On Extreme Events in Banking and Finance

Financial Stability	Risk Management
Tail Risk	Financial Markets

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On Extreme Events in Banking and Finance

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Chapter 1

Introduction

The title of this thesis is 'On extreme events in banking and finance'. Before reading a thesis on extreme events, you have to ask yourself whether 'extreme' events, or events that occur with a small probability only, are fundamentally different from normal events. If both type of events were quite similar, then it is advisable to close this thesis and to read a different one that studies all events in the distant past, and not only those considered to be extreme. By definition, such a thesis would be based on a larger number of observations. Owing to the greater wealth of experience it draws on, such a thesis could provide more revealing insights. In other words, a thesis studying extreme events, such as this one, would be redundant.

Of course, the fact that this thesis has been written strongly suggests that extreme events in banking and finance may be quite different from normal events. And this is also what the results in this thesis suggest. During extreme events, the relationship between two financial variables can differ fundamentally from the relationship observed during normal periods. This central theme runs throughout all chapters in this thesis.

Chapters 2 and 3 stress the importance of studying relationships between financial returns during extreme events to obtain a better understanding of potential losses in investment portfolios. Those chapters also introduce a number of new tools to study extreme events in financial markets. Chapter 4 discusses how securitization may improve risk management on the one hand and may destabilize the financial system during an extreme event such as an adverse shock in depositors' confidence, on the other. Chapter 5 shows why it is important to study bank profitability during extreme events, such as severe recessions. Chapter 6 provides an illustration of how studying extreme events may improve risk estimates. The final chapter of this thesis, Chapter 7, illustrates how extreme observations occurring in financial markets are related to what is happening in the real economy. The remainder of this introduction provides a short overview of each chapter.

Financial risk managers try to limit the likelihood of large losses in their investment portfolios. An important source of portfolio risk is systematic risk, or market risk, which is the volatility in portfolio outcomes due to general market movements. It is common practice to quantify the amount of systematic risk in portfolios by estimating a linear relationship between movements in the portfolio value and the market value. In financial jargon, the slope of such a linear relationship is said to be the portfolio's 'beta'. This beta provides an estimate of the expected change in the portfolios value proportional to changes in the general market index.

Both practitioners and academics have realized the limitations of this approach for risk management purposes. Risk management based on such portfolio betas relies on the assumption that the co-movement of stocks is similar under normal and extremely adverse market conditions. Yet it is well known that relationships in financial markets change during market downturns, see e.g. Longin and Solnik (2001) and Ang and Bekaert (2002). Scholars have partially addressed this issue by estimating the linear relationship conditional on market downturns, see e.g. Price *et al.* (1982) and Ang *et al.* (2006a). However, estimating such a relationship is still based on a large number of observations from relatively normal trading days. A reliable approach to estimating the loadings on systematic risk conditional on *extremely* adverse market conditions, or systematic tail risk, is still absent from the literature.

Chapter 2 attempts to fill this gap. Using Extreme Value Theory, we develop a measure of loading on systematic tail risk (the tail beta) and develop its estimation methodology. Empirically, we find that assets with historically high tail betas are associated with significantly larger losses during future extreme market downturns. This shows that historical tail betas can capture future loadings on systematic tail risk. We also find that stocks with high tail betas earn higher returns, or a 'risk premium', during normal periods. However, based on more than 40 years of stock market data, we find that the higher returns received during normal periods are almost entirely erased by higher losses during market crashes.

To estimate the relationship between individual stocks and the general market index, it is necessary to quantify somehow the amount of dependence between individual stocks and the general market index. In traditional regression analysis, the correlation coefficient plays this role. When estimating tail betas in Chapter 2, we use the concept of 'tail dependence', which measures the proportion of extreme changes in the general market index that coincides with extreme changes in the value of individual stocks. Chapter 3 provides a new approach to estimating tail dependence. We show how the estimates from a regression model coincide with the non-parametric estimates of tail dependence. The advantage of this regression approach is the easy implementation and the straightforward extension of analyzing tail dependence to higher dimensional levels. A short version of this chapter is published as Van Oordt and Zhou (2012).

Another important issue in financial risk management is portfolio diversification, see e.g. Samuelson (1967). Diversification plays a major role in managing financial institutions, such as insurers and banks. Although it is common knowledge that diversification usually decreases the riskiness of individual financial institutions, the relationship between diversification and systemic risk is not so well known. Banking literature identifies a dark side of diversification at financial institutions, which is that diversification tends to increase the probability of joint failures or systemic crises, see e.g. Shaffer (1994) and Wagner (2010). The intuition behind this finding is that diversification increases the similarity of financial institutions. This makes them vulnerable to similar shocks.

Chapter 4 provides a theoretical discussion of diversification at financial institutions through securitization. Loan portfolio securitization has risen in importance during the last few decades. Securitization decreases the entrance cost of investing in certain loan types for financial institutions. Through securitization, investment in specific business or geographical areas becomes available without opening an entire new loan business. For example, European banks can invest in the U.S. home mortgage market by simply buying securities on this type of loans. Securitization thus acts as a catalyst for diversification by offering new prospects for diversification. The question remains whether securitization also amplifies the dark side of diversification. Chapter 4 is the first study to show why tranching, that is the practice of slicing securities on loan portfolios into different classes with different seniority levels, may play a vital role in this discussion. This chapter is forthcoming as Van Oordt (2013).

Another financial stability topic growing in importance is stress testing, i.e., predicting what would happen to the financial system or individual institutions in certain adverse scenarios. The relationship between economic growth and bank profitability plays a considerable role in stress tests, which are used to forecast bank performance in adverse macroeconomic scenarios. Commonly, empirical literature finds that the impact of economic growth on bank profit is relatively small, which suggests that bank performance is hardly affected by recessions. In Chapter 5, we assess whether the impact of economic growth on bank profitability is stronger in deep recessions than under normal economic conditions. Our results suggest that the small impact observed in normal economic conditions cannot be extrapolated unreservedly to severe recessions. We find a much larger impact of output growth on bank profitability during deep recessions. This finding suggests that the necessary capital buffers for banks to weather deep recession are larger than one would expect based on the weak relation observed during normal economic conditions. This chapter is published as Bolt *et al.* (2012).

Chapter 6 studies large agricultural commodity prices shocks. These large shocks occur more frequently than is justified by conventional statistical distributions, such as the normal distribution. Suggestions as to why commodity price shocks exhibit such heavy-tailed behavior have been sparse in economic literature. Chapter 6 suggests how heavy tails in primary commodity prices may arise endogenously in the economy. In our model, spikes in commodity prices arise due to productivity shocks in the agricultural sector, such as droughts. These shocks result in heavy-tailed commodity prices, even if they are not (but can be) heavy-tailed themselves. We also provide evidence on how heavy tails help to better quantify the extreme price risk of agricultural commodities.

Chapter 7 studies the dependence of extreme price shocks in futures markets for crop commodities. These markets, which can be easily accessed by speculators, exhibit frequent large price jumps and price drops from one day to another, see Chapter 6. For some crop commodity pairs, we observe relatively high levels of dependence among these extreme price changes. Chapter 7 focuses on the question of whether the strong dependence observed in potentially speculative financial markets can be related to observations in the real economy. Using U.S. crop data, we quantify the degree in which crop commodity pairs are grown in similar regions, or 'geographical overlap'. We find that a majority of the variation in tail dependence can be explained by this measure of geographical overlap. This result suggests that the strong dependence observed between the extreme price shocks of some crop commodity pairs is not without economic reason.

Chapter 2

Systematic tail risk

We develop a measure of loading on systematic tail risk, the tail beta, defined as the sensitivity of assets to extreme market downturns, and establish its estimation methodology. Empirically, we test the presence of a systematic tail risk premium in the cross-section of expected returns. The evidence supports a significant positive premium for assets with historically high tail betas during normal periods. However, high historical tail betas are also associated with large future losses under extremely adverse market conditions. Historically, these losses almost entirely offset the premium received during normal periods.[1]

Keywords: Tail beta, systematic risk, asset pricing, Extreme Value Theory.

JEL Classification Numbers: C14, G11, G12

2.1 Introduction

Risk managers are concerned with the performance of portfolios in distress events, the so-called tail events in the market. In this chapter we investigate the market risk of assets

[1]This chapter is co-authored by Chen Zhou. The authors thank discussants and participants in various conferences and seminars where previous versions of this chapter have been presented: the Spring Meeting of Young Economists (2011, Groningen), the workshop on Extreme Dependence in Financial Markets (Rotterdam, 2011), the 9th INFINITI Conference on International Finance (Dublin, 2011), the 7th Conference on Extreme Value Analysis (Lyon, 2011), the 65th Econometric Society European Meeting (Oslo, 2011), the 2011 Joint Statistical Meetings (Miami, 2011), the 15th SGF Conference (Zürich, 2012), the 5th International Risk Management Conference (Rome, 2012), the 4th Annual CF-Wharton-Tinbergen Conference (Cambridge, 2012), the 27th Annual Congress of the European Economic Association (Málaga, 2012), the Netherlands Economists Day (Amsterdam, 2012) and seminars in De Nederlandsche Bank, Ca' Foscari University of Venice, and University of Amsterdam. Especially, the authors want to thank Gerry Abdesaken, Kris M.R. Boudt, Francis X. Diebold, Bernd Schwaab and Casper G. de Vries for useful comments and suggestions. An early version of this chapter was circulated under the title 'Systematic risk under extremely adverse market conditions'.

under extremely adverse market conditions, or the loading on systematic tail risk. We develop an additive measure of sensitivity to systematic tail risk, the 'tail beta'. The scope of this chapter comprises the estimation of tail beta, an empirical assessment of its role in explaining the cross-section of expected returns, and the potential application of tail betas in risk management.

Systematic tail risk potentially plays an important role in asset pricing. Arzac and Bawa (1977) derive an asset pricing theory under the safety-first principle of Telser (1955). In their framework investors maximize the expected return under a Value-at-Risk constraint. As a consequence the cross-section of expected returns can be explained by a 'beta' different from the regular market beta. If investors are constrained by a Value-at-Risk with a sufficiently small probability, the beta of Arzac and Bawa (1977) equals to our tail beta under the assumption of a linear model for tail observations.

Using an Extreme Value Theory (EVT) approach, we develop a methodology to estimate the tail beta that exploits the feature of heavy tails and tail dependence in stock returns. Because tail events are rare, tail betas must be estimated from a small number of observations. If tail betas are estimated based on a few tail observations only, the new estimator performs better than simply applying a regression conditional on large market losses. We apply our estimator on daily stock return data.

We test whether tail betas help to explain the cross-section of expected returns. That is, we test whether tail betas estimated on historical data are compensated by a risk premium after accounting for other risk factors. From the asset pricing tests, we observe a risk premium for assets with high historical tail betas during normal periods. However, historically high tail betas are also associated with large losses during future market crashes. We find that these losses almost entirely offset the premium received during normal periods. Hence, from our results the additional role of systematic tail risk in explaining the cross-section of expected returns seems to be limited.

Nevertheless, the empirical results provide evidence that historical tail betas help to determine which assets will take relatively strong hits during future market crashes. This result points in the direction of a useful role for tail betas in risk management. In the last part of this chapter we discuss how tail betas help to calculate traditional measures of portfolio tail risk, such as Value-at-Risk. These theoretical results can be applied to evaluate the tail risk of portfolios.

A potential reason on why we emphasize on tail beta is the changing relation between asset and market returns under different market conditions. It is a well-known stylized fact that equity returns demonstrate higher correlations during periods of high stock market volatility, see e.g. King and Wadhwani (1990), Longin and Solnik (1995), Karolyi and Stulz (1996) and Ramchand and Susmel (1998). In addition, correlations increase especially during periods of severe market downturns, as reported by Longin and Solnik (2001), Ang and Bekaert (2002), Ang and Chen (2002) and Patton (2004). The changing correlation structure may signal that the comovement of assets with the market depends on market conditions.

Several studies discuss this changing comovement in the context of a nonlinear relation between asset returns and market risk. In line with the theoretical work of Rubinstein (1973) and Kraus and Litzenberger (1976), one strand of literature estimates the relation between asset returns and market risk with higher order approximations. Among others, Harvey and Siddique (2000) and Dittmar (2002) add respectively a second and third moment to the relation, and find that *coskewness* and *cokurtosis* play a role in asset pricing. However, room for extensions with additional higher moments may be limited, because the heavy tails observed in stock market returns provide evidence that further higher moments may not exist, see Mandelbrot (1963a) and Jansen and De Vries (1991).

An alternative strand of literature focuses on the comovement of asset returns with the market return under specific market conditions. For example, in line with the theory in Bawa and Lindenberg (1977), several studies investigate the *downside beta*, which is defined as the market beta conditional on below average market returns, see e.g. Price *et al.* (1982), Harlow and Rao (1989) and Ang *et al.* (2006a). Our tail beta fits within this strand of literature, because it focuses on comovement with the market return under specific market conditions. However, in contrast to downside beta, the tail beta measures comovement with the market if market downturns are extreme.

Our study is related to the empirical asset pricing literature on tail risks. A few studies focus on the role of tail risk in the cross-section of expected return irrespective of its relation with market risk, see e.g. Bali *et al.* (2009), Huang *et al.* (2012) and Cholette and Lu (2011). Other studies focus on the fear of tail events in the market index and the corresponding premium in options markets, see e.g. Bates (1991, 2000), Broadie *et al.* (2007) and Bollerslev and Todorov (2011). Interestingly, the premium for aggregate tail risk observed in options markets by the latter two studies does not appear in our cross-sectional analysis of stock returns. Kelly (2011) provides an alternative cross-sectional analysis. After constructing an index on the level of tail risks in the market, Kelly obtains 'tail risk betas' for individual assets by regressing asset returns on innovations in his index. These betas can be considered as a tail equivalent of the volatility betas of Ang *et al.* (2006b), which measure the comovement of stock returns with innovations in market volatility. In contrast to the 'tail risk beta' of Kelly (2011), our 'tail beta' can be considered as a tail equivalent of the regular market beta.

The expression 'tail beta' appears in the literature with other meanings. For example, De Jonghe (2010) estimates 'tail betas' by applying a tail dependence measure from Poon *et al.* (2004) on stock returns. Spitzer (2006) and Ruenzi and Weigert (2012) examine its asset pricing power. This tail dependence measure is defined as the probability of an extreme downward movement of the asset, conditional on the occurrence of a market crash. Hence, instead of measuring the magnitude of the comovement, this measure has the meaning of a conditional probability. Interestingly, the tail dependence measure will turn up as an important building block in the EVT approach to estimate the tail beta. Further, Bali *et al.* (2011) estimate 'hybrid tail betas'. Their aim is to capture the covariance of the asset and the market, given an adverse return of the *asset*. Consequently, they do not focus on comovement in case of extremely adverse market conditions.

Compared to those two measures, our tail beta has two appealing features. First, its interpretation as a measure of comovement with the market is in absolute terms, which is similar to the interpretation of the regular market beta. That is, on a day that the market suffers a loss of 10%, an asset with a tail beta of 2 is expected to suffer a downward movement of 20%. Second, our tail beta is an additive measure of tail risk. Similar to regular market betas, one can obtain the tail beta of a portfolio by taking the weighted average over the tail betas of the individual assets. Consequently, the tail beta provides a clear insight on how each asset contributes to the tail risk of portfolio.

Our empirical results are especially strong among stocks with a relatively high level of idiosyncratic volatility. The intuition behind this finding is as follows. Idiosyncratic volatility is measured by the standard deviation of the residuals from estimating a linear model, see e.g. Ang *et al.* (2006b, 2009) and Fu (2009). A larger deviation from the linear model induces a higher perceived level of idiosyncratic risk. Therefore, we expect to observe a higher level of idiosyncratic volatility in case of a larger deviation between the tail beta and the market beta. This relation causes our results to be stronger among stocks with high levels of idiosyncratic volatility. It is further worth to notify that the results remain significant among stocks with a relatively low level of idiosyncratic volatility.

Our results are not explained by other factors documented in the asset pricing literature. They remain robust after controlling for downside beta, coskewness and cokurtosis. The results are not explained by return characteristics related to past performance, such as momentum (Carhart (1997)), short-term reversal (Jegadeesh (1990)) and long-term reversal (De Bondt and Thaler (1985)). Further, the results are established within all size cohorts (Fama and French (2008)) and do not seem to be related with characteristics such as volume (Gervais *et al.* (2001)) and aggregate liquidity (Pastor and Stambaugh (2003)). Our results are established using both equal and value weighted portfolios, they do not depend on including or excluding NASDAQ and AMEX firms, or on including or excluding the recent crisis period. Finally, the results are not very sensitive to the exact choice on how many observations are used to estimate tail betas.

2.2 Theory

To define tail beta we first introduce a linear model that decomposes the tail risk of an asset into a systematic and an idiosyncratic component as follows. We denote the return on asset j and the market portfolio as R_j and R_m. The excess returns on asset j and the market are given by $R_j^e := R_j - R_f$ and $R_m^e = R_m - R_f$, where R_f denotes the risk free rate. The following linear model decomposes the asset excess return under extremely adverse market condition:

$$R_j^e = \beta_j^T R_m^e + \varepsilon_j, \quad \text{for} \quad R_m^e < -VaR_m(\bar{p}), \tag{2.1}$$

where ε_j is the idiosyncratic error term that is uncorrelated with R_m^e under the condition $R_m^e < -VaR_m(\bar{p})$ and $E\varepsilon_j = 0$. $VaR_m(\bar{p})$ denotes the Value-at-Risk (VaR) of the excess market return that is exceeded with some low probability $\bar{p}$, such that $\Pr(R_m^e \leq -VaR_m(\bar{p})) = \bar{p}$. In line with the interpretation of the regular market beta in the Capital Asset Pricing Model (CAPM), the parameter β_j^T, which measures the sensitivity to systematic tail risk, will be regarded as the 'tail beta'.

Under the linear tail model in equation (2.1), the tail beta plays an important role in asset pricing according to the asset pricing theory under the safety-first principle developed by Arzac and Bawa (1977) (AB1977). Although the linear tail model specifies the comovement between the asset and market excess return under extremely adverse market conditions only, safety-first investors do not need any further assumptions to price the asset. In particular, under the linear tail model in (2.1), the beta that determines expected returns in the asset pricing theory of AB1977 is identical to the tail beta.

AB1977 build an asset pricing theory under the safety-first principle of Telser (1955).[2] They assume that investors maximize the expected return while limiting the probability to suffer a particular large loss below a predetermined admissible probability level p. In other words, investors maximize the expected return under a VaR constraint.

In a distribution-free setup, AB1977 first prove a separation theorem: the relative asset allocation of any safety-first investor is independent of the investor's wealth and borrowing. All investors hold the same optimal portfolio, while the level of investment and borrowing of each investor depends on the tolerated VaR and the level of wealth. Second, they derive the equilibrium price for any asset. In equilibrium the expected return of any asset j is given by

$$E(R_j^e) = \beta_j^{AB} E(R_m^e), \tag{2.2}$$

[2]The initial safety-first principle introduced by Roy (1952) assumes that agents minimize the probability of suffering a large loss. AB1977 adapt the formulation by Telser (1955), which assumes that agents do not want the probability of suffering a particularly large loss to exceed a pre-specified level.

where the calculation of parameter β_j^{AB} is given in AB1977. From their calculation, we further derive the parameter β_j^{AB} in the following proposition. Appendix A provides a formal proof.

Proposition 2.2.1 *The parameter* β_j^{AB} *is given by*

$$\beta_j^{AB} = \frac{E(R_j^e | R_m^e = -VaR_m(p))}{-VaR_m(p)}, \tag{2.3}$$

where $VaR_m(p)$ *is the VaR of* R_m^e *at the admissible probability level* p.

From the proposition we observe that β_j^{AB} is determined by the asset's contribution to the VaR of the market return at probability level p.

Suppose that investors in AB1977 consider losses that occur with a sufficiently small probability in their safety-first principles, such that $p < \bar{p}$. With the linear tail model in (2.1), we can further manipulate the expression of β_j^{AB} in (2.3) as

$$\begin{aligned}\beta_j^{AB} &= \frac{E(\beta_j^T R_m^e + \varepsilon_j | R_m^e = -VaR_m(p))}{-VaR_m(p)} \\ &= \beta_j^T + \frac{E(\varepsilon_j)}{-VaR_m(p)} \\ &= \beta_j^T.\end{aligned}$$

Under the linear tail model in (2.1), we thus obtain that the tail beta, β_j^T, is identical to the beta in the AB1977 asset pricing theory, β_j^{AB}. Plugging this result in the asset pricing relation in (2.2) gives

$$E(R_j^e) = \beta_j^T E(R_m^e). \tag{2.4}$$

Consequently, given the linear tail model, the expected return of assets under the safety-first framework depends on their tail betas.

2.3 Methodology

Our objective is to test whether assets with a relatively high sensitivity to extremely adverse market downturns measured by their tail betas do earn an additional systematic tail risk premium. The presence of such a premium would provide some empirical evidence for the predictions from the safety-first framework of Arzac and Bawa (1977). We start by establishing an estimation methodology on the tail beta, and continue with introducing the framework for the asset pricing test.

2.3.1 Estimating tail beta

The tail beta β^T in the linear tail model in (2.1) could be regarded as a regression coefficient. Consequently, a direct approach to estimate the tail beta is to perform a regression analysis conditional on observations corresponding to large market losses only. Among others, Price *et al.* (1982) and Ang *et al.* (2006a) apply this approach to estimate downside betas while conditioning on below average market returns (or alternatively, market returns below the risk free rate). To estimate tail betas, one must choose a lower threshold, such that only observations corresponding to large market losses are taken into account. We refer to this method as the conditional regression approach.[3]

Two potential drawbacks of applying the conditional regression approach to estimate tail betas arise. First, because the conditional regression is based on a small number of observations, this approach yields potentially a large estimation error. Second, the heavy-tailedness of financial returns may further increase the variance of the estimator. Therefore, we establish an alternative method to estimate tail betas. Based on the same low number of observations, our new estimator exhibits a lower estimation error by exploiting the heavy-tailed feature of the financial returns.

We start from assuming the heavy-tailedness of the returns of financial assets. The definition of heavy-tailedness is given as follows. The tail distribution of the excess return on asset j can be expressed as

$$\Pr(R_j^e < -u) \sim A_j u^{-\alpha_j}, \quad \text{as} \quad u \to \infty, \tag{2.5}$$

where R_j^e denotes the excess asset return. The parameter α_j is called the tail index, and the parameter A_j is the scale.[4] Following the definition of heavy-tailedness in (2.5), we further assume for the excess market return and the idiosyncratic risk that

$$\Pr(R_m^e < -u) \sim A_m u^{-\alpha_m} \text{ and } \Pr(\varepsilon_j < -u) \sim A_{\varepsilon_j} u^{-\alpha_{\varepsilon_j}}, \text{ as } u \to \infty. \tag{2.6}$$

The idiosyncratic risk, ε_j, is assumed to be independent from the excess market risk, R_m^e.

The linear tail model in (2.1) induces a dependence structure between the excess market return and the excess asset return, if extremely adverse market conditions occur, i.e., if $R_m^e < -VaR_m(\bar{p})$. This dependence structure determines the tail dependence between the asset return and market return. By measuring the level of tail dependence

[3]For example, Post and Versijp (2007) provide estimates of tail beta from regressions conditional on markets return below −10%.

[4]Jansen and De Vries (1991) estimate tail indices for the downside tail in the range $3 < \hat{\alpha} < 5$ for US stocks. Loretan and Phillips (1994) find $3 < \hat{\alpha} < 4$ for equity returns. Poon *et al.* (2004) find similar numbers for several international stock market indices. In line with these results we observe $\hat{\alpha}_m = 3.5$ as average estimate for the market index.

between the asset and market returns, we are able to make statistical inference on the tail beta.

For this purpose we introduce the following tail dependence measure from multivariate EVT,

$$\tau_j := \lim_{p\to 0} \tau_j(p) := \lim_{p\to 0} \frac{1}{p} \Pr(R_j^e < -VaR_j(p) \text{ and } R_m^e < -VaR_m(p)),$$

where $VaR_j(p)$ is the VaR of the asset excess return at probability level p. Within a risk management framework, the τ-measure has a clear economic interpretation towards contagion risk. By rewriting it as

$$\tau_j = \lim_{p\to 0} \Pr(R_j^e < -VaR_j(p) | R_m^e < -VaR_m(p)),$$

the measure τ_j indicates the probability of having a large loss on asset j conditional on an extremely adverse market return. The potential values of the τ-measure are bounded by $0 \leq \tau \leq 1$, because it is the limit of a conditional probability. The case $\tau = 0$ is regarded as tail independence, while the case $\tau = 1$ corresponds to complete tail dependence.[5] Moreover, the τ-measure does not depend on the marginal distributions. These features of the τ-measure indicate that it plays a similar role as the correlation coefficient, but focuses only on the dependence in the tails. The τ-measure has been applied to several financial datasets in order to measure tail dependence, see e.g. Straetmans *et al.* (2008), Kang *et al.* (2010) and De Jonghe (2010).

The following proposition shows how the τ-measure is related to β_j^T under the linear tail model in (2.1). We provide the proof in Appendix A.

Proposition 2.3.1 *Under the linear tail model in* (2.1) *and the heavy-tail setup of the downside distributions in* (2.6)*, with* $\alpha_j > \frac{1}{2}\alpha_m$ *and* $\beta_j^T \geq 0$*, we have that as* $p \to 0$*,*

$$\tau_j(p) \sim \left(\beta_j^T \frac{VaR_m(p)}{VaR_j(p)}\right)^{\alpha_m}. \tag{2.7}$$

The relation in (2.7) provides the basis for estimating the tail beta.[6] For that purpose, we estimate each component in (2.7) except β_j^T and invert the relation to obtain an

[5] The τ-measure is closely related to other tail dependence measures, for example, the measure $E(\kappa|\kappa \geq 1)$ introduced by Embrechts *et al.* (2000) and applied in Hartmann *et al.* (2004). Here κ is the number of tail events which is defined as having a large loss that corresponds to tail probability p. Thus, the measure $E(\kappa|\kappa \geq 1)$ is the expected number of tail events given that there is at least one which is also an alternative measure on tail dependence. It is not difficult to verify that in a bivariate case, the two measures are connected by $E(\kappa|\kappa \geq 1) = \frac{2}{2-\tau}$.

[6] The relation in Proposition 2.3.1 holds under the weak condition that $\alpha_j > \frac{1}{2}\alpha_m$. This condition requires a lower bound on the tail index of asset excess returns. Empirical research usually finds that α_m is around 4 (see footnote 4). Given this finding, the condition is equivalent to $\alpha_j > 2$, which is satisfied if the excess returns of individual assets have finite variance. In addition, the relation does not depend on assuming the heavy-tailedness of the idiosyncratic risk, ε_j. The proposition holds even if ε_j follows a thin-tailed distribution, such as the normal distribution.

estimate of β_j^T. As in usual extreme value analysis, to mimic the limit procedure $p \to 0$, with the number of observation denoted by n, one may consider only the k observations in the tail region, such that as $n \to +\infty$, $k := k(n) \to \infty$ and $k/n \to 0$. In other words, for statistical estimation the probability p is set at the level $p = k/n$. Then the estimate of each component is obtained as follows. First, the tail index α_m can be estimated from the k highest losses by the so-called *Hill estimator* proposed in Hill (1975). Second, multivariate EVT provides estimates on the τ measure when $\tau > 0$, see e.g. De Haan and Ferreira (2006). Third, the VaRs of the market return and the asset return at probability k/n are estimated by the $(k+1)-$th highest loss of each of the two return series. Multivariate EVT assures the consistency of the estimators of each component, even under temporal dependence such as volatility clustering. For details on the estimation of these components, see Appendix B. After estimating the components, we invert equation (2.7) to obtain the estimator of β_j^T as

$$\hat{\beta}_j^T := \widehat{\tau_j(k/n)}^{1/\hat{\alpha}_m} \frac{\widehat{VaR_j(k/n)}}{\widehat{VaR_m(k/n)}}. \tag{2.8}$$

From the consistency of the estimators on the components, it is a direct consequence that $\hat{\beta}_j^T$ is a consistent estimator of β_j^T, even under temporal dependence such as volatility clustering. We refer to this method as the EVT approach.

We remark that the structure of the estimator $\hat{\beta}_j^T$ in (2.8) shows similarities with the estimator of the market beta in an ordinary least squares (OLS) regression. When estimating the market beta by a regression analysis, the OLS estimator is given as

$$\hat{\beta} = \hat{\rho}\frac{\hat{\sigma}_j}{\hat{\sigma}_m},$$

where $\hat{\rho}$ is the estimated correlation coefficient between R_j^e and R_m^e, $\hat{\sigma}_j$ and $\hat{\sigma}_m$ are the estimated standard deviations of R_j^e and R_m^e respectively. The estimator $\hat{\beta}$ consists of three components, the dependence measure, and two measures on the individual risks of the asset return and market return. Similarly, the estimator $\hat{\beta}_j^T$ consists of the tail dependence measure, $\hat{\tau}_j$, and two tail risk measures: the VaRs of the asset return and market return. The estimator of the tail beta combines these three components in a similar way as the OLS estimator of the market beta. The only additional element in the estimator of the tail beta is the tail index $\hat{\alpha}_m$.

We run simulations to examine the performance of the procedure to estimate tail beta. The setup of the simulations is as follows. In each sample, we generate 1,250 observations for the series R_m^e and ε from a Student-t distribution with 4 degrees of freedom.[7] The asset returns are constructed by aggregating the simulated R_m^e and ε according to different

[7]The number of generated observations, 1,250, is approximately the number of trading days in a

linear models. First, we consider three regular linear models in which the tail beta equals to the regular market beta as $\beta = \beta^T = 0.5, 1, 1.5$. Second, we consider two segmented linear models, which represent assets with $\beta = 1$ during regular market condition but different tail betas. If the loss on the market is less than a threshold 2.5, then the asset return is generated from a linear model with $\beta = 1$.[8] Otherwise, it is generated from a linear tail model with $\beta^T = 0.5$ and $\beta^T = 1.5$ respectively. For each of the five models, we generate 10,000 samples, and estimate β^T in each sample using both the conditional regression approach and the EVT approach. Then, by comparing the estimates to the real β^T value, we calculate the mean squared error (MSE), the estimation bias and the estimation variance for the two approaches.

The first column of Figure 2.1 compares the MSE between the EVT approach and the conditional regression. Under the heavy-tailed setup, we observe a better performance of the EVT approach relative to the conditional regression, if tail betas are estimated based on a few observations in the tail, i.e., for low levels of k. However, the conditional regression may perform better if more observations from the moderate level are included, i.e., for high levels of k. Nevertheless, the MSE of the EVT approach is not very sensitive to including more observations from the moderate level. The second and third columns of Figure 2.1 shows the decomposition of the MSE into squared bias and variance. We observe that the estimates from the conditional regression bears a large variation, while the estimation error from the EVT approach is mainly due to a positive bias.[9]

To estimate tail beta in the asset pricing test we prefer the EVT approach because of three reasons. First, if the estimation of tail beta is based on a few observations far in the tail, then the EVT approach performs better in terms of MSE. Second, although it is difficult to justify where the tail region starts with real data, the estimation error from the EVT approach is not very sensitive to the choice of the number of observations used in estimation. Third, in the asset pricing test we rank the relative level of the tail beta among different financial assets. This ranking is less contaminated if all estimated tail betas are upwards biased. However, the large variance in the estimates from the conditional regression approach may severely distort the ranking of the assets. Therefore,

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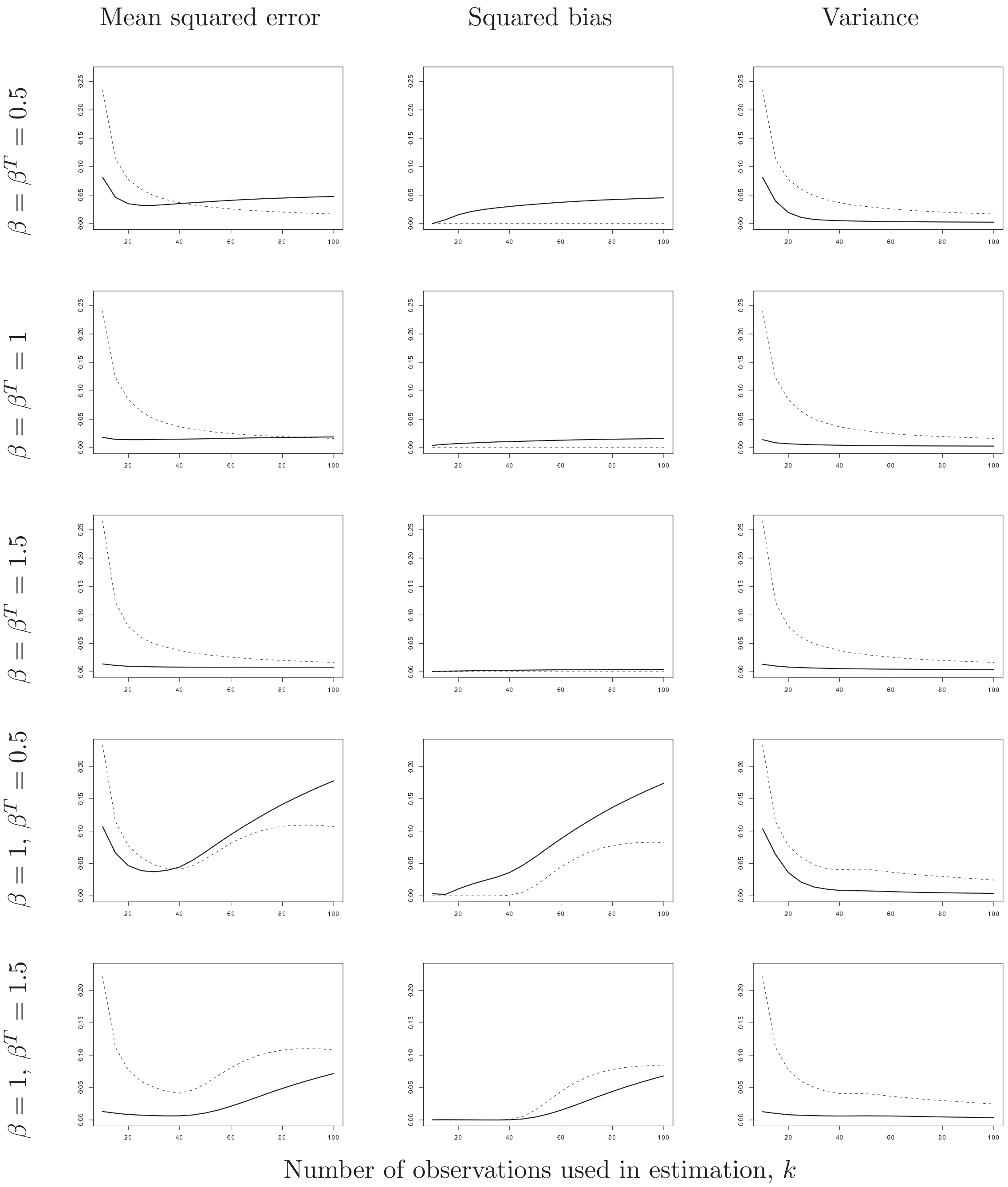
period of five years, which is the length of our estimation window in the empirical exercise. We choose the Student-t distribution because it is known to have heavy tails with the tail index equal to the degrees of freedom. The choice on the parameters α is similar to the estimates from the empirical analysis. We also perform simulations for $\alpha = 3$ and $\alpha = 5$. The results are comparable and omitted.

[8]The choice of this threshold corresponds to the quantile at a probability level 3.3% of a Student-t distribution with degree of freedom 4. Within a 1,250 observation sample, about $1,250 \times 3.3\% \approx 41$ asset returns are expected to be generated from the linear tail model. This is close to our choice of k in the empirical analysis.

[9]To illustrate the decomposition of the MSE, we report the squared bias in the figures, which do not show the sign of the bias. Numerical results show that the bias is consistently positive for all simulated models.

Figure 2.1: Simulations of tail beta estimators



The solid lines report the simulation results for the EVT approach in (2.8), while the dashed lines report those of the conditional regression approach. The simulations are based on 10,000 samples with 1,250 observations each. The asset returns are constructed from the simulated market excess returns and idiosyncratic risks following different linear relations. Market excess returns and idiosyncratic risks are randomly drawn from the Student-t distribution with degrees of freedom 4. In the first three rows, the asset returns follow a linear relation that does not depend on market conditions (i.e., $\beta^T = \beta$). In the fourth and fifth row, the asset returns are generated by a segmented linear model, where the slope changes to β^T if the market return is below -2.5 (i.e., approximately 41 observations in each sample). The estimates from the simulations, $\hat{\beta}^T$, are compared to the true value, β^T. The mean squared error is calculated as $\frac{1}{10{,}000}\sum_{i=1}^{10{,}000}(\beta^T - \hat{\beta}_i^T)^2$, where i refers to the i-th sample in the simulation. The squared bias is calculated as $\frac{1}{10{,}000}\sum_{i=1}^{10{,}000}(\beta^T - \bar{\beta}^T)^2$ and the variance is calculated as $\frac{1}{10{,}000}\sum_{i=1}^{10{,}000}(\bar{\beta}^T - \hat{\beta}_i^T)^2$, where $\bar{\beta}^T = \frac{1}{10{,}000}\sum_{i=1}^{10{,}000}\hat{\beta}_i^T$. The squared bias and squared variance in columns 2 and 3 sum up to the mean squared error in column 1.

we use the EVT approach to estimate the tail beta in our asset pricing test.

2.3.2 Portfolio formation

Our sample contains data on NYSE, AMEX and NASDAQ stocks of non-financials between July 1963 and December 2010. The data are obtained from the stock database of the Center for Research in Security Prices (CRSP). In addition, we download the risk free rate series and the excess return series on the market portfolio from the data library section on Kenneth French's website. The total data sample covers a period of 570 months.

To reliably estimate the tail betas, a large sample of historical returns is necessary. Researchers often estimate regular market betas based on the past 60 historical monthly returns. Such a low number of observations is insufficient for our purpose to estimate tail betas. This is why we obtain both daily and monthly returns from CRSP. During our procedure to estimate the tail betas, we will resort to using daily returns without altering the estimation horizon. To be consistent with most asset pricing literature, we use the monthly returns throughout the other parts of the asset pricing tests.

For each individual firm in the sample we calculate its tail beta in the following way. First, we calculate daily returns in excess of the risk free rate for each firm by subtracting the daily risk free rate from the firm specific daily return. Based on historical firm specific return series and historical market returns, we calculate firm specific tail betas by applying the estimator in (2.8).

The firm specific tail betas are estimated with a monthly rolling window over a period of 60 months, which is on average approximately $1,250$ trading days. Each time window starts at the first day of the first month and ends at the last day of the last month. Within each window we use about 4% of the worst market days to estimate the tail beta. More specifically, following the procedure in Appendix B, in the baseline results we fix parameter k in (2.8) at 50 days for each estimation window of 60 months.

We study the impact of past systematic tail risk loadings on future returns by forming portfolios based on the estimated tail betas at the end of every estimation period. We seek to test the hypothesis that observed high tail betas in the past are expected to earn an additional risk premium in the future. This is tested against the null hypothesis that high tail betas observed in the past do not earn an additional risk premium in the future. To maximize the variation in the potential additional risk premium for systematic tail risk, we form portfolios sorted on the spread between tail betas and regular market betas.[10]

[10] In this respect we follow Ang *et al.* (2006a), who sort on the spread between downside beta and market beta.

At the end of every estimation period we rank the firms based on the spread between the estimated tail betas and regular market betas. Based on these rankings we construct five portfolios, with each portfolio containing the same number of stocks. During the portfolio formation procedure we exclude the firms that do not qualify according to the following two conditions. First, the stock must be trading at a price above 5 USD on the last day of the estimation period. We use this criterion to exclude firms in severe financial distress. Second, stocks should not report zero returns on more than 60% of the trading days in the estimation period. We use this criterion to avoid that our results are affected by estimations based on daily returns of thin traded stocks.

Using several benchmark models, we define the risk-adjusted return for individual stocks as follows:

$$R^*_{j,t} = R_{j,t} - R_{f,t} - \sum_{k=1}^{m} \hat{\beta}_{j,k} F_{k,t}, \tag{2.9}$$

where $R_{j,t}$ is the monthly return on stock j at time t, $R_{f,t}$ is the risk-free rate, and $F_{k,t}$ denotes the of k-th of m risk factors in the benchmark model. We estimate the factor loadings, $\hat{\beta}_{j,k}$ for individual stocks using monthly rolling regressions with the 60-month window prior to t. Risk-adjusted returns on the formed portfolios, $R^*_{p,t}$, are calculated by averaging the risk-adjusted returns for individual stocks, $R^*_{j,t}$, using both equal and value weights. The holding period of the portfolios is the first month after the estimation period. Hence, to summarize, formation of portfolios occurs at the start of each month using information from past daily and monthly return data over a period of 60 months.

Based on the returns of the constructed portfolios we further construct a *zero investment portfolio*. The risk adjusted returns on this portfolio is earned when taking a long position of 1 USD on the portfolio with the 20% highest spreads between tail and market betas, while taking a short position of 1 USD on the portfolio with the 20% lowest spreads.

2.4 Results

2.4.1 Descriptive statistics

Table 2.1 reports the average of some descriptive statistics across the stocks in each of the sorted portfolios. When sorting according to the estimated tail betas in panel (a), we observe that stocks with high tail betas also tend to have high market betas on average. There is a clear trend in the average market betas across the tail beta portfolios. Trends with similar signs can be observed for the downside beta, the standard deviation, the skewness and the excess kurtosis of the returns. These trends indicate that stocks with high tail betas also tend to have higher values for other potential risk measures. Interestingly, the tail dependence measure, τ, which is an ingredient of the tail beta calculation, remains on average at a relatively constant level across the different portfolios. Apparently, there is not a very strong relation between the firms that comove more with the market in extremely adverse market conditions, and those firms that have a high level of tail dependence. Finally, high tail beta firms are relatively small in terms of market capitalization, but tend to have higher trading volumes.

Most trends fade or reverse if we sort on the spread between the tail beta and the market beta in panel (b). Apparently, many of the aforementioned trends are rather driven by high regular market betas than by high tail betas. After sorting on the spread we observe that both the market beta and the downside beta increase as the spread decreases. Further, a clear trend appears for coskewness. However, no strong trend can be observed for the other return characteristics. The trend in trading volume is also reversed: high spread firms are less frequently traded. The relation between the tail beta and the market beta stress the importance of sorting on the spread between tail beta and market beta. Moreover, the relation between the tail beta and other risk measures, especially downside beta and coskewness, stresses the importance of performing robustness checks in our asset pricing tests.

2.4.2 Persistence

First, we verify whether the estimates of tail beta obtained from historical data are persistent over time. In the absence of such persistence, estimating tail betas based on historical data would merely have a descriptive function, and would provide no insight in future comovements during adverse market conditions. To investigate this issue we provide transition matrices in Table 2.2. The transition matrices in Table 2.2, panel (a) are based on firm tail betas and respectively their 12 month lagged value. One concern is that the potential persistence observed in transition matrices based on 12 month lagged values is spurious because those transition matrices are based on tail beta estimates from two overlapping data samples. To address this issue, we also construct transition matrices based on 60 months lagged data samples in Table 2.2, panel (b). Tail betas are estimated following both the EVT approach and the conditional regression approach. The table also provides a similar matrix for market betas estimated from a regression with the CAPM as benchmark model. Higher numbers along and around the diagonal point into the direction of a more persistent sorting.

In Table 2.2 we observe two patterns from the transition matrices based on 12 month lagged values. First, the numbers along the diagonal of the transition matrices are higher if the matrix is constructed from tail betas estimated with the EVT approach. This

Table 2.1: Descriptive statistics

Panel (a): sorting on β^T	High β^T	4	3	2	Low β^T
Return characteristics:					
$\bar{\beta}$	1.70	1.32	1.10	0.91	0.62
$\overline{\beta^T - \beta}$	0.91	0.62	0.48	0.37	0.27
$\bar{\beta}^{DS}$	1.72	1.33	1.11	0.92	0.65
Standard deviation	18.30	13.70	11.12	9.33	7.40
Idiosyncratic volatility	15.20	11.40	9.25	7.77	6.34
$\bar{\tau}$	0.22	0.21	0.21	0.20	0.15
Skewness	0.81	0.52	0.38	0.31	0.35
Coskewness	-0.04	-0.04	-0.04	-0.04	-0.03
Excess kurtosis	2.55	1.80	1.52	1.38	1.65
Cokurtosis	0.03	0.06	0.09	0.11	0.10
Firm characteristics:					
Market capitalization (bln USD)	0.74	1.19	2.15	3.02	2.66
Volume (mln shares)	10.21	8.05	7.97	7.74	5.25

Panel (b): sorting on $\beta^T - \beta$	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$
Return characteristics:					
$\bar{\beta}$	0.91	1.01	1.05	1.16	1.54
$\overline{\beta^T - \beta}$	1.32	0.72	0.47	0.25	-0.11
$\bar{\beta}^{DS}$	1.08	1.05	1.05	1.13	1.42
Standard deviation	14.66	11.58	10.54	10.49	12.59
Idiosyncratic volatility	13.03	9.89	8.76	8.48	9.81
$\bar{\tau}$	0.19	0.21	0.21	0.20	0.18
Skewness	0.71	0.44	0.37	0.36	0.51
Coskewness	-0.10	-0.06	-0.03	-0.02	0.02
Kurtosis	2.22	1.51	1.46	1.53	2.18
Cokurtosis	0.11	0.05	0.05	0.07	0.10
Firm characteristics:					
Market capitalization (bln USD)	0.79	2.03	2.60	2.52	1.81
Volume (mln shares)	4.69	7.30	8.24	8.68	10.32

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas of NYSE, AMEX and NASDAQ stocks by applying the EVT approach in (2.8) on daily returns from the past 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We exclude stocks with more than 60% zero daily returns in the 60 months prior to t, and stocks with a price below 5 USD at the end of month $t-1$. Stocks are sorted into five quintiles: panel (a) considers quintiles sorted on tail betas, panel (b) considers quintiles sorted on the spread between the tail beta and the market beta. The reported numbers are averages for the stocks in each sort. We first average across firms in each month t, and then average over the 510 months in the sample. The market beta, downside beta, standard deviation, idiosyncratic volatility, skewness, coskewness, excess kurtosis and cokurtosis are calculated from the 60 monthly returns prior to t. The market beta is reported as $\bar{\beta}$. The spread between tail beta and market beta is reported as $\overline{\beta^T - \beta}$. Downside beta, $\bar{\beta}^{DS}$, is estimated by a regression conditional on below average market returns. Idiosyncratic volatility is calculated as the standard deviation of the residuals obtained from regressing individual stock returns on the FF3 factors. The tail dependence measure, τ, is based on daily returns from the 60 months prior to t and calculated following the estimator in Appendix 2.B, equation (2.21), with $k = 50$. Coskewness and Cokurtosis are calculated following (2.10) and (2.11). Market capitalization and trading volume are provided at the end of month $t-1$.

Table 2.2: Transition matrices

Panel (a): 12 months			$t+12$				
EVT approach			High β^T	4	3	2	Low β^T
		High β^T	80	18	2	0	0
		4	15	62	21	2	0
	t	3	2	16	59	21	2
		2	0	2	17	64	16
		Low β^T	0	1	2	14	83
Conditional approach			High β^T	4	3	2	Low β^T
		High β^T	74	18	5	2	1
		4	18	52	21	7	2
	t	3	5	20	48	22	6
		2	2	7	21	50	20
		Low β^T	1	2	6	20	71
Market beta			High β	4	3	2	Low β
		High β	76	20	3	1	0
		4	17	54	23	5	1
	t	3	3	21	51	22	3
		2	1	4	21	56	18
		Low β	0	1	3	17	79

Panel (b): 60 months			$t+60$				
EVT approach			High β^T	4	3	2	Low β^T
		High β^T	49	29	14	6	2
		4	21	30	27	17	6
	t	3	9	18	30	30	13
		2	4	11	23	35	26
		Low β^T	2	4	10	22	62
Conditional approach			High β^T	4	3	2	Low β^T
		High β^T	34	24	18	14	10
		4	21	23	21	19	15
	t	3	15	20	22	23	20
		2	12	18	22	24	25
		Low β^T	8	14	19	25	33
Market beta			High β	4	3	2	Low β
		High β	37	25	17	11	6
		4	20	26	25	19	10
	t	3	12	22	26	25	15
		2	8	15	24	28	25
		Low β	4	8	14	26	52

Note: The table provides transition matrices based on 12 month (panel a) and 60 month (panel b) lagged values. At the start of each month t between July 1968 and Dec 2009, we estimate tail betas of NYSE, AMEX and NASDAQ stocks by applying the EVT approach in (2.8) on past daily returns from the 60 months prior to t. Stocks are sorted into five quintiles according to the tail beta estimates. We exclude stocks with more than 60% zero daily returns in the 60 months prior to t, and stocks with a price below 5 USD at the end of month $t-1$. We also determine the allocation of each firm in the tail beta quintiles at the start of month $t+12$. For each tail beta quintile at time t we calculate the percentage of surviving firms allocated in each tail beta quintile at the start of month $t+12$. The results in the transition matrices are averages over time.

We repeat the procedure for tail betas obtained from the conditional regression approach by performing a regression on daily returns conditional on the 50 worst market returns from the 60 months prior to t. We also repeat the procedure for market betas obtained from a regression on monthly returns from the 60 months prior to t with the CAPM as benchmark model. The lower panel provides the same matrices after sorting at the start of month t (between July 1968 and Dec 2005) and at the start of month $t+60$. Higher numbers along and around the diagonal of the transition matrices point into the direction of a more persistent sorting.

suggests that the EVT approach provides a more persistent classification of firms based on the sensitivity to systematic tail risk than the conditional regression approach. The higher persistence is a potential consequence of the lower variance associated with the EVT approach. Second, the numbers along and around the diagonal of the transition matrices based on tail beta are in general above those in the transition matrix constructed from market betas. This pattern suggest that if one is inclined to believe that historical market betas contain information about future comovement with the market, then there seems to be no reason to worry about the information contained by historical tail betas on future comovement with the market under extremely adverse market conditions. Not surprisingly, the overall level of the numbers on the diagonal is lower in the transition matrices based on 60 month lagged values. However, the observed patterns remain: tail betas from the EVT approach provide a more persistent sorting of firms; and the sorting of firms based on tail beta is not less persistent than the sorting of firms on market beta.

2.4.3 Asset pricing tests

Table 2.3 presents the baseline asset pricing result on the sorted portfolios. The unadjusted average excess return on the zero-investment portfolio is slightly below, but not significantly different from zero. For returns adjusted with the CAPM and the Fama and French (1993) three factor model (FF3), the return on the zero-investment portfolio is slightly above, but still not significantly different from zero. This result holds for both equal and value weighted portfolios (the t-statistics are between 0.6 and 1.0). Also if we repeat our procedure within presorted size cohorts and calculate the FF3-adjusted returns, the absence of any significant risk premium remains prevalent. To summarize, we cannot reject the null hypothesis that having an exposure on systematic tail risk did not receive an additional (ex post) risk premium in the market.

The results so far are rather pessimistic about the role of systematic tail risk in explaining the cross-section of expected returns. One potential reason for not finding any significant results might be the failure to reliably estimate a forward looking measure of the sensitivity to systematic tail risk. The definition of tail beta only implies that high tail beta stocks suffer from large in-sample losses under extremely adverse market conditions. Nevertheless, the results in the transition matrices seem to suggest that the sorting based on estimates of tail beta is quite persistent over time. This persistency hints, but does not guarantee that historical tail betas capture future sensitivity towards systematic tail risk. To test this explicitly, we take the (risk-adjusted) portfolio returns from the last exercise, and focus on the months with an excess market return, $R^e_{m,t}$, lower than -5%. We consider these 54 months as representing ‘extremely adverse market conditions’.

Table 2.3: General asset pricing results

	Sort:		High β^T	4	3	2	Low β^T	(5) - (1)
$\bar{R}^e_p$	EW		0.42	0.72	0.75	0.70	0.66	-0.24
			(1.1)	(2.3)	(2.8)	(3.0)	(3.6)	(-0.8)
	VW		0.37	0.53	0.45	0.40	0.50	-0.13
			(0.9)	(1.6)	(1.8)	(1.9)	(3.1)	(-0.4)
	Sort:		High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
$\bar{R}^e_p$	EW		0.56	0.69	0.67	0.69	0.65	-0.09
			(1.8)	(2.6)	(2.7)	(2.7)	(2.2)	(-0.6)
	VW		0.44	0.53	0.48	0.45	0.44	-0.01
			(1.5)	(2.3)	(2.3)	(2.1)	(1.7)	(-0.0)
$\bar{R}^*_{CAPM,p}$	EW		0.24	0.31	0.27	0.24	0.04	0.20
			(1.4)	(2.8)	(3.0)	(2.3)	(0.2)	(0.9)
	VW		0.11	0.19	0.13	0.02	-0.12	0.22
			(0.7)	(2.3)	(2.5)	(0.3)	(-1.0)	(1.0)
$\bar{R}^*_{FF3,p}$	EW		0.03	0.10	0.05	0.03	-0.09	0.11
			(0.3)	(2.3)	(1.2)	(0.6)	(-0.8)	(0.6)
	VW		0.11	0.17	0.08	0.02	-0.03	0.14
			(0.9)	(2.4)	(1.4)	(0.4)	(-0.3)	(0.7)
$\bar{R}^*_{FF3,p}$	EW	Small	-0.15	0.14	0.18	0.06	0.05	-0.19
		2	-0.03	0.03	0.11	0.06	-0.08	0.05
		3	0.09	0.03	0.05	0.05	-0.10	0.19
		4	0.06	0.13	0.07	0.00	-0.10	0.16
		Large	0.14	0.04	0.01	-0.09	-0.08	0.22
		Avg	0.02	0.07	0.08	0.02	-0.06	0.08
t-stat		Small	(-0.9)	(1.3)	(2.1)	(0.7)	(0.4)	(-0.8)
		2	(-0.3)	(0.4)	(1.4)	(0.7)	(-0.5)	(0.2)
		3	(0.7)	(0.5)	(0.8)	(0.6)	(-0.8)	(0.9)
		4	(0.5)	(1.7)	(1.2)	(-0.0)	(-0.8)	(0.9)
		Large	(1.5)	(0.8)	(0.1)	(-1.4)	(-0.8)	(1.3)
		Avg	(0.2)	(1.6)	(1.9)	(0.3)	(-0.6)	(0.5)
$\bar{R}^*_{FF3,p}$	VW	Small	-0.23	0.11	0.14	0.03	0.04	-0.27
		2	-0.04	-0.01	0.09	0.06	-0.08	0.03
		3	0.11	0.06	0.04	0.04	-0.11	0.22
		4	0.06	0.14	0.06	-0.02	-0.09	0.14
		Large	0.17	0.15	0.05	-0.09	-0.03	0.20
		Avg	0.01	0.09	0.08	0.00	-0.05	0.07
t-stat		Small	(-1.4)	(1.2)	(1.6)	(0.3)	(0.3)	(-1.2)
		2	(-0.4)	(-0.1)	(1.1)	(0.6)	(-0.5)	(0.1)
		3	(0.9)	(0.8)	(0.5)	(0.5)	(-0.8)	(1.0)
		4	(0.5)	(2.0)	(1.0)	(-0.2)	(-0.7)	(0.8)
		Large	(1.8)	(2.0)	(0.8)	(-1.3)	(-0.2)	(1.1)
		Avg	(0.1)	(1.9)	(1.8)	(0.1)	(-0.5)	(0.4)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (2.8) on past daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We form 5 equal weighted (EW) and value weighted (VW) portfolios by sorting on either the tail beta (the first row) or the spread between tail beta and market beta (from the second row onwards) and construct a zero-investment portfolio. We calculate risk-adjusted returns by applying (2.9) on monthly stock returns at time t, where the loadings on the risk factors in the benchmark model are estimated for each stock by an OLS regression on monthly returns from the 60 months prior to t.
The first and second row report the average excess portfolio return, $\bar{R}^e_p$. The third and fourth rows, report the average CAPM- and FF3-adjusted portfolio returns, $\bar{R}^*_{CAPM,p}$ and $\bar{R}^*_{FF3,p}$. The fifth and sixth rows report the average FF3-adjusted returns after presorting the equities in five size cohorts and then sorting on the tail beta spread within each size cohort, where size is measured by market capitalization at the end of month $t-1$. Newey-West t-statistics are reported in parentheses.

