A FRAMEWORK FOR CONDITIONAL FAILURE PREDICTION

by

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ABSTRACT

In this paper, a new framework for failure prediction is presented. Its most important feature is that the prediction of corporate failure is conditioned on the values of a series of exogenous risk factors. For instance, a firm may be predicted to go bankrupt in case that the wage rate raises with more than ten percent.

In the framework presented here failure is equated to cash insolvency. Consequently, a model for cash balances is developed in which future cash balances are related to future values of risk factors. The reaction of a firm to a change in the value of a risk factor (its sensitivity to that factor) generally differs from the reaction of other firms. Thus, each firm has its own specific risk profile.

The framework may be applied in several fields. For banks it supports in the monitoring of clients. Given a particular change in the risk factor value, it can be easily seen which clients will suffer mostly and, hence, will be a risk for the bank. Similarly, it may help the government in detecting those firms/industries that will be harmed strongly under various scenarios of risk factor changes. Furthermore, it may help the firm's management to find ways to prevent the problems caused by risk factor changes, since the model indicates how risks are averted by for example a change in the sensitivities. Finally, the framework does not only apply in the case of failure (when cash balances are less than zero), but also in case the analysis is focussed on the risks of reaching a specified target level of cash flow.

Key words: cash insolvency prediction, decision support systems, firm evaluation, multi-factor model, risk analysis.

CONTENTS

1. Introduction

Corporate failure has not failed to attract considerable attention in both the academic and business world. Accordingly, the prediction of corporate failure has been well researched by among others Altman (1968, 1984), Edmister (1972) and Taffler (1982).

The general approach to predict corporate failure is presented in Figure 1. As input a sample containing both failed and non-failed firms is used. Then a statistical classification technique is applied to find a relation between the firm characteristics on the one hand and future corporate failure on the other. This relation is used to obtain insight into the predictability of corporate failure.

Input		**Model**		**Output**
Sample of failed and non-failed firms at time t, and their characteristics at time $t-k$	⟶	Statistical Classification Technique	⟶	Relation between firm characteristics and probability of failure

Figure 1: General approach to corporate failure prediction

As often mentioned in the literature, the use of these kinds of approaches is not without problems and pitfalls. Lev (1974, p. 149) states that the theoretical foundation of these models may need some improvement, and Taffler (1982) mentions as a drawback of corporate failure models that "dramatic changes in the UK economy and major changes in the system of company taxation *inter alia* call its subsequent intertemporal validity into question." In other words, a disadvantage of corporate failure models may be that changes in exogenous risk factors are not taken account of.

To overcome these drawbacks, during the last decade research has been directed to incorporate theoretically determined financial characteristics to explain corporate failure. For example, concerning the firm characteristics Casey and Bartczak (1985) as well as Gentry, Newbold and Whitford (1985) incorporated cash flow ratios in their model as theoretically determined characteristics predicting corporate failure.[1] Also some other classification techniques were used, such as recursive partitioning investigated by Frydman et al. (1985). Lawrence et al. (1992) explicitly take account of the fact that the future economic situation may influence corporate bankruptcy.[2] They adopt economic state variables, such as unemployment rates and retail sales in their model and find a significant influence.

In this paper, a new framework for failure prediction is presented. Its most important feature is that the prediction of corporate failure is conditioned on the values of a series of exogenous risk factors. For instance, a firm may be predicted to go bankrupt in case that the wage rate raises with more than ten percent.

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[1] *Casey and Bartczak found that operating cash flow data did not improve classification accuracy significantly. Gentry, Newbold and Whitford (1985) found that cash-flow-based funds flows offer a viable alternative for classifying failed and nonfailed firms. A study of Gombala (1987) et al. confirmed the findings of Casey and Bartczak that the ratio cash flow from operations divided by total assets is insignificant in predicting corporate failure.*

[2] *In this paper the terms "bankruptcy" and "failure" are interchangeable.*

The remainder of this paper is organized as follows. In Section 2 we discuss the multi-factor approach, which is used to explain future cash balances by among other things a series of risk factors. Next, in Section 3 the framework for conditional failure prediction is presented. Section 4 illustrates the framework numerically and Section 5 summarizes.

2. The multi-factor model

This section starts with a presentation of a view of the firm and its environment. In this view, a firm's performance is not only determined by the firm's decisions but also by its, uncertain, environment. This uncertain part of performance can be attributed to the changes of risk factors and the firm's sensitivities for these changes. This approach to risk, often called the multi-factor approach, has been applied by Spronk and Van der Wijst (1987) in the field of corporate risk analysis. Berry, Burmeister and McElroy (1988) apply the method to analyze stock returns. Van Aalst et al. (1993) use the method for asset liability matching for pension funds. Hallerbach (1994) applies the method to portfolio analysis, Goedhart and Spronk (1991) illustrate the approach in the field of financial planning, and LoCascio and Spronk (1992) investigate the Italian stock market making use of the multi-factor method. The model presented here differs from the model for stock returns since performance measures of firms (in contrast with security portfolios) are analyzed. More details about this model as well as other applications can be found in Vermeulen et al. (1993, 1994) and Vermeulen (1994). In the following, first a verbal description of the model will be provided, after which the relation between performance measures and risk factors will be formalized in the second part of the section.

