

**The effects of decision flexibility**  
**in the hierarchical investment decision process**

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## REPORT SERIES *RESEARCH IN MANAGEMENT*

BIBLIOGRAPHIC DATA AND CLASSIFICATIONS			
Abstract	<p>Large institutional investors allocate their funds over a number of classes (e.g. equity, fixed income and real estate), various geographical regions and different industries. In practice, these allocation decisions are usually made in a hierarchical (top-down), consecutive way. At the higher decision level, the allocation is made on basis of benchmark portfolios (indexes). Such indexes are then set as targets for the lower levels. For example, at the top level the allocation decision is made on the basis of asset class benchmark indexes, on the second level the decisions are made on the basis of sector benchmark indexes, etc. Obviously, the lower levels have considerable flexibility to deviate from these targets. That is the reason why targets often come with limits on the maximally allowed deviation (or 'tracking error') from these targets. The potential consequences of deviations from the benchmark portfolios have received very little attention in the literature.</p> <p>In this paper, we discuss and illustrate this influence. The lower level tracking errors with respect to the benchmark indexes propagate to the top level. As a result the risk-return characteristics of the actual aggregate portfolio will be different from those of the initial benchmark-based portfolio. We illustrate this effect for a two level process to allocate funds over individual US stocks and sectors. We show that the benchmark allocation approaches used in practice yield inferior solutions when compared to a non-hierarchical approach where full information about individual lower level investment opportunities is available. Our results reveal that even small deviations from the benchmark portfolios can cause large shifts in the top-level risk-return space. This implies that the incorporation of lower level information in the initial top-level decision process will lead to a different (possibly better) allocation.</p>		
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## Abstract

Large institutional investors allocate their funds over a number of classes (e.g. equity, fixed income and real estate), various geographical regions and different industries. In practice, these allocation decisions are usually made in a hierarchical (top-down), consecutive way. At the higher decision level, the allocation is made on basis of benchmark portfolios (indexes). Such indexes are then set as targets for the lower levels. For example, at the top level the allocation decision is made on the basis of asset class benchmark indexes, on the second level the decisions are made on the basis of sector benchmark indexes, etc. Obviously, the lower levels have considerable flexibility to deviate from these targets. That is the reason why targets often come with limits on the maximally allowed deviation (or 'tracking error') from these targets. The potential consequences of deviations from the benchmark portfolios have received very little attention in the literature.

In this paper, we discuss and illustrate this influence. The lower level tracking errors with respect to the benchmark indexes propagate to the top level. As a result the risk-return characteristics of the actual aggregate portfolio will be different from those of the initial benchmark-based portfolio. We illustrate this effect for a two level process to allocate funds over individual US stocks and sectors. We show that the benchmark allocation approaches used in practice yield inferior solutions when compared to a non-hierarchical approach where full information about individual lower level investment opportunities is available. Our results reveal that even small deviations from the benchmark portfolios can cause large shifts in the top-level risk-return space. This implies that the incorporation of lower level information in the initial top-level decision process will lead to a different (possibly better) allocation.

## 1. Introduction

Large institutional investors allocate their funds over a number of asset classes (e.g. equity, fixed income and real estate), various geographical regions and different industries. In practice, these allocation decisions are usually made in a hierarchical (top-down), consecutive way. At the higher decision level, the allocation is made on basis of benchmark portfolios (indexes). Such indexes are then set as targets for the lower levels. For example, at the top level the allocation decision is made on the basis of asset class benchmark indexes, on the second level the decisions are made on the basis of sector benchmark indexes, etc. Obviously, the lower levels have considerable flexibility to deviate from these targets. That is the reason why targets often come with limits on the maximally allowed deviation from these targets. This deviation can be measured in two ways. Firstly in terms of differences between the weights of a benchmark and the actual portfolio; the allowed deviations in the weights are termed tactical asset allocation bands, TAA bands henceforth. Secondly, these deviations can be measured in terms of the standard deviation of the return differential between the benchmark and the actual portfolio; this is the statistical tracking error, see for example Grinold and Kahn [1999].

The potential consequences for the top-level portfolio as result of deviations from the benchmark portfolios at a lower level have received very little attention in the literature. To determine the lower level actions Baumol & Fabien [1964] used the Dantzig-Wolfe decomposition algorithm, which does not allow for genuine autonomous decision-making by lower-level management. A similar top-down planning method is also used by Saaty et al. [1981] to determine an optimal portfolio through hierarchies. In this paper, we discuss and illustrate the potential consequences of deviations from benchmark portfolios in a multi-level investment decision process. Ammann & Zimmermann [2001] investigated the relationship between statistical tracking error of a portfolio and tactical asset allocation bands. One of their findings is that deviations from the compositions of the benchmark portfolios at the lower level has a greater effect on top level portfolio characteristics than deviations from the weights of the benchmarks in the top level portfolio. Our analysis is more general in the sense that we do not only focus on the

changes in the standard deviation of the return on the top level portfolio (which defines the tracking error), but also on changes in the expected return of these top level portfolios. So we mould our analysis in risk-return space.

