with common support of $[a,b]$, where $a < b$. Define

$$H_0^A = H_0^D = h, \quad H_j^A(x) = \int_a^x H_{j-1}^A(t) \, dt \quad \text{and} \quad H_j^D(x) = \int_x^b H_{j-1}^D(t) \, dt$$  \hspace{1cm} (1)

for $h = f, g$; $H = F, G$; and $j = 1, 2, 3$.

We call the integral $H_j^A$ the $j$-order ascending cumulative distribution function (ACDF), and the integral $H_j^D$ the $j$-order descending cumulative distribution function (DCDF), for $j = 1, 2$ and 3 and for $H = F$ and $G$.

### 2.2.1. SD for Risk Averters

The most commonly used SD rules corresponding with three broadly defined utility functions are first-, second- and third-order Ascending SD (ASD)\(^3\) for risk averters, denoted FASD, SASD, and TASD, respectively. All investors are assumed to have non-satiation (more is preferred to less) under FASD, non-satiation and risk aversion under SASD, and non-satiation, risk aversion, and decreasing absolute risk aversion (DARA) under TASD. We define the ASD rules as follows (see Quirk and Saposnik, 1962; Fishburn, 1964; Hanoch and Levy, 1969):

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\(^2\) See Wong and Li (1999), Li and Wong (1999), and Sriboonchitta et al. (2009) for further discussion.

\(^3\) We call it Ascending SD as its integrals count from the worst return ascending to the best return.

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\( X \) dominates \( Y \) by FASD (SASD, TASD), denoted by \( X \succ_1 Y \) (\( X \succ_2 Y \), \( X \succ_3 Y \)) if and only if \( F_1^A(x) \leq G_1^A(x) \) (\( F_2^A(x) \leq G_2^A(x) \), \( F_3^A(x) \leq G_3^A(x) \)) for all possible returns \( x \), and the strict inequality holds for at least one value of \( x \).

The theory of SD is important as it is related to utility maximization (see Quirk and Saposnik 1962; Hanoch and Levy, 1969). The theory can be extended to non-differentiable utility (see Wong and Ma (2008) for further details). The existence of ASD implies that risk-averse investors always obtain higher expected utility when holding the dominant asset than when holding the dominated asset, so that the dominated asset would never be chosen. We note that hierarchical relationship exists in ASD: FASD implies SASD which, in turn, implies TASD. However, the converse is not true: the existence of SASD does not imply the existence of FASD. Likewise, a finding of the existence of TASD does not imply that existence of SASD or FASD. Thus, only the lowest dominance order of ASD is reported.

### 2.2.2. SD for Risk Seekers

The SD theory for risk seekers has also been well established in the literature. Whereas SD for risk averters works with the ACDF, which counts from the worst return ascending to the best return, SD for risk seekers works with the DCDF, which counts from the best return descending to the worst return (Wong and Li, 1999; Levy and Levy 2004; Post and Levy, 2005). Hence, SD for risk seekers is called Descending SD (DSD). We have the following definition for DSD (see Hammond, 1974; Wong and Li, 1999).

\( X \) dominates \( Y \) by FDSD (SDSD, TDSD) denoted by \( X \succ^D_1 Y \) (\( X \succ^D_2 Y \), \( X \succ^D_3 Y \)) if and only if \( F_1^D(x) \geq G_1^D(x) \) (\( F_2^D(x) \geq G_2^D(x) \), \( F_3^D(x) \geq G_3^D(x) \)) for all possible returns \( x \), the strict inequality holds for at least one value of \( x \); where FDSD (SDSD, TDSD) denotes first-order
(second-order, third-order) Descending SD.

All investors are assumed to have non-satiation under FDSD, non-satiation and risk seeking under SDSD, and non-satiation, risk seeking and increasing absolute risk seeking under TDSD. Similarly, the theory of DSD is related to utility maximization for risk seekers (Li and Wong 1999). The hierarchical relationship also exists in DSD, so that only the lowest dominance order of DSD is reported.

Typically, risk averters prefer assets that have a smaller probability of losing, especially in downside risk, while risk seekers prefer assets that have a higher probability of gaining especially in upside profit. In order to make a choice between two assets $X$ and $Y$, risk averters will compare their corresponding $j$-order ASD integrals and choose $X$ if $F_j^A$ is smaller. On the other hand, risk seekers will compare their corresponding $j$-order DSD integrals and choose $X$ if $F_j^D$ is bigger (Wong and Chan, 2008).

