Abstract

Providing African truck drivers with adequate access to healthcare is an effective way to reduce the burden and the spread of HIV and other infectious diseases. Therefore, NGO North Star Alliance builds a network of healthcare facilities along major African trucking routes. Choosing the locations of new facilities presents novel and complex optimization problems. This paper considers a general design problem: the Roadside Health Care Facility location Problem (RHFLP). RFHLP entails to select locations for new facilities and to choose for each of these facilities whether or not to add healthcare services for HIV, STIs, Tuberculosis, and/or Malaria to the standard health service package. The objective combines the maximization of the truck driver patient volume at these facilities and the maximization of the extent to which the truck drivers have continuous access to the needed health service packages. We present three measures for continuous access to health services by mobile patients and integrate these measures in a mixed-integer programming formulation for RHFLP. Moreover, we prove the RHFLP to be strongly NP-hard and derive analytical results for the worst-case effects of impreciseness in the input data. We show how large scale real life problem instances can be solved, presenting numerical experiments for the North-South corridor network (Southern and Eastern Africa) and discuss policy implications.

Keywords. roadside healthcare; truck drivers; facility location, continuous access

1 Introduction.

Over the past decades, long distance truck drivers in Sub-Saharan Africa have been extremely vulnerable to several diseases (Apostolopoulos and Sönmez 2007). Their lives are characterized by long separation from their spouses and social-cultural norms, difficult and dangerous working conditions, monotony, and loneliness (IOM/UNAIDS 2003, ILO 2005). This working environment has shown to be conducive to their engagement in high-risk sexual behaviors that are characterized by multiple sexual partnerships and low consistent condom use (Morris and Ferguson 2007, 2006, Nzyuko et al. 1997, Orubuloye et al. 1993). As a result, the prevalence rates of HIV and other Sexually Transmitted Infections (STIs) among the truck drivers and among their sex partners have been extremely high (Matovu and Ssebadduka 2012, Wilson 2005, Mbugua et al. 1995, Bwayo et al. 1994). In addition, the transport sector has been linked with several other communicable diseases, like Tuberculosis (TB) and Malaria (Apostolopoulos and Sönmez 2007).

This has brought about a number of problems. First, the extremely high prevalence of HIV, STIs, Malaria, and TB among the truck drivers has lead to correspondingly high rates of morbidity and mortality. Second, because of their high-risk sexual behavior, it has been suggested that truck drivers play a major role in the spread of HIV and other communicable diseases in Sub-Saharan Africa (Apostolopoulos and Sönmez 2007, Morris and Ferguson 2007, 2006, Laukamm-Josten et al. 2000, Caldwell et al. 1999, Hudson 1996). This role can for instance be illustrated by the facts that 56% of the 310 truck drivers included in a study along a South African highway were HIV positive, whereas 70% of them had wives and girlfriends in rural areas and only 13% of them had used condoms at the last sexual encounter (Ramjee and Gouws 2002). More recent studies confirm this pattern (Frank et al. 2013, Matovu and Ssebadduka 2012). Lastly, the burden and the spread of the high-impact diseases have a huge economic impact. The diseases have had impoverishing effects on patients, decrease labor productivity, and have slowed down economic growth (Rosen et al. 2007, World Bank 2006, Bates et al. 2004, Sachs and Malaney 2002, Ahlburg 2000, Needham et al. 1998). The prevalence of HIV among employees has lead to increased absenteeism and a lack of skilled staff, which in turn lead to rising production costs and declining company profits (Gatignon and Wassenhove 2008, Apostolopoulos and Sönmez 2007, Daly...
The NGO North Star Alliance (North Star) aims to remove these barriers that hinder access to healthcare. In the past years, the NGO located 30 primary healthcare facilities, called Roadside Wellness Centers (RWCs), at busy truck stops and border crossings along the major transport corridors in Sub-Saharan Africa. These RWCs provide truck drivers with a collection of basic health services, including condom distribution, behavior change communication (BCC), voluntary counseling and testing (VCT), and clinical services. The services are grouped into five service packages: Malaria care (MC), TB care (TC), HIV care (HC), STI care (SC), and primary care (PC). The PC package is offered at every RWC, whereas the others are optional (the costs of equipment, salary, and training make these packages expensive). The current network of RWCs provides many truck drivers with the needed health services. Nevertheless, it is far from serving all Sub-Saharan truck drivers who might benefit from the services provided. In addition, it does not provide continuous access to health service for the truck drivers it serves. That is, for many truck drivers, the coverage of the network of health facilities along their routes falls short of providing effective treatment. Therefore, further expansion of the RWC network is needed (and planned).

So far, North Star has been employing a pragmatic yet somewhat myopic expansion strategy. Locations for new RWCs were primarily selected based on the expected number of truck driver visits to an RWC at these locations, which we refer to as the patient volume criterion from now on. In so doing, North Star ignored the continuous access to healthcare for truck drivers. Whereas a greedy algorithm suffices to solve the location allocation problem North Star has so far considered, including the continuous access criterion makes the problem very complex (we will show that the problem formulation presented in this paper is strongly NP-hard).

This paper considers the problem of jointly choosing the locations of new facilities and allocating service packages among them, so as to maximize the patient volume at the new facilities and to maximize the number of truck drivers having continuous access to the needed health services. As scientific literature that defines continuous access for mobile patients is lacking, we present initial measures and models for this criterion. Next, we propose a mixed-integer programming formulation of the resulting novel location allocation problem. It differs from related flow covering problems (see, e.g., Averbakh and Berman 1996, Kuby and Lim 2005) in the sense that the degree of coverage (i.e., the extent to which a truck driver has continuous access to the service) depends on the travel time intervals between adjacent healthcare facilities along the truck driver’s route.

We prove that our problem formulation is strongly NP-hard and derive analytical results for the worst-case effects of impreciseness in the input data. In addition, we perform numerical experiments to test the performance of our model and to gain practical insights for decision makers. Our experiments show that large problem instances can be solved to optimality, and that the model promises to yield considerable improvement with respect to the location decisions taken by the NGO. Finally, we perform a sensitivity analysis to reveal the effects of modelling choices and data inaccuracy on the optimal solution.

This paper is not only relevant to Sub-Saharan Africa. Truck drivers have been reported to contract and spread many (sexually transmitted) diseases and/or have poor access to the needed health services in many countries. Examples include the USA (Lichtenstein et al. 2008, Solomon et al. 2004, Stratford et al. 2000), Brazil (Malta et al. 2006, Lacerda et al. 1997), the Baltic Region (Kulis et al. 2004), India (Pandey et al. 2008, Roa et al. 1997), and China (Wong et al. 2007, Chen et al. 2006). Moreover, the proposed model may also apply to a variety of related problems which locate facilities to serve moving demand, such as the positioning of refueling stations, convenience stores, ambulances, billboards, and detection or inspection stations (e.g., for hazardous vehicles).

The remainder of this paper is organized as follows. In section 2 we describe the problem in detail. This is followed by a description of the model in section 3. The results of the numerical experiments are described in section 4. Finally, in section 5 we summarize our findings and draw some conclusions.

2 Problem Description.

As scientific literature on healthcare facility location problems which strive to service mobile demand appears to be lacking (see Rahman and Smith (2000) for a review of healthcare facility location problems), we have first set out to collect relevant optimization criteria and model requirements by interviewing the NGO’s staff.
(medical staff, staff members from the global headquarter, and staff from a regional headquarter). Through these interviews we learned that the following two criteria are essential: (1) The expected patient volume at the new facilities. (2) The extent to which truck drivers have continuous access to the needed health services. More specifically, since a network can provide truck drivers with different levels of access to different service packages, we split up criterion (2) into multiple sub-criteria, referring to the objectives of ensuring continuous access to each of the service packages (in North Star’s case: PC, MC, TC, HC, SC).

The patient volume criterion relates to the purpose of serving a largest possible population of truck drivers. Optimization with respect to the second criterion refers to the health benefits attained for the truck drivers served, and their (sexual) network. Continuous access enables accessible, person-focused, coordinated, and continuous care (De Vries et al. 2014). These characteristics have been shown to lessen treatment delay and to stimulate treatment adherence, and thereby to decrease disease progression, disease transmission, and drug resistance (De Vries et al. 2014).

An extensive search did not reveal any scientific literature presenting models for continuous access to health care by mobile patients. In fact, commonly accepted definitions fitting all patients groups appear to be lacking, causing the modeling of continuous access to be far from straightforward. We therefore consulted health services experts and staff from North Star (Pinxten et al. 2013) to explore measures for continuous access as befitting the most prevalent diseases. Our discussions led to the classification of healthcare service packages into three different service-types.