Table 2.4: Results under extremely adverse market conditions ($R^e_{m,t} < -5\%$)

	Sort:		High β^T	4	3	2	Low β^T	(5) - (1)
$\bar{R}^e_p$	EW		-13.62	-10.20	-8.39	-7.03	-4.94	-8.68
			(-14.9)	(-12.4)	(-10.9)	(-10.2)	(-8.8)	(-11.8)
	VW		-14.22	-11.46	-9.17	-7.13	-5.12	-9.10
			(-15.0)	(-15.6)	(-15.7)	(-13.4)	(-10.4)	(-9.5)
	Sort:		High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
$\bar{R}^e_p$	EW		-9.93	-8.49	-8.08	-8.26	-9.42	-0.51
			(-11.2)	(-11.0)	(-12.1)	(-12.6)	(-13.5)	(-0.9)
	VW		-9.58	-7.59	-7.69	-8.10	-9.26	-0.33
			(-13.0)	(-12.2)	(-16.6)	(-16.8)	(-15.9)	(-0.4)
$\bar{R}^*_{CAPM,p}$	EW		-2.38	-0.07	0.76	1.64	3.70	-6.07
			(-3.9)	(-0.2)	(2.3)	(4.8)	(8.1)	(-8.7)
	VW		-3.18	-1.03	-0.36	0.67	2.63	-5.81
			(-7.3)	(-3.1)	(-1.7)	(2.7)	(6.8)	(-8.7)
$\bar{R}^*_{FF3,p}$	EW		-2.52	-0.31	0.44	1.19	3.17	-5.69
			(-7.8)	(-1.7)	(3.3)	(6.1)	(9.2)	(-9.5)
	VW		-2.51	-0.71	-0.17	0.78	2.40	-4.90
			(-6.2)	(-2.3)	(-0.8)	(3.3)	(6.4)	(-8.0)
$\bar{R}^*_{FF3,p}$	EW	Small	-3.83	-1.07	0.66	1.36	3.59	-7.43
		2	-3.15	-0.53	0.31	1.35	3.91	-7.05
		3	-2.24	-0.52	0.57	1.20	3.16	-5.40
		4	-1.54	-0.08	0.46	1.34	2.90	-4.44
		Large	-1.52	-0.18	0.56	0.84	2.34	-3.87
		Avg	-2.46	-0.48	0.51	1.22	3.18	-5.64
t-stat		Small	(-8.3)	(-3.1)	(2.7)	(5.1)	(9.1)	(-10.4)
		2	(-7.1)	(-1.9)	(1.4)	(4.6)	(8.2)	(-9.0)
		3	(-5.5)	(-2.1)	(2.4)	(4.9)	(8.2)	(-7.8)
		4	(-4.1)	(-0.3)	(2.0)	(5.2)	(7.5)	(-7.4)
		Large	(-4.5)	(-0.9)	(2.7)	(3.3)	(6.5)	(-7.6)
		Avg	(-7.6)	(-2.8)	(3.7)	(6.3)	(9.3)	(-9.6)
$\bar{R}^*_{FF3,p}$	VW	Small	-3.92	-1.16	0.58	1.35	3.54	-7.45
		2	-3.15	-0.45	0.25	1.38	3.87	-7.02
		3	-2.15	-0.44	0.64	1.20	3.14	-5.29
		4	-1.57	-0.13	0.51	1.34	2.87	-4.44
		Large	-1.59	-0.12	-0.08	0.48	2.30	-3.89
		Avg	-2.48	-0.46	0.38	1.15	3.14	-5.62
t-stat		Small	(-8.7)	(-3.4)	(2.5)	(4.9)	(9.1)	(-10.7)
		2	(-7.4)	(-1.6)	(1.1)	(4.7)	(8.2)	(-9.1)
		3	(-5.3)	(-1.8)	(2.5)	(4.7)	(8.2)	(-7.9)
		4	(-4.1)	(-0.5)	(2.2)	(5.3)	(7.6)	(-7.4)
		Large	(-4.0)	(-0.5)	(-0.3)	(1.8)	(5.7)	(-6.4)
		Avg	(-8.1)	(-2.8)	(3.0)	(6.3)	(9.5)	(-9.9)

Note: We calculate the average of the excess returns and risk-adjusted returns conditional on extremely adverse market conditions for the portfolios sorted on tail beta (the first row) or the spread between the tail beta and the market beta (from the second row onwards). From July 1968 and Dec 2010, we consider the returns from the 54 months in which the market factor lost at least 5% of its value, i.e., $R^e_{m,t} < -5\%$. The first row and second row report the average excess portfolio return, $\bar{R}^e_p$. The third and fourth row, report the CAPM- and FF3-adjusted portfolio return, $\bar{R}^*_{CAPM,p}$ and $\bar{R}^*_{FF3,p}$. The fifth and sixth row report the average FF3-adjusted returns after presorting the equities in five size cohorts based on market capitalization at the end of month $t-1$ and then sorting on the tail beta spread within each size cohort. Standard t-statistics are reported in parentheses.

Table 2.4 reports the results for the subsample of months with extremely adverse market conditions. The first line in Table 2.4 reports the average historical returns on portfolios sorted on tail beta alone. Across the portfolios, we observe a strong downward sloping trend in the average losses if one moves from the portfolios with the highest tail betas to those with low tail betas. For the high tail betas we find an average loss of 13.62%, while the average loss for the portfolio with the lowest tail beta loadings equals to 4.94%. However, this result may merely reflect the underlying difference in terms of the (regular) market betas. Consequently, we continue to the portfolios sorted on the spread between tail beta and market beta.

Considering sorts based on the spread between tail and market beta, the difference in average losses, $\bar{R}_p^e$, seems to disappear. However, from the descriptive statistics we know that portfolios with a relatively high spread tend to have a low market beta. Consequently, we consider the risk-adjusted returns using the CAPM as benchmark model, $\bar{R}^*_{CAPM,p}$. We find that portfolios with a high positive spread between tail beta and market beta strongly underperform during extremely adverse months, while portfolios with a negative spread between tail beta and market beta strongly outperform. The zero investment portfolio, with a long exposure on high spread stocks and a short exposure on low spread stocks, yields significant additional losses during extremely adverse months. The CAPM-adjusted returns during extremely adverse months add up to -6.07% and -5.81% for respectively the value and equal weighted portfolios (t-statistics around -8.7). The FF3-adjusted returns do report similar results. The trend in the losses and the significance of the results are also robust among all size cohorts. The least significant t-statistic for the zero investment portfolios is -6.4 for the value weighted portfolio based on large firms. To summarize, from the results in Table 2.4 we find that stocks with a high spread between tail beta and market beta strongly underperform during extremely adverse market conditions. Hence, it seems implausible that the failure to establish a risk premium on tail beta stems from its potential failure in capturing the future sensitivity to systematic tail risk.

Another potential reason for not observing the additional risk premium for systematic tail risk comes from the double-edged sword impact of high tail betas. Although investors may receive a premium during good times, large losses are suffered during extreme market downturns. These large negative returns may partly cancel out the positive risk premium. Hence, to test for the presence of a positive risk premium during the 'business as usual' months, we also exclude months that coincide with extremely adverse market conditions.

Table 2.5 reports the baseline results for the subsample with the remaining 457 months. Based on these results, we find that stocks with a high spread between tail and market beta, do significantly outperform low spread stocks during 'business as usual' periods.

The average risk-adjusted premium with FF3 as benchmark model is 0.80% and 0.74% per month for respectively the equal and value weighted portfolios (with t-statistics of 5.6 and 4.3). Although these results are weaker for the smallest firms, the results are robust and significant among all size cohorts.

The asset pricing tests show that investors receive a significant premium for having higher loadings on systematic tail risk during normal times. However, historically the premium is barely enough to compensate for the additional losses that occur during extremely adverse market conditions. This seems to be a reason why an additional risk premium on systematic tail risk is not observed over the entire historical sample. To conclude, from our results the additional role of systematic tail risk in explaining the cross-section of expected return seems to be limited.

2.5 Robustness checks

In this section we test whether our results are robust, and, in particular, whether other findings on asset pricing factors in the literature can explain part of our results. In the first subsection, we extend the FF3 benchmark model with several other factors that are relevant in the empirical asset pricing literature. Because Daniel and Titman (1997) find evidence that return premia on stock characteristics are not necessarily due to loadings on pervasive risk factors, we also check whether the findings on tail beta remain robust after presorting on several stock characteristics. We report these results in the second subsection. The final subsection provides robustness checks for deviations in the methodology.[11]

2.5.1 Extending the benchmark model

We extend the FF3 benchmark model with several other asset pricing factors documented in the literature. Table 2.6 reports some summary statistics and the sources of these factors and the period over which each factor is available. The results of the robustness checks are reported in Table 2.7 and Table 2.8. For each extended benchmark model we report three lines of results: we report the average risk-adjusted return of the portfolios, the average risk-adjusted return during adverse market conditions and the average risk-adjusted return during usual periods.

[11] Throughout the robustness checks, we report the results for the value weighted portfolios. For most robustness checks the t-statistics for the equal weighted zero investment portfolios are above those for the value weighted portfolios. Consequently, the reported significance of the premium during normal times and the significance of the loss during adverse market conditions can usually be considered to be at the conservative side.

Table 2.5: Results under usual market conditions ($R^e_{m,t} \geq -5\%$)

	Sort:		High β^T	4	3	2	Low β^T	(5) - (1)
$\bar{R}^e_p$	EW		2.09	2.02	1.83	1.62	1.33	0.76
			(6.4)	(8.4)	(9.1)	(9.5)	(10.2)	(2.9)
	VW		2.10	1.95	1.59	1.29	1.17	0.93
			(6.3)	(7.6)	(8.0)	(7.8)	(8.8)	(3.1)
	Sort:		High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
$\bar{R}^e_p$	EW		1.80	1.78	1.71	1.74	1.84	-0.04
			(7.4)	(8.8)	(9.2)	(9.1)	(8.4)	(-0.3)
	VW		1.62	1.49	1.45	1.46	1.59	0.03
			(7.2)	(8.3)	(9.0)	(8.4)	(7.8)	(0.2)
$\bar{R}^*_{CAPM,p}$	EW		0.55	0.35	0.22	0.08	-0.40	0.94
			(3.6)	(3.5)	(2.6)	(0.9)	(-3.2)	(6.1)
	VW		0.50	0.33	0.19	-0.06	-0.44	0.94
			(3.8)	(4.3)	(3.6)	(-1.0)	(-4.6)	(5.0)
$\bar{R}^*_{FF3,p}$	EW		0.33	0.15	0.01	-0.10	-0.47	0.80
			(3.4)	(3.1)	(0.2)	(-2.0)	(-5.3)	(5.6)
	VW		0.42	0.28	0.11	-0.07	-0.32	0.74
			(3.4)	(3.8)	(2.1)	(-1.3)	(-3.6)	(4.3)
$\bar{R}^*_{FF3,p}$	EW	Small	0.29	0.28	0.12	-0.09	-0.37	0.66
		2	0.34	0.10	0.09	-0.09	-0.55	0.89
		3	0.36	0.10	-0.01	-0.09	-0.49	0.85
		4	0.24	0.15	0.03	-0.16	-0.46	0.70
		Large	0.34	0.07	-0.06	-0.19	-0.37	0.70
		Avg	0.31	0.14	0.03	-0.13	-0.45	0.76
t-stat		Small	(2.0)	(2.7)	(1.2)	(-0.9)	(-3.0)	(3.8)
		2	(2.5)	(1.1)	(1.0)	(-1.1)	(-4.7)	(4.9)
		3	(3.0)	(1.3)	(-0.1)	(-1.2)	(-4.5)	(4.9)
		4	(2.3)	(2.0)	(0.4)	(-2.1)	(-4.2)	(4.5)
		Large	(3.8)	(1.2)	(-1.1)	(-3.1)	(-4.1)	(5.0)
		Avg	(3.4)	(2.8)	(0.7)	(-2.4)	(-5.1)	(5.4)
$\bar{R}^*_{FF3,p}$	VW	Small	0.21	0.26	0.09	-0.13	-0.37	0.59
		2	0.32	0.04	0.07	-0.10	-0.54	0.87
		3	0.37	0.12	-0.03	-0.10	-0.50	0.87
		4	0.25	0.17	0.01	-0.18	-0.44	0.68
		Large	0.38	0.18	0.06	-0.15	-0.30	0.68
		Avg	0.31	0.15	0.04	-0.13	-0.43	0.74
t-stat		Small	(1.4)	(2.5)	(0.9)	(-1.3)	(-3.0)	(3.1)
		2	(2.4)	(0.5)	(0.8)	(-1.1)	(-4.7)	(4.7)
		3	(3.0)	(1.4)	(-0.4)	(-1.4)	(-4.6)	(5.0)
		4	(2.3)	(2.2)	(0.2)	(-2.2)	(-4.0)	(4.3)
		Large	(4.2)	(2.8)	(1.1)	(-2.4)	(-3.2)	(4.4)
		Avg	(3.4)	(3.2)	(0.8)	(-2.5)	(-4.9)	(5.2)

Note: For the portfolios sorted on tail beta (the first row) or the spread between the tail beta and the market beta (from the second row onwards), we calculate average excess returns and average risk-adjusted returns conditional on usual market conditions. From July 1968 and Dec 2010, we consider the returns from the 456 months in which the market factor lost at most 5% of its value, i.e., $R^e_{m,t} \geq -5\%$.
The first row and second row report the average excess portfolio return, $\bar{R}^e_p$. The third and fourth row, report the CAPM- and FF3-adjusted portfolio return, $\bar{R}^*_{CAPM,p}$ and $\bar{R}^*_{FF3,p}$. The fifth and sixth row report the average FF3-adjusted returns after presorting the equities in five size cohorts based on market capitalization at the end of month $t-1$ and then sorting on the tail beta spread within each size cohort. Standard t-statistics are reported in parentheses.

Table 2.6: Summary of factors in the robustness checks

Factors	Start	End	Obs	Average return	St.dev.	Source
Market	196307	201012	570	0.45	4.53	K. French
SMB	196307	201012	570	0.27	3.17	K. French
HML	196307	201012	570	0.40	2.94	K. French
Momentum	196307	201012	570	0.72	4.35	K. French
Short-term Reversal	196307	201012	570	0.54	3.16	K. French
Long-term Reversal	196307	201012	570	0.33	2.54	K. French
Liquidity	196801	201012	516	0.49	3.55	R. Stambaugh
Downside beta	196807	201012	510	0.03	3.48	Calculations
Spread ($\beta^{DS} - \beta$)	196807	201012	510	0.10	1.60	Calculations
Coskewness	196807	201012	510	-0.21	1.69	Calculations
Cokurtosis	196807	201012	510	0.02	1.79	Calculations

Note: For each risk factor in the robustness checks we provide some summary statistics. First we report the start and the end date of its availability, and the number of observations. Then we report the average excess return and its standard deviation. The last column reports the source of the risk factor. 'K. French' and 'R. Stambaugh' refer to the personal homepages of Kenneth French and Robert Stambaugh. 'Calculations' refer to the following procedure.
At the start of each month t between July 1968 and Dec 2010, we estimate the relevant risk measure for all NYSE, AMEX and NASDAQ stocks based on monthly returns from the past 60 months prior to t. Downside beta is estimated by a regression conditional on below average market returns. Coskewness and Cokurtosis are calculated following (2.10) and (2.11). We exclude stocks that have more than 60% zero daily returns in the 60 months prior to t and stocks with a price below 5 USD at the end of month $t-1$. Stocks are sorted based on the relevant risk measure. For downside beta (spread) and cokurtosis, the return on the risk factor in month t is given by calculating the value weighted return of stocks with estimates above the 70th percentile and subtract the value weighted return of stocks below the 30th percentile in month t. For coskewness, we apply the same percentile rule, but subtract the return on the portfolio with high coskewness from the return on the portfolio with low coskewness.

In the asset pricing literature several alternative characteristics are used to document potential nonlinearities in the relation between the asset and market return, such as downside beta, coskewness and cokurtosis. First, we extend the FF3 benchmark model with factors based on those nonlinearities. We construct corresponding risk factors using the following methodology. In correspondence with our estimation window of market beta and tail beta, all return characteristics necessary to construct the risk factors are calculated from returns during the past 60 months. Downside beta, β_j^{DS}, is estimated by performing an OLS regression conditional on the months in which the excess market return is below its average across the estimation period. In accordance with Ang *et al.* (2006a), we also calculate the downside beta spread, defined as $\beta_j^{DS} - \beta_j$. Following Harvey and Siddique (2000), we calculate coskewness as

$$\beta_j^{SKD} = \frac{E[\epsilon_j (R_m^e - \bar{R}_m^e)^2]}{\sqrt{E[\epsilon_j^2]} E[(R_m^e - \bar{R}_m^e)^2]}, \tag{2.10}$$

where $\epsilon_j = R_j^e - \alpha_j - \beta_j R_m^e$, the residual from a regression of the excess asset return on the excess market return. Following Dittmar (2002), we also control for cokurtosis, which we calculate as

$$\beta_j^{KUD} = \frac{E[\epsilon_j(R_m^e - \bar{R}_m^e)^3]}{\sqrt{E[\epsilon_j^2]}(E[(R_m^e - \bar{R}_m^e)^2])^{3/2}}. \tag{2.11}$$

For each characteristic we construct a risk factor by the difference between the value weighted return of stocks with estimates above the 70th percentile and the value weighted returns of stocks below the 30th percentile.[12]

After adding factors on downside beta or the downside beta spread to the FF3 benchmark model in Table 2.7, the results do not change much. During the extremely adverse months, the additional loss of the tail beta spread portfolio remains at a level of about −5%. The significance of the result decreases somewhat after controlling for downside beta spread as the t-statistic increases from −8.0 to −7.3. The robustness of the results is in line with the descriptive statistics. Within the sorts on the (spread of) tail beta, the average regular market beta is very similar to the average downside beta. Consequently, the risk-adjusted returns should not change much after adding downside beta factors.

We also extend the FF3 benchmark model with coskewness and cokurtosis factors. From the descriptive statistics the portfolios with high tail beta spreads have, on average, more negative estimates for coskewness. Adding the coskewness factor may thus potentially weaken our results. However, adding coskewness and cokurtosis does not alter our results much. If both factors are added, the additional loss of the tail beta spread portfolio is estimated at −5.34% with a t-statistic of −7.6 during extremely adverse months, while the premium during usual market conditions is 0.77% with a t-statistic of 4.0.

We further consider several factors related to time dynamics in stock returns. Jegadeesh and Titman (1993) report a persistence in the returns based on the performance during the past 3 to 12 months. Based on this result, Carhart (1997) extends the FF3 model by including a momentum factor. Further, De Bondt and Thaler (1985) find a long term reversal in stock returns based on the performance over the past 3 to 5 years, while Jegadeesh (1990) and Lehmann (1990) report a short-term reversal based on the performance over respectively the last month and last week.

To test whether these findings explain part of our results, we add the momentum factor to the benchmark model in Table 2.8. The additional loss during extremely adverse months on the high spread minus low spread portfolio cannot be explained by momentum. The magnitude of the loss during adverse market conditions hardly changes and remains significant with a t-statistic of −7.7. Interestingly, if the momentum factor is added, the premium during the usual months decreases from 0.74% to 0.44%. However, the premium

[12] Following Harvey and Siddique (2000), we construct the zero investment portfolio for coskewness from a *short* position in the stocks with coskewness estimates above the 70th percentile and a *long* position in the stocks with coskewness estimates below the 30th percentile.

Table 2.7: Risk factors capturing the nonlinear relation with the market

Factors	$R^e_{m,t}$	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
FF3	All	0.17	0.18	0.06	0.02	0.06	0.10
+ Downside beta		(1.2)	(2.2)	(1.0)	(0.3)	(0.5)	(0.5)
	Adverse	-2.76	-1.06	-0.14	0.99	2.95	-5.71
		(-6.1)	(-3.1)	(-0.6)	(3.7)	(6.5)	(-7.5)
	Usual	0.50	0.32	0.08	-0.09	-0.27	0.77
		(3.9)	(4.1)	(1.4)	(-1.6)	(-2.8)	(4.3)
FF3	All	0.16	0.24	0.10	0.02	-0.02	0.18
+ Spread ($\beta^{DS} - \beta$)		(1.1)	(3.0)	(1.6)	(0.3)	(-0.1)	(0.8)
	Adverse	-2.48	-0.75	-0.17	0.79	2.55	-5.03
		(-5.8)	(-2.1)	(-0.7)	(3.0)	(5.8)	(-7.3)
	Usual	0.46	0.35	0.13	-0.07	-0.31	0.77
		(3.5)	(4.3)	(2.3)	(-1.1)	(-3.1)	(4.1)
FF3	All	0.12	0.20	0.07	0.02	-0.02	0.14
+ Coskewness		(0.9)	(2.5)	(1.1)	(0.3)	(-0.2)	(0.6)
	Adverse	-2.72	-0.84	-0.27	0.71	2.62	-5.34
		(-6.2)	(-2.3)	(-1.1)	(2.6)	(6.1)	(-7.8)
	Usual	0.44	0.32	0.11	-0.06	-0.32	0.76
		(3.4)	(4.0)	(1.9)	(-1.0)	(-3.2)	(4.1)
FF3	All	0.14	0.23	0.10	0.03	-0.01	0.14
+ Coskewness		(0.9)	(2.8)	(1.5)	(0.5)	(-0.0)	(0.6)
+ Cokurtosis	Adverse	-2.63	-0.93	-0.29	0.73	2.71	-5.34
		(-5.7)	(-2.7)	(-1.1)	(2.8)	(6.4)	(-7.6)
	Usual	0.45	0.36	0.14	-0.05	-0.32	0.77
		(3.4)	(4.4)	(2.4)	(-0.8)	(-3.0)	(4.0)
FF3	All	0.15	0.27	0.10	0.05	0.03	0.12
+ Spread ($\beta^{DS} - \beta$)		(1.0)	(3.2)	(1.5)	(0.8)	(0.2)	(0.5)
+ Coskewness	Adverse	-2.45	-0.88	-0.25	0.78	2.68	-5.13
+ Cokurtosis		(-5.2)	(-2.5)	(-1.0)	(3.1)	(5.6)	(-6.7)
	Usual	0.45	0.40	0.14	-0.03	-0.27	0.72
		(3.4)	(4.8)	(2.3)	(-0.5)	(-2.7)	(3.8)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (2.8) on past daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We form 5 value weighted (VW) portfolios by sorting on the spread between tail beta and market beta and construct a zero-investment portfolio. We calculate risk-adjusted returns by applying (2.9) on the monthly stock return at time t, where the loadings on the risk factors in the benchmark model are estimated for each stock by an OLS regression on monthly returns from the 60 months prior to t. Three lines of results are reported for each robustness check. First, the average risk-adjusted return across all months; second, the average risk-adjusted return across months with an excess market return below -5%; third, the average risk-adjusted return across the months with market returns above -5%.
The first row reports results after adding the downside beta factor to the FF3 benchmark model. The second row reports the results after adding the factor based on the spread between downside beta and the regular market beta to the FF3 benchmark model. In the third row we include the coskewness factor in the benchmark model. The fourth row reports the results after including the coskewness factor and cokurtosis factor to the FF3 benchmark model. The last row reports the results after adding the coskewness, cokurtosis and the downside beta spread factors to the FF3 benchmark model. The numbers in parentheses are Newey-West corrected t-statistics for the average returns across all months, and standard t-statistics for conditional averages.

remains significant with a t-statistic of 2.6. The decrease in the premium received during usual market days reduces the overall premium on the zero investment portfolio from 0.14 to -0.13 (both insignificant). Adding factors for long-term and short-term reversal does not change this picture much.

Pastor and Stambaugh (2003) cite anecdotical evidence on the withdrawal of liquidity around market crashes. They observe the sharpest troughs in their liquidity measure during months with significant financial and economic events, such as the 1987 crash and the 1998 collapse of LTCM. To check whether the sensitivity to the aggregate liquidity factor can explain part of our results, we add the liquidity factor from Pastor and Stambaugh (2003) to the FF3 benchmark model. The results remain practically unchanged after this addition.

2.5.2 Stock characteristics

To provide further evidence that our results are not due to stock characteristics previously documented in the literature, we provide results after presorting on several characteristics. That is, at $t-1$ we first presort firms based on a certain characteristic, and then we sort on tail beta within each cohort. We report the results of this procedure with size as presorting variable in Table 2.3. In this subsection we focus on other characteristics. In Table 2.9 we report the results after averaging within each tail beta quintile across the different cohorts based on the presorting characteristic.

Table 2.9 shows that the results are qualitatively not affected by presorting on downside beta, coskewness and cokurtosis. To capture short-term reversal, momentum and long-term reversal, we presort stocks on the return accumulated over the past month, the past 2-12 months and the past 13-60 months. The baseline results do not change much after presorting on those characteristics.

Gervais *et al.* (2001) and Kaniel *et al.* (2012) document a high-volume premium. In Table 2.1 we observe that firms with a high spread between tail beta and market beta have a trading volume that is on average twice as low as firms with a low spread. To test whether our findings are related to trading volume, we presort the firms on trading volume. Also after controlling for trading volume the baseline results are unaffected.

Idiosyncratic volatility

We provide results on idiosyncratic volatility in more detail, because we observe a pattern after presorting on this characteristic. Following Ang *et al.* (2006b, 2009) and Fu (2009) we concentrate on idiosyncratic volatility relative to the FF3 benchmark model, which is measured by the standard deviation of the residuals obtained from regressing the indi-

Table 2.8: Robustness for other risk factors

Factors	$R^e_{m,t}$	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
FF3	All	0.00	0.15	0.11	0.07	0.12	-0.13
+ Momentum		(-0.0)	(2.3)	(2.0)	(1.3)	(1.0)	(-0.6)
	Adverse	-2.48	-0.57	-0.09	0.81	2.40	-4.88
		(-6.1)	(-2.0)	(-0.5)	(3.4)	(6.0)	(-7.7)
	Usual	0.29	0.24	0.14	-0.02	-0.15	0.44
		(2.4)	(3.4)	(2.6)	(-0.4)	(-1.7)	(2.6)
FF3	All	0.12	0.20	0.10	0.05	0.00	0.12
+ Reversal		(1.0)	(2.8)	(1.8)	(0.9)	(0.0)	(0.6)
	Adverse	-2.12	-0.46	-0.08	0.71	2.18	-4.30
		(-4.9)	(-1.4)	(-0.4)	(3.0)	(5.5)	(-7.0)
	Usual	0.38	0.28	0.12	-0.03	-0.26	0.64
		(3.2)	(3.7)	(2.4)	(-0.6)	(-2.8)	(3.7)
FF3	All	-0.01	0.18	0.13	0.11	0.15	-0.16
+ Momentum		(-0.1)	(2.7)	(2.3)	(2.0)	(1.2)	(-0.8)
+ Reversal	Adverse	-2.20	-0.39	-0.01	0.84	2.30	-4.50
		(-5.0)	(-1.1)	(-0.1)	(3.5)	(5.4)	(-7.2)
	Usual	0.25	0.25	0.14	0.02	-0.11	0.35
		(2.0)	(3.4)	(2.8)	(0.4)	(-1.1)	(2.0)
FF3	All	0.18	0.21	0.09	0.00	-0.08	0.26
+ Liquidity		(1.5)	(2.8)	(1.5)	(-0.0)	(-0.7)	(1.3)
	Adverse	-2.27	-0.66	-0.16	0.78	2.30	-4.57
		(-5.0)	(-2.0)	(-0.7)	(3.1)	(5.4)	(-6.8)
	Usual	0.47	0.31	0.12	-0.09	-0.35	0.81
		(3.6)	(4.0)	(2.2)	(-1.6)	(-3.6)	(4.5)
FF3	All	0.14	0.26	0.15	0.09	0.12	0.02
+ Momentum		(1.2)	(3.8)	(2.5)	(1.5)	(0.9)	(0.1)
+ Reversal	Adverse	-1.81	-0.29	0.09	0.86	2.26	-4.07
+ Liquidity		(-3.8)	(-0.8)	(0.4)	(3.2)	(4.7)	(-6.2)
	Usual	0.36	0.32	0.16	0.00	-0.13	0.49
		(2.9)	(4.1)	(2.7)	(0.0)	(-1.3)	(2.7)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (2.8) on past daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We form 5 value weighted portfolios by sorting on the spread between tail beta and market beta and construct a zero-investment portfolio. We calculate risk-adjusted returns by applying (2.9) on the monthly stock return at time t, where the loadings on the risk factors in the benchmark model are estimated for each stock by an OLS regression on monthly returns from the 60 months prior to t. Three lines of results are reported for each robustness check. First, the average risk-adjusted return across all months; second, the average risk-adjusted return across months with an excess market return below -5%; third, the average risk-adjusted return across the months with market returns above -5%.
The first row reports results after adding the momentum factor to the FF3 benchmark model. The second row reports the results after adding the long-term en short-term reversal factor to the FF3 benchmark model. In the third row both reversal factors and the momentum factor are included in the benchmark model. The fourth row reports the results after including the liquidity factor to the FF3 benchmark model. The last row reports the results after including all these factors to the FF3 benchmark model. The numbers in parentheses are Newey-West corrected t-statistics for the average returns across all months, and standard t-statistics for conditional averages.

Table 2.9: Results after presorting

Presorting	$R^e_{m,t}$	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
Spread ($\beta^{DS} - \beta$)	All	0.13	0.17	0.08	-0.06	-0.06	0.19
		(1.1)	(2.4)	(1.4)	(-1.1)	(-0.5)	(1.0)
	Adverse	-2.24	-0.65	0.03	0.59	2.47	-4.71
		(-6.1)	(-2.1)	(0.2)	(2.8)	(6.3)	(-7.2)
	Usual	0.42	0.27	0.08	-0.13	-0.36	0.77
		(3.8)	(3.9)	(1.5)	(-2.5)	(-4.2)	(4.8)
Coskewness	All	0.10	0.16	0.11	-0.01	-0.05	0.14
		(0.8)	(2.3)	(2.4)	(-0.2)	(-0.4)	(0.7)
	Adverse	-2.16	-0.66	-0.14	0.64	2.49	-4.65
		(-5.6)	(-2.4)	(-0.6)	(3.0)	(7.4)	(-7.6)
	Usual	0.36	0.25	0.14	-0.09	-0.35	0.71
		(3.3)	(3.7)	(2.6)	(-1.7)	(-4.1)	(4.4)
Cokurtosis	All	0.13	0.15	0.11	-0.02	-0.06	0.19
		(1.1)	(2.2)	(2.2)	(-0.3)	(-0.6)	(1.0)
	Adverse	-2.40	-0.63	-0.09	0.66	2.47	-4.87
		(-5.8)	(-2.4)	(-0.5)	(3.0)	(7.1)	(-7.7)
	Usual	0.43	0.24	0.13	-0.10	-0.36	0.79
		(3.8)	(3.6)	(2.6)	(-1.9)	(-4.4)	(4.8)
Past 1 month	All	0.14	0.10	0.05	0.05	0.03	0.11
performance		(1.2)	(1.5)	(0.8)	(0.8)	(0.3)	(0.6)
	Adverse	-2.38	-0.80	0.15	1.02	2.55	-4.94
		(-6.5)	(-2.9)	(0.7)	(3.9)	(7.8)	(-8.9)
	Usual	0.44	0.21	0.03	-0.07	-0.27	0.70
		(3.7)	(3.2)	(0.6)	(-1.3)	(-3.1)	(4.2)
Past 2-12 months	All	0.07	0.13	-0.03	-0.03	-0.09	0.15
performance		(0.6)	(2.0)	(-0.6)	(-0.5)	(-0.9)	(0.8)
	Adverse	-2.53	-0.55	-0.32	0.79	2.60	-5.12
		(-6.7)	(-2.3)	(-1.4)	(4.1)	(7.8)	(-8.7)
	Usual	0.37	0.21	0.00	-0.13	-0.41	0.78
		(3.4)	(3.1)	(0.0)	(-2.3)	(-4.8)	(4.9)
Past 13-60 months	All	0.10	0.19	0.07	0.02	0.01	0.08
performance		(0.9)	(2.8)	(1.2)	(0.4)	(0.1)	(0.4)
	Adverse	-2.44	-0.62	0.22	0.91	2.79	-5.23
		(-6.9)	(-2.4)	(1.0)	(4.3)	(9.5)	(-10.1)
	Usual	0.40	0.28	0.05	-0.08	-0.31	0.71
		(3.5)	(4.1)	(0.9)	(-1.4)	(-3.5)	(4.4)
Volume	All	0.09	0.11	0.03	0.00	-0.09	0.18
		(0.9)	(1.9)	(0.7)	(-0.1)	(-1.0)	(1.1)
	Adverse	-2.20	-0.77	0.04	0.74	2.43	-4.63
		(-7.1)	(-3.7)	(0.2)	(3.6)	(8.2)	(-8.6)
	Usual	0.36	0.22	0.03	-0.09	-0.39	0.75
		(4.0)	(3.5)	(0.7)	(-1.8)	(-5.3)	(5.7)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (2.8) on daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We first presort the equities into five cohorts according to the stock characteristic specified in the first column. Within each cohort we form 5 value weighted portfolios by sorting on the spread between tail beta and market beta and construct a zero-investment portfolio. We calculate FF3-adjusted portfolio returns at time t by applying (2.9) on monthly stock returns, where the loadings on the risk factors are estimated by an OLS regression on monthly returns from the 60 months prior to t. The reported numbers are the risk-adjusted returns averaged within each tail beta quintile across the different cohorts based on the presorting characteristic.
In the first three rows, we report results after presorting on downside beta spread, coskewness and cokurtosis. The fourth, fifth, sixth and seventh row report results after presorting on the past 1 month return, the past 2-12 months return, the past 13-60 months return, and on trading volume in month $t-1$. The numbers in parentheses are Newey-West corrected t-statistics for the average returns across all months, and standard t-statistics for conditional averages.

vidual stock returns on the FF3 factors. Table 2.1 reports a U-shaped relation between idiosyncratic volatility and the size of the spread between tail beta and market beta. The observed U-shape is intuitive. Both, a relatively high (positive) and a relatively low (negative) spread between tail beta and market beta, indicate a deviation from a linear relation with the market. If idiosyncratic risk is measured by the standard deviation of the residuals obtained from a linear regression, then a larger deviation from the linear model will induce a higher perceived level of idiosyncratic risk. Therefore, everything else being equal, we expect to observe a higher level of idiosyncratic volatility in case of a larger deviation of the tail beta from the market beta. According to this intuition, the level of idiosyncratic volatility may produce a signal of the absolute spread between tail beta and market beta without revealing the sign of the spread. Therefore, in the higher idiosyncratic volatility cohort we expect larger differences between the high and low tail beta firms. This should be reflected by both higher losses on the zero investment portfolio during extremely adverse market conditions and a larger premium during usual market conditions. If this is the case, we would expect that the magnitude of our findings increases with respect to the level of idiosyncratic volatility.

To test this conjecture we provide detailed results of presorting firms on idiosyncratic volatility in Table 2.10. The results confirm our expectation. The difference in losses between high and low tail beta spread stocks during adverse market conditions for firms in the lowest idiosyncratic volatility quintile is 2.95%, while the difference for stocks in the highest idiosyncratic volatility quintile is 9.21%. It is further notable that the difference increases monotonically and is very significant in all idiosyncratic volatility quintiles. The same relation is observed in the premium during usual market days, although the relation is not entirely monotonic. The difference in return between high and low tail beta spread stocks during usual months for firms in the lowest idiosyncratic volatility quintile is 0.73%, while the difference for stocks in the highest idiosyncratic volatility quintile is 1.63%. To summarize, the results confirm that idiosyncratic volatility provides a signal of the magnitude of the spread between tail beta and market beta without revealing its sign.

2.5.3 Methodological changes

Table 2.11 provides the results from robustness checks for several methodological deviations. One potential reason why we do not find a positive premium on systematic tail risk is the market turmoil from 2007 until 2010. The recent crisis may have erased the potential positive premium on systematic tail risk. To test this hypothesis we repeat the tests on a subsample until 2007. The average value weighted FF3-adjusted return on

Table 2.10: Presorting on idiosyncratic volatility

$R^e_{m,t}$		Idiosyncratic Volatility	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
All	$\bar{R}^*_{FF3,p}$	Low	0.21	0.06	0.08	0.01	-0.13	0.34
		2	0.29	0.16	0.11	-0.19	-0.08	0.36
		3	0.18	0.13	0.17	-0.10	0.06	0.12
		4	0.16	-0.18	0.21	0.09	0.02	0.14
		High	0.01	-0.17	-0.12	-0.31	-0.48	0.48
		Avg	0.17	0.00	0.09	-0.10	-0.12	0.29
	t-stat	Low	(2.4)	(0.7)	(1.1)	(0.2)	(-1.4)	(2.4)
		2	(2.2)	(1.5)	(1.0)	(-1.8)	(-0.6)	(1.8)
		3	(1.0)	(0.9)	(1.2)	(-0.8)	(0.3)	(0.5)
		4	(0.8)	(-1.1)	(1.5)	(0.6)	(0.1)	(0.4)
		High	(0.0)	(-0.9)	(-0.6)	(-1.5)	(-1.9)	(1.2)
		Avg	(1.4)	(-0.0)	(1.3)	(-1.2)	(-0.9)	(1.4)
Adverse	$\bar{R}^*_{FF3,p}$	Low	-1.17	-0.01	0.03	0.43	1.78	-2.95
		2	-1.49	-0.07	0.24	0.22	2.08	-3.57
		3	-1.81	-0.65	0.56	1.71	2.92	-4.73
		4	-2.05	-1.36	0.68	1.50	4.06	-6.11
		High	-4.16	-1.32	0.41	1.90	5.05	-9.21
		Avg	-2.14	-0.68	0.39	1.15	3.18	-5.31
	t-stat	Low	(-3.0)	(-0.0)	(0.1)	(1.5)	(5.0)	(-6.0)
		2	(-3.7)	(-0.2)	(0.6)	(0.7)	(4.0)	(-4.9)
		3	(-3.4)	(-1.2)	(1.3)	(3.2)	(4.8)	(-5.9)
		4	(-3.1)	(-2.1)	(1.3)	(2.2)	(5.9)	(-5.9)
		High	(-4.3)	(-1.7)	(0.7)	(2.4)	(5.6)	(-6.6)
		Avg	(-4.9)	(-2.0)	(1.5)	(3.1)	(7.2)	(-7.4)
Usual	$\bar{R}^*_{FF3,p}$	Low	0.38	0.06	0.08	-0.04	-0.35	0.73
		2	0.50	0.18	0.09	-0.24	-0.34	0.83
		3	0.41	0.23	0.12	-0.32	-0.28	0.70
		4	0.42	-0.04	0.16	-0.07	-0.46	0.88
		High	0.50	-0.04	-0.18	-0.57	-1.13	1.63
		Avg	0.44	0.08	0.05	-0.25	-0.51	0.95
	t-stat	Small	(4.3)	(0.8)	(1.1)	(-0.4)	(-3.8)	(5.2)
		2	(4.1)	(1.9)	(1.0)	(-2.3)	(-2.7)	(4.3)
		3	(2.4)	(1.6)	(0.9)	(-2.2)	(-2.0)	(3.0)
		4	(2.1)	(-0.3)	(1.0)	(-0.5)	(-2.8)	(3.2)
		Large	(1.9)	(-0.2)	(-1.0)	(-2.9)	(-4.5)	(4.4)
		Avg	(4.0)	(1.0)	(0.8)	(-3.0)	(-5.0)	(5.5)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (2.8) on daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We first presort the equities into five cohorts based on the level of idiosyncratic volatility. Idiosyncratic volatility is calculated as the standard deviation of the residuals obtained from regressing monthly stock returns from the 60 months prior to t on the FF3 factors. Within each idiosyncratic volatility cohort we form 5 value weighted portfolios by sorting on the spread between tail beta and market beta and construct a zero-investment portfolio. We calculate FF3-adjusted portfolio returns at time t by applying (2.9) on monthly stock returns, where the loadings on the risk factors are estimated by an OLS regression on monthly returns from the 60 months prior to t. The table reports the average risk-adjusted returns of each portfolio. The numbers in parentheses are Newey-West corrected t-statistics for the averages across all months, and standard t-statistics for conditional averages.

the zero investment portfolio increases from 0.14% to 0.19%, but remains insignificantly different from zero with a t-stat of 1.0. Further, the positive premium on systematic tail risk during usual market days and the additional loss during extremely adverse periods remain strongly significant.

A structural break in the CRSP data is the entrance of NASDAQ firms in January 1973. After collecting five years of return data to estimate tail betas, the first NASDAQ firms enter the constructed portfolios in January 1978. To test whether the results are robust for this structural break, we repeat the asset pricing tests on portfolio returns from January 1978 onwards. The results are robust for restricting the sample horizon. Alternatively, we also repeat the asset pricing tests while restricting the sample to NYSE firms only. Although the loss during adverse months for the high minus low tail beta spread decreases from 4.90% to 4.48%, its t-statistic increases from −8.0 to −8.7. These results suggest that our findings are not sample specific.

The next robustness check is on the choice in the definition of extremely adverse market conditions. In the baseline result, we define months with an excess market return below −5% as 'extremely adverse market conditions' and those with a return above −5% as 'usual market conditions'. We test whether our results are robust for this specific choice. In the table we repeat our procedure, but include only the 20 worst months in the sample of extremely adverse months. Our results are robust for this alternative definition. The risk-adjusted loss on the zero investment portfolio increases from −4.9% to −6.5% during these more adverse months. The inclusion of potentially adverse months in the sample with usual months also shrinks the premium received during the other months, both in terms of magnitude and significance level. Nevertheless, the reduced premium remains significant.

The estimation of each tail beta is based on the 50 worst market days during the past 60 months. To test whether our results are robust for alternative choice, we also estimate the tail beta based on the 30 worst market days during the past 60 months, i.e., we set $k = 30$ and estimate tail beta based on about $k/n \approx 2.5\%$ of the worst market days. The results do barely change, although the t-statistic of the premium during usual market days decreases somewhat from 4.3 to 3.5.

Finally, we test whether the results are robust for using an alternative estimator for tail beta. Instead of using the EVT estimator in (2.8), we apply the conditional regression approach on daily returns. That is, we perform an OLS regression based on the 50 days with the worst daily market returns in each estimation window. The results remain practically unchanged, both in terms of magnitude and significance.

To summarize, the robustness checks suggest that our baseline results are robust for several methodological changes, and for controlling several firm characteristics that have

Table 2.11: Methodological robustness

Robustness	$R^e_{m,t}$	Obs	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
Pre-crisis	All	463	0.12	0.17	0.07	-0.01	-0.07	0.19
			(0.9)	(2.3)	(1.3)	(-0.3)	(-0.7)	(1.0)
	Adverse	44	-2.33	-0.72	-0.18	0.54	1.92	-4.25
			(-5.0)	(-2.2)	(-0.8)	(2.1)	(4.9)	(-6.5)
	Usual	419	0.38	0.26	0.10	-0.07	-0.29	0.66
			(3.0)	(3.4)	(1.9)	(-1.3)	(-3.3)	(3.8)
Post NASDAQ Break	All	397	0.14	0.17	0.12	-0.01	-0.01	0.15
			(1.0)	(2.0)	(1.9)	(-0.1)	(-0.1)	(0.6)
	Adverse	40	-2.59	-0.83	-0.07	0.79	2.93	-5.51
			(-5.4)	(-2.2)	(-0.3)	(2.6)	(6.5)	(-7.4)
	Usual	357	0.45	0.29	0.14	-0.10	-0.34	0.79
			(3.1)	(3.3)	(2.3)	(-1.5)	(-3.3)	(3.9)
NYSE stocks only	All	510	0.11	0.15	0.03	-0.03	-0.13	0.23
			(1.0)	(2.6)	(0.6)	(-0.5)	(-1.2)	(1.4)
	Adverse	54	-1.66	-0.32	0.29	0.68	2.43	-4.09
			(-4.1)	(-1.4)	(1.2)	(2.6)	(6.8)	(-7.3)
	Usual	456	0.31	0.21	0.00	-0.12	-0.43	0.75
			(3.2)	(3.7)	(0.1)	(-2.0)	(-4.7)	(4.7)
More adverse months	All	510	0.11	0.17	0.08	0.02	-0.03	0.14
			(0.9)	(2.4)	(1.4)	(0.4)	(-0.3)	(0.7)
	Adverse	20	-3.36	-0.92	-0.32	0.90	3.13	-6.49
			(-5.0)	(-1.9)	(-1.0)	(2.1)	(4.2)	(-5.4)
	Usual	490	0.25	0.22	0.10	-0.02	-0.16	0.41
			(2.1)	(3.0)	(1.8)	(-0.3)	(-1.8)	(2.3)
Lower tail threshold	All	510	0.02	0.17	0.09	-0.01	-0.02	0.04
			(0.2)	(2.5)	(1.7)	(-0.2)	(-0.2)	(0.2)
	Adverse	54	-2.28	-0.57	-0.09	0.63	2.48	-4.77
			(-5.6)	(-2.1)	(-0.4)	(3.0)	(5.3)	(-7.0)
	Usual	456	0.29	0.26	0.11	-0.09	-0.32	0.61
			(2.5)	(3.6)	(2.1)	(-1.5)	(-3.4)	(3.5)
Conditional approach	All	510	0.08	0.13	0.03	-0.13	-0.06	0.14
			(1.1)	(2.6)	(0.5)	(-1.6)	(-0.4)	(0.7)
	Adverse	54	-1.23	0.26	0.78	1.09	3.61	-4.84
			(-4.1)	(1.4)	(3.4)	(4.0)	(6.2)	(-7.3)
	Usual	456	0.24	0.11	-0.06	-0.27	-0.50	0.73
			(3.5)	(2.1)	(-0.9)	(-3.4)	(-4.0)	(4.4)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (2.8) on past daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We form 5 value weighted portfolios by sorting on the spread between tail beta and market beta and construct a zero-investment portfolio. We calculate FF3-adjusted returns by applying (2.9) on the monthly stock return at time t, where the loadings on the risk factors in the benchmark model are estimated for each stock by an OLS regression on monthly returns from the 60 months prior to t. Three lines of results are reported for each robustness check. First, the average FF3-adjusted return across all months; second, the average FF3-adjusted return across months with an excess market return below a certain threshold; third, the average FF3-adjusted return across the months with market returns above this threshold. If the threshold is not specified, then it is fixed at -5%.

The first row reports results based on the pre-crisis sample: from July 1968 to Dec 2006. The second row reports results based on the sample after the entrance of NASDAQ firms: from Jan 1978 to Dec 2010. The third row varies the threshold such that only 20 months are selected in the sample with extremely adverse market conditions (the corresponding threshold level is -8.1%). The fourth row reports the results if tail betas are estimated with $k = 30$ instead of $k = 50$ in (2.8). The fifth row reports the result based on estimating the tail beta by a conditional regression approach, based on the observations corresponding to the 50 worst daily market excess returns during the months between $t - 60$ and $t - 1$. The numbers in parentheses are Newey-West corrected t-statistics for the averages across all months, and standard t-statistics for conditional averages.

been documented in the literature to explain the cross-section of returns. Two findings are notable. First, adding the momentum factor reduces the premium that portfolios with high tail betas receive during usual periods, without reducing the additional losses that these portfolios suffer during extremely adverse months. Second, we find evidence that idiosyncratic risk provides a signal about the magnitude of the deviation between tail beta and market beta.

2.6 Risk management

Since historical tail betas are able to capture future losses under extremely adverse market conditions, the tail beta may help investors to assess the tail risk of portfolios. As an additive measure of loadings on systematic tail risks, the tail beta is a useful measure in a portfolio setup. In this section, we discuss the application of the linear tail model for portfolio tail risk management.

We consider a portfolio consisting of d assets, following the linear tail model in (2.1) with nonnegative tail betas, $\beta_1^T, \cdots, \beta_d^T$. Then, under extremely adverse market conditions, the excess return of a portfolio with non-negative weights, $w_1, \cdots, w_d$, can be written as

$$R_P^e = \left(\sum_{j=1}^{d} w_j \beta_j^T \right) R_m^e + \sum_{j=1}^{d} w_j \varepsilon_j, \quad \text{for} \quad R_m^e < -VaR_m(\bar{p}). \tag{2.12}$$

This shows that the portfolio return also follows a linear tail model. The sensitivity to systematic tail risk can be measured by the portfolio tail beta, which is a weighed average of the tail beta of the individual assets, $\beta_P^T = \sum_{j=1}^{d} w_j \beta_j^T$. The idiosyncratic component is given by $\varepsilon_P = \sum_{j=1}^{d} w_j \varepsilon_j$.

To evaluate the tail risk of a portfolio, it is necessary to investigate how to aggregate the systematic and idiosyncratic tail risks. We start by discussing the aggregation for an individual asset. Suppose the linear tail model in (2.1) and the heavy-tailed setup in (2.6) hold for a larger area, $\min(R_m^e, R_j^e) < -VaR_m(\bar{p})$, given a low probability level $\bar{p}$. Then the probability of a loss on asset j exceeding u can be approximated by

$$\Pr(R_j^e < -u) \sim \Pr(\beta_j^T R_m^e < -u) + \Pr(\varepsilon_j < -u), \quad \text{as} \quad u \to \infty. \tag{2.13}$$

This approximation follows from Feller's convolution theorem on aggregating heavy-tailed risk factors, see Feller (1971, p. 278). When aggregating independent heavy-tailed risk factors, the probability that the sum is above a high threshold can be approximated by the sum of the probabilities of each risk factor being above that threshold.[13] Due to this

[13]Embrechts *et al.* (1997, Lemma 1.3.1.), provides the proof for the case $\alpha_m = \alpha_{\varepsilon_j}$. Along the same lines of proof one can obtain that this relation holds for $\alpha_m \neq \alpha_{\varepsilon_j}$.

approximation, the excess asset return must also follow a heavy-tailed distribution. If we have $\alpha_{\varepsilon_j} > \alpha_m$, then the systematic tail risk dominates the idiosyncratic tail risk, in the sense that $\Pr(\varepsilon_j < -u) = o(\Pr(\beta_j^T R_m^e < -u))$ as $u \to \infty$. Consequently, the downside tail distribution of the excess asset return, R_j^e, follows a heavy-tailed distribution with tail index $\alpha_j = \alpha_m$ and scale $A_j = (\beta_j^T)^{\alpha_m} A_m$. In contrast, if $\alpha_{\varepsilon_j} < \alpha_m$, then idiosyncratic risk dominates the tail risk of the asset, and we have $\alpha_j = \alpha_{\varepsilon_j}$ and $A_j = A_{\varepsilon_j}$. If we have $\alpha_{\varepsilon_j} = \alpha_m$, then both of the two components contribute to the tail risk of the asset, and we have $A_j = (\beta_j^T)^{\alpha_m} A_m + A_{\varepsilon_j}$.

In the portfolio context, we first consider the case $\alpha_{\varepsilon_1} = \cdots = \alpha_{\varepsilon_d} = \alpha_m$. Suppose the assets have independent idiosyncratic tail risks with scales $A_{\varepsilon_1}, \cdots, A_{\varepsilon_d}$. Following Feller's convolution theorem, the downside tail of the portfolio follows a heavy-tailed distribution with tail index $\alpha_P = \alpha_m$ and scale

$$A_P = (\beta_P^T)^{\alpha_m} A_m + \sum_{j=1}^{d} w_j^{\alpha_m} A_{\varepsilon_j}. \tag{2.14}$$

All unknown parameters in equation (2.14) can be statistically estimated. Our EVT approach in equation (2.8) provides an estimate for β_j^T. Then the tail beta of the portfolio, β_P^T, can be obtained by taking a weighed average. The scale of the idiosyncratic tail risk, A_{ε_j}, can be obtained from

$$\hat{A}_{\varepsilon_j} = \hat{A}_j - (\hat{\beta}_j^T)^{\hat{\alpha}_m} \hat{A}_m, \tag{2.15}$$

where the scales of the market return and the asset return, A_m and A_j, can be estimated by univariate EVT analysis, see Appendix B. With equation (2.14), we thus obtain the estimate of the scale of a portfolio. Then the VaR of the portfolio for a low probability level p can be calculated from the approximation

$$VaR_P(p) \approx \left(\frac{A_P}{p}\right)^{1/\alpha_m}. \tag{2.16}$$

Next, consider the case in which some assets in the portfolio correspond to $\alpha_{\varepsilon_j} > \alpha_m$. The idiosyncratic tail risk of those assets will be dominated by their systematic tail risk and will not contribute to the tail risk of the portfolio. In this case, it is still possible to evaluate the scale of the portfolio in equation (2.14), by omitting the idiosyncratic tail risks of those assets. However, it is not necessary to identify those assets or to modify the estimation procedure from equations (2.14) and (2.15). Assets with $\alpha_{\varepsilon_j} > \alpha_m$ exhibit complete tail dependence with the market return, i.e., $\tau_j = 1$ and $A_j = (\beta_j^T)^{\alpha_m} A_m$. The estimator on A_{ε_j} in (2.15) converges to zero under the EVT approach. Thus including the estimate of A_{ε_j} for such assets will not contaminate the estimate of the portfolio scale. Hence, equation (2.14) can be applied for any portfolio consisting of assets with $\alpha_{\varepsilon_j} \geq \alpha_m$.

Finally, we discuss the case in which some assets correspond to $\alpha_{\varepsilon_j} < \alpha_m$. Theoretically, the downside tail risk of the portfolio would be dominated by the idiosyncratic risk of the asset with the lowest tail index. However, in practice this may not be the case. The reason is that the return on many assets is in fact bounded from below by -100%.[14] Such an asset j with investment weight w_j can generate a maximum loss of w_j. Therefore, in a well-diversified portfolio with a sufficiently large number of assets, the idiosyncratic tail risks do not contribute to the tail risk of the portfolio under the condition that their returns have a lower bound. This is achieved even if some assets correspond to the case $\alpha_{\varepsilon_j} < \alpha_m$.[15] In contrast to the idiosyncratic risks, the systematic tail risk cannot be diversified away by investing in a large number of assets, because the tail beta of a portfolio is the weighted average of those of the individual assets. Hence, for any well-diversified portfolio consisting of a sufficiently large number of assets with lower bounded returns, the scale of its downside tail distribution can be approximated by

$$A_P = \left(\sum_{j=1}^{d} w_j \beta_j^T \right)^{\alpha_m} A_m.$$

Subsequently, the VaR can be calculated from equation (2.16).

2.7 Concluding remarks

This chapter proposes a measure of sensitivity to systematic tail risk. In particular, we focus on the estimation methodology and on whether systematic tail risk is compensated in the cross-section of expected returns. Asset pricing theory based on an equilibrium framework with safety-first investors suggests that higher loadings on systematic tail risk should be associated with a positive risk premium. We show theoretically that the risk premium is proportional to the tail beta, which measures the sensitivity of an asset to systematic tail risk. Based on EVT, we establish a methodology to estimate the tail beta. We test empirically whether assets with high tail betas received an additional risk premium.

We find that assets with higher tail betas are associated with significantly larger losses during future extreme market downturns. Hence, historical tail betas are able to capture the sensitivity to systematic tail risk. When considering the entire sample, the large losses under extremely adverse market conditions are offset by the premium received under usual

[14] Examples of assets of which the returns have a lower bound are long positions in stocks and bonds. Counterexamples are short positions in currencies and stocks.

[15] The lower bound of equity returns is not accounted for in the heavy tail approximation, as in (2.6). Instead, one could consider truncated heavy-tailed distributions. Ibragimov and Walden (2007) prove the diversification effects of bounded risk factors from truncated heavy-tailed distributions provided that the number of risk factors is sufficiently large.

market conditions. The asset pricing tests do not report a significant premium or loss for high tail beta stocks over the entire historical sample. There are several possible explanations for these results. A pessimistic view is that systematic tail risk does not play an additional role in explaining the cross-section of expected returns. An alternative view is that systematic tail risk is priced in the cross-section of expected returns, but that time-variation in real tail betas causes a measurement issue. For example, when sorting on historical estimates, time-varying tail betas could weaken the observed premium. Nevertheless, since our historical estimates are sufficient to differentiate future losses under extremely adverse market conditions, this alternative explanation is satisfactory only if the risk premium for loading on systematic tail risk is rather low. Hence, our results suggest that the room for an additional role of systematic tail risk in explaining the cross-section of expected returns is limited.

We also discuss how the tail beta could be applied in risk management. As an additive measure of the sensitivity of asset returns to market risk during extremely adverse market shocks, the interpretation is rather appealing. Similar to the regular market beta, one can obtain the tail beta of a portfolio by taking the weighted average over the tail betas of individual assets. We demonstrate how tail betas help to evaluate traditional measures of portfolio tail risk, such as VaR.