2.3. Stochastic Dominance Tests

The advantages presented by SD have motivated prior studies, which have used SD techniques to analyze many financial puzzles. There are two broad classes of SD tests. One is the minimum/maximum statistic, while the other is based on distribution values computed on a set of grid points. McFadden (1989) develops an SD test using the minimum/maximum statistic, followed by Klecan et al. (1991) and Kaur et al. (1994). Barrett and Donald (2003) develop a Kolmogorov-Smirnov-type test, and Linton et al. (2005) extend their work by relaxing the iid assumption. On the other hand, the SD tests developed by Anderson (1996) and Davidson and Duclos (2000) compare the underlying distributions at a finite number of grid points. The SD test developed by DD has been found to be one of the most powerful approaches, and is also less
conservative in terms of size (see Tse and Zhang, 2004; Lean et al., 2008).

2.3.1. Stochastic Dominance Test for Risk Averters

Let \{(f_i, s_i)\} \(i = 1, ..., n\) be pairs of observations drawn from the random variables \(X\) and \(Y\), with distribution functions \(F\) and \(G\), respectively, and with their integrals \(F_j^A(x)\) and \(G_j^A(x)\) defined in (1) for \(j = 1, 2, 3\). For a grid of pre-selected points \(x_1, x_2, ..., x_k\), the \(j\)-order Ascending DD test statistic for risk averters, \(T_j^A\) is:

\[
T_j^A(x) = \frac{\hat{F}_j^A(x) - \hat{G}_j^A(x)}{\sqrt{\hat{V}_j^A(x)}}
\]

where \(\hat{V}_j^A(x) = \hat{V}_{F_j^A}(x) + \hat{V}_{G_j^A}(x) - 2\hat{V}_{FG_j^A}(x)\);

\[
\hat{H}_j^A(x) = \frac{1}{N(j-1)!} \sum_{i=1}^{N} (x - z_i)^{(j-1)}_+,
\]

\[
\hat{V}_{H_j^A}(x) = \frac{1}{N} \left[ \frac{1}{N((j-1)!^2} \sum_{i=1}^{N} (x - z_i)^{(2(j-1))} - \hat{H}_j^A(x)^2 \right], H = F, G; z = f, s;
\]

\[
\hat{V}_{FG_j^A}(x) = \frac{1}{N} \left[ \frac{1}{N((j-1)!^2} \sum_{i=1}^{N} (x - f_i)^{(j-1)}_+ (x - s_i)^{(j-1)}_+ - \hat{F}_j^A(x)\hat{G}_j^A(x) \right].
\]

It is not possible to test empirically the null hypothesis for the full support of the distributions. Thus, Bishop et al. (1992) propose to test the null hypothesis for a pre-designed finite numbers of values \(x\). Specifically, for all \(i = 1, 2, ..., k\); the following hypotheses are tested:

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4 In this paper, \(f\) denotes the returns of futures prices, while \(s\) denotes the returns of spot prices.
where $M_\alpha^j$ is the bootstrapped critical value of $j$-order DD statistics. The test statistics are compared with $M_\alpha^j$ at each point of the combined sample.\(^5\) However, it is empirically difficult to do so when the sample size is very large. In order to ease the computation, we specify $K$ equal-interval grid points $\{x_k, k = 1, 2, \ldots, K\}$ which cover the common support of random samples $\{X_i\}$ and $\{Y_i\}$. Simulations show that the performance of the modified DD statistics is not sensitive to the number of grid points. Thus, in practice, we follow Fong et al. (2005) and Gasbarro et al. (2007), among others, and choose $K = 100$.

2.3.2. Stochastic Dominance Test for Risk Seekers

\(^5\) Refer to Bai et al. (2011) for the construction of the bootstrapped critical value $M_\alpha^j$. 