**CTL. Services with a critical time-limit of access.**

This service type corresponds to diseases for which access within a time-limit from the moment of (self) diagnosis is crucial. The CTL service-type particularly applies to rapidly progressing diseases for which timely access reduces morbidity or may even be life saving (Kruk and Freedman 2008). For instance, access to antimalarial drugs within 24 hours of onset of symptoms is essential, and is therefore a commonly used health performance indicator (see, e.g., WHO/UNICEF 2003).

**RCTL. Services with a recommended time-limit and a critical time-limit of access.**

The CTL service-type implicitly assumes that one is either “too late” or “on time”. For many diseases however, responsiveness is less dichotomous. The RCTL service-type refers to health services for which a definition of being too late (i.e., a critical time-limit) and on time (i.e., a recommended time-limit) exist, but that are also characterized by a continuous relationship between accessibility and health outcomes. The existence of such *distance-decay relationship* has been confirmed in a variety of studies (see, e.g., Buor 2003, Humphreys and Smith 2009).

**ASAP. Services that need to be accessed as soon as possible after the moment that a truck driver decides that he needs them.**

The ASAP service-type corresponds to health services for which there exists a distance-decay relationship without a clear time-limit of access. For instance, though there does not exist a clear time-limit for diagnosing and treatment services for TB and HIV, a higher level of access is associated with less treatment delay and improved treatment adherence, which lead to lower mortality and morbidity, less drug resistance, and less disease transmission (De Vries et al. 2014).

### 3 Model.

In this section, we formally model the location allocation problem. Section 3.1 models the patient volume criterion and the continuous access criterion. Next, section 3.2 provides a mixed-integer programming (MIP) formulation of the problem. Finally, in section 3.3 we perform an analytical analysis of this model to get insight into the sensitivity of the optimal solution with respect to imprecision in the main input parameters.

Throughout this section, we model a problem instance by means of a graph $G(L, E)$. The set of vertices $L$ is indexed by $k$ and $l$, and is the union of the set of current facility locations, $KC$, the set of potential facility locations, $KP$, the set of truck route origins, $O$, and the set of truck route destinations, $D$. Together, $KC$ and $KP$ make up the total set of facility locations, denoted by $K$. Furthermore, $Q$ denotes the set of long distance truck flows $q$ that need to be provided with roadside healthcare services. A truck flow represents a collection of truck drivers who travel the same long distance route. These routes define paths in the graph: the path corresponding to flow $q$ is an edge progression (i.e., a sequence of roads) which defines an ordered set of vertices as follows. It consists of the start vertex $O_q \in O$, the set of facility locations that are along the path, $K_q \subseteq K$, and the end vertex, $D_q \in D$.

Parameter $p$ denotes the number of health facilities to be newly allocated, and parameters $p_s$ the number of service packages $s$ to be offered among these new RWCs. The set of health service packages is represented by $S$. We use the binary decision variable $x_k$ to indicate whether a health facility is placed at location $k$ ($x_k = 1$)
or not \((x_k = 0)\), and the binary decision variable \(y_{ks}\) to indicate whether service package \(s\) is offered at location \(k\) \((y_{ks} = 1)\) or not \((y_{ks} = 0)\). Finally, parameter \(c_{fks}\) equals 1 if the current roadside healthcare facility at location \(k\) offers service package \(s\) and equals 0 otherwise.

3.1 Optimization Criteria.

Let \(Z_{PV}\) denote the patient volume for a given solution \(\omega\), which specifies a network of roadside healthcare facilities and their service packages. We estimate \(Z_{PV}\) using the average daily number of truck drivers who spend the night at that location. (This number can be easily measured per location - also for locations where no facility is located yet.) We subsequently assume that the number of truck drivers that access a roadside healthcare facility, when available at a certain location, is proportional to the expected daily number of truck drivers who spend the night there. North Star confirms that this is a realistic assumption. Moreover, lacking further information about this relationship, we assume that this number of accessing truck drivers is independent of the service packages offered. Thus, we henceforth denote by \(d_k\) the (expected) patient volume at location \(k\) and derive that:

\[
Z_{PV} = \sum_{k \in K} d_k x_k
\]

For solution \(\omega\), \(Z_s\) denotes the extent to which the entire population of truck drivers who need package \(s\) has continuous access to this package. To measure \(Z_s\), we associate with each truck driver who needs this package a package \(s\) coverage score. This score ranges between 0 and 1, and indicates the level of continuous access to package \(s\) by the truck driver. Thus, for solution \(\omega\), \(Z_s\) equals the sum of the package \(s\) coverage scores of all truck drivers who need this package. Note that each truck driver belonging to the same flow \(q\) has the same level of access to package \(s\). Let \(c_{qs}\) denote the package \(s\) coverage score for each of the \(f_{qs}\) truck drivers in flow \(q\) who need this service package. Then it holds that:

\[
Z_s = \sum_{q \in Q} f_{qs} c_{qs}
\]

The remainder of this section focusses on the definition of \(c_{qs}\). Specifically, based on the observations described in section 2, we propose for each of the three service types (CTL, RCTL, and ASAP) a variable to measure continuous access. We start with explaining the intuition behind each of these variables. Afterwards, we make some assumptions to be able to calculate these variables and give their formal definition. At the end of this section we propose a generic way to transform values of these variables into values of \(c_{qs}\).

3.1.1 Measures of Continuous Access.

Figure 1 shows part of the time-line representing the trip of a truck driver. The boxes (with the cross) at the time-line represent the moments at which some service package \(s\) is passed. Suppose that this package is of type CTL. At a given point of time, we say that a truck driver is “covered” if the travel time to the next facility along his path that offers package \(s\) (i.e., his access time) is at most the critical time-limit. Obviously, the larger the part of the time-line the truck driver is covered, the better it is. Therefore, for service packages of type CTL, we measure continuous access as the fraction of time the truck driver is covered (see the upper part of Figure 1: the truck driver is covered in the light parts of the time-line).

Second, suppose that the package is of type RCTL. Now, we define that a truck driver is “covered” if his access time is at most the recommended time-limit. Furthermore, we say that he is “partially covered” if his access time is larger than the recommended time-limit, but smaller than the critical time-limit. To capture the distance-decay relationship (i.e., the relationship between health outcomes and access time), we define that the degree of coverage decreases (linearly) as the access time increases to the critical time-limit. Next, we measure continuous access as the average degree of coverage the truck driver has during his trip (see the middle part of Figure 1: the lower the truck driver’s degree of coverage, the darker the shade of grey).

Finally, suppose that the package is of type ASAP. Then, for each point of time during the truck drivers trip, it holds that the lower the access time, the better it is. Based on this observation, we measure continuous access as the average value of the truck driver’s access time during his trip (i.e., the average length of the arrow in the lower part of Figure 1).

3.1.2 Formal Definition of Measures of Continuous Access.

Let the set of service-types be represented by \(J = \{\text{CTL}, \text{RCTL}, \text{ASAP}\}\), and let \(S_j \subseteq S\) denote the set of service packages that are of service-type \(j\). In the previous subsection, we introduced for each \(j \in J\) a variable to measure continuous access to service packages \(s\) that are in \(S_j\). We denote this variable by \(a_{qs}^j\). Before we
propose its formal definition, we need to define the trip of a truck driver in flow $q$. We represent this trip by the following vector of vertices: $\pi_{qs} = [\pi_{qs}(1), \pi_{qs}(2), \ldots, \pi_{qs}(n-1), \pi_{qs}(n)]$. Here, $\pi_{qs}(1)$ and $\pi_{qs}(n)$ are the start vertex $O_q$ and the end vertex $D_q$, respectively. The vector $[\pi_{qs}(2), \ldots, \pi_{qs}(n-1)]$ is the sequence of package $s$ locations that are passed during a trip from $O_q$ to $D_q$. A package $s$ location is a location $k$ for which holds that $y_{ks} = 1$. North Star has confirmed that the following assumptions are generally met:

**Assumption 1.** A truck driver cyclically travels from $O_q$ to $D_q$ to $O_q$ to $D_q$ et cetera, as follows: $[\pi_{qs}(1), \pi_{qs}(2), \ldots, \pi_{qs}(n-1), \pi_{qs}(n), \pi_{qs}(n-1), \ldots, \pi_{qs}(2), \pi_{qs}(1), \pi_{qs}(2), \ldots]$ 

**Assumption 2.** A truck driver never changes the route because of illness

Let us consider the case that $|\pi_{qs}| \geq 3$. Because of assumptions 1 and 2, we know that a truck driver in flow $q$ always returns to $\pi_{qs}(1)$ after reaching $\pi_{qs}(n)$, and vice versa. We refer to the path from $\pi_{qs}(1)$ via $\pi_{qs}(n)$ to $\pi_{qs}(1)$ as the flow $q$ cycle. This cycle could be regarded as a set of trips between successively passed package $s$ locations. This set is given by: $\{\pi_{qs}(h) \rightarrow \pi_{qs}(h+1)|2 \leq h \leq n-2\} \cup \{\pi_{qs}(n-1) \rightarrow \pi_{qs}(n) \rightarrow \pi_{qs}(n-1)\} \cup \{\pi_{qs}(h+1) \rightarrow \pi_{qs}(h)|2 \leq h \leq n-2\} \cup \{\pi_{qs}(2) \rightarrow \pi_{qs}(1) \rightarrow \pi_{qs}(2)\}$. We refer to these trips as package $s$ cycle segments from now on. The durations of these trips are obtained from the parameters $t_{kl}$, which denote the travel time between two locations $k$ and $l$.