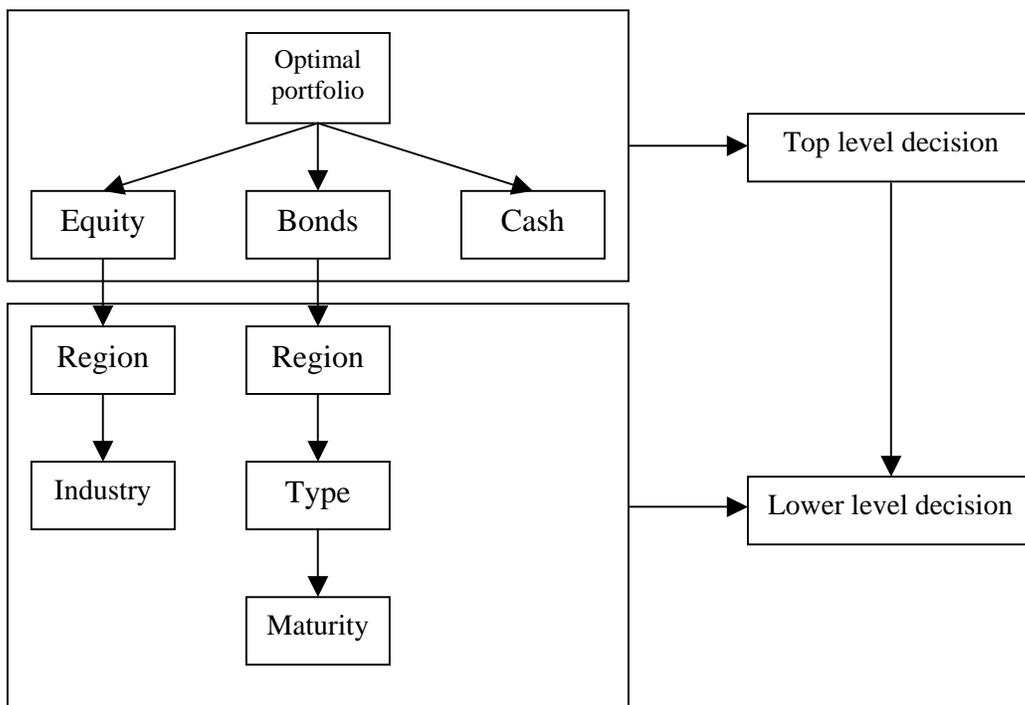
The lower level tracking errors with respect to the benchmark indexes propagate to the top level. As a result the risk-return characteristics of the actual aggregate portfolio will be different from those of the initial benchmark-based portfolio. We illustrate this effect for a two level decision process to allocate funds over individual US stocks and sectors. We show that the benchmark allocation approach used in practice yields inferior solutions when compared to a non-hierarchical approach where full information about individual lower level investment opportunities is available. Although this may not come as a surprise, our results reveal that even small deviations from the benchmark portfolios (small allowed TAA bands) can cause large shifts in the top-level risk-return space. This implies that the incorporation of lower level information in the initial top-level decision process will lead to a different (possibly better) allocation.

The structure of the paper is as follows. In the next section we outline the top-down investment decision structure and discuss the inherent problem in the level-per-level optimization. In section 3 we describe how we investigate the effects of decision flexibility. In section 4 we formulate the optimization models that we use to study the differences between the staged and non-staged portfolio decision problems. We investigate four portfolio decision problems using US equity data. The data set is described in section 5. We present and discuss the results from this study in section 6. Section 7 concludes the paper with a summary and outlines our plans for future research.

## **2. Top-Down Investment Structure**

The traditional institutional investment process has a top-down structure with managers at the top who decide the investment strategy in the (near) future. For example, based on the major equity and bond benchmark indices, they decide what proportion of their available funds should be invested in equities and how much in bonds. On the lower level

it is up to the local managers to find the particular portfolio of equities or bonds that satisfies the mandate given by the upper level. These portfolios may deviate from the benchmark portfolios; the flexibility in deviating from the benchmark is controlled by the tracking error constraints. These constraints may be formulated in terms of deviations from portfolio weights (the so-called Tactical Asset Allocation bands, TAA bands) or in terms of standard deviation of return differentials between the benchmark and the actual portfolio, i.e. the statistical tracking error. Figure 1 presents a schematic representation of the entire decision process.



**Figure 1:** An example of traditional multi-level Top-Down investment decision process.

The top-level decision in Figure 1 is based on the different benchmark indexes that are available for the different investment possibilities. In the equity case, the top-level investment decision is made on the basis of an equity benchmark index, for example the *Morgan Stanley Capital International World Index*. At the lower level, the portfolio managers can deviate from the benchmark composition by changing the allocation of funds of geographical regions and/or industry sectors. Likewise, in the bond case the top-level decision is made starting from an overall bond index. In the following decision step,

the managers on the lower level must decide in which country to invest and in what type of bond. Within the mandate provided by the top level, they must also decide on the maturity or duration profile of the portfolio. Local expertise is of invaluable importance in this kind of interregional investments. We stress that Figure 1 is only an illustration of a possible order of action.

### **3. Investigating the effect of decision flexibility**

In this paper we illustrate the potential consequences of decision flexibility in the hierarchical investment decision process. We do so by introducing a two-stage investment process in which a portfolio has to be selected from a total of 125 stocks stemming from 5 different sectors. We will define a utopia solution in which the portfolio is created in one step, thus assuming that all necessary information for this choice is available. Next we will define a benchmark solution in which first an allocation is made over the different sectors, based on the characteristics of the sector indexes, followed by allocation decisions within the sectors, assuming no decision flexibility to deviate from the sector index. (Actually we will be using two different benchmarks since we use two types of indexes: equally weighted indexes and capitalization weighted indexes.) The benchmark procedures are often used in practice, so that the comparison of the benchmark solutions with the utopia solution gives an impression of what decision flexibility might deliver. Next we introduce two simple two-stage investment procedures. The results of the latter are compared with both the utopia and the benchmark solutions.