A familiar view represents the firm as an organized chain of input, throughput, and output. The inputs have to be paid for and, hence, lead to cash outflows, whereas the outputs generate cash inflows. The cash flows are defined as the difference between cash inflows and cash outflows.

The supply of input factors and their prices, as well as the demand for output products and their prices are uncertain causing the cash inflows and cash outflows to be stochastic. Examples are the wage rate, oil price and the exchange rate of the dollar. The risk factors influence the performance measures of the firm. The magnitude of a risk factor's influence on a performance measure of the firm is called the *sensitivity* of the firm's performance measure to the risk factor.[3]

Changes in performance are the combined result of changes of the risk factors and the sensitivities to these changes. The higher the sensitivities, the greater the impact of an unexpected risk factor change on performance will be, and the greater the risks the firm runs.[4] The vector of sensitivities, called the risk profile, describes the risks the firm faces and thus can be seen as a multi-dimensional risk measure.

The sensitivities are related to the firm's characteristics.[5] For instance, the interest rate sensitivity is highly dependent on the firm's level of debt, and the wage rate sensitivity depends, among other things, on the number of employees. Similarly, the business cycle sensitivity is influenced by the firm's product range, and so on.

To formalize this view on the firm, a one period model is constructed that can be used to analyze the risk and expected level of a firm's performance.[6] The following relation between performance and the risk factor is assumed (later on, the model will be extended to more risk factors.):[7]

$$\tilde{R}_t = a_t + \tilde{b}_t\tilde{f}_t + \eta_t \qquad (1)$$

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[3] *The risk concept used here is largely in line with that of Cooper and Chapman (1987, p. 2) who define risk as: "exposure to the possibility of economic or financial loss or gain, physical damage or injury, or delay, as a consequence of the uncertainty associated with pursuing a particular course of action."*

[4] *Of course, the variance of the risk factors should also be taken into account when evaluating the risk profile.*

[5] *Actually, they may also depend on industry characteristics, such as the degree of competition in an industry.*

[6] *An advantage of the one-period model is that it facilitates easy presentation. Extension to more than one period is possible. Since the main principles of risk management shown in the one period model are equal to those of the multi-period model, we shall concentrate on the one period model only.*

[7] *This model differs from the arbitrage pricing theory (APT) as developed by Ross (1976) in several aspects. The APT describes the pricing of assets in a general equilibrium framework. The approach presented in this article limits itself to postulating a functional relation between a performance measure and a multitude of stochastic risk factors. The multi-factor approach does not assume a general equilibrium framework nor intends to derive market prices for the risk factors involved, see Hallerbach (1994, p. 33 ff).*

where

$\tilde{R}_t$ = performance,

a_t = a fixed term,

b_t = the sensitivity,

$\tilde{f}_t$ = the value of the risk factor,

$\tilde{\eta}_t$ = the error term,

t = index for time,

$\tilde{.}$ = random variable.

The fixed term a_t and the sensitivity b_t differ per firm, because of differences in firm characteristics, such as product range and management style.

From expression (1) the following expression is derived for performance at time $t+1$; defining Δ as the difference operator, i.e., $\Delta x_t = x_t - x_{t-1}$.

$$\tilde{R}_{t+1} = a_{t+1} + b_{t+1}\tilde{f}_{t+1} + \tilde{\eta}_{t+1}$$

$$= a_t + \Delta a_{t+1} + (b_t + \Delta \tilde{b}_{t+1})(f_t + \Delta \tilde{f}_{t+1}) + \tilde{\eta}_{t+1}$$

which, by assuming that the factor values at time t have been realized, and thus are not stochastic anymore can be rewritten as:

$$= a_t + b_t f_t + \Delta a_{t+1} + \Delta b_{t+1} f_t + b_t \Delta \tilde{f}_{t+1} + \Delta b_{t+1} \Delta \tilde{f}_{t+1} + \tilde{\eta}_{t+1} \qquad (2)$$

Apart from changes in risk factors, expression (2) leaves room for changes in the fixed term a_t and in the sensitivity b_t. These changes are caused by instruments, which may vary from a change in firm characteristics to the purchase of a financial contract. The fixed term may change because of the costs associated with the application of the instruments. Here we assume that no instruments are used and, thus, that $\Delta a_{t+1} = \Delta b_{t+1} = 0$. Expression (2) then comes down to:

$$\tilde{R}_{t+1} = a_t + b_t f_t + b_t \Delta \tilde{f}_{t+1} + \tilde{\eta}_{t+1} \qquad (3)$$