2.A Appendix A. Proofs

Proof of Proposition 2.2.1

We start by introducing the notation from Arzac and Bawa (1977). Denote the initial and future market value of asset j by V_j and X_j. Then each asset j generates a return $R_j = X_j/V_j$. The value weighted market portfolio has the initial and future value $V_m = \sum_j V_j$ and $X_m = \sum_j X_j$, and therefore the market return is defined as $R_m = \frac{\sum_j X_j}{\sum_j V_j} = \sum_j w_j^* R_j$, with weights $w_j^* = V_j/(\sum_j V_j)$.

Investor i holds a portfolio with fractions of the risky assets $(\gamma_{i,1}, \gamma_{i,2}, \cdots)$ which generates a future value as $\sum_j \gamma_{i,j} X_j = \sum_j \gamma_{i,j} V_j R_j$. Denote the p-quantile of the future value of investor i and the market portfolio as Q^i and Q_m respectively. Then the p-quantile of the market return is $q_m = Q_m/V_m$.

With this notation, Arzac and Bawa (1977, equation 14) give the formula to calculate the parameter β_j^{AB} as

$$\beta_j^{AB} = \frac{q_j - r_f}{q_m - r_f}.$$

Here q_j is given by

$$q_j := \frac{\frac{\partial Q^i}{\partial \gamma_{i,j}} \mid_{(\gamma_{i,j})=(\gamma_i)}}{V_j},$$

where (γ_i) is the optimal portfolio holding for investor i on all assets. The right hand side is the same across all investors.

Because $q_m - r_f$ is the $p-$quantile of the market excess return, we get that $q_m - r_f = -VaR_m(p)$. Therefore, to prove the proposition, it is only necessary to prove that $q_j = E(R_j | R_m = Q_m(p))$, where $Q_m(p) = q_m$ is the $p-$quantile of the return of the market portfolio.

In order to relate the quantile of the future value of investors' portfolio to that of the market return, we define for any positive investments $(u_1, u_2, \cdots)$ the $p-$quantile of $\sum_j u_j R_j$ as $f(u_1, u_2, \cdots)$. Notice that $Q_m = f(V_1, V_2, \cdots)$, $Q^i = f(\gamma_{i,1}V_1, \gamma_{i,2}V_2, \cdots)$. We calculate q_j as follows,

$$q_j = \frac{\frac{\partial f(\gamma_{i,1}V_1, \gamma_{i,2}V_2, \cdots)}{\partial \gamma_{i,j}} \mid_{(\gamma_{i,j})=(\gamma_i)}}{V_j} = \frac{V_j \frac{\partial f}{\partial u_j} \mid_{(u_j)=(\gamma_i V_j)}}{V_j} = \frac{\partial f}{\partial u_j} \mid_{(u_j)=(\gamma_i V_j)} .$$

The function f is homogeneous with degree one, which implies that its partial derivative $\frac{\partial f}{\partial u_j}$ is a homogeneous function with degree zero. Consequently, we have

$$\frac{\partial f}{\partial u_j} \mid_{(u_j)=(\gamma_i V_j)} = \frac{\partial f}{\partial u_j} \mid_{(u_j)=(w_j^*)} .$$

To derive the partial derivative of the f function, we use the expression that

$$f(u_1, u_2, \cdots) = E(\sum_j u_j R_j | \sum_j u_j R_j = f(u_1, u_2, \cdots,))$$

Thus,

$$\begin{aligned}
&\frac{\partial f}{\partial u_j} |_{(u_j)=(w_j^*)} \\
=&\frac{\partial}{\partial u_j} E(\sum_j u_j R_j | \sum_j u_j R_j = f(u_1, u_2, \cdots,)) |_{(u_j)=(w_j^*)} \\
=&\frac{\partial}{\partial u_j} \sum_j u_j E(R_j | \sum_j u_j R_j = f(u_1, u_2, \cdots,)) |_{(u_j)=(w_j^*)} \\
=&E(R_j | \sum_j u_j R_j = f(u_1, u_2, \cdots,)) |_{(u_j)=(w_j^*)} \\
=&E(R_j | R_m = Q_m(p)).
\end{aligned}$$

The last equality comes from the fact that $R_m = \sum_j w_j^* R_j$ and $Q_m(p) = f(w_1^*, w_2^*, \cdots)$. The q_j thus quantifies the contribution of asset j to the $p-$quantile of the market return. The proposition is thus proved.□

Proof of Proposition 2.3.1

In the case $\beta_j^T = 0$, we have that R_m^e and R_j^e are tail independent, which implies that $\tau_j = 0$. Thus equation (2.7) holds automatically.

In the rest of the proof, we consider $\beta_j^T > 0$ in the linear tail model. We first compare the marginal VaRs of the asset excess return and the market excess return as in the following lemma.

Lemma 2.A.1 *Under the single factor model in* (2.1) *and the heavy-tail setup of the downside distributions* (2.6)*, we have that, for sufficiently low probability* p*,*

$$\beta^T VaR_m(p) \leq VaR_j(p). \tag{2.17}$$

Proof of Lemma 2.A.1

As $p \to 0$, the VaR of the market return $VaR_m(p)$ converges to infinity. Thus, when the tail probability p is sufficiently low such that $VaR_m(p)$ exceeds the threshold in the linear tail model, the linear relation (2.1) is valid for $R_m^e < -VaR_m(p)$. Hence, we have that for any $\delta > 0$,

$$\begin{aligned}
\Pr(R_j^e < -\beta_j^T VaR_m(p)) &\geq \Pr(R_j^e < -\beta_j^T VaR_m(p) \text{ and } R_m^e < -VaR_m(p)) \\
&\geq \Pr(\beta_j^T R_m^e + \varepsilon_j < -\beta_j^T VaR_m(p) \text{ and }
\end{aligned}$$

$$\begin{aligned}&\beta_j^T R_m^e < -\beta_j^T VaR_m(p)(1+\delta))\\ &\geq \Pr(\beta_j^T R_m^e < -\beta_j^T VaR_m(p)(1+\delta) \ \text{ and } \ \varepsilon_j < \delta\beta_j^T VaR_m(p))\\ &= \Pr(R_m^e < -VaR_m(p)(1+\delta)) \Pr(\varepsilon_j < \delta\beta_j^T VaR_m(p)).\end{aligned}$$

The last equality is due to the independency between R_m^e and ε_j. From the heavy-tail setup, we obtain that

$$\lim_{p\to 0} \frac{\Pr(R_m^e < -VaR_m(p)(1+\delta))}{\Pr(R_m^e < -VaR_m(p))} = (1+\delta)^{-\alpha_m}.$$

Moreover, it is obvious that $\lim_{p\to 0} \Pr(\varepsilon_j < \delta\beta_j^T VaR_m(p)) = 1$. Thus, we have that

$$\liminf_{p\to 0} \frac{\Pr(R_j^e < -\beta_j^T VaR_m(p))}{p} \geq (1+\delta)^{-\alpha_m}.$$

Notice that the above inequality holds for all $\delta > 0$. By taking $\delta \to 0$, we get that

$$\liminf_{p\to 0} \frac{\Pr(R_j^e < -\beta_j^T VaR_m(p))}{p} \geq 1.$$

From the definition of VaR, $\Pr(R_j^e < -VaR_j(p)) = p$. Hence, for sufficiently low probability p, inequality (2.17) holds. □

Based on the linear tail model (2.1), we calculate the tail dependence measure $\tau_j(p)$. When the tail probability p is sufficiently low such that $VaR_m(p)$ exceeds the threshold in linear tail model, we have that $\tau_j(p) = \frac{\Pr(C)}{p}$, with C denoting a joint tail event as

$$\begin{aligned}C &:= \left\{R_j^e < -VaR_j(p) \ \text{ and } \ R_m^e < -VaR_m(p)\right\}\\ &= \left\{\beta_j^T R_m^e + \varepsilon_j < -VaR_j(p) \ \text{ and } \ R_m^e < -VaR_m(p)\right\}.\end{aligned}$$

Define $C_0 = \left\{\beta^T R_m^e < -VaR_j(p)\right\}$. We shall show that $\Pr(C) \sim \Pr(C_0)$ as $p \to 0$. For any $0 < \delta < 1$, consider two sets C_1 and C_2 defined as

$$\begin{aligned}C_1 &:= \left\{\beta_j^T R_m^e < -VaR_j(p)(1+\delta) \ \text{ and } \ \varepsilon_j < \delta VaR_j(p)\right\},\\ C_2 &:= C_{21} \bigcup C_{22}\\ &:= \left\{\beta_j^T R_m^e < -VaR_j(p)(1-\delta)\right\} \bigcup \left\{\varepsilon_j < -\delta VaR_j(p) \ \text{ and } \ R_m^e < -VaR_m(p)\right\}.\end{aligned}$$

It is obvious that $C \subset C_2$. Moreover, from Lemma 2.A.1, we get that $VaR_j(p)(1+\delta) > VaR_j(p) \geq \beta_j^T VaR_m(p)$, which implies that $C_1 \subset C$. Hence $\Pr(C)$ is bounded by $\Pr(C_1)$ and $\Pr(C_2)$. We calculate the two probabilities as follows.

Since R_m^e and ε_j are independent, we get that as $p \to 0$,

$$\Pr(C_1) = \Pr(\beta_j^T R_m^e < -VaR_j(p)(1+\delta)) \Pr(\varepsilon_j < \delta VaR_j(p)) \sim \Pr(C_0)(1+\delta)^{-\alpha_m}.$$

Here we use the fact that as $p \to 0$, $\Pr(\varepsilon_j < \delta VaR_j(p)) \to 1$ and the heavy-tailedness of the downside distribution of R_m^e. It implies that

$$\liminf_{p\to 0} \frac{\Pr(C)}{\Pr(C_0)} \geq (1+\delta)^{-\alpha_m}. \tag{2.18}$$

Similarly, we get that

$$\lim_{p\to 0} \frac{\Pr(C_{21})}{\Pr(C_0)} = (1-\delta)^{-\alpha_m}.$$

For the set C_{22}, we have that

$$\begin{aligned}
\limsup_{p\to 0} \frac{\Pr(C_{22})}{\Pr(C_0)} &= \limsup_{p\to 0} \frac{\Pr(R_m^e < -VaR_m(p))\Pr(\varepsilon_j < -\delta VaR_j(p))}{\Pr(C_0)} \\
&= \limsup_{p\to 0} \frac{p \cdot A_{\varepsilon_j}(\delta VaR_j(p))^{-\alpha_{\varepsilon_j}}}{A_m(VaR_j(p)/\beta_j^T)^{-\alpha_m}} \\
&= \limsup_{p\to 0} \frac{A_{\varepsilon_j}}{A_m \delta^{\alpha_{\varepsilon_j}}(\beta_j^T)^{\alpha_m}} p \left(\frac{A_j}{p}\right)^{\frac{\alpha_m - \alpha_{\varepsilon_j}}{\alpha_j}} \\
&= 0.
\end{aligned}$$

The last step comes from combining the assumption $\alpha_j > \frac{1}{2}\alpha_m$ with the fact that $\alpha_j \leq \min\{\alpha_m, \alpha_{\varepsilon_j}\}$: this implies that $\alpha_m - \alpha_{\varepsilon_j} < \alpha_j$. Therefore,

$$\limsup_{p\to 0} \frac{\Pr(C)}{\Pr(C_0)} \leq \limsup_{p\to 0} \frac{\Pr(C_{21})}{\Pr(C_0)} + \frac{\Pr(C_{22})}{\Pr(C_0)} = (1-\delta)^{-\alpha_m} \tag{2.19}$$

Since inequalities (2.18) and (2.19) hold for any $0 < \delta < 1$, by combining the two and taking $\delta \to 0$, we get that $\lim_{p\to 0} \frac{\Pr(C)}{\Pr(C_0)} = 1$. Thus, as $p \to 0$,

$$\tau_j(p) = \frac{\Pr(C)}{p} \sim \frac{\Pr(C_0)}{\Pr(R^m < -VaR_m(p))} \sim \left(\frac{\beta^T VaR_m(p)}{VaR_j(p)}\right)^{\alpha_m}. \square$$

2.B Appendix B. Details on the estimation procedure

The EVT approach estimates the tail beta β_j^T via the estimator in equation (2.8). In this appendix we provide the details on how each component is estimated.

First, to estimate the tail index of the market return α_m, a widely-applied estimator is the so-called *Hill estimator* proposed in Hill (1975). Suppose the observed market excess returns are $R^e_{m,1}, \cdots, R^e_{m,n}$. Consider the losses defined as $X_t^{(m)} = -R^e_{m,t}$, for $t = 1, \cdots, n$. By ranking them as $X_{n,1}^{(m)} \leq X_{n,2}^{(m)} \leq \cdots \leq X_{n,n}^{(m)}$, the Hill estimator is defined as

$$\frac{1}{\hat{\alpha}_m} := \frac{1}{k} \sum_{i=1}^{k} \log X_{n,n-i+1}^{(m)} - \log X_{n,n-k}^{(m)}, \tag{2.20}$$

where $k := k(n)$ is an intermediate sequence such that as $n \to \infty$, $k \to \infty$ and $k/n \to 0$. A side result is the estimation of the scale parameter A_m:

$$\hat{A}_m = \left(X_{n,n-k}^{(m)}\right)^{\hat{\alpha}_m} \frac{k}{n}.$$

Secondly, for the τ_j parameter, multivariate EVT provides a nonparametric estimate by a counting measure as follows,

$$\hat{\tau}_j := \frac{1}{k} \sum_{t=1}^{n} 1_{\left\{X_t^{(j)} > X_{n,n-k}^{(j)} \text{ and } X_t^{(m)} > X_{n,n-k}^{(m)}\right\}}, \tag{2.21}$$

where $X_{n,n-k}^{(j)}$ is the $k+1-$th highest order statistic of the losses of the asset excess returns: $X_t^{(j)} := -R^e_{j,t}$, $t = 1, \cdots, n$.

Finally, the VaRs of the market and asset return at a tail probability level k/n are estimated by their $k + 1-$th highest loss.

In all above estimators, it is necessary to specify an intermediate sequence $k = k(n)$ such that $k \to \infty$ and $k/n \to 0$ as $n \to \infty$. Practically, these theoretical conditions on k are not relevant for a finite sample size n. Consequently, how to choose a proper k in the estimator is a major issue in estimation. Instead of taking an arbitrary k, a usual procedure is to calculate the estimates of β_j^T under different values of k and draw a line plot against the values of k. For low values of k, the estimate exhibits a large variance, while for a high values of k, it bears a potential bias, because also observations from the moderate level are included in the estimation. Therefore, k is usually chosen by picking the first stable part of the line plot starting from low k, which balances the tradeoff between the variance and the bias. Because k is chosen from a stable part of the line plot, a small variation in the value of k does not change the estimated value of β_j^T. Thus, estimation of the tail beta is not sensitive to the exact value of k. This procedure has been applied in univariate and multivariate EVT to estimate the tail index α and the tail dependence measure τ.

The temporal dependence in financial data, such as volatility clustering, does not affect the consistency of the estimators for each component. For the Hill estimator, see Hsing (1991), for the VaR and the tail dependence measure, see Hill (2009). In general, under weak conditions, EVT analysis can be applied to temporal dependent data without modification, see Drees (2008) for a general discussion. Therefore the estimator in equation (2.8) remains consistent under temporal dependence. However, temporal dependence does affect the variance of the estimates. In financial applications, one may rely on a block bootstrap procedure to obtain adjusted standard errors.

In our empirical exercise, we estimate tail betas from a 60 months window, which contains roughly 1,250 daily returns. From performing the selection procedure above on several series, we fix k at 50. This corresponds to estimating tail betas based on about $k/n \approx 4\%$ of the observations.

Chapter 3

The simple econometrics of tail dependence

This chapter provides a regression approach to estimate tail dependence measures. The estimates coincide with the non-parametric estimates following Extreme Value Theory. The approach can easily be extended to higher dimensional analysis. We provide an example on international stock markets.[1]

Keywords: Tail dependence, regression analysis, Extreme Value Theory.

JEL Classification Numbers: C14, C58.

3.1 Introduction

Tail dependence refers to the dependence among extreme events. This dependence structure is not necessarily similar to the dependence structure among ordinary observations. A fascinating example is observed in financial markets: the dependence among asset returns increases during volatile periods, and especially during strong market downturns, see Longin and Solnik (1995, 2001), Ang and Chen (2002), Ang and Bekaert (2002) among others.

The major difficulty of measuring tail dependence is the scarcity of observations on extreme events. The developments in multivariate Extreme Value Theory (EVT) allow assessing the tail dependence structure based on observations close to the tail. For example, in the applications by Poon *et al.* (2004) and Hartmann *et al.* (2004), measuring

[1]This chapter is co-authored by Chen Zhou. A shorter version of this chapter was published in Economics Letters, 116, 371–373, The simple econometrics of tail dependence, Van Oordt and Zhou, Copyright Elsevier (2012). Available online at http://dx.doi.org/10.1016/j.econlet.2012.04.016.

tail dependence reveals which stock market pairs are likely to crash jointly or which are likely to exhibit *flight-to-quality* phenomena.

One often applied measure on tail dependence is the τ-measure, see e.g. Straetmans *et al.* (2008), De Jonghe (2010), Pais and Stork (2011) and Beine *et al.* (2010).[2] The pairwise τ-measure compares the probability of a joint tail event to that of a tail event of one variable, where 'tail events' occur only with very low probability. Formally, the τ-measure for the pairwise dependence between the left tails of two random variables x and y is given by

$$\tau_{y|x} = \lim_{p \to 0} \frac{\Pr(y < Q_y(p) \quad \text{and} \quad x < Q_x(p))}{p} = \lim_{p \to 0} \Pr(y < Q_y(p) | x < Q_x(p)),$$

where $Q_y(p)$ denotes the quantile of the distribution of y at probability level p.[3] Although the notation $\tau_{y|x}$ can be read as the conditional probability of a tail event of y given the occurrence of a tail event of x, it does not necessarily imply any causality. Similar to correlation, the τ-measure ranges from 0 to 1, which indicates the level of tail dependence. The case $\tau_{y|x} = 0$ corresponds to tail independence, while the case $\tau_{y|x} = 1$ corresponds to complete tail dependence.

The τ-measure can be estimated non-parametrically under the multivariate EVT framework, see e.g. Embrechts *et al.* (2000) and Schmidt and Stadtmüller (2006). This chapter provides a simple ordinary least squares (OLS) regression approach to estimate τ. Our estimator exactly coincides with the non-parametric estimator. Because pairwise tail dependence is only a projection of the multidimensional tail dependence structure, it is crucial to have an accessible instrument to investigate multidimensional tail dependence. The advantage of our regression approach is contained by the straightforward extension of analyzing tail dependence to higher dimensional levels. We provide an empirical example on multidimensional tail dependence among international stock markets.

3.2 Indicator regression

The non-parametric estimator of the $\tau_{y|x}$-measure is calculated as the ratio between the number of observations in which both x and y are extreme and those in which variable x is extreme. Here being *extreme* is defined as having a value below the k-th lowest observation in the sample, given a sufficiently low k, see Embrechts *et al.* (2000). With

[2] Some studies report manipulations of τ: Hartmann *et al.* (2004, 2010) and De Vries (2005) report $2/(2-\tau)$; Garita and Zhou (2009) report $\tau/(2-\tau)$.

[3] The definition of τ is sufficiently flexible to define tail dependence among all tail pairs, i.e. to vary among the left and right tails. In addition, several studies choose to replace $Q_y(p)$ and $Q_x(p)$ in the definition of τ by a sufficiently low threshold $-u$. The proposed regression approach can deal with both definitions of τ.

observations x_t and y_t for $t = 1, 2, ..., n$, the non-parametric estimator of $\tau_{y|x}$ is given by

$$\hat{\tau}_{y|x} = \frac{\sum_{t=1}^{n} I_{y \text{ and } x,t}}{\sum_{t=1}^{n} I_{x,t}},$$

where $I_{y \text{ and } x,t} = \mathbf{1}(y_t < Q_y(k/n) \textit{ and } x_t < Q_x(k/n))$ and $I_{x,t} = \mathbf{1}(x_t < Q_x(k/n))$ with $\mathbf{1}()$ denoting the indicator function, and where the quantile function $Q(k/n)$ is usually estimated by the k-th lowest observation. With a similar notation for $I_{y,t}$, we have $I_{y \text{ and } x,t} = I_{y,t}I_{x,t}$, and rewrite the estimator $\hat{\tau}_{y|x}$ as

$$\hat{\tau}_{y|x} = \frac{\sum_{t=1}^{n} I_{y,t}I_{x,t}}{\sum_{t=1}^{n} I_{x,t}I_{x,t}}.$$

Hence, the non-parametric estimator is computationally equivalent to the OLS estimate of the slope coefficient when the indicator for extreme values of y is regressed on the indicator for extreme values of x without intercept. In other words, the OLS estimate of β in the indicator regression

$$I_{y,t} = \beta \times I_{x,t} + \varepsilon_t$$

equals to the non-parametric estimate $\hat{\tau}_{y|x}$.[4]

Studies in the literature usually focus on pairwise tail dependence only. However, this is in general not sufficient to reproduce the higher dimensional tail dependence structure. For example, consider a three-dimensional case with random variables y, x_1 and x_2 with all pairwise τ-measures equal to 1/2, i.e. $\tau_{y|x_1} = \tau_{y|x_2} = \tau_{x_2|x_1} = 1/2$.[5] Several potential tri-variate dependence structures can lead to such pairwise τ-measures. One possibility is that y is extreme if and only if x_1 and x_2 are both extreme. Another one is that y is extreme if and only if one of x_1 and x_2 is extreme. The two tri-variate dependence structures are very different, and so are their economic consequences. Hence, it is necessary to have an instrument to investigate multidimensional tail dependence.

The indicator regression can be extended to multidimensional analysis. To analyze the tail dependence among random variable y and multiple random variables $x_1, \cdots, x_m$, one should include all possible interactions among the indicators $I_{x_i,t}$ in the indicator regression. The regression then gives a complete figure on tail dependence in the multi-dimensional case. To illustrate this idea we consider the three-dimensional dependence structure between variable y and variables x and z by estimating the following indicator regression:

$$I_{y,t} = \beta_x I_{x,t} + \beta_z I_{z,t} + \beta_{x,z} I_{x,t} I_{z,t} + \varepsilon_t.$$

[4] We remark that the usual standard errors of the OLS regression are meaningless. Correct standard errors for $\hat{\tau}$ can be obtained by using a (block) bootstrap. To obtain the standard errors reported in Table 3.1, we followed Hartmann *et al.* (2010) and set the optimal block length equal to $n^{1/3}$.

[5] This example is close to the findings of Poon *et al.* (2004) on pairwise tail dependence among the stock markets of UK, Germany and France.

Similar to the pairwise case, the interpretation of the multidimensional tail dependence structure follows the interpretation of a usual regression. That is, to get the non-parametric estimate of the conditional probability of y being extreme given a certain scenario on x and z, one has to calculate the predicted expectation of the left hand side indicator given the value of the right hand side indicators under the scenario.[6] For example, for the scenario in which x is extreme and z is not extreme, the conditional probability that y is extreme, $\hat{\tau}_{y|x,\bar{z}}$, equals to $\hat{\beta}_x \cdot 1 + \hat{\beta}_z \cdot 0 + \hat{\beta}_{x,z} \cdot 1 \cdot 0 = \hat{\beta}_x$.[7] Similarly, the estimate of the probability that y is extreme, conditional on both x and z being extreme, $\hat{\tau}_{y|x,z}$, is given by $\hat{\beta}_x + \hat{\beta}_z + \hat{\beta}_{x,z}$.

The indicator regression in high dimensional cases may involve a large number of higher dimensional interactions. The estimation of a full tail dependence structure suffers from a dimensional curse: with scenarios defined on m variables, $2^m - 1$ parameters must be estimated. Nevertheless, both complete tail dependence and tail independence among regressors provide potential relief to this dimensional curse. First, if two regressors are recognized as completely tail dependent, then the individual regressors can be left out. For instance, suppose that in the three-dimensional example, x and z are completely tail dependent. Then a perfect collinearity problem will occur in the indicator regression, since we have $I_{x,t} = I_{z,t} = I_{x,t}I_{z,t}$. Because the scenario in which only one of x and z is extreme has probability zero, the only meaningful model is to regress $I_{y,t}$ on $I_{x,t}I_{z,t}$. The indicator regression coefficient gives the estimate of $\tau_{y|x,z}$. Second, if two regressors are recognized as tail independent, then the interaction of the two is by definition zero. In this case all interaction terms that include the two regressors can be omitted from the indicator regression. To summarize, identifying completely tail dependent or tail independent regressors before performing the full indicator regression provides relief to the dimensional curse problem.

3.3 Tail dependence among stock markets

We provide an empirical example of the indicator regression at work by extending some of the pairwise results from Poon *et al.* (2004) to the multidimensional case. Poon *et al.* (2004) estimate the pairwise tail dependence among major stock indices.[8] We replicate

[6]The proof is provided in Appendix 3.A.

[7]The absence (presence) of the bar on the subscript of $\hat{\tau}$ denotes a scenario with a particular variable being (not) extreme.

[8]The stock market indices are S&P500 for the U.S., FTSE100 for the U.K., DAX30 for Germany and CAC40 for France. Our analysis regards the results on unfiltered data in the third subperiod: from November 28, 1990 to November 12, 2001. The total sample size is $n = 2,859$. With an exception for $\tau_{UK|FR}$ our pairwise estimates equal those in Poon *et al.* (2004, Table 3). The exception is due to a different choice of k. To report estimates that are comparable to the multidimensional analysis, we

Table 3.1: Tail dependence among the U.K. and other major equity markets.

Regression	Coefficient	Tail dependence	Probability (s.e.)
$\hat{\beta}_{US}$	0.105	$\hat{\tau}_{UK\|US,\overline{GE},\overline{FR}}$	0.105 (0.041)
$\hat{\beta}_{GE}$	0.188	$\hat{\tau}_{UK\|\overline{US},GE,\overline{FR}}$	0.188 (0.078)
$\hat{\beta}_{FR}$	0.200	$\hat{\tau}_{UK\|\overline{US},\overline{GE},FR}$	0.200 (0.090)
$\hat{\beta}_{US,GE}$	0.253	$\hat{\tau}_{UK\|US,GE,\overline{FR}}$	0.545 (0.221)
$\hat{\beta}_{US,FR}$	-0.055	$\hat{\tau}_{UK\|US,\overline{GE},FR}$	0.250 (0.196)
$\hat{\beta}_{GE,FR}$	0.128	$\hat{\tau}_{UK\|\overline{US},GE,FR}$	0.515 (0.096)
$\hat{\beta}_{US,GE,FR}$	0.182	$\hat{\tau}_{UK\|US,GE,FR}$	1.000 (0.110)

Note: The estimates are obtained from an indicator regression based on U.K. stock market returns and U.S., German and French stock market returns. Each returns series runs from November 28, 1990 until November 12, 2001, containing $2,859$ daily observations. We choose k at 87. The first column presents the regression coefficients. The second column presents the conditional probabilities of observing an extreme loss in the U.K. market conditional on a scenario in the other three markets. The absence of the bar in the conditioning scenario in the subscript of $\hat{\tau}$ denotes an extreme loss in that particular market.

their pairwise results on the tail dependence between the U.K. stock market and those of France, Germany and the U.S. with the indicator regression, which gives

$$\hat{\tau}_{UK|US} = 0.29 \quad , \quad \hat{\tau}_{UK|GE} = 0.46 \quad \text{and} \quad \hat{\tau}_{UK|FR} = 0.43.$$

The results on pairwise tail dependence suggest that the U.K. market suffers the least co-crashes with the U.S. stock market, while its tail dependence with the German and French stock market appears to be on a comparable level.

Subsequently, we estimate the full tail dependence structure among the four countries by an indicator regression that includes all possible interactions, which gives

$$I_{UK,t} \approx 0.11 \cdot I_{US,t} + 0.19 \cdot I_{GE,t} + 0.20 \cdot I_{FR,t} + 0.25 \cdot I_{US,t} I_{GE,t} +$$

$$-0.06 \cdot I_{US,t} I_{FR,t} + 0.13 \cdot I_{GE,t} I_{FR,t} + 0.18 \cdot I_{US,t} I_{GE,t} I_{FR,t}.$$

In Table 3.1 we calculate the conditional probabilities from the regression coefficients for all possible scenarios as discussed in Section 2.

Three results follow from the multidimensional analysis. First, considering the scenario in which *only one particular* market suffers an extreme loss, then the U.K. market has the weakest link with the U.S. market, while those with the German and French market are comparable. This result matches the result from the pairwise analysis.

Second, given an extreme loss in the U.S. market, the link of the U.K. market with the German market is stronger than the link with the French market. Hence, from

choose a common $k = 87$ for all market pairs.

the multidimensional analysis we observe a distinction between the French and German market that does not appear from the pairwise analysis.

Third, we observe that the U.K. market is expected to suffer an extreme loss with almost certainty, if the other three markets are observed to suffer an extreme loss. In contrast to the moderate levels of pairwise tail dependence, the multidimensional analysis reveals a strong linkage among the four markets.

3.4 Systemic importance of financial institutions

With the failure of the investment bank Lehman Brothers in 2008, the financial system in the US and the EU came close to a complete meltdown. This phenomenon falls in line with the discussion on “tail dependence”: crashes of financial institutions are likely to happen simultaneously. Financial institutions that are more likely to co-crash with other institutions within the financial system are sometimes considered as more systemically important. For policymakers, measuring the systemic importance of financial institutions turns to be the key issue in both financial stability assessment and macro-prudential supervision. Recent studies propose a few measures on the systemic importance. For example, Segoviano and Goodhart (2009) propose the Probability of Cascade Effects (PCE), which measures the probability of having at least one other bank crash given the crash of one specific bank. Zhou (2010) proposes the Systemic Impact Index (SII), which measures the expected number of crashes within the financial system given the crash of one specific bank. We demonstrate that these two systemic importance measures can be estimated with the indicator regression method.

Consider a financial system consisting of d financial institutions, with variables $x_{1,t}, ..., x_{d,t}$ indicating their statuses in period t. An extremely low value in x_i indicates that bank i is distressed. Examples of such status variables are given by the returns on bank i's equity or by the returns on providing insurance on bank i's debt, as reflected by the changes in the price of credit default swaps (CDS). In correspondence with the foregoing, we use $I_{i,t}$ to indicate whether bank i is in distress at period t, i.e. $I_{i,t} = \mathbf{1}(x_{i,t} < Q_i(p))$.

We start with the PCE-measure of Segoviano and Goodhart (2009). Segoviano and Goodhart (2009) apply the CIMDO copula approach to estimate the PCE-measure, while Zhou (2010) provides a non-parametric estimate of the PCE-measure. We show how to apply the indicator regression to produce the non-parametric estimate. It is obvious that the event ”having at least one other bank crash” can be indicated by the indicator $I_{\neq i,t} = \max_{j \neq i} I_{j,t}$. Similar to the estimation of τ, one can estimate the OLS regression

$$I_{\neq i,t} = PCE_i \cdot I_{i,t} + \varepsilon_t,$$

to obtain the estimate of the PCE-measure for bank i. It is straightforward to see that such an estimate is exactly equal to the non-parametric estimate of the PCE-measure.

Next, we discuss the SII-measure in Zhou (2010). The non-parametric estimate of the SII-measure for bank i equals to the sum of the estimates on $\tau_{j|i}$ which is the τ-measure between bank i and any bank j, $j = 1, 2, ..., d$. From the indicator regression, $\tau_{j|i}$ can be estimated by the OLS regression

$$I_{j,t} = \tau_{j|i} I_{i,t} + \varepsilon_t.$$

By aggregating the regressions, we get that performing a single OLS regression

$$\sum_{j=1}^{d} I_{j,t} = SII_i \cdot I_{i,t} + \varepsilon_t$$

leads to the non-parametric estimate of the SII-measure for bank i. Notice that this is beyond the aforementioned concept of "indicator regressions", because the dependent variable is not an indicator, but a measure for the status of the system.

3.5 Concluding remark

Because the tail dependence measure is a regression coefficient in an indicator regression, it is possible to incorporate other covariates. De Jonghe (2010) investigates the determinants of the τ-measure between bank equity returns and a banking index by regressing the estimated τ-measure on bank level characteristics. Such a two-step approach may result in a loss of efficiency. Differently, it is possible to combine the indicator regression with the cross-sectional model to construct a one-step approach. This is left for future research.

3.A Appendix A. Proof

Define $N_x = \sum I_{x,t}$, $N_{x,z} = \sum I_{x,t}I_{z,t}$, etc, where the sum is over all t. Then the non-parametric estimate that y is extreme, conditional on x and/or z being extreme, is given by

$$\hat{\tau}_{y|x,\bar{z}} = \frac{N_{y,x} - N_{y,x,z}}{N_x - N_{x,z}}, \quad \hat{\tau}_{y|\bar{x},z} = \frac{N_{y,z} - N_{y,x,z}}{N_z - N_{x,z}} \quad \text{and} \quad \hat{\tau}_{y|x,z} = \frac{N_{y,x,z}}{N_{x,z}}.$$

We prove that OLS estimates from

$$I_{y,t} = \beta_x I_{x,t} + \beta_z I_{z,t} + \beta_{x,z} I_{x,t} I_{z,t} + \varepsilon_t$$

provides the non-parametric estimators as $\hat{\tau}_{y|x,\bar{z}} = \hat{\beta}_x$, $\hat{\tau}_{y|\bar{x},z} = \hat{\beta}_z$ and $\hat{\tau}_{y|x,z} = \hat{\beta}_x + \hat{\beta}_z + \hat{\beta}_{x,z}$.

Proof. Write $\mathbf{Y} = \begin{pmatrix} I_{y,1} \\ \vdots \\ I_{y,n} \end{pmatrix}$ and $\mathbf{X} = \begin{pmatrix} I_{x,1} & I_{z,1} & I_{x,1}I_{z,1} \\ \vdots & \vdots & \vdots \\ I_{x,n} & I_{z,n} & I_{x,n}I_{z,n} \end{pmatrix}$. Then the vector $\beta = (\beta_x, \beta_z, \beta_{x,z})'$ is estimated by $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$. We start with calculating $(\mathbf{X}'\mathbf{X})^{-1}$:

$$\mathbf{X}'\mathbf{X} = \begin{pmatrix} N_x & N_{x,z} & N_{x,z} \\ N_{x,z} & N_z & N_{x,z} \\ N_{x,z} & N_{x,z} & N_{x,z} \end{pmatrix}.$$

The inverted matrix $(\mathbf{X}'\mathbf{X})^{-1}$ is

$$\frac{1}{N_{x,z}(N_x - N_{x,z})(N_z - N_{x,z})} \begin{pmatrix} N_{x,z}(N_z - N_{x,z}) & 0 & -N_{x,z}(N_z - N_{x,z}) \\ 0 & N_{x,z}(N_x - N_{x,z}) & -N_{x,z}(N_x - N_{x,z}) \\ -N_{x,z}(N_z - N_{x,z}) & -N_{x,z}(N_x - N_{x,z}) & N_x N_z - (N_{x,z})^2 \end{pmatrix}.$$

Multiplying $(\mathbf{X}'\mathbf{X})^{-1}$ with

$$\mathbf{X}'\mathbf{Y} = \begin{pmatrix} N_{y,x} & N_{y,z} & N_{y,x,z} \end{pmatrix}'$$

gives the estimates:

$$\hat{\beta} = \begin{pmatrix} \frac{N_{y,x} - N_{y,x,z}}{N_x - N_{x,z}} \\ \frac{N_{y,z} - N_{y,x,z}}{N_z - N_{x,z}} \\ \frac{N_{y,x,z}}{N_{x,z}} - \frac{N_{y,x} - N_{y,x,z}}{N_x - N_{x,z}} - \frac{N_{y,z} - N_{y,x,z}}{N_z - N_{x,z}} \end{pmatrix} = \begin{pmatrix} \hat{\tau}_{y|x,\bar{z}} \\ \hat{\tau}_{y|\bar{x},z} \\ \hat{\tau}_{y|x,z} - \hat{\tau}_{y|x,\bar{z}} - \hat{\tau}_{y|\bar{x},z} \end{pmatrix}.$$

■

In the proof we do not assume a common number of tail observations k. This justifies the remark in footnote 3.

Chapter 4

Securitization and the dark side of diversification

Diversification by banks affects the systemic risk of the sector. Importantly, Wagner (2010) shows that linear diversification increases systemic risk. We consider the case of securitization, whereby loan portfolios are sliced into tranches with different seniority levels. We show that tranching offers nonlinear diversification strategies, which can reduce the failure risk of individual institutions beyond the minimum level attainable by linear diversification without increasing systemic risk.[1]

Keywords: Diversification, risk management, securitization, systemic risk, tranching.

JEL Classification Numbers: G11, G21.

4.1 Introduction

In this paper, we examine the relationship between systemic risk and diversification by financial institutions through securitization. We investigate this in a framework in which the securitization of loan portfolios is explicitly modeled as a diversification strategy. That is, financial institutions are allowed to structure securities on loan portfolios into a junior and a senior tranche. We use this framework to answer two questions. First, does

[1]This chapter will be published in the Journal of Financial Intermediation, forthcoming, Van Oordt, Securitization and the dark side of diversification, Copyright Elsevier (2013). Available online at http://dx.doi.org/10.1016/j.jfi.2013.05.001. Thanks are due to Casper G. de Vries, Job Swank, Chen Zhou and Wilko Bolt for valuable advice and comments. The author is also grateful for comments and suggestions from Günter Franke, Gabriele Galati, Sherrill Shaffer, Razvan E. Vlahu, Wolf Wagner, the editor Viral V. Acharya, an anonymous referee, and participants in the Financial Innovation Workstream (2011) of De Nederlandsche Bank, the lunch seminars at De Nederlandsche Bank (2012) and the Marie Curie Conference on Financial Risk Management & Risk Reporting in Konstanz (2013).

diversification through securitization provide better diversification strategies than linear diversification? And second, what is the relationship between diversification through securitization and systemic risk?

Although it is generally known that diversification usually decreases the riskiness of individual institutions, the relationship between diversification and systemic risk is less commonly known. Banking literature finds that diversification increases the similarity among institutions. It therefore tends to increase the probability of joint failures or systemic crises, which is the dark side of diversification. Shaffer (1985, 1994) and Ibragimov *et al.* (2011) find this result in case of full diversification, or full risk sharing. Wagner (2010) establishes the result that any degree of diversification increases the probability of joint failures.

The theoretical result regarding the dark side of diversification is based on linear diversification strategies. That is, financial institutions diversify asset holdings by exchanging shares in their projects. Although this result has several important applications, it does not apply unconditionally to securitization. This is because securities on loan portfolios are usually sliced into tranches with different seniority levels. The payoff of those tranches is nonlinear in the return of the underlying loan portfolio. Diversification through securitization is therefore not the same as linear diversification.

Interestingly, taking tranching into account proves to have a substantial impact from both a microprudential and macroprudential point of view. From a microprudential perspective, tranching facilitates a decrease in the probability of individual failures beyond the minimum level that could be achieved by linear diversification strategies. From a macroprudential viewpoint, diversification through securitization may help avoid the dark side of diversification. In contrast to the result on linear diversification in the literature, we find that diversification through securitization does not necessarily increase the probability of systemic failures.

The problem with linear diversification is that profits and losses are always shared among investors. Therefore, if two banks share the ownership of two loan portfolios, the losses generated by one portfolio may trigger the insolvency of both banks. This could happen even if one of the two portfolios performs relatively well. Tranching helps to avoid such scenarios by introducing seniority levels that determine the order of payment. Suppose that each bank owns the junior tranche of one portfolio and the senior tranche of the other portfolio. If the maximum payoff of a senior tranche is set sufficiently high, then any return on a portfolio above this threshold will guarantee the solvency of the bank that owns the senior tranche. Nevertheless, risk sharing is still in place, because any return on top of this threshold benefits the owner of the junior tranche and may counterbalance potential losses on the other portfolio.

However, the benefits of securitization come at a cost. Our analysis reveals that structuring claims on loan portfolios into different seniority classes introduces nonlinear effects in the financial system. If financial institutions adopt the optimal strategy, even a small unanticipated confidence shock can strongly increase the risks in the financial system due to those nonlinearities. Nevertheless, the diversification benefits of tranching appear to be relatively robust if institutions anticipate the possibility of confidence shocks. Tranching strategies perform better than linear diversification strategies provided that the level of uncertainty regarding depositor confidence is limited.

Many studies on systemic risk focus on contagion through, for example, the interbank market, the payment system or asset prices. By contrast, the present results do not depend on contagion. Whether a bank fails does not depend on other banks. In line with the mechanism of Wagner (2010), the conclusions follow automatically from the similarity in risk exposures. This channel has been the focus of several other studies. Acharya and Yorulmazer (2007) and Acharya (2009) model an increase in joint failure risk if financial institutions invest in similar projects. De Vries (2005) discusses the frequency of joint tail events in the case of risk sharing. When more than two institutions are involved, Slijkerman *et al.* (2013) show that mergers (as a form of diversification) do not necessarily increase the risk of all institutions failing jointly. Zhou (2010) investigates the relationship between the similarity in portfolio holdings of other institutions and the expected number of institutions that simultaneously face financial distress. Allen *et al.* (2012) discuss how the frequency of joint failures can be affected by the interaction between asset commonality and the network of cross holdings. The contribution of the present paper is to show that the positive relationship between risk sharing and joint failures may not hold true in a world with tranching.

Exchanging tranches is a way of transferring credit risk. Several studies examine the effects of loan sales and credit risk transfer on screening and monitoring incentives in the presence of asymmetric information, see e.g. Pennacchi (1988), Gorton and Pennacchi (1995), Duffee and Zhou (2001), Morrison (2005), Chiesa (2008), Parlour and Plantin (2008) and Hakenes and Schnabel (2010).[2] In particular, in contrast to previous findings in the literature, Chiesa (2008) finds that credit risk transfer may enhance loan monitoring. Some studies motivate the creation of tranches with different seniority levels by the presence of information asymmetries, see e.g. Boot and Thakor (1993), Riddiough (1997), DeMarzo and Duffie (1999) and DeMarzo (2005).[3] In these studies, informed

[2]Allen and Carletti (2006) and Wagner and Marsh (2006) show that credit risk transfer can have both beneficial and harmful effects in terms of financial stability without considering the possibility of tranching.

[3]See Gorton and Pennacchi (1990) for a similar argument in the context of liquidity creation by financial institutions.

agents typically trade the most informationally sensitive tranche. Our analysis suggests a complementary justification for tranching based on risk management considerations without information asymmetries.

Although the focus of this study is on the securitization of loan portfolios, the conclusions can be applied to several other fields. The conclusions on the benefits and risks of securitization directly apply to the syndicated loan business. The 'loan portfolios' can be given a much broader interpretation, such as investment in regions or business lines. In this context, it is notable that the hierarchy in payoffs due to seniority classes also arises in the distinction between debt and equity. Among financial institutions, obtaining an equity stake can be regarded as obtaining a junior claim on the assets of another firm, while providing an interbank loan can be seen as obtaining a senior claim. Finally, the result on the improvement in risk management due to tranching can be applied by any two investors who wish to avoid losses beyond a certain level.

The remainder of the paper is organized as follows. Section 4.2 describes the model and introduces the possibility of tranching. Section 4.3 presents the main results. Diversification through securitization in the presence of confidence shocks is discussed in Section 4.4. Section 4.5 provides a discussion of several other extensions. Conclusions are set out in Section 4.6. The proofs are included in the Appendix.

4.2 Model

Before modeling the securitization of loan portfolios, we first give a short description of the general framework. We build on the framework of Wagner (2010). In this framework, two banks each manage one unit of funds from risk-neutral investors. The share of deposits is denoted by d and the remaining share of funds is equity capital. The investment opportunities are given by assets X and Y. We will refer to these assets as loan portfolios. The gross returns on the loan portfolios, x and y, are identical and independently distributed with density function $\phi(\cdot)$, which has full support on $[0, s]$, with $s > d$. The joint density function is denoted by $\phi(x, y)$ if independence is not assumed.

There are three periods. At date 1, the two banks make an investment decision. At date 2, loan portfolio returns x and y are revealed. If the return on a bank's investments is insufficient to cover the deposits, d, then a bank run will occur on that particular bank. Consequently, it must liquidate its investments. If no run occurs on the other bank, then the investments can be transferred to the surviving bank at a cost c. However, if a systemic crisis occurs, that is if a bank run occurs on both banks, then there is no bank to continue the investments. Therefore, the investments must be (physically) liquidated at a greater cost cq ($q > 1$).[4] The loan portfolios mature at date 3.

Banks maximize their value by maximizing the expected return for the risk-neutral providers of funds. The probability of a systemic crisis is denoted by π^S and the probability of a run on bank i only by π_i^I. Further, the bank's value if not liquidated is denoted by v_i. So the expected value, W_i, can be written as

$$W_i = E[v_i] - c(\pi_i^I + q\pi^S), \tag{4.1}$$

where the last part reflects the expected default costs. Total welfare in the economy equals the sum of the expected returns of both banks

$$W_1 + W_2 = E[v_1] + E[v_2] - c(\pi_1^I + \pi_2^I + 2q\pi^S). \tag{4.2}$$

Diversification in the original framework is restricted to linear diversification strategies. That is, banks are allowed to diversify asset holdings by constructing linear combinations of the loan portfolios X and Y. Each bank i is allowed to invest a share of their funds, $r_i \in [0, 1]$, in the other bank's loan portfolio.[5] Hence, bank i's (gross) return on investment, if not liquidated, is given by

$$v_1(x, y) = (1 - r_1)x + r_1 y, \tag{4.3}$$

$$v_2(x, y) = (1 - r_2)y + r_2 x. \tag{4.4}$$

The restriction to linear diversification is covered in these equations because the proceeds of the bank's investments are defined as linear combinations of returns x and y.

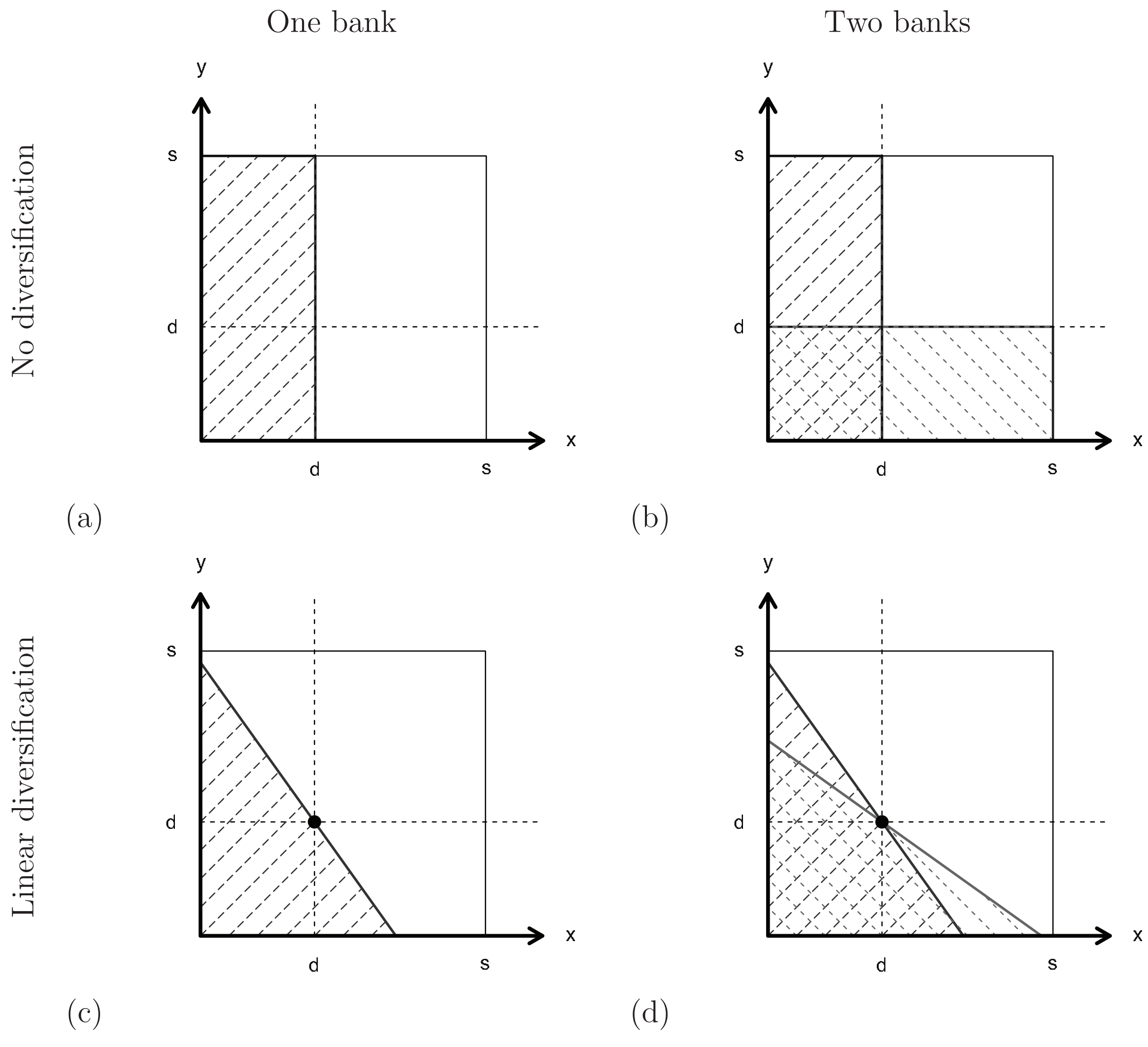
Without diversification, a bank run occurs whenever the return on the bank's own loan portfolio is below d. These bank run outcomes are represented for bank 1 by the dashed area in Figure 4.1, panel (a). Bank 2 has been added in Figure 4.1, panel (b). The set of outcomes under which a systemic crisis occurs is represented by the double dashed square. If banks are allowed to enter into linear diversification, then they can turn the border at which no bank run occurs in point (d,d), see Figure 4.1, panel (c). The probability of a run on bank 1 depends on the probability mass of the dashed area, and can be obtained by taking the surface integral of $\phi(x, y)$ over the dashed area. The probability of a run on bank 1 may decrease due to linear diversification.[6] However, the probability of a

[4]The propositions in our study also hold true under the assumption $q \geq 1$. Further, if it is assumed that the liquidations cost, $cq(n)$, increases in the number of simultaneous failures, n, then the propositions also hold true in a framework with pairwise risk sharing among four banks, where banks 1 and 2 (3 and 4) are endowed with portfolio X (Y).

[5]Wagner (2010) restricts without loss of generality $r_i \in [0, 1/2]$. In the new model, this restriction is not without loss of generality.

[6]With linear diversification this is, for example, the case if $\phi(\cdot)$ denotes a uniform distribution on interval $[0, s]$ with $s > 2d$.

Figure 4.1: No diversication and linear diversification



The dashed areas in panel (a) and (c) represent the outcomes for which a run on bank 1 occurs under, respectively, no diversification and linear diversification. Bank 2 has been added in panel (b) and (d). The outcomes that correspond to joint failures are represented by the double dashed area.

joint failure increases simultaneously because the set of outcomes which correspond to a systemic crisis strictly increases with diversification. Diversification increases the double dashed square in Figure 4.1, panel (b), by two triangles, see Figure 4.1, panel (d). This is the core mechanism behind the dark side of diversification in the framework of Wagner (2010).

4.2.1 Model innovation

The restriction to linear diversification does not allow financial institutions to securitize loan portfolios into tranches with different seniority levels because the return on investment in tranches is nonlinear in the return on the underlying loan portfolios. The innovation is to introduce tranching to the model.

The feature of tranching loan portfolios is modeled as follows. When making the investment decision, banks 1 and 2 securitize their loan portfolios into a senior and a junior tranche. Banks fix the maximum payoff of the senior tranche at $k \in [0, s]$. At date 3, the senior tranche is served first, up to this maximum k. The junior tranche then receives the residual of the return on the underlying loan portfolio. The junior tranches thus receive a nonzero payment only if the proceeds from the underlying portfolio are sufficient to honor the maximum payoff k to the senior tranches. Hence, for securities on portfolio X, the payoff of the senior tranche equals $x^s(k) = \min\{x, k\}$, while the payoff of the junior tranche is $x^j(k) = \max\{x - k, 0\}$. Equivalently, for portfolio Y we have $y^s(k) = \min\{y, k\}$ and $y^j(k) = \max\{y - k, 0\}$. It is assumed that banks keep the junior tranche on their own balance sheet, and choose to exchange fraction r_i of the senior tranches. Finally, the proportion of the senior tranche that is not exchanged, $(1 - r_i)$, remains on bank i's balance sheet.[7]

Following this securitization process, bank i's return on investment, if not liquidated, is given by

$$v_1(x, y) = (1 - r_1)x + r_1 \cdot \max\{x - k, 0\} + r_1 \cdot \min\{y, k\}, \tag{4.5}$$

$$v_2(x, y) = (1 - r_2)y + r_2 \cdot \max\{y - k, 0\} + r_2 \cdot \min\{x, k\}. \tag{4.6}$$

Banks maximize their expected value by setting $k \in [0, s]$ and $r_i \in [0, 1]$. Following Wagner (2010), we focus on symmetric equilibria, such that tranching is essentially a redistribution of the cash flows from loan portfolios X and Y among banks 1 and 2.

It is important to note that linear diversification is captured by equations (4.5) and (4.6) as a special case. If banks set the maximum payoff to the senior tranche equal to the

[7] It is irrelevant to the results whether we assume that banks exchange senior tranches and keep the junior tranche or vice versa.

maximum return on the loan portfolio, i.e., if $k = s$, then the portfolio returns in equations (4.3) and (4.4) are obtained. In this case, the payoff of the junior tranche always equals zero, while the senior tranche replicates the return of the original loan portfolio.

4.2.2 Tranching and bank runs

Because bank runs generate liquidation costs, banks try to avoid bank run outcomes by optimizing their investment strategy, that is by setting $k \in [0, s]$ and $r_i \in [0, 1]$. Given the investment strategy, one can obtain the set of bank run outcomes for both banks. This is done by deriving a 'no bank run' border, $\bar{y}_i(x)$, which provides the minimum return y to prevent a run on bank i given the realization of x. The surface below the function $\bar{y}_i(x)$ on the xy-plane represents the set of bank run outcomes. The probability of a bank run depends on the amount of probability mass in the area with bank run outcomes and can be obtained by taking the corresponding surface integral over the joint density function, $\phi(x, y)$.

The 'no bank run' border, $\bar{y}_i(x)$, is derived by setting the return given no default equal to the level of deposits, i.e., $v_i(x, y) = d$, and solving for y. Due to the nonlinearities, it is expedient to write the solution as three cases that each represent a possible class of investment strategies.[8]

Class 1: conditional on $k < d$ we have

$$\bar{y}_1(x) = \begin{cases} \infty, & \text{if } x \in [0, d), \\ (d + r_1 k - x)/r_1, & \text{if } x \in [d, s]. \end{cases} \tag{4.7}$$

Class 2: conditional on $k \geq d$ and $r_1 k < d$ we have

$$\bar{y}_1(x) = \begin{cases} \infty, & \text{if } x \in [0, (d - r_1 k)/(1 - r_1)), \\ (d + r_1 x - x)/r_1, & \text{if } x \in [(d - r_1 k)/(1 - r_1), k), \\ (d + r_1 k - x)/r_1, & \text{if } x \in [k, s]. \end{cases} \tag{4.8}$$

Class 3: conditional on $k \geq d$ and $r_1 k \geq d$ we have

$$\bar{y}_1(x) = \begin{cases} (d + r_1 x - x)/r_1, & \text{if } x \in [0, k), \\ (d + r_1 k - x)/r_1, & \text{if } x \in [k, s]. \end{cases} \tag{4.9}$$

The set of bank run outcomes within Class 1 is illustrated for bank 1 by the dashed area in Figure 4.2, panel (a).[9] In comparison with the dashed area in Figure 4.1, panel (a), the set of bank run outcomes within Class 1 is increased relative to the no diversification case. The intuition is as follows. Within Class 1, bank 1 sets the maximum payoff of the senior

[8] Due to space considerations, we provide $\bar{y}_i(x)$ for bank 1 only. Because of symmetry, the 'no bank run' border for bank 2, $\bar{y}_2(x)$, can be obtained by mirroring $\bar{y}_1(x)$ in $y = x$.

[9] The notation $\bar{y}_i(x) = \infty$ is used to specify that any return y would be insufficient to avoid a run on bank i (given the support of the density function $[0, s]$).

tranches below the amount of deposits on its balance sheet, i.e., $k < d$. Consequently, the bank's losses on its loan portfolio are shared, only after losses generated by the junior tranches are sufficiently large to trigger a run on the bank. Therefore, bank runs are not avoided. On the contrary, for $r_1 > 0$, the bank is also exposed to a senior tranche on loan portfolio Y. This means bank 1 is also vulnerable to bad realizations of y, which is represented by the additional dashed triangle in Figure 4.2, panel (a).