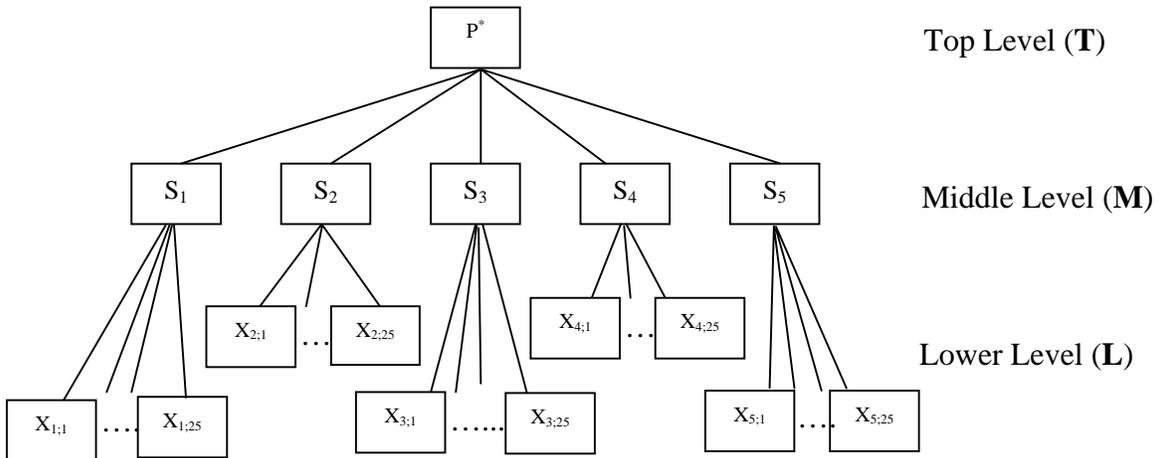
In this paper we use a one-period Markowitz [1959] mean-variance portfolio model (henceforth MV model), assuming that the investor's preferences can be represented by a preference functional defined over the mean and the variance of a portfolio's return,  $V(\mu_p, \sigma_p^2)$ . The expected return and variance on a portfolio are linear and quadratic functions of the weights of the individual stocks included in the portfolio:

$$\mu_p = \sum_{i=1}^N w_i \mu_i$$

$$\sigma^2 = \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij}$$

where  $\mu_p$  is the expected portfolio return and  $\sigma_p^2$  is the variance of portfolio  $p$ ,  $w_i$  represents the weight of security  $i$  in portfolio  $p$ ,  $\mu_i$  is the expected return on security  $i$  and  $\sigma_{ij}$  denotes the covariance between the returns on securities  $i$  and  $j$ . In the sequel of this paper we will choose expected excess returns as return measure. We have chosen for the MV model because it is widely known. However, other portfolio selection models could have been used just as well to show the effect of decision flexibility.

The structure of the assumed underlying hierarchical decision process is depicted in Figure 2.



**Figure 2:** The hierarchical structure in the investment decision process.

In Figure 2,  $P^*$  denotes the optimal portfolio at the top level,  $S_i$  represents sector  $i$  and  $X_{ij}$  is the  $j$ -th stock in sector  $i$ . Figure 2 is a slight simplification of the investment decision process illustrated in Figure 1, but the analogy still holds. Instead of multiple asset classes Figure 2 only shows the workings of the model in the one asset class case: equity. The entire framework can be easily extended to include other asset classes like bonds, real estate and cash.

Before going into technical details of the methodology we first introduce the following notations. Here we distinguish three decision levels: Top, Middle and Lower. In the lower level we consider  $N = 125$  individual stocks, allocated to  $S = 5$  sectors in the middle level. The weight of an individual stock  $i$  in the overall portfolio  $P$  at the top level is denoted by  $w_{LT,i}$ . Hence we have the budget restriction

$$\sum_{i=1}^{125} w_{LT,i} = 1$$

The weight of sector  $S$  in portfolio  $P$  is  $w_{MT,i}$ , hence:

$$\sum_{i=1}^5 w_{MT,i} = 1$$

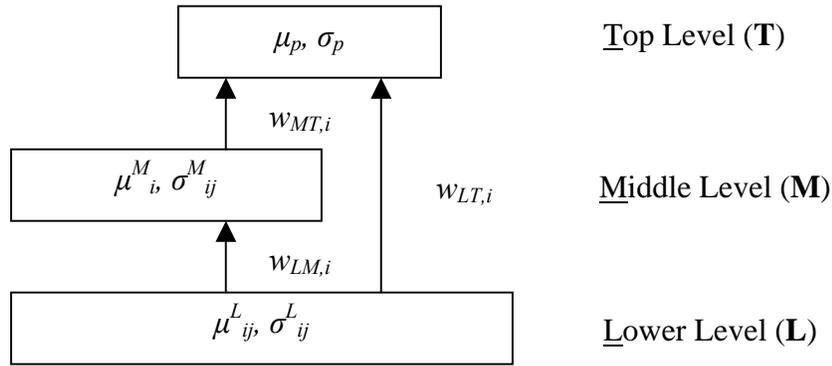
Finally, the weight of stock  $i$  in sector  $S$  is  $w_{LM,i}$ , hence:

$$\sum_{i=1}^{25} w_{LM,i} = 1$$

Therefore, the weight of stock  $i$  in  $P$  can be expressed as:

$$w_{LT,i} = w_{MT,i} \cdot w_{LM,i}$$

Figure 3 gives the schematic abstraction of the optimization processes in which the relevant notations are given in each level.