Subtracting expression (1) from expression (3), leads to an expression for the change in performance

$$\Delta \tilde{R}_{t+1} = b_t \Delta \tilde{f}_{t+1} + \tilde{\varepsilon}_{t+1} \qquad (4)$$

where

$\tilde{\varepsilon}_{t+1} = \tilde{\eta}_{t+1} - \tilde{\eta}_t.$

This expression can easily be extended to more than one risk factor. In that case we get:

$$\Delta\tilde{R}_{t+1} = \sum_{i=1}^{k} b_{it}\Delta\tilde{f}_{it+1} + \tilde{\varepsilon}_{t+1} \tag{5}$$

This model is further extended by Vermeulen et al. (1993) who explain the sensitivities in terms of firm characteristics:

$$b_{it} = \sum_{m=1}^{M} fc_{tnm}\gamma_{im} \tag{6}$$

where

fc_{tnm} = firm characteristic m of firm n at time t,

γ_{im} = the influence of firm characteristic m on the sensitivity to risk factor i.

Substituting expression (6) into expression (5) leads to:

$$\Delta\tilde{R}_{t+1} = \sum_{i=1}^{k} \left(\sum_{m=1}^{M} fc_{tnm}\gamma_{im}\right)\Delta\tilde{f}_{it+1} + \varepsilon_{t+1} \tag{7}$$

The parameters γ_{im} can be estimated using panel techniques. In order to estimate expression (7) the data of various firms can be pooled, i.e., in to estimate γ_{im} only one regression is needed in which data of a couple of firms over a number of time periods are used.[8]

3. The conditional failure prediction model

The model that is used to predict bankruptcy is based on a model for future cash balances. This model is extended by means of the multi-factor model that

[8] *Moreover estimating expression (5) directly is more difficult because of the possibly low number of observations, since in practice most financial firm characteristics are only observed on a yearly basis for smaller firms, and quarterly for larger corporations.*

was discussed in Section 2. Consequently, a more detailed view on future cash flows is obtained.

For reasons of simplicity we equate bankruptcy and cash insolvency. The latter occurs when the cash flows become negative and the deficit flow is large enough and lasts long enough to exhaust the cash balances. This relationship may be expressed in general terms as follows:

$$\tilde{CB}_t = CB_{t-1} + \tilde{NOCF}_t + \tilde{CF}_t \qquad (8)$$

where

$\tilde{CB}_t$ = cash balances at time t,

$\tilde{NOCF}_t$ = non operational cash flow at time t,

$\tilde{CF}_t$ = the cash flow at time t.

As shown in the preceding section the uncertain cash flows depend on the uncertain development of the risk factors.[9] In order to analyze future (operational) cash flows we apply the framework developed earlier and arrive at a model in which the cash flow of year t can be explained by last year's cash flow and by the changes of some risk factors. Thus, total risk is broken up in the separate influences of the underlying risk factors. In that way not only the magnitude of risk is obtained, but also the source of risk and the firm's exposure to this specific kind of risk. This detailed analysis may help in preventing bankruptcy, since it clarifies which risks to concentrate on. Moreover, if the decision maker provides a set of future factor values, he obtains an indication of failure conditional on this specific set.

The empirical multi-factor model used to estimate the sensitivities, which was formulated in expression (7), can be rewritten in cash flow terms as:

$$\tilde{CF}_t = CF_{t-1} + \sum_{i=1}^{k} b_i \Delta\tilde{f}_{it} + \tilde{\varepsilon}_t \qquad (9)$$

where

[9] *In addition to the multi-factor analysis of the future cash flows, an analysis of the determinants of the past cash flow and the future non operational cash flows is necessary. The future non operational cash flows will among other things be related to the debt ratio of the firm, and its past cash flow to the firm's profitability.*

$\tilde{CF}_t$ = the stochastic cash flow in year t,

CF_{t-1} = the realized cash flow last year,

$\Delta\tilde{f}_{it}$ = the change in risk factor i,

b_i = the sensitivity to risk factor i,

$\tilde{\varepsilon}_t$ = an error term for which the usual assumptions hold.[10]

After substitution of equation (9) in (8) we obtain:

$$\tilde{CB}_t = CB_{t-1} + \tilde{NOCF}_t + CF_{t-1} + \sum_{i=1}^{k} b_i \Delta\tilde{f}_{it} + \tilde{\varepsilon}_t \qquad (10)$$

If the stochastic specification of the non operating cash flows and the risk factors was precisely known, then the probability density function of CB_t could be obtained and thus its expected value and variance. Then, clearly the probability of failure could be calculated, i.e., the probability that CB_t < 0. Unfortunately, in practice information of such a precise nature on the uncertain risk factors is generally not available.