**Example 1.** For sake of conciseness, we omit the subscripts $q$ and $s$ in this example. Figure 2 describes the path of a truck driver travelling from origin $\pi(1)$ to destination $\pi(4)$. During his trip, he passes two package $s$ locations: $\pi(2)$ and $\pi(3)$. Hence, the set of package $s$ cycle segments is given by $\{\pi(2) \rightarrow \pi(3), \pi(3) \rightarrow \pi(4) \rightarrow \pi(3), \pi(3) \rightarrow \pi(2), \pi(2) \rightarrow \pi(1) \rightarrow \pi(2)\}$. Consequently, the set of travel time intervals between successively passed package $s$ locations are given by: $\{t_{\pi(2),\pi(3)}, t_{\pi(3),\pi(4)} + t_{\pi(4),\pi(3)}, t_{\pi(3),\pi(2)}, t_{\pi(2),\pi(1)} + t_{\pi(1),\pi(2)}\}$.

Using these definitions, we now define $a^C_{qs}$. First, let us consider the definition of $a^C_{qs}$. Let $T_q$ denote the duration of a complete trip along the flow $q$ cycle, and let $\tau^C_{qs}$ be the critical time-limit of accessing package $s$. Next, consider a given moment $t \in [0, T_q]$ during a truck driver’s trip along this cycle. Let $ats_{qs}(t)$ denote the truck driver’s access time: the travel time to the next package $s$ location he will pass. Furthermore, let $ats^{C}_{qs}$ denote the truck driver’s access time score at time $t$, which equals 1 if $ats_{qs}(t) \leq \tau^C_{qs}$, and equals 0 otherwise. Then $a_{qs}^{C}$, the fraction of time the truck driver is covered, is calculated as:

$$a_{qs}^{C} = \frac{1}{T_q} \int_{t=0}^{T_q} ats^{C}_{qs}(t) dt$$ (3)
Figure 3: Time-slots during which truck drivers are at most $\tau = 5$ time-units from the next package's location.

If $|\pi_q| = 2$, then $a_{qs}^{CTL}$ equals 0. Otherwise, the integral can be calculated per package $s$ cycle segment. For a segment $(k \rightarrow l)(k, l) \in K_q \times K_q)$, the total time a truck driver is covered is given by $t_{e_{kl}}^{CTL} = \min(t_{kl}, \tau_q^{CTL})$. Similarly, for the segments $k \rightarrow D_q \rightarrow k$ and $l \rightarrow O_q \rightarrow l$, the value of the integral is given by $t_{e_{kD_q}}^{CTL} = \min(t_{kD_q} + t_{D_q}, \tau_q^{CTL})$ and $t_{e_{O_q}}^{CTL} = \min(t_{O_q} + t_{O_q}, \tau_q^{CTL})$, respectively. Hence, $a_{qs}^{CTL}$ can be calculated as:

$$a_{qs}^{CTL} = \frac{1}{T_q} \left( t_{e_{\pi_q(1)\pi_q(2)}}^{CTL} + t_{e_{\pi_q(n-1)\pi_q(n)}}^{CTL} + \frac{\sum_{h=2}^{n-2} t_{e_{\pi_q(h)\pi_q(h+1)}}^{CTL}}{2} \right)$$

Example 1 (cont’d) (Example 1 (cont’d).) Let the travel times in the path introduced in Example 1 be $t_{\pi(1)\pi(2)} = t_{\pi(2)\pi(1)} = 4$, $t_{\pi(2)\pi(3)} = t_{\pi(3)\pi(2)} = 7$, $t_{\pi(3)\pi(4)} = t_{\pi(4)\pi(3)} = 3$. This implies that $T = 28$. Next, let $\tau$ be equal to 5. The light arcs in Figure 3 illustrate the time slots during which a truck driver is covered. For each of the segments, the duration of this time-slot equals 5. Therefore $a^{CTL} = \frac{20}{28}$.

Second, let us consider the definition of $a_{qs}^{RCTL}$. The recommended time-limit and the critical time-limit of access are denoted by $\tau_q^{RCTL}$ and $\tau_q^{RCTL}$, respectively. As defined in the previous subsection, the access time score $a_{qs}^{RCTL}(t)$ (i.e., the degree of coverage) increases linearly from 0 to 1 when the access time $a_{qs}(t)$ decreases from $\tau_q^{RCTL}$ to $\tau_q^{RCTL}$.

$$a_{qs}^{RCTL}(t) = \begin{cases} 0 & \text{if } a_{qs}(t) > \tau_q^{RCTL} \\ \frac{\tau_q^{RCTL} - a_{qs}(t)}{\tau_q^{RCTL} - \tau_q^{RCTL}} & \text{if } \tau_q^{RCTL} \leq a_{qs}(t) \leq \tau_q^{RCTL} \\ 1 & \text{if } a_{qs}(t) < \tau_q^{RCTL} \end{cases}$$

Next, $a_{qs}^{RCTL}$, the average value of the access time score during the truck driver’s trip, is calculated as:

$$a_{qs}^{RCTL} = \frac{1}{T_q} \int_{t=0}^{T_q} a_{qs}^{RCTL}(t)dt$$

Observe that $a_{qs}^{RCTL} = a_{qs}^{CTL}$ if $\tau_q^{RCTL} = \tau_q^{RCTL} = \tau_q^{RCTL}$. Again, $a_{qs}^{RCTL}$ equals 0 if $|\pi_q| = 2$. In case that $|\pi_q| \geq 3$, the integral can be calculated per package $s$ cycle segment. For the segment $(k \rightarrow l)(k, l) \in K_q \times K_q)$, the value of this integral is calculated as:

$$t_{e_{kl}}^{RCTL} = \begin{cases} t_{kl} & \text{if } t_{kl} < \tau_q^{RCTL} \\ t_{kl} - \frac{1}{2} \left( \frac{t_{kl} - \tau_q^{RCTL}}{\tau_q^{RCTL} - \tau_q^{RCTL}} \right)^2 & \text{if } \tau_q^{RCTL} \leq t_{kl} \leq \tau_q^{RCTL} \\ \tau_q^{RCTL} - \frac{1}{2} \left( \tau_q^{RCTL} - \frac{\tau_q^{RCTL}}{\tau_q^{RCTL}} \right)^2 & \text{if } t_{kl} > \tau_q^{RCTL} \end{cases}$$

Parameters $t_{e_{kD_q}}^{RCTL}$ and $t_{e_{O_q}}^{RCTL}$ denote the value of the integral for the segments $k \rightarrow D_q \rightarrow k$ and $l \rightarrow O_q \rightarrow l$, respectively. These values are obtained by replacing $t_{kl}$ in equation 7 by the duration. Using these parameters, $a_{qs}^{RCTL}$ can be calculated as:

$$a_{qs}^{RCTL} = \frac{1}{T_q} \left( t_{e_{\pi_q(1)\pi_q(2)}}^{RCTL} + t_{e_{\pi_q(n-1)\pi_q(n)}}^{RCTL} + \frac{\sum_{h=2}^{n-2} t_{e_{\pi_q(h)\pi_q(h+1)}}^{RCTL}}{2} \right)$$
Finally, let us consider the definition of $a_{qs}^{ASAP}$. As defined, $a_{qs}^{ASAP}$ denotes the average value of $at_{qs}(t)$ during the truck driver’s trip:

$$a_{qs}^{ASAP} = \frac{1}{T_q} \int_{t=0}^{T_q} at_{qs}(t) dt$$

(9)

It is possible to interpret $a_{qs}^{ASAP}$ as the expected travel time to the next package $s$ location from the moment a truck driver in flow $q$ decides that he needs this service package. This interpretation is valid under the assumption that the probability that this moment takes place is equal for each point of time during his trip. $a_{qs}^{ASAP}$ equals $\infty$ if $|\pi_{qs}|$ equals 2. For the case that $|\pi_{qs}| \geq 3$, De Vries (2011) showed that:

$$a_{qs}^{ASAP} = \frac{1}{2T_q} \left( (t_{\pi_{qs}(1),\pi_{qs}(2)} + t_{\pi_{qs}(2),\pi_{qs}(1)})^2 + (t_{\pi_{qs}(n-1),\pi_{qs}(n)} + t_{\pi_{qs}(n),\pi_{qs}(n-1)})^2 + \sum_{h=2}^{n-2} (t_{\pi_{qs}(h),\pi_{qs}(h+1)} + t_{\pi_{qs}(h+1),\pi_{qs}(h)}) \right)$$

(10)

Example 1 (cont’d). In our example, the average value of $at_{qs}(t)$ during a trip along the cycle is given by $\frac{\tau_1^{RCTL} + \tau_2^{RCTL}}{28} \approx 3.5$.