**Figure 3:** Schematic representation of the optimization processes.

$\mu_i^L$  is stock  $i$ 's expected excess return in the lower level and  $\sigma_{ij}^L$  is the covariance between stock  $i$  and  $j$ . When  $i = j$ , then  $\sigma_{ij}^L$  is stock  $i$ 's variance. In level (M) the same analogy applies. The difference between level (M) and level (L) is that the expected excess return vector contains sector returns instead of individual stock returns. The covariance matrix at the (M) level contains the sector (co-)variances. At the very top there is only expected portfolio excess return and variance.

#### 4. Finding Utopia and other portfolios

In the extreme case where the market is totally transparent and the top managers have detailed knowledge about the individual stocks, we have a single-stage optimization problem in which efforts are exerted to find the  $w_{LT,i}$  weights, i.e. the individual stock weights in the optimal portfolio. However, in practice it is difficult, if not impossible, for top management to have the full picture. Therefore the results obtained from this problem after optimization (being an efficient frontier) is a *utopian* one: in theory (i.e. assuming complete and reliable information) it cannot be surpassed in terms of performance, but it is unlikely to be realized in practice. In addition to the utopia case, we take as benchmark the fairly common practice in which top managers have only the aggregated sector information available to them. The top managers' optimization problem is then to choose  $w_{MT,i}$  for the different sectors, i.e. in which sector to invest in order to maximize excess

return while minimizing portfolio risk. So also this benchmark comes as an efficient frontier.

As one may expect (and as will be illustrated by our experiments) there is room between the utopia and the benchmark efficient frontiers. Given that the benchmark efficient frontier is derived while assuming no decision flexibility at the lower decision levels and assuming full flexibility in the utopia case, one may expect the utopia frontier to dominate the benchmark frontier. Obviously, allowing for tracking errors at the lower levels provides decision flexibility that might be used to shift the efficient frontier from the benchmark frontier in the direction of the utopia frontier. In this exploratory paper we propose two simple two-staged optimization procedures to get closer to the utopian frontier. However, *ex ante* we know that the results obtained using the staged optimization cannot excel or equal the utopian case. After all, the staged optimization case has additional constraints when compared to the utopian case. Referring to Figure 3, we here investigate two times four optimization problems: four in the case of capitalization-weighted indexes and another four in the case of equally weighted indexes:

1. The first problem is the utopian case in which a direct optimization is performed between the lower level and the top level in order to find the  $w_{LT,i}$  weights. There is no hierarchical decision process here. Top management has full and direct overview of all investment opportunities.
2. In problem 2 we keep  $w_{LM,i}$  fixed according to the benchmark weights selected (either equal- or market capitalization weights) and solve the optimization problem in order to find the optimal  $w_{MT,i}$  weights. We call this the benchmark case.
3. In the first staged optimization problem the first step is to find optimal  $w_{LM,i}$  weights between level (**L**) and (**M**) instead of using the equal- or market capitalization weights used in the benchmark case. Using these optimized  $w_{LM,i}$  weights we then view two sub cases in constructing optimal portfolios. First we use the  $w_{MT,i}$  weights found in the benchmark case to construct an optimal portfolio. This optimization is designated as staged optimization I.

4. In the last problem we also optimize between level **(M)** and **(T)** to find new  $w_{MT,i}$  weights that give an optimal portfolio based on the new sector returns. We call this staged optimization II.

The motivation for proposing two distinct two-staged optimization problems is that it allows us to quantify any allocation error when decisions are made using a top-down decision structure.

In the next two sections we describe the four multi-level optimizations in detail, both for the case in which capitalization weighted indexes are used and for the case where equally weighted portfolios are used.

#### 4.1 Capitalization Weighted portfolios

The returns on the capitalization weighted sector portfolios on the intermediate level are constructed by multiplying each stock's return with its capitalization weight ( $w_{cap}$ ). This cap weight is calculated by taking the average of that stock's market value divided by the sector market value over the past 10 years. The cross-sectional arithmetic average of the stock return is then the sector index return. We construct the efficient frontier of the benchmark by repeatedly minimizing the sectors' total risk for various levels of expected portfolio return. In other words we minimize the portfolio variance under the budget constraint and no short selling condition for various level of expected portfolio excess return  $R$ :

$$\begin{aligned}
 & \min \sum_{i=1}^5 \sum_{j=1}^5 w_{MT,i} w_{MT,j} \sigma_{ij}^m \\
 & \text{st. } \sum_{i=1}^5 w_{MT,i} = 1 \qquad \qquad \qquad \text{(CWBenchmark)} \\
 & \qquad 0 \leq w_{MT,i} \leq 1 \qquad \forall i \\
 & \qquad \sum_{i=1}^5 w_{MT,i} r_{M,i} = R
 \end{aligned}$$

Here  $r_{M,i}$  represents the expected excess return of sector  $i$ . Since the weight of an individual stock in each sector is fixed we allow the optimization between level (**M**) and (**T**) to be totally free. This control is very weak and as a consequence very unbalanced portfolios may result. But that is not a problem since an investor may actually choose to invest in a single sector. From the resulting efficient frontier we determine the optimal portfolio by using the Sharpe [1966, 1994] ratio.<sup>1</sup>