\(H_0 : F_j^A(x_i) = G_j^A(x_i), \text{ for all } x_i; \)
\(H_A : F_j^A(x_i) \neq G_j^A(x_i) \text{ for some } x_i; \)
\(H_{A1} : F_j^A(x_i) \leq G_j^A(x_i) \text{ for all } x_i, F_j^A(x_i) < G_j^A(x_i) \text{ for some } x_i; \)
\(H_{A2} : F_j^A(x_i) \geq G_j^A(x_i) \text{ for all } x_i, F_j^A(x_i) > G_j^A(x_i) \text{ for some } x_i. \)

We note that in the above hypotheses, $H_A$ is set to be exclusive of both $H_{A1}$ and $H_{A2}$. This means that if the test does not reject $H_{A1}$ or $H_{A2}$, it will not be classified as $H_A$. Therefore, Bai et al. (2011) modify the decision rules to be:

\[
\max_{1 \leq k \leq K} |T_j^A(x_k)| < M_\alpha^j, \text{ accept } H_0 : X = Y \\
\max_{1 \leq k \leq K} T_j^A(x_k) > M_\alpha^j \text{ and } \min_{1 \leq k \leq K} T_j^A(x_k) < -M_\alpha^j, \text{ accept } H_A : X \neq Y \\
\max_{1 \leq k \leq K} T_j^A(x_k) < M_\alpha^j \text{ and } \min_{1 \leq k \leq K} T_j^A(x_k) < -M_\alpha^j, \text{ accept } H_{A1} : X \succeq Y \\
\max_{1 \leq k \leq K} T_j^A(x_k) > M_\alpha^j \text{ and } \min_{1 \leq k \leq K} T_j^A(x_k) > -M_\alpha^j, \text{ accept } H_{A2} : Y \succeq X
\]
In order to test SD for risk seekers, the DD statistics for risk averters are modified to be the Descending DD test statistic, \( T^D_j \), such that:

\[
T^D_j(x) = \frac{\hat{F}^D_j(x) - \hat{G}^D_j(x)}{\sqrt{\hat{J}^D_j(x)}},
\]

(5) where \( \hat{F}^D_j(x) = \hat{V}^D_j(x) + 2\hat{V}^D_{FG,j}(x); \)

\[
\hat{H}^D_j(x) = \frac{1}{N(j-1)!} \sum_{i=1}^{N} (z_i \cdot x)^{j-1},
\]

\[
\hat{V}^D_{F,j}(x) = \frac{1}{N} \left[ \frac{1}{N((j-1)!)^2} \sum_{i=1}^{N} (z_i \cdot x)^{2(j-1)} - \hat{H}^D_j(x)^2 \right], H = F, G; z = f, s;
\]

\[
\hat{V}^D_{FG,j}(x) = \frac{1}{N} \left[ \frac{1}{N((j-1)!)^2} \sum_{i=1}^{N} (f_i - x)^{j-1} (s_i - x)^{j-1} - \hat{F}^D_j(x)\hat{G}^D_j(x) \right];
\]

where the integrals \( F^D_j(x) \) and \( G^D_j(x) \) are defined in (1) for \( j = 1, 2, 3 \). For \( i = 1, 2, \ldots, k \), the following hypotheses are tested for risk seekers:

\[
H_0: F^D_j(x_i) = G^D_j(x_i), \text{ for all } x_i;
\]

\[
H_D: F^D_j(x_i) \neq G^D_j(x_i) \text{ for some } x_i;
\]

\[
H_{D1}: F^D_j(x_i) \geq G^D_j(x_i) \text{ for all } x_i, F^D_j(x_i) > G^D_j(x_i) \text{ for some } x_i;
\]

\[
H_{D2}: F^D_j(x_i) \leq G^D_j(x_i) \text{ for all } x_i, F^D_j(x_i) < G^D_j(x_i) \text{ for some } x_i.
\]

and the critical rule for risk seekers can be obtained as in Bai et al. (2011).

Similarly to the situation in testing the Ascending SD test, we follow Fong et al. (2005, 2008) and Lean et al. (2007) to make 10 major partitions with 10 minor partitions within any two consecutive major partitions in each comparison, and use the simulated critical values in Bai et
al. (2011). This allows the consistency of both the magnitude and sign of the DD statistics between any two consecutive major partitions to be examined.

Not rejecting either $H_0$ or $H_A$ or $H_D$ implies the non-existence of any SD relationship between $X$ and $Y$, non-existence of any arbitrage opportunity between these two markets, and that neither of these markets is preferred to the other. If $H_{AI}$ ($H_{A2}$) of order one is accepted, $X$ ($Y$) stochastically dominates $Y$ ($X$) at first order, while if $H_{DI}$ ($H_{D2}$) of order one is accepted, asset $X$ ($Y$) stochastically dominates $Y$ ($X$) at first order. In this situation, and under certain regularity conditions, an arbitrage opportunity exists and any non-satiated investors will be better off if they switch from the dominated to the dominant asset. On the other hand, if $H_{AI}$ ($H_{A2}$) [$H_{DI}$ ($H_{D2}$)] is accepted at order two (three), a particular market stochastically dominates the other at second- (third-) order. In this situation, arbitrage opportunity does not exist, and switching from one asset to another will only increase the risk averters’ [seekers’] expected utility, though not their expected wealth (Jarrow, 1986; Falk and Levy, 1989; Wong et al. 2008). These results could be used to infer that market efficiency and market rationality could still hold in these markets (see Chan et al. (2012) and the references contained therein for further information).