3.1.3 Relation between $c_{qs}$ and $a_{qs}^{i}$.

Now we have provided the definitions of $a_{qs}^{i}$, we need to show how to transform values of these variables into values of $c_{qs}$. We define $c_{qs}$ as a piecewise linear function $g_{qs}^i(\cdot)$ of $a_{qs}^{i}$. The rationale behind this choice is that a piecewise linear function provides decision makers with a flexible way to define the relationship between $c_{qs}$ and $a_{qs}^{i}$, and keeps the model linear. In case that $a_{qs}^{i}$ is to be maximized (i.e., when $j \in \{CTL,RCTL\}$), we define that $c_{qs}$ increases linearly from 0 to 1 when $a_{qs}^{i}$ increases from some lower bound threshold $\alpha L_{qs}^j$ to some upper bound threshold $\alpha U_{qs}^j$. In case that this variable is to be minimized (i.e., when $j = ASAP$), $c_{qs}$ decreases linearly from 1 to 0 when $a_{qs}^{i}$ increases from some lower bound threshold $\alpha L_{qs}^j$ to some upper bound threshold $\alpha U_{qs}^j$.

3.2 Mixed Integer Programming Formulation.

The model presented in section 3.1 allows us to formulate the location allocation problem as a multi-objective mixed-integer linear programming problem. The parameters $w_{PV}$ and $w_s$ denote the weights assigned to $Z_{PV}$ and $Z_s$, respectively. We refer to the problem formulation presented next as the Roadside Healthcare Facility Location Problem (RHFLP).
calculated by means of the following auxiliary (linear) constraints:

\[ \text{maximize } Z = w_{PV} \sum_{k \in K} d_k x_k + \sum_{s \in S} w_s \sum_{q \in Q} f_{qs} v_{qs} \]  

s.t. \[ c_{qs} = g_{qs}^l(a_{qs}^l) \quad q \in Q, j \in J, s \in S_j \]  

\[ \sum_{k \in KP} x_k = p \]  

\[ \sum_{k \in KP} y_{ks} = p_s \quad s \in S \]  

\[ x_k \geq y_{ks} \quad k \in K, s \in S \]  

\[ x_k = 1 \quad k \in KC \]  

\[ y_{ks} = cf_{ks} \quad k \in KC, s \in S \]  

\[ x_k, y_{ks} \in \{0, 1\} \quad k \in K, s \in S \]  

The objective function 11 maximizes a weighted sum of \( Z_{PV} \) and \( Z_s \). The package \( s \) coverage scores are defined in constraint 12. We elaborate on this constraint later. Constraints 13 and 14 impose the number of new facilities and the number of service packages \( s \) to be allocated to a potential facility location. Next, constraint 15 stipulates that such service package can only be allocated to location \( k \) if a health facility is located there. The current network of health facilities is described in constraints 16 and 17. Last, constraint 18 defines our decision variables as binary variables.

The RHFLP can be regarded as a facility location problem which balances the objectives to maximize the total node demand covered by the facilities (i.e., patient volume), and to maximize the total flow demand covered by the facilities (i.e., truck drivers who need continuous access to a given service). The second objective makes the RHFLP a multi-coverage flow interception facility location problem (FIFLP). Namely, it is (generally) beneficial to serve a truck driver by multiple facilities. To our knowledge, there are only two other multi-coverage FIFLPs: the billboard location problem (Averbakh and Berman 1996) and the flow refueling location problem (Kuby and Lim 2005). Though these problems show much similarity to the RHFLP, there are significant differences. The RHFLP defines that truck drivers are covered to some degree, and that this degree of coverage depends on the travel time intervals between adjacent facilities along his route. Instead, the billboard location problem defines the degree of coverage based on the number of billboards along a route. The flow refueling location problem does consider the driving times between adjacent facilities, but differs from the RHFLP because it uses a binary coverage definition (i.e., vehicles are either covered or not). For a comprehensive literature review on related facility location problems, we refer to De Vries (2011).

To linearize constraint 12, let us re-encode the vector \( \pi_{qs} \) by introducing the variables \( i_{klqs} \). This variable equals 1 if location \( l \) is the immediate successor of location \( k \) in vector \( \pi_{qs} \) and equals 0 otherwise. Constraints 19-23, which can be regarded as flow conservation constraints, ensure that \( i_{klqs} \) takes the values as defined above. The proof is given by De Vries (2011). In these constraints, \( L_q \) denotes the total set of vertices in path \( q \) (i.e., \( L_q = \{O_q \cup K_q \cup D_q\} \)), and \( L_{kq} \) is the set of locations that are passed after passing location \( k \) during a trip from \( O_q \) to \( D_q \).

\[ \sum_{l \in L_q} i_{klqs} = y_{ks} \quad q \in Q, k \in K_q, s \in S \]  

\[ \sum_{l \in L_q} i_{klqs} = 1 \quad q \in Q, k \in O_q, s \in S \]  

\[ \sum_{k \in L_q} i_{klqs} = y_{ls} \quad q \in Q, l \in K_q, s \in S \]  

\[ \sum_{k \in L_q} i_{klqs} = 1 \quad q \in Q, l \in D_q, s \in S \]  

\[ i_{klqs} \in [0, 1] \quad q \in Q, k \in L_q, l \in L_{kq}, s \in S \]  

Note that constraint 23 defines \( i_{klqs} \) as continuous variables. Using these variables, the value of \( a_{qs}^l \) is calculated by means of the following auxiliary (linear) constraints:
\[ a_{qs}^{CTL} = \frac{1}{T_q} \left( \sum_{l \in K_q} i_{O_l} q_{ls} t_c^{CTL}_{O_l} + \sum_{k \in D_q} i_{kD_q} q_{ls} t_c^{CTL}_{kD_q} + \sum_{k \in K_q} \sum_{l \in K_q} i_{klq} (t_c^{CTL}_{kl} + t_c^{CTL}_{l_k}) \right) \] 

\[ a_{qs}^{RCTL} = \frac{1}{T_q} \left( \sum_{l \in K_q} i_{O_l} q_{ls} t_c^{RCTL}_{O_l} + \sum_{k \in D_q} i_{kD_q} q_{ls} t_c^{RCTL}_{kD_q} + \sum_{k \in K_q} \sum_{l \in K_q} i_{klq} (t_c^{RCTL}_{kl} + t_c^{RCTL}_{l_k}) \right) \] 

\[ a_{qs}^{ASAP} = \frac{1}{2T_q} \left( \sum_{l \in K_q} i_{O_l} q_{ls} (t_{O_l} + t_{D_q})^2 + \sum_{k \in D_q} i_{kD_q} q_{ls} (t_{kD_q} + t_{D_k})^2 \right) + \sum_{k \in K_q} \sum_{l \in K_q} i_{klq} (t_{l_k}^2 + t_{k_l}^2) \] 

\[ + i_{O_s} d_{qs} M \] 

Here, \( M \) is a large number that sets \( a_{qs}^{ASAP} \) to a large constant if service package \( s \) is not offered along the path of flow \( q \). Namely, constraints 20 and 22 impose that \( i_{O_s} d_{qs} = 1 \) if this is the case. The set of auxiliary (linear) constraints that model \( c_{qs} \), as a piecewise linear function of \( a_{qs} \), can be formulated using the so-called Lambda Method (Lee and Wilson 2001). These constraints are given in Appendix A.

### 3.3 Worst-case effect of impreciseness in \( f_{qs}, d_k, w_{PV} \), and \( w_s \).

The model described in section 3.2 makes use of the parameters \( f_{qs} \) and \( d_k \), which tend to be imprecise in the data-scarce environment North Star operates in. Though this does not affect the feasible solution space, it does affect the value of each solution. As a result, the optimal solution found using the presently used parameter values may deviate from the “true” optimal solution.

Proposition 1 provides an upper bound on the effects of incorrect values of the parameters \( f_{qs} \) and \( d_k \) on the optimal solution (see Appendix B for the proof). Here, \( \omega^\# \) denotes the true optimal solution and \( \omega^* \) the optimal solution based on the presently used parameter values. We denote the value function resulting from using the true parameters by \( v^\# \), and the value function using the presently used parameters by \( v^* \). For instance, \( v^\#(\omega^*) \) denotes the value of the solution \( \omega^* \) in case that the true parameter values were used.

**Proposition 1.** If the true values of \( f_{qs} \) and \( d_k \) deviate at most a fraction \( \delta \) from the presently used parameter values, then \( v^\#(\omega^*) \leq \frac{1+\delta}{1-\delta} v^*(\omega^*) \). This bound is tight.