Within Class 2 and Class 3, bank 1 sets the maximum payoff of the senior tranches at greater than or equal to the amount of deposits on the balance sheet, i.e., $k \geq d$. However, within Class 2, the maximum payoff of the investment in senior tranches of loan portfolio Y is not sufficient to avoid a run on bank 1 for every possible outcome of loan portfolio X (because $r_1 k < d$). Therefore, the minimum return $\bar{y}_1(x)$ in equation (4.8) equals infinity for low realizations of x, which is represented by the dashed rectangle along the y-axis in Figure 4.2, panel (b). By contrast, within Class 3 any adverse return on loan portfolio X can be absorbed by a sufficiently high return on loan portfolio Y, see Figure 4.2, panel (c).

4.3 Results

Systemic crises cannot be avoided by linear diversification. In particular, Wagner (2010) shows that linear diversification cannot decrease the set of outcomes with a joint failure beyond the square to the left and below point (d, d) in Figure 4.1, panel (a). This result proves to be robust if tranching loan portfolios is allowed.

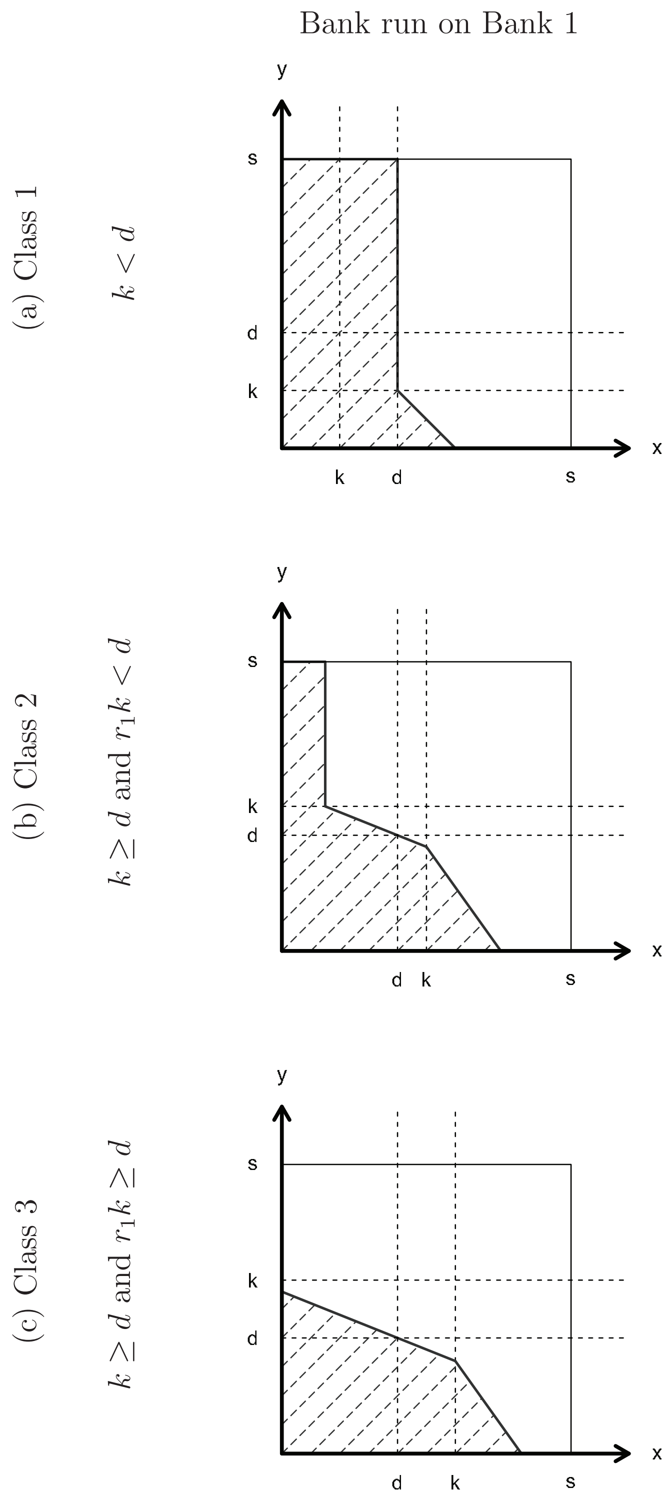
Proposition 4.3.1 *If banks maintain the junior tranches of their loan portfolios, as in equations* (4.5) *and* (4.6)*, then the probability of a joint failure cannot be decreased by tranching or linear diversification.*

Although tranching and linear diversification do not decrease the probability of systemic crises, they may decrease the probability of a run on each individual bank. Interestingly, the investment strategy that minimizes the probability of individual bank failures is independent of the marginal return distribution, $\phi(\cdot)$.

Proposition 4.3.2 *The probability of a run on bank i is minimized if and only if bank i sets the maximum payoff of the senior tranche equal to the level of deposits, i.e., $k = d$, and exchanges its senior tranche entirely, i.e., $r_i = 1$.*

Corollary 4.3.1 *If bank i adopts the investment strategy with tranching in Proposition 4.3.2, then the probability of a run on bank i is strictly smaller than the minimum bank run probability attainable by any linear diversification strategy.*

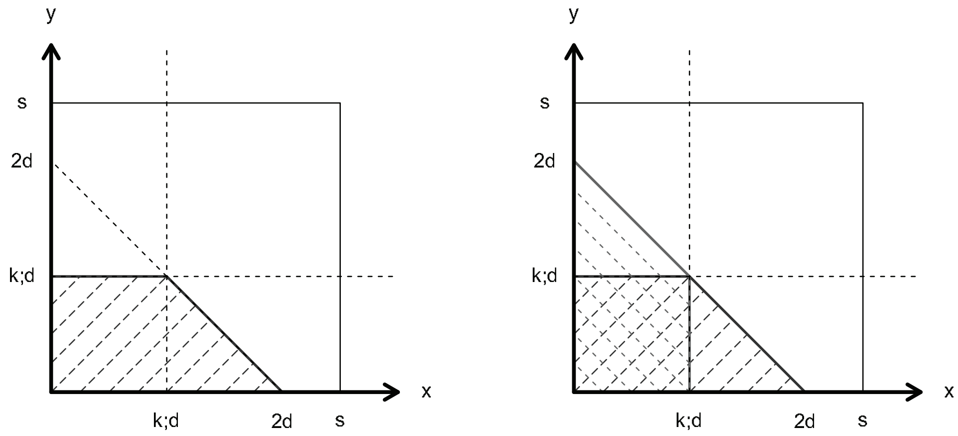
Figure 4.2: The three classes of investment strategies with tranching



The dashed areas in panel (a), (b) and (c) give the possible outcomes for which a run on bank 1 occurs for each of the three classes of possible investment strategies described in, respectively, equations (4.7), (4.8) and (4.9).

Figure 4.3: Diversification through securitization

(a) One Bank (b) Two Banks



The dashed area in panel (a) represents the set of outcomes for which a run on bank 1 occurs under the investment strategy that minimizes the probability of runs on bank 1 in Proposition 4.3.2. Bank 2 has been added to the figure in panel (b). The set of outcomes that correspond to joint failures is represented by the double dashed area.

The effectiveness of the investment strategy in Proposition 4.3.2 is illustrated in Figure 4.3. Following equation (4.9), the 'no bank run' border for bank 1 under the strategy in Proposition 4.3.2 is given by

$$\bar{y}_1^*(x) = \begin{cases} d, & \text{if } x = [0, d), \\ 2d - x, & \text{if } x = [d, s]. \end{cases}$$

The probability that bank 1 will become insolvent under the strategy in Proposition 4.3.2 is

$$\pi_1^* = \int_0^d \int_0^d \phi(x)\phi(y)\, dy\, dx + \int_d^{\min\{2d,s\}} \int_0^{2d-x} \phi(x)\phi(y)\, dy\, dx, \tag{4.10}$$

which is the probability of observing an outcome in the dashed area in Figure 4.3, panel (a). The first and second double integral in equation (4.10) correspond, respectively, to the dashed square and the dashed triangle in the set of bank run outcomes.

From Corollary 4.3.1, it follows that the strategy with the smallest bank run probability involves tranching, regardless of the distribution function $\phi(\cdot)$. It is worth noting that there are always nonnegative diversification benefits with tranching possible. By contrast, in some cases every degree of linear diversification increases the probability of bank runs.[10] This is because the strategy in Proposition 4.3.2 shrinks the set of bank

[10] For example, if $\phi(\cdot)$ denotes a uniform distribution on interval $[0, s]$ with $d < s < 2d$, then every degree of linear diversification increases the probability of a bank run. In the case of the tranching strategy in Proposition 4.3.2, the bank run area would be characterized by a dashed square and a triangle that is truncated on the right side for values above s.

run outcomes relative to the no diversification case without introducing new bank run outcomes.

The probability of simultaneous bank runs is not increased by the tranching strategy in Proposition 4.3.2. This follows from the size of the double dashed area in Figure 4.3, panel (b). Therefore, the strategy that achieves the smallest bank run probability also minimizes the expected costs due to systemic crises. This ensures both the private and social optimality of the strategy.

Proposition 4.3.3 *The optimal investment strategy is to set the maximum payoff of the senior tranches equal to the level of deposits, i.e., $k = d$, while exchanging the senior tranche entirely, i.e., $r_1 = r_2 = 1$.*

According to Proposition 4.3.3, banks with high leverage ratios should invest the majority of their funds in securities issued against one of the two underlying loan portfolios. Suppose, for example, that both banks in the model have a leverage ratio of 10. In other words, 90% of the principal in their loan portfolios is funded by deposits, while the remaining 10% is funded by equity. It is then optimal, according to Proposition 4.3.3, for each bank to hold the senior tranche of one loan portfolio and the junior tranche of the other portfolio. The maximum payoff of the senior tranche should be fixed at 90% of the principal, while the junior tranche should be eligible only for any return above this threshold (which potentially includes the remaining 10% of the principal and agreed interest payments). Although the 'degree of diversification' in this investment strategy seems relatively low, it results in fewer bank failures and systemic crises than if banks were to spread their investments more evenly over the two loan portfolios.

4.4 Securitization and confidence shocks

The findings in the previous section are derived under the assumption that banks have precise knowledge about the condition under which bank runs occur. Based on this knowledge, banks can perfectly calibrate the payoffs of the senior tranches by setting $k = d$. In practice, however, banks do not possess this knowledge. Especially during periods of low depositor confidence, bank runs may occur even if the return on the bank's investments is sufficient to repay all depositors.

In this section, we discuss whether tranching is still optimal in the presence of confidence shocks. The shock in depositor confidence is modeled as follows. After banks set the investment strategy at date 1, a shock to depositor confidence occurs. Due to this confidence shock, depositors will participate in a run if the return on the bank's investments at date 2 is smaller than δ. Hence, if $\delta > d$, depositors may run on solvent banks.

First, we consider the expected liquidation costs of three different investment strategies for specific levels of δ. To quantify the results, we assume that the returns on the loan portfolios, x and y, are uniformly distributed over the interval $[0, s]$, with $s > \delta(1 + \sqrt{2q-1})$. This distributional assumption does not qualitatively affect the results. For a given minimum return to avoid bank runs, δ, the expected default costs in case of the no tranching and no diversification strategy are

$$c(\pi_1^I + \pi_2^I + 2q\pi^S) = 2c\frac{s\delta - \delta^2}{s^2} + 2cq\frac{\delta^2}{s^2}. \tag{4.11}$$

Further, following Proposition 1 of Wagner (2010), the social optimal strategy with linear diversification is $r^* = 1/(1 + \sqrt{2q-1})$. If banks follow this strategy, the expected default costs equal

$$c(\pi_1^I + \pi_2^I + 2q\pi^S) = 2c\left(q + \sqrt{2q-1}\right)\frac{\delta^2}{s^2}. \tag{4.12}$$

Finally, if banks adopt the tranching strategy in Proposition 4.3.3, the expected default costs are given by

$$c(\pi_1^I + \pi_2^I + 2q\pi^S) = \begin{cases} 2c\frac{k\delta - (1/2)\delta^2}{s^2} + 2cq\frac{\delta^2}{s^2}, & \text{if } \delta \in [0, k], \\ 2c\frac{s\delta - \delta^2}{s^2} + 2cq\frac{\delta^2}{s^2} + c(2q-1)\frac{k^2}{s^2}, & \text{if } \delta \in (k, s]. \end{cases} \tag{4.13}$$

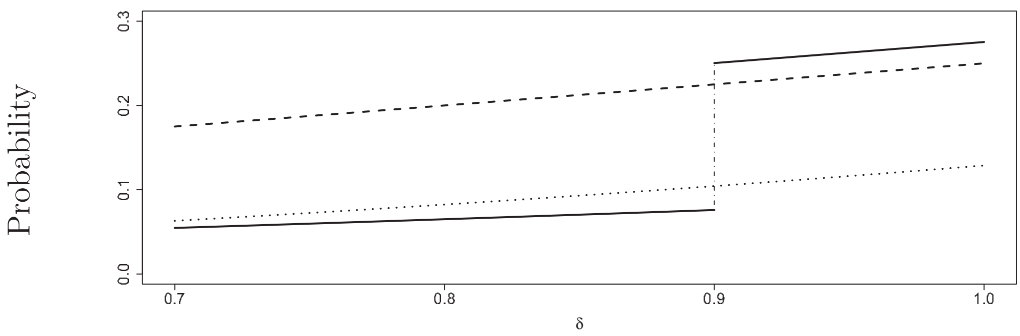
Figure 4.4 illustrates the expected default costs for different levels of δ. It reflects the probability that each bank fails in panel (a), the probability of a systemic crisis in panel (b), and the expected liquidation costs in panel (c). From Figure 4.4 it follows that the bank run probabilities will react smoothly to a change in depositor confidence if there is no diversification (dashed line) or linear diversification (dotted line), while the bank run probabilities may react quite erratically in case of the tranching strategy in Proposition 4.3.3 (solid line). Although the tranching strategy yields the lowest expected default costs in the absence of adverse confidence shocks ($\delta = d = 0.9$), even a small adverse shock may considerably increase the risks in the financial system (whenever $\delta > d = k$).

The origin of the jump in expected default costs in equation (4.13) is a shift in the class of investment strategies discussed in Subsection 4.2.2. The case $k \geq \delta$ corresponds to an investment strategy in Class 3, equation (4.9), while the case $k < \delta$ corresponds to an investment strategy in Class 1, equation (4.7). The strategies in Class 1 are known to result in a larger set of bank run outcomes than the no diversification strategy, see also Figure 4.2, panel (a). Hence, if $\delta > k$, the situation in a banking system with diversification through securitization is worse compared to a banking system without any diversification.

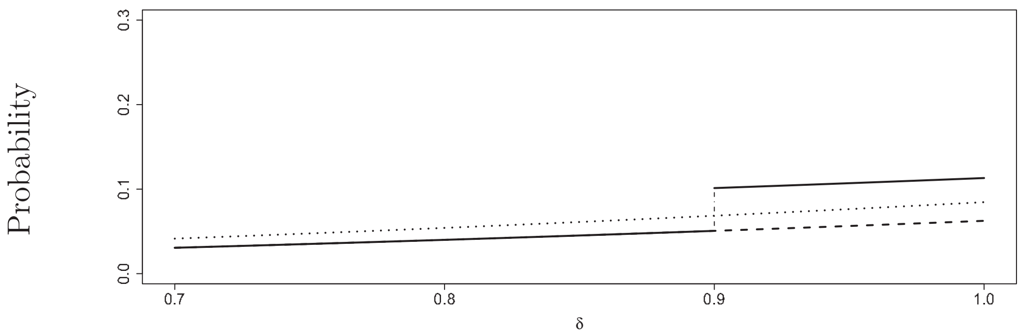
From a risk management point of view, the scenario $k < \delta$ is particularly unfavorable. Hence, a strategy of naively setting $k = d$ in a situation with uncertainty is not recommended. Instead, prudent banks that anticipate the possibility of a confidence shock

Figure 4.4: Diversification outcomes after a confidence shock

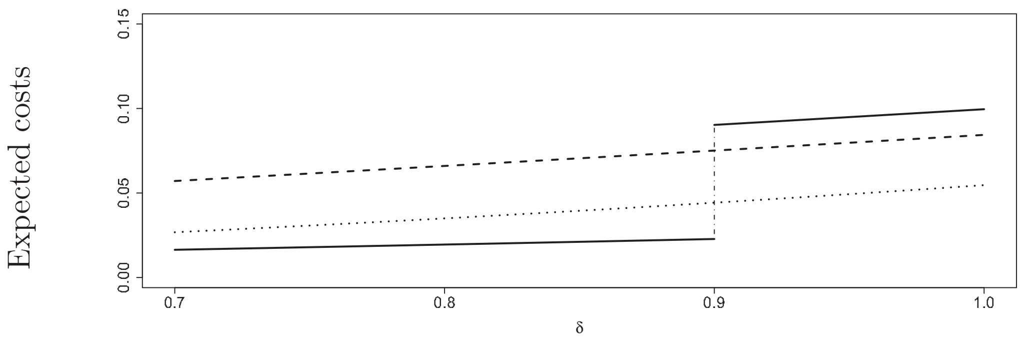
(a) Probability of a run on each bank: $\pi_i^I + \pi^S$



(b) Probability of a system failure: π^S



(c) Expected liquidation costs: $c(\pi_1^I + \pi_2^I + 2q\pi^S)$



The panels reflect the outcomes of different investment strategies given the minimum gross return on investment necessary to avoid a bank run, δ. The solid lines represent the results of the strategy with tranching in Proposition 4.3.3. The dashed (dotted) lines correspond to the results with no tranching or diversification (optimal linear diversification). Panel (a) shows the failure probability of each bank, panel (b) the probability of a joint failure and panel (c) the expected liquidation costs. The parameter choices are $c = 15\%$, $cq = 22.5\%$, $d = 0.9$ and $s = 4$.

may choose to move away from the strategy in Proposition 4.3.3 by setting k at some level above d. The question remains whether such a prudent tranching strategy would perform better than the optimal linear diversification strategy. Our focus is on deriving a condition on the range of possible values of δ, i.e., $[\delta_{MIN}, \delta_{MAX}]$, under which a prudent tranching strategy performs better than the optimal linear diversification strategy.[11] From equations (4.12) and (4.13) we conclude that a prudent tranching strategy that sets $k = \delta_{MAX}$ and $r = 1$ is preferable to the optimal linear diversification strategy, if

$$\delta_{MAX} \leq \left(\frac{1}{2} + \sqrt{2q-1}\right)\delta_{MIN}, \tag{4.14}$$

regardless of the distribution of δ.

The condition on the range of δ can be considered relatively weak.[12] Hence, even if there is significant dispersion in the possible values of δ, a prudent tranching strategy performs better than the optimal linear diversification strategy. Therefore, the diversification benefits of tranching are relatively robust, and do not depend on having perfect knowledge of when bank runs occur. Interestingly, the better performance of the prudent tranching strategy is especially robust in the case of relatively high liquidation costs during systemic crises, i.e., for high values of q. This is because the linear diversification strategy increases the probability of systemic crises relative to the no diversification case, while a prudent tranching strategy with $k = \delta_{MAX}$ does not. Therefore, tranching may be especially important in a system with relatively high liquidation costs in the event of systemic crises.

It is further worth noting that tranching is not necessarily optimal in all situations. It is sometimes preferable for banks to choose a linear diversification strategy. However, this is only the case if there is a potential for runs on very solvent banks, i.e., if banks have to anticipate a very wide range of values for δ. From a policy perspective, this may serve as an argument to promise liquidity support in the event of a run on 'very solvent' banks. Providing liquidity support if bank runs occur on very solvent banks removes the impact of exceptional high values of δ (relative to d). Therefore, the ex ante knowledge of liquidity support in the event of runs on very solvent banks encourages these banks to opt for the more efficient tranching strategy. Consequently, in addition to avoiding the liquidation of solvent banks ex post, promising liquidity assistance to very solvent banks may also prevent financial institutions from becoming insolvent.

[11] Although it may be unrealistic to assume that banks know the exact distribution function of δ, it is probably not unrealistic to assume that they have some idea of the range of its possible values.

[12] Following the condition in (4.14), δ_{MAX} could be 50% higher than δ_{MIN} as $q \to 1$.

4.5 Discussion

4.5.1 Dependence

Although the returns on X and Y are assumed to be independent in the model, the propositions hold true under the more general assumption of a symmetric return distribution, i.e., $\phi(x, y) = \phi(y, x)$. For the *strict* optimality of the optimal tranching strategy, it additionally has to be assumed that any (x, y) belonging to $[0, s] \times [0, s]$ may occur as a realization of (X, Y). Further, it is notable that it is not necessary to assume that the return distribution is bounded from above, i.e., $s \to \infty$. Therefore, many popular distributions satisfy the two conditions on the return distribution, including for example the bivariate (log)normal distribution with a correlation coefficient $-1 < \rho_{x,y} < 1$ and equal marginals, i.e., $\sigma_x = \sigma_y$.

4.5.2 More assets

The diversification effect of tranching is not limited by the number of assets in the model. Although the model is based on two assets, X and Y, the diversification strategy with tranching can be applied on two portfolios that are formed out of any (even) number of assets. Suppose there are four assets, $X_1, ..., X_4$. Consider the following two portfolios: $X = X_1 + X_2$ and $Y = X_3 + X_4$. Let each bank hold a junior tranche on one portfolio and a senior tranche on the other portfolio. From both an individual and macroprudential point of view, this strategy performs better than full diversification, in which both banks hold 50% of each portfolio, which boils down to both banks holding 50% of each asset.

4.5.3 Optimal risk sharing

Thus far, the discussion has not answered the question of whether optimal risk sharing is achieved by the optimal securitization strategy in Proposition 4.3.3, or whether it is possible to improve on the strategy by adopting more sophisticated tranching strategies. In the context of the model, optimal risk sharing means avoiding liquidation costs whenever possible. Hence, in the case of optimal risk sharing, the number of surviving banks should depend solely on the total resources available to repay depositors, i.e., $x + y$. It follows from this that a contract exhibits optimal risk sharing if, and only if, it satisfies the following conditions for any (x, y):[13]

(i) If $x + y \geq 2d$: both banks survive;

[13] We thank an anonymous referee for mentioning that Conditions 2 and 3 can be combined in a single condition as ‘If $d \leq x + y < 2d$: one bank survives’.

(ii) If $d \leq x + y < 2d$, and either $x \geq d$ or $y \geq d$: one bank survives;

(iii) If $d \leq x + y < 2d$, and both $x < d$ and $y < d$: one bank survives;

(iv) If $x + y < d$: neither bank survives.

Whether an investment strategy satisfies the optimal risk sharing conditions can be verified by diagrams such as those presented in Figures 1 and 3. The no diversification strategy in Figure 1, panel (b), satisfies Conditions 2 and 4, but not Conditions 1 and 3, while a full risk-sharing strategy satisfies Conditions 1 and 4, but not Conditions 2 and 3. Furthermore, of all the possible risk sharing strategies in equations (4.5) and (4.6), the optimal tranching strategy is the only one simultaneously satisfying Conditions 1, 2 and 4.

The optimal tranching strategy does not satisfy Condition 3, however. Condition 3 considers the scenarios in which both banks would fail in the absence of risk sharing, i.e., $x < d$ and $y < d$, but where the total resources would, in principle, be sufficient to save one of them, i.e., $x + y \geq d$. In such a scenario, it would be optimal to allocate the resources of the two failing institutions to one surviving institution. Nevertheless, it follows from Proposition 4.3.1 that this does not occur under any tranching or diversification strategy. Therefore, the optimal tranching strategy does not result in optimal risk sharing.

Importantly, the reason that optimal risk sharing is not achieved is not because of restrictions on the possible tranching strategies in the model, but instead because of a characteristic of tranching. Once a tranche has been defined, its payoff depends solely on the uncertain payoff of the underlying. In this respect, tranches are comparable to simple option contracts.[14] By contrast, in order to decide which bank should obtain the resources required to survive in the scenarios considered in Condition 3, an instrument where the payoff depends on additional information is needed. One example would be an instrument that pays out all the proceeds to the bank whose loan portfolio has the highest return (if both $x < d$ and $y < d$), while another example would be an instrument that pays out all the proceeds to the bank winning a coin toss. To determine the payoffs, such 'instruments' need information in addition to the return on the underlying (namely, the return on the components of the underlying, or the outcome of a coin toss). Tranches do not facilitate such payoff structures. Hence, even if more sophisticated tranching strategies involving more than two seniority classes were considered, it would still not be possible to satisfy Condition 3. In summary, although tranching improves risk sharing, optimal risk sharing is not attainable through any symmetric tranching strategy.

[14]This is also the case with more sophisticated tranching strategies, involving several layers of seniority. The payoff structure of a tranche is always similar to that of a put or call option, or a combination of the two.

4.5.4 Shareholders

The model assumes that financial institutions seek to maximize the total value of equity and debt. There is no conflict between shareholders and depositors. But do banks continue to follow the optimal tranching strategy if they seek to maximize the total value of equity? The answer to this question is no. From the shareholders' perspective, the no tranching and no diversification strategy is the best strategy. Not engaging in any tranching or diversification maximizes the option value of equity capital.[15]

The reason that banks that maximize the total value of equity do not adopt the optimal tranching strategy is because avoiding defaults, in the context of the model, always implies transferring funds from the shareholders of a solvent bank to the debt holders of an otherwise insolvent bank. Consequently, avoiding defaults is always costly for shareholders. In addition, because liquidation costs represent an *ex post* burden on debt holders, these costs do not provide shareholders with an incentive to avoid insolvencies (the costs of borrowing are given). In summary, the optimal tranching strategy (or any other diversification strategy) results in additional costs for shareholders, without any compensating benefits.

Owing to the internal agency conflict between depositors and shareholders, regulators cannot expect banks to follow the socially desirable tranching strategies in equilibrium. Therefore, although financial institutions often diversify their activities, the question remains as to whether the observed degree of diversification is socially beneficial. The answer to this question depends on the underlying investment strategy, which is in turn affected by banks' incentives.

Besides the fact that shareholder value is maximized by banks adopting a strategy without tranching, the wrong incentives may encourage banks to adopt harmful tranching strategies (i.e., with $k < d$). Suppose, for instance, that a regulator realizes that banks will not diversify at their own initiative if their objective is to maximize shareholder value and so forces them to hold a diversified investment portfolio. Although this may be socially optimal in the case of linear diversification, it may be counterproductive if tranching is possible. This is because banks can achieve the required degree of diversification by investing their funds in the senior tranche of one loan portfolio and the junior tranche of another portfolio. If $k < d$, such a strategy results in more bank failures and systemic crises than the no tranching and no diversification strategy. Nevertheless, depending on the model parameters, shareholders may prefer such a strategy to the linear diversification strategy because it offers them a higher expected return.[16] In summary, given the internal

[15] The formal proof of this statement is available from the author upon request.

[16] This is illustrated by the following numerical example in the case of uniformly distributed portfolio returns with $s = 3.2$. In this case, investing in the senior tranche of one loan portfolio and the junior

agency conflict, regulators cannot presume the tranching strategies adopted by banks to be free of harmful effects.

Although tranching allows financial institutions to adopt improved risk management strategies, it also enhances the potential adverse impact that investment strategies may have on society in the case of internal agency conflicts. When seeking to establish whether current tranching and diversification practices at financial institutions are socially beneficial, it is insufficient for regulators solely to consider general indicators such as the degree of diversification. Our results suggest that regulators need to examine the payoff structure of the underlying securities in greater depth in order to establish whether the financial stability objectives are actually achieved.

4.6 Concluding remarks

Securitization of loan portfolios has gained in importance in recent decades. It decreased the entrance cost to financial institutions of investing in other loan types and increased the availability of investments in specific business or geographical areas without the need to set up entirely new loan operations. Securitization thus catalyzed diversification by offering new prospects to diversify asset holdings. However, did securitization also increase the dark side of diversification? Not necessarily, according to the conclusions drawn in the present paper. Nevertheless, our conclusions show that securitization by financial institutions may have cast shadows of its own on the stability of the financial system and that of individual institutions.

Previous theoretical findings suggest that the design of junior and senior securities is driven by information asymmetries, see, for example, Boot and Thakor (1993), Riddiough (1997) and DeMarzo and Duffie (1999). The present study suggests an alternative motivation. It shows that risk management considerations alone may provide sufficient incentives to design securities with different seniority levels. Although the two explanations may be complementary, they have different empirical implications. Following the notion of information asymmetry, junior securities are more likely to be traded by investors with an information advantage. By contrast, the risk management aspect examined in our paper indicates that market participants' decision to invest in junior or senior tranches depends on the combination of assets held in their portfolios.

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tranche of another loan portfolio with $k = 0.64$ corresponds to a degree of diversification of 36% (in terms of expected returns). If $d = 0.90$, shareholders will prefer a tranching strategy with $k = 0.64$ and $r = 1$ to a linear diversification strategy with $r = 0.36$ (the expected returns for the shareholders in the case of tranching and linear diversification are 0.7848 and 0.7515 respectively). By contrast, if $d = 0.65$, shareholders will prefer the linear diversification strategy with $r = 0.36$ (the expected returns are 0.9693 and 0.9694 respectively).

Several interesting directions remain unexplored in the current study. One involves analyzing individual and systemic risk due to diversification through securitization in a system with more banks and more assets. This could trigger potential network effects or draw attention to the default costs in the event of a partial systemic failure. Our final remark concerns the similarity in the payoff structure of tranches and options. Under additional assumptions it may be possible to derive the prices of tranches from an option pricing framework. These prices could be useful in a study on potential contagion due to tranching.

4.A Appendix A. Proofs

4.A.1 Derivation of the no bank run border $\bar{y}_1(x)$

The set of possible returns (x, y) for bank i to be solvent is derived from solving $v_i(x, y) \geq d$. Following (4.5), for bank 1 we must have

$$(1 - r_1)x + r_1 \cdot \max\{x - k, 0\} + r_1 \cdot \min\{y, k\} \geq d.$$

The solution is given by the union of the following subsets:
Subset A, defined on $x \in [0, k)$ and $y \in [0, k]$, is the set where $y \geq \frac{d}{r_1} - \frac{x - r_1 x}{r_1}$.
Subset B, defined on $x \in [0, k)$ and $y \in (k, s]$, is the set where $x \geq \frac{d - r_1 k}{1 - r_1}$.
Subset C, defined on $x \in [k, s]$ and $y \in [0, k]$, is the set where $y \geq \frac{d}{r_1} - \frac{x - r_1 k}{r_1}$.
Subset D, defined on $x \in [k, s]$ and $y \in (k, s]$, is the set where $x \geq d$.

We separate the solution into three cases: case 1 in which $k < d$, case 2 in which $k \geq d$ and $r_1 k < d$ and case 3 in which $k \geq d$ and $r_1 k \geq d$. These three cases refer to the three different classes of investment strategies in Subsection 4.2.2.

Case 1: subset A is empty. To see this, we rewrite the condition as $r_1(y - x) \geq d + x$. In subset A, $y - x$ is bounded by k. Because we have $k < d$, the condition is never satisfied for $r_1 \in [0, 1]$. Also, B is empty. Rewriting the condition gives $(1 - r_1)x \geq d - r_1 k$, which is never satisfied for $x < k < d$. The condition for subset C can be rewritten as $r_1 y \geq d - x + r_1 k$. For $y \leq k$, the condition cannot be satisfied, unless we have $x \geq d$. If $x \geq d$, then the left border of the set is given by $y \geq \frac{d}{r_1} - \frac{x - r_1 k}{r_1}$ (until we have $y \geq 0$ for $x \geq d + r_1 k$). Within subset D, the condition is $x \geq d$. Hence, equation (4.7) provides the minimum return y that is necessary for bank 1 to be solvent given return x for case 1.

Case 2: in subset A, rewriting the condition gives $x \geq (d - r_1 k)/(1 - r_1)$. Because we have $r_1 k < d$, it follows from the condition that subset A is empty for $x \in [0, \frac{d - r_1 k}{1 - r_1})$, while subset A is nonempty for $x \in [\frac{d - r_1 k}{1 - r_1}, k)$ and has the left border $y \geq \frac{d}{r_1} - \frac{x - r_1 x}{r_1}$. The left border of subset B is given by $x \geq \frac{d - r_1 k}{1 - r_1}$, because $r_1 k < d$. Solving the condition for subset C to zero gives $x = d + r_1 k$. Hence, if $k \leq d + r_1 k$, then the left border of C is described by $y \geq \frac{d}{r_1} - \frac{x - r_1 k}{r_1}$ for $x \in [k, d + r_1 k)$. If $k > d + r_1 k$, then the condition on subset C is not binding. The condition for subset D is not binding because $k \geq d$. Hence, for case 2, the return $\bar{y}_1$ is given by equation (4.8).

Case 3: following the derivation for case 2, subset A is nonempty for $x \in [0, k)$, because the condition $x \geq (d - r_1 k)/(1 - r_1)$ is always satisfied provided that $r_1 k \geq d$. The left border of subset A is described by $y \geq \frac{d}{r_1} - \frac{x - r_1 x}{r_1}$. The conditions on B and D are not binding because, respectively, $r_1 k \geq d$ and $d < k (\leq x)$. For subset C, the derivation of the left border is the same as for case 2. Hence, for case 3 the return $\bar{y}_1$ is given by equation (4.9).

4.A.2 Proof of Proposition 4.3.1

The no tranching and no diversification case is given by $r_1 = r_2 = 0$. For $r_1 = 0$, the minimum return for bank 1 to be solvent, $\bar{y}_1(x)$, is given by either equation (4.7) or (4.8). Given $r_1 = 0$, both equations give $\bar{y}_1(x) = \infty$ for $x \in [0, d)$ and $\bar{y}_1(x) \leq 0$ for $x \in [d, s]$. Hence, if $r_1 = 0$, bank 1 is solvent if and only if $x \geq d$. Conversely, if $r_2 = 0$, bank 2 is solvent if and only if $y \geq d$. Hence, without tranching or diversification, a system failure occurs if and only if both $x < d$ and $y < d$, see also the double dashed area in Figure 4.1, panel (b).

When allowing for tranching and diversification, first we have that $\bar{y}_1 = \infty$ if $x \in [0, d)$ for any $k \in [0, d)$ and $r_1 \in [0, 1]$ from equation (4.7). Second, from equations (4.8) and (4.9), it follows that $\bar{y}_1(d) = d$ for any $k \in [d, s]$ and $r_1 \in [0, 1]$. Note that $\bar{y}_1(x)$ in equations (4.8) and (4.9) are non-increasing functions in x for any $k \in [d, s]$ and $r_1 \in [0, 1]$. Combining the three gives $\bar{y}_1(x) \geq d$ if $x \in [0, d)$ for any $k \in [0, s]$ and $r_1 \in [0, 1]$. Consequently, a run on bank 1 must occur if both $x < d$ and $y < d$ for any $k \in [0, s]$ and $r_1 \in [0, 1]$. From symmetry along the 45-degree line, $y = x$, a run on bank 2 occurs if both $y < d$ and $x < d$ for any $k \in [0, s]$ and $r_2 \in [0, 1]$. Consequently, irrespective of the tranching or diversification strategy, a system failure occurs if both $x < d$ and $y < d$. □

4.A.3 Proof of Proposition 4.3.2

Proposition 4.3.2 is proven if $\pi_1 > \pi_1^*$ for any $k \in [0, s]$ and $r_1 \in [0, 1]$, except $k = d$ and $r_1 = 1$. The probability of a bank run for each strategy can be obtained by taking the surface integral of $\phi(x, y)$ over the area below the no bank run border

$$\pi_1 = \int_0^\infty \int_0^{\bar{y}_1(x)} \phi(x, y)\, dy\, dx. \tag{4.15}$$

From Proposition 4.3.1 it follows that all strategies associate the square of outcomes with both x and y below d with a run on bank 1. Therefore, the double integral can be written out as

$$\pi_1 = \int_0^d \int_0^d \phi(x, y)\, dy\, dx + \int_d^\infty \int_0^{\bar{y}_1(x)} \phi(x, y)\, dy\, dx + \int_0^d \int_d^{\bar{y}_1(x)} \phi(x, y)\, dy\, dx \tag{4.16}$$

$$=: I_A^* + I_B + I_C. \tag{4.17}$$

Similarly, the optimal strategy can be written as

$$\pi_1^* = \int_0^d \int_0^d \phi(x, y)\, dy\, dx + \int_d^{2d} \int_0^{2d-x} \phi(x, y)\, dy\, dx \tag{4.18}$$

$$=:I_A^* + I_B^*. \tag{4.19}$$

Further, the symmetry of the distribution function, $\phi(x, y) = \phi(y, x)$, gives

$$\pi_1^* = \int_0^d \int_0^d \phi(x, y)\, dy\, dx + \int_0^d \int_d^{2d-x} \phi(x, y)\, dy\, dx \tag{4.20}$$
$$=:I_A^* + I_C^*, \tag{4.21}$$

with $I_B^* = I_C^*$. To prove $\pi_1 > \pi_1^*$ for a certain strategy, it is sufficient to prove either $I_B > I_B^*$ or $I_C > I_C^*$. From comparing (4.16) with (4.18) and (4.20), it follows that it is sufficient to prove, respectively, either

$$\bar{y}_1(x) > 2d - x \text{ for } d < x < 2d, \tag{4.22}$$

or

$$\bar{y}_1(x) > 2d - x \text{ for } 0 < x < d. \tag{4.23}$$

To prove Proposition 4.3.2, it is necessary to show this for any $k \in [0, s]$ and $r_1 \in [0, 1]$, except $k = d$ and $r_1 = 1$. This describes the general line of the proof.

First, we consider $k < d$, i.e., the strategies in Class 1. The corresponding no bank run border is given in equation (4.7). For $x \leq d$ we have $\bar{y}_1(x) = \infty > 2d - x$, which proves $k < d$ yields $\pi_1 > \pi_1^*$ via (4.23). An illustration of this case is provided in Figure 4.5, panel (a).

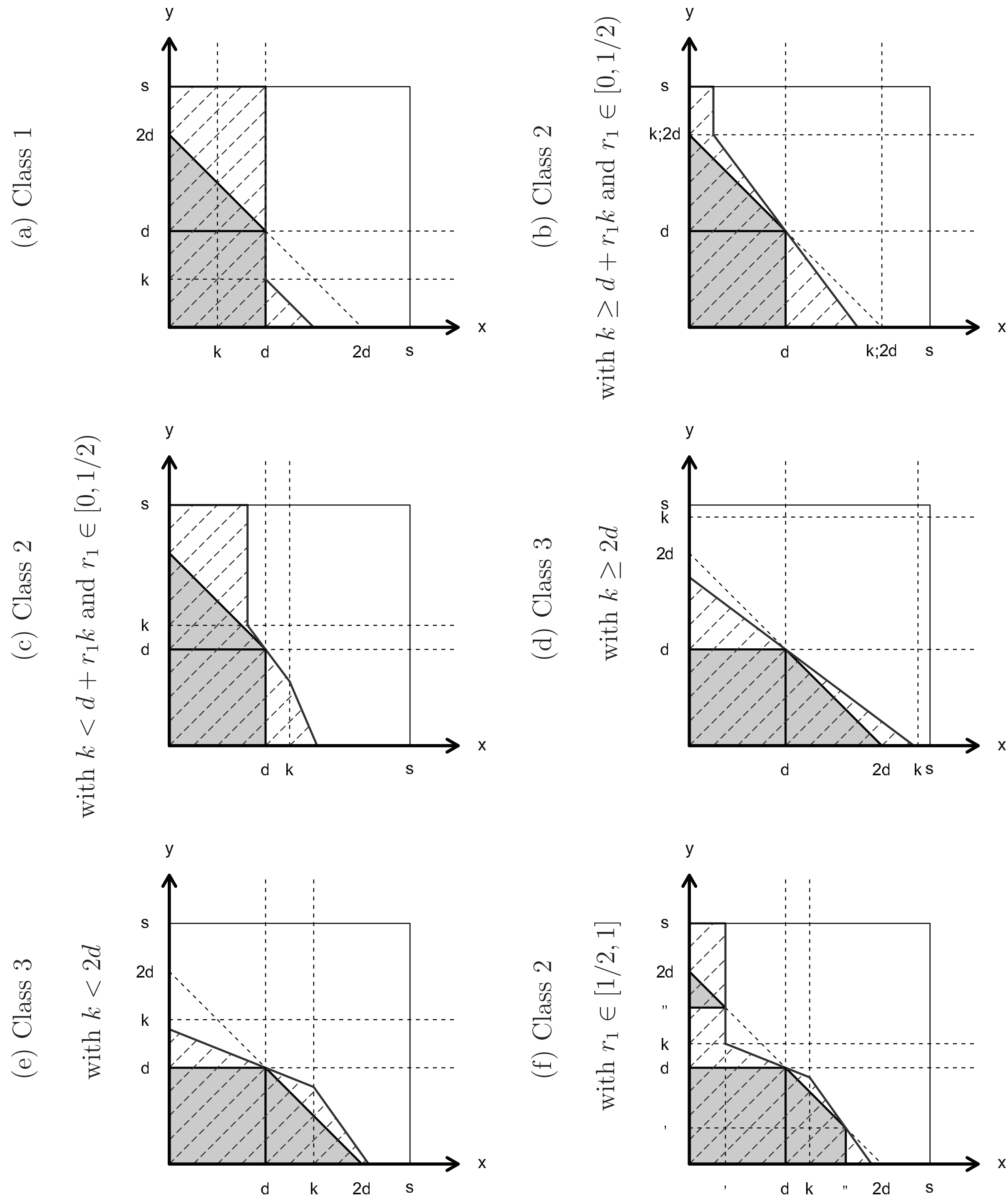
Second, we consider the case $r_1 \in [0, 1/2)$ and $k \geq d$. In this case, the no bank run border is given by either (4.8) or (4.9). Following these equations, for $x < d$ and $k \geq d$, we either have $\bar{y}_1(x) = \infty$ or $\bar{y}_1(x) = (d + r_1x - x)/r_1$. Hence, for $x < d$ we have $\bar{y}_1(x) \geq (d + r_1x - x)/r_1$. For the proof it must be shown that for any $x < d$

$$(d + r_1x - x)/r_1 > 2d - x, \tag{4.24}$$
$$d + r_1x - x > 2r_1d - r_1x, \tag{4.25}$$
$$(1 - 2r_1)d > (1 - 2r_1)x. \tag{4.26}$$

It follows from inequality (4.26) that (4.24) is implied by $x < d$ provided that $r_1 \in [0, 1/2)$. This proves $r_1 \in [0, 1/2)$ and $k \geq d$ yields $\pi_1 > \pi_1^*$ via (4.23). An illustration of this case with $k \geq d + r_1k$ is given in Figure 4.5, panel (b). Figure 4.5, panel (c) provides an illustration of this case for $k < d + r_1k$.

Third, when concentrating on Class 3, i.e., $k \geq d$ and $r_1k \geq d$, we first consider the case $k \geq 2d$ and $r_1 \in [1/2, 1]$. From equation (4.9) we have $\bar{y}_1(x) = (d + r_1x - x)/r_1$ for $0 \leq x < 2d$. It follows from inequality (4.26) that $\bar{y}_1(x) > 2d - x$ is implied by $d < x < 2d$ provided that $r_1 \in (1/2, 1]$. This proves that Class 3 with $k > 2d$ and $r_1 \in (1/2, 1]$ yields

Figure 4.5: Comparison with the optimal strategy



(a) Class 1

(b) Class 2 with $k \geq d + r_1 k$ and $r_1 \in [0, 1/2)$

(c) Class 2 with $k < d + r_1 k$ and $r_1 \in [0, 1/2)$

(d) Class 3 with $k \geq 2d$

(e) Class 3 with $k < 2d$

(f) Class 2 with $r_1 \in [1/2, 1]$

The figures help to interpret the proof of Proposition 4.3.2. The dashed areas correspond to the set of bank run outcomes given a certain investment strategy. The faded area represents a set of outcomes with the same probability mass as the investment strategy in Proposition 4.3.2.

$\pi_1 > \pi_1^*$ via (4.22). Figure 4.5, panel (d) provides an illustration of this case. We left out the special case $r_1 = 1/2$. In this case, we have $\bar{y}_1(x) = 2d - x$ for $0 < x < 2d$. Since we have $\bar{y}_1(x) = 2d - x$ for $d < x < 2d$ and $\bar{y}_1(x) = 2d - x > d$ for $0 < x < d$, this case also yields $\pi_1 > \pi_1^*$ ($I_B = I_B^*$ and $I_C > 0$). To summarize, Class 3 with $k > 2d$ and $r_1 \in [1/2, 1]$ yields $\pi_1 > \pi_1^*$.

Fourth, we consider Class 3, $k \geq d$ and $r_1 k \geq d$, with $k < 2d$ and $r_1 \in [1/2, 1]$. It follows from equation (4.9) and inequality (4.26) that $\bar{y}_1(x) \geq 2d - x$ for $d < x < k$, where the inequality holds with equality only if $r_1 = 1/2$. From equation (4.9) we have $\bar{y}_1(x) = (d + r_1 k - x)/r_1$ for $k \leq x < 2d$. Hence, for $k \leq x < 2d$ we have to prove

$$(d + r_1 k - x)/r_1 \geq 2d - x. \tag{4.27}$$

This is implied by $r_1 k \geq d$: Substituting $r_1 k \geq d$ and further manipulating gives

$$\begin{aligned} (2d - x)/r_1 &\geq 2d - x, & (4.28)\\ 2d - x &\geq r_1(2d - x). & (4.29) \end{aligned}$$

It follows from (4.29) that (4.27) is implied by $k \leq x < 2d$. Note that (4.27) holds with equality only if $r_1 = 1$ and $k = d$. Via (4.22), this proves that Class 3 with $d \leq k < 2d$ and $r_1 \in [1/2, 1]$ yields $\pi_1 > \pi_1^*$, unless $r_1 = 1$ and $k = d$. Figure 4.5, panel (e) provides an illustration of this case. Note that the strategy $r_1 = 1$ and $k = d$ is also the strategy in Proposition 4.3.2.

Finally, we consider Class 2, $k \geq d$ and $r_1 k < d$, with $r_1 \in [1/2, 1]$. Figure 4.5, panel (f) provides an illustration of this case. The proof that this case is suboptimal follows a slightly different line. For this case, we prove that $\bar{y}_1(x) > 2d - x$ for $0 < x < a$ and $d < x < 2d - a$, where $a = \frac{d - r_1 k}{1 - r_1}$. This is sufficient to prove $\pi_1 > \pi_1^*$. To see this, consider the following inequality

$$\int_0^a \int_d^{2d-x} \phi(x,y)\,dy\,dx > \int_0^a \int_{2d-a}^{2d-x} \phi(x,y)\,dy\,dx = \int_{2d-a}^{2d} \int_0^{2d-x} \phi(x,y)\,dy\,dx, \tag{4.30}$$

where the inequality holds because $a = \frac{d - r_1 k}{1 - r_1} < d$ and where the equality holds due to the symmetry of $\phi(x,y)$. Further, consider

$$\int_d^{2d-a} \int_0^{2d-x} \phi(x,y)\,dy\,dx + \int_{2d-a}^{2d} \int_0^{2d-x} \phi(x,y)\,dy\,dx = \int_d^{2d} \int_0^{2d-x} \phi(x,y)\,dy\,dx = I_C^*. \tag{4.31}$$

From the first double integral in (4.30) and the first double integral in (4.31) it follows that $I_B + I_C > I_C^*$ if $\bar{y}_1(x) > 2d - x$ for both $0 < x < a$ and $d < x < 2d - a$.

The first part of the proof follows directly from equation (4.8), which gives $\bar{y}_1(x) = \infty$ for $x < \frac{d-r_1k}{1-r_1}$. Consequently, we must have $\bar{y}_1(x) > 2d - x$ for $0 < x < \frac{d-r_1k}{1-r_1}$. The second part of the proof is more complicated. From equation (4.8) and inequality (4.26) with $r_1 \in (1/2, 1]$ we have that $\bar{y}_1(x) > 2d - x$ for $d < x < k$. Following (4.8), we further need to prove

$$(d + r_1k - x)/r_1 > 2d - x, \tag{4.32}$$

for $k \leq x < 2d - a$. Some manipulation gives

$$d - 2r_1d + r_1k > x - r_1x, \tag{4.33}$$

$$d + \frac{r_1}{1 - r_1}(k - d) > x. \tag{4.34}$$

Note that the condition in (4.34) is implied by $x < 2d - a$ and $a = \frac{d-r_1k}{1-r_1}$. Hence, (4.32) holds for $k \leq x < 2d - a$. Consequently, the second part, i.e., $\bar{y}_1(x) > 2d - x$ for $d < x < 2d - a$, is also proven. Therefore, Class 2 with $r_1 \in [1/2, 1]$ yields $\pi_1 > \pi_1^*$.□

Chapter 5

Bank profitability during recessions

This chapter contributes to the literature on the relation between bank profitability and economic activity. When allowing for stronger co-movement of bank profit with economic activity during deep recessions, we find a much larger impact of output growth on bank profitability than commonly found in the literature. Among the different components of bank profit, loan losses are the main driver of this result. We also find long-term interest rates in previous years to be important determinants of bank profit in times of high economic growth. Our findings are robust to the use of aggregate or individual bank data.[1]

Keywords: Bank profitability, business cycle.

JEL Classification Numbers: E32, G21.

5.1 Introduction

The banking crisis of 2008 and the ensuing deep recession highlight the importance of understanding the main drivers of bank profitability. The recent Basel III Accord, urging banks to retain more profits and pay out fewer dividends when Tier 1 capital buffers are below required levels, also calls for more research into the main determinants of bank

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profit.

Demirgüç-Kunt and Huizinga (1999) are among the first to relate bank profit of individual banks in different countries to a set of macroeconomic indicators. Subsequent studies use aggregate rather than individual bank data (e.g. Bikker and Hu (2002), Albertazzi and Gambacorta (2009)) or focus on individual banks in only one particular country (e.g. Athanasoglou *et al.* (2008) for Greece). Other studies focus on one particular factor affecting bank profit, such as loan loss provisioning (Laeven and Majnoni (2003), Bikker and Metzemakers (2005), Bouvatier and Lepetit (2008)), bad bank loans (e.g. Salas and Saurina (2002), Quagliariello (2007), Louzis *et al.* (2012)), loan defaults (e.g. Jacobson *et al.* (2005), Pesaran *et al.* (2003), Duffie *et al.* (2007), Castrén *et al.* (2010)) or bank loan recovery rates (e.g. Khieu *et al.* (2011)).

We contribute to this literature in several respects.

First, existing studies typically find only small effects of real output growth on bank profitability. However, it can be shown that the effects are much larger when allowing for asymmetric effects through the business cycle. In particular, we find that pro-cyclicality of bank profit is stronger for deep recessions than under normal economic conditions. We find that each percent contraction of real GDP during severe recessions leads to a quarter of a percentage point decrease in the return on total bank assets.

Second, most empirical studies of bank profitability include current market interest rates in the regressions to model that net interest income constitutes a substantial part of bank profit. Our approach, instead, takes the banks' balance sheet as starting point and acknowledges that net interest income depends on the complete history of outstanding loans and deposits and their associated interest rates. Consequently, our regressions include past and current interest rates, as well as past and current stocks of loans and deposits, subject to survival rates that capture both amortization and loan losses. This modeling approach, which gives rise to several cross-terms in the regressions, enables us to analyze business cycle effects on bank profitability in a consistent and dynamic way. The empirical results confirm the importance of including these cross-terms in the regressions. In particular, the size of the positive effect of long-term interest rates on bank profit depends on the growth rate of the real economy. Also, the transmission of interest and economic growth rates to bank profit is conditional upon the specific balance sheet structure.

In addition, instead of focusing on either total profit or loan loss provisioning, we look at both total profit and the components that define it: net interest income, other income (such as fees), net provisioning for loan losses and all other costs. In our analysis, net provisioning and other costs are taken together. The reason is that when net provisioning falls short of total loan losses (which they often do according to Laeven and Majnoni

(2003)), part of the losses are directly incurred as ‘costs’ on the income statement. Moreover, we also allow for an asymmetric effect of the business cycle on loan losses. Among the different components of bank profit, provisioning and other costs are found to be the driver behind the asymmetric effect of the business cycle on total profit.

Finally, while earlier studies use either aggregate bank data or individual bank data, we use both. We have aggregate bank data for 17 countries over three decades and individual bank data for 19 countries over a period of 18 years. Aggregate and individual data have their own merits. Aggregate data are typically available for longer time periods, but are dominated by the largest banks. Individual bank data are typically available for a shorter time period, but this drawback is made up by a large cross-sectional dimension. As a result, aggregate data tend to capture the country’s ‘dominant’ bank, while individual bank data account for the heterogeneity in a country’s banking population. Therefore, the results for aggregate and individual data differ as a rule. In fact, we find differences in the timing of business cycle effects between both samples, but the asymmetric business cycle effect is found for both aggregate and bank specific data, suggesting that this result is robust.

The setup of this chapter is as follows. The data and some stylized facts are discussed in Section 5.2. Section 5.3 sets out our model, while Section 5.4 presents the estimation results. Section 5.5 concludes.

5.2 Data and stylized facts

Two types of bank data are used in the empirical part of this chapter: aggregate bank data and individual bank data. The aggregate bank data have been taken from the OECD Bank Profitability Statistics and cover 17 countries.[2] This is an unbalanced panel dataset over the period 1979-2007. The number of observations ranges from 13 for Australia to 28 for Germany, Netherlands, Spain and Switzerland (Table 5.1). Figure 5.1 shows bank profitability as well as the decomposition for the eight countries for which at least 21 observations are available. As it turns out, the component net provisioning and other costs is an important driver of the variability in profitability, although this does not hold for all countries.[3] The contribution of net interest income shows a downward trend, indicating a shift to fee-based banking activities.

[2]Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland and United States. Appendix 5.A provides variable names, symbols (for the model in Section 5.3), definitions and data sources.

[3]Although data are available for the whole sample, Sweden and Belgium have been excluded from Figure 5.1. The OECD Bank Profitability Statistics reports huge bank profitability numbers for Sweden in 1991-92 which seems to be inconsistent with the Swedish banking crisis. In the empirical analysis we exclude Swedish observations from 1991 to 2001.

Figure 5.1: Decomposition of bank profit by country. Source: OECD.