In the above analysis, in order to minimize the Type II error and to accommodate the effect of almost SD (Leshno and Levy, 2002; Guo et al., 2013), we follow Gasbarro et al. (2007) and use a conservative 5% cut-off point in checking the proportion of test statistics for statistical inference. Using a 5% cut-off point, we conclude that one prospect dominates another prospect only if we find that at least 5% of the statistics are significant.

3. Empirical Results and Discussion

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6 Refer to Bai et al. (2011) for further details.
7 Refer to Jarrow (1986) for the conditions.
3.1. Mean Variance Analysis

Table 1 provides the descriptive statistics for the daily returns of oil spot prices and oil futures prices for the entire sample period. The means of their daily returns are about 0.04%, significant at 10% for the oil spot but not significant for oil futures. From the unreported paired t-tests, the mean return of oil spot is not significantly higher than that of futures whereas, as expected, its standard deviation is not significantly smaller than that of futures. As both the means and standard deviations are not significantly different for the two returns, the MV criterion is unable to indicate any preference between these two assets.

[Table 1 here]

For the CAPM measures, the absolute value of beta of oil spot returns is smaller than that of futures, both being negative and less than one. Both returns have similar Sharpe ratios, Treynor and Jensen indices, with no significant differences between the returns for each statistic. Thus, the information drawn from the CAPM statistics do not lead to any preference between spot and futures prices. In addition, the highly significant Kolmogorov-Smirnov (K-S) and Jarque-Bera (J-B) statistics in Table 1 indicate that both returns are non-normal. Moreover, both daily returns are negatively skewed. As expected, oil futures have much higher kurtosis than spot prices, with both being higher than under normality. Both skewness and kurtosis indicate non-normality in the returns distributions, and lead to the conclusion that the normality condition for the traditional MV and CAPM measures is violated.

3.2. SD Analysis for Risk Averters

8 The results of other normality tests, such as Shapiro-Wilk, lead to the same conclusion. The results are available on request.
We consider the CDFs of the returns for both oil spot and futures prices and their corresponding first three orders of the Ascending DD statistics, $T_j^A$, for risk averters in Figure 1. If oil futures dominate spot in the sense of FASD, then the CDF of futures returns should lie significantly below that of spot prices over the entire range. However, Figure 1 shows that the CDF of spot lies below that of futures in downside risk, while the CDF of futures lies below that of spot on upside profit. This indicates that there is no FASD between the two returns, and that spot dominates futures on downside risk while futures dominate spot on upside profit.

In order to verify this finding formally, we use the first three orders of the Ascending DD statistics, $T_j^A$ ($j = 1,2,3$), for the two series, with the results reported in Table 2. DD shows that the null hypothesis can be rejected if any of the test statistics $T_j^A$ is significant, and of the wrong sign.

[Figure 1 here]

The values of $T_1^A$ in Figure 1 move from positive to negative along the distribution of returns. The percentage of significant values reported in Table 2 show that 8% of $T_1^A$ is significantly positive, whereas 9% of $T_1^A$ is significantly negative. Thus, the hypotheses that futures stochastically dominate spot, or vice-versa, at first order are rejected, implying no arbitrage opportunity between these two series. We can, however, state that oil spot prices dominate futures in downside returns, while oil futures dominate spot in upside profit.

[Table 2 here]

The SD criterion enables a comparison of utility interpretations in terms of investors’ risk
aversion and decreasing absolute risk aversion, respectively, by examining the higher-order SD relationships. The Ascending DD statistics, $T_2^A$ and $T_3^A$, in Figure 1 are positive over the entire range of the return distribution, with 10% of $T_2^A$ (8% of $T_3^A$) being significantly positive and no $T_2^A$ ($T_3^A$) being significantly negative. Thus, the oil spot marginally SASD (TASD) dominates futures, and risk-averse investors would prefer investing in oil spot than in futures to maximize expected utility. This result is different from Lean et al. (2010), who find investors are indifferent from investing in oil spot or futures.