To illustrate this bound, let \( \delta \) be equal to 0.1. Then the true value of \( \omega^* \) is at most 22% lower than the true value of \( \omega^\# \). This bound is only attained with equality under two very specific circumstances. First, \( v^*(\omega^*) = v^*(\omega^\#) \). Second, \( K^* \cap K^\# = \emptyset \) and \( Q^* \cap Q^\# = \emptyset \), where \( K^* = \{k|x_k = 1, \omega^*\}, K^\# = \{k|x_k = 1, \omega^\#\}, Q^* = \{q|c_{qs} > 0, \omega^*\} \) and \( Q^\# = \{q|c_{qs} > 0, \omega^\#\} \). Generally this second condition is not met, since it is likely that the solutions \( \omega^* \) and \( \omega^\# \) both locate facilities at the busiest truck stops (i.e., \( K^* \cap K^\# \neq \emptyset \)), and provide the busiest truck flows with continuous access to service package \( s \) (i.e., \( Q^* \cap Q^\# \neq \emptyset \)).

Given that North Star’s ultimate objective is to maximize the total health benefits gained by the truck drivers, the objective weights \( w_{PV} \), and \( w_s \) should indicate the amount of health benefits that are gained when the sub-scores \( Z_{PV} \) and \( Z_s \) increase by 1, respectively. However, reaching the objectives seems to bring about many types of health benefits that are hard to quantify (e.g., less transmission, less drug resistance). This shows that it is likely that the weights are imprecise. Let \( OB \) denote the set of objectives \( ob \), and let \( w_{ob}^* \) denote the presently used objective weight for objective \( ob \). Next, \( Z_{ob} \) denotes a lower bound on the sub-score \( Z_{ob} \) (e.g., the value of \( Z_{ob} \) in the network without the new facilities). Furthermore, we define parameter \( \gamma \) as follows (note: \( \gamma \leq 1 \)):

\[ \gamma = \frac{\sum_{ob \in OB} w_{ob} Z_{ob}}{v^*(\omega^*)} \]  

An upper bound on the effects of impreciseness in the objective weights on the optimal solution is given in proposition 2 (see Appendix C for the proof).

**Proposition 2.** If the true values of \( w_{PV} \), and \( w_s \) deviate at most a fraction \( \delta \) from the presently used parameter values, then \( v^\#(\omega^*) \leq \frac{(1+\delta)^2}{(1-\delta)(1+\delta+2\gamma)} v^*(\omega^*) \).

For example, if the value of the optimal solution obtained using the “incorrect” parameter values is twice as high as the lower bound on the optimal solution (i.e., if \( \gamma = 0.5 \)), and if \( \delta = 0.1 \), then the true value of of this solution is at most 12% lower than the true value of \( \omega^\# \).
4 Performance Analysis.

This section illustrates how our model can be used for strategic planning and reveals practical insights for decision makers. Section 4.1 describes our baseline case study, and shows the solution to a problem instance. The RHFLP is a strongly $\mathcal{NP}$-hard problem, even in case that $|S| = 1$ (see Appendix D for the proof). In section 4.2, we apply our model to a variety of cases to get more insight in the tractability of the model. Finally, section 4.3 explores how changes in the objective weights and in the parameter $p$ affect the optimal solution.

4.1 Baseline Case Description and Results.

We base our test case on the North-South Corridor (NSC) network in Sub-Saharan Africa. The required data are obtained from North Star (current and some of the potential RWC locations), several of its partner organizations (road network, truck routes, and travel times), public resources (disease prevalence and incidence data for HIV (Frank et al. 2013, Azuonwu et al. 2011, Ramjee and Gouws 2002, Mbugua et al. 1995, Bwayo et al. 1994), for TB (WHO 2013), for STIs (Matovu and Ssebadduka 2012, Morris and Ferguson 2007, Jackson et al. 1997), and for Malaria (WHO 2012)), and expert opinion (demand data estimates (Hontelez 2013) and model parameters (Pinxten et al. 2013)). We make use of the modeling platform AIMMS for the implementation of the MIP model, and use CPLEX 12.5 as the solver engine, on a PC with a 2.4 GHz Intel Core i5 processor and 6 GB RAM.

The NSC road network spans 8 Sub-Saharan countries and 10647 kilometers of road (TradeMark Southern Africa 2013). It connects the main ports in East and Southern Africa with the main inland cities and other areas of economic importance (e.g., the Copper Belt in DR Congo). The roads are densely populated by long-distance trucks. It is estimated that the daily number of vehicle-kilometers by medium and heavy load trucks at this road network approaches 13 million and that every day over 19 thousand medium and heavy load trucks pass one of the 12 main country border crossings in this network (we derive these estimates from the data presented in the report by Odoki et al. (2009)).

Figure 5 depicts the NSC network as a graph. The set of nodes consists of 19 $O-D$ nodes, 12 current RWC locations, and 59 potential RWC locations. The latter are strategic locations like border crossings, harbors, crossroads, and large cities. The edges in the network are the roads connecting these 90 locations. We consider 30 long distance truck flows and assume that truck drivers take the shortest path from the origin to the destination.

Patient volume data $d_k$ and service package needs per flow $f_{qs}$ data are partly incomplete. Therefore, we generate the parameters such that the resulting scenario reflects the trade-offs decision makers in practice. First, we generate $d_k$ and $f_{qs}$ such that the patient volume term in the objective does not dominate the continuous access criterion or vice versa. Second, we generate $f_{qs}$ such that differences in demand for different service packages are reflected. More specifically, we draw parameters $d_k$ from a uniform distribution $U(0, 1)$ first, and normalize them such that $\sum_{k \in K} d_k = 10000$ afterwards. We calculate the flow demand as $f_{q \theta s} = f_q \theta_{qs}$. Here, $f_q$ denotes the size of flow $q$, which we generate in exactly the same way as $d_k$. $\theta_{qs}$ represents the fraction of truck drivers in flow $q$ who need service package $s$. These parameters are given in table 1. This table also describes the definitions of $c_{qs}$ that are used in this case study. As mentioned, we base these definitions and

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**Figure 5: Map of the North-South Corridor Network.**
Figure 6: Solution balanced strategy.

Figure 7: Solution patient volume strategy.

Parameter values on interviews with experts (Pinxten et al. 2013) (note, though, that they do not necessarily represent the actual values North Star is going to use). A full description of the sets $K, K_P, K_C, O, D, Q$, and the parameters $d_k$, $f_{qs}$, $c_{f_{qs}}$, and $t_{kl}$ is provided in the online appendix. Finally, we use following values for the objective weight parameters: $w_{PV} = 1$, $w_{PC} = 1$, $w_{MC} = 2$, $w_{TC} = 6$, $w_{HC} = 5$, $w_{SC} = 3$.

Table 1: Value of $\theta_{qs}$ and the definition of $c_{qs}$ for each package $s$

<table>
<thead>
<tr>
<th>Package</th>
<th>Value of $\theta_{qs}$</th>
<th>Service-type</th>
<th>Service-type parameters</th>
<th>$\alpha L_s^j$</th>
<th>$\alpha U_s^j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC</td>
<td>1.000</td>
<td>ASAP</td>
<td></td>
<td>0.50</td>
<td>1.50</td>
</tr>
<tr>
<td>MC</td>
<td>0.150</td>
<td>CTL</td>
<td>$\tau_{MC}^{CTL} = 1.50$</td>
<td>0.75</td>
<td>0.90</td>
</tr>
<tr>
<td>TC</td>
<td>0.015</td>
<td>CTL</td>
<td>$\tau_{TC}^{CTL} = 2.00$</td>
<td>0.75</td>
<td>0.90</td>
</tr>
<tr>
<td>HC</td>
<td>0.275</td>
<td>ASAP</td>
<td></td>
<td>1.00</td>
<td>2.00</td>
</tr>
<tr>
<td>SC</td>
<td>0.175</td>
<td>RCTL</td>
<td>$\tau_{1SC}^{RCTL} = 1.50$</td>
<td>$\tau_{2SC}^{RCTL} = 3.00$</td>
<td>0.50</td>
</tr>
</tbody>
</table>

In our baseline case, we optimize the locations of 4 new RWCs, and optimize the allocation of 2 of each optional service package among them (i.e., $p = p_{PC} = 4$, $p_{MC} = p_{TC} = p_{HC} = p_{SC} = 2$). For tractability reasons, we initially only include the first 20 truck flows listed in the online appendix. Based on this case, we analyze two investment strategies. An investment strategy is characterized by the relative importance it assigns to the two main objectives. To capture this relative importance in one parameter $r \in [0, 1]$, we rewrite our objective function to $Z = rZ_{PV} + (1 - r)Z_{CA}$, where $Z_{CA} = \sum_{s \in S} w_s Z_s$. We refer to $Z_{CA}$ as the continuous access score from now on. Higher values of parameter $r$ indicate that the decision maker assigns a higher value to fulfilling the objective of maximizing the patient volume and a lower value to fulfilling the objective of ensuring continuous access. The first strategy we consider is the “balanced strategy”, which we model by taking $r = 0.5$ (this strategy thereby balances both objectives). The second investment strategy is North Star’s current strategy. The main focus of this strategy lies on maximizing the patient volume served (the effects in terms of continuous access are not explicitly considered). We model this strategy as a strategy in which $r = 0.999$, and refer to this strategy as the “patient volume strategy”.