In the utopian case the top managers are aware of all the individual stocks' properties and will thus select the "best" when making investment decisions. The optimization problem then is a direct optimization between level (**L**) and level (**T**). The diversification effect is here at its biggest:

$$\begin{aligned}
 & \min \sum_{i=1}^{125} \sum_{j=1}^{125} w_{LT,i} w_{LT,j} \sigma_{ij}^L \\
 & \text{st. } \sum_{i=1}^{125} w_{LT,i} = 1 \\
 & \quad w_{cap} - 0.04 \leq w_{LT,i} \leq w_{cap} + 0.04 \quad \forall i \\
 & \quad w_{LT,i} \geq 0 \\
 & \quad \sum_{i=1}^{125} w_{LT,i} r_{L,i} = R
 \end{aligned} \tag{CWUtopia}$$

Here  $r_{L,i}$  represents the expected excess return of stock  $i$ . The tracking error introduced here is  $\pm 4\%$ : we may deviate from the historical capitalization weights by at most 4%. By using these  $\pm 4\%$  TAA bands we allow for some elbowroom in the optimization.

Theoretically we know that any solution emerging from (CWUtopia) signifies best results because only the most suitable stocks will be selected in this optimization process. However, as mentioned before it is difficult, if not impossible, in practice for the top management to have a full overview of the entire investment possibility space in different markets. Local expertise is an indispensable attribute in the asset allocation decision process. Also it is questionable whether the top managers have the time and energy to evaluate the entire investment possibility space. Thus the extreme situation in

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<sup>1</sup> Since we use excess returns throughout, the next-month expected return is the average excess return plus the average historical risk free rate minus the current one-month risk free rate.

which the top management directly chooses an investment portfolio from the entire collection of stocks is unrealistic.

Therefore we turn our attention to a staged optimization procedure and try to replicate the utopian results by decentralization. The idea here is that it is usually the lower level management that has detailed information about specific parts of the investment possibility space. Hence the first step in this staged optimization process is to find the (sub-) optimal portfolio for each individual sector at the middle level (**M**). The top management then chooses the global optimal portfolio from these optimized sector indices. However, instead of just looking at the new top portfolio optimized in stages, we also look at the portfolio that can be constructed using the weights from solving (CWBenchmark). In this way we can obtain an idea of the size of the possible asset allocation error that emerges when allocation decisions are based on the sub optimal information set:

$$\begin{aligned}
 & \min \sum_{i=1}^{25} \sum_{j=1}^{25} w_{LM,i} w_{LM,j} \sigma_{ij}^L \\
 & st. \quad \sum_{i=1}^{25} w_{LM,i} = 1 \\
 & \quad w_{cap} - 0.04 \leq w_{LM,i} \leq w_{cap} + 0.04 \quad \forall i \\
 & \quad w_{LM,i} \geq 0 \quad \forall i \\
 & \quad \sum_{i=1}^{25} w_{LM,i} r_{L,i} = R
 \end{aligned}
 \tag{CWStaged II}$$

Optimization between level (**L**) and level (**M**)

$$\begin{aligned}
 & \min \sum_{i=1}^5 \sum_{j=1}^5 w_{MT,i} w_{MT,j} \sigma_{ij}^{m'} \\
 & st. \quad \sum_{i=1}^5 w_{MT,i} = 1 \\
 & \quad 0 \leq w_{MT,i} \leq 1 \quad \forall i \\
 & \quad \sum_{i=1}^5 w_{MT,i} r_{M,i} = R
 \end{aligned}
 \tag{CWStaged I}$$

Optimization between level (**M**) and level (**T**)

Here we also introduce a tracking error of  $\pm 4\%$  in the lower level (**L**). The optimal portfolio in the lower level is again determined by maximizing the Sharpe ratio and the sector index is based on the optimized weights. This optimization procedure is a staged one in which diversification effects among stocks in different sectors are ignored.

## 4.2 Equally Weighted Portfolios

In order to see whether the effects illustrated in the last sub section depend on the starting weights in the portfolios, we repeat the entire experiment using totally different starting weights. The benchmark model here is constructed by setting each individual stock weight in the sector indexes at level (**M**) to 4% (since there are 25 stocks in each sector). The optimization problem between the top level (**T**) and intermediate level (**M**) remains the same as in the market value portfolio case and the efficient frontier that results by solving (EWBenchmark) for different level of portfolio excess returns  $R$  is then the equally weighted benchmark:

$$\begin{aligned}
 & \min \sum_{i=1}^5 \sum_{j=1}^5 w_{MT,i} w_{MT,j} \sigma_{ij}^m \\
 & \text{st. } \sum_{i=1}^5 w_{MT,i} = 1 \\
 & \quad 0 \leq w_{MT,i} \leq 1 \quad \forall i \\
 & \quad \sum_{i=1}^5 w_{MT,i} r_{M,i} = R
 \end{aligned} \tag{EWBenchmark}$$

For the utopian situation where the top level management has all the information, we introduced a domain space of 4% for each stock weight in the optimization problem:

$$\begin{aligned}
& \min \sum_{i=1}^{125} \sum_{j=1}^{125} w_{LT,i} w_{LT,j} \sigma_{ij}^L \\
& \text{st. } \sum_{i=1}^{125} w_{LT,i} = 1 \\
& \quad 0 \leq w_{LT,i} \leq 0.04 \quad \forall i \\
& \quad \sum_{i=1}^{125} w_{LT,i} r_{L,i} = R
\end{aligned} \tag{EWUtopia}$$

Here the maximum weight of an individual stock is set to 4% because an investor may want to invest in a single sector. By relaxing the individual stock weight in the top level portfolio to a maximum of 4% we can obtain a higher portfolio return than in the equally weighted case while keeping the portfolio risk at the same level. Intuition here is that instead of considering poor performing stocks, the investor can invest a big share of his wealth in the stocks with higher performance. Again the practical applicability of this utopian situation is questionable, we therefore next consider the staged optimization case.