3.3. Will Risk Seekers Have Different Preferences?

If we apply the existing ASD tests, we would draw the conclusion regarding the preferences of risk-averse investors, but not of risk seekers. Nonetheless, the results also show that futures dominate spot for upside profit. However, using the ASD test alone could not yield any inference based on this information. Thus, an extension of the SD test for risk seekers is necessary, as discussed above. Subsequent discussions illustrate the applicability of DSD test for risk seekers in this section.

It is well known that investors could be risk seeking (see, for example, Markowitz, 1952a; Kahneman and Tversky, 1979; Tversky and Kahneman, 1992; Levy and Levy, 2004; Post and Levy, 2005). In order to examine the possible risk-seeking behaviour, DSD theory for risk seeking has been developed. In this paper, we put the theory into practice by extending the DD test for risk seekers, namely Descending DD statistics, $T_j^D$ ($j = 1, 2$ and $3$), of the first three orders for risk seekers with the correspondence statistics, as discussed above.

[Figure 2 here]
Figure 2 shows the descending cumulative density functions (DCDFs) for the daily returns of both oil spot and futures prices over the entire distribution range for the whole sample period. The crossing of the two DCDF curves suggests that there is no FDSD between futures and spot returns. The DCDF of futures lies above that of spot for upside profit, while the DCDF of spot lies above that of futures for downside risk. Therefore, futures may be preferred to spot for upside profit, while spot may be preferred to futures for downside risk.

In order to test this phenomenon formally, we plot the Descending DD statistics, $T_j^D$, of the first three orders in Figure 2, and report the percentages of their significant positive and negative portions in Table 3. Figure 2 shows that $T_1^D$ is positive in the upside profit range and negative in the downside risk range. Table 3 shows that 9% (8%) of the positive (negative) values of $T_1^D$ is significant, indicating that there is no FDSD relationship between the two series for the entire period.

As there is no FDSD, we examine $T_j^D$ for the second and third orders. Both $T_2^D$ and $T_3^D$ in Figure 2 are positive for the entire range, implying that risk-seeking investors could prefer futures to spot. In order to verify this statement statistically, we use the results in Table 3 that 10% (12%) of $T_2^D$ ($T_3^D$) are significantly positive, while no $T_2^D$ ($T_3^D$) is significantly negative. Therefore, oil futures SDSD and TDSD oil spot, so that risk-seeking investors prefer oil futures to spot to maximize their expected utility.

In addition, neither FASD nor FDSD leads to the conclusion that market efficiency or market rationality could hold in the oil spot and futures markets. The preferences of risk-averse and risk-seeking investors towards spot and futures do not violate market efficiency unless the
oil market has only one type of investor. These results are consistent with many previous studies in the literature (see, for example, Fong et al. (2005), who examine the momentum profits in stocks markets).

From the unreported paired t-tests, the mean return of oil spot is not significantly higher than that of futures whereas, as expected, its standard deviation is not significantly smaller than that of futures. As both the means and standard deviations are not significantly different for the two returns, the MV criterion is unable to indicate any preference between these two assets for risk seekers.

3.4. The Impact of Oil Crises

Tables 4A and 4B provide the descriptive statistics of the daily returns of oil spot and futures prices for the OPEC and Iraq War sub-periods. As most of the results of the MV criterion and CAPM statistics for all sub-periods are similar to those for the full sample period, we discuss only the results that are different from the full sample period. However, compared with the pre-OPEC sub-period, the means for both spot and futures returns in the OPEC sub-period dramatically increased five-fold. On the other hand, compared with the pre-Iraq-War sub-period, both spot and futures returns in the Iraq-War sub-period were reduced by 90%. Nonetheless, the differences between the means of spot and futures in each sub-period is still not significant. In addition, the standard deviations for the returns of spot and futures are also not significantly different in each of the sub-periods. Thus, similar to the inference to the entire sample, both the MV criterion and the CAPM statistics are unable to indicate any dominance between the spot and futures markets.