Figure 6 describes the optimal solution for the balanced strategy. We see that 4 additional RWCs can bring about a vast improvement in terms of continuous access and patient volume. Namely, the value of $Z_{PV}$ increases from 1737 to 2546, and the value of $Z_{CA}$ increases from 1570 to 10605. Zooming in on the allocation decisions, we see that the optimal solution for the balanced strategy allocates RWCs and service packages to locations that are characterized by (1) a high expected patient volume, and (2) a large impact on the extent to which truck drivers have continuous access to the needed service packages. The first can be illustrated by the fact that the value of $d_k$ at the locations the RWCs are allocated to (i.e., Dedza, Tete, Chimoio, and Beitbridge South) is 175, 205, 254, and 175, respectively, whereas the average value of $d_k$ among all potential RWC locations equals 144. The second characteristic can be illustrated by considering the effects of the new RWCs and service packages on the travel time intervals between package $s$ locations. Next, we analyze these effects for the PC package.
For a comprehensive overview of the effects for the additional service packages, we refer to appendix E.

First, let us analyze the effects of the new RWCs at Dedza, Tete, and Chimoio. These RWCs are located along the “Beira Corridors”: Beira Port - Harare, Beira Port - Blantyre, and Beira Port - Lilongwe. Figure 8 shows the travel time intervals between package PC locations at these paths both before and after the new RWCs are placed. The changes in these intervals reveal the rationale behind the decision to locate three RWCs along these three paths. The Beira corridors are relatively short (a return trip takes 5.7 - 7.6 driving days), so that locating 2 or 3 RWCs along them suffices to provide the truck drivers in the corresponding flows with a high level of access to the needed health services. For example, the RWC at Chimoio (48) reduces the gap between Beira Port and the first package PC locations along the paths Beira Port - Harare and Beira Port - Blantyre from 1.9 to 0.7 days, and from 3.0 to 0.7 days, respectively. Furthermore, whereas the initial network had no RWCs along the path Beira Port - Lilongwe, the three new RWCs ensure that truck drivers who travel along this path pass the PC package at time intervals 1.3 (48→50→48), 1.0 (48→35, 35→48), 1.6 (35→24, 24→35), and 1.0 (24→23→24). Consequently, the average value of the access time score during a trip along these paths decreases significantly, causing a large gain in terms of continuous access to the PC package (see appendix E). Another reason for locating RWCs at Dedza and Tete is that these locations are along the path Durban Port - Lilongwe. As a result, the average value of the access time during a return trip at this path decreases from 3.98 to 1.16, which makes the corresponding package PC coverage score increase from 0 to 0.34. Since the flow volume at this path is very large (552), this increase is very beneficial.

The fourth new RWC is placed at Beithbridge South, which is at one of the main borders between South Africa and Zimbabwe. It has been estimated that truck drivers spend an average of two days at this border to wait for clearance. Hence, truck drivers who depart from Durban or Johannesburg, and pass this border to enter Zimbabwe currently face a large gap between the time of departure and the time at which the first RWC – at Beithbridge North – is passed. These gaps are 3.9 (Durban) and 3.0 (Johannesburg) days, respectively. The new RWC at Beithbridge South decreases these gaps to 1.9 and 1.0, respectively, which brings about large gains in terms of continuous access for the truck flows Johannesburg - Bulawayo, Durban Port - Harare, and Durban Port – Lilongwe.

Figure 7 describes the optimal solution for the patient volume strategy. This strategy can be regarded as a two-stage decision strategy. In the first stage, it simply allocates the four new RWCs to the four locations with the highest expected patient volume: Songwe ($d_k = 255$), Chimoio ($d_k = 254$), Francistown ($d_k = 267$), and Mbabane ($d_k = 256$). Thereby, $Z_{PV}$ increases from 1737 to 2769 (note that $Z_{PV}$ increases to 2546 when the balanced strategy is used). In the second stage, it allocates the service packages among the new RWCs so as to
maximize $Z_{CA}$. Because the continuous access criterion is not considered in the first stage decisions, the effects in terms of $Z_{CA}$ are relatively small: $Z_{CA}$ increases from 1570 to 6862 (note that this score increases to 10605 when the balanced strategy is used). This is mainly due to the new RWCs at Songwe and Mbabane. These have almost no impact in terms of continuous access, since only some small truck flows pass these locations. The additional service packages are therefore allocated to the RWCs at Chimoio and Francistown, which are passed by several large truck flows. Chimoio is passed by Beira Port - Harare and Beira Port - Blantyre. As mentioned, this RWC significantly reduces the gap between Beira Port and the first RWC along these paths. Similarly, the RWC at Francistown considerably decreases the gap to the first RWC for several flows departing from Durban (Durban Port - Livingstone, Durban Port - Lusaka, Durban Port - Harare, and Durban Port - Francistown). For a comprehensive overview of the effects of the new RWCs and service packages in terms of continuous access, we again refer to appendix E.

Finally, the optimal solutions illustrate an important property of the RHFLP: adding multiple packages to the network can bring about synergy effects. For instance, when adding the HC package to Tete or to Beitbridge South only, the value of $Z_{HC}$ increases by 281 and 341, respectively, whereas $Z_{HC}$ increases by 722 when allocating this package to both locations. This can be explained by the fact that (generally) multiple service packages need to be placed along the path of flow $q$ to cause a significant increase in the value of $c_{qs}$.

4.2 Model Statistics.

Table 2 describes the solution times and model statistics of several problem instances. We vary the number of network flows, $|Q|$, and the structure of the network. Two alternative network structures are introduced: sparse, which is generated by selecting the minimum spanning tree of our graph $G(L,E)$, and dense, which is generated by adding for each vertex two edges that connect this vertex with the two closest neighbor vertices it was not connected to in the baseline network. We generate the time needed to traverse such edge by dividing the Euclidian distance of the edge by the average speed of 40 km/hour. All other variables are kept constant.

| Instance | $|Q|$ | Structure | Variables (Integer) | Constraints | CPU (sec.) |
|----------|------|-----------|--------------------|-------------|-----------|
| D20      | 20   | Dense     | 5086 (655)         | 2732        | 44        |
| D25      | 25   | Dense     | 6576 (730)         | 3417        | 69        |
| D30      | 30   | Dense     | 7646 (805)         | 1002        | 226       |
| B20      | 20   | Baseline  | 7671 (655)         | 3182        | 70        |
| B25      | 25   | Baseline  | 10316 (730)        | 4047        | 872       |
| B30      | 30   | Baseline  | 12311 (805)        | 4792        | 4909      |
| S20      | 20   | Sparse    | 11306 (655)        | 3692        | 1899      |
| S25      | 25   | Sparse    | 15241 (730)        | 4717        | 10328     |
| S30      | 30   | Sparse    | 17491 (805)        | 5492        | 25012     |

We observe a rapid growth of the problem size and of the CPU time needed to solve an instance when the network structure becomes sparser and when $|Q|$ increases. The first can be explained from the fact that the number of RWC locations in a path, $|K_q|$, increases when the network gets sparser, which brings about many variables in 23 and many constraints in 19-22.

4.3 Effect of $p$ and objective weights.

Figures 9 and 10 summarize the results when solving the RHFLP for $p \in \{2, 4, \ldots, 30\}$, and for $r \in \{0, 0.2, 0.5, 0.8, 1\}$. The value of $p_s$ equals $\frac{p}{2}$ for all additional packages. All other parameters are kept constant.

The results illustrate that focusing solely on one criterion (i.e., choosing $r = 0$ or $r = 1.0$) results in significant sub-optimality in terms of the other criterion. By setting $r = 0.8$ ($r = 0.2$), large gains can be made in terms of continuous access (patient volume) at a marginal loss in terms of patient volume (continuous access). Notice that the 0.8 results are close to optimal for both patient volume and continuous access, and that scores lower than 0.5 apparently yield negligible further improvements in continuous access.

5 Discussion and conclusions.

Moving forward, North Star intends to expand its network of roadside healthcare facilities. Though there exist many studies that deal with healthcare facility location problems (see Rahman and Smith (2000) for a review) the problem of locating healthcare facilities that serve moving patients appears to have been disregarded in
scientific literature. In this paper, we have introduced a mixed-integer linear programming model to make recommendations on the locations of new RWCs and on the allocation of health service packages among them, taking North Star’s two main criteria – the number of truck driver visits to the RWCs, and the extent to which truck drivers have continuous access to the needed health services – into account. In order to quantify the access provided along the routes the truck drivers travel, three measures are proposed. These indicate to what extent a truck driver can reach the needed service package within the recommended/critical time-limit ($a^{CTL}_{qs}$ and $a^{RCTL}_{qs}$), and reach the health service package soon after the moment at which he realizes that he needs it ($a^{ASAP}_{qs}$).