However, in order to obtain insight into the odds of bankruptcy, a specific set of factor values can be chosen and it can be investigated whether or not such a scenario would lead to cash insolvency. In other words, when we neglect the error term, net cash balances are involuntary reduced to zero and cash insolvency occurs if the realized values CB_t, $NOCF_t$ and Δf_{it} are such that

$$CB_t = CB_{t-1} + NOCF_t + CF_{t-1} + \sum_{i=1}^{k} b_i \Delta f_{it} < 0 \qquad (11)$$

In that case the plane distinguishing between bankrupt and non-bankrupt firms is:

$$CB_t = CB_{t-1} + NOCF_t + CF_{t-1} + \sum_{i=1}^{k} b_i \Delta f_{it} = 0 \Rightarrow \qquad (12)$$

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[10] *I.e. zero expected value, constant variance, no autocorrelation in the residuals and independence between the residual term and the independent variables.*

$$CB_{t-1} + NOCF_t + CF_{t-1} = - \sum_{i=1}^{k} b_i \Delta f_{it}$$

Hence, the set of firms which are expected to become cash insolvent is:

$$S^- = \{ CB_{t-1} + NOCF_t + CF_{t-1} + \sum_{i=1}^{k} b_i \Delta f_{it} < 0 \} \quad (13)$$

And the set of firms which are not expected to become cash insolvent is:

$$S^+ = \{ CB_{t-1} + NOCF_t + CF_{t-1} + \sum_{i=1}^{k} b_i \Delta f_{it} \geq 0 \} \quad (14)$$

Thus, whether or not a firm belongs to a particular set firstly depends on its last year's cash balances and cash flow as well as its expected non-operational cash flows and sensitivities. Secondly, it depends on the factor values that were provided by the decision maker.

In order to predict corporate failure these characteristics and the future course of the risk factors must be known. The sensitivities can be estimated using expression (7).

After estimation of the sensitivities, for each possible combination of future risk factors, it can be seen whether or not the firm fails. In this way, a failure prediction is obtained that is conditional on the scenario of future factor values. Good candidates for possible scenarios should be provided by the decision maker, who may use a macro-economic model for this purpose.

4. Illustration

To illustrate the model presented in Section 3 we may restrict ourselves to one risk factor, simplifying equation (12) to:

$$CB_{t-1} + \tilde{NOCF}_t + CF_{t-1} = -b\Delta\tilde{f}_t \quad (15)$$

For four fictive firms the values of CF_{t-1} and b are found in Table 1. Furthermore, we assume $(CB_{t-1} + NOCF_t)$ to be equal to zero and the decision maker wants to investigate the consequences of a drop in the factor value by four. Hence, $\Delta f_t = -4$.

Of course, when the method is operationalized in practice some parts of the model should be worked out in greater detail. For example, last year's cash balances, CB_{t-1}, should be determined more precisely.

Table 1: The cash flow and sensitivity of some firms

	Cash flow	Sensitivity, b	Cash flow/b
Firm A	30	3	10
Firm B	5	1	5
Firm C	15	5	3
Firm D	2	1	2

Note: amounts are in millions of guilders.

Also the change of the risk factor, Δf_t, should be a well educated guess based on macro-econometric models.

The so-called *critical line* in the (CF_{t-1},b)-plane is:

$$CF_{t-1} = 4.b \tag{16}$$

Figure 2 presents this critical-line as well as the position of the four firms. The vertical axis presents last year's cash flow (CF_{t-1}), and the horizontal axis the sensitivity to the risk factor (b). All firms are plotted in accordance with their (CF_{t-1},b)-values. The firms below the critical line, such as Firm C (denoted by C), are expected to become cash insolvent.

Although good forecast models exist, the risk factors may still be subjected to much uncertainty. Then it is interesting to investigate not only one particular change of the risk factor, but also other possible changes. of it. Thus one obtains different critical lines for different changes in the risk factor. Assuming the following possible percentual changes of the risk factor: Δf = 0, -4, -8, -12, the lines in Figure 2 were drawn.

Figure 2 indicates which firms have negative cash flows given particular changes of the risk factor. For instance, it is easily seen that Firm A will become cash insolvent if the change of the risk factor is between -8 and -12, whereas Firm D already performs badly if the risk factor changes by -4.