[Table 4 here]
From the DD test, we find that all the values of $T_j^A$ and $T_j^D$ ($j = 1, 2$ and $3$) for both risk averters and risk seekers are not significant at the 5% level for the first three orders in the pre-OPEC sub-period. Therefore, there is no arbitrage opportunity in these markets, and both risk averters and risk seekers are indifferent between these two indices in the pre-OPEC sub-period. However, in the OPEC sub-period, Table 2 shows that 20% (19%) of $T_2^A$ (3$A$) are significantly positive, and none of the $T_2^A$ (3$A$) is significantly negative. Table 3 reveals that 25% (33%) of $T_2^D$ (3$D$) are significantly positive, and none of the $T_2^D$ (3$D$) is significantly negative at the 5% level. Similar inferences can be drawn for the Iraq War sub-period. Hence, we conclude that, compared with the full sample period, risk-averse investors prefer the spot index and risk seekers are attracted to the futures index to maximize their expected utility, though not their expected wealth, in both the OPEC and Iraq War sub-periods.

3.5. Arbitrage Opportunity, Market Rationality, and Market Efficiency

Without identifying any risk index or any specific model, the SD rules can be used to determine whether there is any opportunity for arbitrage, and whether markets are efficient and investors are rational. It is well known (Jarrow, 1986; Falk and Levy, 1989) that, under certain conditions, if either FASD or FDSD exists, arbitrage opportunities exist, and investors will increase their wealth and expected utility if they shift from holding the dominated to the dominant asset. However, Wong et al. (2008) have shown that, if FASD or FDSD exists statistically, arbitrage opportunities may not exist, but investors can increase their expected wealth as well as their expected utility if they shift from holding the dominated to the dominant asset. It is well known from the market efficiency hypothesis that if one is able to earn abnormal returns, the market is considered to be inefficient. Market efficiency can be tested using SD rules as follows: if investors can switch their asset choice and increase their expected wealth, independently of their specific preferences, then market inefficiency is implied.
In our analysis, we find that oil spot FSD dominates futures in downside returns, while futures FSD dominate spot prices in upside returns, spot does not FSD dominate futures over the entire distribution, and vice-versa, for the entire periods, as well as in all sub-periods. As our analysis concludes that investors will not increase their expected wealth by switching their investment from oil futures to spot, or vice-versa, there is no arbitrage opportunity in the oil spot and futures markets. This implies that there is no arbitrage opportunity in the oil spot and futures markets for the entire period, as well as in all sub-periods.

Falk and Levy (1989) have shown that, given two assets, X and Y, if by switching from X to Y (or by selling X short and holding Y long), an investor can increase expected utility, the market is inefficient. SASD from one prospect, say spot, over another prospect, say futures, does not imply any arbitrage opportunity, but it does imply the preference of spot over futures by risk-averse investors. Nonetheless, risk averters would not make an expected profit by switching from futures to spot, but switching would allow investors to increase their expected utility. A similar argument can be made for the TASD criterion. This is exactly what is found in the oil spot and futures markets. Should we claim that the oil spot and futures markets are inefficient and investors are irrational?

Such a claim could be made if markets have only risk-averse investors. However, it is well known that markets could have other types of investors (see, for example, Friedman and Savage, 1948; Markowitz, 1952b; Broll et al., 2010; Egozcue et al., 2011 for further discussion). Under the assumption that markets could contain more than one type of investor, such as risk averters and risk seekers, it is possible that one asset dominates another asset by ASD but is dominated by DSD. These are precisely the findings obtained in this paper. Oil spot stochastically dominates oil futures strictly in the sense of SASD and TASD, while futures stochastically dominate spot strictly in the sense of SDSD and TDSD.
Therefore, risk averters could prefer to invest in the spot market rather than in futures, while risk seekers could prefer to invest in futures rather than in the spot market. In equilibrium, the number of trades that risk averters, who go long in spot and/or short sell futures, would match the number of trades that risk seekers, who go long in futures and/or short sell spot. In this situation, there is no upward or downward pressure on prices in the spot or futures market, while both risk averters and risk seekers could obtain what they want. Under these conditions, we agree with the assessment in Qiao et al. (2013) that the market remains efficient and that investors are rational.

4. Conclusion

This paper offered a robust decision tool for investment decisions with uncertainty in oil markets. The SD tests revealed the existence of arbitrage opportunities, identified the preferences for both risk averters and risk seekers over different investment prospects, and enabled inferences regarding market rationality and market efficiency. We applied the DD tests to examine the behaviour of both risk averters and risk seekers with regard to the Brent crude oil spot and futures markets, and compared the performances in the two markets.