Numerical and analytical results reveal three key implications for decision makers. First our model is generally quite robust to deviations in the main input data. Nevertheless, worst case analysis shows that it remains important for decision makers to focus on consistent estimates of the parameters used in the model. Second, our experiments show that considerable gains can be made by balancing the weights assigned to the two optimization criteria instead of focusing on one of them. Namely, it is possible to obtain solutions that are close to optimal for both patient volume and continuous access. Last, synergy effects can be obtained by locating multiple service packages. Decision makers can exploit this property in two ways. First, instead of simply fixing the values of $p$ and $p_s$, they can investigate whether large synergy effects can be gained by (slightly) increasing these values. Second, by making long-term investment plans for the network of RWCs instead of taking investment decisions sequentially, they can make optimal use of the synergy effects between all future RWCs. Hence, such plan avoids the sub-optimality resulting from sequential decision making (even when the sequential problems are solved to optimality).

The value of the model is currently being assessed by North Star, which is expanding its network in several parts of Sub-Saharan Africa. A simplified version of the model (with one service package, using $a^{ASAP}_{qs}$) was implemented in a software package, and tested on large problem instances; see De Vries (2011) for the results. The NGO’s current investment strategy can be regarded as a strategy with $r$ equal (or very close) to 1: the main focus lies on maximizing the patient volume served, whereas the effects in terms of continuous access are not explicitly considered. Our analysis suggests that, by decreasing the value of $r$ to 0.8, North Star can considerably improve the impact of its investments in terms of continuous access at a marginal loss in the patient volume served.

We showed that the RHFLP is a strongly $\mathcal{NP}$-hard problem. Numerical experiments confirm that solving this problem becomes extremely difficult when the number of flows and the number of facility locations along the path of a flow increase, particularly in case of a sparse network structure. Future research is needed to develop solution methods that provide good solutions for large problem instances. Another important direction for future research is to (empirically) investigate what definition of the coverage score (including the corresponding parameters) is most appropriate for a given service package. Furthermore, as we use several criteria to measure the quality of location allocation decisions, it is relevant to investigate the health value of improvements in terms of such criterion. Next to insight in the effectiveness of North Star’s services, this also shows how the objective weights should be chosen such that the total health value of the investments is maximized. Last, our model does not consider equity in health delivery. This criterion, however, will get more and more relevant as capacity increases. Investigating the tradeoff between equity and total health value is certainly an interesting research direction (see, e.g., McCoy and Lee 2013).

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A  Linearization of $c_{qs} = g^{i}_{s}(a^{i}_{qs})$.

Let $\lambda_{iqs} \geq 0$ denote some fractional variable, with $i \in \{1, 2, 3, 4\}$. We define that $g^{i}_{s}(a^{i}_{qs}) = \lambda_{1qs} + \lambda_{2qs}$ if the service-type of package $s$ defines that $c_{qs}$ is a non-increasing function of $a^{i}_{qs}$ (i.e., ASAP). Furthermore, $g^{i}_{s}(a^{i}_{qs})$ equals $\lambda_{3qs} + \lambda_{4qs}$ if the service-type of package $s$ defines that $c_{qs}$ is a non-decreasing function of $a^{i}_{qs}$ (i.e., CTL and RCTL). Next, let $M_{qs}$ represent an upper bound on $a^{i}_{qs}$, and let $AS$ denote the set of all vectors of 4 binary variables for which hold that only binary variables $i$ and $i + 1$ are equal to 1, for some $i \in \{1, 2, 3\}$. Finally, variables $z_{iqs}$ are auxiliary binary variables. Then the values of the lambdas are obtained through the following set of constraints:

\[
\begin{align*}
\lambda_{1qs} + \lambda_{2qs} + \lambda_{3qs} + \lambda_{4qs} &= 1 & q \in Q, j \in J, s \in S_j \\
\lambda_{1qs}0 + \lambda_{2qs}a L_s + \lambda_{3qs}a U_s + \lambda_{4qs}M_{qs} &= a^{i}_{qs} & q \in Q, j \in J, s \in S_j \\
\lambda_{iqs} \leq z_{iqs} & i \in \{1, 2, 3, 4\}, q \in Q, j \in J, s \in S_j \\
\{z_{1qs}, z_{2qs}, z_{3qs}, z_{4qs}\} & \in AS & q \in Q, j \in J, s \in S_j
\end{align*}
\]

B  Proof of Proposition 1.

**Proof.** By definition, $v^{\ast}(\omega^{\ast}) \geq v^{\ast}(\omega^{\ast})$. In addition, the objective function is linear in $f_{qs}$ and $d_{k}$, which implies that $v^{\#}(\omega^{\ast}) \geq (1 - \delta) v^{\ast}(\omega^{\ast})$ and that $v^{\#}(\omega^{\ast}) \leq (1 + \delta) v^{\ast}(\omega^{\ast})$. Hence, $\frac{v^{\#}(\omega^{\ast})}{1 - \delta} \geq v^{\ast}(\omega^{\ast}) \geq \frac{v^{\#}(\omega^{\ast})}{1 + \delta}$. This completes the proof. \hfill \Box

C  Proof of Proposition 2.

**Proof.** Let us rewrite $v^{\#}(\omega^{\ast}) - v^{\ast}(\omega^{\ast})$ to $\sum_{ob \in OB^{+}} w^{\ast}_{ob}(Z^{\#}_{ob} - Z^{\ast}_{ob}) + \sum_{ob \in OB^{-}} w^{\ast}_{ob}(Z^{\#}_{ob} - Z^{\ast}_{ob})$. Here, $OB^{+} = \{ob|Z^{\#}_{ob} > Z^{\ast}_{ob}\}$ and $OB^{-} = \{ob|Z^{\#}_{ob} \leq Z^{\ast}_{ob}\}$. Note that $v^{\#}(\omega^{\ast}) - v^{\ast}(\omega^{\ast})$ is maximized if $w^{\ast}_{ob} = (1 - \delta) w^{\ast}_{ob}$ for all $ob \in OB^{-}$ and if $w^{\ast}_{ob} = (1 + \delta) w^{\ast}_{ob}$ for all $ob \in OB^{+}$. Next, observe that the fact that $v^{\ast}(\omega^{\ast}) \geq v^{\ast}(\omega^{\ast})$ implies $\sum_{ob \in OB^{+}} w^{\ast}_{ob}(Z^{\#}_{ob} - Z^{\ast}_{ob}) \leq \sum_{ob \in OB^{-}} w^{\ast}_{ob}(Z^{\#}_{ob} - Z^{\ast}_{ob})$. Consequently,

\[
\begin{align*}
\begin{align*}
\frac{v^{\#}(\omega^{\ast})}{1 + \delta} + v^{\ast}(\omega^{\ast}) & \leq (1 + \delta) \sum_{ob \in OB^{+}} w^{\ast}_{ob}(Z^{\#}_{ob} - Z^{\ast}_{ob}) + (1 - \delta) \sum_{ob \in OB^{-}} w^{\ast}_{ob}(Z^{\#}_{ob} - Z^{\ast}_{ob}) \\
& \leq 2\delta \sum_{ob \in OB^{-}} w^{\ast}_{ob}(Z^{\#}_{ob} - Z^{\ast}_{ob}) \\
& \leq 2\delta \sum_{ob \in OB^{-}} w^{\ast}_{ob}(Z^{\#}_{ob} - Z^{\ast}_{ob})
\end{align*}
\end{align*}
\]

Using the fact that $v^{\ast}(\omega^{\ast}) \leq \frac{v^{\#}(\omega^{\ast})}{1 + \delta}$, the bound can be rewritten to:

\[
\frac{v^{\#}(\omega^{\ast})}{1 - \delta} - 2\delta \sum_{ob \in OB} w^{\ast}_{ob}Z_{ob}
\]

By rewriting this inequality, we obtain the bound given in proposition 2. \hfill \Box

continuous access and for providing the data. We would also like to thank J. Hontelez on his support on the use of disease-related data for our case study. We also acknowledge K. Dalmeijer for inspiring the strongly NP-completeness proof. Finally, we would like to thank T. van Dijk, F. van Helden, F. van der Wal, and E. Even from the ORTEC Consulting Group for their support on the implementation of this research in the software-package POLARIS.
D Proof of strongly \( \mathcal{NP} \)-hardness RHFLP.