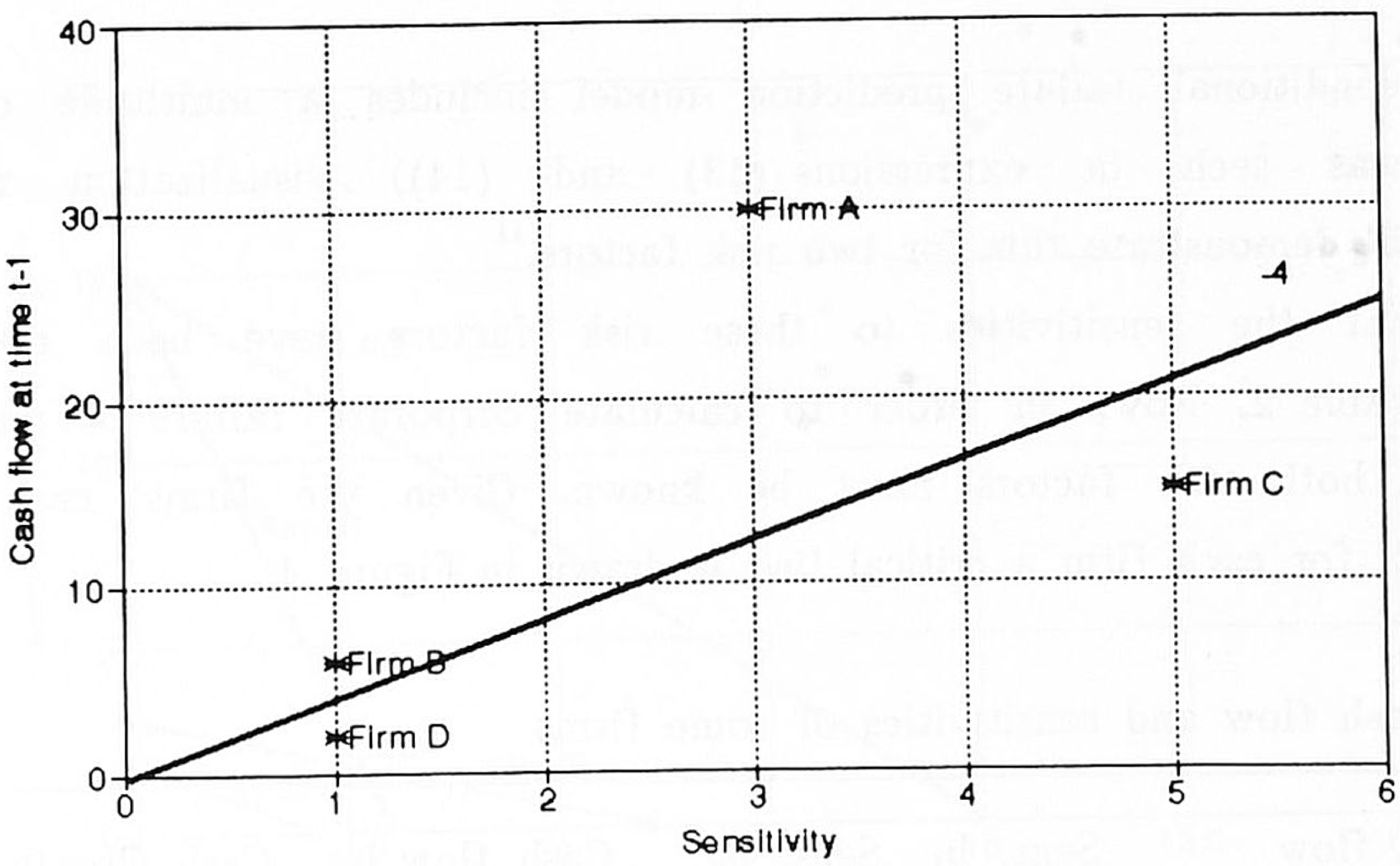

Figure 2: The critical line

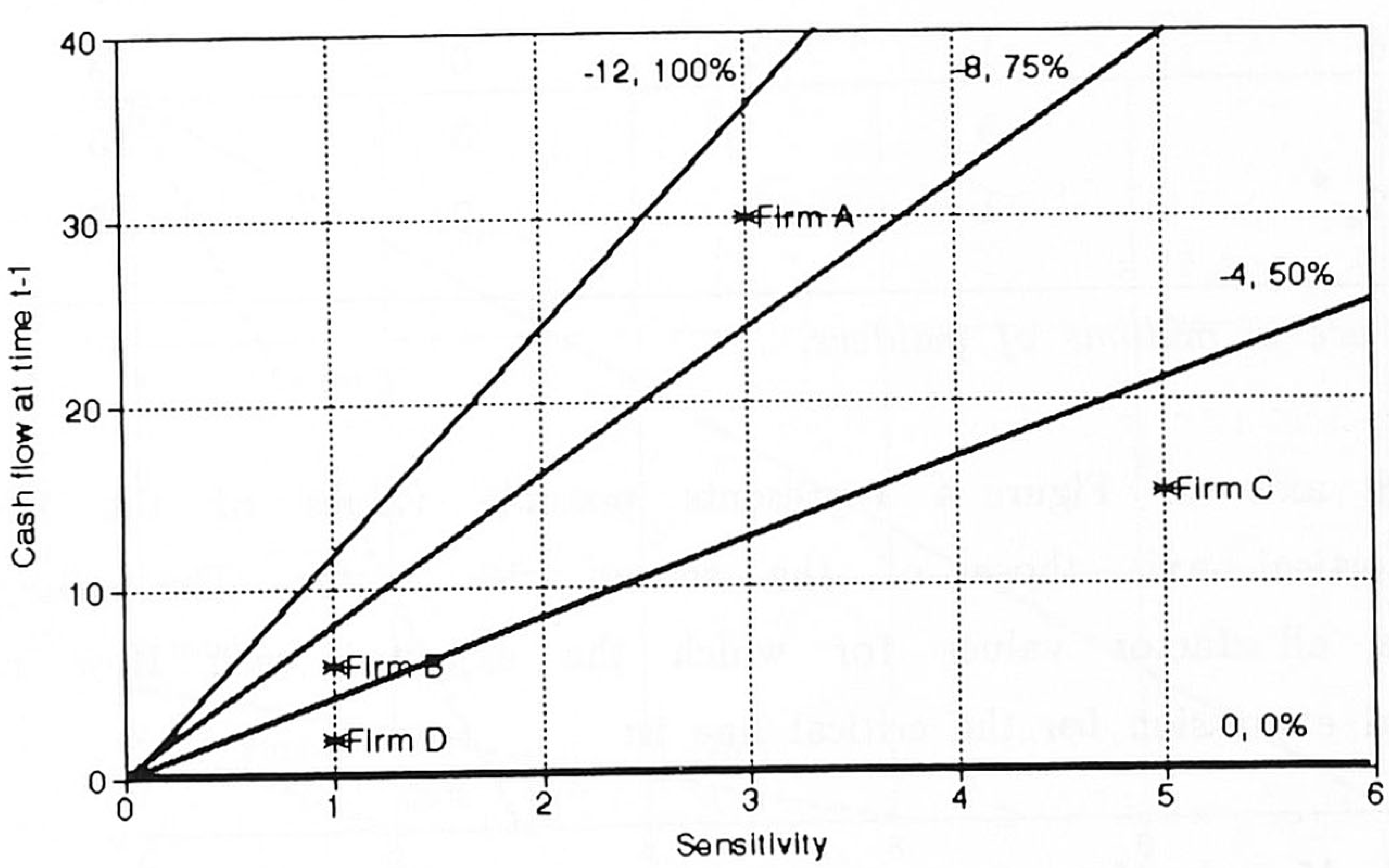

Figure 3: The failure dial

Using this method, it is easily computed what the percentage of firms suffering cash insolvency will be, given a particular change in the risk factor. For example, if the risk factor decreases 12, we see that all firms, i.e. 100%, become cash insolvent, whereas this percentage reduces to 50% (2 of the 4 firms) if the risk factor drops by 4.

To the left of the critical lines two numbers can be seen. The first number indicates the change of the risk factor, and the second number denotes the percentage of firms that have negative cash flows given that particular change of the risk factor.

In case the conditional failure prediction model includes a multitude of risk factors (as was seen in expressions (13) and (14)) visualization remains possible. We will demonstrate this for two risk factors.[11]

Assume that the sensitivities to these risk factors have been estimated as given in Table 2. Now, in order to calculate corporate failure a guess of the values of both risk factors must be known. Given the firms' cash flows and sensitivities, for each firm a critical line is drawn in Figure 4.

Table 2: The cash flow and sensitivities of some firms

	Cash flow	Sens. b_1	Sens. b_2	Cash flow/b_1	Cash flow/b_2
Firm A	30	3	2	10	15
Firm B	6	1	2	6	3
Firm C	15	5	1	3	15
Firm D	2	1	1	2	2

Note: amounts are in millions of guilders.

The horizontal axis of Figure 4 represents possible values of the first risk factor, the vertical axis those of the second risk factor. The critical line simply denotes all factor values for which the expected cash flow is zero, thus the general expression for the critical line is:

$$CF_t = CF_{t-1} + b_1\Delta f_1 + b_2\Delta f_2 = 0 \qquad (17)$$

For example, the critical line of Firm B is:

$$6 + 1*\Delta f_1 + 2*\Delta f_2 = 0. \qquad (18)$$

In Figure 4 the values on the axes are negative. If the realized factor values appear to be to the left of the critical line, the firm is expected to go bankrupt. For instance, if both risk factors take the value -5, then Firm B fails, since $CF_{t-1} + b_1\Delta f_1 + b_2\Delta f_2 = 6 + (1*-5) + (2*-5) = -9 < 0$.