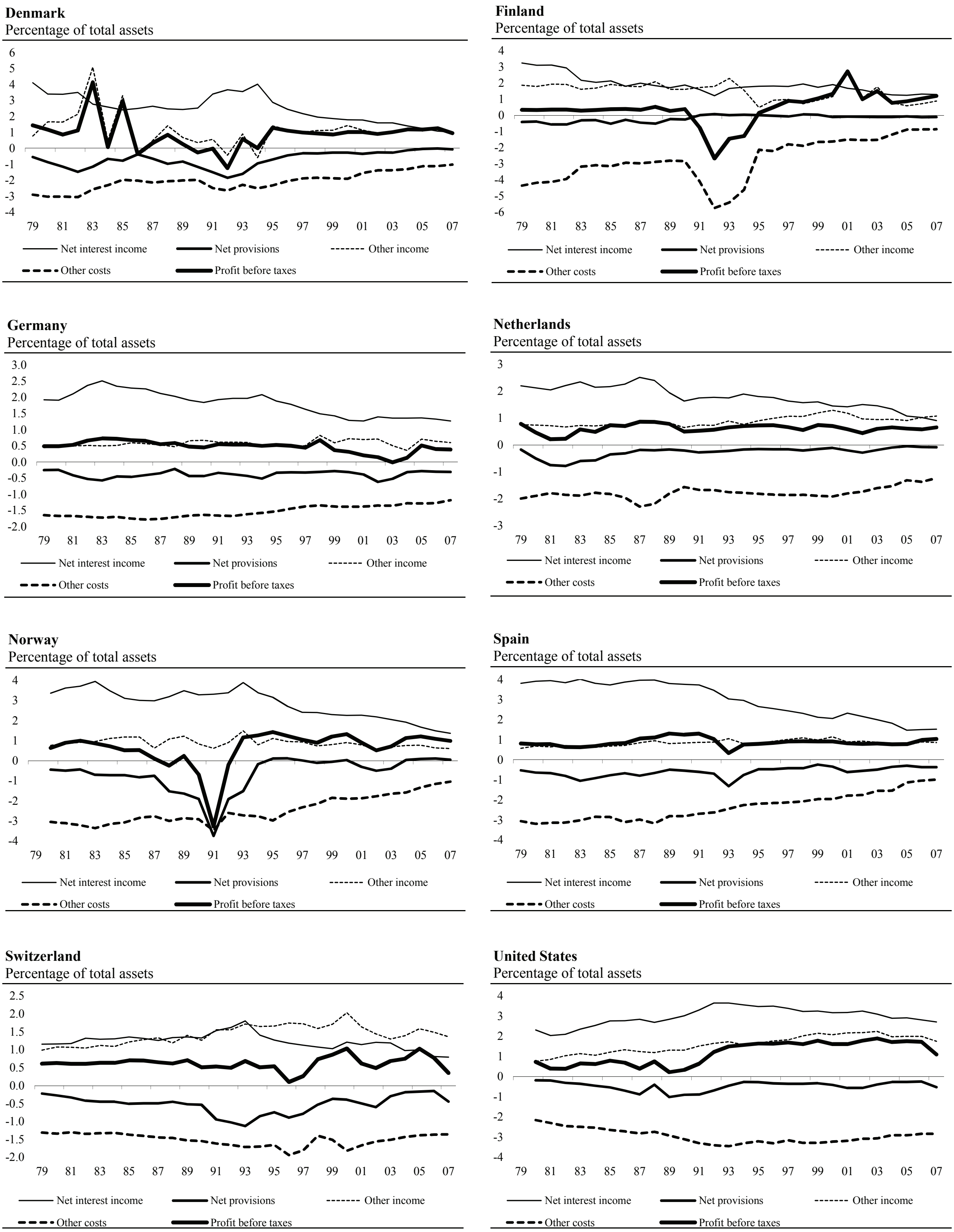
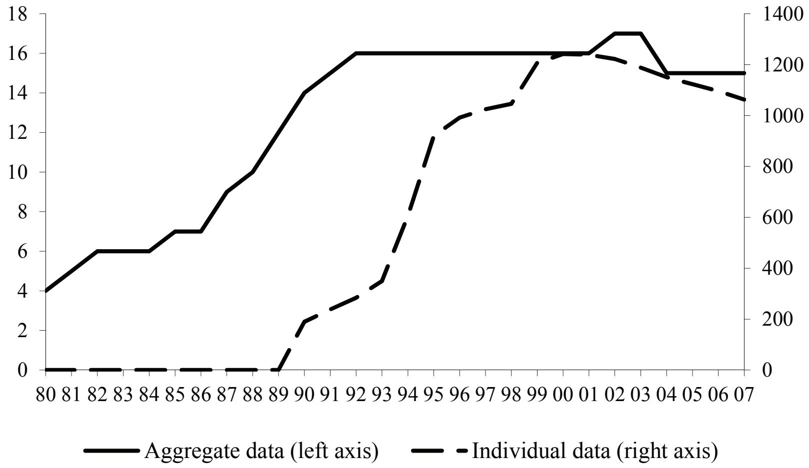


Figure 5.2: Frequency distribution of observations over time



The individual bank data are from BankScope. We select commercial banks, savings banks, cooperative banks, real estate/mortgage banks and investment banks, leaving out bank holdings to avoid double counting. After inspection of the histograms of the bank-specific variables, we trim profit before tax, net interest income and other income at the 0.2nd and 99th percentiles to exclude outliers (all scaled by total assets at the beginning of the period). For the other variables additional trimming is not necessary. Next, as the panel data are unbalanced (Figure 5.2), we select banks for which at least six observations of bank data are available (after scaling by total assets at the beginning of the period). This selection procedure results in an unbalanced panel dataset of 16,182 bank-year observations over the period 1990-2007 across 19 different countries (Table 5.1). Commercial banks form the bulk of the sample (Table 5.2). The median ratio of profit before tax over (the lag of) total assets is 0.75% for the individual bank sample, while it is 0.72% for the aggregate sample (Table 5.2).[4]

Figure 5.3 shows the stylized bank balance sheet that is the starting point of our analysis. On the asset side we have loans and non-interest earning assets. On the liability side are deposits, bank capital and other net interest bearing liabilities (net borrowing from banks and the central bank and net outstanding securities).

The correlation matrix in Table 5.4 shows that the pairwise correlation coefficients between variables in the aggregate data set can differ substantially from those in the individual data set. For example, deposits and loans are strongly correlated in the aggregate data set, whereas they are hardly correlated in the individual data set. The main reason is

[4]In the individual bank dataset, some countries, such as the United States, Germany and Japan, are more represented than some smaller countries. In contrast, in the aggregate sample, both large and small countries get equal weight. Still, our main results hold for both datasets.

Table 5.1: Distribution over country

Country	Aggregate bank data	Individual bank data	
	Number of obs.	Number of obs.	Average number of obs. per year
Australia	13	313	17.4
Austria	18	309	17.2
Belgium	26	336	18.7
Canada	19	277	15.4
Denmark	21	247	13.7
Finland	20	58	3.2
France	19	1,853	102.9
Germany	28	2,977	165.4
Italy	16	1,274	70.8
Japan	14	2,129	118.3
Netherlands	28	352	19.6
New Zealand	17	98	5.4
Norway	23	185	10.3
Portugal	0	191	10.6
Spain	28	986	54.8
Sweden	10	233	12.9
Switzerland	28	278	15.4
United Kingdom	0	1,104	61.3
United States	27	2,982	165.7
Total	355	16,182	899.0

that interbank loans and deposits cancel out in the aggregate but can differ substantially for an individual bank.

The macroeconomic data used in the empirical part, such as real GDP growth and inflation, are from the OECD Main Economic Indicators database, see Appendix 5.A.

5.3 Model

Our starting point of the analysis is a bank's income statement and balance sheet. We explicitly model the dynamic build-up of a bank's loan portfolio and its effect on net interest income, taking into account differences in lending rates, maturities and write-offs across time. Omitting any subscripts for individual banks in our notation throughout the chapter, we may write for bank profit (before tax):

$$\Pi_t = NII_t + OI_t - (BL_t + OC_t), \tag{5.1}$$

Table 5.2: Distribution over bank type: individual bank data

Bank type	Number of obs.
Commercial banks	8,479
Cooperative bank	2,226
Investment banks	646
Real estate / mortgage banks	1,294
Savings banks	3,537
Total	16,182

Figure 5.3: Stylized bank balance sheet

Assets	Symbol	Liabilities	Symbol
Loans	L	Deposits	D
Non-interest earning assets		Other net interest bearing liabilities	R
		Bank capital	
Total	A	Total	A

where NII_t denotes net interest income, OI_t other income, BL_t bad loan losses and OC_t operating cost at time t. Other income includes net fees and commission, net trading income, and net returns from financial transactions. All variables are scaled by total assets at the beginning of the period, A_{t-1}, to avoid potential endogeneity problems that might arise when total assets are affected by current macroeconomic and financial variables.[5] Four equations are estimated in our analysis: one for bank profit, Π_t, and three for the individual components, NII_t, OI_t, and $(BL_t + OC_t)$.

5.3.1 Net interest income

Net interest income is given by interest revenues minus interest expenses:

$$NII_t = r_{L,t}L_t - r_{D,t}D_t, \tag{5.2}$$

where L_t and D_t denote the outstanding amounts of loans and deposits on the balance sheet. We assume that the maturity of deposits is short, so that the funding cost, $r_{D,t}$, is only based on the current deposit rate, $r_{d,t}$. Profit-maximizing banks set their deposit rate at a level equal to the short-term risk-free rate $r_{s,t}$ minus a compensation for the marginal operating cost of managing deposits, c_D:

$$r_{d,t} = r_{s,t} - c_D. \tag{5.3}$$

[5]Adrian and Shin (2010) observe that banks tend to actively manage the amount of total assets on their balance sheets in response to business cycle fluctuations. Scaling by lagged balance sheet items is also advocated by Foos *et al.* (2010).

Table 5.3: Summary statistics

Variables	Symbol	Mean	Median	Standard deviation	Number of obs.
Aggregate bank data					
Profit before tax	Π_t	0.0076	0.0072	0.0065	355
Net interest income	NII_t	0.0214	0.0201	0.0090	366
Other income	OI_t	0.0116	0.0110	0.0062	366
Loan losses and costs	$(BL_t + OC_t)$	0.0252	0.0229	0.0101	355
Loans	L_{t-1}	0.4462	0.4417	0.1541	368
Deposits	D_{t-1}	0.5097	0.5039	0.1430	368
Other net interest bearing liabilities	R_{t-1}	-0.2047	-0.2007	0.1037	368
Individual bank data					
Profit before tax	Π_t	0.0094	0.0075	0.0107	16,126
Net interest income	NII_t	0.0238	0.0220	0.0137	16,182
Other income	OI_t	0.0130	0.0082	0.0186	16,182
Loan losses and costs	$(BL_t + OC_t)$	0.0271	0.0244	0.0197	16,126
Loans	L_{t-1}	0.5810	0.6153	0.2135	16,158
Deposits	D_{t-1}	0.5607	0.6179	0.2773	16,167
Other net interest bearing liabilities	R_{t-1}	-0.0263	-0.0554	0.3334	16,182

Note: All variables have been scaled by total assets at the beginning of the period (A_{t-1}).

In contrast to the rate paid to deposit holders, the rate of return on the loan portfolio, $r_{L,t}$, is equal to a weighted average of lending rates on loans in current and past years, $r_{l,t}, r_{l,t-1}, ..., r_{l,t-i}$. For a loan extended in a particular year, the lending rate is assumed to be set as a mark-up over the risk-free capital market rate, $r_{f,t}$, where the mark-up compensates for operating expenses, c_L, expected default losses, f^e, and the associated risk k.[6] Following Cavallo and Majnoni (2001), we specify for the lending rate in year t:

$$r_{l,t} = r_{f,t} + c_L + f^e + k. \tag{5.4}$$

The weights of the lending rates for the different loan vintages, $r_{l,t}, r_{l,t-1}, ..., r_{l,t-i}$, in the average lending rate of the loan portfolio, $r_{L,t}$, depend on the fractions of the different loan vintages in the current total loan portfolio, $\omega_t,\ \omega_{t-1},\ ..., \omega_{t-i}$. These fractions, ω_{t-i}, are determined by the amount of new loans, NL_{t-i}, originating in period $t-i$ and the annual survival rates of these loans. We specify that the annual survival rate, λ_t, depends on both the "natural" maturity structure of a bank's loan portfolio, $\overline{\lambda}$, which we assume

[6]For a derivation of optimal bank lending rates under monopolistic competition, see e.g. Swank (1999).

Table 5.4: Correlation matrix

Variable	Symbol	Π_t	NII_t	OI_t	$(BL_t + OC_t)$	L_{t-1}	D_{t-1}	R_{t-1}
Aggregate bank data								
Profit before tax	Π_t	1						
Net interest income	NII_t	0.32***	1					
Other income	OI_t	0.57***	0.22***	1				
Loan losses and costs	$(BL_t + OC_t)$	-0.01	0.82***	0.43***	1			
Loans	L_{t-1}	0.22***	0.55***	0.26***	0.51***	1		
Deposits	D_{t-1}	0.18***	0.50***	0.13**	0.42***	0.86***	1	
Other net interest bearing liabilities	R_{t-1}	-0.07	0.12**	-0.16***	0.05	0.13**	-0.04	1
Individual bank data								
Profit before tax	Π_t	1						
Net interest income	NII_t	0.53***	1					
Other income	OI_t	0.39***	0.17***	1				
Loan losses and costs	$(BL_t + OC_t)$	0.17***	0.55***	0.80***	1			
Loans	L_{t-1}	0.13***	0.32***	-0.17***	-0.01	1		
Deposits	D_{t-1}	-0.00	0.25***	-0.09***	0.10***	0.04***	1	
Other net interest bearing liabilities	R_{t-1}	0.06***	0.01*	-0.05***	-0.08***	0.61***	-0.74***	1

Note: Pearson correlation coefficients. ***, **, * denote significance at the 1%, 5% and 10% level, respectively. All variables have been scaled by total assets at the beginning of the period (A_{t-1}).

to be constant over time, and the amount of bad loans that is written off, BL_t. As shown in Appendix 5.B, net interest income on the loan portfolio then equals:[7]

$$\begin{aligned} NII_t &= \tfrac{1}{2} r_{l,t} \cdot \omega_t \cdot L_t + \textstyle\sum_{i=1}^{\infty} r_{l,t-i} \cdot \omega_{t-i} \cdot L_t - r_{D,t} D_t \\ &= \tfrac{1}{2} r_{l,t} \left(NL_t \right) + \textstyle\sum_{i=1}^{\infty} r_{l,t-i} \left(NL_{t-i} \times \prod_{j=1}^{i} \left(\bar{\lambda} - \frac{BL_{t-j+1}}{L_{t-j}} \right) \right) - r_{D,t} D_t. \end{aligned} \tag{5.5}$$

Note that, given interest rates and natural maturity, net interest income typically depends on the flows of new loans and bad loans in current and past years and on the current stock of deposits. We will specify simple reduced-form relations that link these variables to the business cycle through current and lagged values of real GDP growth, $y_t, ..., y_{t-i}$, and/or unemployment rates, $u_t, ..., u_{t-i}$.

First, for new loans origination, we assume:

$$\frac{NL_t}{L_{t-1}} = g \left(y_t, s_t \right). \tag{5.6}$$

This reduced form equation describes the rate of new loan origination as a function of economic growth, y_t, and the slope of the yield curve, s_t, defined as the difference between the long-term interest rate and the short-term interest rate, $r_{f,t} - r_{s,t}$. For real GDP growth a positive sign is expected. A positive relation between new loan origination and real GDP growth is confirmed in, e.g., Berger and Udell (2004), Calza *et al.* (2006), Sørensen *et al.* (2009), and Jiménez *et al.* (2012). The slope of the yield curve reflects the relative price of credit extension, where the short-term interest rate may be viewed as an alternative price for making a money market loan. Since bank loans are typically long-term financed, we expect a negative sign here.

The second reduced form equation relates deposit growth to the short-term interest rate, $r_{s,t}$, and the inflation rate, i_t:

$$\frac{D_t - D_{t-1}}{D_{t-1}} = h \left(r_{s,t}, i_t \right). \tag{5.7}$$

We expect a positive sign for the short-term interest rate and a negative sign for the inflation rate, which is regarded as the opportunity cost of holding deposits.

5.3.2 Other income

Other income – mainly including fees and income from trading in financial markets – is related to local stock market returns and current long and short-term interest rates. Following Albertazzi and Gambacorta (2009), we also add stock market volatility, which

[7] Assuming a uniform distribution for new loan origination, the expected interest income on new loans is *half* the lending rate for the current year.

positively affects trading volumes. Finally, economic growth is expected to have a positive influence on other income:

$$\frac{OI_t}{A_{t-1}} = f^o\left(r_{m,t}, CV_{m,t}, r_{f,t}, r_{s,t}, y_t, y_{t-1}\right), \tag{5.8}$$

where $r_{m,t}$ is the average local stock market return (excluding dividends) and $CV_{m,t}$ the coefficient of variation in monthly stock returns.

5.3.3 Loan losses and operating cost

In contrast to net interest income and other income, losses on bad loans are generally not observed directly from the income statement. Instead of losses on bad loans, we observe net provisions. However, under adverse economic conditions, provisions tend to fall short of loan losses (e.g. Laeven and Majnoni (2003)) and, hence, part of these losses are incurred directly as costs in the income statement. Therefore, the effect of severe recessions on loan losses will be underestimated if only net provisions are considered. To adjust for this bias, we will therefore consider the sum of net provisions and all other costs in the empirical specification.

For loan losses, we postulate:

$$\frac{BL_t}{L_{t-1}} = f\left(y_t, ..., y_{t-i}, u_t, ..., u_{t-i}\right). \tag{5.9}$$

Since firms' profitability declines during recessions, we expect more defaults on business loans for lower real GDP growth rates. Also, lower income growth and higher unemployment rates increase the number of defaults on consumer loans.[8] Thus the expected signs are positive for real GDP growth rates and negative for unemployment rates. Rescaling equation (5.9) by total assets implies:

$$\frac{BL_t}{A_{t-1}} = \frac{L_{t-1}}{A_{t-1}} f\left(y_t, ..., y_{t-i}, u_t, ..., u_{t-i}\right). \tag{5.10}$$

From equation (5.3) and (5.4), we have that total operating cost increases with the amount of deposits and loans through the unit cost of managing deposits, c_D, and loans, c_L. The net effect of macroeconomic conditions on operating cost is ambiguous. While unfavourable economic conditions raise the costs of collecting repayments on loans, fewer

[8]For example, Rinaldi and Sanchis-Arellano (2006) show that unemployment – serving as an indicator of future income uncertainty – is a significant determinant of non-performing loans. Gross and Souleles (2002) report that unemployment is significant for explaining household delinquencies. In the empirical section we find significant results for both unemployment and real GDP growth. Omitting unemployment does not qualitatively affect the estimated coefficients of the recession dummies, while the relation with real GDP growth generally becomes stronger. This is because real GDP growth and unemployment are correlated, so that real GDP growth partly captures the influence of the unemployment rate.

new loans will be extended under such conditions. Therefore, we simply estimate the following empirical relation, without any a priori sign predictions. Again, we scale operating cost by bank assets.

$$\frac{OC_t}{A_{t-1}} = f^c\left(\frac{L_{t-1}}{A_{t-1}}, \frac{D_{t-1}}{A_{t-1}}, y_t, u_t, i_t\right). \tag{5.11}$$

5.3.4 Empirical specifications

Four equations are estimated in our analysis: one for bank profit, and three for individual components. First, after some algebraic manipulations (see Appendix 5.B for a full derivation), net interest income is obtained by substituting equation (5.6), (5.7), and (5.9) into equation (5.5). This yields:

$$\frac{NII_t}{L_{t-1}} = \left\{\frac{1}{2}g_t \cdot r_{l,t} + \sum_{i=1}^{\infty} r_{l,t-i}\left(g_{t-i} \times \prod_{j=1}^{i}\frac{\lambda_{t-j+1}}{g_{t-j}+\lambda_{t-j}}\right)\right\} - \frac{D_{t-1}}{L_{t-1}} \times \left\{\left(1+\frac{1}{2}h_t\right) r_{d,t}\right\}. \tag{5.12}$$

Equation (5.12) illustrates that the long-term interest rate has a more persistent effect on net interest income than the short-term interest rate: a drop in the current long-term interest rate depresses net interest income as long as loans originated during the current period are part of the bank's loan portfolio. However, since in our analysis all liabilities (i.e., deposits) are assumed to be short-term financed, a change in the current short-term interest rate is likely to have a larger direct impact on net interest income than a change in the current long-term interest rate.

To estimate equation (5.12) we assume several simplifications. That is, we simplify equation (5.12) by replacing the product term $\prod_{j=1}^{i} \lambda_{t-j+1}/\left(g_{t-j}+\lambda_{t-j}\right)$ with some constant C^i. Hence, we abstract from the fact that large loan losses during a certain period decrease the loan survival rate in that period. We also abstract from the 'indirect effect' of loan growth, which implies that high loan growth in period t-1 decreases the fraction of loans originated during period t-2. However, the 'direct effect' of the business cycle on new loan origination is kept intact, since $g_t(\cdot)$ and $g_{t-i}(\cdot)$ are still allowed to vary through the business cycle.[9] Taking first-order approximations for $g_{t-i}(\cdot)$ and $h_t(\cdot)$ in equation (5.12) and including lags up to four periods yield the following empirical

[9] Although abstracting from the indirect effect is obviously a simplification, the impact of this simplification should not be overemphasized. Since bank loans have on average a long maturity, the size of $g_{t-j}(\cdot)$ is small relative to the survival rate λ, which is the other term in the denominator in equation (5.12).

specification (see Appendix 5.B for more details):

$$\begin{aligned}\frac{NII_t}{L_{t-1}} &= \alpha_{11} + \alpha_{12}\frac{D_{t-1}}{L_{t-1}} + \alpha_{13}\frac{R_{t-1}}{L_{t-1}} \\ &+ \textstyle\sum_{i=0}^{4}\left[\beta_{i1}r_{f,t-i} + \beta_{i2}y_{t-i} + \beta_{i3}s_{t-i} + \beta_{i4}(r_{f,t-i}\cdot y_{t-i}) + \beta_{i5}(r_{f,t-i}\cdot s_{t-i})\right] \\ &+ \frac{D_{t-1}}{L_{t-1}}\times\left[\beta_{51}r_{s,t}+\beta_{52}r_{s,t}^2+\beta_{53}i_t + \beta_{54}r_{s,t}\cdot i_t\right] + \beta_{61}\frac{R_{t-1}}{L_{t-1}}r_{s,t} + \varepsilon_t,\end{aligned} \tag{5.13}$$

where R_{t-1}/L_{t-1} is added to correct for interest expenses due to 'other interest-earning bank liabilities', see Figure 5.3.[10]

Second, the empirical specification for other income is simply a linear approximation of equation (5.8):

$$\frac{OI_t}{A_{t-1}} = \alpha_{21} + \delta_{01}y_t + \delta_{02}y_{t-1} + \delta_{03}r_{f,t} + \delta_{04}r_{s,t} + \delta_{05}r_{m,t} + \delta_{06}CV_{m,t} + \xi_t. \tag{5.14}$$

Third, as mentioned in Subsection 5.3.3, we estimate the sum of net provisions and all other costs so as to avoid underestimation of the recession effect due to "ill-provisioning". Moreover, we allow for non-linearity in the relation between loan losses and the business cycle. Marcucci and Quagliariello (2009) report a more pronounced relation between the business cycle and credit risk during severe economic downturns. By including so-called "recession slope dummies", we allow for such an asymmetric relation in the following equation:

$$\begin{aligned}\frac{BL_t+OC_t}{A_{t-1}} &= \alpha_{31} + \alpha_{32}\frac{L_{t-1}}{A_{t-1}} + \alpha_{33}\frac{D_{t-1}}{A_{t-1}} + \frac{L_{t-1}}{A_{t-1}}\times\textstyle\sum_{i=0}^{2}\left[\gamma_{i1}y_{t-i} + \gamma_{i2}\mathbf{1}_{y_{t-i}<a}\cdot(y_{t-i}-a)\right] \\ &+ \gamma_{31}\frac{L_{t-1}}{A_{t-1}}u_t + \gamma_{41}y_t + \gamma_{42}r_{f,t} + \gamma_{43}r_{s,t} + \gamma_{44}r_{m,t} + \gamma_{45}CV_{m,t} + \gamma_{46}i_t + \nu_t,\end{aligned} \tag{5.15}$$

where $\mathbf{1}_{y_{t-i}<a}$ is an indicator function for severe recessions, which equals 1 if $y_{t-i} < a$ and 0 otherwise (a will be determined in the empirical part of this chapter). To our knowledge, allowing for severe recessions in this way when explaining bank profit is a novel feature, and the significance of $\mathbf{1}_{y_{t-i}<a}$ would suggest that, foremost, loan losses really start to bite because of insufficient financial buffers when recessions get severe. Instead, small economic downturns may be absorbed by the accumulated buffers in the form of financial reserves held by businesses and households, or loan provisions by banks.[11]

Fourth, the relation for total profit before tax (scaled by total assets) is given by summing up equations (5.13), (5.14) and (5.15), see also Appendix 5.C:

$$\frac{\Pi_t}{A_{t-1}} = \frac{L_{t-1}}{A_{t-1}} \times \text{ eq. (5.13) } + \text{ eq. (5.14) } - \text{ eq. (5.15).} \tag{5.16}$$

[10]When estimating equation (5.13), we include all lower order terms that constitute the interaction terms. See Brambor *et al.* (2006) for an account of why inclusion of all first-order that constitute the interaction terms is important for a robust interpretation of the marginal effects. This procedure is also followed when estimating all other equations.

[11]In the estimation of equation (5.15), using aggregate bank data, we find that including up to two lags yields significant coefficients.

Note that it is likely that the relation for total profit is noisier than the three individual equations. Thus, when estimating this relation one may expect larger standard errors.

5.4 Estimation results

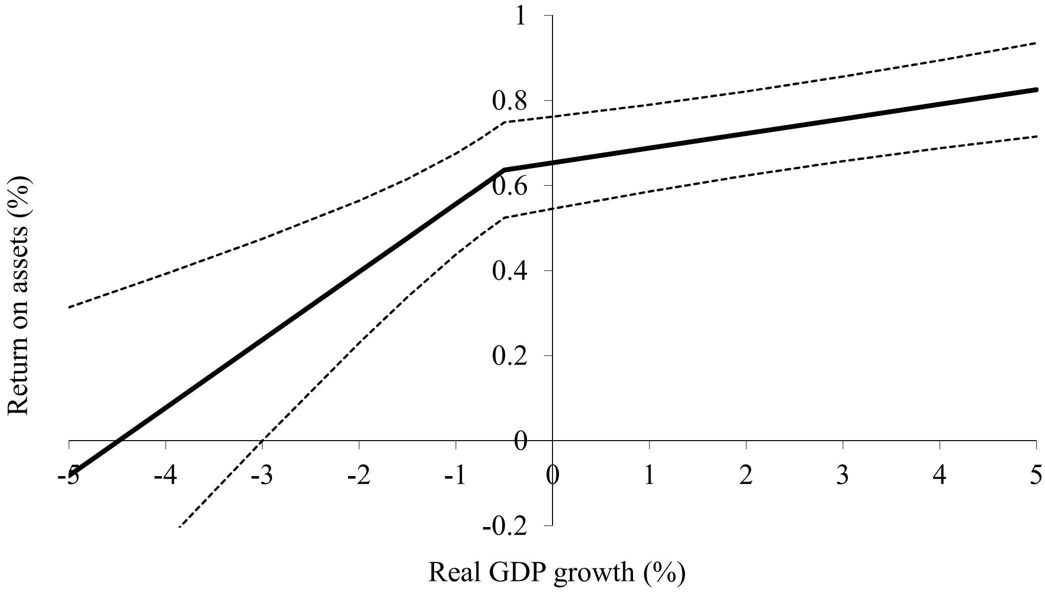
For severe recessions, defined as a drop in real GDP by more than 0.5%, we find that the pro-cyclicality of bank profit is stronger than under normal economic conditions. This result is obtained from both aggregate bank data and individual bank data. Tables 5.5 and 5.6, model (3), show the estimation results for the relationship between bank profitability and the business cycle. The coefficient for the contemporaneous recession slope dummy is similar for both data samples and amounts to around 0.4. Hence, the fall in the return on assets due to an additional percentage point decline in the current real GDP growth rate during a severe recession is about 0.4 percentage point multiplied by the loans-to-assets ratio. For illustration purposes we present the asymmetric effect of severe recessions on bank profitability in Figure 5.4, which is based on the aggregate data estimations. In Figure 5.4 we have chosen a loans-to-assets ratio of 0.40, which is close to the mean in the aggregate data sample. The figure shows a kinked line, suggesting that the negative effect of economic downturns on profit is stronger during severe recessions (i.e., a rate of real GDP growth less than −0.5%).[12]

The above-mentioned definition of a severe recession – a drop of real GDP by more than 0.5% – has been based on optimal breakpoint analysis. Specifically, we have to specify breakpoint a for the 'severe recession dummy' $\mathbf{1}_{y_{t-i}<a}$ in equation (5.15), which has a value of 1 for $y_{t-i} < a$ and 0 otherwise. We determine the optimal breakpoint a as follows. First, for the aggregate bank data we apply the following break test: we calculate Wald test statistics for different breakpoints under the null hypothesis that severe recessions do not have any additional impact. More formally, given some breakpoint a, we perform a Wald test to check whether the sum of the coefficients of the recession slope dummies equals zero. Second, following Andrews (1993), we choose the optimal breakpoint a that maximizes the test statistic associated with the break test. The maximum test statistic corresponds with $a \approx -0.5\%$, see Figure 5.5.

In the following, we discuss the results for the three profit components (Tables 5.5, 5.6 and 5.7, models (1) and (2)). Tables 5.5 and 5.6 present the estimates for (i) other income, i.e., equation (5.14), (ii) net provisions plus costs, i.e., equation (5.15), and (iii)

[12] Using individual bank data, the mean marginal effect of an additional percentage point decline in economic activity during severe recessions on return on assets amounts to a quarter of a percent. This number is calculated from the estimation results in Table 5.6: $-0.01 \cdot [0.0281 + (0.5810) \cdot (-0.0302 + 0.397)] \approx 0.0024$, where 0.5810 is the mean loans-to-assets ratio in the individual data sample. The result is not affected if the median loans-to-assets ratio is used instead.

Figure 5.4: Relation between real economic growth and return on assets.



The estimated relation between real GDP growth and bank profitability in the Netherlands using aggregate bank data. The dotted lines represent the 95% confidence interval.

profit before tax, i.e., equation (5.16). Table 5 is for the aggregate bank data, Table 5.6 for the individual bank data.[13] Table 5.7 presents the estimates for net interest income, i.e., equation (5.13), for both aggregate and individual bank data.

For the aggregate bank data (17 countries over 3 decades) we have applied generalized least squares (GLS), allowing for the presence of panel specific autocorrelation in the error terms and heteroskedasticity across panels. For the individual bank dataset, for which the number of banks is large and the number of observations per bank relatively small, the within estimator has been used, allowing for first-order autocorrelation in the disturbances.[14,15]

[13]Typical bank specific variables, such as log(assets), log(loans) and a CR3 concentration ratio (i.e., the aggregate market share of the three largest banks), do not follow from our theoretical derivation. Nevertheless, we did experiment with these variables in the empirical specification using individual bank data. Although inclusion of those variables alters the significance of some regressors (notably the effect of inflation and unemployment turns significant at the 5% level), the influence of the recession slope dummies remains intact. Moreover, including recession slope dummies in the model for other income and net interest income does not yield significant coefficients.

[14]It may be argued that individual bank samples in particular suffer from survivorship bias as a result of the fact that failing banks drop out of such samples in quite an early stage of the bankruptcy process. Assuming that some survivorship bias is present in our dataset, this will lead to an underestimation of the asymmetric effect of recessions on profits, because especially banks with severe losses (being the reason of failure) are underrepresented. Hence, the presence of survivorship bias would strengthen our finding of a significant asymmetric business cycle effect.

[15]The Baltagi-Wu LBI statistics are around 1.4, indicating significant serial correlation in the residuals for the individual bank data. This is why we apply the estimation technique developed by Baltagi and Wu (1999) for unequally spaced panel data with AR(1) disturbances. Our results are not very sensitive to

Table 5.5: Estimation results for other income, net provisions and costs, and profit before tax: aggregate bank data.

Model:	(1)		(2)		(3)	
VARIABLES	$\frac{\text{Other Income}_t}{\text{Assets}_{t-1}}$	(s.e.)	$\frac{\text{Net provisions}_t + \text{Costs}_t}{\text{Assets}_{t-1}}$	(s.e.)	$\frac{\text{Profit before tax}_t}{\text{Assets}_{t-1}}$	(s.e.)
Real GDP growth$_t$	-0.00377	(0.0073)	-0.0706***	(0.024)	0.0228	(0.023)
Real GDP growth$_{t-1}$	0.0134*	(0.0071)	-0.0320	(0.025)	0.0464**	(0.023)
Real GDP growth$_{t-2}$			-0.0394*	(0.023)	0.0192	(0.022)
$(L_{t-1}/A_{t-1})\cdot$ Real GDP growth$_t$			0.0958*	(0.056)	0.0289	(0.052)
$(L_{t-1}/A_{t-1})\cdot$ Real GDP growth$_{t-1}$			0.0520	(0.058)	-0.0734	(0.053)
$(L_{t-1}/A_{t-1})\cdot$ Real GDP growth$_{t-2}$			0.0795	(0.054)	-0.0200	(0.050)
$(L_{t-1}/A_{t-1})\cdot$ Recession slope dummy$_t$			-0.200*	(0.115)	0.314***	(0.115)
$(L_{t-1}/A_{t-1})\cdot$ Recession slope dummy$_{t-1}$			-0.475***	(0.122)	0.571***	(0.116)
$(L_{t-1}/A_{t-1})\cdot$ Recession slope dummy$_{t-2}$			-0.274**	(0.114)	0.383***	(0.111)
Unemployment rate$_t$			-0.0221	(0.033)	-0.00825	(0.024)
$(L_{t-1}/A_{t-1})\cdot$ Unemployment rate$_t$			0.139*	(0.080)	0.0598	(0.061)
Inflation$_t$	-0.00231	(0.0081)	-0.00462	(0.013)	0.0176	(0.012)
Long-term interest rate$_t$	-0.0665***	(0.0167)	0.0583***	(0.020)	-0.0422**	(0.018)
Short-term interest rate$_t$	0.0187*	(0.0112)	0.0376***	(0.014)	-0.00584	(0.013)
$(L_{t-1}/A_{t-1})\cdot$ Long-term interest rate$_t$					—	
(. . .)					—	
Local stock market index return$_t$	0.00169***	(0.0006)	—		—	
Local stock market index volatility$_t$	-0.000526	(0.0024)	—		—	
L_{t-1}/A_{t-1}	0.00706*	(0.0041)	0.0131	(0.009)	0.00956	(0.007)
D_{t-1}/A_{t-1}	-0.00769***	(0.0027)	—		—	
R_{t-1}/A_{t-1}	0.00453	(0.0038)	—		-0.00529**	(0.003)
Constant	0.00715***	(0.0015)	0.0111***	(0.003)	0.00225	(0.003)
Number of observations	350		347		347	
Number of countries	17		17		17	
Adjusted pseudo-R^2	0.022		0.517		0.305	
Wald Chi^2 [d.o.f.]	63.07***	[10]	323.07***	[15]	154.38***	[16]

Note: Generalized least squares with random country effects, heteroskedastic panels and panel specific autocorrelation. ***, **, * denote significance at the 1%, 5% and 10% level, respectively. For models (2) and (3) we apply backward elimination according to the following rules: insignificant regressors are removed unless one of their lags is significant. Inflation is always included to allow for the possibility that real rather than nominal interest rates are the relevant drivers. For methodological reasons, variables are included if they are significantly interacted with other variables and real GDP growth is retained if a recession slope dummy is included. (. . .) = list of regressors in equation (5.16) and – = regressor excluded due to insignificance in the backward elimination procedure. Blank = not in model.

Table 5.6: Estimation results for other income, net provisions and costs, and profit before tax: individual bank data.

Model:	(1)		(2)		(3)	
VARIABLES	$\frac{\text{Other Income}_t}{\text{Assets}_{t-1}}$	(s.e.)	$\frac{\text{Net provisions}_t + \text{Costs}_t}{\text{Assets}_{t-1}}$	(s.e.)	$\frac{\text{Profit before tax}_t}{\text{Assets}_{t-1}}$	(s.e.)
Real GDP growth$_t$	-0.00915	(0.0081)	0.0251	(0.022)	0.0281	(0.020)
Real GDP growth$_{t-1}$	0.0202***	(0.0071)	0.0657***	(0.020)	0.0119	(0.016)
Real GDP growth$_{t-2}$			0.0546***	(0.019)	-0.0192	(0.014)
(L_{t-1}/A_{t-1}) · Real GDP growth$_t$			-0.0434	(0.034)	-0.0302	(0.030)
(L_{t-1}/A_{t-1}) · Real GDP growth$_{t-1}$			-0.104***	(0.032)	0.0482*	(0.026)
(L_{t-1}/A_{t-1}) · Real GDP growth$_{t-2}$			-0.0345	(0.030)	0.0294	(0.023)
(L_{t-1}/A_{t-1}) · Recession slope dummy$_t$			-0.440***	(0.104)	0.397***	(0.081)
(L_{t-1}/A_{t-1}) · Recession slope dummy$_{t-1}$			0.105	(0.101)	-0.0182	(0.078)
(L_{t-1}/A_{t-1}) · Recession slope dummy$_{t-2}$			-0.0852	(0.070)	-0.0401	(0.053)
Unemployment rate$_t$			0.0778***	(0.019)	0.0147	(0.015)
(L_{t-1}/A_{t-1}) · Unemployment rate$_t$			0.0769**	(0.031)	-0.0517**	(0.025)
Inflation rate$_t$	-0.00205	(0.0093)	-0.00540	(0.010)	0.0104	(0.008)
Long-term interest rate$_t$	-0.00455	(0.0142)	0.0282*	(0.017)	-0.0242	(0.034)
Long-term interest rate$_{t-1}$					-0.0366	(0.029)
Short-term interest rate$_t$	0.00500	(0.0101)	0.0675***	(0.012)	0.0333	(0.020)
(L_{t-1}/A_{t-1}) · Long-term interest rate$_t$					0.103*	(0.057)
(L_{t-1}/A_{t-1}) · Long-term interest rate$_{t-1}$					0.0903*	(0.047)
(...)					—	
(D_{t-1}/A_{t-1}) · Short-term interest rate$_t$					-0.243***	(0.043)
(D_{t-1}/A_{t-1}) · Short-term interest rate$_t^2$					0.882***	(0.245)
(R_{t-1}/A_{t-1}) · Short-term interest rate$_t$					-0.117***	(0.033)
Local stock market index return$_t$	0.00250***	(0.0006)	-0.00269***	(0.001)	0.00473***	(0.001)
Local stock market index volatility$_t$	0.00249	(0.0022)	0.00118	(0.002)	0.00483***	(0.002)
L_{t-1}/A_{t-1}	-0.0110***	(0.0020)	0.0351***	(0.004)	0.00816**	(0.003)
D_{t-1}/A_{t-1}	0.0197***	(0.0023)	-0.0199***	(0.002)	0.000539	(0.002)
R_{t-1}/A_{t-1}	-0.0162***	(0.0019)	-0.0268***	(0.002)	-0.00681***	(0.002)
Constant	0.00727***	(0.0005)	0.00263***	(0.001)	0.00352***	(0.001)
Number of observations	14,829		14,061		14,294	
R^2 (within; between; overall)	0.010; 0.012; 0.005		0.044; 0.012; 0.023		0.029; 0.018; 0.027	
Number of banks	1,398		1,394		1,403	
AR coefficient; Baltagi-Wu LBI	0.525; 1.326		0.536; 1.342		0.408; 1.556	

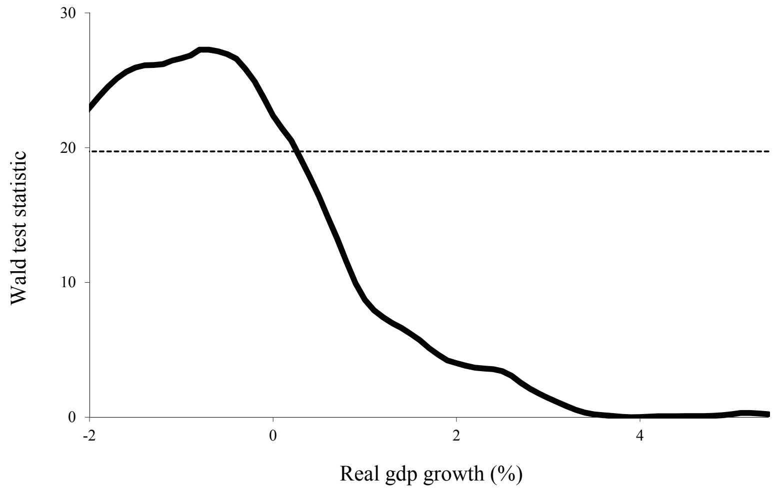
Note: Least Squares with fixed bank effects and an AR(1) error term. ***, **, * denote significance at the 1%, 5% and 10% level, respectively. For model (3) we apply backward elimination according to the following rules: insignificant regressors are removed unless one of their lags is significant. Inflation is always included to correct for nominal interest rates and recession slope dummy variables are always included. For methodological reasons, variables are included if they are significantly interacted with other variables and real GDP growth is retained if a recession slope dummy is included. (...) = list of regressors in equation (5.16) and – = regressor excluded due to insignificance in the backward elimination procedure. Blank = not in model.

Table 5.7: Estimation results for net interest income.

Model:	(1) *Aggregate bank data:*		(2) *Individual bank data:*	
VARIABLES	$\frac{\text{Net Interest Income}_t}{\text{Loans}_{t-1}}$	(s.e.)	$\frac{\text{Net Interest Income}_t}{\text{Loans}_{t-1}}$	(s.e.)
Long-term interest rate$_t$	0.0228	(0.037)	0.260***	(0.091)
Long-term interest rate$_{t-1}$	0.0959***	(0.030)	0.129***	(0.028)
Long-term interest rate$_{t-2}$	0.0671**	(0.030)	0.0255	(0.024)
Long-term interest rate$_{t-3}$	0.0810***	(0.026)	0.0613***	(0.022)
Long-term interest rate$_{t-4}$	–		0.0896***	(0.019)
Real GDP growth$_t$	-0.125***	(0.040)	-0.0479*	(0.028)
Real GDP growth$_{t-1}$	-0.0310	(0.038)	-0.0888***	(0.025)
Real GDP growth$_{t-2}$	-0.0753**	(0.036)	-0.00345	(0.024)
Real GDP growth$_{t-3}$	-0.0547	(0.035)	-0.0567***	(0.019)
Real GDP growth$_{t-4}$	–		0.0174	(0.020)
Slope yield curve$_{t-1}$	–		0.110**	(0.049)
Slope yield curve$_{t-2}$	–		0.0609	(0.042)
Slope yield curve$_{t-3}$	–		0.0464	(0.039)
Slope yield curve$_{t-4}$	–		0.00514	(0.033)
Long-term rate · Real GDP growth$_t$	2.168***	(0.569)	1.520***	(0.515)
Long-term rate · Real GDP growth$_{t-1}$	0.609	(0.454)	1.854***	(0.456)
Long-term rate · Real GDP growth$_{t-2}$	1.347***	(0.441)	0.834**	(0.403)
Long-term rate · Real GDP growth$_{t-3}$	0.723*	(0.418)	1.055***	(0.339)
Long-term rate · Real GDP growth$_{t-4}$	–		-0.0967	(0.285)
Long-term rate · Slope yield curve$_t$	–		-2.236*	(1.299)
Long-term rate · Slope yield curve$_{t-1}$	–		-1.851**	(0.749)
Long-term rate · Slope yield curve$_{t-2}$	–		-1.440**	(0.579)
Long-term rate · Slope yield curve$_{t-3}$	–		-0.682	(0.529)
Long-term rate · Slope yield curve$_{t-4}$	–		-0.442	(0.427)
Short-term interest rate$_t$	-0.325***	(0.063)	0.248***	(0.078)
Inflation$_t$	0.0137	(0.014)	0.102**	(0.043)
Short-term interest rate$_t^2$	—		0.274	(0.541)
Short-term interest rate$_t$ · Inflation$_t$	–		-1.231*	(0.725)
(D_{t-1}/L_{t-1}) · Short-term interest rate$_t$	0.260***	(0.052)	-0.389***	(0.057)
(D_{t-1}/L_{t-1}) · Short-term interest rate$_t^2$	–		-1.183***	(0.412)
(D_{t-1}/L_{t-1}) · Inflation$_t$	–		-0.142***	(0.041)
(D_{t-1}/L_{t-1}) · Short-term rate$_t$· Inflation$_t$	–		2.470***	(0.796)
(R_{t-1}/L_{t-1}) · Short-term interest rate$_t$	–		-0.493***	(0.049)
D_{t-1}/L_{t-1}	0.0133***	(0.005)	0.00980***	(0.002)
R_{t-1}/L_{t-1}	0.00113	(0.002)	-0.00347*	(0.002)
Constant	0.0140**	(0.006)	0.00581***	(0.001)
Number of observations	331		13,294	
Number of countries; banks	17		1,329	
Wald Chi^2 [d.o.f.]	442.19***	[17]	–	
R^2	0.484		0.197; 0.273; 0.296	
AR-coefficient; Baltagi-Wu LBI	–		0.442; 1.509	

Note: ***, **, * denote significance at the 1%, 5% and 10% level, respectively. We drop the current slope of the yield curve, s_t, because it equals the long-term minus the short-term interest rate, which are already both included as separate regressors.
Aggregate bank data: Generalized least squares with random country effects, heteroskedastic panels and panel-specific autocorrelation. R^2 is the (adjusted) pseudo-R^2. For aggregate bank data we apply backward elimination according to the following rules: insignificant regressors are removed unless one of their lags is significant. Inflation is always included to allow for the possibility that real rather than nominal interest rates are the relevant drivers. For methodological reasons, variables are included if they are significantly interacted with other variables. – = regressor excluded due to insignificance in the backward elimination procedure.
Individual bank data: Least squares with fixed bank effects and an AR(1) error term. R^2 is given for within, between, and overall, respectively. We include banks with a loans-to-assets ratio above 0.2.

Figure 5.5: Optimal breakpoint analysis for severe recessions.



The figure shows the Wald test statistic for the null hypothesis that severe recessions do not have any additional impact on bank profit. The test has been applied to the model for return on assets estimated with aggregate bank data (Table 5.5, model (3)). Following Hansen (1997), the horizontal dotted line reports above which value the joint insignificance of the recession dummies can be rejected at a 1% level.

5.4.1 Net interest income

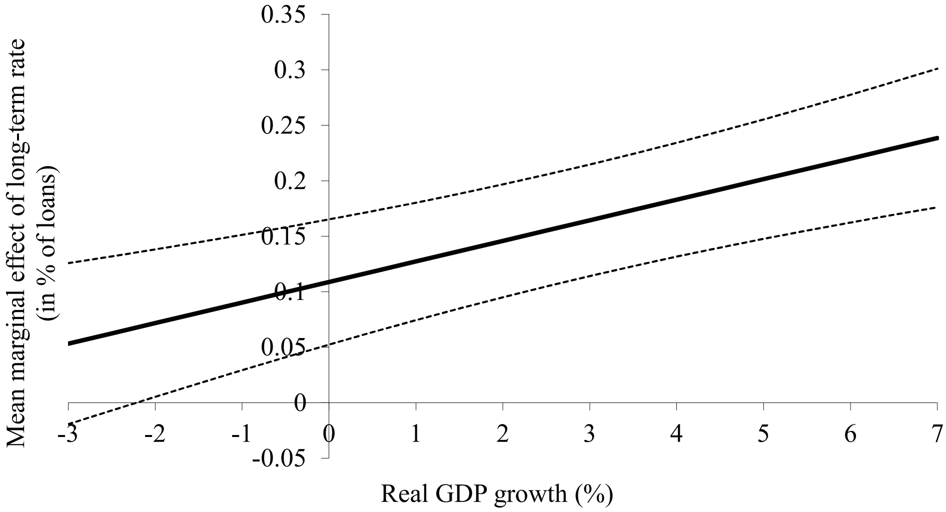
The results indicate a protracted and significant positive relation between net interest income and the long-term interest rate. The reported coefficients in Table 5.7 are significant up to 4 lags. Real GDP growth is also found to have a significant influence on net interest income, both as a stand-alone regressor and interacted with the long-term interest rate, which confirms our theoretical prior. Figure 5.6 reports the marginal effect of the lagged long-term interest rate given lagged real GDP growth.[16] As it turns out, a 1.0 percentage point increase in the long-term interest rate increases net interest income by 0.11% of the loan portfolio during periods of no growth. However, if real activity expands by 5.0%, an identical increase in the long-term interest rate increases net interest income by as much as 0.20% of the loan portfolio. Finally, the slope of the yield curve is found to restrain the positive effect of higher long-term interest rates on net interest income, see Figure 5.7.

The short-term interest rate has a negative marginal effect on net interest income, as expected. Based on aggregate data, the size of the marginal effect is reported in Figure

this correction for autocorrelation. We further note that a Wald test rejects the replacement of individual bank dummies by country dummies.

[16]Marginal effects are computed by taking the derivative of the model in equation (5.13) with respect to the lagged long-term interest rate, $r_{f,t-1}$. After substituting the coefficients from the individual data in Table 5.7, the marginal effect is estimated by $\partial \frac{NII_t}{L_{t-1}} / \partial r_{f,t-1} \approx 0.129 + 1.854 y_{t-1} - 1.851 s_{t-1}$. The mean marginal effects reported in Figures 5.6 and 5.7 can be obtained by respectively replacing s_{t-1} and y_{t-1} with their sample means $\bar{s}_{t-1} \approx 0.011$ and $\bar{y}_{t-1} \approx 0.022$.

Figure 5.6: Marginal effect of the lagged long-term interest rate on interest income.



Marginal effect of the lagged long-term interest rate on interest income in relation to lagged real GDP growth. As real GDP growth increases, the long-term interest rate becomes more important. The figure is based on individual bank data. The dotted lines represent the 95% confidence interval. Figures based on aggregated bank data yield similar pictures.

5.8 for different funding structures.[17] Figure 5.8 shows that net interest income is strongly affected by changes in the short-term interest rate if the loan portfolio is completely funded in the wholesale market (i.e., D_{t-1}/L_{t-1}= 0%). However, the mean marginal effect of the short-term interest rate approaches zero as the proportion of loans funded by deposits increases, which decreases the proportion funded in the wholesale market given the level of equity. The right side of Figure 5.8 suggests that a bank could in theory neutralize its sensitivity to the short-term rate by taking a net long position in the wholesale market.

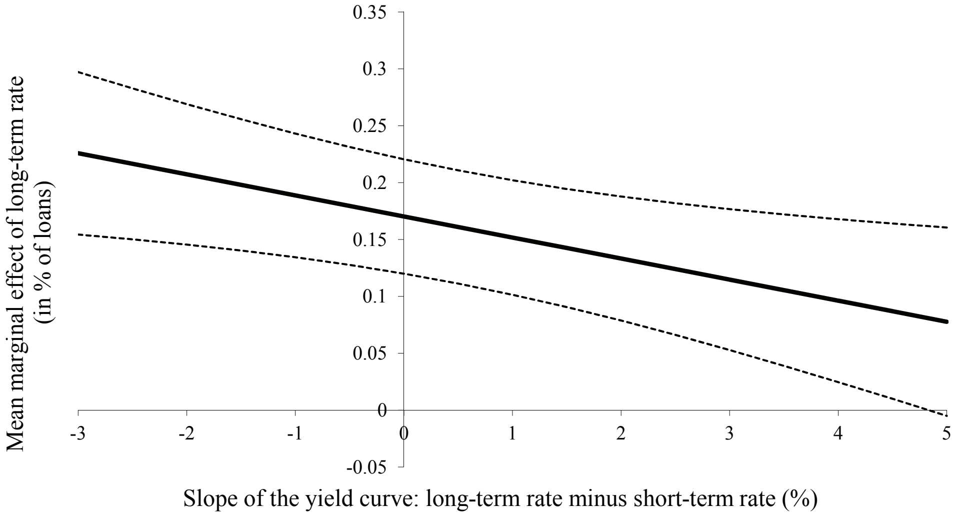
5.4.2 Net provisions and costs

The recession slope dummy variables turn out to be significant in both samples, giving support to the notion of asymmetric business cycle effects on loan losses (Table 5.5 and 5.6). A decline in real economic activity increases net provisioning and costs stronger during severe recessions. Further, in line with expectations, loan losses are significantly increased by a rise in unemployment.

We note that the timing of the asymmetric effect is different for the individual and the aggregate data. According to the aggregate data estimates, the asymmetric effect

[17]Similar to Figures 5.6 and 5.7, the marginal effect reported in Figure 5.8 is obtained by taking the derivative of the model in equation (5.13) with respect to the short-term interest rate, $r_{s,t}$, and substituting the estimated coefficients from the aggregate data in Table 5.7 to obtain the relation $\partial \frac{NII_t}{L_{t-1}} / \partial r_{s,t} \approx -0.325 + 0.260 D_{t-1}/L_{t-1}$.

Figure 5.7: Marginal effect of the lagged long-term interest rate on interest income



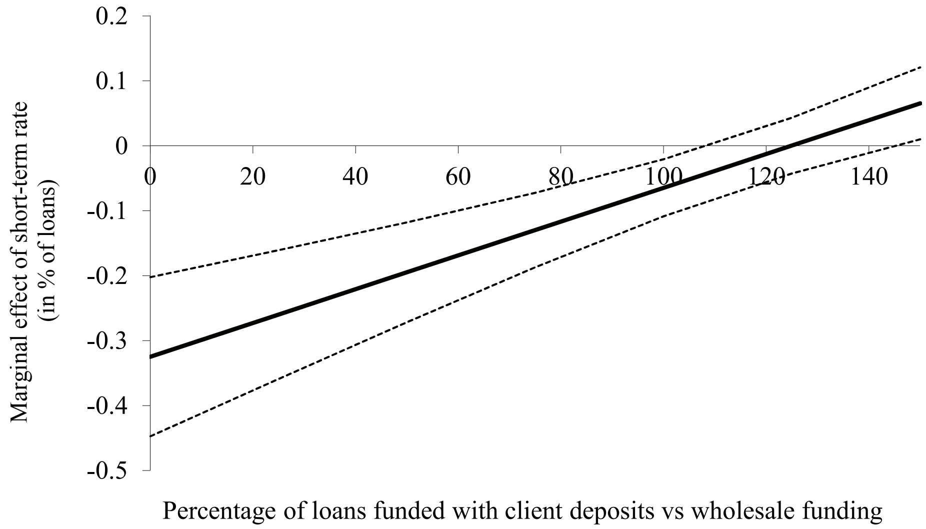
Marginal effect of the lagged long-term interest rate on net interest income in relation to the lagged slope of the yield curve. As one moves to the right, the yield curve becomes steeper and the long-term interest rate becomes less important. The figure is based on individual bank data. The dotted lines represent the 95% confidence interval.

is strongest in the first year *after* the recession, whereas the individual data estimates put emphasis on the asymmetric effect during the current period. Also its magnitude, measured by the sum of the asymmetric effects over three years, is different for the micro and macro samples: calculations suggest that each additional percentage point of a fall in real GDP during a severe recession results in additional loan losses of 0.9% or 0.4% of the current loan portfolio according to the aggregate and individual data, respectively.[18]

There is some evidence that unemployment affects net provisions and costs, presumably due to the incidence of problem loans. Table 5.5, model (2), reports a (weakly significant) positive coefficient for the interaction between the loans to assets ratio and the unemployment rate. Apparently, the effect of unemployment on net provisions and costs increases with the proportion of loans on the bank balance sheet. Table 5.6, model (2), confirms this finding for the individual bank sample. Moreover, the coefficient for unemployment itself is found to be significant as well.

[18] For the aggregate data the additional effect during severe recessions equals $-0.200 - 0.475 - 0.274 \approx -0.94$ and for the individual data the effect equals $-0.440 + 0.105 - 0.0852 \approx -0.42$. As mentioned in the introduction, it is to be expected that the two datasets give different results. Experiments with several weighting schemes, with proxies for bank size, such as the log of the average or median number of employees, or the log of one plus the ranking based on total assets in 2007 (converted to a common currency), or inverses of these, indicate that these differences are persistent.

Figure 5.8: Marginal effect of short-term interest rate and the funding structure



Marginal effect of short-term rate on net interest income for different funding structures. As one moves from the left to the right along the x-axis, deposit funding becomes a more important source of funding for the bank. On the x-axis we denote deposits as percentage of loans, D/L. As funding through deposits increases, net interest income becomes less sensitive to movements in the short-term interest rate. The figure is based on aggregate bank data. The dotted lines represent the 95% confidence interval. Figures based on individual bank data yield a less distinctive picture.

5.4.3 Other income

The fit of the model for this rather heterogeneous profit component (equation (5.14)) is weak, both for the aggregate sample and the individual bank sample. However, in both samples, the coefficient of the local stock market index is significant and positive. This confirms that other income, which partly consists of investment banking fees, moves with the tide of stock market sentiments. Further, a positive effect of (lagged) economic growth is found in both datasets.

5.5 Conclusion

The current banking crisis and the concurrent severe recession revive the interest in the issue of pro-cyclicality of bank profitability. This chapter contributes to existing literature in several respects. First, we assess whether the degree of pro-cyclicality of bank profitability is stronger for deep recessions than under normal economic conditions. Second, we derive a theoretical model for bank profit, which takes into account that the composition of all outstanding loans at the current period results from the accumulation of loans extended in previous periods, on the one hand, and the survivor rate of these loans (depending on both amortization and loan losses), on the other. Further, we do not only estimate the pro-cyclicality of total profit, but also of the three components that define it: net interest income, other income, and net provisioning plus all other costs. Finally, we test this relationship using both aggregate and individual bank data.

This approach yields two main empirical results:

First, we find evidence for our theoretical prediction that a bank's lending history should also be taken into account when explaining its current net interest income. Specifically, long-term interest rates in previous years are found to be important determinants, especially when economic growth (and, hence, lending activity) was relatively high at the time.

Second, we find evidence that bank profit behaves pro-cyclically and that this co-movement is especially strong during severe recessions. Among the different profit components, loan-loss provisioning is found to be the driver of this asymmetry. We find evidence that each percentage point of a contraction in real GDP during severe recessions leads to a quarter of a percentage point decrease in the return on bank assets.

5.A Appendix A. Variable definitions and data sources

See Table 5.8.