**Proof.** First, note that the decision version of the RHFLP is in \( \mathcal{NP} \): our MIP formulation allows us to verify a YES answer to this problem in polynomial time. Next, we show how to construct a polynomial transformation of the CLIQUE problem to the decision-version of the RHFLP. Since the CLIQUE problem is strongly \( \mathcal{NP} \)-Complete (Papadimitriou and Steiglitz 1998), this shows that the decision-version of the RHFLP is \( \mathcal{NP} \)-Complete too.

We consider a graph \( G(V, E) \). CLIQUE corresponds to the decision problem “does \( G \) contain a complete sub-graph consisting of \( k \) nodes?”. To perform the transformation, we introduce for every node \( v \in V \) a potential RWC location \( k \in K_P \). Next, for every edge in \( E \), we introduce a path \( q \). This is a direct path from a potential RWC location to another potential RWC locations. These two locations correspond to the two nodes the corresponding edge connects. This transformation can be performed in polynomial time since \(|K_P| = |V|\) and \(|Q| = |E|\).

We use the following parameter values: \(|S| = 1, p = p_s = k, f_{qs} = 1 \forall q, d_k = 0, w_s = 1\), and \( t_{kl} = 1 \ \forall (k, l)\). Furthermore, we choose the remaining parameters such that the package \( s \) coverage score of flow \( q \) equals 1 if and only if RWCs are placed at both locations passed by this flow. In case that \( j = CTL \), it suffices to choose parameter values \( \tau_{r_{s}^{CTL}} = 1, \alpha_{r_s}^{L} = 1, \alpha_{U_s}^{CTL} = 1 \). In case that \( j = RCTL \), it suffices to choose parameter values \( \tau_{r_{s}^{RCTL}} = 1, \tau_{s}^{RCTL} = 1, \alpha_{L_s}^{RCTL} = 1, \alpha_{U_s}^{RCTL} = 1 \). Finally, in case that \( j = ASAP \), the parameter values \( \alpha_{L_s}^{ASAP} = 0.5, \alpha_{U_s}^{ASAP} = 0.5 \) satisfy this condition.

Next, we argue that CLIQUE has a YES-answer if and only if the corresponding instance of the decision version of the RHFLP has a solution of value \( k \cdot (k - 1)/2 \). First, suppose the CLIQUE has a YES-answer (i.e., the graph contains a clique of size \( k \)). Note that there are at most \( k \cdot (k - 1)/2 \) flow paths connecting each pair of the \( k \) RWC locations we allocate the \( p_s \) service packages to. Hence, at most \( k \cdot (k - 1)/2 \) flow paths have a package \( s \) coverage score that is larger than 0. Since the solution value equals the number of flow paths for which the package \( s \) coverage score equals 1, this implies that the RHFLP instance has a solution value that is at most \( k \cdot (k - 1)/2 \). Next, observe that, in case that CLIQUE has a YES-answer, there are exactly \( k \cdot (k - 1)/2 \) flow paths connecting each pair of the \( k \) RWC locations corresponding to the \( k \)-clique. Then allocating the \( p_s \) service packages to these \( k \) RWC locations makes sure that the package \( s \) coverage score of each of the corresponding \( k \cdot (k - 1)/2 \) flows has the value 1. This implies that the corresponding solution value is exactly \( k \cdot (k - 1)/2 \).

Second, suppose that the RHFLP instance has a value of \( k \cdot (k - 1)/2 \). Then this means that at least \( k \cdot (k - 1)/2 \) flows have a strictly positive package \( s \) coverage score. Next, observe that the fact that we are only allowed to allocate \( k \) service packages \( s \) implies that at most \( k \cdot (k - 1)/2 \) flows have a strictly positive package \( s \) coverage score. Hence, exactly \( k \cdot (k - 1)/2 \) flows have a strictly positive package \( s \) coverage score. This condition is only met if there are exactly \( k \cdot (k - 1)/2 \) unique flows that visit two of the \( k \) allocated service packages. This implies that the \( k \) vertices in the corresponding CLIQUE instance form a clique of size \( k \).

E Effects of optimal solution for baseline case on \( c_{qs} \) and \( \alpha_{qs}^j \).

<table>
<thead>
<tr>
<th>( D_q - D_d )</th>
<th>( T_q )</th>
<th>( L_d )</th>
<th>( c_{q,PC} )</th>
<th>( c_{q,MC} )</th>
<th>( c_{q,TC} )</th>
<th>( c_{q,HC} )</th>
<th>( c_{q,SC} )</th>
<th>Initial network</th>
<th>Optimal network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dar es Salaam Port - Lusaka</td>
<td>15.88</td>
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<td>0.87</td>
<td>0.56</td>
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<td>1.00</td>
<td>0.42</td>
<td>0.56</td>
</tr>
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<td>0.56</td>
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<td>1.00</td>
<td>0.42</td>
<td>0.56</td>
</tr>
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<td>1.00</td>
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</tr>
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<td>1.00</td>
<td>0.42</td>
<td>0.56</td>
</tr>
<tr>
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<td>1.00</td>
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<td>0.56</td>
</tr>
<tr>
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<td>0.56</td>
<td>0.74</td>
<td>1.00</td>
<td>1.00</td>
<td>0.42</td>
<td>0.56</td>
</tr>
<tr>
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<td>0.74</td>
<td>1.00</td>
<td>1.00</td>
<td>0.42</td>
<td>0.56</td>
</tr>
<tr>
<td>Durban Port - Harare</td>
<td>10.34</td>
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<td>0.56</td>
<td>0.74</td>
<td>1.00</td>
<td>1.00</td>
<td>0.42</td>
<td>0.56</td>
</tr>
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<td>Durban Port - Francistown</td>
<td>5.26</td>
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<td>1.00</td>
<td>0.42</td>
<td>0.56</td>
</tr>
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<td>1.00</td>
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<td>0.56</td>
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<td>1.00</td>
<td>1.00</td>
<td>0.42</td>
<td>0.56</td>
</tr>
<tr>
<td>Johannesburg - Bulawayo</td>
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<td>0.56</td>
</tr>
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<td>1.00</td>
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<td>0.56</td>
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<td>1.00</td>
<td>0.57</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Table 3: Truck flows and the effects of new RWCs on \( c_{qs} \) in the baseline case, using the balanced strategy.
### Table 4: The effects of new RWCs on $a^j_{iq}$ in the baseline case, using the balanced strategy.

<table>
<thead>
<tr>
<th>$G_q - D_q$</th>
<th>Initial network</th>
<th>Optimal network</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a^A_{AP}$</td>
<td>$a^C_{TL}$</td>
</tr>
<tr>
<td>Dar es Salaam Port - Lilongwe</td>
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</tr>
<tr>
<td>Nacala Port - Luanda</td>
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<td>0.22</td>
</tr>
<tr>
<td>Beira Port - Blantyre</td>
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</tr>
<tr>
<td>Durban Port - Lilongwe</td>
<td>2.65</td>
<td>0.36</td>
</tr>
<tr>
<td>Gaborone - Lilongwe</td>
<td>6.27</td>
<td>0.16</td>
</tr>
<tr>
<td>Durban Port - Livingstone</td>
<td>3.84</td>
<td>0.29</td>
</tr>
<tr>
<td>Durban Port - Harare</td>
<td>1.78</td>
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<tr>
<td>Durban Port - Francistown</td>
<td>2.63</td>
<td>0.39</td>
</tr>
<tr>
<td>Durban Port - Kolwezi</td>
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<td>0.18</td>
</tr>
<tr>
<td>Durban Port - Lubumbashi</td>
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</tr>
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<tr>
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</table>

### Table 5: Truck flows and the effects of new RWCs on $c_{iq}$ in the baseline case, using the patient volume strategy.

<table>
<thead>
<tr>
<th>$G_q - D_q$</th>
<th>$T_q$</th>
<th>$D_q$</th>
<th>$c_{q,PC}$</th>
<th>$c_{q,MC}$</th>
<th>$c_{q,TC}$</th>
<th>$c_{q,HC}$</th>
<th>$c_{q,SC}$</th>
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<td>0.00</td>
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<tr>
<td>Beira Port - Harare</td>
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<tr>
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<td>0.00</td>
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</tr>
</tbody>
</table>

### Table 6: The effects of new RWCs on $a^j_{iq}$ in the baseline case, using the patient volume strategy.

<table>
<thead>
<tr>
<th>$G_q - D_q$</th>
<th>Initial network</th>
<th>Optimal network</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a^A_{AP}$</td>
<td>$a^C_{TL}$</td>
</tr>
<tr>
<td>Dar es Salaam Port - Luanda</td>
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<tr>
<td>Nacala Port - Luanda</td>
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<tr>
<td>Beira Port - Blantyre</td>
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<td>0.36</td>
</tr>
<tr>
<td>Durban Port - Lilongwe</td>
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<td>Durban Port - Livingstone</td>
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</tr>
<tr>
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<td>1.78</td>
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<tr>
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17