[11] *Of course the more risk factors are taken account of, the harder visualization becomes, although many "solutions" can be found to overcome the associated problems.*

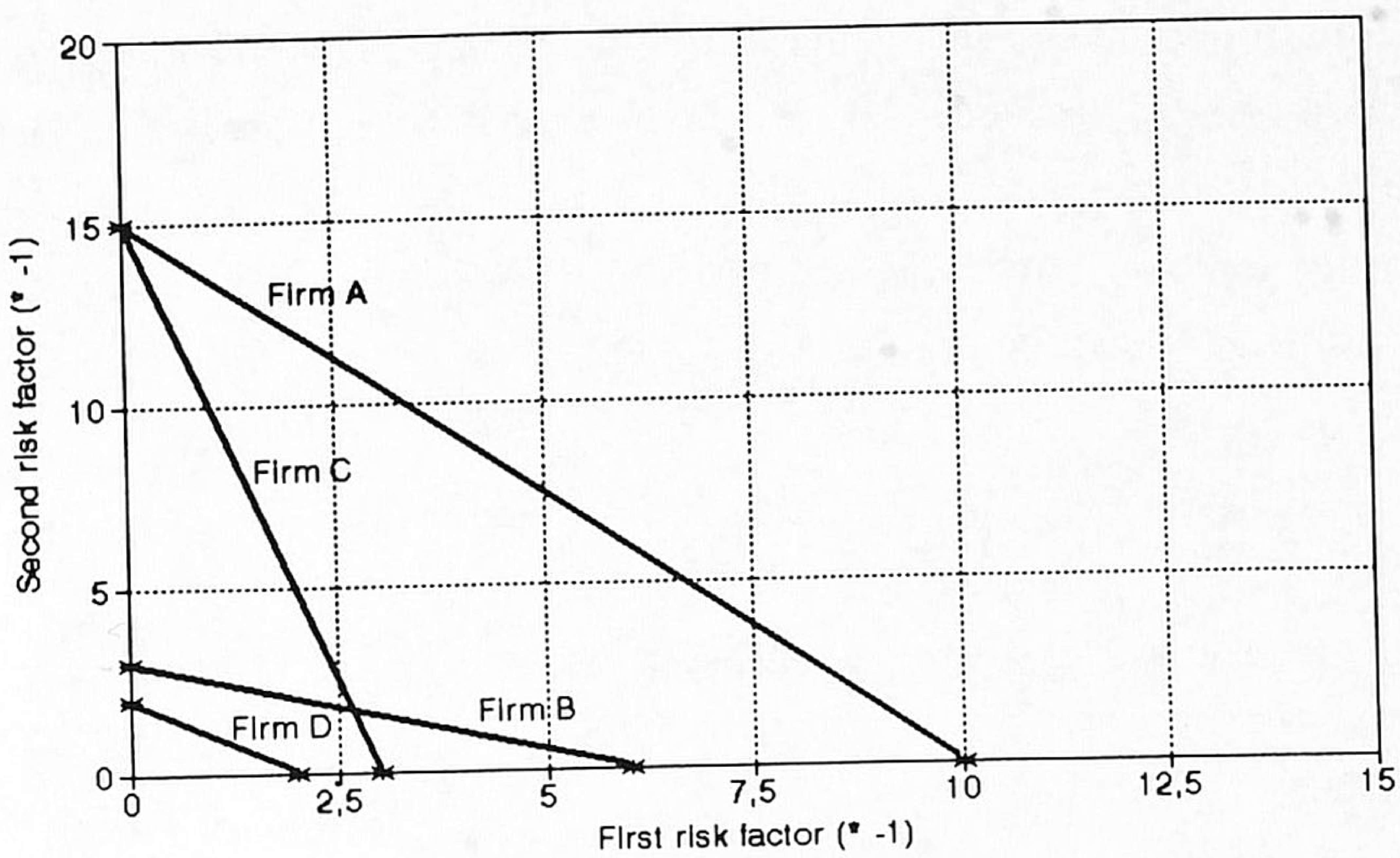

Figure 4: The critical lines in a two-factor case

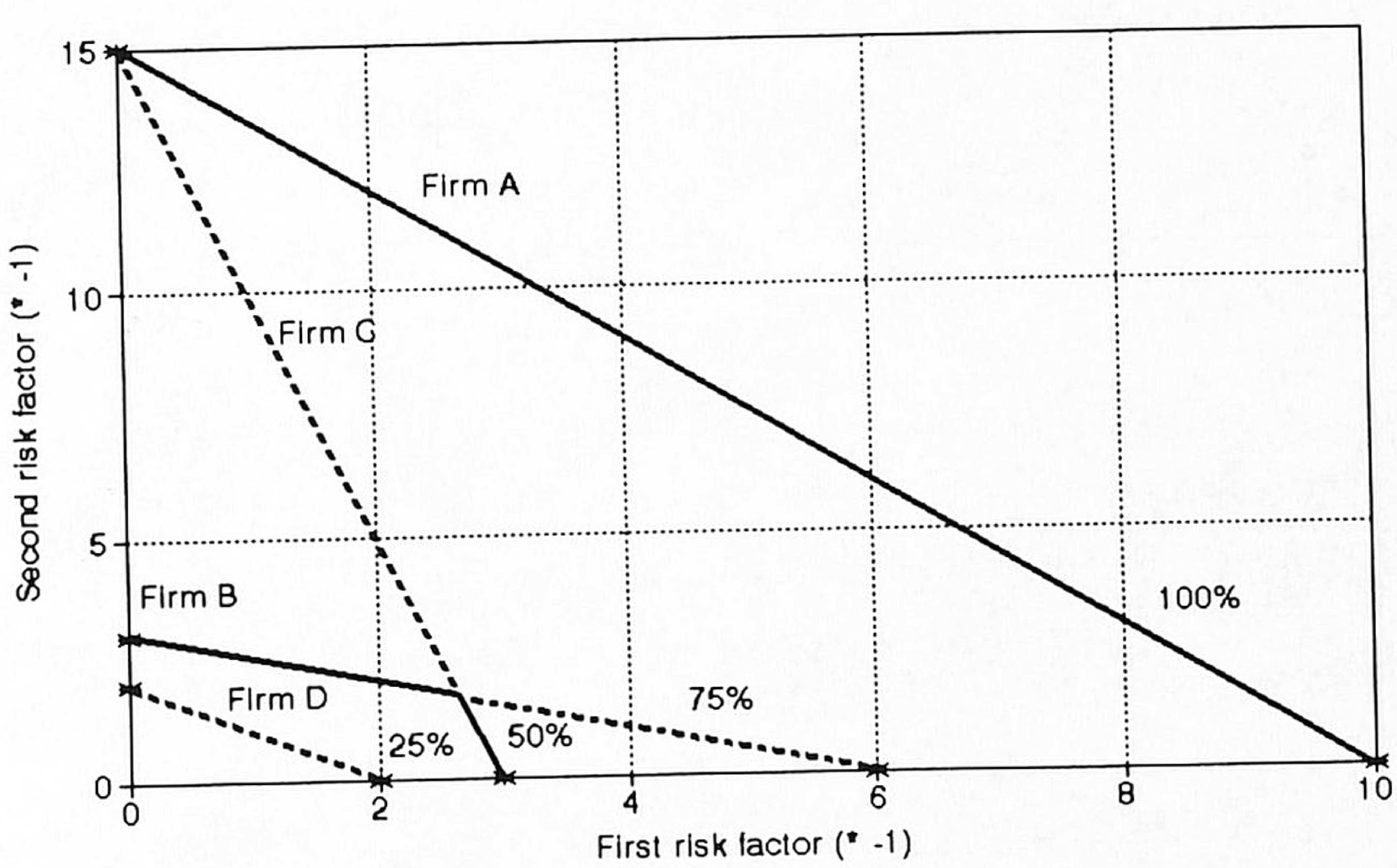

Figure 5: The failure polylines

Given the critical lines, failure polylines can be drawn, which are the two-dimensional equivalents of the failure dial. The failure polylines divide the plane of the risk factors in various sets. The first set refers to those combinations of risk factor values for which no firm goes bankrupt. The second set indicates those combination of risk factor values for which only one firm goes bankrupt, and so on. The last set indicates combinations of risk factors for which all firms go bankrupt. Of course, the critical lines and the failure polylines are related.

Figure 5 presents the failure polylines. Should the combination of the values of the risk factors lie to the left of the failure polyline, then this line indicates what percentage of firms goes bankrupt. For instance, are the changes of the first and second risk factor -5 and -8 respectively, then all firms will go bankrupt. Similar to the failure dial, the failure polylines indicate what percentage of firms will go bankrupt for the specified combination of risk factor values.

5. Summary

In this paper, a framework for conditional failure prediction is presented. It is assumed that failure occurs when current cash balances plus future cash flows are less than zero. The future operational cash flows are analyzed by the multi-factor model. Consequently, corporate failure is explicitly related to the future course of risk factors. Since this future course of risk factors is not certain, the prediction of corporate failure is conditional on the assumed future values of the risk factors. As a result, the framework not only helps in indicating *that*, but also in explaining *why* a firm is predicted to go bankrupt. Consequently, also solutions to prevent bankruptcy are provided.

Also in the more fortunate situation that the firm is not predicted to go bankrupt, the framework still might be of help in managing the firm's cash flow and risks. Then the firm's management is not searching for ways to keep cash balances positive, but rather to reach a specified target cash flow level.

In this paper, we also presented several ways to visualize this relation between the future cash flow and the possible risk factor values. In a nutshell, these visualizations provide a clear picture of the odds a firm runs. Such a picture may be useful for banks when they monitor their clients. When the method is applied at industry level, it may be useful for the government to determine which industry is most vulnerable given the future scenario of the risk factors.

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