In the staged optimization case some freedom of choices in the domain of the stock weights is introduced in the form of a TAA band. On the lower level, instead of a 4% fixed weight for each stock in the equally weighted sector index, we allow the weights to vary between 0% and 8%. The optimization problem in the top level remains the same as before. The advantage that results from decision freedom in the lower level can now be quantified:

$$\begin{aligned}
& \min \sum_{i=1}^{25} \sum_{j=1}^{25} w_{LM,i} w_{LM,j} \sigma_{ij}^L \\
& \text{st. } \sum_{i=1}^{25} w_{LM,i} = 1 \\
& \quad 0 \leq w_{LM,i} \leq 0.08 \quad \forall i \\
& \quad \sum_{i=1}^{25} w_{LM,i} r_{L,i} = R
\end{aligned} \tag{EWStaged II}$$

Optimization between level **(L)** and level **(M)**

Optimization between  
level (**M**) and level (**T**)

$$\begin{aligned}
 & \min \sum_{i=1}^5 \sum_{j=1}^5 w_{MT,i} w_{MT,j} \sigma_{ij}^m \\
 & \text{st. } \sum_{i=1}^5 w_{MT,i} = 1 \\
 & \quad 0 \leq w_{MT,i} \leq 1 \quad \forall i \\
 & \quad \sum_{i=1}^5 w_{MT,i} r_{M,i} = R
 \end{aligned}
 \tag{EWStaged I}$$

## 5. Data

From the DataStream database we extracted the monthly data of five U.S. sectors and the 3-month T-Bill rate over the period of December 1990 till May 2002. These sectors are Aerospace (AEROS), real estate development (RLDEV), food and drugs retailers (FDRET), oil (OIL) and life assurances (LIFEA). The data consist of the stocks' total return index and market value. The stocks in each sector have data that span the entire time interval. From each sector we randomly pick 25 stocks and using these stocks we construct five sector indices<sup>2</sup>. More detailed information concerning the data is available from the authors upon request.

## 6. Results

In this section we outline the potential consequences of flexibility in decision-making. Recall that we first created two benchmarks based on the market capitalization weights and equality weights of 125 stocks spread over 5 sectors. By minimizing the portfolio variance for various levels of portfolio excess return under the budget and no short selling constraints we constructed the efficient frontier of the benchmarks. In step two we assumed that the information set over the individual companies in the sample is openly available such that investment decisions can be directly taken based on this information

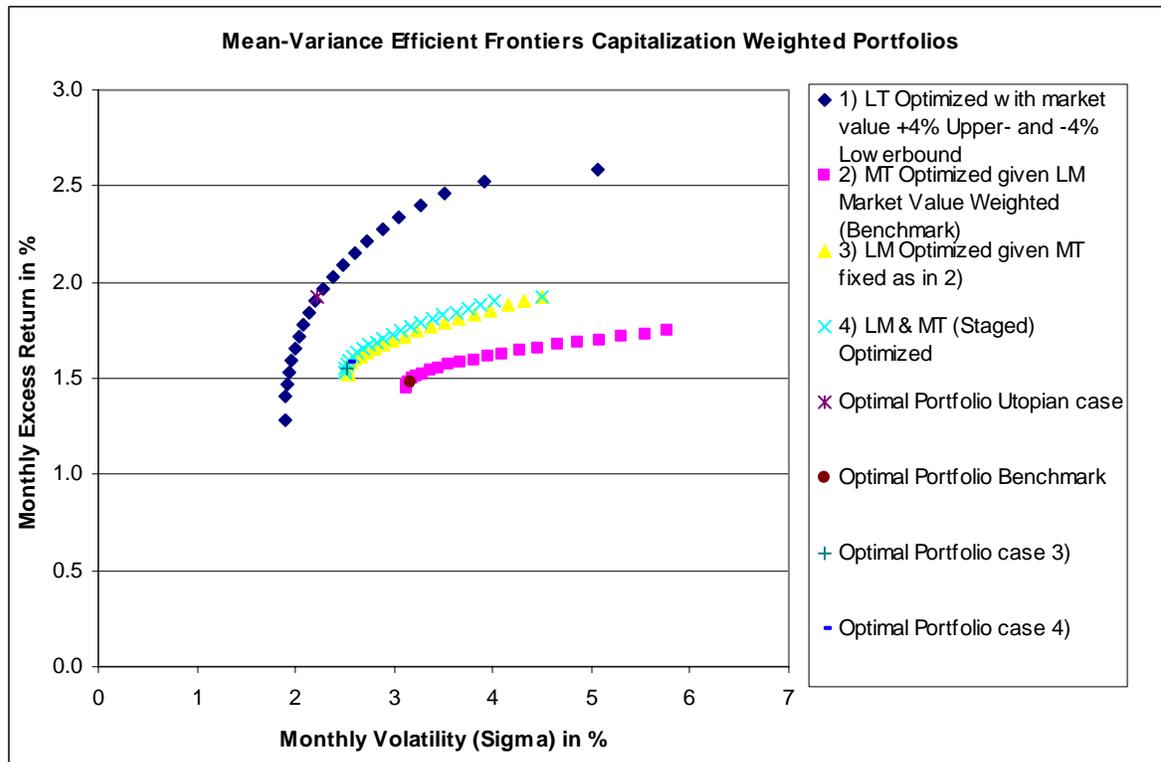
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<sup>2</sup> We realize that this procedure introduces survivorship bias. However, this does not concern us here since our only objective is to show that information aggregation may cause loss of opportunities.

set; hence the hierarchy in the decision structure is eliminated totally. We call this the utopian case since it is difficult to apply in practice. The discrepancy between the benchmark and utopia efficient frontiers can be viewed as room for the lower level managers to manoeuvre. We model this elbowroom by introducing a tracking error in the lower level. Using optimization in stages we mimic the utopian outcomes. In section 6.1 we present the results for the capitalization weighted benchmarks and in section 6.2 for the equally weighted benchmarks.