The empirical results showed conclusively that oil spot dominates oil futures on downside risk, whereas futures dominate spot on upside profit. We concluded that there is no arbitrage opportunity and the markets are efficient. In addition, it was shown that oil spot dominates futures in downside returns, while oil futures dominate spot in upside profit. In addition, we provided evidence that risk-averse investors prefer oil spot, while risk seekers are attracted to oil futures in order to maximize their expected utility, though not their expected wealth, for the entire period, as well as for both the OPEC and Iraq War sub-periods.
### Table 1: Descriptive Statistics of Oil Spot and Futures Returns, 1989 – 2008

<table>
<thead>
<tr>
<th>Variable</th>
<th>Oil Spot Returns</th>
<th>Oil Futures Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>0.04354*</td>
<td>0.04323</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.01864</td>
<td>0.02193</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.9201***</td>
<td>-1.6782***</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>12.9542***</td>
<td>32.0111***</td>
</tr>
<tr>
<td>Jarque-Bera (J-B)</td>
<td>21711.86***</td>
<td>180710.47***</td>
</tr>
<tr>
<td>Kolmogorov-Smirnov (K-S)</td>
<td>0.06536***</td>
<td>0.07046***</td>
</tr>
<tr>
<td>Beta</td>
<td>-0.0153</td>
<td>-0.1617</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>3.68</td>
<td>3.04</td>
</tr>
<tr>
<td>Treynor Index</td>
<td>-0.96252</td>
<td>-0.08788</td>
</tr>
<tr>
<td>Jensen Index</td>
<td>0.014768</td>
<td>0.014404</td>
</tr>
<tr>
<td>F Statistics</td>
<td>0.7221</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>5085</td>
<td>5085</td>
</tr>
</tbody>
</table>

Note: *** significant at 1% level, ** significant at 5% level, * significant at 10%. F Statistics test the equality of variances. Refer to footnote 4 for the formula of Sharpe Ratio, Treynor Index, and Jensen Index, and further information about these statistics. The values of the Sharpe Ratio, Treynor Index and Jensen Index are annualized.
### Table 2: Results of DD Test for Risk Averters

<table>
<thead>
<tr>
<th>Sample</th>
<th>FASD</th>
<th>SASD</th>
<th>TASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$%T_1^A &gt; 0$</td>
<td>$%T_1^A &lt; 0$</td>
<td>$%T_2^A &gt; 0$</td>
</tr>
<tr>
<td>Whole Period</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Pre-OPEC</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>OPEC</td>
<td>17</td>
<td>17</td>
<td>20</td>
</tr>
<tr>
<td>Pre-Iraq</td>
<td>7</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Iraq War</td>
<td>6</td>
<td>19</td>
<td>10</td>
</tr>
</tbody>
</table>

Note: DD test statistics are computed over a grid of 100 daily oil spot and futures returns. The table reports the percentage of DD statistics which are significantly negative or positive at the 5% level, based on the simulated critical values recommended by Bai et al. (2011). Refer to equation in (2) for the definition of $T_j^A$ for $j = 1, 2$ and 3, where $F_j^A$ and $G_j^A$ represent the $j^{th}$ ACDFs for the returns of futures and spot, respectively.

### Table 3: Results of DD Test for Risk Seekers

<table>
<thead>
<tr>
<th>Sample</th>
<th>FDSD</th>
<th>SDSD</th>
<th>TDSD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$%T_1^D &gt; 0$</td>
<td>$%T_1^D &lt; 0$</td>
<td>$%T_2^D &gt; 0$</td>
</tr>
<tr>
<td>Whole Period</td>
<td>9</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Pre-OPEC</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>OPEC</td>
<td>17</td>
<td>17</td>
<td>25</td>
</tr>
<tr>
<td>Pre-Iraq</td>
<td>7</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Iraq War</td>
<td>19</td>
<td>6</td>
<td>24</td>
</tr>
</tbody>
</table>

Note: DD test statistics are computed over a grid of 100 daily oil spot and futures returns. The table reports the percentage of DD statistics which are significantly negative or positive at the 5% level, based on the simulated critical values recommended by Bai et al. (2011). Refer to equation in (3) for the definition of $T_j^D$ for $j = 1, 2$ and 3, where $F_j^D$ and $G_j^D$ represent the $j^{th}$ DCDFs for the returns of futures and spot, respectively.
Table 4A: Descriptive Statistics of Oil Spot Prices and Oil Futures Prices for Sub-Periods