5.B Appendix B. Derivation for net interest income

We define the rate of return on the loan portfolio, $r_{L,t}$, as:

$$r_{L,t} = \frac{1}{2}\omega_t \times r_{l,t} + \sum_{i=1}^{\infty} \omega_{t-i} \times r_{l,t-i}. \tag{5.17}$$

Assuming the probability of a loan turning into a bad loan to be independent of the loan's maturity, we can write the weight of loans from year $t-i$ in the total current loan portfolio, ω_{t-i}, as:

$$\omega_{t-i} = \frac{NL_{t-i}}{L_t} \times \prod_{j=1}^{i} \lambda_{t-j+1} \text{ for } i = 1, 2, \cdots, \text{ and } \omega_t = \frac{NL_t}{L_t}, \tag{5.18}$$

where NL_{t-i} denotes the amount of loans originated during year $t-i$ and where λ_{t-j+1} denotes the survival rate of loans from year $t-j$ to year $t-j+1$ as:

$$\lambda_{t-j+1} = \overline{\lambda} - \frac{BL_{t-j+1}}{L_{t-j}}, \tag{5.19}$$

with $\overline{\lambda}$ the survival rate due to the 'natural' maturity structure of bank loans. Substituting definitions (5.18) and (5.19) into (5.17) and multiplying by L_t gives:

$$r_{L,t}L_t = \left(\frac{1}{2}NL_t\right) r_{l,t} + \sum_{i=1}^{\infty} r_{l,t-i} \left(NL_{t-i} \times \prod_{j=1}^{i} \lambda_{t-j+1}\right). \tag{5.20}$$

Substituting equation (5.19) and subtracting interest expenses, $r_{D,t}D_t$, from equation (5.20) yields the intermediate result in equation (5.5) as in (5.21):

$$NII_t = \frac{1}{2}r_{l,t}\left(NL_t\right) + \sum_{i=1}^{\infty} r_{l,t-i} \left(NL_{t-i} \times \prod_{j=1}^{i} \left(\bar{\lambda} - \frac{BL_{t-j+1}}{L_{t-j}}\right)\right) - r_{D,t}D_t. \tag{5.21}$$

Multiplying equation (5.20) by $\frac{1}{L_{t-1}}$, which is equal to $\frac{1}{L_{t-i-1}} \prod_{j=1}^{i} \frac{L_{t-j-1}}{L_{t-j}}$, gives:

$$\frac{r_{L,t}L_t}{L_{t-1}} = \left(\frac{1}{2}\frac{NL_t}{L_{t-1}}\right) r_{l,t} + \sum_{i=1}^{\infty} r_{l,t-i} \left(\frac{NL_{t-i}}{L_{t-i-1}} \times \prod_{j=1}^{i} \lambda_{t-j+1} \frac{L_{t-j-1}}{L_{t-j}}\right). \tag{5.22}$$

Substituting $L_{t-j} = NL_{t-j} + \lambda_{t-j}L_{t-j-1}$ into the product term in equation (5.22) results in:

$$\frac{r_{L,t}L_t}{L_{t-1}} = \left(\frac{1}{2}\frac{NL_t}{L_{t-1}}\right) r_{l,t} + \sum_{i=1}^{\infty} r_{l,t-i} \left(\frac{NL_{t-i}}{L_{t-i-1}} \times \prod_{j=1}^{i} \frac{\lambda_{t-j+1}L_{t-j-1}}{NL_{t-j} + \lambda_{t-j}L_{t-j-1}}\right). \tag{5.23}$$

Table 5.8: Variable definitions and data sources.

Variable	Symbol	Definition	Source
Macroeconomic data			
Real GDP growth	y	Δlog(GDP at constant prices)	OECD MEI
Inflation	i	Δlog(GDP at current prices) – Δlog(GDP at constant prices)	OECD MEI
Long-term interest rate	r_f	Interest rate on government bonds (10 years)	OECD MEI
Short-term interest rate	r_s	Money market interest rate (3 months)	OECD MEI
Local stock market index return	r_m	Δlog(Stock market index)	OECD MEI
Local stock market index volatility	CV_m	$\hat{\sigma}(r_m)/\hat{\mu}$(Stock market index)	OECD MEI
Unemployment rate	u	Unemployment / (Unemployment + Employment)	OECD MEI
Slope yield curve	s	Long-term interest rate – Short-term interest rate	OECD MEI
Aggregated bank data			
Profit before tax	Π	Income before tax (9)	OECD BPS
Net interest income	NII	Net interest income (3)	OECD BPS
Other income	OI	Net non-interest income (4)	OECD BPS
Net provisions and costs	$BL + OC_t$	Operating expenses (6) + Net provisions (8)	OECD BPS
Loans	L	Loans (16) – Bonds (23)	OECD BPS
Deposits	D	Customer deposits (22)	OECD BPS
Other net interest bearing liabilities	R	Borrowing from central bank (18) + Interbank deposits (19) – Cash and balance with central bank (14) – Interbank deposits (15) – Securities (17)	OECD BPS
Assets	A	Balance sheet total (25)	OECD BPS
Individual bank data			
Profit before tax	Π	Profit before tax	BankScope
Net interest income	NII	Net interest revenue	BankScope
Other income	OI	Other income	BankScope
Net provisions and costs	$BL + OC_t$	Profit before tax – Net interest revenue – Other income	BankScope
Loans	L	Loans	BankScope
Deposits	D	Total customer deposits	BankScope
Other net interest bearing liabilities	R	Other funding + Deposits from banks + Other deposits and short-term borrowing – Other earning assets	BankScope
Assets	A	Total assets	BankScope

Note: MEI refers to *Main Economic Indicators*, BPS refers to *Bank Profitability Statistics*. For BPS, item numbers are given within parentheses.

Dividing the numerator and denominator of the fraction in the product term by L_{t-j-1}:

$$\frac{r_{L,t}L_t}{L_{t-1}} = \left(\frac{1}{2}\frac{NL_t}{L_{t-1}}\right) r_{l,t} + \sum_{i=1}^{\infty} r_{l,t-i} \left(\frac{NL_{t-i}}{L_{t-i-1}} \times \prod_{j=1}^{i} \frac{\lambda_{t-j+1}}{\frac{NL_{t-j}}{L_{t-j-1}} + \lambda_{t-j}}\right). \tag{5.24}$$

Similar to the derivation of gross interest income above, we also derive an expression for interest expenses under the assumption that deposits flow in uniformly during the period,

$$\frac{r_{D,t}D_t}{D_{t-1}} = \left(1 + \frac{1}{2}\frac{D_t - D_{t-1}}{D_{t-1}}\right) r_{d,t}. \tag{5.25}$$

We subtract interest expenses in equation (5.25) from interest income in equation (5.24) to derive net interest income, NII_t. We substitute equations (5.6), (5.7) and (5.9) into net interest income which yields equation (5.12) as in equation (5.26):

$$\frac{NII_t}{L_{t-1}} = \left\{\frac{1}{2}g_t \cdot r_{l,t} + \sum_{i=1}^{\infty} r_{l,t-i} \left(g_{t-i} \times \prod_{j=1}^{i} \frac{\lambda_{t-j+1}}{g_{t-j} + \lambda_{t-j}}\right)\right\} - \frac{D_{t-1}}{L_{t-1}} \times \left\{\left(1 + \frac{1}{2}h_t\right) r_{d,t}\right\}. \tag{5.26}$$

We add the term R_{t-1} to the equation in order to correct for interest expenses on other net interest bearing liabilities in the bank balance sheet data (see Appendix 5.A for details on R_{t-1}). Simplifying the model in equation (5.26) by replacing the product term $\prod_{j=1}^{i} \lambda_{t-j+1}/\left(g_{t-j} + \lambda_{t-j}\right)$ with C^i gives:

$$\frac{NII_t}{L_{t-1}} = \left[\frac{1}{2}g_t \cdot r_{l,t} + \sum_{i=1}^{\infty} r_{l,t-i} g_{t-i} C^i\right] - \frac{D_{t-1}}{L_{t-1}} \times \left[\left(1 + \frac{1}{2}h_t\right) r_{d,t}\right] - \frac{R_{t-1}}{L_{t-1}} \left[r_{R,t}\right]. \tag{5.27}$$

Take a first-order approximation for $g_{t-i} \approx a_0 + a_1 \cdot y_{t-i} + a_2 \cdot s_{t-i}$ and $h_t \approx b_0 + b_1 r_{s,t} + b_2 i_t$:

$$\begin{aligned} \tfrac{NII_t}{L_{t-1}} \approx & \left[\tfrac{1}{2}\left(a_0 + a_1 \cdot y_t + a_2 \cdot s_t\right) \cdot r_{l,t} + \textstyle\sum_{i=1}^{\infty} r_{l,t-i}\left(a_0 + a_1 \cdot y_{t-i} + a_2 \cdot s_{t-i}\right) C^i\right] \\ & - \tfrac{D_{t-1}}{L_{t-1}} \times \left[\left(1 + \tfrac{1}{2}\left(b_0 + b_1 r_{s,t} + b_2 i_t\right)\right) r_{d,t}\right] - \tfrac{R_{t-1}}{L_{t-1}}\left[r_{R,t}\right]. \end{aligned} \tag{5.28}$$

Assume that the financing rate for 'other net interest bearing liabilities', R_{t-1}, is given by some linear function of the short-term interest rate ($r_{R,t} = d_0 + d_1 \cdot r_{s,t}$). Substitute equation (5.3) and (5.4) for $r_{l,t-i}$ and $r_{d,t}$:

$$\begin{aligned} \tfrac{NII_t}{L_{t-1}} \approx & \left[\begin{array}{l} \tfrac{1}{2}\left(a_0 + a_1 \cdot y_t + a_2 \cdot s_t\right) \cdot \left(r_{f,t} + k + c_L + f^e\right) + \\ + \sum_{i=1}^{\infty}\left(r_{f,t-i} + k + c_L + f^e\right)\left(a_0 + a_1 \cdot y_{t-i} + a_2 \cdot s_{t-i}\right) C^i \end{array}\right] \\ & - \tfrac{D_{t-1}}{L_{t-1}} \times \left[\left(1 + \tfrac{1}{2}\left(b_0 + b_1 r_{s,t} + b_2 i_t\right)\right)\left(r_{s,t} - c_D\right)\right] \\ & - \tfrac{R_{t-1}}{L_{t-1}}\left[d_0 + d_1 \cdot r_{s,t}\right]. \end{aligned} \tag{5.29}$$

When estimating the model on net interest income separately, we find that including up to four lags yields significant coefficients. For $i = 1, 2, 3, 4$, renaming the estimation parameters and adding an error term yields the empirical specification in equation (5.13) as:

$$\begin{aligned}\frac{NII_t}{L_{t-1}} &= \alpha_{11} + \alpha_{12}\frac{D_{t-1}}{L_{t-1}} + \alpha_{13}\frac{R_{t-1}}{L_{t-1}} \\ &+ \sum_{i=0}^{4}\left[\beta_{i1}r_{f,t-i} + \beta_{i2}y_{t-i} + \beta_{i3}s_{t-i} + \beta_{i4}(r_{f,t-i}\cdot y_{t-i}) + \beta_{i5}(r_{f,t-i}\cdot s_{t-i})\right] \\ &+ \frac{D_{t-1}}{L_{t-1}}\times\left[\beta_{51}r_{s,t}+\beta_{52}r_{s,t}^2+\beta_{53}i_t + \beta_{54}r_{s,t}\cdot i_t\right] + \beta_{61}\frac{R_{t-1}}{L_{t-1}}r_{s,t} + \varepsilon_t.\end{aligned} \tag{5.30}$$

5.C Appendix C. Specification of profit before tax

By substituting equation (5.13), (5.14) and (5.15) into (5.16), we obtain:

$$\begin{aligned}\frac{\Pi_t}{A_{t-1}} &= (\delta_{01}-\gamma_{41})\,y_t + \delta_{02}y_{t-1} - \frac{L_{t-1}}{A_{t-1}}\times\sum_{i=0}^{2}\left[\gamma_{i1}y_{t-i} + \gamma_{i2}1_{y_{t-i}<a}\cdot(y_{t-i}-a)\right] \\ &- \gamma_{31}\frac{L_{t-1}}{A_{t-1}}u_t - \gamma_{46}i_t + (\delta_{03}-\gamma_{42})\,r_{f,t} + (\delta_{04}-\gamma_{43})\,r_{s,t} \\ &+ \frac{L_{t-1}}{A_{t-1}}\times\sum_{i=0}^{4}\left[\beta_{i1}r_{f,t-i} + \beta_{i2}y_{t-i} + \beta_{i3}s_{t-i} + \beta_{i4}(r_{f,t-i}\cdot y_{t-i}) + \beta_{i5}(r_{f,t-i}\cdot s_{t-i})\right] \\ &+ \frac{D_{t-1}}{A_{t-1}}\times\left[\beta_{51}r_{s,t}+\beta_{52}r_{s,t}^2+\beta_{53}i_t + \beta_{54}r_{s,t}\cdot i_t\right] + \beta_{61}\frac{R_{t-1}}{A_{t-1}}r_{s,t} \\ &+ (\delta_{05}-\gamma_{44})\,r_{m,t} + (\delta_{06}-\gamma_{45})\,CV_{m,t} \\ &+ (\alpha_{11}-\alpha_{32})\,\frac{L_{t-1}}{A_{t-1}} + (\alpha_{12}-\alpha_{33})\,\frac{D_{t-1}}{A_{t-1}} + \alpha_{13}\frac{R_{t-1}}{A_{t-1}} + (\alpha_{21}-\alpha_{31}) + \frac{L_{t-1}}{A_{t-1}}\varepsilon_t + \xi_t - \nu_t.\end{aligned}$$

When estimating the empirical specification on profit before tax, we restrict the number of lags on the determinants of net interest income to one.

Chapter 6

On agricultural commodities' extreme price risk

Price risk is among the most substantial risk factors for farmers. We assess how power law tails in agricultural prices may arise endogenously in economic systems. Using thirty years of daily futures price data, we show that the returns of nine agricultural commodities closely follow a power law in the tail of their distributions. We apply Extreme Value Theory to estimate Value-at-Risk and Expected Shortfall risk measures. Back-testing verifies the validity of these risk measurement methods.[1]

Keywords: Agricultural commodities, Extreme Value Theory, heavy tails, risk management.

JEL Classification Numbers: C14, Q11, Q14.

6.1 Introduction

Extreme movements in agricultural commodity prices are anything but uncommon. For instance, between August 2007 and March 2008, the price of wheat almost doubled. However, before the end of 2008, the wheat price had again returned to its original level. Another example is the price of corn, which fell a massive 55% in the second half-year of 2008 alone. More recently, in the summer of 2012, price increases of corn, soybeans and other field crops of more than 50% coincided with severe droughts. These whopping returns illustrate the highly volatile behavior of agricultural commodity prices.

Large price movements negatively impact many of the industry's stakeholders, among

[1]This chapter is co-authored by Philip A. Stork and Casper G. de Vries. Comments and suggestions from Pierre M. Lafourcade, Job Swank and Chen Zhou are gratefully acknowledged.

which are producers, traders and processing firms. Also, the developing countries' economic prosperity often relies on the price development of raw material commodities. Total exports of more than fifty countries consist for more than half of only three or fewer commodities.[2] This reliance makes these countries very vulnerable to price decreases and volatility, see e.g. Balagtas and Holt (2009), Deaton (1999) or Lence (2009).

In the same vein, individual farmers also are highly vulnerable to commodity price risk, see e.g. Lence (2009). All price decreases translate fully and directly in a loss of the farmer's income. In this chapter we take the perspective of an individual U.S. farmer when analyzing agricultural commodities price risk. Nevertheless, many of the risk management findings in this chapter are also relevant to other affected parties.

A good understanding of the most extreme commodity returns is instrumental in any commodity risk management application. Small changes in revenues should not affect the farmer much. Large price drops however, may result in bankruptcy. Using a standard two-sector macro model, we describe how fat tails in agricultural commodity prices may arise endogenously. In our general equilibrium model, commodity price spikes occur as a result of factor productivity shocks, due to e.g. hurricanes and droughts. These shocks feed through the system and render the equilibrium price distribution fat-tailed.

As the typical farmer is risk-averse, see e.g. Coyle (2007), he is inclined to hedge his price risks. He may for instance remove his price risk by entering into forward or futures contracts, see e.g. Lence (2009). Alternatively, he may choose to hedge only against large price declines by buying out-of-the-money put options. He would then retain the potential profit from sudden price increases. In order to decide upon the optimal risk mitigation strategy, the farmer needs a risk quantification methodology.[3] He should be able to answer questions like: How likely is a 10% fall in the corn price over the next two days? How likely is a price change which may result in bankruptcy? Is either the corn or the wheat price more likely to be involved with extreme price movements? What is the expected size of the maximum loss due to price risk during the next decade?

Such questions may be addressed by the use of Extreme Value Theory (EVT). This technique is particularly suited for estimating the likelihood of extreme returns when the probability distribution functions are non-normal. Mills (1927) was one of the first to discuss the non-normality of commodity returns as he reported a higher kurtosis, and thus implying more extreme returns. Mandelbrot (1963b) modeled the returns in the spot market for cotton by means of the stable distributions to capture the heavy-tail

[2] Based on the UNCTAD 1995 Commodity Yearbook. We refer to Bidarkota and Crucini (2000) for an extensive analysis of the relationship between the terms of trade of developing nations and world prices of internationally traded primary commodities.

[3] For discussion on the use of derivatives to hedge commodity price risk we refer to Lu and Neftci (2009).

phenomenon. More recently, Ai *et al.* (2006) also discuss the non-normality of commodity returns, which they find to be characterized by frequent price jumps and fat tails. We refer to Wang and Tomek (2007), and Kat and Oomen (2006) for thorough studies of the time series properties of agricultural product prices.

Over the last years, the popularity of EVT to assess the risk of an extreme event has increased considerably. For example, EVT has been used to examine the severity of stock market crashes, the pricing of catastrophic loss risk in reinsurance or the extent of operational risk in banks.[4] EVT is particularly suitable for the analysis of extremely rare events when sample sizes are too small for determining the probability, extent or cause of the extreme returns using conventional statistical techniques. The semi-parametric EVT approach exploits the functional regularities that probability distributions display far away from the center.

Interestingly, in spite of its growing recognition, EVT applications to agricultural price risk management have so far been sparse in the academic literature. Kofman and De Vries (1990) estimate the tail distribution parameters for potato futures. Matia *et al.* (2002) estimate the parameters of a large number of general commodities and find the tails to be fat. Their paper provides no risk management applications, however. Krehbiel and Adkins (2005) apply EVT to four complex NYSE energy futures contracts to estimate various risk measures. Even so, their analysis is limited to oil and gas contracts, whose return distribution may be very different from renewable agricultural commodities. More recently, Morgan *et al.* (2012) use EVT on weekly data to estimate three different tail risk measures for corn and soybeans. Their thorough study is evidence of the growing interest in this topic.

The main contribution of this chapter is twofold. The first contribution is to show how the heavy-tailedness of agricultural prices may arise endogenously in an economic model. The second contribution is the use of EVT to measure the extreme price risk of nine different agricultural commodities. We use a back-testing procedure to provide empirical evidence on the accuracy of the proposed risk measures. We show that the non-normality of the return distribution strongly influences the level of the risk measures. Our empirical estimates provide a good indication of the size of the risks as measured by widely used and easily interpretable risk measures. This chapter provides farmers and other stakeholders with a reliable toolset to quantify their price risks and to answer the questions above.

The remainder of this chapter is as follows. Section 6.2 provides a model in which commodity price spikes arise endogenously, as a result of productivity shocks. Section 6.3 discusses how to apply EVT to estimate the Value-at-Risk and Expected Shortfall risk

[4] Based on the ECB June 2006 Financial Stability Review.

measures. The data are described in Section 6.4. Empirical estimates of the distributions' tails are presented in Section 6.5. Value-at-Risk and Expected Shortfall estimates, as well as back-testing results are provided in Section 6.6. We conclude in Section 6.7.

6.2 Theory

Mandelbrot (1963a,b), using Houthakker's cotton price series, is probably the first who documented that the tails of the distribution of logarithmic commodity price changes diminishes by a power instead of an exponent, as is the case under the more common (log)normal assumption. If the tail of a distribution diminishes by a power, then the probability of variable x exceeding threshold u, if u is large, is distributed as:

$$\Pr(x > u) \sim Cu^{-\alpha}, \tag{6.1}$$

where $C > 0$ and $\alpha > 0$ are respectively the *scale* and the *shape* parameter. The distribution is named after Pareto who discovered that the tail of the income distribution follows a power law. Distributions with tails that obey the functional form in (6.1) are classified as *heavy-tailed.* Tails which follow a power law are in the end always fatter than tails that decrease by an exponent.

To explain the heavy-tail nature, Mandelbrot advances that the physical world is full of heavy-tailed phenomena, which may trigger the heavy-tailedness of commodity price changes.[5] But how a power law may arise endogenously in this type of market has not been investigated.

Below we develop a small standard macro model with an agricultural sector to study the agricultural prices in equilibrium. The equilibrium agricultural price distribution is the result of factor productivity shocks that feed through the system. These shocks themselves do not have to be (but can be) heavy-tailed, in contrast to what is assumed in the literature discussed above. This is the extra kick that our economic analysis provides. We show how the power law spikes observed in agricultural commodity prices can arise endogenously in the economy. The model describes how adverse productivity shocks, such as drought and hurricanes, affect the tail distribution of commodity prices.

[5] See e.g. Newman (2005) and Salvadori *et al.* (2007) for a number of natural hazards that follow a power law distribution, among which the magnitude of earthquakes, the volume of air-fall material from volcanic eruptions, various drought measures, flood levels, and the scale of wars. Several of the above events influence agricultural prices in one way or another. Spikes or sudden drops in prices can be triggered by, for instance, a drought or bumper crop.

6.2.1 Model

We use a standard off-the-shelf two-sector macro model.[6] The agricultural sector is modeled as the competitive sector. The other sector produces differentiated goods in the spirit of Dixit and Stiglitz (1977) (subsequently referred to as DS1977). Exogenous shocks affect the productivity of both sectors. In the agricultural sector, these shocks can be best thought of as changes in weather and other natural hazards. For the differentiated goods sector, which also captures the services industry, the shocks mostly represent changes in productivity.

The macro literature has focused almost exclusively on the DS1977 specification for the differentiated goods demand, see e.g. Walsh (2008). The familiar DS1977 specification with endogenous labor supply derives from the following utility function

$$U = Z^{1-\theta}\left[\frac{1}{n}\sum_{i=1}^{n} Q_i^{\rho}\right]^{\theta/\rho} - \frac{1}{1+\gamma}L^{1+\gamma}, \tag{6.2}$$

where Z is the competitive good, the Q_is are the differentiated goods and L is labor. To guarantee concavity and allow for zero demand for a particular Q_i, the parameter ρ is constrained to $\rho \in (0,1)$. We envision the Z good to be the staple of agricultural produce, while the Q_i goods capture the production of other goods and services. Parameter $\theta \in (0,1)$ determines the relative importance of the other goods and services to the agricultural produce in the consumer's consumption bundle. The higher the level of θ, the smaller the share of income the consumer is willing to spend on agricultural goods. Parameter γ is the inverse of the Frisch elasticity of labor supply. In general, the higher the level of γ, the less responsive the labor supply is to changes in the wage rate.

The budget constraint reads

$$wL + \Pi(Q) = qZ + \frac{1}{n}\sum_{i=1}^{n} p_i Q_i, \tag{6.3}$$

where w is the wage rate and q, p_i are the goods prices, while $\Pi(Q)$ are the profits received from the differentiated goods sector.[7]

For the supply side we assume Ricardian technologies for all the goods, where

$$Z = BN, \tag{6.4}$$

and

$$Q_i = AN_i. \tag{6.5}$$

[6] See Ardeni and Freebairn (2002) for a discussion on the interaction between agricultural prices and the macro economy.

[7] The quantities of the differentiated goods, the Q_i, are normalized by the number of differentiated goods, n. This notation is in analogy with the common continuous good notation often used in theoretical macro literature.

Here A and B are the productivity coefficients while N and N_i are the respective labor inputs. Both A and B are random variables. In the case of A, these are the familiar supply side total factor productivity shocks. In the case of the agricultural sector, the B captures the random element due to nature. We assume that the market for the agricultural product is perfectly competitive. The producer of the differentiated product exploits his pricing power, but ignores his pricing effect on the consumer income $wL+\Pi(Q)$ and the price index of the differentiated goods,

$$P = \left(\frac{1}{n} \sum_{i=1}^{n} p_i^{\rho/(\rho-1)} \right)^{\frac{\rho-1}{\rho}}. \tag{6.6}$$

Finally, to determine the price level we assume a simple quantity type relation

$$M = wL. \tag{6.7}$$

6.2.2 Equilibrium price distribution

With the above preparations, we can now obtain the implications for the equilibrium prices.

Proposition 6.2.1 *The prices of the differentiated goods are*

$$p_i = M \frac{1/\rho^{\theta/\gamma+1}}{A \left(\theta^\theta (1-\theta)^{1-\theta} A^\theta B^{1-\theta} \right)^{1/\gamma}}. \tag{6.8}$$

For the agricultural good the price is

$$q = M \frac{1/\rho^{\theta/\gamma}}{B \left(\theta^\theta (1-\theta)^{1-\theta} A^\theta B^{1-\theta} \right)^{1/\gamma}}. \tag{6.9}$$

Proof. See Appendix 6.A. ■

Most macro models consider shocks to M, A and B. Let us focus on the natural shocks B.[8] Assuming M and A to be constant, we can write the price of the agricultural good as

$$q(B) = \Theta B^{-\frac{1+\gamma-\theta}{\gamma}}, \tag{6.10}$$

[8]For our results to hold it is not necessary to assume a constant A and M. For example, the heavy-tailedness of the equilibrium price distribution due to natural shocks is preserved if the productivity of the differentiated sector does not collapse completely, which implies that the support of A is bounded away from zero. Further, the heavy-tailedness of the equilibrium price distribution is not affected if the distribution of M has exponential tails, such as the lognormal distribution.

where

$$\Theta = M \frac{1/\rho^{\theta/\gamma}}{\left(\theta^{\theta}\left(1-\theta\right)^{1-\theta} A^{\theta}\right)^{1/\gamma}}.$$

For illustrative purposes we assume that B follows a beta distribution (we relax this assumption later):

$$\Pr\left\{B < t\right\} = t^{\beta} \tag{6.11}$$

on $[0, 1]$ and $\beta > 0$. Consider the implication for the price distribution of the agricultural product. Denote the randomness in q by $\tilde{q}$. Then

$$\begin{aligned} \Pr\left\{\tilde{q} > u\right\} &= \Pr\left\{\Theta B^{-\frac{1+\gamma-\theta}{\gamma}} > u\right\} \\ &= \Pr\left\{B < \Theta^{\frac{\gamma}{1+\gamma-\theta}} u^{-\frac{\gamma}{1+\gamma-\theta}}\right\} \\ &= \Theta^{\frac{\beta\gamma}{1+\gamma-\theta}} u^{-\frac{\beta\gamma}{1+\gamma-\theta}}, \end{aligned} \tag{6.12}$$

with support on $[\Theta, \infty)$. The distribution of equilibrium prices in equation (6.12) has the same functional form as the heavy-tailed distribution in equation (6.1). But naturally, this result is subject to the qualification that it crucially relies on the restrictive assumption of the beta distribution for shocks from nature in (6.11).

We proceed with relaxing the assumption of the beta distribution for B in (6.11). More specific, we derive a general condition on the density function of B, $f_B(B)$, such that the equilibrium prices follow a heavy-tailed distribution if this condition hold. This condition is provided in the following proposition.

Proposition 6.2.2 *Given the price-productivity relation for agricultural products in equation* (6.10)*, we have that*

$$\Pr(\tilde{q} > u) \sim \mathcal{L}(u) u^{-\alpha} \text{ as } u \to \infty, \tag{6.13}$$

with

$$\alpha = \xi \frac{\gamma}{1+\gamma-\theta}, \tag{6.14}$$

if

$$\lim_{s \downarrow 0} \frac{w f_B(sw)}{f_B(s)} = w^{\xi} \text{ with } \xi \in \mathbb{R}^{+}. \tag{6.15}$$

Proof. See Appendix 6.B. ■

Following the condition in (6.15), the shape of the density function of B close to zero determines whether the condition on the distribution of B is satisfied. It is not difficult to verify that a broad range of distribution functions with positive support satisfy the condition in the proposition. For instance, the standard uniform distribution and the exponential distribution satisfy the condition in (6.15) with $\xi = 1$, the Chi-squared distribution with k degrees of freedom satisfies the condition with $\xi = k/2$, the Gamma distribution with shape parameter k satisfies the condition with $\xi = k$, and the (heavy-tailed) Burr (Type XII) distribution with parameters (c, k) satisfies the condition with $\xi = c$. All these distributions would result in a heavy-tailed equilibrium price distribution.

However, not every possible distribution yields heavy-tailed prices in the macroeconomic framework. An example of a popular exception is the lognormal distribution: Its limit in (6.15) converges to 0.[9]

From Proposition 6.2.2 we have that the shape parameter of the tail distribution is not only affected by the properties of the statistical distribution of productivity shocks, but also by the value of economic parameters θ and γ. Given the distribution function of productivity shocks, it follows from equation (6.14) that a high value of $1 - \theta$ results in a low shape parameter of the equilibrium price distribution of agricultural goods, α, and hence in a fatter tail. This finding has the following intuition. The importance of the share of the agricultural good in the consumption bundle of the agents is represented by $1 - \theta$, see equation (6.2). The larger the role of the agricultural good for the agents' utility, the more extreme price reactions one may expect if supply falls. This is reflected in a fatter tail of the equilibrium price distribution, i.e., a lower α.

It also follows from equation (6.14) that a high value of the parameter γ results in a high value of the shape parameter, α. Adverse technology shocks have a twofold effect on the output of the competitive sector. First, given the amount of labor used, an adverse technology shock in the competitive sector directly reduces output. Second, low productivity decreases the equilibrium amount of labor used in the competitive sector, which further reduces output. Parameter γ determines the sensitivity of the wage rate to changes in the amount of labor used in production. With a higher value of γ, a reduction in the amount of labor results in a larger drop of the wage rate, which translates in a smaller change in the equilibrium amount of labor. Therefore, the change in production of the competitive good is smaller for high values of parameter γ, which results in thinner tails of the equilibrium price distribution, i.e., a higher α.

[9] The statistical distribution of crop yields has been the topic of an extant literature, see e.g., Day (1965), Gallagher (1986), Nelson and Preckel (1989), Moss and Shonkwiler (1993), Ramírez (1997), Just and Weninger (1999), Atwood *et al.* (2003), Ramírez *et al.* (2003), Harri *et al.* (2009) and Koundouri and Kourogenis (2011).

6.3 Methodology

The previous section discussed the plausibility of fitting the tail distribution of changes in food prices to a power law. Next, we apply EVT to determine the parameters of the power law.

6.3.1 Fitting the power law

As a first step we calculate n returns, r_t, from the observed prices as $(P_t - P_{t-1})/P_{t-1}$. Secondly, the returns are ordered from high to low: $X_1 \geq \ldots \geq X_n$. The number of returns in the tail of the distribution is set equal to k. This implies that X_{k+1} approaches the threshold, which is the minimum u for which the distribution in (6.1) applies. All returns above this threshold are assumed to be distributed by a power law. Next, we estimate the shape and the scale parameter (α and C) by following Hill (1975):

$$\frac{1}{\hat{\alpha}} = \frac{1}{k}\sum_{j=1}^{k} \ln \frac{X_j}{X_{k+1}} \tag{6.16}$$

and

$$\hat{C} = \frac{k}{n} X_{k+1}^{\hat{\alpha}}, \tag{6.17}$$

where equation (6.16) is generally referred to as the Hill estimator.

Although the concept and the estimation of the parameters are straightforward, the choice of k is not. The optimum depends on the sample size T and the tail-thickness α; the further one moves out into the tails, the better becomes the Pareto approximation of those tails. However, this reduces the number of observations available for estimation and increases the uncertainty of the estimate.

In practice, one may resort to visual inspection of the so-called Hill plots to determine the optimal level of threshold k. The number of observations included in the tail, k, is along the x-axis. For each number of observations, k, the Hill estimate for α is calculated. The optimal threshold is selected from the region in which the Hill estimate for α is more or less stable, see also Drees *et al.* (2000).

6.3.2 Risk measures

Both risk measures introduced in this subsection measure the probability of extremely low returns (or extremely high returns in case of a short position), also called ‘tail risks’. The first one, Value-at-Risk (VaR), is one of the most widely used risk measures in financial risk modeling.[10] VaR plays an important role in the safety-first framework developed by Roy (1952) and Telser (1955). Agents with the safety-first principle of Telser (1955) in their utility functions maximize their expected return, while limiting the probability that a loss larger than some disaster level occurs at some admissible level p. Basically, such agents maximize their expected return under a VaR constraint with probability p.

The VaR (in terms of returns) is simply defined as a quantile estimate. After fitting a power law distribution to the data, the VaR is estimated by inverting the power law in equation (6.1). To derive a VaR estimator, the parameter estimates for C and α in equations (6.16) and (6.17) are substituted into the power law, which gives $\Pr(x > u) \sim \frac{k}{n} X_{k+1}^{\hat{\alpha}} u^{-\hat{\alpha}}$. Following the definition of VaR, $VaR(p)$ can considered as the threshold u which is exceeded with probability p. Hence, we replace u by $VaR(p)$ and $\Pr(X > VaR(p))$ by p. After rewriting, the following VaR estimator is obtained:

$$\widehat{VaR}(p) = X_{k+1} \left(\frac{k}{np} \right)^{1/\hat{\alpha}}. \tag{6.18}$$

One of the shortcomings of VaR is that it contains no information on the size of the losses beyond the $p\%$ worst case. A risk measure which overcomes this problem is the Expected Shortfall (ES). The $ES(p)$ measures the *expected* amount one looses in the $p\%$ worst cases. In case of a power law tail, the estimated Expected Shortfall is a multiplication of VaR and a constant, which depends on the shape parameter only, see also Danielsson *et al.* (2006):

$$\lim_{p \downarrow 0} \frac{ES(p)}{VaR(p)} = \frac{\alpha}{\alpha - 1}. \tag{6.19}$$

6.4 Data

In many financial risk management applications it is common practice to use relatively high-frequency return series. For our purpose, daily data are preferable over weekly or monthly data for two reasons. First, the use of high frequency data, implying more observations, improves the quality of the parameter estimates. Second, the choice for daily data corresponds to the time required to hedge a farmer's price risk on the financial markets, which can typically be accomplished within one day, or even less.

In this chapter we employ futures prices instead of spot prices. One reason for this choice is the lack of reliable high frequency commodity spot prices. In general, historical daily commodity spot prices contain a high number of zero returns and are more

[10] One of the reasons that VaR is so widespread is that a bank's capital requirements by the Basel II accords, depends on the size of its VaR, see e.g. Danielsson *et al.* (1998), or Giot and Laurent (2003).

likely to be affected by bid-ask bounces because of a lack of liquidity. By contrast, for many agricultural commodities, futures contracts are highly liquid and exchange-traded instruments for which reliable historical high frequency data is available.

Futures contracts for delivery at a particular date are usually traded for a relatively short period, ranging from several months up to several years. To obtain long-term futures returns series, or so-called *continuous* series, we therefore need to combine consecutive data from several futures contracts, see e.g. De Roon *et al.* (2000). We take considerable effort to construct high-quality continuous futures return series. Our procedure is as follows. First, we download daily open interest and price series of all available futures contracts from DataStream for each commodity. Those time series are available over a period of 33 years: from January 1979 until December 2011. Subsequently, daily returns are calculated for all futures price series. Finally, we construct the continuous futures returns series from the individual return series. In January 1979 we start with the futures contract that has the largest open interest. For each day we include its returns in the new continuous series until six weeks before the contract's last trading day. At this date we switch to the futures contract with the largest open interest and a later last trading day. Again we include the returns until six weeks before the last trading day and repeat the last step. This procedure results for each commodity in a continuous futures returns series with 8,609 daily observations from on average 153 different futures contracts.

Our method has an important advantage compared to DataStream's procedure to construct continuous futures series. By calculating returns prior to constructing the continuous series, no returns are calculated over price observations from two different futures series. Therefore, our series represents the return that investors could achieve by rolling over futures contracts as opposed to the continuous DataStream series which includes price jumps due to changes in the underlying futures series. The extreme returns in our series thus represents true financial risks to market participants.[11]

From all commodity futures traded in the United States, the following seven crop commodities and two animal commodities are investigated: corn, oats, soybeans, wheat, cotton, sugar, orange juice, live cattle and lean hogs.[12]

[11] In addition, shifts in the roll-over date often occur in the DataStream continuous series. By means of an extreme example: the second largest daily price fall during the last 30 years in the unadjusted DataStream series for cotton (NCTCS00) is caused by a delayed roll-over date. The return of -26.3% is caused by the difference between 113.6, which is the price for delivery in July 1995 listed on the 4th of July, 1995, and 83.75, which is the price for delivery in October 1995 listed on the 5th of July, 1995. Such extreme observations may distort the assessment of the actual tail distribution.

[12] See Appendix 6.C for details on the selection process of the commodities. The continuous futures returns are available from the corresponding author on request.

Table 6.1: Descriptive statistics

Commodity	Mean	St.dev.	Min.	Min. Date	Series	Max.	Max. Date	Series
Corn	-0.01	1.4	-7.6	2009-06-30	1009	9.0	2009-09-15	0310
Cotton	0.01	1.4	-6.7	2008-10-20	0309	7.2	2008-12-08	0309
Oats	-0.01	1.7	-11.3	2005-03-31	0705	11.1	2005-03-30	0705
Soybeans	0.00	1.4	-7.1	2009-07-07	1009	6.9	1999-08-02	1099
Wheat	-0.01	1.6	-9.5	2009-01-12	0309	9.2	2008-10-29	0309
Lean hogs	-0.01	1.4	-6.7	1998-12-11	0399	7.1	1998-12-14	0399
Live cattle	0.01	0.9	-6.2	2003-12-30	0304	4.2	1989-07-19	1089
Orange juice	0.00	1.7	-12.8	2010-01-11	0310	16.3	2006-10-12	0307
Sugar	0.02	2.3	-16.7	1988-07-26	1088	15.3	1985-07-26	1085

Note: The first two columns report the mean and the standard deviation of the daily returns series. The other columns report the minima and maxima of the return series, the date of these observations and the code of the futures series in which these were observed. The code of the futures series refers to the month of delivery with format MMYY.

6.5 Empirical tail estimates

Table 6.1 reports the descriptive statistics of the daily futures return series. A quick overview of the data confirms the non-normality of the returns. Six out of nine series contain at least one observation with a distance of at least six standard deviations from the mean. The probability of such a return occurring equals about 2.0×10^{-9} under the assumption of normality, or roughly once every 2 million years.[13] The other tree series contain at least one observation with a distance of five standard deviations from the mean, an observation that would happen roughly once every 7 thousand years under the assumption of a normal distribution.

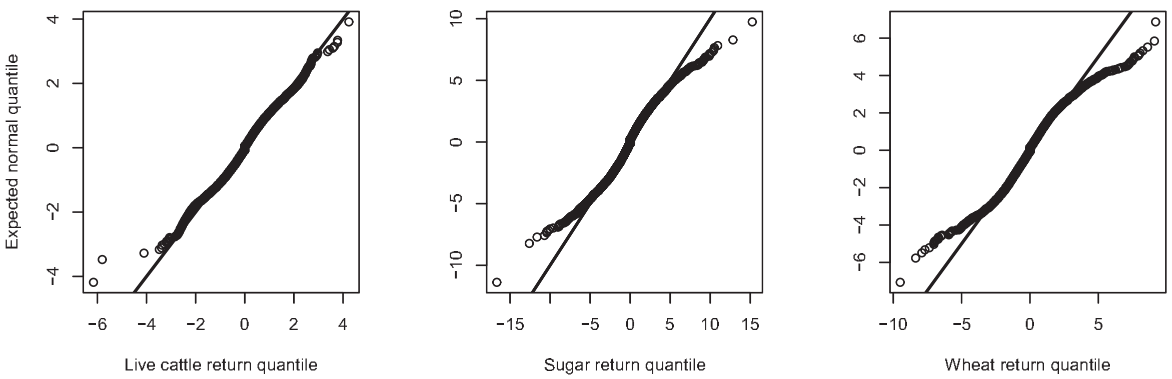
As an illustration, Figure 6.1 reports QQ-plots of three arbitrarily chosen daily return series (live cattle, sugar and wheat). The non-normality is strongly confirmed by QQ-plots of the return series against the normal distribution. Only the daily return series for live cattle seems to be an exception. Except for a few extreme tail observations, the distribution of the live cattle return series is in general quite close to the normal distribution.

Table 6.2 documents the estimated tail parameters. Unreported Hill plots show that the Hill estimates are relatively stable when a total of 150 tail observations are used, i.e., approximately 1.75% of all observations. The value of most shape parameters is estimated to equal around 4. The most risky commodity with respect to the shape parameter is Orange Juice with estimates of around 3.5 and 3.1 for respectively the left and the right tail.[14] Live Cattle and Lean Hogs are the commodities with the highest estimates for the

[13]If $X \sim N(0,1)$, then $\Pr(X \leq -6) \approx 1.0 \times 10^{-9}$.

[14]Subsections 6.3.1 and 6.3.2 describes how a power law is fitted to the right tail of the return distri-

Figure 6.1: QQ-plots of agricultural commodity returns.



QQ-plots of the daily wheat, live cattle and sugar returns against a normal distribution with the same mean and standard deviation.

shape parameters, implying thinner tails. For Live Cattle this finding is not remarkable: The QQ-plots already showed that the Live Cattle futures return distribution is quite similar to the thin-tailed normal distribution.

Standard errors of the shape parameters are obtained from a bootstrap procedure. De Haan *et al.* (1994) demonstrate the asymptotic normality for the Hill estimator and the VaR estimate for independent and identically distributed (iid) returns. Recent research demonstrates that the asymptotic normality of the Hill estimator holds in the presence of serial dependence, see e.g. Hsing (1991) and Drees (2008). Following Hartmann *et al.* (2006), we refrain from assumptions on the specific dependence structure and apply a bootstrap procedure with fixed block length and 10,000 replications. Following Hall *et al.* (1995), the optimal block length is set equal to $n^{1/3}$.

6.5.1 Alternative data frequencies

In this subsection, the shape parameters are tested for sensitivity to changes in the data frequency. In principle, the estimated shape parameters should be robust for changes in the data frequency in case of independent and identically distributed returns.[15] Nevertheless, volatility clustering or daily price limits may result in different tail behavior for different data frequencies. To this end, two-day returns and weekly returns are calculated from the daily return series. In order to test for equality of the shape parameters, we es-

bution. To fit a power law to the left tail multiply the return series with -1.

[15]Mandelbrot (1963a) shows that power law distributions are invariant with respect to the shape parameter under several basic transformations. The shape parameter is invariant with regard to summation of random variables, mixing random variables with different scale parameters and selection of maxima. It follows that the power law distribution is independent of data frequency choices, distribution mixture assumptions and missing data. As a consequence, sample-specific data problems are unlikely to affect the observed shape parameter.

Table 6.2: Tail parameter estimates.

Commodity	Left Tail			Right Tail		
	Shape	(s.e.)	Scale	Shape	(s.e.)	Scale
Corn	4.26	0.38	3.03	4.41	0.35	4.57
Cotton	4.52	0.42	4.09	4.55	0.38	4.65
Oats	4.64	0.35	10.55	4.27	0.33	6.36
Soybeans	4.44	0.34	4.07	4.47	0.34	3.75
Wheat	3.63	0.28	1.58	3.80	0.29	2.53
Lean hogs	7.06	0.78	80.79	5.64	0.54	12.41
Live cattle	7.98	1.19	9.45	7.84	0.72	7.51
Orange juice	4.29	0.34	6.54	3.61	0.29	2.47
Sugar	4.58	0.36	34.78	4.15	0.33	16.89

Note: The first column of the left and right side report the estimated shape parameter from equation (6.16) for respectively the left and right tail. The second column reports the corresponding standard error from the bootstrap procedure described in Section 6.5. The third column reports the estimated scale parameter from equation (6.17). Both tails consist of 150 observations, i.e., for each tail approximately 1.75% of 8,609 observations.

timate the shape parameters from the two-day returns and weekly returns. Subsequently, we calculate the following t-statistic

$$T = \frac{\hat{\alpha}_1 - \hat{\alpha}_2}{\hat{\sigma}(\alpha_1 - \alpha_2)}, \tag{6.20}$$

where $\hat{\sigma}(\alpha_1 - \alpha_2)$ denotes the standard deviation of the difference between the estimated shape parameters from two different frequencies, and where the t-statistic converges to a standard normal distribution under the null hypothesis of equal shape parameters. The standard deviation, $\hat{\sigma}(\alpha_1 - \alpha_2)$, is obtained from a block bootstrap procedure, in which each bootstrapped sample is obtained from daily returns, but subsequently also transformed into lower frequency returns to calculate the two-day or weekly shape parameter.

Table 6.3, panels (a) and (b) report the results of the robustness test for changing the data frequency. The differences in the shape parameters between daily and two-day return distributions in panel (a) are statistically significant at a 5% level for three commodities. Besides live cattle, the tails of the corn and the lean hogs distribution have significantly lower shape parameters for two-day returns. In panel (b) we test whether the shape parameter changes significantly if one further extends the data frequency from two-day returns to weekly returns. We do not find significant differences between the shape parameters of two-day and weekly returns. Apparently, the issues that potentially cause differences between the daily and two-day parameter estimates in our sample do not play a large role if one turns to estimation at lower data frequencies.

Table 6.3: Tail parameter estimates with other data frequencies.

Panel (a): Two-day returns

Commodity	Left Tail				Right Tail			
	Shape	(s.e.)	Scale	t-stat	Shape	(s.e.)	Scale	t-stat
Corn	3.36	0.31	2.81	-2.17**	3.43	0.42	3.96	-2.44**
Cotton	3.85	0.35	6.64	-1.23	4.24	0.41	13.44	-0.47
Oats	4.04	0.39	17.64	-1.31	3.53	0.33	7.49	-1.79
Soybeans	3.86	0.37	6.24	-1.52	4.11	0.45	8.66	-0.10
Wheat	3.73	0.42	5.77	0.28	3.49	0.35	5.27	-0.53
Lean hogs	4.78	0.56	27.98	-3.17***	4.97	0.44	24.42	-0.80
Live cattle	4.84	0.48	3.62	-2.91***	4.77	0.44	3.53	-3.67***
Orange juice	4.07	0.38	18.29	-0.51	3.15	0.30	4.02	-0.72
Sugar	4.33	0.45	79.64	-0.62	4.03	0.43	56.17	-0.11

Panel (b): Weekly returns

Commodity	Left Tail				Right Tail			
	Shape	(s.e.)	Scale	t-stat	Shape	(s.e.)	Scale	t-stat
Corn	3.21	0.43	10.27	-0.29	3.84	0.54	51.11	1.87*
Cotton	4.39	0.71	78.10	0.87	3.02	0.46	7.64	-1.34
Oats	4.19	0.58	169.25	0.20	4.03	0.92	122.05	1.50
Soybeans	3.49	0.44	17.44	-0.75	3.49	0.50	17.02	-1.18
Wheat	3.86	0.61	32.09	0.20	3.50	0.40	25.43	-0.13
Lean hogs	4.13	0.70	46.75	-1.04	3.99	0.55	33.89	-1.14
Live cattle	4.12	0.72	9.94	-1.18	4.60	0.65	22.60	-0.11
Orange juice	3.54	0.50	30.03	-0.87	2.78	0.43	9.76	-1.01
Sugar	5.05	0.70	2805.17	1.07	4.08	0.77	338.67	1.04

Note: Shape parameter are estimated from equation (6.16) for the left and right tail of two-day returns in panel (a) and weekly returns in panel (b). For two-day (weekly) returns we set k=100 (k=50). Reported standard errors (s.e.) are generated by the bootstrap procedure described in Section 6.5. Scale parameters are calculated from equation (6.17). For the two-day estimates in panel (a) we provide t-statistics for testing against the null hypothesis of equal shape parameters in the daily return and two-day return data, see equation (6.20). For the weekly estimates in panel (b) we provide t-statistics for testing the null hypothesis of equal shape parameters in the two-day return and weekly return data. Significance at the 10%, 5% and 1% level is denoted by respectively *, ** and ***.

Table 6.4: Risk estimates

Panel (a): Left Tail

Commodity	Probability level: 0.10%				Probability level: 0.01%			
	VaR	Min.	Max.	ES	VaR	Min.	Max.	ES
Corn	6.56	5.72	7.40	8.57	11.26	8.90	13.63	14.72
Cotton	6.28	5.71	6.86	8.07	10.45	8.75	12.16	13.42
Oats	7.36	6.66	8.05	9.38	12.08	10.11	14.05	15.40
Soybeans	6.51	5.80	7.21	8.40	10.94	9.03	12.85	14.12
Wheat	7.60	6.56	8.64	10.49	14.34	11.11	17.57	19.79
Lean hogs	4.95	4.44	5.47	5.77	6.87	5.66	8.07	8.00
Live cattle	3.15	2.81	3.49	3.60	4.20	3.41	4.99	4.80
Orange juice	7.76	6.94	8.57	10.12	13.27	10.93	15.62	17.31
Sugar	9.83	8.75	10.91	12.58	16.26	13.25	19.26	20.80

Panel (b): Right Tail

Commodity	Probability level: 0.10%				Probability level: 0.01%			
	VaR	Min.	Max.	ES	VaR	Min.	Max.	ES
Corn	6.76	6.12	7.41	8.75	11.41	9.42	13.40	14.75
Cotton	6.41	5.69	7.12	8.21	10.63	8.60	12.67	13.63
Oats	7.78	6.90	8.66	10.16	13.34	10.86	15.81	17.41
Soybeans	6.30	5.70	6.90	8.11	10.53	8.78	12.28	13.57
Wheat	7.88	6.88	8.88	10.70	14.45	11.40	17.51	19.62
Lean hogs	5.32	4.76	5.88	6.47	8.00	6.60	9.41	9.73
Live cattle	3.12	2.95	3.30	3.58	4.19	3.76	4.62	4.80
Orange juice	8.71	7.55	9.87	12.05	16.49	12.76	20.22	22.81
Sugar	10.46	9.19	11.74	13.79	18.24	14.39	22.08	24.03

Note: Value-at-Risk (VaR) estimates for the left tail in panel (a) and the right tail in panel (b) are calculated from equation (6.18) for the 0.10% and the 0.01% probability level. We also provide the 95% confidence bands (Min.; Max.) of the VaR estimates. The Expected Shortfall (ES) estimates are calculated from equation (6.19).

6.6 Risk estimates

This Section discusses the empirical results of the risk estimates. Table 6.4 reports the 0.1% and 0.01% VaR and Expected Shortfall estimates, following equations (6.18) and (6.19). The 0.1% VaR is expected to be exceeded about once every four years, and the 0.01% VaR about once every 40 years, or about once during a farmer's full career.

The relevant information for farmers with respect to risk management is contained in the left tail of the distribution, shown in panel (a). We find that sugar has the highest price risk of all commodities studied. Once every four (forty) years the sugar price is expected to fall by more than 9.8% (16.3%) within a one-day period. The safest commodity, in terms of price development, appears to be live cattle. Over a 40-year period a farmer may expect the cotton price to fall only once more than 4.8% within a single day.

In the introduction the question was raised on the likelihood of a price change occurring of such a size that it could result in the farmer's bankruptcy. It is possible to answer this question with the results in Sections 5 and 6. Suppose a farmer who grows wheat and who would have serious solvency problems after a 15% fall in the wheat price from its current level. What is the probability that an unhedged farmer will default from one day to the next?

To answer this question, we apply equation (6.1) to the left tail, and substitute the absolute value of the threshold return for u. Subsequently, the parameter estimates from Table 6.2 for C and α are substituted into equation (6.1). Because the farmer is worried about price falls (as opposed to price increases), the parameters for the left tail are employed. Calculating the result from equation 1 renders the probability of solvency problems: $\Pr(x > u) \approx 1.58 \times 15^{-3.63} \approx 8.50 \times 10^{-5}$. The inverse of this number yields the number of days in which a price fall of at least 15% is expected to occur. The outcome is around 11,750 (trading) days. So once every 45 years we expect to see such a large fall in the price of wheat.

6.6.1 Back-testing

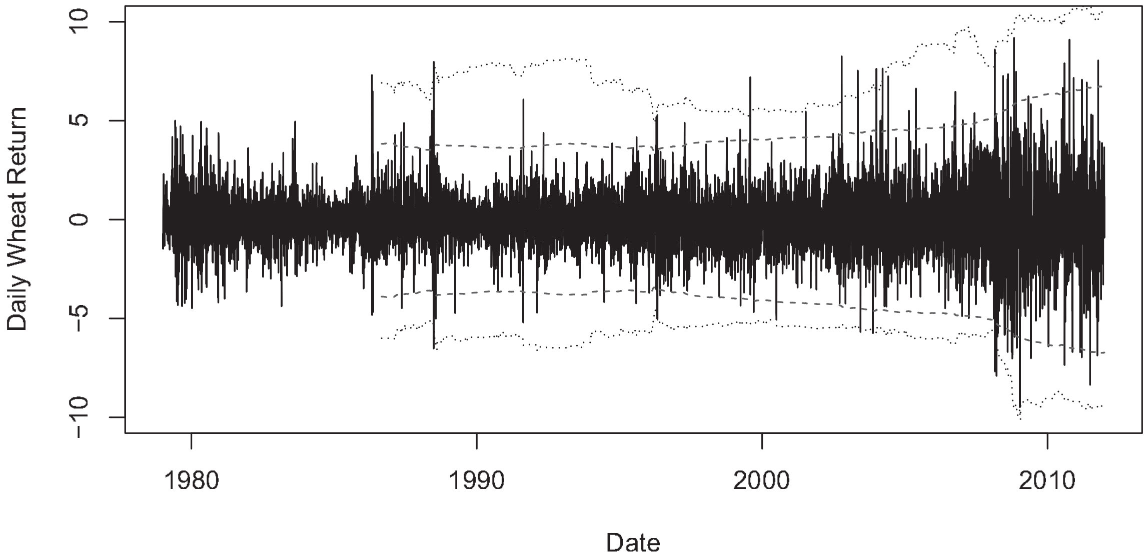
To examine the accuracy of the $VaR(p)$ estimates, we employ an out-of-sample back-testing procedure. In this method, the $VaR(p)$ estimates based on historical price changes are compared to the realized price changes. Thus, first $VaR_t(p)$ is estimated using a horizon of m preceding returns: $\{r_{t-m}, ..., r_{t-1}\}$. If the realized return r_t exceeds the estimated $VaR_t(p)$, then a VaR-violation is registered. Subsequently the procedure above is repeated at time $t + 1$ etc.

For a dataset containing n returns, the procedure is repeated $n-m$ times. According to the Value-at-Risk definition, if the $VaR(p)$ estimate is of high quality, then the proportion of VaR-violations should have a value close to p. Thus, if the underlying distribution does not change over time, $\frac{1}{n-m} \sum_{t=m+1}^{n} \mathbf{1}(r_t > VaR_t(p)) = p$ holds approximately for accurate VaR estimates, where $\mathbf{1}(\cdot)$ denotes the indicator function.

The procedure is implemented as follows. The back-testing procedure is performed under both the assumption of power law tails and under the assumption of the more conventional normal distribution. The estimation 'horizon' m is set at 2,000 daily observations. Each tail is assumed to contain the most extreme 2.5% of all observations.

A visual representation of the procedure's results is given in Figure 6.2. The spikes show daily wheat returns. The dotted and dashed lines show respectively the 0.1% VaR estimates from the power law tail and the normal distribution, using the preceding 2,000 daily returns. The test procedure boils down to counting the number of spikes that exceed either the dotted or the dashed line.

Figure 6.2: Backtesting agricultural Value-at-Risk.



The spikes show the daily wheat returns. The dotted (dashed) line shows the 0.1% VaR estimates according to the power law tail (normal) distribution. The risk measure is estimated from the preceding 2,000 daily returns. From the figure it is clear that the VaR estimate from the normal distribution is exceeded at a higher frequency than 0.001.

Table 6.5: Back-testing agricultural Value-at-Risk.