### 6.1 Results Capitalization-Weighted portfolios

Figure 4 gives the efficient frontiers of the four optimization cases with the market capitalization weighted portfolios as benchmark.



**Figure 4:** *MV efficient frontiers for the market capitalization weighted benchmarks.*

A first glance at Figure 4 confirms our prediction that the efficient frontiers of staged optimizations lie between the benchmark and the utopian frontiers. At closer inspection Figure 4 reveals that staged optimization improves the performance both in risk and return level with respect to the benchmark. Table 1 shows the annualized expected excess returns (Mu) and volatilities (Sigma) of the optimal portfolios for all the optimization cases. Also we provide the expected excess return for a specific level of volatility as well as the volatility for a specific level of expected excess return.

%	Annualized			
	Mu	Sigma	Mu   Sigma = 4	Sigma   Mu = 1.70
1)	23.05	7.68	30.31	7.02
2)	17.80	10.95	19.41	17.62
3)	18.63	8.71	22.32	10.37
4)	19.05	8.81	22.82	9.89

**Table 1:** Percentage expected excess return, volatility of the optimal portfolios, the expected excess return for certain level of volatility and volatility for some level  $\mu$  in the market capitalization weighted benchmark case.

Clearly, at the same level of risk the benchmark yields an inferior portfolio return and it is much riskier at the same level of return.

Another interesting point in Figure 4 is how close the staged optimizations' frontiers are situated with respect to each other. Clearly risk minimization at certain level of return *within* a sector has great influence on the portfolio selection process since the staged optimization efficient frontiers greatly surpass that of the benchmark. But from Figure 4 it seems that optimization *between* the sectors has little or negligible influence. This is consistent with the findings of Ammann & Zimmermann [2001]. We hypothesize that the reason for this can be found in the risk structure of the sectors. Table 2 gives the numerical details.

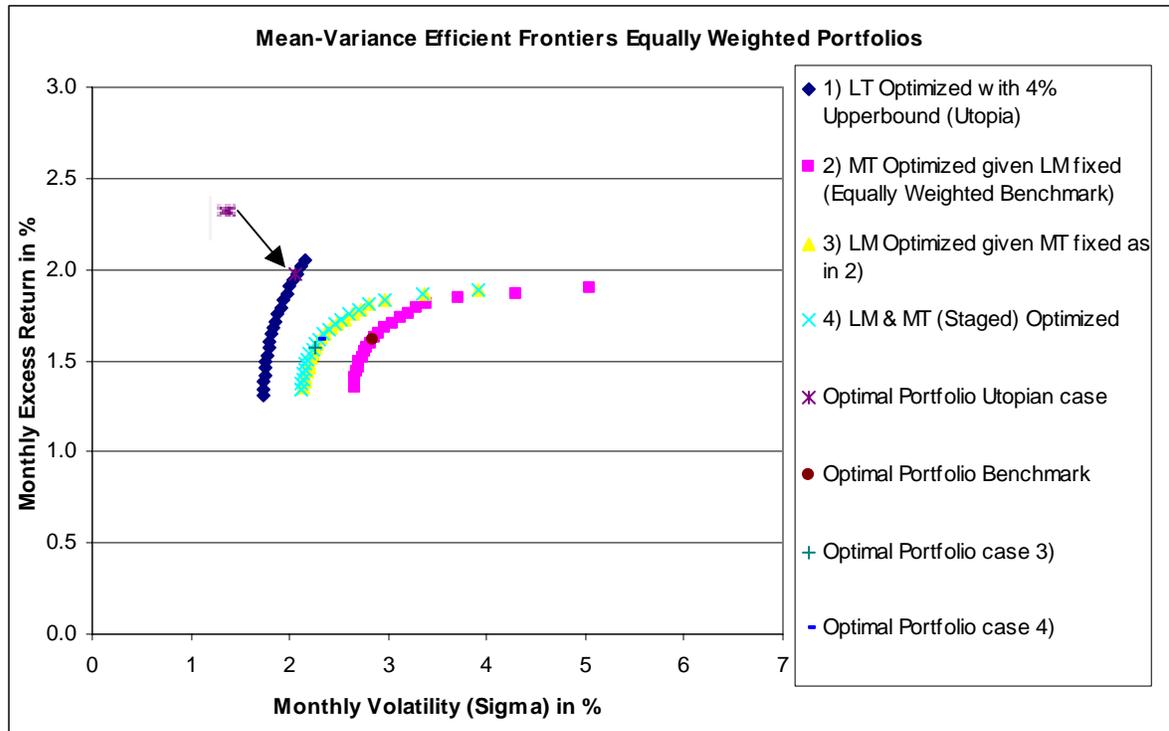
Average Excess Return	AEROS	FDRET	LIFEA	OIL	RLDEV
Benchmark	1.46	1.43	1.74	1.16	1.49
	6.42	4.88	5.78	7.54	3.44
Optimized	1.89	1.51	1.92	1.43	1.53
	5.56	3.81	4.50	7.54	2.68

**Table 2:** *Monthly percentage expected excess return and volatility of the market valued benchmark and the optimized sectors.*

Ranking the sectors' volatility in Table 2 in ascending order, we see that in the benchmark case the RLDEV sector has the lowest volatility and OIL the highest. In the optimized case the ranking order is exactly the same as in the benchmark case. Recall that the goal function in our optimization process is minimizing the volatility, thus it is not surprising that the optimal sector weights found in the benchmark case produce an efficient frontier similar to the optimized case.