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pre-OPEC</th>
<th>OPEC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Oil Spot Prices</td>
<td>Oil Futures Prices</td>
</tr>
<tr>
<td>Mean (%)</td>
<td>0.01287</td>
<td>0.01185</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.01969</td>
<td>0.02240</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.08807***</td>
<td>-2.6108***</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>17.4315***</td>
<td>51.5032**</td>
</tr>
<tr>
<td>J-B</td>
<td>25063.69*</td>
<td>280027.11***</td>
</tr>
<tr>
<td>K-S</td>
<td>0.08918*</td>
<td>0.1069**</td>
</tr>
<tr>
<td>Beta</td>
<td>0.01124</td>
<td>-0.3738</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>-0.8875</td>
<td>-1.0375</td>
</tr>
<tr>
<td>Treynor Index</td>
<td>-0.33592</td>
<td>0.01326</td>
</tr>
<tr>
<td>Jensen Index</td>
<td>-4108</td>
<td>-0.00203</td>
</tr>
<tr>
<td>F Statistics</td>
<td>0.7726</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>2824</td>
<td>2824</td>
</tr>
</tbody>
</table>

Note: *** significant at 1% level, ** significant at 5% level, * significant at 10%. F Statistics test the equality of variances. Refer to footnote 4 for the formula of the Sharpe Ratio, Treynor Index, and Jensen Index, and further information about these statistics. The values of the Sharpe Ratio, Treynor Index and Jensen Index are annualized.

Table 4B: Descriptive Statistics of Oil Spot Prices and Oil Futures Prices for Sub-Periods

<table>
<thead>
<tr>
<th>Variable</th>
<th>Pre-Iraq War</th>
<th>Iraq War</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Oil Spot Prices</td>
<td>Oil Futures Prices</td>
</tr>
<tr>
<td>Mean (%)</td>
<td>0.01566</td>
<td>0.01339</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.01956</td>
<td>0.02284</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.01998***</td>
<td>-2.03499***</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>14.2659***</td>
<td>37.07080***</td>
</tr>
<tr>
<td>J-B</td>
<td>20252.50***</td>
<td>181905.92***</td>
</tr>
<tr>
<td>K-S</td>
<td>0.07501***</td>
<td>0.09179***</td>
</tr>
<tr>
<td>Beta</td>
<td>0.02377</td>
<td>0.1861</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.3443</td>
<td>-0.65</td>
</tr>
<tr>
<td>Treynor Index</td>
<td>0.001172</td>
<td>0.0003267</td>
</tr>
<tr>
<td>Jensen Index</td>
<td>-2.66*10^-5</td>
<td>-7.09*10^-5</td>
</tr>
<tr>
<td>F Statistics</td>
<td>0.7338</td>
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<tr>
<td>N</td>
<td>3708</td>
<td>3708</td>
</tr>
</tbody>
</table>

Note: *** significant at 1% level, ** significant at 5% level, * significant at 10%. F Statistics test the equality of variances. Refer to footnote 4 for the formula of the Sharpe Ratio, Treynor Index, and Jensen Index, and further information about these statistics. The values of the Sharpe Ratio, Treynor Index and Jensen Index are annualized.
Figure 1: Ascending Distribution of Returns and DD Statistics for Risk Averters – Whole Period

Notes: ASD1 (ASD2, ASD3) refers to the first (second, third)-order ascending DD statistics, $T_j^A$, for $j = 1, 2$ and 3.

Refer to equation (2) for the definition of $T_j^A$. The right-hand side Y-axis is used for the ascending CDF of the spot and futures returns, whereas the left-hand side Y-axis is used for $T_j^A$ for $j = 1, 2$ and 3.
Figure 2: Descending Distribution of Returns and DD Statistics for Risk Seekers – Whole Period

Notes: Refer to the right hand side Y-axis for the descending CDF of the spot and futures returns. DSD1 refers to the first-order descending DD statistics, DSD2 refers to the second-order descending DD statistics, and DSD3 refers to the third-order descending DD statistics. DSD1 (DSD2, DSD3) refers to the first (second, third)-order descending DD statistics, $T^D_j$, for $j = 1, 2$ and 3. Refer to equation (3) for the definition of $T^D_j$. The right-hand side Y-axis is used for the descending CDF of the spot and futures returns, whereas the left-hand side Y-axis is used for $T^D_j$ for $j = 1, 2$ and 3.
References