Violations	Left Tail						Right Tail					
Probability	1%		0.1%		0.05%		1%		0.1%		0.05%	
	EVT	Nrm	EVT	Nrm	EVT	Nrm	EVT	Nrm	EVT	Nrm	EVT	Nrm
Corn	125	181	3	68	2	52	121	193	6	75	1	60
Cotton	124	175	9	33	5	21	106	165	8	53	3	41
Oats	101	145	7	41	3	28	86	145	6	49	2	36
Soybeans	99	158	4	65	0	52	86	138	4	58	1	44
Wheat	107	129	8	49	2	36	103	158	8	59	3	43
Lean hogs	91	127	11	24	6	14	85	110	6	30	2	23
Live cattle	80	107	4	28	3	15	85	108	8	20	3	12
Orange juice	99	174	5	65	3	49	97	165	9	65	4	47
Sugar	73	111	6	42	3	31	72	123	3	27	0	20
Average	99.9	145.2	6.3	46.1	3.0	33.1	93.4	145.0	6.4	48.4	2.1	36.2
Expected	66.1	66.1	6.6	6.6	3.3	3.3	66.1	66.1	6.6	6.6	3.3	3.3

Note: The table reports the number of VaR-violations in a back-testing procedure. The columns report the number of VaR-violations for different VaR levels. The two bottom lines report respectively the expected number of VaR-violations and the average number of VaR-violations. Although the normal distribution (Nrm) and the power law tail distribution (EVT) both seem to underestimate the 1% VaR, the power law tail distribution turns out to be quite accurate for more extreme events.

The results of the test procedure are summarized in Table 6.5. For each commodity, the number of VaR-violations is provided for three different VaR probabilities. The first and fourth column in Table 6.5 show that the 1% VaR estimates from both distributions are violated too often. In other words, the 1% VaR estimates are too low for both distributions, although the error is markedly smaller for the power law tail VaR estimates.

The predictions from the normal distribution underestimate the risk when one moves further into the tail. The third and sixth column in Table 6.5 show that the number of 0.05% VaR-violations is more than 10 times too high under the assumption of the normal distribution. However, the power law tail estimates for VaR at smaller probabilities (e.g., 0.1% and 0.05%) turn out to be quite accurate. The 0.05% VaR for the left (right) tail is on average exceeded by 3.0 (2.1) observations per commodity, which is not far from the expected number of violations in case of a perfectly accurate VaR prediction (i.e., 3.3). Those estimates concern the very extreme events (about 1 observation in respectively 4 and 8 years), which are most important from a farmer's risk management perspective.

6.7 Conclusion

A good understanding of extreme commodity returns is instrumental in any commodity risk management application. Especially knowledge regarding large price swings is most relevant for risk management purposes. We construct a two-sector general equilibrium model which describes how productivity shocks affect agricultural commodity prices. In our model, extreme price spikes arise endogenously as a result of productivity shocks in the agricultural sector, which results in a heavy-tailed equilibrium price distribution.

The economic literature on real business cycles reasons that productivity shocks are a dominant source of fluctuations in economic aggregates. Agricultural producers experience a relative large amount of those shocks through their exposure to weather conditions and other natural forces. Prior studies show that those shocks have a relatively large impact on agricultural commodity price behavior, see e.g. Deaton and Laroque (1992, 1996, 2003) and Ai *et al.* (2006). Our study shows why such productivity shocks may result in heavy-tailed price distributions, even if they are not heavy-tailed themselves.

We build on prior work to provide further empirical evidence that agricultural commodity price returns are heavy-tailed. We use Extreme Value Theory to estimate the parameters or the power law in the tail of their distribution. These estimates are used to measure an agricultural producer's extreme price risks. We calculate Value-at-Risk and Expected Shortfall measures to provide estimates of the likelihood and size of the largest losses a farmer may encounter. Back-testing shows that this methodology is superior to risk measures that are based on the conventional normal distribution assumption.

6.A Appendix A. Derivation of equilibrium prices

The first order conditions for optimality entail

$$(1-\theta)\, Z^{-\theta} n^{-\theta/\rho} \left[\sum_{i=1}^{n} Q_i^{\rho}\right]^{\theta/\rho} - \lambda q = 0,$$

$$\theta \left(\frac{Z}{\left[\sum_{i=1}^{n} Q_i^{\rho}\right]^{1/\rho}}\right)^{1-\theta} n^{-\theta/\rho} \left[\sum_{i=1}^{n} Q_i^{\rho}\right]^{\frac{1}{\rho}-1} Q_j^{\rho-1} - \lambda \frac{1}{n} p_j = 0,$$

$$-L^{\gamma} + \lambda w = 0,$$

and

$$wL + \Pi(Q) = qZ + \frac{1}{n}\sum_{i=1}^{n} p_i Q_i.$$

The first order conditions imply the familiar price and wage ratios

$$\frac{p_i}{p_j} = \frac{Q_i^{\rho-1}}{Q_j^{\rho-1}},$$

$$\frac{p_j}{q} = \frac{\theta}{1-\theta}\frac{Z}{Q_j}\frac{p_j^{\rho/(\rho-1)}}{\frac{1}{n}\sum_{i=1}^{n} p_i^{\rho/(\rho-1)}},$$

and

$$\frac{w}{q} = (qP)^{\theta}\, \frac{L^{\gamma}}{(1-\theta)^{1-\theta}\, \theta^{\theta}},$$

where the price index for differentiated goods is defined as in equation (6.6).

Then the labor supply can be written as

$$L = \left((1-\theta)^{1-\theta}\, \theta^{\theta} \frac{w}{q^{1-\theta} P^{\theta}}\right)^{1/\gamma}. \tag{6.21}$$

The competitive goods demanded can be expressed as

$$Z = (1-\theta)\, \frac{wL + \Pi(Q)}{q}. \tag{6.22}$$

The differentiated goods demanded can be expressed as

$$Q_i = \theta \frac{wL + \Pi(Q)}{p_i} \left(\frac{p_i}{P}\right)^{\rho/(\rho-1)}. \tag{6.23}$$

6.A.1 Supply

From the perfectly competitive agricultural market we have that

$$\Pi(Z) = qZ - wN = \left(q - \frac{w}{B}\right) Z = 0,$$

so that

$$q = w/B. \tag{6.24}$$

The differentiated goods profit function reads

$$\begin{aligned}\Pi(Q_i) &= p_i Q_i - wN_i = \left(p_i - \frac{w}{A}\right) Q_i \\ &= \left(p_i - \frac{w}{A}\right) \theta \frac{wL + \Pi(Q)}{p_i} \left(\frac{p_i}{P}\right)^{\rho/(\rho-1)}.\end{aligned}$$

The producer exploits his pricing power, but ignores his pricing effect on the price index P of the differentiated goods and the consumer income $wL+\Pi(Q)$.[16] Differentiation gives

$$\frac{\partial \Pi(Q_i)}{\partial p_i} = \frac{1}{\rho - 1} Q_i \left\{\rho - \frac{1}{A}\frac{w}{p_i}\right\}.$$

Exploiting the pricing power therefore implies setting prices

$$p_i = \frac{w}{\rho A}. \tag{6.25}$$

Hence, $P = w/\rho A$ as all prices are identical. Total profits in the differentiated goods sector equal

$$\begin{aligned}\Pi(Q) &= \frac{1}{n}\sum_{i=1}^{n} \Pi(Q_i) = \sum_{i=1}^{n} \left(1 - \frac{w/p_i}{A}\right) \theta \left[wL + \Pi(Q)\right] \left(\frac{p_i}{P}\right)^{\rho/(\rho-1)} \\ &= (1-\rho)\,\theta \left[wL + \Pi(Q)\right].\end{aligned}$$

Solve for the total sectorial profits as

$$\Pi(Q) = \frac{(1-\rho)\,\theta}{1-(1-\rho)\,\theta} wL. \tag{6.26}$$

[16] One can easily incorporate this effect as well, if desired, see Yang and Heijdra (1993). For two reasons we do not follow this route. One may doubt that producers take this macro effect of their pricing behavior into account. Moreover, it adds little to the insights derived form specifying the differentiated goods sector.

6.A.2 Equilibrium

It follows in equilibrium, after substituting the price levels into the labor supply equation (6.21), that

$$L = \left(\theta^{\theta}(1-\theta)^{1-\theta}A^{\theta}B^{1-\theta}\right)^{1/\gamma}\rho^{\theta/\gamma} = \varphi\rho^{\theta/\gamma}, \tag{6.27}$$

say, and where

$$\varphi = \left(\theta^{\theta}(1-\theta)^{1-\theta}A^{\theta}B^{1-\theta}\right)^{1/\gamma}.$$

Furthermore, from (6.22), (6.26) and (6.27)

$$Z = (1-\theta)\frac{B}{1-(1-\rho)\theta}\varphi\rho^{\theta/\gamma}. \tag{6.28}$$

Similarly, using (6.23), (6.26) and (6.27)

$$Q_j = \theta\frac{A}{1-(1-\rho)\theta}\rho\varphi\rho^{\theta/\gamma}.$$

Hence,

$$\frac{1}{n}\sum_{j=1}^{n}Q_j = \theta\frac{A}{1-(1-\rho)\theta}\varphi\rho^{\theta/\gamma+1}. \tag{6.29}$$

With the above preparations, we now derive the implications for the equilibrium prices. From (6.24), combined with (6.7) and (6.27), we obtain

$$q = \frac{w}{B} = \frac{M}{B}\frac{1}{L} = M\frac{1/\rho^{\theta/\gamma}}{B\left(\theta^{\theta}(1-\theta)^{1-\theta}A^{\theta}B^{1-\theta}\right)^{1/\gamma}}.$$

Similarly, using (6.25) combined with (6.7) and (6.27) yields

$$p_i = p = \frac{w}{\rho A} = \frac{M}{\rho A}\frac{1}{L} = M\frac{1/\rho^{\theta/\gamma+1}}{A\left(\theta^{\theta}(1-\theta)^{1-\theta}A^{\theta}B^{1-\theta}\right)^{1/\gamma}}.$$

6.B Appendix B. Proof of Proposition 6.2.2

Given (6.10), we want to find the condition on the density for B such that probability distribution of the price $\tilde{q}$ follows a heavy-tailed distribution. We have that $\Pr(\tilde{q} > u) \sim \mathcal{L}(u)u^{-\alpha}$ as $u \to \infty$ if $\tilde{q}$ is regularly α-varying at infinity with $0 < \alpha < \infty$, i.e., if

$$\lim_{t\to\infty}\frac{1-F_q(tu)}{1-F_q(t)} = u^{-\alpha} \text{ with } \alpha \in \mathbb{R}^+, \tag{6.30}$$

where F_q denotes the cumulative distribution function of $\tilde{q}$, see also De Haan (1970). We thus need to find the condition such that F_q it is regularly varying at infinity. Rewriting (6.30) with L'Hôpital's Rule gives the condition

$$\lim_{t\to\infty} \frac{u f_q(tu)}{f_q(t)} = u^{-\alpha} \text{ with } \alpha \in \mathbb{R}^+, \tag{6.31}$$

where f_q denotes the density of $\tilde{q}$. Given equation (6.10), we have that the equilibrium price $q(B)$ is a strictly decreasing function of B for $\theta \in (0, 1)$. Therefore, by a transformation of variable we have that

$$f_q(\tilde{q}) = \left| \frac{dB(\tilde{q})}{d\tilde{q}} \right| f_B(B(\tilde{q})), \tag{6.32}$$

where $B(\tilde{q})$ denotes the inverse of $\tilde{q}(B)$. With the inverse of equation (6.10) and the derivative of the inverse of equation (6.10) this gives

$$f_q(\tilde{q}) = \frac{1}{\eta} \Theta^{1/\eta} \tilde{q}^{-(1/\eta+1)} f_B(\Theta^{1/\eta} \tilde{q}^{-1/\eta}), \tag{6.33}$$

where

$$\eta = \frac{1+\gamma-\theta}{\gamma}.$$

Hence, from (6.31) we seek

$$\begin{aligned}
&\lim_{t\to\infty} \frac{u \frac{1}{\eta} \Theta^{1/\eta} (tu)^{-(1/\eta+1)} f_B(\Theta^{1/\eta} (tu)^{-1/\eta})}{\frac{1}{\eta} \Theta^{1/\eta} (t)^{-(1/\eta+1)} f_B(\Theta^{1/\eta} (t)^{-1/\eta})} = u^{-\alpha} \text{ with } \alpha \in \mathbb{R}^+; \\
&\lim_{t\to\infty} \frac{u^{-1/\eta} f_B(\Theta^{1/\eta} t^{-1/\eta} u^{-1/\eta})}{f_B(\Theta^{1/\eta} t^{-1/\eta})} = u^{-\alpha} \text{ with } \alpha \in \mathbb{R}^+; \\
&\lim_{s\downarrow 0} \frac{w f_B(sw)}{f_B(s)} = w^{\eta\alpha} \text{ with } \alpha \in \mathbb{R}^+.
\end{aligned} \tag{6.34}$$

where $w = u^{-1/\eta}$ and $s = \Theta^{1/\eta} t^{-1/\eta}$. Hence, given (6.10), if the condition in (6.34) holds for the density function of B, we have that $\Pr(\tilde{q} > u) \sim \mathcal{L}(u) u^{-\alpha}$ as $u \to \infty$. Proposition 6.2.2 is then obtained by writing $\xi = \eta\alpha$ in the condition in (6.34) and using $\eta > 0$.

6.C Appendix C. Commodity selection

In this appendix we explain how the commodities for this research are selected. The employed futures series need to satisfy two conditions: availability and relevance from the perspective of a US farmer. Initially, our sample contains all traded commodity futures within the US. A list of 19 commodities remains after removing non-agricultural and identical commodities. Four commodities from this list are removed because of data availability (Butter, Milk, Dry Wey and Rice are only available from 1996 onwards or even later). Next, six of the remaining fifteen series are removed because of low relevance. Because soybeans is included, soy meal and soybeanoil are removed. Because live cattle is included, cattle feeder is removed. Because lean hogs is included, frozen pork bellies is dropped. Cocoa and coffee are removed because of relatively low relevance for US farmers. This leaves us with corn (CC.), wheat (CW.), oats (CO.), soybeans (CS.), live cattle (CLC), lean hogs (CLH), cotton (NCT), sugar (NSB) and orange juice (NJO).[17]

[17] Thomson Datastream codes between brackets.

Chapter 7

Geography and extreme crop commodity returns

This paper studies the dependence of extreme shocks in the prices of crop commodities on futures markets. We observe relatively high levels between U.S. crop commodity pairs that are grown in similar regions. The strong positive relation between our measure of geographical overlap and the level of tail dependence among daily returns on futures markets suggests that a majority of the observed tail dependence can be explained by common productivity shocks, such as weather conditions. This puts a limit on diversification possibilities for individual farmers through crop diversification and underlines the importance of hedging via financial markets.[1]

Keywords: Tail dependence, commodity prices, geographical overlap, futures contracts.

JEL Classification Numbers: G13, Q13, Q24.

7.1 Introduction

This paper studies the dependence of extreme shocks in the prices of crop commodities on futures markets. Pindyck and Rotemberg (1990) advanced an interesting hypothesis that has become known as the 'excessive comovement hypothesis' on commodity prices. This hypothesis poses commodity prices to move together in excess of what can be explained by economic rationale. Pindyck and Rotemberg (1990) support this hypothesis with evidence that seemingly unrelated commodities move together, even after controlling for

[1]This chapter is co-authored by Philip A. Stork and Casper G. de Vries. Comments and suggestions from Dirk Schoenmaker and seminar participants at De Nederlandsche Bank (Amsterdam, 2012) are gratefully acknowledged.

macroeconomic variables. They advance the possibility that the excessive comovement is caused by herd behavior of traders who are bearish and bullish on all commodities without economic explanation. The existence of excessive comovement casts doubts on the rationality of competitive commodity markets.

The study of Pindyck and Rotemberg (1990) aroused the interest of the economic profession in the matter, see e.g. the subsequent studies of Palaskas and Varangis (1991), Leybourne *et al.* (1994), Deb *et al.* (1998), Cashin *et al.* (1999), Ai *et al.* (2006), Lescaroux (2009) and Byrne *et al.* (2013). These studies approach the excessive comovement hypothesis on commodity prices using different methodologies and data sources. In general, these subsequent studies find no evidence or only weak evidence in favor of the 'excess comovement hypothesis'. In the light of these findings one may conclude that empirical evidence in favor of excess comovement can be considered as thin. Nevertheless, there are several reasons to remain doubtful about the absence of excess comovement.

A first reason to question the absence of excess comovement is that irrational herding behavior is primarily believed to result in excessive comovement in the short run. Especially in the short run there is insufficient time to collect adequate information on the causes of price movements. However, most empirical studies analyze prices at a monthly, quarterly or annual frequency, and thus focus on comovement at a relatively long horizon, when the dust might have settled down and prices are restored to their correct levels. A second reason is that especially uncertain environments characterized by extreme price jumps and crashes triggers irrational herding behavior of speculators. By contrast, empirical studies focus on correlation-based measures. These measures may provide a correct description of the comovement in all regimes, but do not necessarily provide a correct picture of the comovement of extreme shocks. A final reason is that a majority of the empirical work focuses on comovement in spot markets, while irrational herding is believed to be stronger in markets prone to speculation, such as futures markets.

This chapter adds a new perspective to the literature on the excess comovement hypothesis, by studying the comovement of *extreme* price shocks observed among daily returns in futures markets. In contrast to existing empirical literature, we focus on dependence among the tails of commodity return distributions. Our focus is exclusively on crop commodities. Theoretically, we show that tail dependence among crop commodity returns should be positively affected by the degree in which commodities are grown in similar regions if the dependence structure is determined by fundamentals. Based on U.S. crop data we calculate a related measure on the degree of geographical overlap. We observe a strong and persistent positive link between the tail dependence among crop commodity returns and the degree of geographical overlap. Our results suggest that the majority of variation in tail dependence can be explained by the degree of geographical overlap. Further, we do not find much evidence of tail dependence among commodities not grown in similar regions. To summarize, we find no support of excess comovement among crop commodity returns in highly uncertain crisis periods.

Our results put a limit on the possibilities for risk reduction through crop diversification by individual farmers. Although crop diversification may reduce price risk to some extent, it leaves farmers vulnerable to the simultaneous extreme price shocks that characterize commodity pairs which are grown in the same regions. Therefore, our results underline the importance of hedging via financial markets. The concern of simultaneous extreme price shocks also applies to small banks providing loans to several local crop producers. The loan portfolio of such a bank may contain a considerable concentration of risk, even if the borrowers do not produce the same commodity. Diversification across non-agricultural industries, or diversification across different regions, may help to further enhance the risk profile of such local banks.

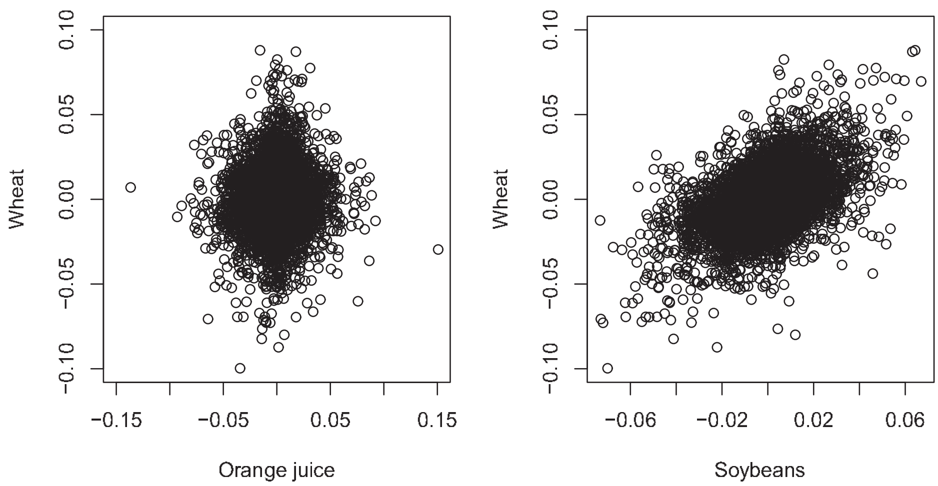
The study connects to a growing body of literature exploring the links between extreme observations in financial markets by asymptotic dependence measures. Some of these studies assess which international financial markets exhibit more or less joint extreme observations, see e.g. Poon *et al.* (2004), Hartmann *et al.* (2004) and Cholette *et al.* (2012). Other studies measure or explain the level of tail dependence among stock returns of financial institutions, see e.g. De Jonghe (2010), Zhou (2010) and Pais and Stork (2011), the joint occurrence of currency crisis, see e.g. Hartmann *et al.* (2010), or on the level of tail dependence among the returns on stocks and the returns on a general market index, see e.g. Straetmans *et al.* (2008). To our best knowledge, assessing and explaining the level of tail dependence among crop commodities is new to the literature. We find that the level of tail dependence observed among some crop commodity pairs is at a similar level as the level of tail dependence among stock markets of strongly linked economies.

The remainder of this chapter is organized as follows. Section 7.2 introduces the tail dependence measure and discusses its relation with the degree of geographical overlap. Section 7.3 discusses the methodology. Results are provided in Section 7.4. Section 7.5 concludes.

7.2 Theory

In our empirical application we focus on the comovement of extreme negative price shocks among crop commodities. For illustrative purposes, Figure 7.1 shows scatters of daily price changes of orange juice and soybeans and daily price changes of wheat. The comovement of large downward price movements manifests itself by the extent in which observations cluster in the lower left corner of the scatters. This type of clustering seems to be much

Figure 7.1: Dependence among daily futures return series



Continuous return series based on daily futures prices from 1979 until 2011 (8,609 observations).

higher in the scatter with price changes of soybeans and wheat than in the scatter with price changes of orange juice and wheat.

Multivariate extreme value theory has developed tail dependence measures to formally assess the level of comovement among extreme price shocks. Following Poon *et al.* (2004), the tail dependence among negative price shocks of two crop commodities can be measured as

$$\tau_{x,y} =: \lim_{p \downarrow 0} \tau_{x,y}(p) = \lim_{p \downarrow 0} \frac{\Pr\{X \leq Q_x(p) \ \& \ Y \leq Q_y(p)\}}{p}, \tag{7.1}$$

where $Q_x(\cdot)$ and $Q_y(\cdot)$ denote the quantile functions of the price shocks X and Y. The measure in equation (7.1) compares the probability of observing extremely low values of two variables simultaneous to the probability of observing extreme downward movements in a single variable. Here, an extremely low value means a large negative price shock that happens with some small probability p only, where $p \to 0$. The interpretation of the measure is as follows. The case $\tau_{x,y} = 0$ is known as tail independence – variables X and Y are never simultaneously extremely low – and the case $\tau_{x,y} = 1$ is known as perfect tail dependence. By rewriting the measure as

$$\tau_{x,y} = \lim_{p \downarrow 0} \Pr\{Y \leq Q_y(p) | X \leq Q_x(p)\} = \lim_{p \downarrow 0} \Pr\{X \leq Q_x(p) | Y \leq Q_y(p)\}, \tag{7.2}$$

it can be interpreted as a conditional probability, where it provides the chance of observing Y to be extremely low, conditional on observing an extremely low X, and vice versa. Finally, it is worth to mention that $\tau_{x,y}$ is equivalent to $\lim_{p \downarrow 0} C(p,p)/p$, where $C(\cdot,\cdot)$ denotes the copula among X and Y. So (7.2) captures the limit copula. It has the characteristic that it is not being driven by the dependence in the center (as the fully parametric copula approach is).

Dependence among extreme price shocks of crop commodities can have several potential causes. First, the prices of the commodities may be vulnerable to changes in the macro-economic environment. Second, the demands for commodities may be directly related (nonzero cross price elasticities). Third, the supply of both commodities may be related. Finally, potential herding on financial markets without economic reason may cause common price movements.

In a stylized framework we analyze how related supply factors may cause tail dependence among the negative price shocks of two seemingly unrelated crop commodities (with zero cross price elasticity). We discuss the tail dependence of crop commodities x and y in a two region set up. Let q_i and v_i denote the size of the area planted with respectively commodity x and y in region i. Then the production of crop commodities x and y is given by

$$\begin{aligned} s_x &= q_1\tilde{\theta}_1 + q_2\tilde{\theta}_2, \\ s_y &= v_1\tilde{\theta}_1 + v_2\tilde{\theta}_2, \end{aligned} \tag{7.3}$$

where θ_i denotes the productivity in region i. The uncertainty about θ_i due to natural shocks such as the weather conditions, droughts and hurricanes is denoted by $\tilde{\theta}_i$. We assume that the innovations in productivity are independently distributed across regions and exhibit heavy tails with tail index α as

$$\Pr(\tilde{\theta}_i > u) = \mathcal{L}(u)u^{-\alpha} \text{ as } u \to \infty, \tag{7.4}$$

where $\mathcal{L}(u)$ denotes a slowly varying function in the sense that $\mathcal{L}(tu)/\mathcal{L}(u)$ converges to 1 for any $t > 0$ as $u \to \infty$. Finally, we assume strictly positive price-quantity relations as $P_x(s_x) > 0$ and $P_y(s_y) > 0$, with respectively $\frac{\partial P_x}{\partial s_x} < 0$ and $\frac{\partial P_y}{\partial s_y} < 0$. The production functions in (7.3) in combination with the price-quantity relations can be considered as a reduced-form model with inelastic supply and (hyperbolic or exponential) demand.

The tail dependence of extremely negative price shocks is not affected by the precise functional forms of the price-quantity relations. This holds true because the tail dependence measure in equation (7.1) is not affected by transformations of the marginal distributions. Hence, the translation of productivity shocks to changes in prices through the price-quantity relations affects the marginal distributions, but does not affect the dependence structure. In other words, the (tail) copula is not affected by the marginal distribution of the prices. The only change is that the dependence structure among *high* levels of supply corresponds to the dependence structure among *low* price levels, because low crop commodity prices in the framework are due to bumper crops.

The level of tail dependence among negative price shocks of two crop commodities depends on the question whether their production is exposed to similar productivity shocks, the $\tilde{\theta}_i$s. This in turn depends on the geographical location where commodities are grown. In Appendix 7.A we derive the theoretical level of tail dependence as

$$\tau_{x,y} = \left(\frac{q_1^\alpha}{q_1^\alpha + q_2^\alpha} \wedge \frac{v_1^\alpha}{v_1^\alpha + v_2^\alpha}\right) + \left(\frac{q_2^\alpha}{q_1^\alpha + q_2^\alpha} \wedge \frac{v_2^\alpha}{v_1^\alpha + v_2^\alpha}\right). \tag{7.5}$$

This theoretical result has several implications.[2] First, from the result in (7.5) we have that $\tau_{x,y} = 1$ if the ratio between the planted crops is exactly the same across all regions, i.e., if $q_i = cv_i$ with $c > 0$ for $i = 1, 2$. For example, if the area planted with wheat covers exactly three times the area planted with soybeans in each region, then we expect the returns of soybeans and wheat to be perfectly tail dependent. Second, we have $\tau_{x,y} = 0$ if two commodities are grown exclusively in separate regions, i.e., if $q_1 > 0 = v_1$ and $v_2 > 0 = q_2$.

All intermediate outcomes between tail independence and perfect tail dependence follow from situations in which both crops are grown together in at least one region, but where the ratio between the two crops is not constant across all regions where the commodities are grown. Loosely said, from the result in (7.5), it follows that tail dependence among seemingly unrelated crop commodities increases with the geographical overlap in the regions where crop commodities are grown. Crops grown in similar regions exhibit positive tail dependence, while crops grown in different regions do not.

The intuition behind these theoretical results is that the production of two crop commodities has to be exposed to the same shocks to be tail dependent. In the Appendix we provide an extension of the framework with macro shocks, which affect the production of all commodities across all regions, and commodity specific shocks, which affect the production of one commodity across all regions. The extension shows that the presence of macro shocks may result in positive tail dependence, even if two commodities are not grown together in any region. Further, the presence of commodity specific shocks tends to decrease the level of tail dependence. Nevertheless, the extension of equation (7.5) also shows that geographical overlap remains a relevant determinant of the level of tail dependence in the presence of macro shocks and commodity specific shocks, conditional upon the assumption that the region specific shocks, the θ_is, bear some relevance for large shocks in the production of agricultural commodities.[3]

[2] The minimum of a and b is denoted as $(a \wedge b)$.

[3] Formally, it is sufficient to have that $\frac{\Pr(\tilde{\varepsilon}>u)}{\Pr(\tilde{\theta}_i>u)}$ is finite as $u \to \infty$ for each newly introduced shock $\tilde{\varepsilon}$. If this condition would not hold true, then the fraction of large shocks in the production that is due to a large region specific shock tends to zero. In other words, region specific shocks would be irrelevant for large shocks in the production. See Subsection 7.A.1 in the Appendix for more details.

Further, the stylized framework does not allow for differences in productivity across different commodities or for differences in productivity across different regions for a specific commodity. The q_is and v_is simply measure the size of the area in region i where respectively commodity x and y are grown. Nevertheless, the theoretical framework and theoretical level of tail dependence in equation (7.5) can easily be adjusted to take region and commodity specific differences in productivity into account. More specifically, if the framework is adjusted to accommodate differences in productivity, then the planted areas in equation (7.5) for the theoretical level of tail dependence, i.e., the q_is and v_is, are replaced commodity specific expected production in each region, i.e., $E(\theta_{i,x})q_i$ and $E(\theta_{i,y})v_i$. In our robustness tests we come back to this productivity correction.

Returning to the discussion on excess comovement, our study focuses on the hypothesis that there is excess comovement among extreme price shocks of crop commodities. From the stylized framework above it follows that one may expect to observe positive tail dependence among crop commodities if there is overlap in the regions in which these are grown. Therefore, finding positive tail dependence among such commodities does not necessarily justify to use of the expression *excess* comovement. By contrast, not observing tail dependence among commodities which are not grown together, would show an apparent lack of evidence for excess comovement among those crop commodities. In our empirical exercise we will assess whether the returns of commodity pairs with negligible overlap exhibit tail dependence.

7.3 Methodology

7.3.1 Tail dependence

For seven crop commodities we collect daily price data of all futures contracts traded over the period 1979-2011 from Thomson Datastream. The commodities are corn, cotton, oats, orange juice, soybeans, sugar and wheat. To obtain long-term futures return series, or so-called *continuous* series, we have to combine consecutive data from several futures contracts, see e.g. De Roon *et al.* (2000). In January 1979 we start with the futures contract that has the largest open interest. For each day we copy its log return into the new continuous series until six weeks before the contract's last trading day. At this date we switch to the futures contract with the largest open interest and a later last trading day. This procedure results for each commodity in a continuous futures return series with 8,609 daily observations. Table 7.1 provides some descriptive statistics.[4]

The pairwise tail dependence measure in (7.1) can be estimated non-parametrically by

[4]Depending on the number of delivery dates available throughout the year and the liquidity of individual contracts, continuous series are constructed from data of 114 until 180 contracts.

Table 7.1: Descriptive statistics of daily futures returns

	Mean	St.dev.	Skewness	Minimum	Maximum	Contracts
Corn	-0.02	1.41	0.03	-7.85	8.66	119
Cotton	0.00	1.44	-0.01	-6.91	6.93	114
Oats	-0.03	1.74	-0.07	-11.94	10.52	141
Orange juice	-0.01	1.66	-0.03	-13.66	15.08	180
Soybeans	-0.01	1.40	-0.17	-7.32	6.70	154
Sugar	-0.01	2.28	-0.12	-18.22	14.20	119
Wheat	-0.02	1.56	0.08	-9.97	8.79	124

Note: Descriptive statistics of daily log returns on futures contracts from continuous series running from 1979 until 2011 (8,609 observations). Returns are multiplied by 100. The last column reports the number of different futures contracts the continuous return series is constructed from.

counting the number of observations in which both variables have extremely low values and dividing this by the number of observations in which one specific variable has an extremely low values, see e.g. Embrechts *et al.* (2000) and Schmidt and Stadtmüller (2006). Extremely low refers to values of X and Y below the $k+1$-lowest observation of respectively X and Y, where k is chosen such that $k(n) \to \infty$ and $k(n)/n \to 0$ as the number of observations $n \to \infty$. Formally, after ordering the observations X_t and Y_t from low to high as $\overline{X}_1, \ldots, \overline{X}_n$ and $\overline{Y}_1, \ldots, \overline{Y}_n$, the estimator is given as

$$\hat{\tau}_{x,y} = \frac{\sum_{t=1}^{n} \mathbf{1}_{X_t < \overline{X}_{k+1} \ \& \ Y_t < \overline{Y}_{k+1}}}{\sum_{t=1}^{n} \mathbf{1}_{X_t < \overline{X}_{k+1}}}, \tag{7.6}$$

where $\mathbf{1}$ denotes the indicator function for the condition in the subscript. The temporal dependence in financial data, such as volatility clustering, does not affect the consistency of the estimator in equation (7.6), see e.g. Drees (2008) and Hill (2009). However, temporal dependence does affect the variance of the estimates. Following Hartmann *et al.* (2010), standard errors are obtained from a block bootstrap procedure with a block length equal to $n^{1/3}$. We calculate the estimator in (7.6) by estimating the model

$$\mathbf{1}_{Y_t < \overline{Y}_{k+1}} = \beta \mathbf{1}_{X_t < \overline{X}_{k+1}} + \varepsilon_t \tag{7.7}$$

with ordinary least squares (OLS). Van Oordt and Zhou (2012) show that $\hat{\beta}^{OLS} = \hat{\tau}_{x,y}$.

Table 7.2 provides the tail dependence estimates and their standard errors for the 21 commodity pairs over the entire period for $k = 100$. The pairs orange juice-sugar and oats-orange juice exhibit the least dependence among extremely negative price shocks. For both commodity pairs $\tau_{x,y} = 0$ falls within the 95% confidence interval of the estimate. Nevertheless, in general we observe strong tail dependence among crop commodity pairs. The pairs corn-soybeans and corn-wheat exhibit the strongest tail dependence

Table 7.2: Dependence among extremely adverse shocks in futures prices

	Corn	Cotton	Oats	Orange juice	Soybeans	Sugar	Wheat
Corn	1	0.18	0.30	0.05	0.46	0.06	0.44
		(0.05)	(0.04)	(0.02)	(0.06)	(0.03)	(0.06)
Cotton		1	0.11	0.06	0.16	0.07	0.14
			(0.03)	(0.02)	(0.04)	(0.03)	(0.04)
Oats			1	0.03	0.25	0.05	0.22
				(0.02)	(0.04)	(0.02)	(0.04)
Orange juice				1	0.04	0.02	0.05
					(0.02)	(0.02)	(0.02)
Soybeans					1	0.08	0.31
						(0.03)	(0.05)
Sugar						1	0.08
							(0.03)
Wheat							1

Note: Estimates of the tail dependence among negative returns on futures contracts on different commodities. The estimates are based on daily returns on future contracts over the period 1979 - 2011. The reported numbers are obtained from the estimator in (7.6) with $k = 100$ ($k/n \approx 1.2\%$). Standard errors are obtained from a block bootstrap method with a block length of $n^{1/3} \approx 20$ and $10,000$ replications.

with estimates of $\tau_{x,y}$ close to 0.5. Such a high level of tail dependence is comparable to what is generally observed among the returns on stock market indices of strongly linked economies.[5]

7.3.2 Geographical overlap

Following equation (7.5) the overlap in the regions where commodities are grown can cause tail dependence among crop commodity pairs. To see whether the strong tail dependence of some commodity pairs in Table 7.2 is justified by overlap in regions where commodities are grown, we first have to quantify the degree of overlap in the regions where commodities are grown.

One potential measure of the degree of geographical overlap among two crop commodities is the correlation coefficient of the observed area planted with each commodity across different regions. However, although correlation across regions is related to geographical overlap, it has difficulties to distinguish which commodity pairs exhibit overlap and which do not. First, a correlation coefficient across regions of -1 for two commodities does not necessarily imply no overlap. Compare for example the case in which the observed area planted with each commodity equals respectively $(q_1, q_2) = (2, 0)$ and $(v_1, v_2) = (0, 2)$

[5]For example, for the France and German stock market, Poon *et al.* (2004, Table 3) report $\hat{\tau} = 0.52$ for the period 1990–2001. Cholette *et al.* (2012, Table A.4) report $\hat{\tau} = 0.58$ for the period 1990–2006.

and the case in which $(q_1, q_2) = (2, 1)$ and $(v_1, v_2) = (1, 2)$. Calculating the correlation coefficient of the observed areas planted with the two commodities across regions yields a correlation coefficient $\varrho(q_i, v_i) = -1$ in both cases. Yet, the conclusion of no overlap, in the sense that the two commodities are not grown together in any region, is only justified in the first case. Second, a correlation coefficient across regions larger than -1 does not necessarily imply overlap. Compare for example the case in which $(q_1, q_2) = (2, 0)$ and $(v_1, v_2) = (0, 2)$ and the case in which $(q_1, q_2, q_3) = (2, 0, 0)$ and $(v_1, v_2, v_3) = (0, 2, 0)$. Both cases exhibit no overlap, but only the first case has a correlation coefficient of -1.[6]

Because distinguishing between overlap and no overlap is important in our application, we derive an alternative measure which does not suffer from the problems described above and is closely connected to our theoretical framework. Formally, we measure the degree of geographical overlap by

$$O_{x,y} = \sum_{s \in S} \left(\frac{q_s}{\sum_{i \in S} q_i} \right) \wedge \left(\frac{v_s}{\sum_{i \in S} v_i} \right), \tag{7.8}$$

where $S = \{1, 2, \ldots, m\}$ denotes the m regions where the measure is calculated over.[7]

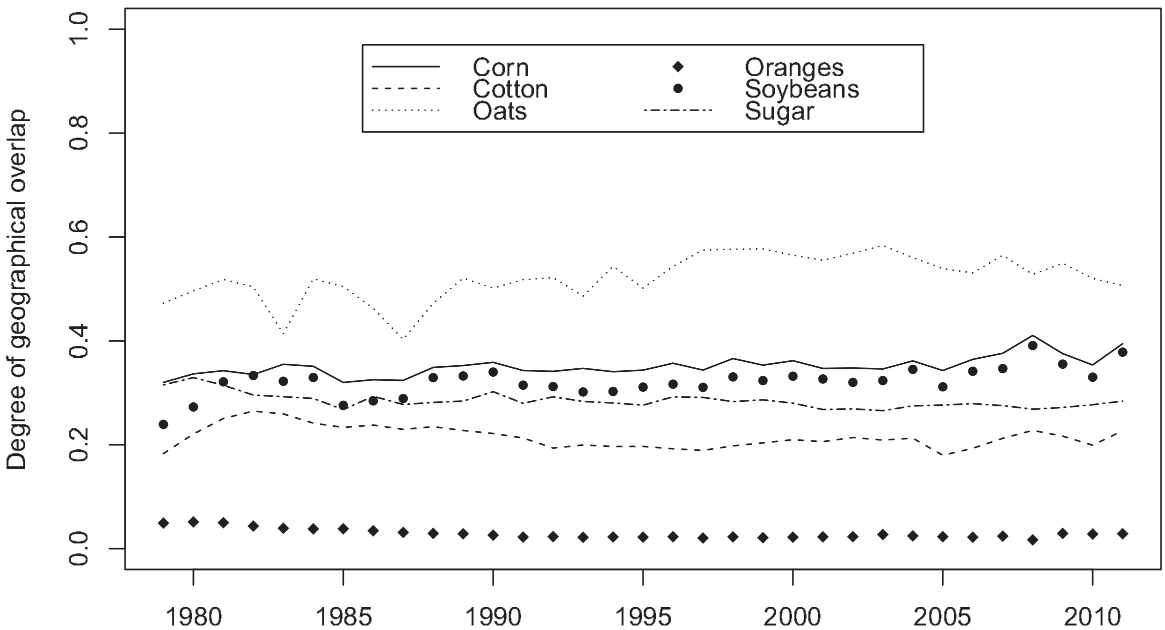
To calculate the degree of geographical overlap among crop commodity pairs we need region specific data on agricultural activity. We obtain those data from the National Agricultural Statistics Service (NASS) of the United States Department of Agriculture. For each commodity we obtain annual survey data on acres planted, acres harvested and the yield on the state level.[8] Figure 7.2 shows the degree of geographical overlap among several crop commodities and wheat. In general the level of geographical overlap between commodities is relative stable over time. Irrespective of the observed year, the strongest degree of geographical overlap with wheat is reported for oats, while the lowest is reported for oranges. The largest increase in the degree of geographical overlap is observed among soybeans and wheat which increased from 0.24 in 1979 to 0.38 in 2011.

[6]The case $(q_1, q_2, q_3) = (2, 0, 0)$ and $(v_1, v_2, v_3) = (0, 2, 0)$ yields a correlation coefficient $\varrho(q_i, v_i) = -1/2$. Another potential problem of the correlation coefficient across different regions is that it cannot be calculated if one of the crop commodities is planted on an area of exactly the same size across all regions, for example if $(q_1, q_2) = (1, 1)$ and $(v_1, v_2) = (2, 1)$.

[7]In the examples mentioned before the overlap measure is calculated as follows. The case $(q_1, q_2) = (2, 0)$ and $(v_1, v_2) = (0, 2)$ gives $O_{x,y} = (2/2 \wedge 0/2) + (0/2 \wedge 2/2) = 0$. The case $(q_1, q_2) = (2, 1)$ and $(v_1, v_2) = (1, 2)$ gives $(2/(2+1) \wedge 1/(2+1)) + (1/(2+1) \wedge 2/(2+1)) = (1/3) + (1/3) = 2/3$. The case $(q_1, q_2, q_3) = (2, 0, 0)$ and $(v_1, v_2, v_3) = (0, 2, 0)$ gives $O_{x,y} = 0 + 0 + 0 = 0$.

[8]For oranges the NASS provides survey data since ‘09. For oranges we supplement the survey data with census data from ‘78, ‘82, ‘87, ‘92, ‘97, ‘02 and ‘07 and linearly interpolate the missing years. For sugar we combine data on sugar beets and sugar cane. For these two series acres planted is not available and we use acres harvested instead.

Figure 7.2: Geographical overlap of crop commodities with wheat



The figure reports the degree of geographical overlap of different crop commodities with wheat. The overlap measure is calculated from equation (7.8) using annual data of area planted on the state level.

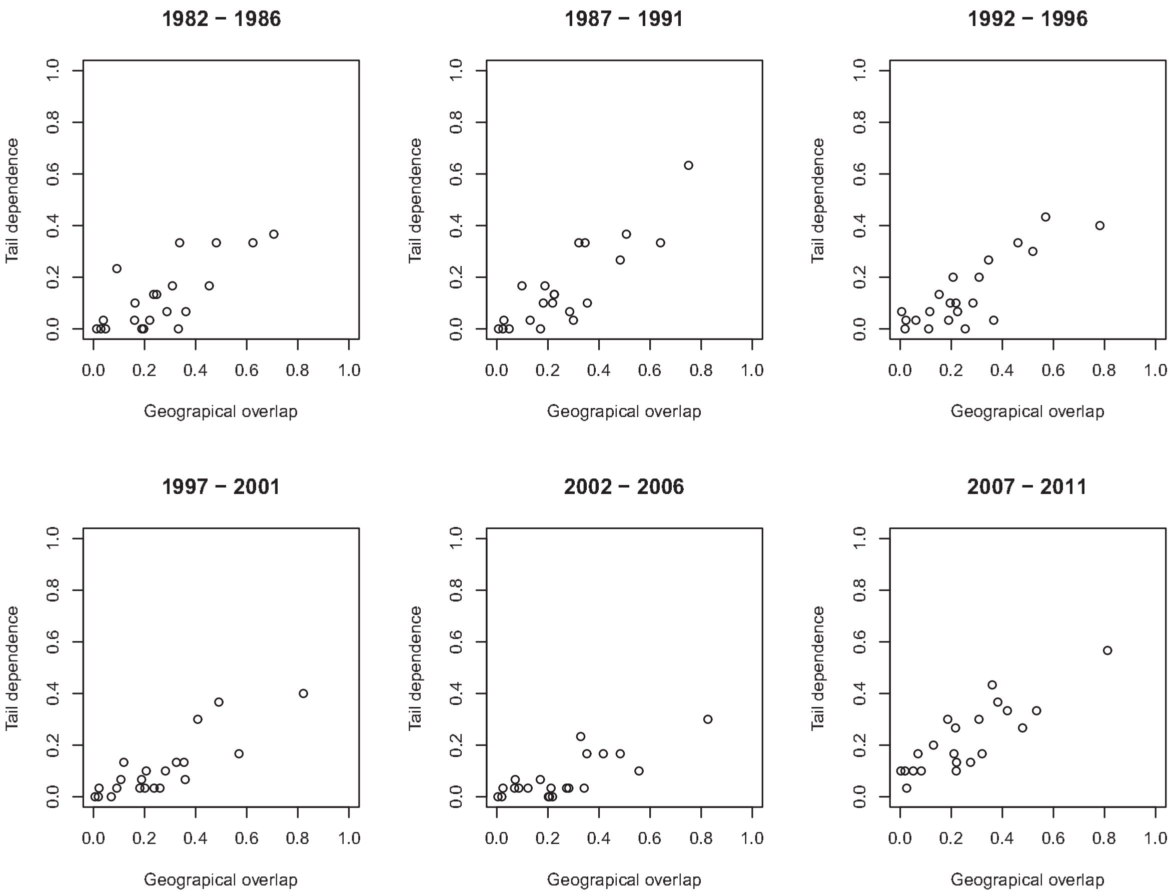
7.4 Results

We calculate the average degree of geographical overlap among crop commodity pairs from equation (7.2) during six different periods. For each period we also estimate the tail dependence among the daily future returns of commodity pairs from the estimator in equation (7.6). Figure 7.3 shows a scatter of the tail dependence and the average degree of geographical overlap for each period. Each of these scatters shows a positive association between tail dependence among daily futures return. Table 7.3 reports the corresponding correlation coefficients.[9] Over the different periods the range of the correlation coefficients is between 0.74 for 1982 - 1986 and 0.86 for 1987 - 1991. Following these numbers, the degree of geographical overlap approximately explains between 55% and 74% of the variance in the tail dependence. Each correlation coefficient is highly significant.

We try several changes to our baseline methodology to assess whether the positive association between geographical overlap and tail dependence in high frequency returns in futures markets is robust. The first change is to take the region and commodity specific differences in productivity into account in the calculation in the degree of geographical overlap. To this end we estimate the average productivity for each commodity in each

[9]Besides the Pearson's correlation coefficients in Table 7.3, we also calculated Spearman's rank correlation coefficients. These rank correlation coefficients are between 0.58 - 0.75 and each of them is significantly different from zero at a 1% significance level.

Figure 7.3: Geographical overlap and tail dependence of returns among commodity pairs



The figure shows scatters of the average degree of geographical overlap and the pairwise tail dependence of daily returns on futures contracts. The average degree of geographical overlap is calculated as the simple average of the measure in equation (7.8) for each year. Tail dependence estimates are obtained from the estimator in equation (7.6) with $k = 30$ ($k/n \approx 2.3\%$).

Table 7.3: Correlation among geographical overlap and tail dependence

Period	Correlation	t-statistic	p-value
1982 - 1986	0.74	4.8	0.000
1987 - 1991	0.86	7.3	0.000
1992 - 1996	0.84	6.8	0.000
1997 - 2001	0.84	6.7	0.000
2002 - 2006	0.78	5.4	0.000
2007 - 2011	0.85	6.9	0.000

Note: Pearson's correlation coefficients between the average degree of geographical overlap and the pair-wise tail dependence of daily returns on futures contracts. The average degree of geographical overlap is calculated as the simple average of the measure in equation (7.8) for each year. Tail dependence estimates are obtained from the estimator in equation (7.6) with $k = 30$ ($k/n \approx 2.3\%$). The second column provides t-statistics for testing against the null hypothesis of no correlation. The third column provides the corresponding p-values (critical values for rejection of the null hypothesis at a 5% and 1% significance level are respectively 2.44 and 2.87).

particular state over the five years prior as a proxy for the region-commodity specific productivity of the year under observation.[10] This correction does not affect our measure for geographical overlap much. The correlation coefficient between the overlap measures before and after the productivity correction is about 0.99 in each period. Not surprisingly, after the productivity correction, the correlation coefficients between the degree of geographical overlap and tail dependence remain in a range between 0.73 and 0.86.

Another potential concern is the sensitivity of the relation for changing the number of tail observations in the estimation of tail dependence, i.e., the choice of k in equation (7.6). The correlations coefficient for different levels of k range from 0.78 to 0.86 for $k = 25$ and from 0.68 to 0.85 for $k = 35$. We also test whether the observed relation is sensitive for the specification of the subperiods. If we look at decades instead of five year periods, then we observe correlation coefficients between 0.77 and 0.89 (for $k = 30$ and $k = 50$). Alternatively, if we alter the first year of the analysis to 1979, i.e., 1979-1983, 1984-1978, etc., then we observe correlations in a range of 0.76 and 0.87. Finally, we also test whether the relation holds for dependence among extreme price increases. After estimating the tail dependence among positive price shocks we find correlation coefficients between 0.64 and 0.86. The correlations remain highly significant after all these changes. We conclude that the positive association between the degree of overlap in regions where commodities are grown and the tail dependence among daily futures returns seems to be quite robust across different subperiods, different choices of k, and for making corrections for productivity.

[10]We require at least three years of historical yield data. For oranges we assume equal productivity across all regions due to lack of yield data.

Table 7.4: Tail dependence in case of negligible and material geographical overlap

Group	Negligible geographical overlap ($O < 0.05$)		Material geographical overlap ($O \geq 0.05$)	
	Observations	Tail dependent	Observations	Tail dependent
1982 - 1986	4	0	17	8
1987 - 1991	4	0	17	9
1992 - 1996	3	0	18	7
1997 - 2001	3	0	18	6
2002 - 2006	3	0	18	5
2007 - 2011	3	0	18	14
Total:	20	0	106	49

Note: Crop commodity pairs are classified in pairs with negligible and material geographical overlap. The degree of geographical overlap is calculated from equation (7.8). The threshold value is fixed at 0.05. We further calculate tail dependence estimates of daily future returns from equation (7.6) with $k = 30$ ($k/n \approx 2.3\%$). The table reports within each class the number of crop commodity pairs for which zero tail dependence among daily future returns, i.e., $\tau_{x,y} = 0$, falls outside the 95% confidence interval.

The question remains whether commodity pairs without geographical overlap exhibit positive tail dependence. Unfortunately, we have no commodity pairs with no geographical overlap, i.e., with $O = 0$, because commodity pairs in our data are always grown together in at least one state. Instead, we divide the crop commodity pairs for each period in two groups with negligible geographical overlap ($O < 0.05$) and material overlap ($O \geq 0.05$). In both groups we calculate a 95% confidence interval around the tail dependence estimate for each commodity pair within each period. Table 7.4 reports the number of commodity pairs within each group and the number of commodity pairs for which $\tau = 0$ falls outside the 95% confidence interval. Each period 3 to 4 and 17 to 18 crop commodity pairs are classified as having respectively negligible and material geographical overlap. Table 7.4 shows that zero dependence falls outside the tail dependence confidence interval in 49 of the 106 cases in the group with material geographical overlap, while zero tail dependence is never outside the confidence interval in the group with negligible geographical overlap. These results raise some confidence that there is no comovement and thus no *excess* comovement among extreme price shocks of crop commodities if there is no geographical overlap (although we cannot determine whether the observed tail dependence is exclusively due to geographical overlap).

The result that $\tau = 0$ falls inside the 95% confidence interval for crop commodities with negligible geographical overlap is subject to several qualifications. Although the result is robust for changes in methodology, such as making corrections for productivity or having the five year periods starting earlier, its outcome partially depends on the somewhat arbitrary choice of the length of the estimation horizon and the choice of setting the threshold for negligible geographical overlap at $O < 0.05$. Theoretically, the higher the threshold value and the longer the time period under considerations, i.e., the larger n, the more likely it is that $\tau = 0$ falls outside the 95% confidence interval for some crop commodity pair with a 'negligible' degree of geographical overlap. For example, the geographical overlap measure for wheat and orange in Figure 7.2 is on average 0.03, while the tail dependence estimate in Table 7.2 equals $\hat{\tau} = 0.05$ with a standard error of 0.02 (which ensures that $\tau = 0$ falls outside the 95% confidence interval). Indeed, if the time horizon is sufficiently long, any minor degree of geographical overlap could be considered as 'material', because the standard error of the tail dependence estimates decreases with the number of observations. Nevertheless, the results show that the current estimation horizon is sufficiently long to observe that $\tau = 0$ falls outside the 95% confidence interval for commodity pairs with 'material' overlap.

7.5 Conclusion

This chapter documents the comovement of crop commodity prices from the perspective of the extreme price shocks observed among daily returns in futures markets. For some crop commodity pairs we observe similar levels of tail dependence as the literature documents for stock market returns of strongly linked economies. Theoretically, we show that the level of tail dependence among crop commodity returns is positively affected by the degree in which commodities are grown in similar regions. Empirically, we observe a strong and persistent positive link between the tail dependence among crop commodity returns and the degree of geographical overlap. Our results suggest that a vast majority of variation in tail dependence can be explained by the degree of geographical overlap. Further, we do not find evidence of tail dependence among commodities not grown in similar regions, which suggest that our analysis on extreme crop commodity returns does not provide support for the excess comovement hypothesis.

Our results can be interpreted as supportive evidence for the hypothesis that a majority of the tail dependence among crop commodity prices in high frequency data can be explained by common productivity shocks, such as weather conditions, due to geographical overlap. Nevertheless, in this context we admit that our results are subject to the qualification that other economic factors may play a role in the observed relation. For example, it might be the case that those agricultural product pairs that can be considered as substitutes are also those commodity pairs that are grown in similar regions. Although such factors might explain part of the presented evidence, our analysis reveals a strong and persistent pattern that requires a justification from economic theory. The hypothesis that the relations stems from common productivity shocks due to geographical overlap cannot unreservedly be rejected.