## 6.2 Results Equally-Weighted portfolios

We repeat the experiment by replacing the capitalization weights in the benchmark by equal weights. Figure 5 presents the efficient frontiers in the equally weighted benchmark case. Here instead of using the Sharpe ratio to determine the optimal sector portfolio we purely look at portfolio with the same return as the historical sector average. Thus we are focusing on risk reduction through sector optimization.



**Figure 5:** *MV efficient frontiers for the equally weighted benchmark.*

Figure 5 shows similar results as Figure 4. Again we see that staged optimizations do provide better overall results than the benchmark. Also the staged optimization utilizes only part of the diversification effect that is associated with lower level decision freedom. Following this trend we observe that just like in Figure 4 the two staged optimization cases are almost identical in performance. Table 3 gives the numerical details and the expected excess return for certain level of volatility and volatility for some level of the optimal portfolios.

%	Annualized			
	Mu	Sigma	Mu   Sigma = 2.70	Sigma   Mu = 1.50
1)	23.65	7.16	N.A	6.12
2)	19.39	9.83	17.66	9.42
3)	18.82	7.81	21.28	7.67
4)	19.36	7.95	21.37	7.55

**Table 3:** *Percentage expected excess return and volatility of the optimal portfolios in case 2, 3 and 4.*

Again the benchmark results are clearly surpassed by the other portfolios.

Average Excess Return	AEROS	FDRET	LIFEA	OIL	RLDEV
Benchmark	1.89	0.95	1.77	1.49	1.40
	5.05	3.58	3.53	9.05	2.82
Optimized	1.89	0.95	1.77	1.49	1.40
	3.92	2.87	3.07	7.69	2.37

**Table 4:** Monthly percentage expected excess return and volatility of the equally weighted benchmark and the optimized sectors.

Just as before we believe that the similarity in performance of the two staged optimizations is due to the risk structure of the data set. Overall we conclude that allowing for decision flexibility greatly improves attainable performance. Reversely, restricting decision flexibility impairs attainable performance. Although the utopian case is really “utopian”, our results clearly reveal the influence of various levels of decision flexibility.

Both in the case of capitalization-weighted portfolios and in the case of equally weighted portfolios, the differences between the four types of portfolios are remarkable. However, the difference tends to increase with higher return levels. For instance, in Figure 4, fixing the return level at 1,5% yields a risk difference of around 1%. Fixing the return level at 2% gives a risk difference of over 2%.

Finally, we observe that all four equally weighted portfolios dominate each of the respective capitalization-weighted portfolios.

## 7. Conclusion

In practice large institutional investors allocate their funds over a number of asset classes, various geographical regions and different industries in a consecutive, hierarchical way. At the higher decision level, the allocation is often made on the basis of asset class benchmark indexes, on the second level the decisions are made on the basis of sector benchmark indexes, etc. Obviously, the lower levels have considerable flexibility to deviate from these targets, for example within tactical asset allocation bands. That is the

reason why targets often come with limits on the maximally allowed deviation from these targets.

In this paper we explored the possible consequences of allowing flexibility in the lower levels. In total four cases were considered in our experiment. We first constructed the utopian portfolio in which the hierarchical nature of the asset allocation process is totally eliminated; hence the optimal portfolio was constructed using information of all individual stocks, instead of using an index. The benchmark portfolios were constructed using either market capitalization weights or equal weights. In between the boundaries of the two extreme cases we experimented by introducing a tracking error in the lower level and consequently we improved the sector performance. With the optimized sector values we extended our experiment by using two sets of weights to construct the top optimal portfolio. First we use the weights obtained when the optimal portfolio was constructed using the fixed sector benchmark. Then we solved the optimization problem for the sectors again using the newly optimized sector index. In theory, the resulting portfolio must be the best portfolio that can be constructed. In this way we were able to evaluate the size of the allocation error of the top-down allocation process in mean-variance terms.

The results confirm that by introducing TAA bands the benchmark can easily be beaten in terms of performance. Also by eliminating decision hierarchy we see that the overall portfolio performance skyrocketed. However, complete elimination of hierarchy is such an impractical notion that it remains a truly utopian goal. The surprising part of the result is that the efficient frontiers of the two staged optimization cases lie very close to each other. This implies that the allocation error among sectors is very small or even negligible. We believe that this finding is not conclusive because it heavily depends on the specific correlation structure in the dataset.

Our next step in research is to extend the dataset to a more general asset allocation setting, to consider alternative tracking error specifications and to analyze the problem in an analytical framework. This may shed a more detailed light on the role of the correlation structure on the influence of decision flexibility. However, given the limitations of our current experiment, we are convinced that we have revealed the potential influence of various levels of decision flexibility. We hope that our findings

may serve as an eye opener for practitioners who as a standard apply hierarchical procedures and thereby tacitly – or unknowingly - accept their limitations.

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