7.A Appendix A. Derivation of tail dependence

We start with deriving the tail dependence among bumper crops, τ_{s_x,s_y}, defined as

$$\tau_{s_x,s_y} =: \lim_{p\downarrow 0} \frac{\Pr\{s_x > Q_{s_x}(1-p) \;\&\; s_y > Q_{s_y}(1-p)\}}{p}, \tag{7.9}$$

where $Q_{s_x}(\cdot)$ and $Q_{s_y}(\cdot)$ denote the quantile function of respectively s_x and s_y. Given the amount of land planted in each area we can derive the probability of bumper crops. More specific, from (7.3) and (7.4) we have for commodity x and y as $u \to \infty$

$$\begin{aligned}
\Pr(s_x > u) &= \mathcal{L}(u)(q_1^\alpha + q_2^\alpha)u^{-\alpha} + o(u^{-\alpha}),\\
\Pr(s_y > u) &= \mathcal{L}(u)(v_1^\alpha + v_2^\alpha)u^{-\alpha} + o(u^{-\alpha}),
\end{aligned} \tag{7.10}$$

where s_x and s_y denote the production of respectively commodity x and y.[11] Further, substitution of the production functions in (7.3) into equation (7.9) gives

$$\tau_{s_x,s_y} = \lim_{p\downarrow 0} \frac{\Pr\{q_1\tilde{\theta}_1 + q_2\tilde{\theta}_2 > Q_{s_x}(1-p) \;\&\; v_1\tilde{\theta}_1 + v_2\tilde{\theta}_2 > Q_{s_y}(1-p)\}}{p}. \tag{7.11}$$

Using the heavy-tailedness of the innovations we obtain

$$\begin{aligned}
\tau_{s_x,s_y} = &\lim_{p\downarrow 0} \frac{\Pr\{\tilde{\theta}_1 > \frac{Q_{s_x}(1-p)}{q_1} \;\&\; \tilde{\theta}_1 > \frac{Q_{s_y}(1-p)}{v_1}\}}{p}\\
&+ \lim_{p\downarrow 0} \frac{\Pr\{\tilde{\theta}_2 > \frac{Q_{s_x}(1-p)}{q_2} \;\&\; \tilde{\theta}_2 > \frac{Q_{s_y}(1-p)}{v_2}\}}{p} + \lim_{p\downarrow 0} \frac{o(p)}{p},
\end{aligned} \tag{7.12}$$

which is equivalent to

$$\begin{aligned}
\tau_{s_x,s_y} &= \lim_{p\downarrow 0} \frac{\Pr\{\tilde{\theta}_1 > \frac{Q_{s_x}(1-p)}{q_1} \vee \frac{Q_{s_y}(1-p)}{v_1}\}}{p} + \lim_{p\downarrow 0} \frac{\Pr\{\tilde{\theta}_2 > \frac{Q_{s_x}(1-p)}{q_2} \vee \frac{Q_{s_y}(1-p)}{v_2}\}}{p}\\
&\quad + \lim_{p\downarrow 0} \frac{o(p)}{p},\\
&=: \lim_{p\downarrow 0} A_1(p) + \lim_{p\downarrow 0} A_2(p) + \lim_{p\downarrow 0} \frac{o(p)}{p}.
\end{aligned} \tag{7.13}$$

Focusing on $A_1(p)$, we have that

$$\begin{aligned}
\lim_{p\downarrow 0} A_1(p) &= \lim_{p\downarrow 0} \frac{\Pr\{\tilde{\theta}_1 > \frac{Q_{s_x}(1-p)}{q_1}\}}{p} \wedge \frac{\Pr\{\tilde{\theta}_1 > \frac{Q_{s_y}(1-p)}{v_1}\}}{p}\\
&= \lim_{p\downarrow 0} \frac{\mathcal{L}(\frac{Q_{s_x}(1-p)}{q_1})(\frac{Q_{s_x}(1-p)}{q_1})^{-\alpha}}{\mathcal{L}(Q_{s_x}(1-p))(q_1^\alpha + q_2^\alpha)(Q_{s_x}(1-p))^{-\alpha} + o(Q_{s_x}(1-p)^{-\alpha})}
\end{aligned}$$

[11] This follows from Feller's convolution theorem, see Feller (1971), p. 278. De Vries (2005) provides an intuitive proof.

$$
\begin{aligned}
&\wedge \lim_{p\downarrow 0} \frac{\mathcal{L}(\frac{Q_{s_y}(1-p)}{v_1})(\frac{Q_{s_y}(1-p)}{v_1})^{-\alpha}}{\mathcal{L}(Q_{s_y}(1-p))(v_1^\alpha+v_2^\alpha)(Q_{s_y}(1-p))^{-\alpha}+o(Q_{s_y}(1-p)^{-\alpha})}\\
&=\frac{q_1^\alpha}{q_1^\alpha+q_2^\alpha}\wedge\frac{v_1^\alpha}{v_1^\alpha+v_2^\alpha},
\end{aligned} \tag{7.14}
$$

where we use (7.4) and (7.10) to obtain the first equality and the definition of a slowly varying function to obtain the second equality. Following a similar line of argument we find for $A_2(p)$ that

$$
\lim_{p\downarrow 0} A_2(p)=\frac{q_2^\alpha}{q_1^\alpha+q_2^\alpha}\wedge\frac{v_2^\alpha}{v_1^\alpha+v_2^\alpha}. \tag{7.15}
$$

By definition we have $\frac{o(p)}{p}=0$ as $p\to 0$. Hence, after substituting (7.14) and (7.15) into (7.13) we obtain the intermediate result

$$
\tau_{s_x,s_y}=\left(\frac{q_1^\alpha}{q_1^\alpha+q_2^\alpha}\wedge\frac{v_1^\alpha}{v_1^\alpha+v_2^\alpha}\right)+\left(\frac{q_2^\alpha}{q_1^\alpha+q_2^\alpha}\wedge\frac{v_2^\alpha}{v_1^\alpha+v_2^\alpha}\right). \tag{7.16}
$$

Further, let $P_x(s_x)$ and $P_y(s_y)$ denote the price-quantity relations for commodities x and y with $P_x(s_x)>0$, $P_y(s_y)>0$ and $\frac{\partial P_x}{\partial s_x}<0$, $\frac{\partial P_y}{\partial s_y}<0$ for any $s_x,s_y\in[0,\infty)$. With these price-quantity relations, we have that $X(s_x)=\log(P_x(s_x)/P_{x,-1})$ and $Y(s_y)=\log(P_y(s_y)/P_{y,-1})$, which denote the log price difference relative to yesterday's price, are strictly decreasing functions in s_x and s_y. It follows that

$$
\begin{aligned}
\tau_{s_x,s_y}&=\lim_{p\downarrow 0}\frac{\Pr\{s_x>Q_{s_x}(1-p)\ \&\ s_y>Q_{s_y}(1-p)\}}{p}\\
&=\lim_{p\downarrow 0}\frac{\Pr\{X(s_x)<X(Q_{s_x}(1-p))\ \&\ Y(s_y)<Y(Q_{s_y}(1-p))\}}{p}\\
&=\lim_{p\downarrow 0}\frac{\Pr\{X<Q_x(p)\ \&\ Y<Q_y(p)\}}{p}=\tau_{x,y}.
\end{aligned} \tag{7.17}
$$

Hence, the marginal transformations affect the univariate distributions, but not the dependence structure. In other words, the copula is unaffected. The only deviation is that the dependence structure among *high* levels of supply corresponds to the dependence structure among *low* prices. This is why the inequalities in the probability turn around after transforming s_x and s_y to X and Y respectively. Finally, combining the intermediate results in (7.16) and (7.17) gives the final result in (7.5).

7.A.1 Extension

In this subsection, we extend the framework with commodity specific and macro shocks. Let $\tilde{\varepsilon}_x$ and $\tilde{\varepsilon}_y$ represent commodity specific productivity shocks for commodity x and y, and let $\tilde{m}$ denote a macro factor that represents productivity shocks across all regions and all commodities. With these new sources of uncertainty, the production functions of crop commodities x and y in (7.3) are formulated as

$$\begin{aligned} s_x &= q_1\tilde{\theta}_1 + q_2\tilde{\theta}_2 + (q_1+q_2)(\tilde{m}+\tilde{\varepsilon}_x); \\ s_y &= v_1\tilde{\theta}_1 + v_2\tilde{\theta}_2 + (v_1+v_2)(\tilde{m}+\tilde{\varepsilon}_y). \end{aligned} \tag{7.18}$$

We define the intensity of large commodity specific and large macro shocks relative to the intensity of large area specific shocks as

$$\begin{aligned} \sigma_{\varepsilon_x} &=: \lim_{u\to\infty} \frac{\Pr(\tilde{\varepsilon}_x > u)}{\Pr(\tilde{\theta}_i > u)}; \\ \sigma_{\varepsilon_y} &=: \lim_{u\to\infty} \frac{\Pr(\tilde{\varepsilon}_y > u)}{\Pr(\tilde{\theta}_i > u)}; \\ \sigma_m &=: \lim_{u\to\infty} \frac{\Pr(\tilde{m} > u)}{\Pr(\tilde{\theta}_i > u)}. \end{aligned} \tag{7.19}$$

For the present we assume σ_{ε_x}, σ_{ε_y} and σ_m to be finite. We drop this assumption later. If we have $\sigma_{\varepsilon_x} \in \mathbb{R}^+$, $\sigma_{\varepsilon_y} \in \mathbb{R}^+$, and $\sigma_{\varepsilon_m} \in \mathbb{R}^+$, then we must have that the commodity specific shocks and macro shocks are heavy-tailed with tail index α. By contrast, with $\sigma_{\varepsilon_x} = 0$ and $\sigma_{\varepsilon_y} = 0$, we either have the case in which there are no large commodity specific shocks, or the case in which commodity specific shocks exhibit a thinner tail than area specific shocks, $\tilde{\theta}_i$. Similarly, with $\sigma_m = 0$, we either have that there are no large macro shocks, or that macro shocks exhibit a thinner tail than area specific shocks. The area specific, commodity specific and macro shocks are assumed to be independent. From (7.4), (7.18) and (7.19), we have for the production of commodities x and y as $u \to \infty$

$$\begin{aligned} \Pr(s_x > u) &= \mathcal{L}(u)\left(q_1^\alpha + q_2^\alpha + (\sigma_m + \sigma_{\varepsilon_x})(q_1+q_2)^\alpha\right) u^{-\alpha} + o(u^{-\alpha}); \\ \Pr(s_y > u) &= \mathcal{L}(u)\left(v_1^\alpha + v_2^\alpha + (\sigma_m + \sigma_{\varepsilon_y})(v_1+v_2)^\alpha\right) u^{-\alpha} + o(u^{-\alpha}). \end{aligned} \tag{7.20}$$

Now we continue along the lines of the original proof. After substituting the production functions in (7.18) in the definition of τ_{s_x,s_y} in (7.9), and after exploiting the heavy-tailedness of the area specific innovations we obtain

$$\begin{aligned} \tau_{s_x,s_y} &= \lim_{p\downarrow 0} \frac{\Pr\{\tilde{\theta}_1 > \frac{Q_{s_x}(1-p)}{q_1} \ \&\ \tilde{\theta}_1 > \frac{Q_{s_y}(1-p)}{v_1}\}}{p} \\ &+ \lim_{p\downarrow 0} \frac{\Pr\{\tilde{\theta}_2 > \frac{Q_{s_x}(1-p)}{q_2} \ \&\ \tilde{\theta}_2 > \frac{Q_{s_y}(1-p)}{v_2}\}}{p} \\ &+ \lim_{p\downarrow 0} \frac{\Pr\{\tilde{m} > \frac{Q_{s_x}(1-p)}{q_1+q_2} \ \&\ \tilde{m} > \frac{Q_{s_y}(1-p)}{v_1+v_2}\}}{p} + \lim_{p\downarrow 0} \frac{o(p)}{p}, \end{aligned} \tag{7.21}$$

which is equivalent to

$$\tau_{s_x,s_y} = \lim_{p\downarrow 0} \frac{\Pr\{\tilde{\theta}_1 > \frac{Q_{s_x}(1-p)}{q_1} \vee \frac{Q_{s_y}(1-p)}{v_1}\}}{p} + \lim_{p\downarrow 0} \frac{\Pr\{\tilde{\theta}_2 > \frac{Q_{s_x}(1-p)}{q_2} \vee \frac{Q_{s_y}(1-p)}{v_2}\}}{p}$$

$$
\begin{aligned}
&+\lim_{p\downarrow 0}\ \frac{\Pr\{\tilde{m} > \frac{Q_{s_x}(1-p)}{q_1+q_2} \vee \frac{Q_{s_y}(1-p)}{v_1+v_2}\}}{p} + \lim_{p\downarrow 0}\ \frac{o(p)}{p},\\
&=: \lim_{p\downarrow 0}\ B_1(p) + \lim_{p\downarrow 0}\ B_2(p) + \lim_{p\downarrow 0}\ B_m(p) + \lim_{p\downarrow 0}\ \frac{o(p)}{p}. \qquad (7.22)
\end{aligned}
$$

Focusing on $B_1(p)$, we have that

$$
\begin{aligned}
\lim_{p\downarrow 0}\ B_1(p) &= \lim_{p\downarrow 0}\ \frac{\Pr\{\tilde{\theta}_1 > \frac{Q_{s_x}(1-p)}{q_1}\}}{p} \wedge \frac{\Pr\{\tilde{\theta}_1 > \frac{Q_{s_y}(1-p)}{v_1}\}}{p}\\
&= \lim_{p\downarrow 0}\ \frac{\mathcal{L}(\frac{Q_{s_x}(1-p)}{q_1})(\frac{Q_{s_x}(1-p)}{q_1})^{-\alpha}}{\mathcal{L}(Q_{s_x}(1-p))\,(q_1^\alpha + q_2^\alpha + (\sigma_m + \sigma_{\varepsilon_x})(q_1+q_2)^\alpha)\,(Q_{s_x}(1-p))^{-\alpha} + o(Q_{s_x}(1-p)^{-\alpha})}\\
&\wedge \lim_{p\downarrow 0}\ \frac{\mathcal{L}(\frac{Q_{s_y}(1-p)}{v_1})(\frac{Q_{s_y}(1-p)}{v_1})^{-\alpha}}{\mathcal{L}(Q_{s_y}(1-p))\,\left(v_1^\alpha + v_2^\alpha + (\sigma_m + \sigma_{\varepsilon_y})(v_1+v_2)^\alpha\right)\,(Q_{s_y}(1-p))^{-\alpha} + o(Q_{s_y}(1-p)^{-\alpha})}\\
&= \frac{q_1^\alpha}{q_1^\alpha + q_2^\alpha + (\sigma_m + \sigma_{\varepsilon_x})(q_1+q_2)^\alpha} \wedge \frac{v_1^\alpha}{v_1^\alpha + v_2^\alpha + (\sigma_m + \sigma_{\varepsilon_y})(v_1+v_2)^\alpha}, \qquad (7.23)
\end{aligned}
$$

where we use (7.4) and (7.20) to obtain the first equality. Following a similar line of argument we obtain

$$
\lim_{p\downarrow 0}\ B_2(p) = \frac{q_2^\alpha}{q_1^\alpha + q_2^\alpha + (\sigma_m + \sigma_{\varepsilon_x})(q_1+q_2)^\alpha} \wedge \frac{v_2^\alpha}{v_1^\alpha + v_2^\alpha + (\sigma_m + \sigma_{\varepsilon_y})(v_1+v_2)^\alpha}. \qquad (7.24)
$$

Further, for $B_m(p)$ we have that

$$
\begin{aligned}
\lim_{p\downarrow 0}\ B_m(p) &= \lim_{p\downarrow 0}\ \frac{\Pr\{\tilde{m} > \frac{Q_{s_x}(1-p)}{q_1+q_2}\}}{p} \wedge \frac{\Pr\{\tilde{m} > \frac{Q_{s_y}(1-p)}{v_1+v_2}\}}{p}\\
&= \lim_{p\downarrow 0}\ \frac{\sigma_m\mathcal{L}(\frac{Q_{s_x}(1-p)}{q_1+q_2})(\frac{Q_{s_x}(1-p)}{q_1+q_2})^{-\alpha}}{\mathcal{L}(Q_{s_x}(1-p))\,(q_1^\alpha + q_2^\alpha + (\sigma_m + \sigma_{\varepsilon_x})(q_1+q_2)^\alpha)\,(Q_{s_x}(1-p))^{-\alpha} + o(Q_{s_x}(1-p)^{-\alpha})}\\
&\wedge \lim_{p\downarrow 0}\ \frac{\sigma_m\mathcal{L}(\frac{Q_{s_y}(1-p)}{v_1+v_2})(\frac{Q_{s_y}(1-p)}{v_1+v_2})^{-\alpha}}{\mathcal{L}(Q_{s_y}(1-p))\,\left(v_1^\alpha + v_2^\alpha + (\sigma_m + \sigma_{\varepsilon_y})(v_1+v_2)^\alpha\right)\,(Q_{s_y}(1-p))^{-\alpha} + o(Q_{s_y}(1-p)^{-\alpha})}\\
&= \frac{\sigma_m(q_1+q_2)^\alpha}{q_1^\alpha + q_2^\alpha + (\sigma_m + \sigma_{\varepsilon_x})(q_1+q_2)^\alpha} \wedge \frac{\sigma_m(v_1+v_2)^\alpha}{v_1^\alpha + v_2^\alpha + (\sigma_m + \sigma_{\varepsilon_y})(v_1+v_2)^\alpha}, \qquad (7.25)
\end{aligned}
$$

where we use (7.4), (7.19) and (7.20) to obtain the first equality. After using (7.23), (7.24), (7.25) and $\tau_{s_x,s_y} = \tau_{x,y}$ in (7.22), we obtain the extended version of the equation for tail dependence in (7.5) as

$$
\begin{aligned}
\tau_{x,y} &= \left(\frac{q_1^\alpha}{q_1^\alpha + q_2^\alpha + (\sigma_m + \sigma_{\varepsilon_x})(q_1+q_2)^\alpha} \wedge \frac{v_1^\alpha}{v_1^\alpha + v_2^\alpha + (\sigma_m + \sigma_{\varepsilon_y})(v_1+v_2)^\alpha}\right)\\
&+ \left(\frac{q_2^\alpha}{q_1^\alpha + q_2^\alpha + (\sigma_m + \sigma_{\varepsilon_x})(q_1+q_2)^\alpha} \wedge \frac{v_2^\alpha}{v_1^\alpha + v_2^\alpha + (\sigma_m + \sigma_{\varepsilon_y})(v_1+v_2)^\alpha}\right)\\
&+ \left(\frac{\sigma_m(q_1+q_2)^\alpha}{q_1^\alpha + q_2^\alpha + (\sigma_m + \sigma_{\varepsilon_x})(q_1+q_2)^\alpha} \wedge \frac{\sigma_m(v_1+v_2)^\alpha}{v_1^\alpha + v_2^\alpha + (\sigma_m + \sigma_{\varepsilon_y})(v_1+v_2)^\alpha}\right). \qquad (7.26)
\end{aligned}
$$

Consider the case in which large macro shocks and large commodity specific shocks do not occur or are thinner tailed than the area specific shocks, i.e., $\sigma_{\varepsilon_x} = \sigma_{\varepsilon_x} = \sigma_m = 0$. In this case equation (7.26) is identical to original equation in (7.5).

Equation (7.26) consists of three components. The value of the first and second component measure the impact of geographical overlap on the level of tail dependence. The third component measures the impact of macro shocks on the level of tail dependence. With $\sigma_m = 0$, the third component of the equation equals zero and macro shocks are irrelevant in determining the level of tail dependence. With $\sigma_m > 0$, the third component in equation (7.26) has a positive value and macro shocks are relevant for the level of tail dependence (provided that the amount of area planted with commodities x and y is nonzero). Simultaneously, an increase in σ_m decreases the relative impact of geographical overlap on the level of tail dependence by increasing the denominator of the first two components in equation (7.26) (with the extreme case $\sigma_m \to \infty$, we have that geographical overlap is irrelevant for the level of tail dependence).

Finally, the relative intensities of large commodity specific shocks, σ_{ε_x} and σ_{ε_y}, appear in the denominators of equation (7.26) only. Hence, in general it holds that the more likely it is that large changes in productivity originate from commodity specific shocks, i.e., the higher σ_{ε_x} and σ_{ε_y}, the lower the level of tail dependence among the commodities. Nevertheless, equation (7.26) also shows that geographical overlap remains a determinant of the level of tail dependence in the presence of macro shocks and commodity specific shocks, conditional upon the assumption that σ_{ε_x}, σ_{ε_y} and σ_m are finite.

Samenvatting (Summary in Dutch)

De titel van dit proefschrift luidt 'Over extreme gebeurtenissen in het bankwezen en de financiële wereld'. Dit proefschrift legt dus de nadruk op 'extreme' gebeurtenissen, of, met andere woorden, gebeurtenissen die met een zeer kleine kans plaatsvinden. Voordat iemand begint met het lezen van een proefschrift met een focus op extreme gebeurtenissen, moet eerst de vraag worden gesteld of zulke gebeurtenissen wel fundamenteel verschillend zijn van reguliere gebeurtenissen. Anders zou het af te raden zijn om vanaf dit punt verder te lezen. Het zou dan nuttiger zijn om onderzoek te bestuderen dat zich baseert op alle waarnemingen uit het verleden, en niet alleen op gebeurtenissen die door de onderzoeker worden beschouwd als 'extreem'. Zulk onderzoek is immers per definitie gebaseerd op een groter aantal waarnemingen, waardoor het mogelijk is om krachtigere conclusies te trekken. Een proefschrift met een focus op extreme gebeurtenissen, zoals u nu in handen heeft, zou in dat geval van ondergeschikt belang zijn.

Het feit dat dit proefschrift is geschreven suggereert dat extreme gebeurtenissen in het bankwezen en de financiële wereld wel degelijk verschillen ten opzichte van normale gebeurtenissen. Dit wordt ook bevestigd door de resultaten van het onderzoek in dit proefschrift. Het centrale thema van de bevindingen is dat het gebruikelijke verband tussen twee factoren in de financiële wereld sterk kan verschillen van de geldende relatie in extreme omstandigheden.

De resultaten in Hoofdstukken 2 en 3 benadrukken het belang van het bestuderen van de samenhang tussen rendementen op financiële markten in extreme marktomstandigheden om een goed begrip te krijgen van de potentiële verliezen op beleggingsportefeuilles. Hoofdstuk 4 bespreekt hoe het verpakken van leningen in effecten aan de ene kant kan helpen om het risicomanagement van banken te verbeteren, maar aan de andere kant ook destabiliserend kan werken ten tijde van een extreme gebeurtenis waarbij het vertrouwen van depositohouders afneemt. Hoofdstuk 5 toont aan waarom het belangrijk is om de relatie tussen de macro-economie en bankwinstgevendheid apart te bestuderen in extreme omstandigheden, zoals ten tijde van diepe recessies. Hoofdstuk 6 illustreert hoe het afzonderlijk bestuderen van extreme waarnemingen kan helpen om te komen tot betere inschattingen van financiële risico's. Het laatste hoofdstuk van dit proefschrift, Hoofdstuk 7, laat zien hoe de samenhang tussen extreme gebeurtenissen in financiële markten gerelateerd kan worden aan wat zich afspeelt in de reële economie. Het vervolg van deze samenvatting geeft een kort overzicht van elk hoofdstuk.

Financiële risicomanagers proberen de kans op grote verliezen op hun beleggingsportefeuilles te beperken. Een belangrijke bron van de risico's in beleggingsportefeuilles wordt gevormd door schommelingen in de algemene koers van aandelenmarkten, het zogenaamde systematische risico. In de praktijk meet men de hoeveelheid systematisch risico van een portefeuille door een lineaire relatie te schatten tussen de waardeveranderingen van de beleggingsportefeuille en de schommelingen in de algemene koers van de aandelenmarkt. In het financiële jargon wordt de helling van zo'n lineaire relatie de 'portefeuillebèta' genoemd. De portefeuillebèta geeft een inschatting van de proportionele verandering in de waarde van de beleggingsportefeuille ten opzichte van veranderingen in de algemene aandelenkoers. Als de koers van de aandelenmarkt daalt met 2%, dan daalt de waarde van een beleggingsportefeuille met een portefeuillebèta van 3 naar verwachting met 6%.

In de praktijk en in de wetenschap is men zich ervan bewust dat deze methode beperkingen heeft voor risicomanagement. Risicomanagement gebaseerd op het inschatten van portefeuillebèta's steunt op de aanname dat de relatie tussen aandelen in de beleggingsportefeuille en de algemene aandelenkoers gelijk is onder normale en extreem ongunstige marktomstandigheden. Een aanname die niet gestaafd wordt door de feiten. Zo is het bijvoorbeeld welbekend dat relaties in financiële markten in een omgeving van een dalende aandelenkoersen verschillend zijn van de geldende relaties in een omgeving van stijgende koersen, zie onder andere de studies van Longin and Solnik (2001) en Ang and Bekaert (2002). Academici hebben dit probleem gedeeltelijk opgelost door inschattingen te maken van lineaire relaties die exclusief gebaseerd zijn op prijsmutaties in een omgeving van dalende koersen, zie bijvoorbeeld de studies van Price *et al.* (1982) en Ang *et al.* (2006a). Echter, zulke inschattingen zijn nog steeds gebaseerd op een groot aantal waarnemingen ten tijde van relatief rustige marktomstandigheden. Betrouwbare methoden om de hoeveelheid systematisch risico in te schatten voor *extreem* ongunstige marktomstandigheden, het 'systematische staartrisico', zijn nog niet voorhanden.

Hoofdstuk 2 is een eerste poging om dit gemis op te vullen. In dit hoofdstuk ontwikkelen wij een maatstaf voor systematisch staartrisico, de 'tail bèta', en ontwikkelen wij de methodologie om een inschatting te maken van deze maatstaf. Empirisch constateren wij dat aandelen met historisch hoge tail bèta's geassocieerd worden met significant grotere verliezen bij een toekomstige beurskrach. Dit toont aan dat tail bèta's gemeten op basis van historische data inderdaad helpen om een betere inschatting te maken van het toekomstige systematische staartrisico. Wij registreren ook dat aandelen met een hoge tail bèta meer rendement opleveren gedurende relatief rustige marktomstandigheden. Echter, gebaseerd op meer dan 40 jaar aan historische koersinformatie blijkt dat deze hogere rendementen gedurende relatief normale marktomstandigheden bijna geheel teniet worden gedaan door de grotere verliezen tijdens een beurskrach.

Om een inschatting te maken van de relatie tussen twee variabelen is het noodzakelijk om op één of andere manier te meten hoe sterk de samenhang tussen die twee is. In traditionele regressie analyse vervult de correlatiecoëfficiënt deze rol. Bij het schatten van de 'tail bètas' in Hoofdstuk 2 gebruiken wij het concept 'staartafhankelijkheid' om de mate van samenhang tussen extreme waardeveranderingen van individuele aandelen en algemene marktschommelingen te meten. Deze maatstaf wordt geïnterpreteerd als de proportie extreme dalingen in de algemene aandelenkoers dat samengaat met extreme waardedalingen in de koers van individuele aandelen. Hoofdstuk 3 voorziet in een nieuwe manier om staartafhankelijkheid te berekenen. Wij laten zien dat de berekeningen die gebeuren bij het inschatten van een bepaald regressiemodel equivalent zijn aan de niet-parametrische schatters van staartafhankelijkheid. Een voordeel van onze methode is dat het relatief eenvoudig is om het regressiemodel verder uit te breiden om ook de staartafhankelijkheid tussen meer dan twee variabelen te meten. Een korte versie van dit hoofdstuk is gepubliceerd als Van Oordt and Zhou (2012).

Een ander belangrijk onderwerp in financieel risicomanagement is de diversificatie van beleggingen, zie bijvoorbeeld Samuelson (1967). Diversificatie speelt een belangrijke rol in het risicomanagement van grote financiële instellingen, zoals verzekeraars en banken. Hoewel het algemeen bekend is dat diversificatie meestal leidt tot een afname van het risico voor individuele instellingen, geniet de relatie tussen diversificatie en systeemrisico veel minder bekendheid. De theoretische literatuur met betrekking tot banken waarschuwt echter voor een schaduwzijde van diversificatie door financiële instellingen. Zo laten de studies van Shaffer (1994) en Wagner (2010) zien dat diversificatie in het algemeen leidt tot een toename van de kans dat verschillende instellingen gelijktijdig falen (en dus niet tot een afname). Het idee achter dit resultaat is dat financiële instellingen door diversificatie meer op elkaar gaan lijken, waardoor zij ook kwetsbaar worden voor dezelfde schokken. Diversificatie door financiële instellingen gaat daarom samen met een toename van het systeemrisico.

Hoofdstuk 4 geeft een theoretische discussie omtrent diversificatie door financiële instellingen in de context van securitisatie. De securitisatie van portefeuilles met leningen heeft een vogelvlucht genomen gedurende de afgelopen decennia. Een gevolg van securitisatie is dat de kosten om te beleggen in bepaalde typen leningen sterk zijn afgenomen. Door securitisatie wordt het mogelijk om als bank te beleggen in leningen in bepaalde sectoren of bepaalde regio's zonder daar een volledig leenbedrijf voor op te zetten. Zo kunnen Europese banken sinds de opkomst van securitisatie beleggen in hypotheekleningen in de Verenigde Staten door eenvoudigweg effecten te kopen waarin zulke leningen zijn ondergebracht. Securitisatie heeft de mogelijkheden voor diversificatie door financiële instellingen dus aanzienlijk vergroot. De vraag blijft of met securitisatie ook de bovengenoemde schaduwzijde van diversificatie is toegenomen. Hoofdstuk 4 is de eerste studie die aantoont waarom het bij het beantwoorden van deze vraag van essentieel belang is om rekening te houden met de praktijk om gebundelde leningen op te knippen in effecten die verschillen in de aansprakelijkheid voor eventuele verliezen. Dit hoofdstuk zal verschijnen als Van Oordt (2013).

Een ander onderwerp dat van groeiend belang is voor de analyse van financële stabiliteit zijn de zogenoemde 'stress testen'. Bij deze 'stress testen' probeert men te voorspellen wat er zou gebeuren met het financiële systeem of met individuele instellingen bij een bepaalde ongunstig verloop van gebeurtenissen. De relatie tussen economische groei en bankwinstgevendheid speelt een belangrijke rol in stress testen die worden gebruikt om te voorspellen wat er met de prestaties van banken gebeurt in ongunstige macro-economische scenario's. De meeste empirische studies vinden een relatief klein effect van economische groei op bankwinstgevendheid, wat suggereert dat bankwinsten nauwelijks worden getroffen door recessies. In Hoofdstuk 5 onderzoeken we of de relatie tussen economische groei en bankwinstgevendheid sterker is ten tijde van diepe recessies dan ten tijde van normale economische omstandigheden. Onze resultaten suggereren dat dit inderdaad het geval is. Het verband tussen economische groei en bankwinstgevendheid dat normaalgesproken vrij zwak is, is dus een stuk sterker ten tijde van ernstige recessies. De buffers die banken nodig hebben om ongeschonden door een diepe recessie te komen zijn dus eigenlijk hoger dan men zou verwachten op basis van het vrij zwakke verband ten tijde van normale economische omstandigheden. Dit hoofdstuk is gepubliceerd als Bolt *et al.* (2012).

Hoofdstuk 6 bestudeert de schokken in de prijzen van agrarische basisproducten. Grote prijsmutaties komen bij deze producten veel vaker voor dan gerechtvaardigd is op basis van conventionele statische kansverdelingen, zoals de normale verdeling. De prijsschokken van agrarische producten worden daarom ook wel 'dikstaartig' genoemd. Verklaringen voor de dikstaartigheid van de prijsschokken van agrarische basisproducten zijn echter maar dun gezaaid in de economische literatuur. Hoofdstuk 6 suggereert hoe deze dikstaartigheid kan ontstaan in een economisch systeem. In ons model verschijnen prijsschokken als gevolg van productiviteitsschokken in de agrarische sector. Een voorbeeld van zo'n productiviteitsschok is een periode van langdurige droogte. Uit ons model blijkt dat deze schokken kunnen leiden tot een dikstaartige verdeling voor de prijzen van agrarische basisproducten, zelfs als de oorspronkelijke schokken niet dikstaartig verdeeld zijn. Daarnaast laten we ook zien hoe een dikstaartige verdeling helpt om het prijsrisico van agrarische basisproducten beter te kwantificeren.

Hoofdstuk 7 bestudeert de samenhang van extreme prijsschokken in de termijnmarkten voor agrarische producten. Deze markten, die eenvoudig toegankelijk zijn voor speculanten, vertonen grote sprongen en dalingen van de ene dag op de andere, zie Hoofdstuk 6. Voor sommige combinaties van agrarische producten observeren we dat extreme sprongen en dalingen vaak op dezelfde dag samenvallen. In Hoofdstuk 7 stellen we de vraag wat de achterliggende reden is voor deze sterke 'staartafhankelijkheid' tussen sommige producten. Met behulp van oogstdata uit de Verenigde Staten meten we de mate waarin bepaalde agrarische producten in dezelfde gebieden verbouwd worden. Wij vinden dat deze maatstaf een meerderheid van de variatie in de staartafhankelijkheid kan verklaren. Dit resultaat suggereert dat er een achterliggende economische reden is die kan verklaren waarom de extreme prijsschokken van bepaalde agrarische producten op termijnmarkten zo sterk met elkaar samenhangen.

Bibliography

Acharya, V. (2009), A theory of systemic risk and design of prudential bank regulation, *Journal of Financial Stability*, **5**, 224–255.

Acharya, V. and T. Yorulmazer (2007), Too many to fail. An analysis of time-inconsistency in bank closure policies, *Journal of Financial Intermediation*, **16**, 1–31.

Adrian, T. and H. Shin (2010), Liquidity and leverage, *Journal of Financial Intermediation*, **19**, 418–437.

Ai, C., A. Chatrath, and F. Song (2006), On the comovement of commodity prices, *American Journal of Agricultural Economics*, **88**, 574–588.

Albertazzi, U. and L. Gambacorta (2009), Bank profitability and the business cycle, *Journal of Financial Stability*, **5**, 393–409.

Allen, F., A. Babus, and E. Carletti (2012), Asset commonality, debt maturity and systemic risk, *Journal of Financial Economics*, **104**, 519–534.

Allen, F. and E. Carletti (2006), Credit risk transfer and contagion, *Journal of Monetary Economics*, **53**, 89–111.

Andrews, D. (1993), Tests for parameter instability and structural change with unknown change point, *Econometrica*, **61**, 821–856.

Ang, A. and G. Bekaert (2002), International asset allocation with regime shifts, *Review of Financial Studies*, **15**, 1137–1187.

Ang, A. and J. Chen (2002), Asymmetric correlations of equity portfolios, *Journal of Financial Economics*, **63**, 443–494.

Ang, A., J. Chen, and Y. Xing (2006a), Downside risk, *Review of Financial Studies*, **19**, 1191–1239.

Ang, A., R. Hodrick, Y. Xing, and X. Zhang (2006b), The cross-section of volatility and expected returns, *Journal of Finance*, **61**, 259–299.

Ang, A., R. Hodrick, Y. Xing, and X. Zhang (2009), High idiosyncratic volatility and low returns: International and further U.S. evidence, *Journal of Financial Economics*, **91**, 1–23.

Ardeni, P. and J. Freebairn (2002), The macroeconomics of agriculture, in G. Rausser and B. Gardner (eds.), *Handbook of Agricultural Economics*, vol. 2, Elsevier Science B.V., pp. 1455–1485.

Arzac, E. and V. Bawa (1977), Portfolio choice and equilibrium in capital markets with safety-first investors, *Journal of Financial Economics*, **4**, 277–288.

Athanasoglou, P., S. Brissimis, and M. Delis (2008), Bank-specific, industry-specific and macroeconomic determinants of bank profitability, *Journal of International Financial Markets, Institutions and Money*, **18**, 121–136.

Atwood, J., S. Shaik, and M. Watts (2003), Are crop yields normally distributed? A reexamination, *American Journal of Agricultural Economics*, **85**, 888–901.

Balagtas, J. and M. Holt (2009), The commodity terms of trade, unit roots, and non-linear alternatives: A smooth transition approach, *American Journal of Agricultural Economics*, **91**, 87–105.

Bali, T., N. Cakici, and R. Whitelaw (2011), Hybrid tail risk and expected stock returns: When does the tail wag the dog?, *Working Paper*.

Bali, T., K. Demirtas, and H. Levy (2009), Is there an intertemporal relation between downside risk and expected returns?, *Journal of Financial and Quantitative Analysis*, **44**, 883–909.

Baltagi, B. and P. Wu (1999), Unequally spaced panel data regressions with AR (1) disturbances, *Econometric Theory*, **15**, 814–823.

Bates, D. (1991), The crash of ’87: Was it expected? The evidence from options markets, *Journal of Finance*, **46**, 1009–1044.

Bates, D. (2000), Post-’87 crash fears in the S&P 500 futures option market, *Journal of Econometrics*, **94**, 181–238.

Bawa, V. and E. Lindenberg (1977), Capital market equilibrium in a mean-lower partial moment framework, *Journal of Financial Economics*, **5**, 189–200.

Beine, M., A. Cosma, and R. Vermeulen (2010), The dark side of global integration: Increasing tail dependence, *Journal of Banking & Finance*, **34**, 184–192.

Berger, A. and G. Udell (2004), The institutional memory hypothesis and the procyclicality of bank lending behavior, *Journal of Financial Intermediation*, **13**, 458–495.

Bidarkota, P. and M. Crucini (2000), Commodity prices and the terms of trade, *Review of International Economics*, **8**, 647–666.

Bikker, J. and H. Hu (2002), Cyclical patterns in profits, provisioning and lending of banks and procyclicality of the new Basel capital requirements, *Banca Nazionale del Lavoro Quarterly Review*, **55**, 143–175.

Bikker, J. and P. Metzemakers (2005), Bank provisioning behaviour and procyclicality, *Journal of International Financial Markets, Institutions and Money*, **15**, 141–157.

Bollerslev, T. and V. Todorov (2011), Tails, fears, and risk premia, *Journal of Finance*, **6**, 2165–2211.

Bolt, W., L. De Haan, M. Hoeberichts, M. van Oordt, and J. Swank (2012), Bank profitability during recessions, *Journal of Banking & Finance*, **36**, 2552–2564.

Boot, A. and A. Thakor (1993), Security design, *Journal of Finance*, **48**, 1349–1378.

Bouvatier, V. and L. Lepetit (2008), Banks procyclical behavior: Does provisioning matter?, *Journal of International Financial Markets, Institutions and Money*, **18**, 513–526.

Brambor, T., W. Clark, and M. Golder (2006), Understanding interaction models: Improving empirical analyses, *Political Analysis*, **14**, 63–82.

Broadie, M., M. Chernov, and M. Johannes (2007), Model specification and risk premia: Evidence from futures options, *The Journal of Finance*, **62**, 1453–1490.

Byrne, J., G. Fazio, and N. Fiess (2013), Primary commodity prices: Co-movements, common factors and fundamentals, *Journal of Development Economics*, **101**, 16–26.

Calza, A., M. Manrique, and J. Sousa (2006), Credit in the euro area: An empirical investigation using aggregate data, *Quarterly Review of Economics and Finance*, **46**, 211–226.

Carhart, M. (1997), On persistence in mutual fund performance, *Journal of Finance*, **52**, 57–82.

Cashin, P., C. McDermott, and A. Scott (1999), The myth of comoving commodity prices, *IMF Working Papers*, **99**, 169.

Castrén, O., S. Dées, and F. Zaher (2010), Stress-testing euro area corporate default probabilities using a global macroeconomic model, *Journal of Financial Stability*, **6**, 64–78.

Cavallo, M. and G. Majnoni (2001), Do banks provision for bad loans in good times? Empirical evidence and policy implications, *World Bank Policy Research Paper*, **2619**.

Chiesa, G. (2008), Optimal credit risk transfer, monitored finance, and banks, *Journal of Financial Intermediation*, **17**, 464–477.

Cholette, L., V. de la Pena, and C. Lu (2012), International diversification: An extreme value approach, *Journal of Banking & Finance*, **36**, 871–885.

Cholette, L. and C. Lu (2011), The market premium for dynamic tail risk, *Working Paper*.

Coyle, B. (2007), Aggregation of price risk over commodities: An economic index number approach, *American Journal of Agricultural Economics*, **89**, 1085–1097.

Daniel, K. and S. Titman (1997), Evidence on the characteristics of cross sectional variation in stock returns, *Journal of Finance*, **52**, 1–33.

Danielsson, J., P. Hartmann, and C. de Vries (1998), The cost of conservatism: Extreme returns, Value-at-Risk, and the Basle 'multiplication factor', *Risk*, **11**, 101–103.

Danielsson, J., B. Jorgensen, M. Sarma, and C. De Vries (2006), Comparing downside risk measures for heavy tailed distributions, *Economics Letters*, **92**, 202–208.

Day, R. (1965), Probability distributions of field crop yields, *Journal of Farm Economics*, **47**, 713–741.

De Bondt, W. and R. Thaler (1985), Does the stock market overreact?, *Journal of Finance*, **40**, 793–805.

De Haan, L. (1970), *On regular variation and its application to the weak convergence of sample extremes*, Ph.D. thesis, Mathematisch Centrum Amsterdam, Amsterdam.

De Haan, L. and A. Ferreira (2006), *Extreme Value Theory: An Introduction*, Springer.

De Haan, L., D. Jansen, K. Koedijk, and C. de Vries (1994), Safety first portfolio selection, extreme value theory and long run asset risks, in J. Galambos (ed.), *Proceedings from a Conference on Extreme Value Theory and Applications*, Kluwer Academic: Boston.

De Jonghe, O. (2010), Back to the basics in banking? A micro-analysis of banking system stability, *Journal of Financial Intermediation*, **19**, 387–417.

De Roon, F., T. Nijman, and C. Veld (2000), Hedging pressure effects in futures markets, *Journal of Finance*, **55**, 1437–1456.

De Vries, C. (2005), The simple economics of bank fragility, *Journal of Banking & Finance*, **29**, 803–825.

Deaton, A. (1999), Commodity prices and growth in Africa, *Journal of Economic Perspectives*, **13**, 23–40.

Deaton, A. and G. Laroque (1992), On the behavior of commodity prices, *Review of Economic Studies*, **59**, 1–23.

Deaton, A. and G. Laroque (1996), Competitive storage and commodity price dynamics, *Journal of Political Economy*, **104**, 896–923.

Deaton, A. and G. Laroque (2003), A model of commodity prices after Sir Arthur Lewis, *Journal of Development Economics*, **71**, 289–310.

Deb, P., P. Trivedi, and P. Varangis (1998), The excess co-movement of commodity prices reconsidered, *Journal of Applied Econometrics*, **11**, 275–291.

DeMarzo, P. (2005), The pooling and tranching of securities: A model of informed intermediation, *Review of Financial Studies*, **18**, 1–35.

DeMarzo, P. and D. Duffie (1999), A liquidity-based model of security design, *Econometrica*, **67**, 65–99.

Demirgüç-Kunt, A. and H. Huizinga (1999), Determinants of commercial bank interest margins and profitability: Some international evidence, *World Bank Economic Review*, **13**, 379–408.

Dittmar, R. (2002), Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns, *Journal of Finance*, **57**, 369–403.

Dixit, A. and J. Stiglitz (1977), Monopolistic competition and optimum product diversity, *American Economic Review*, **67**, 297–308.

Drees, H. (2008), Some aspects of extreme value statistics under serial dependence, *Extremes*, **11**, 35–53.

Drees, H., L. De Haan, and S. Resnick (2000), How to make a Hill plot, *Annals of Statistics*, **28**, 254–274.

Duffee, G. and C. Zhou (2001), Credit derivatives in banking: Useful tools for managing risks?, *Journal of Monetary Economics*, **48**, 25–54.

Duffie, D., L. Saita, and K. Wang (2007), Multi-period corporate default prediction with stochastic covariates, *Journal of Financial Economics*, **83**, 635–665.

Embrechts, P., L. de Haan, and X. Huang (2000), Modelling multivariate extremes, in *Extremes and integrated risk management*, Risk Waters Group, pp. 59–67.

Embrechts, P., C. Klüppelberg, and T. Mikosch (1997), *Modelling extremal events: for insurance and finance*, Springer.

Fama, E. and K. French (1993), Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, **33**, 3–56.

Fama, E. and K. French (2008), Dissecting anomalies, *Journal of Finance*, **63**, 1653–1678.

Feller, W. (1971), *An Introduction to Probability Theory and Its Applications*, vol. 2, Wiley & Sons, New York.

Foos, D., L. Norden, and M. Weber (2010), Loan growth and riskiness of banks, *Journal of Banking & Finance*, **34**, 2929–2940.

Fu, F. (2009), Idiosyncratic risk and the cross-section of expected stock returns, *Journal of Financial Economics*, **91**, 24–37.

Gallagher, P. (1986), US corn yield capacity and probability: estimation and forecasting with nonsymmetric disturbances, *North Central Journal of Agricultural Economics*, **8**, 109–122.

Garita, G. and C. Zhou (2009), Can open capital markets help avoid currency crises?, *Working Paper*.

Gervais, S., R. Kaniel, and D. Mingelgrin (2001), The high-volume return premium, *Journal of Finance*, **56**, 877–919.

Giot, P. and S. Laurent (2003), Market risk in commodity markets: a VaR approach, *Energy Economics*, **25**, 435–457.

Gorton, G. and G. Pennacchi (1990), Financial intermediaries and liquidity creation, *Journal of Finance*, **45**, 49–71.

Gorton, G. and G. Pennacchi (1995), Banks and loan sales. Marketing nonmarketable assets, *Journal of Monetary Economics*, **35**, 389–411.

Gross, D. and N. Souleles (2002), An empirical analysis of personal bankruptcy and delinquency, *Review of Financial Studies*, **15**, 319–347.

Hakenes, H. and I. Schnabel (2010), Credit risk transfer and bank competition, *Journal of Financial Intermediation*, **19**, 308–332.

Hall, P., J. Horowitz, and B. Jing (1995), On blocking rules for the bootstrap with dependent data, *Biometrika*, **82**, 561–574.

Hansen, B. (1997), Approximate asymptotic p values for structural-change tests, *Journal of Business & Economic Statistics*, **15**, 60–67.

Harlow, W. and R. Rao (1989), Asset pricing in a generalized mean-lower partial moment framework: Theory and evidence, *Journal of Financial and Quantitative Analysis*, **24**, 285–311.

Harri, A., C. Erdem, K. Coble, and T. Knight (2009), Crop yield distributions: a reconciliation of previous research and statistical tests for normality., *Review of Agricultural Economics*, **31**, 163–182.

Hartmann, P., S. Straetmans, and C. de Vries (2004), Asset market linkages in crisis periods, *Review of Economics and Statistics*, **86**, 313–326.

Hartmann, P., S. Straetmans, and C. de Vries (2006), Banking system stability: A Cross-Atlantic perspective, in M. Carey and R. Stulz (eds.), *The Risks of Financial Institutions*, University of Chicago Press, pp. 133–192.

Hartmann, P., S. Straetmans, and C. de Vries (2010), Heavy tails and currency crises, *Journal of Empirical Finance*, **17**, 241–254.

Harvey, C. and A. Siddique (2000), Conditional skewness in asset pricing tests, *Journal of Finance*, **55**, 1263–1295.

Hill, B. (1975), A simple general approach to inference about the tail of a distribution, *Annals of Statistics*, **3**, 1163–1174.

Hill, J. (2009), On functional central limit theorems for dependent, heterogeneous arrays with applications to tail index and tail dependence estimation, *Journal of Statistical Planning and Inference*, **139**, 2091–2110.

Hsing, T. (1991), On tail index estimation using dependent data, *Annals of Statistics*, **19**, 1547–1569.

Huang, W., Q. Liu, G. Rhee, and F. Wu (2012), Extreme downside risk and expected stock returns, *Journal of Banking and Finance*, **36**, 1492–1502.

Ibragimov, R., D. Jaffee, and J. Walden (2011), Diversification disasters, *Journal of Financial Economics*, **99**, 333–348.

Ibragimov, R. and J. Walden (2007), The limits of diversification when losses may be large, *Journal of Banking & Finance*, **31**, 2551–2569.

Jacobson, T., J. Lindé, and K. Roszbach (2005), Exploring interactions between real activity and the financial stance, *Journal of Financial Stability*, **1**, 308–341.

Jansen, D. and C. De Vries (1991), On the frequency of large stock returns: Putting booms and busts into perspective, *Review of Economics and Statistics*, **73**, 18–24.

Jegadeesh, N. (1990), Evidence of predictable behavior of security returns, *Journal of Finance*, **45**, 881–898.

Jegadeesh, N. and S. Titman (1993), Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance*, **48**, 65–91.

Jiménez, G., S. Ongena, J. Peydró, and J. Saurina (2012), Credit supply and monetary policy: Identifying the bank balance-sheet channel with loan applications, *American Economic Review*, **102**, 2301–2326.

Just, R. and Q. Weninger (1999), Are crop yields normally distributed?, *American Journal of Agricultural Economics*, **81**, 287–304.

Kang, B., F. In, G. Kim, and T. Kim (2010), A longer look at the asymmetric dependence between hedge funds and the equity market, *Journal of Financial and Quantitative Analysis*, **45**, 763–789.

Kaniel, R., A. Ozoguz, and L. Starks (2012), The high volume return premium: Cross country evidence, *Journal of Financial Economics*, **103**, 255–279.

Karolyi, G. and R. Stulz (1996), Why do markets move together? An investigation of US-Japan stock return comovements, *Journal of Finance*, **51**, 951–986.

Kat, H. and R. Oomen (2006), What every investor should know about commodities I: A univariate return analysis, *Journal of Investment Management*, **5**, 16–40.

Kelly, B. (2011), Tail risk and asset prices, *Chicago Booth Research Paper, Fama-Miller Paper Series*, **11-17**.

Khieu, H., D. Mullineaux, and H. Yi (2011), The determinants of bank loan recovery rates, *Journal of Banking & Finance*, **36**, 923–933.

King, M. and S. Wadhwani (1990), Transmission of volatility between stock markets, *Review of Financial Studies*, **3**, 5–33.

Kofman, P. and C. De Vries (1990), Potato futures returns: A tail investigation, *Review of Futures Markets*, **8**, 244–258.

Koundouri, P. and N. Kourogenis (2011), On the distribution of crop yields: Does the central limit theorem apply?, *American Journal of Agricultural Economics*, **93**, 1341–1357.

Kraus, A. and R. Litzenberger (1976), Skewness preference and the valuation of risk assets, *Journal of Finance*, **31**, 1085–1100.

Krehbiel, T. and L. Adkins (2005), Price risk in the NYMEX energy complex: an extreme value approach, *Journal of Futures Markets*, **25**, 309–337.

Laeven, L. and G. Majnoni (2003), Loan loss provisioning and economic slowdowns: Too much, too late?, *Journal of Financial Intermediation*, **12**, 178–197.

Lehmann, B. (1990), Fads, martingales, and market efficiency, *Quarterly Journal of Economics*, **105**, 1–28.

Lence, S. (2009), Do futures benefit farmers?, *American Journal of Agricultural Economics*, **91**, 154–167.

Lescaroux, F. (2009), On the excess co-movement of commodity prices. A note about the role of fundamental factors in short-run dynamics, *Energy Policy*, **37**, 3906–3913.

Leybourne, S., T. Lloyd, and G. Reed (1994), The excess comovement of commodity prices revisited, *World Development*, **22**, 1747–1758.

Longin, F. and B. Solnik (1995), Is the correlation in international equity returns constant: 1960-1990?, *Journal of International Money and Finance*, **14**, 3–26.

Longin, F. and B. Solnik (2001), Extreme correlation of international equity markets, *Journal of Finance*, **56**, 649–676.

Loretan, M. and P. Phillips (1994), Testing the covariance stationarity of heavy-tailed time series: An overview of the theory with applications to several financial datasets, *Journal of Empirical Finance*, **1**, 211–248.

Louzis, D., A. Vouldis, and V. Metaxas (2012), Macroeconomic and bank-specific determinants of non-performing loans in Greece: A comparative study of mortgage, business and consumer loan portfolios, *Journal of Banking & Finance*, **36**, 1012–1027.

Lu, Y. and S. Neftci (2009), Financial instruments to hedge commodity price risk for developing countries, *Journal of Financial Transformation*, **24**, 137–143.

Mandelbrot, B. (1963a), New methods in statistical economics, *Journal of Political Economy*, **71**, 421–440.

Mandelbrot, B. (1963b), The variation of certain speculative prices, *Journal of Business*, **36**, 394–419.

Marcucci, J. and M. Quagliariello (2009), Asymmetric effects of the business cycle on bank credit risk, *Journal of Banking & Finance*, **33**, 1624–1635.

Matia, K., L. Amaral, S. Goodwin, and H. Stanley (2002), Different scaling behaviors of commodity spot and future prices, *Physical Review E*, **66**, 1–4.

Mills, F. (1927), *The behavior of prices*, National Bureau of Economic Research.

Morgan, W., J. Cotter, and K. Dowd (2012), Extreme measures of agricultural financial risk, *Journal of Agricultural Economics*, **63**, 65–82.

Morrison, A. (2005), Credit derivatives, disintermediation, and investment decisions, *Journal of Business*, **78**, 621–648.

Moss, C. and J. Shonkwiler (1993), Estimating yield distributions with a stochastic trend and nonnormal errors, *American Journal of Agricultural Economics*, **75**, 1056–1062.

Nelson, C. and P. Preckel (1989), The conditional beta distribution as a stochastic production function, *American Journal of Agricultural Economics*, **71**, 370–378.

Newman, M. (2005), Power laws, Pareto distributions and Zipf's law, *Contemporary Physics*, **46**, 323–351.

Pais, A. and P. Stork (2011), Contagion risk in the Australian banking and property sectors, *Journal of Banking & Finance*, **35**, 681–697.

Palaskas, T. and P. Varangis (1991), Is there excess co-movement of primary commodity prices? A co-integration test, *World Bank Working Paper Series*, **758**.

Parlour, C. and G. Plantin (2008), Loan sales and relationship banking, *Journal of Finance*, **63**, 1291–1314.

Pastor, L. and R. Stambaugh (2003), Liquidity risk and expected stock returns, *Journal of Political Economy*, **111**, 642–685.

Patton, A. (2004), On the out-of-sample importance of skewness and asymmetric dependence for asset allocation, *Journal of Financial Econometrics*, **2**, 130.

Pennacchi, G. (1988), Loan sales and the cost of bank capital, *Journal of Finance*, **43**, 375–396.

Pesaran, M., T. Schuermann, B. Treutler, and S. Weiner (2003), Macroeconomic dynamics and credit risk: A global perspective, *Journal of Money, Credit and Banking*, **38**, 1211–1261.

Pindyck, R. and J. Rotemberg (1990), The excess co-movement of commodity prices, *The Economic Journal*, **100**, 1173–1189.

Poon, S., M. Rockinger, and J. Tawn (2004), Extreme value dependence in financial markets: Diagnostics, models, and financial implications, *Review of Financial Studies*, **17**, 581–610.

Post, T. and P. Versijp (2007), Multivariate tests for stochastic dominance efficiency of a given portfolio, *Journal of Financial and Quantitative Analysis*, **42**, 489–515.

Price, K., B. Price, and T. Nantell (1982), Variance and lower partial moment measures of systematic risk: some analytical and empirical results, *Journal of Finance*, **37**, 843–855.

Quagliariello, M. (2007), Banks' riskiness over the business cycle: A panel analysis on Italian intermediaries, *Applied Financial Economics*, **17**, 119–138.

Ramchand, L. and R. Susmel (1998), Volatility and cross correlation across major stock markets, *Journal of Empirical Finance*, **5**, 397–416.

Ramírez, O. (1997), Estimation and use of a multivariate parametric model for simulating heteroskedastic, correlated, nonnormal random variables: The case of corn belt corn, soybean, and wheat yields, *American Journal of Agricultural Economics*, **79**, 191–205.

Ramírez, O., S. Misra, and J. Field (2003), Crop-yield distributions revisited, *American Journal of Agricultural Economics*, **85**, 108–120.

Riddiough, T. (1997), Optimal design and governance of asset-backed securities, *Journal of Financial Intermediation*, **6**, 121–152.

Rinaldi, L. and A. Sanchis-Arellano (2006), Household debt sustainability: what explains household non-performing loans? An empirical analysis, *ECB Working Paper*, **570**.

Roy, A. (1952), Safety first and the holding of assets, *Econometrica*, **20**, 431–449.

Rubinstein, M. (1973), The fundamental theorem of parameter-preference security valuation, *Journal of Financial and Quantitative Analysis*, **8**, 61–69.

Ruenzi, S. and F. Weigert (2012), Extreme dependence structures and the cross-section of expected stock returns, *Working Paper*.

Salas, V. and J. Saurina (2002), Credit risk in two institutional regimes: Spanish commercial and savings banks, *Journal of Financial Services Research*, **22**, 203–224.

Salvadori, G., C. De Michele, N. Kottegoda, and R. Rosso (2007), *Extremes in nature: An approach using copulas*, Springer-Verlag.

Samuelson, P. (1967), General proof that diversification pays, *Journal of Financial and Quantitative Analysis*, **2**, 1–13.

Schmidt, R. and U. Stadtmüller (2006), Non-parametric estimation of tail dependence, *Scandinavian Journal of Statistics*, **33**, 307–335.

Segoviano, M. and C. Goodhart (2009), Banking stability measures, *IMF Working Papers*, **09**, 4.

Shaffer, S. (1985), Counterproductive risk pooling, *Atlantic Economic Journal*, **13**, 99.

Shaffer, S. (1994), Pooling intensifies joint failure risk, *Research in Financial Services: Private and Public Policy*, **6**, 249–280.

Slijkerman, J., D. Schoenmaker, and C. de Vries (2013), Systemic risk and diversification across European banks and insurers, *Journal of Banking and Finance*, **37**, 773–785.

Sørensen, C., D. Marques-Ibáñez, and C. Rossi (2009), Modelling loans to non-financial corporations in the euro area, *ECB Working Paper*, **989**.

Spitzer, J. (2006), Market crashes, market booms and the cross-section of expected returns, *Working Paper*.

Straetmans, S., W. Verschoor, and C. Wolff (2008), Extreme US stock market fluctuations in the wake of 9/11, *Journal of Applied Econometrics*, **23**, 17–42.

Swank, J. (1999), A general equilibrium analysis of short-term interest rate policies and the role of reserve requirements, *Journal of Policy Modeling*, **21**, 389–411.

Telser, L. (1955), Safety first and hedging, *Review of Economic Studies*, **23**, 1–16.

Van Oordt, M. (2013), Securitization and the dark side of diversification, *Journal of Financial Intermediation*, **forthcoming**.

Van Oordt, M. and C. Zhou (2012), The simple econometrics of tail dependence, *Economics Letters*, **116**, 371–373.

Wagner, W. (2010), Diversification at financial institutions and systemic crises, *Journal of Financial Intermediation*, **19**, 373–386.

Wagner, W. and I. Marsh (2006), Credit risk transfer and financial sector stability, *Journal of Financial Stability*, **2**, 173–193.

Walsh, C. (2008), *Monetary Theory and Policy*, 3rd edn., M.I.T. Press.

Wang, D. and W. Tomek (2007), Commodity prices and unit root tests, *American Journal of Agricultural Economics*, **89**, 873–889.

Yang, X. and B. Heijdra (1993), Monopolistic competition and optimum product diversity: Comment, *American Economic Review*, **83**, 295–301.

Zhou, C. (2010), Are banks too big to fail? Measuring systemic importance of financial institutions, *International Journal of Central Banking*, **6**, 205–250.

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